LINEAR EQUATION IN ONE & TWO VARIABLES

LINEAR EQUATIONS IN ONE VARIABLE

A statement of equality of two algebraic expressions, which involve one or more unknown quantities is known as an equation.

A linear equation is an equation which involves linear polynomials.

A value of the variable which makes the two sides of the equation equal is called the solution of the equation.

Same quantity can be added/subtracted to/from both the sides of an equation without changing the equality.

Both the sides of an equation can be multiplied/divided by the same non-zero number without changing the equality.

SOLVING LINEAR EQUATIONS :

(a) Transpose the terms involving the variable to the left hand side and constant terms to the right hand side.

(b) Simplify the two sides in their simplest form.

(c) Solve the equation obtained in step (b) by dividing both sides by the coefficient of variable

❖ EXAMPLES ❖

Ex.1 Solve : $3x + 2 = 11$

Sol. We have : $3x + 2 = 11$

$\Rightarrow 3x + 2 + (-2) = 11 + (-2)$

[Adding –2 to both sides]

$\Rightarrow 3x = 9$

$\Rightarrow \frac{1}{3}(3x) = \frac{1}{3}(9)$ [Dividing both sides by 3]

$\Rightarrow x = 3$

Thus, $x = 3$ is the solution of the given equation.

Ex.2 Solve : $2x + 6 = x - 8$

Sol. We have : $2x + 6 = x - 8$

$\Rightarrow 2x + 6 - 6 = x - 8 - 6$

[Subtracting 6 from both sides]

$\Rightarrow 2x - x = x - 14 - x$

[Subtracting x from both sides]

$\Rightarrow x = -14$

Thus, $x = -14$ is the solution of the given equation.

Ex.3 Solve : $2x + \sqrt{2} = 3x - 4 - 3\sqrt{2}$

Sol. The given equation is

$2x + \sqrt{2} = 3x - 4 - 3\sqrt{2}$

$\Rightarrow 2x - 3x = -4 - 3\sqrt{2} - \sqrt{2}$

$\Rightarrow -x = -4 - 4\sqrt{2}$ [By simplifying both sides] $\Rightarrow$ $-x = -4(1 + \sqrt{2})$

$\Rightarrow x = 4(1+\sqrt{2})$ [Multiplying both sides by–1]

Thus, $4(1+\sqrt{2})$ is the solution of the given equation.

Ex.4 Solve the following equation:

$$-\frac{3.4x}{3} = \frac{1.7}{9}$$

Sol. The given equation is

$$-\frac{3.4x}{3} = \frac{1.7}{9}$$

$\Rightarrow 9 \times -3.4x = 3 \times 1.7$ [By cross-multiplication]

$\Rightarrow -30.6x = 5.1$

$\Rightarrow x = \frac{5.1}{-30.6} = -\frac{5.1}{30.6} = -\frac{1}{6}$

Thus, $x = -\frac{1}{6}$ is the solution of the given equation.

Ex.5 Solve the following equation :

$$\frac{3x-5}{7x-5} = \frac{1}{9}, x \neq \frac{5}{7}$$

Sol. The given equation is $\frac{3x-5}{7x-5} = \frac{1}{9}$

$\Rightarrow 9 \times (3x - 5) = 1 \times (7x - 5)$

[By cross-multiplication]

$\Rightarrow 27x - 45 = 7x - 5$

$\Rightarrow 27x - 7x = 45 - 5$

$\Rightarrow 20x = 40 \quad \Rightarrow x = \frac{40}{20} = 2$

Thus, x = 2 is the solution of the given equations.

Ex.6 Solve each of the following equations :

(i) $\frac{5x-4}{8} - \frac{x-3}{5} = \frac{x+6}{4}$

(ii) $\frac{x-2}{4} + \frac{1}{3} = x - \frac{2x-1}{3}$

Sol. (i) The given equation is

$$\frac{5x-4}{8} - \frac{x-3}{5} = \frac{x+6}{4}$$

The LCM of 8, 5 and 4 is 40. Multiplying both sides by 40 the given equation reduces to

$5(5x - 4) - 8(x - 3) = 10(x + 6)$

$\Rightarrow 25x - 20 - 8x + 24 = 10x + 60$

$\Rightarrow 17x + 4 = 10x + 60$

$\Rightarrow 17x - 10x = 60 - 4 \Rightarrow 7x = 56$

$\Rightarrow x = \frac{56}{7} = 8$

Thus, x = 8 is the solution of the given equation.

(ii) The given equation is $\frac{x-2}{4}+\frac{1}{3} = \frac{x}{1}-\frac{2x-1}{3}$

Multiplying both sides by LCM of 4, 3, 1 and 3 i.e., 12, the given equation reduces to

$3(x - 2) + 4 = 12x - 4(2x - 1)$

$\Rightarrow 3x - 6 + 4 = 12x - 8x + 4$

$\Rightarrow 3x - 2 = 4x + 4$

$\Rightarrow 3x - 4x = 4 + 2$

$\Rightarrow -x = 6$ or $x = -6$

Thus, $x = -6$ is the solution of the given equation.

Ex.7 Solve the equation : $\frac{2}{x-5}=\frac{x+3}{x-5}, x \neq 5$

Sol. The given equation is $\frac{2}{x-5}=\frac{x+3}{x-5}$

$\Rightarrow 2 = x + 3$ [Multiplying both sides by $x - 5$]

$\Rightarrow x + 3 = 2$

$\Rightarrow x = 2 - 3 \Rightarrow x = -1$

Thus, $x = -1$ is the solution of the given equation.

Ex.8 Solve the equation

$2(x + 1)(x + 3) + 8 = (2x + 1)(x + 5)$

Sol. The given equation is

$2(x + 1)(x + 3) + 8 = (2x + 1)(x + 5)$

$\Rightarrow 2(x^2 + x + 3x + 3) + 8 = 2x^2 + 10x + x + 5$

$\Rightarrow 2(x^2 + 4x + 3) + 8 = 2x^2 + 11x + 5$

$\Rightarrow 2x^2 + 8x + 6 + 8 = 2x^2 + 11x + 5$

$\Rightarrow 2x^2 + 8x + 14 = 2x^2 + 11x + 5$

$\Rightarrow 2x^2 + 8x - 2x^2 - 11x = 5 - 14$

$\Rightarrow -3x = -9$

$\Rightarrow x = \frac{-9}{-3} = 3$

Thus, $x = 3$ is the solution of the given equation.

Ex.9 Solve the equation :

$\frac{5x-5}{4x+7} = \frac{5x-31}{4x-23}, \ x \neq \frac{7}{4}, \ x \neq \frac{23}{4}$

Sol. The given equation is

$\frac{5x-5}{4x+7} = \frac{5x-31}{4x-23}$

$\Rightarrow (5x - 5)(4x - 23) = (4x + 7)(5x - 31)$

[By cross multiplication]

$\Rightarrow 20x^2 - 115x - 20x + 115$

$= 20x^2 - 124x + 35x - 217$

$\Rightarrow 20x^2 - 135x + 115 = 20x^2 - 89x - 217$

$\Rightarrow 20x^2 - 135x - 20x^2 - 89x - 217$

$\Rightarrow 20x^2 - 135x - 20x^2 + 89x = -217 - 115$

$\Rightarrow -46x = -332$

$\Rightarrow x = \frac{-332}{-46} = \frac{166}{23}$

Thus, $x = \frac{166}{23}$ is the solution of the given equation.

Applications of linear equations

(i) **Step I :** Read the problem carefully and denote the unknown quantity (or the quantity to be determined) by some variable x (say). If there are more than one unknown quantities, then denote one of them by x (say) and write the others in terms of x.

(ii) **Step II :** From the information given in the problem, formulate a linear equation for x.

(iii) **Step III :** Solve the linear equation obtained in step II to find x.

❖ EXAMPLES ❖

Ex.10 Find a number which, when added to its half, gives 33.

Sol. Let the required number be x. Then, half of the number x is x/2.

It is given that the number added to its half gives 33.

$\therefore\ x + \frac{x}{2} = 33 \quad \Rightarrow \frac{2x + x}{2} = 33$

$\Rightarrow \frac{3x}{2} = 33 \quad \Rightarrow 3x = 66$

So, the required number = 22

Ex.11 A number added to its two-third is equal to 35. Find the number.

Sol. Let the required number be x. Then two third of this number is $\frac{2}{3}x$. We are given that

Number + Two-third of the number = 35

$\Rightarrow x + \frac{2}{3}x = 35 \Rightarrow \frac{3x + 2x}{3} = 35$

$\Rightarrow \frac{5x}{3} = 35 \quad \Rightarrow 5x = 3 \times 35$

$\Rightarrow x = \frac{3 \times 35}{5} = 3 \times 7 = 21$

So, the required number = 21

Ex.12 Find the two numbers whose sum and difference are 25 and 5, respectively.

Sol. Let, one number be x. Then,

second number = 25 – x

[∵ Sum of the numbers = 25]

Now, difference of the numbers = 5

$\Rightarrow x - (25 - x) = 5 \quad \Rightarrow x - 25 + x = 5$

$\Rightarrow 2x = 30 \quad \Rightarrow x = \frac{30}{2} = 15$

$\therefore$ One number = 15

Second number = 25 – x = 25 – 15 = 10

Ex.13 What number increased by 6% of itself gives 2544 ?

