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Probability

TOPIC 1 Random Experiment, Sample Space, Events, Probability of an Event, Mutually Exclusive & Exhaustive Events, Equally Likely

1. Out of 11 consecutive natural numbers if three numbers are selected at random (without repetition), then the probability that they are in A.P. with positive common difference, is: **[Sep. 06, 2020 (I)]**

(a) $\frac{15}{101}$ (b) $\frac{5}{101}$

(c) $\frac{5}{33}$ (d) $\frac{10}{99}$

2. If 10 different balls are to be placed in 4 distinct boxes at random, then the probability that two of these boxes contain exactly 2 and 3 balls is : **[Jan. 9, 2020 (II)]**

(a) $\frac{965}{2^{11}}$ (b) $\frac{965}{2^{10}}$

(c) $\frac{945}{2^{10}}$ (d) $\frac{945}{2^{11}}$

3. If three of the six vertices of a regular hexazon are chosen at random, then the probability that the triangle formed with these chosen vertices is equilateral is : **[April 12, 2019 (I)]**

(a) $\frac{1}{10}$ (b) $\frac{1}{5}$

(c) $\frac{3}{10}$ (d) $\frac{3}{20}$

4. Let S = {1, 2, ..., 20}. A subset B of S is said to be "nice", if the sum of the elements of B is 203. Than the probability that a randomly chosen subset of S is "nice" is : **[Jan. 11, 2019 (II)]**

(a) $\frac{7}{2^{20}}$ (b) $\frac{5}{2^{20}}$

(c) $\frac{4}{2^{20}}$ (d) $\frac{6}{2^{20}}$

5. Two different families A and B are blessed with equal number of children. There are 3 tickets to be distributed amongst the children of these families so that no child gets more than one ticket.
If the probability that all the tickets go to the children of the family B is $\frac{1}{12}$, then the number of children in each family is? **[Online April 16, 2018]**

(a) 4 (b) 6

(c) 3 (d) 5

6. A box $'A'$ contanis 2 white, 3 red and 2 black balls. Another box $'B'$ contains 4 white, 2 red and 3 black balls. If two balls are drawn at random, without replacement, from a randomly selected box and one ball turns out to be white while the other ball turns out to be red, then the probability that both balls are drawn from box $'B'$ is **[Online April 15, 2018]**

(a) $\frac{7}{16}$ (b) $\frac{9}{32}$

(c) $\frac{7}{8}$ (d) $\frac{9}{16}$

7. If the lengths of the sides of a triangle are decided by the three throws of a single fair die, then the probability that the triangle is of maximum area given that it is an isosceles triangle, is : **[Online April 11, 2015]**

(a) $\frac{1}{21}$ (b) $\frac{1}{27}$

(c) $\frac{1}{15}$ (d) $\frac{1}{26}$

8. A number x is chosen at random from the set $\{1, 2, 3, 4, ..., 100\}$. Define the event: A = the chosen number x satisfies

$$\frac{(x-10)(x-50)}{(x-30)} \geq 0$$

Then P (A) is: **[Online April 12, 2014]**

(a) 0.71 (b) 0.70
(c) 0.51 (d) 0.20

9. A set S contains 7 elements. A non-empty subset A of S and an element x of S are chosen at random. Then the probability that $x \in A$ is: **[Online April 11, 2014]**

(a) $\frac{1}{2}$ (b) $\frac{64}{127}$
(c) $\frac{63}{128}$ (d) $\frac{31}{128}$

10. There are two balls in an urn. Each ball can be either white or black. If a white ball is put into the urn and there after a ball is drawn at random from the urn, then the probability that it is white is **[Online May 26, 2012]**

(a) $\frac{1}{4}$ (b) $\frac{2}{3}$
(c) $\frac{1}{5}$ (d) $\frac{1}{3}$

11. If six students, including two particular students A and B, stand in a row, then the probability that A and B are separated with one student in between them is **[Online May 19, 2012]**

(a) $\frac{8}{15}$ (b) $\frac{4}{15}$
(c) $\frac{2}{15}$ (d) $\frac{1}{15}$

12. A number n is randomly selected from the set $\{1, 2, 3, ..., 1000\}$. The probability that $\frac{\sum_{i=1}^{n} i^2}{\sum_{i=1}^{n} i}$ is an integer is **[Online May 12, 2012]**

(a) 0.331 (b) 0.333
(c) 0.334 (d) 0.332

13. Four numbers are chosen at random (without replacement) from the set $\{1, 2, 3, ...20\}$. **[2010]**

Statement -1: The probability that the chosen numbers when arranged in some order will form an AP is $\frac{1}{85}$.

