Short Answer Questions (PYQ)

[2 Mark]

Q.1. The money to be spent for the welfare of the employees of a firm is proportional to the rate of change of its total revenue (marginal revenue). If the total revenue (in rupees) received from the sale of x units of a product is given by $R(x) = 3x^2 + 36x + 5$, find the marginal revenue when $x = 5$.

Ans.

Given: $R(x) = 3x^2 + 36x + 5$

$\Rightarrow R'(x) = 6x + 36$

$\therefore$ Marginal revenue $(\text{when } x = 5) = R'(x)\big]_{x = 5}$

$= 6 \times 5 + 36 = ₹\ 66.$

Q.2. The amount of pollution content added in air in a city due to x-diesel vehicles is given by $P(x) = 0.005x^3 + 0.02x^2 + 30x$. Find the marginal increase in pollution content when 3 diesel vehicles are added.

Ans.

We have to find $[P(x)]x = 3$

Now, $P(x) = 0.005x^3 + 0.02x^2 + 30x$

$\therefore P(x) = 0.015x^2 + 0.04x + 30$

$\Rightarrow [P(x)]x = 3 = 0.015 \times 9 + 0.04 \times 3 + 30$

$= 0.135 + 0.12 + 30 = 30.255$

Q.3. If $C = 0.003x^3 + 0.02x^2 + 6x + 250$ gives the amount of carbon pollution in air in an area on the entry of x number of vehicles, then find the marginal carbon pollution in the air, when 3 vehicles have entered in the area.

Ans.

We have to find $[C(x)]x = 3$

Now $C(x) = 0.003x^3 + 0.02x^2 + 6x + 250$

$\therefore C(x) = 0.009x^2 + 0.04x + 6$

$[C(x)]x = 3 = 0.009 \times 9 + 0.04 \times 3 + 6 = 0.081 + 0.12 + 6 = 6.201$

Q.4. The contentment obtained after eating *x*-units of a new dish at a trial function is given by the Function $C(x) = x^3 + 6x^2 + 5x + 3$. If the marginal contentment is defined as rate of change of $C(x)$ with respect to the number of units consumed at an instant, then find the marginal contentment when three units of dish are consumed.

Ans.

Given, $C(x) = x^3 + 6x^2 + 5x + 3$

$\Rightarrow \ C(x) = 3x^2 + 12x + 5$

$\Rightarrow \ [C(x)]x = 3 = 3 \times 9 + 36 + 5 = 27 + 36 + 5 = 68$ units

$\therefore$ Required marginal contentment $[C(x)]x = 3 = 68$ units

Q.5. If the radius of a sphere is measured as 9 cm with an error of 0.03 cm, find the approximate error in calculating its surface area.

Ans.

Here, radius of the sphere $r = 9$ cm.

Error in calculating radius, $\delta r = 0.03$ cm.

Let δs be approximate error in calculating surface area.

If S be the surface area of sphere, then $S = 4\pi r^2$

$\Rightarrow \ \frac{dS}{dr} = 4\pi . \ 2r = 8\pi r$

Now by definition, approximately

$$\frac{\mathrm{dS}}{\mathrm{dr}} = \frac{\delta S}{\delta r}$$

$$\left[\because \frac{dS}{dr} = \lim_{\delta r \to 0} \frac{\delta s}{\delta r}\right]$$

$$\Rightarrow \delta s = \left(\frac{\mathrm{dS}}{\mathrm{dr}}\right).\delta r \quad \Rightarrow \quad \delta s = 8\pi r \,.\, \delta r$$

$= 8\pi \times 9 \times 0.03\ \mathrm{cm}^2 = 2.16\mathrm{p}\ \mathrm{cm}^2$

$[\because r = 9\ \mathrm{cm}]$

Q.6. Using differentials, find the approximate value of √49.5.

Ans.