Sol. Let the required number be x. Then,

6% of $x = \frac{6x}{100}$

We are given that, $x + \frac{6x}{100} = 2544$

$\Rightarrow x + \frac{3}{50}x = 2544$

$\Rightarrow \frac{50x+3x}{50} = 2544$

$\Rightarrow \frac{53x}{50} = 2544 \qquad \Rightarrow 53x = 2544 \times 50$

$\Rightarrow x = \frac{2544\times50}{53} \Rightarrow x = 48 \times 50 = 2400$

Thus, the required number is 2400.

Ex.14 A number consists of two digits of which the ten's digit exceeds the unit's digit by 6. The number itself is equal to ten times the sum of the digits. Find the number.

Sol. Let the unit's digit be x, Then,

Ten's digit = x + 6

$\therefore$ Number $= 10(x + 6) + x = 10x + 60 + x$

$= 11x + 60$

and, sum of the digits $= x + x + 6 = 2x + 6$

Now, Number = 10 (Sum of the digits)

$\Rightarrow 11x + 60 = 10(2x + 6)$

$\Rightarrow 11x + 60 = 20x + 60$

$\Rightarrow 11x - 20x = 60 - 60$

$\Rightarrow -9x = 0 \qquad \Rightarrow x = 0$

$\therefore$ Number $= 11 \times 0 + 60 = 60$

Ex.15 The perimeter of a rectangular field is 80 m. If the length of the field is decreased by 2m and its breadth is increased by 2m, the area is increased by 36 m^2. Find the length and breadth of the rectangular field.

Sol. Let the length and breadth of the rectangular field be x and y metres respectively. Then,

Perimeter = 2(x + y)

$\Rightarrow 2(x + y) = 80$ [$\because$ perimeter = 80 m]

$\Rightarrow x + y = 40$ [Dividing both sides by 2] $\Rightarrow$ $y = 40 - x$ ...(i)

Area = Length × Breadth = xy = x(40 – x) [Using (i)]

We are given that the area of the field is increased by 36 m^2 if the length of the field is decreased by 2m and breadth is increased by 2m.

$\therefore$ New length = (x – 2)m

New breadth = (y + 2)m

= (40 – x + 2)m [Using (i)]

$= (42 - x)m$

New area $= (x - 2)(42 - x)m^2$

Now, New Area = old Area + 36

$\Rightarrow (x - 2)(42 - x) = x(40 - x) + 36$

$\Rightarrow 42x - x^2 - 84 + 2x = 40x - x^2 + 36$

$\Rightarrow 44x - x^2 - 84 = 40x - x^2 + 36$

$\Rightarrow 44x - x^2 - 40x + x^2 = 84 + 36$

$\Rightarrow 4x = 120 \quad \Rightarrow x = 30$

$\therefore$ Length $= x = 30$ m

And, Breadth $= 40 - x = (40 - 30)m = 10m$

Ex.16 In an isosceles triangle, each of the two equal sides is 4 cm more than the base. If the perimeter of the triangle is 29 cm, find the sides of the triangle.

Sol. Let the base of the triangle be x cm. Then the remaining two equal sides are (x + 4) cm.

$\therefore$ Perimeter $= (x + 4) + x + (x + 4)$

$= (3x + 8)$ cm.

Now, Perimeter = 29 cm

$\Rightarrow 3x + 8 = 29$ cm

$\Rightarrow 3x = 29 - 8 \quad \Rightarrow 3x = 21$

$\Rightarrow x = \frac{21}{3} = 7$

$\therefore x + 4 = 7 + 4 = 11$

Thus the sides of the triangle are 7 cm, 11 cm and 11 cm.

Ex.17 The measures of angles of a quadrilateral in degrees are x, 3x – 40, 2x and 4x + 20. Find the measures of angles.

Sol. Since the sum of the angles of a quadrilateral is 360º. Therefore,

$x + (3x - 40) + 2x + (4x + 20) = 360^\circ$

$\Rightarrow 10x - 20 = 360^\circ \Rightarrow 10x = 360 + 20$

$\Rightarrow x = \frac{380}{10} = 38$

Now, $x = 38 \quad \Rightarrow 2x = 2 \times 38 = 76$,

$3x - 40 = 3 \times 38 - 40 = 74$ and

$4x + 20 = 4 \times 38 + 20 = 172$

Thus, the measures of the angles are 38º, 74º, 76º and 172º.

Ex.18 Kareem is three times as old as his son. After ten years, the sum of their ages will be 76 years. Find their present ages.

Sol. Let the present age of Kareem's son be x years. Then, Kareem's age = 3x years

After 10 year;

Kareem's age = (3x + 10) years

Kareem's son age = (x + 10) years

$\therefore\ (3x + 10) + (x + 10) = 76$

$\Rightarrow 4x + 20 = 76 \quad \Rightarrow 4x = 76 - 20$

$\Rightarrow 4x = 56 \quad \Rightarrow x = \frac{56}{4} = 14$

$\therefore$ Kareem's present age = 3x

= 3 × 14 = 42 years

Kareem's son's age = x = 14 years.

Ex.19 Ten years ago a father was six times as old as his daughter. After 10 years, he will be twice as old as his daughter. Determine their present ages.

Sol. Suppose ten years ago the age of the daughter was x years. Then, ten years ago the age of her father was 6x years.

$\therefore$ Present age of father = (6x + 10) years

Present age of daughter = (x + 10) years

Ten years after :

Father's age = (6x + 10 + 10) years

= (6x + 20) years

Daughter's age= (x + 10 + 10) years

= (x + 20) years

$\therefore\ 6x + 20 = 2(x + 20) \Rightarrow 6x + 20 = 2x + 40$

$\Rightarrow 6x - 2x = 40 - 20 \quad \Rightarrow 4x = 20$

$\Rightarrow x = \frac{20}{4} = 5$

$\therefore$ Present age of father = (6 × 5 + 10) = 40 years

Present age of daughter = (5 + 10) = 15 years

Ex.20 A and B together can do a piece of work in 4 days, but A alone can do it in 12 days. How many days would B alone take to do the same piece of work ?

Sol. Suppose B alone takes x days to do the work.

Since, A alone can complete the work in 12 days's. Therefore, in one day A completes $\frac{1}{12}$ th part of the work.

Similarly, in one day B completes $\left(\frac{1}{x}\right)$ th part of work.

A and B together complete $\left(\frac{1}{4}\right)$ th part of the work in one day.

Now, A's one day's work + B′s one day's work = (A + B)′s one day work

$\Rightarrow \frac{1}{12} + \frac{1}{x} = \frac{1}{4} \Rightarrow \frac{1}{x} = \frac{1}{4} - \frac{1}{12}$

$\Rightarrow \frac{1}{x} = \frac{3-1}{12} \quad \Rightarrow \frac{1}{x} = \frac{1}{6} \quad \Rightarrow 6 = x$

Thus, B alone can do the work in 6 days.

Graph of Linear Equation in One Variable

(i) **Step I :** Obtain the linear equation.

(ii) **Step II :** If the equation is of the form ax = b, a ≠ 0, then plot the point $\left(\frac{b}{a}, 0\right)$ and one more point $\left(\frac{b}{a}, \alpha\right)$, where α is any real number, on the graph paper. If the equation is of the form ay = b, a ≠ 0, then plot the point $\left(0, \frac{b}{a}\right)$ and $\left(\beta, \frac{b}{a}\right)$, where β is any real number, on the graph paper.

(iii) **Step III :** Join the points plotted in step II to obtain the required line.

❖ EXAMPLES ❖

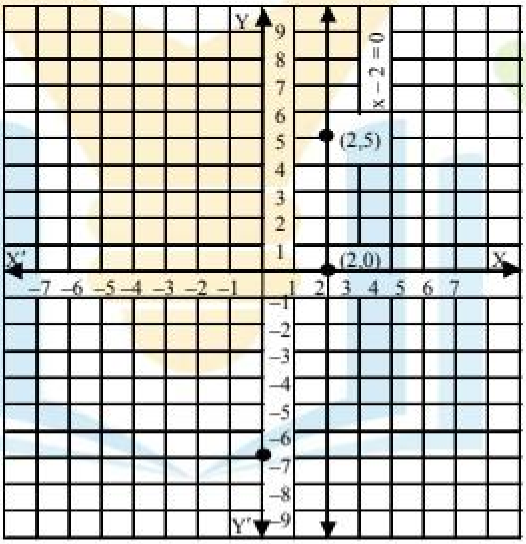
Ex.21 Draw the graph of the following linear equation: x – 2 = 0

Sol. We have, x – 2 = 0 ⇒ x = 2

Clearly, it does not contain y. So, its graph is a line parallel to y-axis passing through the point (2, 0). In fact, it passes through every point whose x-coordinate is 2. Thus, we have the following table exhibiting the coordinates of points on the line represented by x = 2.

x	2	2	2	2
y	0	1	5	-3

Plotting any two points, say (2, 0) and (2, 5) given by the above table on the graph paper and joining them, we obtain the straight line as shown in fig.