Statement -2 : If the four chosen numbers form an AP, then the set of all possible values of common difference is $(\pm 1, \pm 2, \pm 3, \pm 4, \pm 5)$.

(a) Statement -1 is true, Statement -2 is true; Statement -2 is **not** a correct explanation for Statement -1
(b) Statement -1 is true, Statment -2 is false
(c) Statement -1 is false, Statment -2 is true.
(d) Statement -1 is true, Statement -2 is true ; Statement -2 is a correct explanation for Statement -1.

14. An urn contains nine balls of which three are red, four are blue and two are green. Three balls are drawn at random without replacement from the urn. The probability that the three balls have different colours is **[2010]**

(a) $\frac{2}{7}$ (b) $\frac{1}{21}$
(c) $\frac{2}{23}$ (d) $\frac{1}{3}$

15. Five horses are in a race. Mr. A selects two of the horses at random and bets on them. The probability that Mr. A selected the winning horse is **[2003]**

(a) $\frac{2}{5}$ (b) $\frac{4}{5}$
(c) $\frac{3}{5}$ (d) $\frac{1}{5}$

TOPIC 2 Odds Against & Odds in Favour of an Event, Addition Theorem, Boole's Inequality, Demorgan's Law

16. The probabilities of three events A, B and C are given by P(A) = 0.6, P(B) = 0.4 and P(C) = 0.5. If $P(A \cup B) = 0.8$, $P(A \cap C) = 0.3$, $P(A \cap B \cap C) = 0.2$, $P(B \cap C) = \beta$ and $P(A \cup B \cup C) = \alpha$, where $0.85 \leq \alpha \leq 0.95$, then β lies in the interval: **[Sep. 06, 2020 (II)]**

(a) $[0.35, 0.36]$ (b) $[0.25, 0.35]$
(c) $[0.20, 0.25]$ (d) $[0.36, 0.40]$

17. Let A and B be two events such that the probability that exactly one of them occurs is $\frac{2}{5}$ and the probability that A or B occurs is $\frac{1}{2}$, then the probability of both of them occur together is: **[Jan. 8, 2020 (II)]**

(a) 0.02 (b) 0.20
(c) 0.01 (d) 0.10

18. In a class of 60 students, 40 opted for NCC, 30 opted for NSS and 20 opted for both NCC and NSS. If one of these students is selected at random, then the probability that the student selected has opted neither for NCC nor for NSS is : **[Jan. 12, 2019 (II)]**

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$

(c) $\frac{2}{3}$ (d) $\frac{5}{6}$

19. For three events A, B and C,
P(Exactly one of A or B occurs)
= P(Exactly one of B or C occurs)
= P(Exactly one of C or A occurs) = $\frac{1}{4}$ and

P(All the three events occur simultaneously) = $\frac{1}{16}$.

Then the probability that at least one of the events occurs, is : **[2017]**

(a) $\frac{3}{16}$ (b) $\frac{7}{32}$

(c) $\frac{7}{16}$ (d) $\frac{7}{64}$

20. From a group of 10 men and 5 women, four member committees are to be formed each of which must contain at least one woman. Then the probability for these committees to have more women than men, is : **[Online April 9, 2017]**

(a) $\frac{21}{220}$ (b) $\frac{3}{11}$

(c) $\frac{1}{11}$ (d) $\frac{2}{23}$

21. If 12 identical balls are to be placed in 3 identical boxes, then the probability that one of the boxes contains exactly 3 balls is : **(2015)**