Let $\quad f(x) = \sqrt{x},$ where $x = 49$ and $\delta x = 0.5$

$$\therefore\ f(x + \delta x) = \sqrt{x + \delta x} = \sqrt{49.5}$$

Now by definition, approximately we can write

$$f'(x) = \frac{f(x+\delta x) - f(x)}{\delta x} \qquad \text{...(i)}$$

Here $\quad f(x) = \sqrt{x} = \sqrt{49} = 7$ and $\delta x = 0.5$

$$\Rightarrow \quad f'(x) = \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{49}} = \frac{1}{14}$$

Putting these values in (i), we get

$$\frac{1}{14} = \frac{\sqrt{49.5}-7}{0.5}$$

$$\sqrt{49.5} = \frac{0.5}{14} + 7$$

$$= \frac{0.5+98}{14} = \frac{98.5}{14} = 7.036$$

Q.7. Show that the function *f* given by $f(x) = \tan^{-1}(\sin x + \cos x)$ is decreasing for all.

Ans.

We have

$$f(x) = \tan^{-1}(\sin x + \cos x)$$

$$\Rightarrow \quad f'(x) = \frac{1}{1+(\sin x+\cos x)^2} \times (\cos x - \sin x)$$

$$f'(x) = \frac{\cos x - \sin x}{1+(\sin x + \cos x)^2}$$

$$\because \quad 1 + (\sin x + \cos x)^2 > 0 \ \forall x \in R$$

Also, $\forall x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ $\sin x > \cos x$ $\quad \Rightarrow \cos x - \sin x < 0$

$\therefore \ f'(x) = \frac{-\text{ve}}{+\text{ve}} = -\text{ ve}$ $\quad$ *i.e.,* $f'(x) < 0$

$\Rightarrow f(x)$ is decreasing in $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

Q.8. The volume of a cube is increasing at the rate of 9 cm^3/*s*. How fast is its surface area increasing when the length of an edge is 10 cm?

Ans.

Let V and S be the volume and surface area of a cube of side x cm respectively.

Given $\frac{dV}{dt} = 9 \text{ cm}^3/\text{sec}$

We require $\left.\frac{dS}{dt}\right]_{x=10 \text{ cm}}$

Now $V = x^3$

$\Rightarrow \frac{dV}{dt} = 3x^2 . \frac{dx}{dt}$

$\Rightarrow 9 = 3x^2 . \frac{dx}{dt}$

$\Rightarrow \frac{dV}{dt} = \frac{9}{3x^2} = \frac{3}{x^2}$

Again, $\because S = 6x^2$ [By formula for surface area of a cube]

$\Rightarrow \frac{dS}{dt} = 12.x.\frac{dx}{dt}$

$= 12x.\frac{3}{x^2} = \frac{36}{x}$

$\Rightarrow \left.\frac{dS}{dt}\right]_{x=10 \text{ cm}} = \frac{36}{10} = 3.6 \text{ cm}^2/\text{sec.}$

Short Answer Questions (OIQ)

[2 Mark]

Q.1. The length *x* of a rectangle is decreasing at the rate of 3 cm/minute and the width *y* is increasing at the rate of 2 cm/minute, when *x* = 10 cm and *y* = 6 cm, find the rates of change of the perimeter.

Ans.

Let P be the perimeter of rectangle.

$P = 2(x + y)$

Differentiating w.r.t. 't' we get

$$\frac{dP}{dt} = 2\left(\frac{dx}{dt} + \frac{dy}{dt}\right)$$

$$\frac{dP}{dt} = 2(-3 + 2)$$

$$\left[\begin{array}{l}\text{Given} \\ \frac{dx}{dt} = -3\,\text{cm/min} \\ \frac{dy}{dt} = 2\,\text{cm/min}\end{array}\right.$$

$$\Rightarrow \quad \frac{dP}{dt} = 2 \times (-1)$$

$= -2$ cm/minute.

$\Rightarrow$ Perimeter of rectangle is decreasing at the rate of 2 cm/minute.

Q.2. A particle moves along the curve 6*y* = *x*^{3} + 2. Find the points on the curve at which the *y*-coordinate is changing 8 times as fast as the *x*-coordinate.

Ans.