Ex.22 Draw a graph of the equation :

2y + 3 = 9

Sol. We have,

2y + 3 = 9 ⇒ 2y = 9 – 3

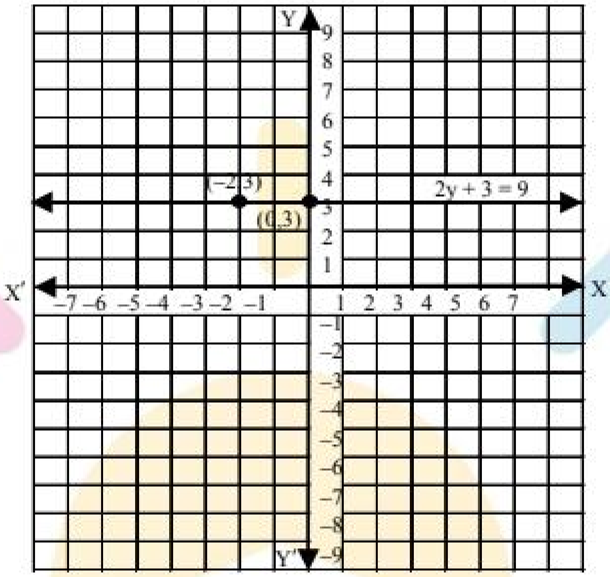
⇒ 2y = 6 ⇒ y = 3

The equation y = 3 does not contain x. So, its graph is a line parallel to x-axis passing through the point (0, 3). Clearly, y = 3 means that for all values of the abscissa x, the ordinate y is 3.

Thus, we have the following table exhibiting the abscissa and ordinates of the points lying on the line represented by the given equation.

x	0	–2
y	3	3

Plotting the points (0, 3) and (–2, 3) on the graph paper and joining them by a line, we obtain the graph of the line represented by the given equation as shown in fig.



Linear equations in Two Variables

An equation of the form ax + by + c = 0 or ax + by = d, where a, b, c, d are real numbers, a $\neq$ 0, b $\neq$ 0 and x, y are variables, is called linear equation in two variables.

Solution :

Any pair of values of x and y which satisfies the equation ax + by + c = 0 is called a solution of it.

❖ EXAMPLES ❖

Ex.23 Which of the following points lie on the x-axis ?

A(1, 1), B(1, 0), C(0, 1), D(0, 0) E(–1, 0), F(0, –1), G(4, 0), H(0, –7).

Sol. Points of the form (a, 0), i.e., the points in which ordinate is 0, those points lie on the x-axis and the points in which abscissa is 0, lie on the y-axis.

B(1, 0), D(0, 0), E(–1, 0), G(4, 0).

All these points have their ordinate 0, so they lie on x-axis.

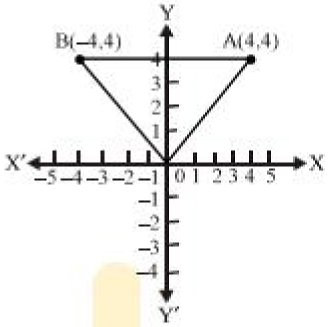
Ex.24 Which of the points in Example. 24 lie on the y-axis ?

Sol. C(0, 1), D(0, 0), F(0, –1), H(0, –7)

All these points have their abscissa 0, so they lie on the y-axis.

Ex.25 Plot the points A(4, 4) and B(–4, 4) and join the lines OA, OB and BA. What figure do you obtain ?

Sol. On joining OA, OB and BA, we get a triangle



Ex.26 Find out which of the following equations have x = 2, y = 1 as a solution:

(a) $2x + 5y = 9$ (b) $5x + 3y = 14$

(c) $2x + 3y = 7$

Sol. (a) $2x + 5y = 9$

Putting x = 2 and y = 1 on the L.H.S.

$2(2) + 5(1) = 4 + 5 = 9 = \text{R.H.S.}$

$\therefore$ x = 2, y = 1 is a solution of the given equation.

(b) $5x + 3y = 14$

Putting x = 2 and y = 1 on the L.H.S.

$5(2) + 3(1) = 10 + 3 = 13 \neq \text{R.H.S}$

$\therefore$ x = 2, y = 1 is not a solution of the given equation.

(c) $2x + 3y = 7$

Putting x = 2, y = 1 on the L.H.S.

$2(2) + 3(1) = 4 + 3 = 7 = \text{R.H.S}$

x = 2, y = 1 is a solution of the given equation.

Graph of Linear Equation ax + by + c = 0 in Two Variables, where a ≠ 0, b ≠ 0

(i) **Step I :** Obtain the linear equation, let the equation be $ax + by + c = 0$.

(ii) **Step II :** Express y in terms of x to obtain

$$y = -\left(\frac{ax + c}{b}\right)$$

(iii) **Step III :** Give any two values to x and calculate the corresponding values of y from the expression in step II to obtain two solutions, say (α_1, β_1) and (α_2, β_2).

If possible take values of x as integers in such a manner that the corresponding values of y are also integers.

(iv) **Step IV :** Plot points (α_1, β_1) and (α_2, β_2) on a graph paper.

(v) **Step V :** Join the points marked in step IV to obtain a line.

The line obtained is the graph of the equation

$ax + by + c = 0$.

❖ EXAMPLES ❖

Ex.27 Draw the graph of the equation y – x = 2.

Sol. We have,

$y - x = 2$

$\Rightarrow y = x + 2$

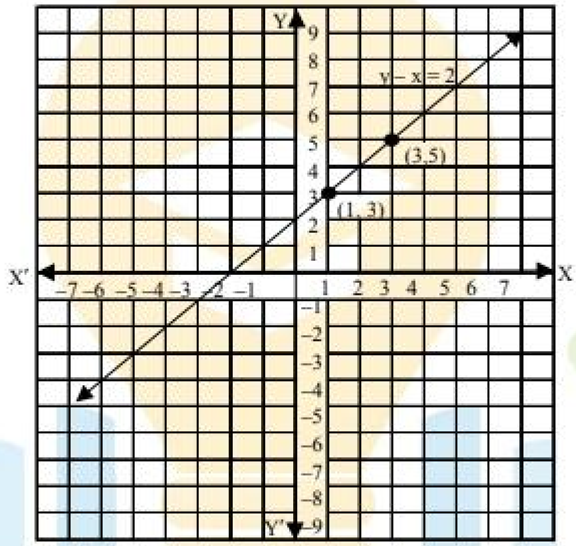
When x = 1, we have : $y = 1 + 2 = 3$

When x = 3, we have : $y = 3 + 2 = 5$

Thus, we have the following table exhibiting the abscissa and ordinates of points on the line represented by the given equation.

x	1	3
y	3	5

Plotting the points (1, 3) and (3, 5) on the graph paper and drawing a line joining them, we obtain the graph of the line represented by the given equation as shown in Fig.



Ex.28 Draw a graph of the line x – 2y = 3. From the graph, find the coordinates of the point when (i) x = – 5(ii) y = 0.

Sol. We have $x - 2y = 3$

$\Rightarrow y = \frac{x-3}{2}$

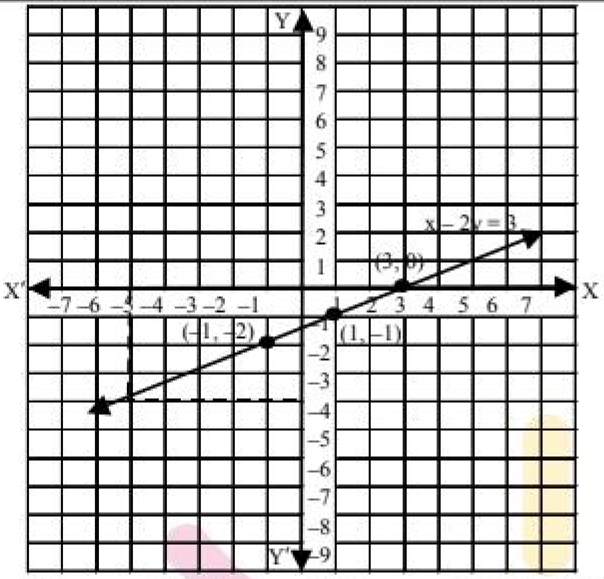
When x = 1, we have : $y = \frac{1-3}{2} = -1$

When x = –1, we have : $y = \frac{-1-3}{2} = -2$

Thus, we have the following table :

x	1	–1
y	–1	–2

Plotting points (1, –1) & (–1, –2) on graph paper & joining them, we get straight line as shown in fig. This line is required graph of equation x – 2y = 3.



To find the coordinates of the point when $x = -5$, we draw a line parallel to y-axis and passing through (–5, 0). This line meets the graph of $x - 2y = 3$ at a point from which we draw a line parallel to x-axis which crosses y-axis at $y = -4$. So, the coordinates of the required point are (–5, –4).

Since $y = 0$ on x-axis. So, the required point is the point where the line meets x-axis. From the graph the coordinates of such point are (3, 0).

Hence, required points are (–5, –4) and (3, 0).

Simultaneous Linear Equations in Two Variables

Definition :

A pair of linear equations in two variables is said to form a system of simultaneous linear equations.

For Example :

Each of the following pairs of linear equations forms a system of two simultaneous linear equations in two variables -

(i) $x + 2y = 3$ (ii) $2u + 5v + 1 = 0$

$2x - y = 5$ $u - 2v + 9 = 0$

(iii) $\frac{3}{x} + \frac{2}{y} = 9$ (iv) $2a + b - 1 = 0$

$\frac{1}{x} - \frac{1}{y} = 5$ $a + b + 5 = 0$

Solution :

A pair of values of the variables x and y satisfying each one of the equations in a given system of two simultaneous linear equations in x and y is called a solution of the system.

So, $x = 2, y = -1$ is a solution of the system of simultaneous linear equations

$x + y = 1$; $2x - 3y = 7$.

❖ EXAMPLES ❖

Ex.29 Check, whether x = 2, y = 3 is a solution of the system of simultaneous linear equations :

$2x + y = 7; \quad 3x + 2y = 12$

Sol. The given equations are

$2x + y = 7$ ....(1)

$3x + 2y = 12$ ....(2)

Putting x = 2 and y = 3 in (1), we get

L.H.S. $= 2 \times 2 + 3 = 4 + 3 = 7 =$ R.H.S.