(a) $220\left(\frac{1}{3}\right)^{12}$ (b) $22\left(\frac{1}{3}\right)^{11}$

(c) $\frac{55}{3}\left(\frac{2}{3}\right)^{11}$ (d) $55\left(\frac{2}{3}\right)^{10}$

22. If A and B are two events such that $P(A \cup B) = P(A \cap B)$, then the incorrect statement amongst the following statements is: **[Online April 9, 2014]**

(a) A and B are equally likely

(b) $P(A \cap B') = 0$

(c) $P(A' \cap B) = 0$

(d) $P(A) + P(B) = 1$

23. If the events A and B are mutually exclusive events such that $P(A) = \frac{3x+1}{3}$ and $P(B) = \frac{1-x}{4}$, then the set of possible values of x lies in the interval : **[Online April 25, 2013]**

(a) [0, 1] (b) $\left[\frac{1}{3}, \frac{2}{3}\right]$

(c) $\left[-\frac{1}{3}, \frac{5}{9}\right]$ (d) $\left[-\frac{7}{9}, \frac{4}{9}\right]$

24. Let X and Y are two events such that $P(X \cup Y) = P(X \cap Y)$.

Statement 1: $P(X \cap Y') = P(X' \cap Y) = 0$

Statement 2: $P(X) + P(Y) = 2P(X \cap Y)$

[Online May 7, 2012]

(a) Statement 1 is false, Statement 2 is true.

(b) Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation of Statement 1.

(c) Statement 1 is true, Statement 2 is false.

(d) Statement 1 is true, Statement 2 is true; Statement 2 is a correct explanation of Statement 1.

25. A die is thrown. Let A be the event that the number obtained is greater than 3. Let B be the event that the number obtained is less than 5. Then $P(A \cup B)$ is **[2008]**

(a) $\frac{3}{5}$ (b) 0

(c) 1 (d) $\frac{2}{5}$

26. Events A, B, C are mutually exclusive events such that $P(A) = \frac{3x+1}{3}$, $P(B) = \frac{1-x}{4}$ and $P(C) = \frac{1-2x}{2}$ The set of possible values of x are in the interval. **[2003]**

(a) [0 , 1] (b) $\left[\frac{1}{3}, \frac{1}{2}\right]$

(c) $\left[\frac{1}{3}, \frac{2}{3}\right]$ (d) $\left[\frac{1}{3}, \frac{13}{3}\right]$

27. A and B are events such that $P(A \cup B)$=3/4, $P(A \cap B)$=1/4, $P(\overline{A})$=2/3 then $P(\overline{A} \cap B)$ is **[2002]**

(a) 5/12 (b) 3/8

(c) 5/8 (d) 1/4

Hints & Solutions

1. **(c)** For an A.P. $2b = a + c$ (even), so both a and c even numbers or odd numbers from given numbers and b number will be fixed automatically.

Required probability $= \frac{{}^6C_2 + {}^5C_2}{{}^{11}C_3} = \frac{25}{165} = \frac{5}{33}$

2. **(Bonus)** Total number of ways placing 10 different balls in 4 distinct boxes = 4^{10}
Since, two of the 4 distinct boxes contains exactly 2 and 3 balls.
Then, there are three cases to place exactly 2 and 3 balls in 2 of the 4 boxes.
Case-1: When boxes contains balls in order 2, 3, 0, 5
Then, number of ways of placing the balls

$$= \frac{10!}{2! \times 3! \times 0! \times 5!} \times 4!$$

Case-2: When boxes contains ball in order 2, 3, 1, 4.
Then, number of ways of placing the balls

$$= \frac{10!}{2! \times 3! \times 1! \times 4!} \times 4!$$

Case-3: When boxes contains ball in order 2, 3, 2, 3
Then, number of ways of placing the balls

$$= \frac{10!}{(2!)^2 \times (3!)^2 \times 2! \times 3!} \times 4!$$

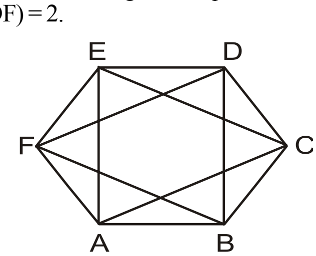
Therefore, number of ways of placing the balls that contains exactly 2 and 3 balls.