Given $6y = x^3 + 2$ $\qquad\qquad \ldots(i)$

Differentiating w.r.t. t, we get

$$6\frac{dy}{dt} = 3x^2 . \frac{dx}{dt}$$

$$2\frac{dy}{dt} = x^2\frac{dx}{dt} \qquad \ldots(ii)$$

From question $\frac{dy}{dt} = 8.\frac{dx}{dt}$

$\therefore (ii) \Rightarrow 2 \times 8\frac{dx}{dt} = x^2\frac{dx}{dt}$

$\Rightarrow x^2 = 16$ $\Rightarrow x = \pm 4$

If $x = 4,\ \ y = \frac{64+2}{6} = 11$

If $x = -4,\ y = \frac{-64+2}{6} = \frac{-62}{6} = \frac{-31}{3}$

Hence, the required points are $(4, 11)$ and $\left(-4, \frac{-31}{3}\right)$.

Q.3. The surface area of a spherical bubble is increasing at the rate of 2 cm^2/s. Find the rate at which the volume of the bubble is increasing at the instant if its radius is 6 cm.

Ans.

Let r be the radius of bubble, S the surface area, and V be the volume of bubble at time t.

Then $\frac{dS}{dt} = 2\,\text{cm}^2/\text{s}$ (given), $r = 6$ cm, $\frac{dV}{dt} = ?$

As $S = 4\pi r^2$ for spherical bubble.

$\therefore \quad \frac{dS}{dt} = \frac{d}{dt}(4\pi r^2) = 8\pi r\frac{dr}{dt}$

$\Rightarrow \quad 2\ \text{cm}^2/s = 8\pi r\frac{dr}{dt} \quad \Rightarrow \quad \frac{dr}{dt} = \frac{2}{8\pi r} = \frac{1}{4\pi r}\text{cm}/s$

Since, $\quad V = \frac{4}{3}\pi r^3$

$\Rightarrow \frac{dV}{dt} = \frac{4}{3} \cdot 3\pi r^2\frac{dr}{dt} = 4\pi r^2 \cdot \frac{1}{4\pi r}\ \text{cm}^3/\text{s} = r\ cm^3/s = 6\ \text{cm}^3/\text{s}$
$[\because r = 6 \text{ cm given}]$

Hence, the volume of the bubble is increasing at the rate of 6 cm^3/s.

Q.4. Find the point at which the tangent to the curve $y = \sqrt{4x-3} - 1$ has its slope $\frac{2}{3}$.

Ans.

Slope of tangent to the given curve

$y = \sqrt{4x-3} - 1$ at $(x, y) = \frac{dy}{dx}$

$$\Rightarrow \quad \frac{2}{3} = \frac{1 \times 4}{2\sqrt{4x-3}} - 0$$

$$\Rightarrow \quad 4\sqrt{4x-3} = 3 \times 4$$

$$\Rightarrow \quad \sqrt{4x-3} = 3$$

$$\Rightarrow \quad 4x - 3 = 9$$

$$\Rightarrow \quad x = \frac{12}{4}$$

$$\Rightarrow \quad x = 3$$

If $x = 3$ then $y = \sqrt{4 \times 3 - 3} - 1 = 3 - 1 = 2$

Therefore, required point is $(3, 2)$.

Q.5. In a competition, a brave child tries to inflate a huge spherical balloon bearing slogans against child labour at the rate of 900 cubic centimeter of gas per second. Find the rate at which the radius of the balloon is increasing when its radius is 15 cm.

Ans.

$\frac{\mathrm{dr}}{\mathrm{dt}} = ?, \quad \text{when } r = 15 \text{ cm}$

$V = \frac{4}{3}\pi r^3$

Differentiating both sides with respect to 't', we get

$\frac{\mathrm{dV}}{\mathrm{dt}} = \frac{4}{3}\pi\, 3r^2 \frac{\mathrm{dr}}{\mathrm{dt}}$

$900 = 4\pi(15)^2 \frac{\mathrm{dr}}{\mathrm{dt}}$

$\Rightarrow \quad \frac{\mathrm{dr}}{\mathrm{dt}} = \frac{900}{225 \times 4 \times \pi} = \frac{1}{\pi} \mathrm{cm} / \mathrm{sec}$