Putting x = 2 and y = 3 in (2), we get

L.H.S. $= 3 \times 2 + 2 \times 3 = 6 + 6 = 12 =$ R.H.S.

Thus, values x = 2 and y = 3 satisfy both the equations (1) and (2).

Hence, x = 2, y = 3 is a solution of the given equations.

Ex.30 Show that x = 2 and y = –1 is not the solution of the given system of simultaneous equations

$3x + 2y = 4 ; \quad 2x + y = 2$

Sol. The given equations are

$3x + 2y = 4$ ....(1)

$2x + y = 2$ ....(2)

On putting x = 2 and y = – 1 in (1), we get

L.H.S. $= 3 \times 2 + 2 \times (-1) = 6 - 2 = 4 =$ R.H.S.

On putting x = 2 and y = – 1 in (2), we get

L.H.S. $= 2 \times 2 + (-1) = 4 - 1 = 3 \neq$ R.H.S.

Thus, x = 2 and y = – 1 satisfy equation (1), but not equation (2).

Therefore, x = 2, y = – 1 is not the solution of the given system of simultaneous equations.

Ex.31 Show that x = 1, y = 1 and x = 2, y = 3 are solutions of the system of equations

$2x - y = 1; \quad 4x - 2y = 2$

Sol. Here, the given equations are

$2x - y = 1$.... (1)

$4x - 2y = 2$.... (2)

On putting x = 1 and y = 1 in (1), we get

L.H.S. $= 2 \times 1 - 1 = 2 - 1 = 1 =$ R.H.S.

On putting x = 1, y = 1 in (2), we get

L.H.S. $= 4 \times 1 - 2 \times 1 = 4 - 2 = 2 =$ R.H.S.

Thus x = 1, y = 1 is the solution of the given system of equations.

On putting x = 2 and y = 3 in (1), we get

L.H.S. $= 2 \times 2 - 3 = 4 - 3 = 1 =$ R.H.S.

On putting x = 2 and y = 3 in (2), we have

L.H.S. $= 4 \times 2 - 2 \times 3 = 8 - 6 = 2 =$ R.H.S.

Thus, x = 2, y = 3 is also the solution of the given system of equations.

Graphical Representation of Pair of Linear Equations

Let the system of pair of linear equations be

$$a_1x + b_1y = c_1 \quad \text{....(1)}$$

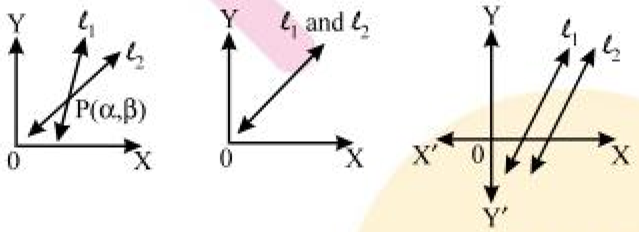
$$a_2x + b_2y = c_2 \quad \text{....(2)}$$

We know that given two lines in a plane, only one of the following three possibilities can happen -

(i) The two lines will intersect at one point.

(ii) The two lines will not intersect, however far they are extended, i.e., they are parallel.

(iii) The two lines are coincident lines.



❖ EXAMPLES ❖

Ex.32 The path of highly number 1 is given by the equation x + y = 7 and the highway number 2 is given by the equation 5x + 2y = 20. Represent these equations geometrically.

Sol. We have, $x + y = 7$

$\Rightarrow y = 7 - x$ ....(1)

In tabular form

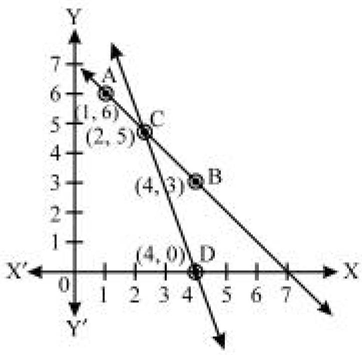
x	1	4
y	6	3
Points	A	B

and $5x + 2y = 20$

$\Rightarrow y = \frac{20-5x}{2}$ ....(2)

In tabular form

x	2	4
y	5	0
Points	C	D



Plot the points A (1, 6), B(4, 3) and join them to form a line AB.

Similarly, plot the points C(2, 5). D (4, 0) and join them to get a line CD. Clearly, the two lines intersect at the point C. Now, every point on the line AB gives us a solution of equation (1). Every point on CD gives us a solution of equation (2).

Ex.33 Aftab tells his daughter, " Seven years ago, I was seven times as old as you were then. Also, three years from now, I shall be three times as old as you will be." (Isn't this interesting ?) Represent this situation algebraically and graphically.

Sol. Let the present age of father be x-years and that of daughter = y years

Seven years ago father's age = (x – 7) years

Seven years ago daughter's age = (y – 7) years

According to the problem

$(x - 7) = 7(y - 7)$ or $x - 7y = -42$ ....(1)

After 3 years father's age = (x + 3) years

After 3 years daughter's age = (y + 3) years

According to the condition given in the question

$x + 3 = 3(y + 3)$ or $x - 3y = 6$ ....(2)

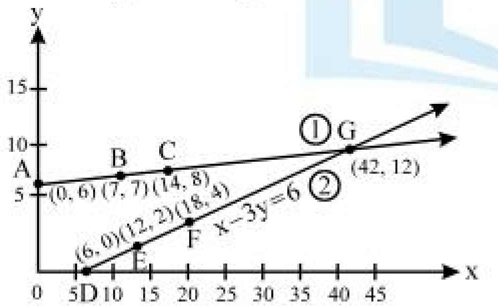
$x - 7y = -42$

x	0	7	14
$y = \frac{x+42}{7}$	6	7	8
Points	A	B	C

$x - 3y = 6$

x	6	12	18
$y = \frac{x-6}{3}$	0	2	4
Points	D	E	F

Plot the points A(0, 6), B(7, 7), C(14, 8) and join them to get a straight line ABC. Similarly plot the points D(6, 0), E(12, 2) and F(18, 4) and join them to get a straight line DEF.



Ex.34 10 students of class X took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz.

Sol. Let the number of boys be x and the number of girls be y.

Then the equations formed are

$x + y = 10$ ....(1)

and $y = x + 4$ ....(2)

Let us draw the graphs of equations (1) and (2) by finding two solutions for each of the equations. The solutions of the equations are given.

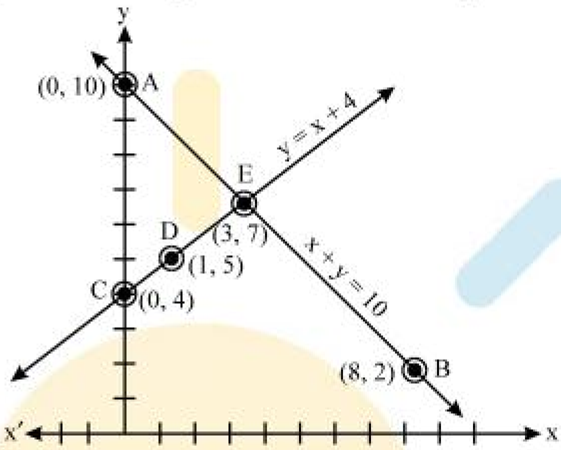
$x + y = 10$

x	0	8
$y = 10 - x$	10	2
Points	A	B

$y = x + 4$

x	0	1	3
$y = x + 4$	4	5	7
Points	C	D	E

Plotting these points we draw the lines AB and CE passing through them to represent the equations. The two lines AB and Ce intersect at the point E (3, 7). So, x = 3 and y = 7 is the required solution of the pair of linear equations.



i.e. Number of boys = 3

Number of girls = 7.

Verification :

Putting x = 3 and y = 7 in (1), we get

L.H.S. = 3 + 7 = 10 = R.H.S.,

(1) is verified.

Putting x = 3 and y = 7 in (2), we get

$7 = 3 + 4 = 7$, (2) is verified.

Hence, both the equations are satisfied.

Ex.35 Half the perimeter of a garden, whose length is 4 more than its width is 36m. Find the dimensions of the garden.

Sol. Let the length of the garden be x and width of the garden be y.

Then the equation formed are

$x = y + 4$ ....(1)

Half perimeter = 36

$x + y = 36$ ....(2)

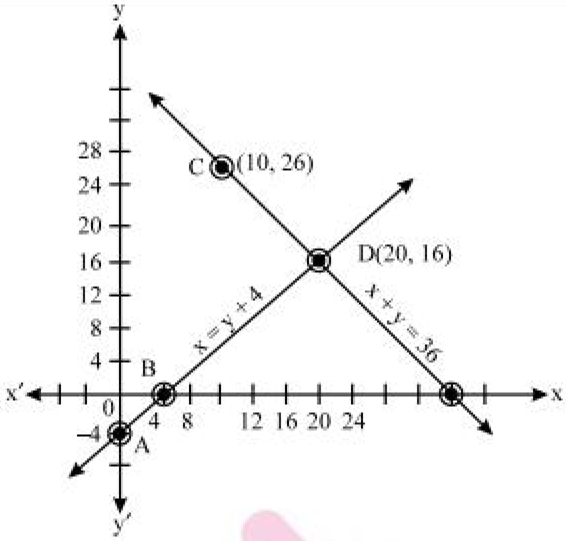
$x = y + 4$

x	0	4
y	−4	0
Points	A	B

$x + y = 36$

x	10	20
$y = 36 - x$	26	16
Points	C	D

Plotting these points we draw the lines AB and CD passing through them to represent the equations.