$$= \frac{10!}{2! \times 3! \times 0! \times 5!} \times 4! + \frac{10!}{2! \times 3! \times 1! \times 4!} \times 4! + \frac{10!}{(2!)^2 \times 2! \times (3!)^2 \times 2!} \times 4!$$

$= 2^5 \times 17 \times 945$
Hence, the required probability

$$= \frac{2^5 \times 17 \times 945}{4^{10}} = \frac{17 \times 945}{2^{15}}$$

3. **(1)** Total no. of triangles = ${}^6C_3$
Favorable no. of triangle i.e, equilateral triangles (ΔAEC and ΔBDF) = 2.



Hence, required probability $= \frac{2}{{}^6C_3} = \frac{1}{10}$

4. **(2)** Since total number of subsets of the set $S = 2^{20}$

Now, the sum of all number from 1 to 20 $= \frac{20 \times 21}{2} = 210$

Then, find the sets which has sum 7.
(1) {7}
(2) {1, 6}
(3) {2, 5}
(4) {3, 4}
(5) {1, 2, 4}
Then, there is only 5 sets which has sum 203

Hence required probability $= \frac{5}{2^{20}}$

5. **(d)** Let the number of children in each family be x.
Thus the total number of children in both the families are $2x$
Now, it is given that 3 tickets are distributed amongst the children of these two families.
Thus, the probability that all the three tickets go to the children in family B

$$= \frac{{}^xC_3}{{}^{2x}C_3} = \frac{1}{12}$$

$$\Rightarrow \frac{x(x-1)(x-2)}{2x(2x-1)(2x-2)} = \frac{1}{12}$$

$$\Rightarrow \frac{(x-2)}{(2x-1)} = \frac{1}{6}$$

$\Rightarrow x = 5$
Thus, the number of children in each family is 5.

6. **(a)** Probability of drawing a white ball and then a red ball from bag B is given by $\frac{{}^4C_1 \times {}^2C_1}{{}^9C_2} = \frac{2}{9}$

Probability of drawing a white ball and then a red ball from bag A is given by $\frac{{}^2C_1 \times {}^3C_1}{{}^7C_2} = \frac{2}{7}$

Hence, the probability of drawing a white ball and then a red ball from bag $B = \frac{\frac{2}{9}}{\frac{2}{7} + \frac{2}{9}} = \frac{2 \times 7}{18 + 14} = \frac{7}{16}$

7. **(b)** Favourable case = (6, 6, 6)
Total case = {(1, 1, 1) (2, 2, 1), (2, 2, 2), (2, 2, 3), (3, 3, 1) ..... (3, 3, 5) (4, 4, 1).... (4, 4, 6) (5, 5, 1).... (5, 5, 6) (6, 6, 1).... (6, 6, 6)}

which satisfies condition a + b > c
Number of total case = 27

Probability = $\frac{1}{27}$

8. **(a)** Given $\frac{(x-10)(x-50)}{(x-30)} \geq 0$

Let $x \geq 10,\ x \geq 50$ equation will be true $\forall x \geq 50$

as $\left(\frac{x-50}{x-30}\right) \geq 0,\ \forall x \in [10, 30)$

$\frac{(x-10)(x-50)}{x-30} \geq 0 \ \forall x \in [10, 30)$

Total value of x between 10 to 30 is 20.
Total values of x between 50 to 100 including 50 and 100 is 51.
Total values of x = 51 + 20 = 71

$P(A) = \frac{71}{100} = 0.71$

9. **(b)** Let S = $\{x_1, x_2, x_3, x_4, x_5, x_6, x_7\}$
Let the chosen element be x_i.
Total number of subsets of S = 2^7 = 128
No. of non-empty subsets of S = 128 – 1 = 127
We need to find number of those subsets that contains x_i.

2	2	2	2	1	2	2

$x_1\ x_2$ ------- x_i ---- x_7

For those subsets containing x_i, each element has 2 choices.
i.e., (included or not included) in subset,
However as the subset must contain x_i, x_i has only one choice. (included one)
So, total no. of subsets containing
x_i = 2 × 2 × 2 × 2 × 1 × 2 × 2 = 64

Required prob $= \frac{\text{No. of subsets containing } x_i}{\text{Total no. of non-empty subsets}}$

$= \frac{64}{127}$

10. **(b)** Total possible event when one ball is taken out = $^3C_1$
Let E : The event of 1 white ball coming out
No. of ways to 1 white ball coming out = $^2C_1$

$\therefore P(E) = \frac{^2C_1}{^3C_1} = \frac{2}{3}$

11. **(b)** Consider a group of three students A, B and an other student in between A and B. Choice for a student between A and B is 4. A and B can interchange their places in the group in 2 ways.

Now the group of three students (student A, student B and a student in between A and B) and the remaining 3 students can be stand in a row in 4! ways.
Hence total number of ways to stand in a row such that A and B are separated with one student in between them = 4 × 2 × 4!
Now total number of ways to stand 6 student stand in a row without any restriction = 6!
Hence required probability

$= \frac{4 \times 2 \times 4!}{6!} = \frac{4 \times 2}{6 \times 5} = \frac{4}{15}$

12. **(c)** $\frac{\sum_{i=1}^{n} i^2}{\sum_{i=1}^{n} i} = \frac{\frac{n(n+1)(2n+1)}{6}}{\frac{n(n+1)}{2}} = \frac{2n+1}{3}$

For n = 1, 2, 3, ..., 1000

Value of $\frac{2n+1}{3} = \frac{3}{3}, \frac{5}{3}, \frac{7}{3}, ..., \frac{2001}{3}$ respectively. Out of $\frac{3}{3}, \frac{5}{3}, \frac{7}{3}, ..., \frac{2001}{3}$ only first term $\left(\frac{3}{3} = 1\right)$, fourth term $\left(\frac{9}{3} = 3\right)$, 667th term $\left(\frac{2001}{3} = 667\right)$ are integers.

Hence, out of 1000 values of $\frac{2n+1}{3}$,

total number of integral values of $\frac{2n+1}{3}$

= 333 + 1 = 334

$\therefore$ Required probability $= \frac{334}{1000} = 0.334$

13. **(b)** Four numbers are chosen from {1, 2, 3...20}
n(S) = $^{20}C_4$
Statement-1:
Common difference is 1; total number of ways = 17
common difference is 2; total number of ways = 14
common difference is 3; total number of ways = 11
common difference is 4; total number of ways = 8
common difference is 5; total number of ways = 5
common difference is 6; total number of ways = 2

Prob. = $\frac{17+14+11+8+5+2}{^{20}C_4} = \frac{1}{85}$

Statement -2 is false, because common difference can be 6 also.

14. **(a)** $n(S)={}^9C_3$

$n(E)={}^3C_1\times{}^4C_1\times{}^2C_1$

Probability $=\dfrac{3\times4\times2}{{}^9C_3}=\dfrac{24\times3!}{9!}\times6!=\dfrac{24\times6}{9\times8\times7}=\dfrac{2}{7}$

15. **(a)** Let 5 horses are H_1, H_2, H_3, H_4 and H_5.
Total ways of selecting pair of horses be

$={}^5C_2=10$[i.e. $H_1H_2, H_1H_3, H_1H_4, H_1H_5,$

$H_2H_3, H_2H_4, H_2H_5, H_3H_4, H_3H_5, H_4H_5$]

Any horse can win the race in 4 ways
(e.g. for H_1 : H_1H_2, H_1H_3, H_1H_4, H_1H_5)