The two lines AB and CD intersect at the point (20, 16), So, x = 20 and y = 16 is the required solution of the pair of linear equations i.e. length of the garden is 20 m and width of the garden is 16 m.

Verification :

Putting x = 20 and y = 16 in (1).

We get

$20 = 16 + 4 = 20$, (1) is verified.

Putting x = 20 and y = 16 in (2). we get

$20 + 16 = 36$

$\Rightarrow 36 = 36$, (2) is verified.

Hence, both the equations are satisfied.

Ex.36 Draw the graphs of the equations $x - y + 1 = 0$ and $3x + 2y - 12 = 0$. Determine the coordinates of the vertices of the triangle formed by these lines and the x-axis, and shade the triangular region.

Sol. Pair of linear equations are :

$x - y + 1 = 0$ ....(1)

$3x + 2y - 12 = 0$ ....(2)

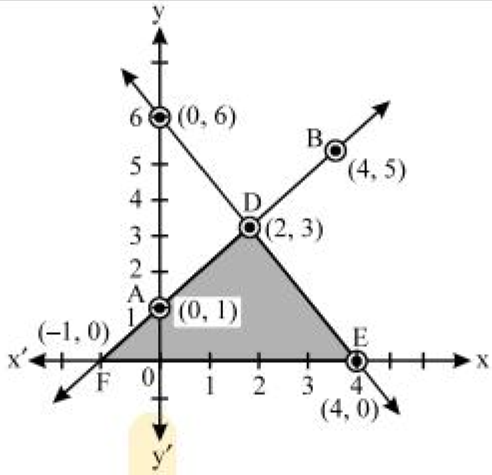
In tabular form

x	0	4
$y = x + 1$	1	5
Points	A	B

In tabular form

x	0	2
$y = \frac{12 - 3x}{2}$	6	3
Points	C	D

Plot the points A(0, 1), B(4, 5) and join them to get a line AB. Similarly, plot the points C(0, 6), D(2, 3) and join them to form a line CD.



Clearly, the two lines intersect each other at the point D(2, 3). Hence x = 2 and y = 3 is the solution of the given pair of equations.

The line CD cuts the x-axis at the point E (4, 0) and the line AB cuts the x-axis at the point F(–1, 0).

Hence, the coordinates of the vertices of the triangle are ; D(2, 3), E(4, 0), F(–1, 0).

Verification :

Both the equations (1) and (2) are satisfied by x = 2 and y = 3. Hence, Verified.

Ex.37 Draw the graph of $x - y + 1 = 0$, $3x + 2y - 12 = 0$. Calculate, the area bounded by these lines and x-axis.

Sol. We have the following equations

$x - y + 1 = 0$

$\Rightarrow y = x + 1$ ....(1)

In tabular form

x	–1	2
y	0	3
Points	A	B

$3x + 2y - 12 = 0$

$\Rightarrow 2y = 12 - 3x$ ....(2)

$y = \frac{12-3x}{2}$

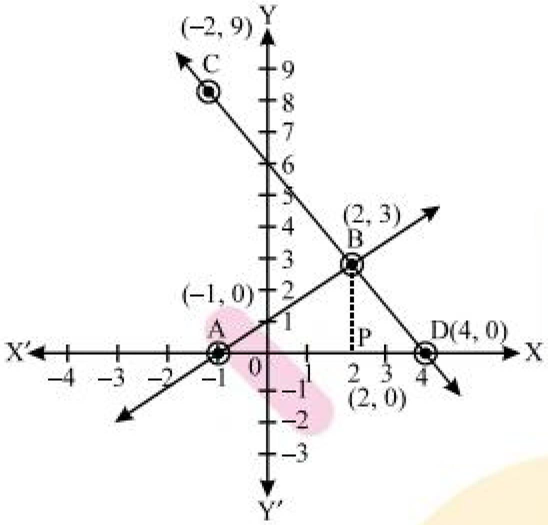
In tabular form

x	–2	4
y	9	–
Points	C	D

Plot the points A (–1, 0), B (2, 3) and join them to get a line AB.

Similarly, plot points C(–2, 9), D (4, 0) and join them to form a line CD. The line AB cuts the x-axis at the point A (–1, 0) and the line CD cuts it at the point line AB cuts the x-axis at the point

A (–1, 0) and the line CD cuts it at the point D(4, 0). Also, the lines AB and CD cut each other at the point B(2, 3) Hence, A (–1, 0), B (2, 3) and D(4, 0) are the vertices of the triangle, so formed. From B (2, 3) draw ⊥ on the x-axis meeting the x-axis at P.



Now, clearly PB is the altitude of the ΔBAD and AD being the base of this triangle. Base AD = Distance between A and D = 4 – (–1) = 5 units.

Similarly, altitude PB = distance between P and B = 3 – 0 = 3 units.

∴ Area of ΔBAD = $\frac{1}{2}$ × Base × Altitude

= $\frac{1}{2} \times 5 \times 3 = \frac{15}{2} = 7.5$ sq. units.

Types of Solutions

There are three types of solutions :

1. Unique solution.
2. Infinitely many solutions
3. No solution.

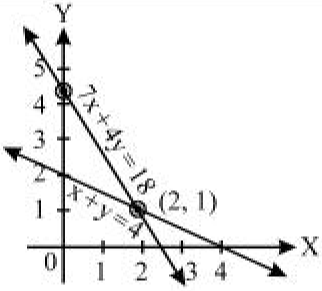
(A) Consistent :

If a system of simultaneous linear equations has at least one solution then the system is said to be consistent.

(i) Consistent equations with unique solution : The graphs of two equations intersect at a unique point. **For example**. Consider

$x + 2y = 4$

$7x + 4y = 18$



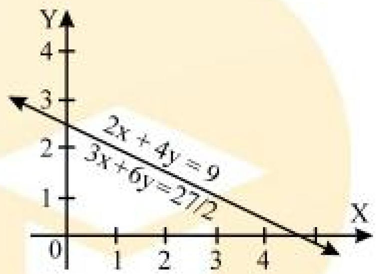
The graphs (lines) of these equations intersect each other at the point (2, 1) i.e., x = 2, y = 1. Hence, the equations are consistent with unique solution.

(ii) Consistent equations with infinitely many solutions : The graphs (lines) of the two equations will be coincident.

For example. Consider

$$2x + 4y = 9$$

$$3x + 6y = \frac{27}{2}$$



The graphs of the above equations coincide. Coordinates of every point on the lines are the solutions of the equations. Hence, the given equations are consistent with infinitely many solutions.

(B) Inconsistent Equation :

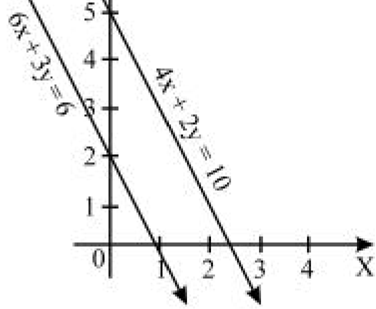
If a system of simultaneous linear equations has no solution, then the system is said to be inconsistent.

No Solution : The graph (lines) of the two equations are parallel.

For example. Consider

$$4x + 2y = 10$$

$$6x + 3y = 6$$



The graphs (lines) of the given equations are parallel. They will never meet at a point. So, there is no solution. Hence, the equations are inconsistent.

S.No	Graph of Two Equations	Types of Equations
1	Intersecting lines	Consistent, with unique solution
2	Coincident	Consistent with infinite solutions
3	Parallel lines	Inconsistent (No solution)

❖ EXAMPLES ❖

Ex.38 Show graphically that the system of equations

$x - 4y + 14 = 0$; $3x + 2y - 14 = 0$

is consistent with unique solution.

Sol. The given system of equations is

$x - 4y + 14 = 0$ ....(1)

$\Rightarrow y = \frac{x+14}{4}$

When $x = 6$, $y = \frac{6+14}{4} = 5$

When $x = -2$, $y = \frac{-2+14}{4} = 3$

In tabular form

x	6	−2
y	5	3
Points	A	B

$3x + 2y - 14 = 0$ ....(2)

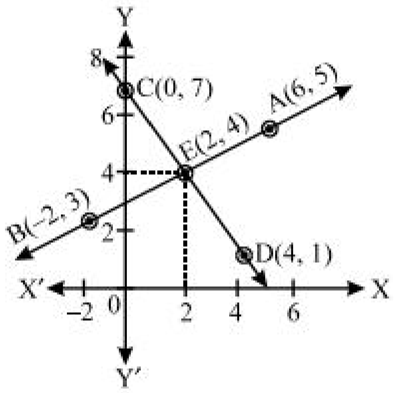
$\Rightarrow y = \frac{-3x+14}{2}$

When $x = 0$, $y = \frac{0+14}{2} = 7$

When $x = 4$, $y = \frac{-3\times4+14}{2} = 1$

In tabular form

x	0	4
y	7	1
Points	C	D



The given equations representing two lines, intersect each other at a unique point (2, 4). Hence, the equations are consistent with unique solution.

Ex.39 Show graphically that the system of equations

$2x + 5y = 16$; $3x + \frac{15}{2}y = 24$

has infinitely many solutions.

Sol. The given system of equations is

$2x + 5y = 16$ ....(1)

$\Rightarrow y = \frac{16-2x}{5}$

When $x = 3$, $y = \frac{16-6}{5} = 2$

When $x = -2$, $y = \frac{16-2\times(-2)}{5} = 4$

In tabular form

x	−2	3
y	4	2
Points	A	B

$3x + \frac{15}{2}y = 24$ ....(1)

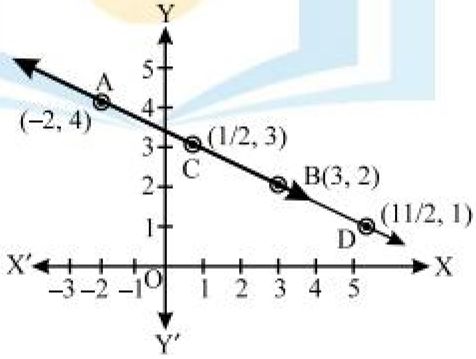
$\Rightarrow y = \frac{48-6x}{15}$ ....(2)

When $x = \frac{1}{2}$, $y = \frac{48-3}{15} = 3$

When $x = \frac{11}{2}$, $y = \frac{48-6\times\left(\frac{11}{2}\right)}{15} = 1$

In tabular form

x	$\frac{1}{2}$	$\frac{11}{2}$
y	3	1
Points	C	D



The lines of two equations are coincident. Coordinates of every point on this line are the solution.

Hence, the given equations are consistent with infinitely many solutions.

Ex.40 Show graphically that the system of equations

$2x + 3y = 10$, $4x + 6y = 12$ has no solution.

Sol. The given equations are

$2x + 3y = 10$

$\Rightarrow 3y = 10 - 2x \qquad \Rightarrow y = \frac{10-2x}{3}$

When $x = -4$, $y = \frac{10-2\times(-4)}{3} = \frac{10+8}{3} = 6$

When $x = 2$, $y = \frac{10-2\times2}{3} = \frac{10-4}{3} = 2$

In tabular form

x	–4	2
y	6	2
Points	A	B

$4x + 6y = 12$

$\Rightarrow 6y = 12 - 4x \qquad \Rightarrow 6y = 12 - 4x$

$\Rightarrow y = \frac{12-4x}{6}$

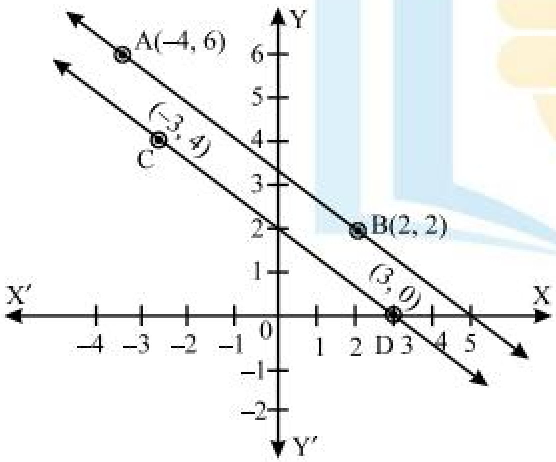
When $x = -3$, $y = \frac{12-4\times(-3)}{6} = \frac{12+12}{6} = 4$

When $x = 3$, $y = \frac{12-4\times(3)}{6} = \frac{12-12}{6} = 0$

In tabular form

x	–3	3
y	4	0
Points	C	D

Plot the points A (–4, 6), B(2, 2) and join them to form a line AB. Similarly, plot the points C(–3, 4), D(3, 0) and join them to get a line CD.



Clearly, the graphs of the given equations are parallel lines. As they have no common point, there is no common solution. Hence, the given system of equations has no solution.

Ex.41 Given the linear equation $2x + 3y - 8 = 0$, write another linear equation in two variables such that the geometrical representing of the pair so formed is :

(i) intersecting lines

(ii) parallel lines

(iii) coincident lines

Sol. We have,

$$2x + 3y - 8 = 0$$

(i) Another linear equation in two variables such that the geometrical representation of the pair so formed is intersecting lines is

$$3x - 2y - 8 = 0$$

(ii) Another parallel lines to above line is

$$4x + 6y - 22 = 0$$

(iii) Another coincident line to above line is

$$6x + 9y - 24 = 0$$

Ex.42 Which of the following pairs of linear equations are consistent obtain the solution in such cases graphically.

(i) $x + y = 5,\ 2x + 2y = 10$

(ii) $x - y = 8,\ 3x - 3y = 16$

(iii) $2x + y - 6 = 0,\ 4x - 2y - 4 = 0$

(iv) $2x - 2y - 2 = 0,\ 4x - 4y - 5 = 0$

Sol. (i) We have,

$x + y = 5$ ....(1)

$\Rightarrow y = 5 - x$

If $x = 0,\ y = 5$

If $x = 5,\ y = 0$

x	0	5
$y = 5 - x$	5	0
Points	A	B

$2x + 2y = 10$ ....(2)

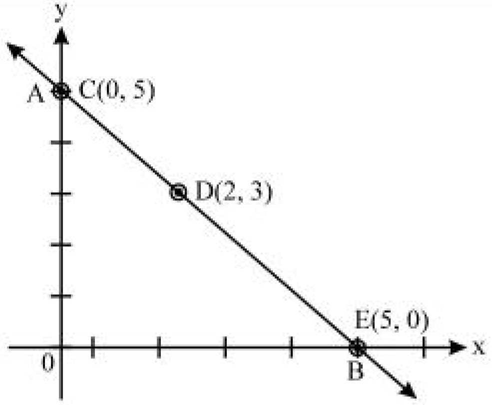
$\Rightarrow y = \frac{10-2x}{2}$

If $x = 0,\ y = 5$

If $x = 2,\ y = 3$

x	0	2	5
$y = \frac{10-2x}{2}$	5	3	0
Points	C	D	E

Plotting the points A(0, 5) and B(5, 0) we get the line AB. Again plotting the points C(0, 5). D(2, 3) and E(5, 0), we get the line CD.



We observe that the lines represented by equations (1) and (2) are coincident. Therefore, equations (1) and (2) have infinitely many common solutions.

Hence, the pair is a dependent consistent pair of linear equations

(ii) $x - y = 8 \quad \Rightarrow \quad y = x - 8$

When $x = 0, \quad y = -8$

When $x = 8, \quad y = 0$

x	0	8
$y = x - 8$	-8	0
Points	A	B

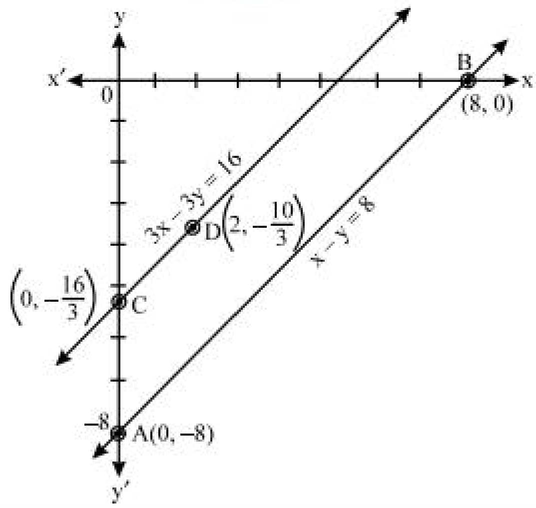
$3x - 3y = 16 \quad \Rightarrow y = \frac{3x-16}{3}$

When $x = 0$, $y = -\frac{16}{3}$

When $x = 2$, $y = -\frac{10}{3}$

x	0	2
$y - \frac{3x-16}{3}$	$-\frac{16}{3}$	$-\frac{10}{3}$
Points	C	D

Plotting the points A(0, –8), B(8, 0) we get the line AB. Plotting the points $C\left(0, -\frac{16}{3}\right)$ and $D\left(2, -\frac{10}{3}\right)$, we get the line CD.



We observe that there is no common point between them; these are parallel lines.

Hence, the pair of linear equations has no solutions. It is an inconsistent pair of linear equations.

(iii) We have,

$$2x + y - 6 = 0 \Rightarrow y = 6 - 2x$$

when $x = 0, y = 6$

when $x = 3, y = 0$

x	0	3
y	6	0
Points	A	B

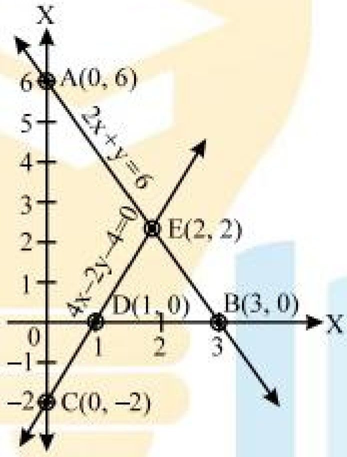
and $4x - 2y - 4 = 0 \Rightarrow y = 2x - 2$

when $x = 0, \quad y = -2$

when $x = 1, \quad y = 0$

x	0	1
y	−2	0
Points	C	D

Plotting the points A(0, 6) and B(3, 0), We get the straight line AB. Plotting C(0, –2) and D(1, 0) we get the straight line CD.



The lines AB and CE intersect at E(2, 2).

We observe that there is a point E common to both the lines AB and CD. Therefore, the pair of equations is consistent and this point (2, 2) gives us the solution of the pairs $2x + y - 6 = 0$ and $4x - 2y - 4 = 0$.

So, the solution of the linear pair is $x = 2$ and $y = 2$.

Hence, the given pair of equations is consistent.