Hence required probability $=\dfrac{4}{10}=\dfrac{2}{5}$

16. **(b)** $\because P(A\cap B)=P(A)+P(B)-P(A\cup B)$

$=1-0.8=0.2$

Now,

$\because P(A\cup B\cup C)=P(A)+P(B)+P(C)-P(A\cap B)$

$-P(B\cap C)-P(C\cap A)+P(A\cap B\cap C)$

$\Rightarrow \alpha=0.6+0.4+0.5-0.2-\beta-0.3+0.2$

$\Rightarrow \beta=1.2-\alpha$

$\because \alpha\in[0.85, 0.95]$ then $\beta\in[0.25, 0.35]$

17. **(4)** $P(\text{exactly one})=\dfrac{2}{5}$

$\Rightarrow\ P(A)+P(B)-2P(A\cap B)=\dfrac{2}{5}$

$P(A \text{ or } B)=P(A\cup B)=\dfrac{1}{2}$

$\Rightarrow\ P(A)+P(B)-P(A\cap B)=\dfrac{1}{2}$

$\therefore\ P(A\cap B)=\dfrac{1}{2}-\dfrac{2}{5}=\dfrac{5-4}{10}=\dfrac{1}{10}=0.10$

18. **(1)** P = Set of students who opted for NCC
Q = Set of Students who opted for NSS
$n(P)=40, n(Q)=30, n(P\cap Q)=20$
$n(P\cup Q)=n(P)+n(Q)-n(P\cap Q)$
$=40+30-20$
$=50$

$\therefore$ Hence, required probability $=1-\dfrac{50}{60}$

$=\dfrac{1}{6}$

19. **(c)** P (exactly one of A or B occurs)

$=P(A)+P(B)-2P(A\cap B)=\dfrac{1}{4}$ ...(1)

P (Exactly one of B or C occurs)

$=P(B)+P(C)-2P(B\cap C)=\dfrac{1}{4}$ ...(2)

P (Exactly one of C or A occurs)

$=P(C)+P(A)-2P(C\cap A)=\dfrac{1}{4}$ ...(3)

Adding (1), (2) and (3), we get

$2\Sigma P(A)-2\Sigma P(A\cap B)=\dfrac{3}{4}$

$\therefore\ \Sigma P(A)-\Sigma P(A\cap B)=\dfrac{3}{8}$

Now, $P(A\cap B\cap C)=\dfrac{1}{16}$

$\therefore P(A\cup B\cup C)$
$=\Sigma P(A)-\Sigma P(A\cap B)+P(A\cap B\cap C)$

$=\dfrac{3}{8}+\dfrac{1}{16}=\dfrac{7}{16}$

20. **(c)** Probability of 4 member committee which contain atleast one woman.

$\Rightarrow$ P(3M, 1W) + P(2M, 2W) + P(1M, 3W) + P(0M, 4W)

$\Rightarrow\ \dfrac{{}^{10}C_3\ {}^5C_1}{{}^{15}C_4}+\dfrac{{}^{10}C_2\ {}^5C_2}{{}^{15}C_4}+\dfrac{{}^{10}C_1\ {}^5C_3}{{}^{15}C_4}+\dfrac{{}^{10}C_0\ {}^5C_4}{{}^{15}C_4}$

$\Rightarrow\ \dfrac{600}{1365}+\dfrac{450}{1365}+\dfrac{100}{1365}+\dfrac{5}{1365}$

$\Rightarrow\ \dfrac{1155}{1365}$

$\therefore$ Probability of committees to have more women than men.

$=\dfrac{P(1M,3W)+P(0M,4W)}{P(3M,1W)+P(2M,2W)+P(1M,3W)+P(0M,4W)}$

$=\dfrac{\frac{105}{1365}}{\frac{1155}{1365}}=\dfrac{1}{11}$

21. **(c)** **Note:-** The question should state '3 different' boxes instead of '3 identical boxes' and one particular box has 3 balls. Then the solution would be:

Required probability $=\dfrac{{}^{12}C_3\times2^9}{3^{12}}$

$=\dfrac{55}{3}\left(\dfrac{2}{3}\right)^{11}$

22. **(d)** Let A and B be two events such that
$P(A\cup B)\ =\ P(A\cap B)$

and $P(A\cup B) = P(A) + P(B) - P(A\cap B)$

option (a) : since $P(A\cup B) = P(A\cap B)$ (given)
therefore A and B are equally likely
Suppose option (b) and option (c) are correct.