(iv) We have,

$$2x - 2y - 2 = 0 \Rightarrow y = x - 1$$

when $x = 0, \; y = -1$

when $x = 1, \; y = 0$

x	0	1
y	–1	0
Point	A	B

$4x - 4y - 5 = 0 \Rightarrow y = x - \frac{5}{4}$

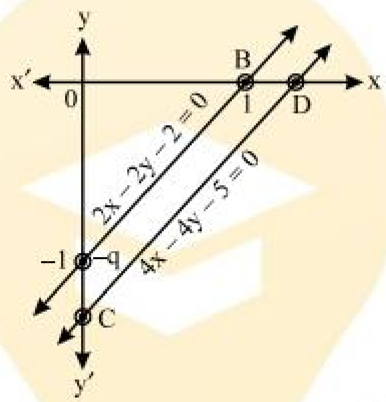
when $x = 0, y = -\frac{5}{4}$

when $x = \frac{5}{4}, \quad y = 0$

x	0	$\frac{5}{4}$
$y = x - \frac{5}{4}$	$-\frac{5}{4}$	0
Point	C	D

Plotting the points A(0, –1) and B(1, 0), we get the line AB.

Plotting the points $C\left(0, -\frac{5}{4}\right)$, $D\left(\frac{5}{4}, 0\right)$ we get the line CD.



We observe that as there is no common point between them, these are parallel lines.

Hence, the pair of linear equations has no solution.

It is an inconsistent pair of linear equations.

Ex.43 Determine graphically the vertices of the triangle, the equations of whose sides are given as,

$y = x$, $y = 0$ and $3x + 2y = 10$

Sol. (a) Graph of $y = x$

We have; $y = x$

When $x = 1, \; y = 1$

When $x = 2, \; y = 2$

Then, we have the following table :

x	1	2
y	1	2

Plotting the points P(1, 1) and Q(2, 2) on the graph paper and drawing a line joining between them, we get the graph of the equation y = x as shown in fig.

(b) Graph of the equation, $y = 0$;

This equation is located at the origin. All points lie at the origin. This is shown in fig.

(c) Graph of the $3x + 2y = 10$

We have, $3x + 2y = 10$

$\Rightarrow y = \frac{10-3x}{2}$

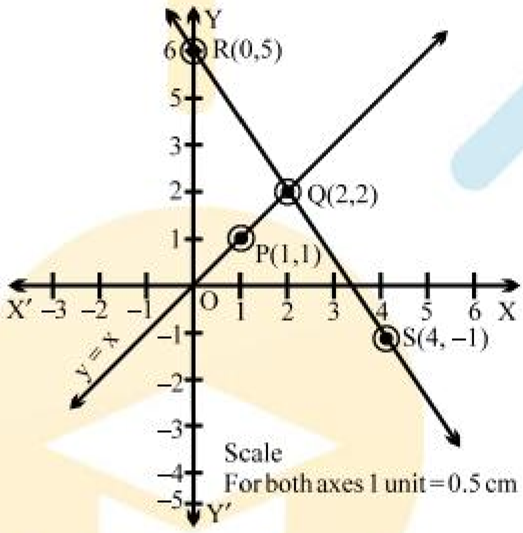
When, $x = 0, y = 5$

When, $x = 4, y = -1$

Then, we have the following table :

x	0	4
y	5	–1

Plotting the points R(0, 5) and S(4, – 1) on the same graph paper and drawing a line joining between them, we get the graph of the equation $3x + 2y = 10$ as shown in fig.



From the graph of the three given equations, we find that the three lines taken in pairs intersect each other at points O(0, 0), T(10/3, 0) and U(2, 2).

Hence, the vertices of the required triangle are (0, 0), (10/3, 0) and (2, 2).

Ex.44 Solve graphically the following system of linear equations.

$2x + y - 11 = 0$; $x - y - 1 = 0$

Also, find the co-ordinates of the points where the lines meet y-axis.

Sol. We have;

$2x + y - 11 = 0$

$x - y - 1 = 0$

(a) Graph of the equation $2x + y - 11 = 0$

We have; $2x + y - 11 = 0$

$\Rightarrow y = -2x + 11$

When, $x = 0, y = -2 \times 0 + 11 = 11$

When, $x = 1, y = -2 \times 1 + 11 = 9$

Then, we have the following table :

x	0	1
y	11	9

Plotting the points P(0, 11) and Q(1, 9) on the graph paper and drawing a line joining between them, we get the equation $2x + y - 11 = 0$ as shown in fig.

(b) Graph of the equation x – y – 1 = 0;

We have ; $x - y - 1 = 0$

$\Rightarrow y = x - 1$

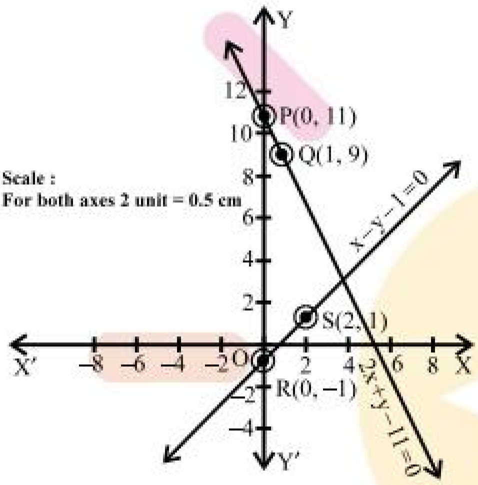
When, $x = 0$, $y = -1$

When, $x = 2$, $y = 1$

Then, we have the following table :

x	0	2
y	–1	1

Plotting points R(0, 1) & S(2, 1) on same graph paper & drawing a line joining between them, we get graph of equation x – y – 1 = 0 as shown in fig.



From the graph we find that the lines represented by the equations 2x + y – 11 and x – y – 1 = 0 meet y-axis at P(0, 11) and R(0, –1) respectively.

Ex.45 Solve the following equations graphically.

$2x - y = 5$; $x + 3y = 6$

Also, find the value of 'a', if 5x – 2y = a.

Sol. We have;

$2x - y = 5$ and $x + 3y = 6$

(a) Graph of the equation 2x – y = 5;

We have, $2x - y = 5$

$\Rightarrow y = 2x - 5$

When, $x = 2$, $y = 2 \times 2 - 5 = -1$

When, $x = -2$, $y = 2 \times (-2) - 5 = -9$

Then, we have the following table :

x	2	–2
y	–1	–9

Plotting the points P(2, – 1) and Q(–2, – 9) on the graph paper and drawing a line joining between them, we get the graph of the equation 2x – y = 5 as shown in fig.

(b) Graph of the equation x + 3y = 6;

We have; $x + 3y = 6$

$\Rightarrow y = \frac{6 - x}{3}$

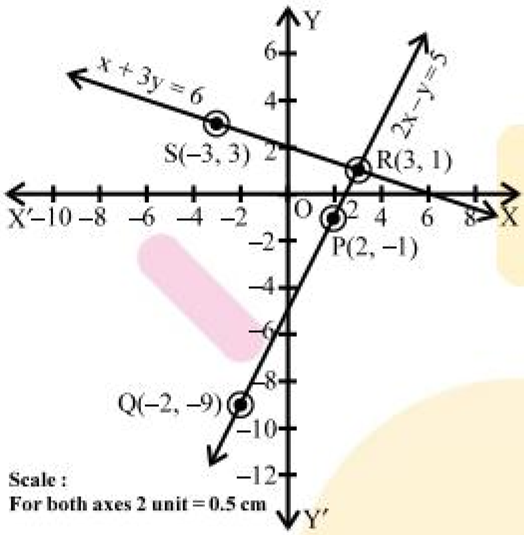
When, $x = 3$, $y = \frac{6 - 3}{3} = 1$

When, x = – 3, $y = \frac{6-(-3)}{3} = 3$

Then, we have the following table :

x	3	– 3
y	1	3

Plotting the points R(3, 1) and S (–3, 3) on the same graph paper and drawing a line joining between them, we get the graph of the equation x + 3y = 6 as shown in fig.



It is seen clearly from fig. that two lines intersect at R(3, 1), which determines x = 3and y = 1.

Putting x = 3 and y = 1 in 5x – 2y = a, we get;

$5 \times 3 - 2 \times 1 = a$

$\Rightarrow a = 15 - 2 = 13$

$\therefore a = 13$

Ex.46 Solve graphically the following system of linear equations;

2x – y = 2; 4x – y = 8

Also, find the co-ordinates of the points where the lines meet x-axis.

Sol. We have;

2x – y = 2 and 4x – y = 8

(a) Graph of the equation 2x – y = 2

We have, 2x – y = 2

$\Rightarrow$ y = 2x – 2

When, x = 1, y = 0

When, x = 0, y = – 2

Then, we have the following table :

x	1	0
y	0	– 2

Plotting the points P(1, 0) and Q(0, – 2) on the same graph paper and drawing a line joining between them, we get the graph of the equation 2x – y = 2 as shown in fig.

(b) Graph of the equation 4x – y = 8

We have, $4x - y = 8$

$\Rightarrow y = 4x - 8$

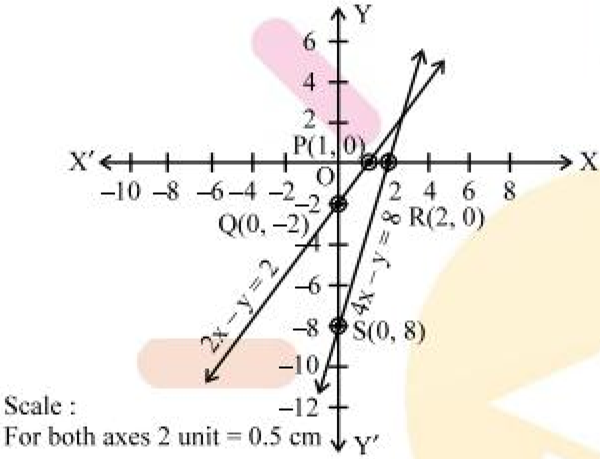
When, $x = 2$, $y = 4 \times 2 - 8 = 0$;

When $x = 0$, $y = 4 \times 0 - 8 = -8$

Then, we have the following table :

x	2	0
y	0	–8

Plotting the points R(2, 0) and S(0, –8) on the same paper and drawing a line joining between them, we get the graph of the equation $4x - y = 8$ as shown in fig.