$\therefore P(A\cap B') = 0$ and $P(A'\cap B) = 0$

$\Rightarrow P(A) - P(A\cap B) = 0$ and $P(B) - P(A\cap B) = 0$

$\Rightarrow P(A) = P(A\cap B)$ and $P(B) = P(A\cap B)$

Thus $P(A) = P(B) = P(A\cap B) = P(A\cup B)$

[$\because$ Given $P(A\cap B) = P(A\cup B)$]

Also, we know

$P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$= P(A\cap B) + P(A\cap B) - P(A\cap B)$

$= P(A\cap B)$

which is true from given condition
Hence, option (a), (b) and (c) are correct.

23. **(c)** Since events A and B are mutually exclusive

$\therefore P(A)+P(B)=1$

$\Rightarrow \frac{3x+1}{3}+\frac{1-x}{4}=1$

$\Rightarrow 12x+4+3-3x=12$

$\Rightarrow 9x=5 \Rightarrow x=\frac{5}{9}$

$\therefore x\in\left[-\frac{1}{3},\frac{5}{9}\right]$

24. **(b)** Let X and Y be two events such that

$P(X\cup Y)=P(X\cap Y)$ ...(1)

We know

$P(X\cup Y)=P(X)+P(Y)-P(X\cap Y)$

$P(X\cap Y)=P(X)+P(Y)-P(X\cap Y)$ (from (1)

$\Rightarrow P(X)+P(Y)=2P(X\cap Y)$

Hence, Statement - 2 is true.

Now, $P(X\cap Y')=P(X)-P(X\cap Y)$

and $P(X'\cap Y)=P(Y)-P(X\cap Y)$

This implies statement-1 is also true.

25. **(c)** A (number is greater than 3) = {4, 5, 6}

$\Rightarrow P(A)=\frac{3}{6}=\frac{1}{2}$

B (number is less than 5) = {1, 2, 3, 4} $\Rightarrow P(B)=\frac{4}{6}=\frac{2}{3}$

$\therefore A\cap B=\{4\}$

$\Rightarrow P(A\cap B)=\frac{1}{6}$

$\therefore P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$=\frac{1}{2}+\frac{2}{3}-\frac{1}{6}=\frac{3+4-1}{6}=1$

26. **(b)** Given that $P(A)=\frac{3x+1}{3}$, $P(B)=\frac{1-x}{4}$ and

$P(C)=\frac{1-2x}{2}$

We know that $0\le P(E)\le 1$

$\Rightarrow 0\le\frac{3x+1}{3}\le 1, \ge -1\le 3x\le 2$

$\Rightarrow -\frac{1}{3}\le x\le\frac{2}{3}$ ...(i)

$0\le\frac{1-x}{4}\le 1\Rightarrow -3\le x\le 1$ ...(ii)

and $0\le\frac{1-2x}{2}\le 1\Rightarrow -1\le 2x\le 1$

$\Rightarrow -\frac{1}{2}\le x\le\frac{1}{2}$ ...(iii)

Also for mutually exclusive events A, B, C,

$P(A\cup B\cup C)=P(A)+P(B)+P(C)$

$\Rightarrow P(A\cup B\cup C)=\frac{3x+1}{3}+\frac{1-x}{4}+\frac{1-2x}{2}$

$\therefore 0\le\frac{1+3x}{3}+\frac{1-x}{4}+\frac{1-2x}{2}\le 1$

$0\le 13-3x\le 12\Rightarrow 1\le 3x\le 13$

$\Rightarrow\frac{1}{3}\le x\le\frac{13}{3}$ ...(iv)

From (i), (ii), (iii) and (iv), we get

$\frac{1}{3}\le x\le\frac{1}{2}\Rightarrow x\in\left[\frac{1}{3},\frac{1}{2}\right]$

27. **(a)** We know that $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\Rightarrow\frac{3}{4}=1-P(\bar{A})+P(B)-\frac{1}{4}$ $[\because P(A)=1-P(\bar{A})]$

$\Rightarrow 1=1-\frac{2}{3}+P(B)\Rightarrow P(B)=\frac{2}{3}$;

Now, $P(\bar{A}\cap B)=P(B)-P(A\cap B)$

$=\frac{2}{3}-\frac{1}{4}=\frac{5}{12}$.