Here, we see that the lines represented by the equations $2x - y = 2$ and $4x - y = 8$ meet x-axis at P(1, 0) and R (2, 0) respectively.

Ex.47 Solve the following system of linear equations graphically;

$3x + y - 11 = 0$; $x - y - 1 = 0$

Shade the region bounded by these lines and also y-axis. Then, determine the areas of the region bounded by these lines and y-axis.

Sol. We have ;

$3x + y - 11 = 0$ and $x - y - 1 = 0$

(i) Graph of the equation $3x + y - 11 = 0$

We have, $3x + y - 11 = 0$

$\Rightarrow y = -3x + 11$

When, $x = 2$, $y = -3 \times 2 + 11 = 5$

When, $x = 3$, $y = -3 \times 3 + 11 = 2$

Then, we have the following table :

x	2	3
y	5	2

Plotting the points P (2, 5) and Q(3, 2) on the graph paper and drawing a line joining between them, we get the graph of the equation $3x + y - 11 = 0$ as shown in fig.

(b) Graph of the equation x – y – 1 = 0

We have,

$x - y - 1 = 0$

$\Rightarrow y = x - 1$

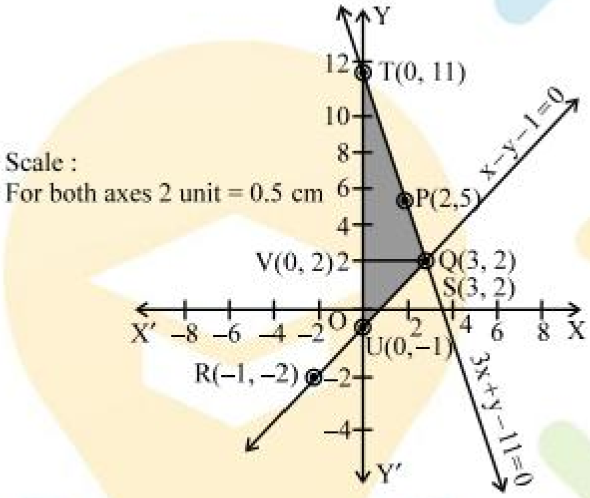
When, x = – 1, y = –2

When, x = 3, y = 2

Then, we have the following table :

x	–1	3
y	–2	2

Plotting the points R(–1, –2) and S(3, 2) on the same graph paper and drawing a line joining between them, we get the graph of the equation x – y – 1 = 0 as shown in fig.



You can observe that two lines intersect at Q(3, 2). So, x = 3 and y = 2. The area enclosed by the lines represented by the given equations and also the y-axis is shaded.

So, the enclosed area = Area of the shaded portion

= Area of ΔQUT = $\frac{1}{2}$ × base × height

$= \frac{1}{2} \times (TU \times VQ) = \frac{1}{2} \times (TO + OU) \times VQ$

$= \frac{1}{2}(11 + 1)\,3 = \frac{1}{2} \times 12 \times 3 = 18$ sq.units.

Hence, required area is 18 sq. units.

Ex.48 Draw the graphs of the following equations ;

$2x - 3y = -6\ ;\ 2x + 3y = 18;\ y = 2$

Find the vertices of the triangles formed and also find the area of the triangle.

Sol. (a) Graph of the equation 2x – 3y = – 6;

We have, $2x - 3y = -6$

$\Rightarrow y = \frac{2x+6}{3}$

When , $x = 0, y = \frac{2 \times 0 + 6}{3} = 2$

When, $x = 3,\ y = \frac{2 \times 3 + 6}{3} = 4$

Then, we have the following table :

x	0	3
y	2	4

Plotting the poinst P(0, 2) and Q(3, 4) on the graph paper and drawing a line joining between them we get the graph of the equation $2x - 3y = -6$ as shown in fig.

(b) Graph of the equation $2x + 3y = 18$;

We have $2x + 3y = 18$

$\Rightarrow y = \frac{-2x+18}{3}$

When, $x = 0,\ y = \frac{-2 \times 0 + 18}{3} = 6$

When, $x = -3,\ y = \frac{-2 \times (-3) + 18}{3} = 8$

Then, we have the following table :

x	0	−3
y	6	8

Plotting the points R(0, 6) and S(−3, 8) on the same graph paper and drawing a line joining between them, we get the graph of the equation $2x + 3y = 18$ as shown in fig.

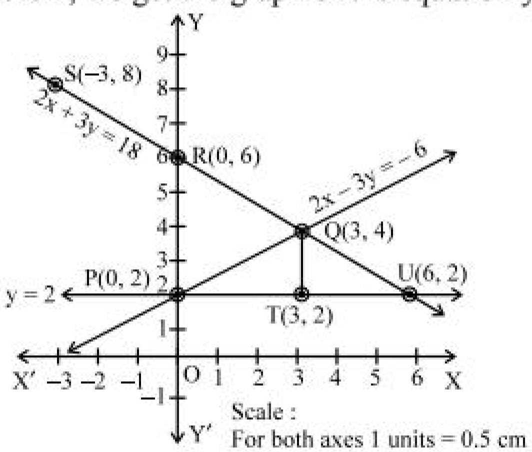
(c) Graph of the equation $y = 2$

It is a clear fact that $y = 2$ is for every value of x. We may take the points T (3, 2), U(6, 2) or any other values.

Then, we get the following table :

x	3	6
y	2	2

Plotting the points T(3, 2) and U(6, 2) on the same graph paper and drawing a line joining between them, we get the graph of the equation $y = 2$ as shown in fig.



From the fig., we can observe that the lines taken in pairs intersect each other at points Q(3, 4), U (6, 2) and P(0, 2). These form the three vertices of the triangle PQU.

To find area of the triangle so formed

The triangle is so formed is PQU (see fig.)

In the ΔPQU

QT (altitude) = 2 units

and PU (base) = 6 units

so, area of $\Delta PQU = \frac{1}{2}$ (base × height)

$= \frac{1}{2}$ (PU × QT) $= \frac{1}{2} \times 6 \times 2$ sq. untis

= 6 sq. units.

Important Points To Be Remembered

Pair of lines $a_1x+b_1y+c_1=0$ $a_2x+b_2y+c_2=0$	$\frac{a_1}{a_2}$	$\frac{b_1}{b_2}$	$\frac{c_1}{c_2}$	Compare the ratio	Graphical representation	Algebraic interpretation
$2x+3y+4=0$ $5x+6y+9=0$	$\frac{2}{5}$	$\frac{3}{6}$	$\frac{4}{9}$	$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Intersecting lines	Exactly one solution (unique)
$x+2y+5=0$ $3x+6y+15=0$	$\frac{1}{3}$	$\frac{2}{6}$	$\frac{5}{15}$	$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	Coincident lines	Infinitely many solutions
$2x-3y+4=0$ $4x-6y+10=0$	$\frac{2}{4}$	$\frac{-3}{-6}$	$\frac{4}{10}$	$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	Parallel lines	No solution

From the table above you can observe that if the line.

$a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are

(i)	for the intersecting lines then $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$
(ii)	for the coincident lines then $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
(iii)	for the parallel lines then $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Ex.49 On comparing the ratios $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$ and without drawing them, find out whether the lines representing the following pairs of linear equations intersect at a point, are parallel or coincide.

(i) $5x - 4y + 8 = 0$, $7x + 6y - 9 = 0$

(ii) $9x + 3y + 12 = 0$, $18x + 6y + 24 = 0$

(iii) $6x - 3y + 10 = 0$, $2x - y + 9 = 0$

Sol. Comparing the given equations with standard forms of equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ we have,

(i) $a_1 = 5$, $b_1 = -4$, $c_1 = 8$;

$a_2 = 7$, $b_2 = 6$, $c_2 = -9$

$\therefore \frac{a_1}{a_2} = \frac{5}{7}, \frac{b_1}{b_2} = \frac{-4}{6}$

$\Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Thus, the lines representing the pair of linear equations are intersecting.

(ii) $a_1 = 9,\ b_1 = 3,\ c_1 = 12;$

$a_2 = 18,\ b_2 = 6,\ c_2 = 24$

$\therefore\ \frac{a_1}{a_2} = \frac{9}{18} = \frac{1}{2},\ \frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}$

and $\frac{c_1}{c_2} = \frac{12}{24} = \frac{1}{2}$

$\Rightarrow\ \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Thus, the lines representing the pair of linear equation coincide.

(iii) $a_1 = 6,\ b_1 = -3,\ c_1 = 10;$

$a_2 = 2,\ b_2 = -1,\ c_2 = 9$

$\therefore\ \frac{a_1}{a_2} = \frac{6}{2} = 3,\ \frac{b_1}{b_2} = \frac{-3}{-1} = 3,\ \frac{c_1}{c_2} = \frac{10}{9}$

$\Rightarrow\ \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Thus, the lines representing the pair of linear equations are parallel.

Algbraic Solution of a System of Linear Equations

Sometimes, graphical method does not give an accurate answer. While reading the coordinates of a point on a graph paper, we are likely to make an error. So, we require some precise method to obtain accurate result. Algebraic methods given below yield accurate answers.

(i) Method of elimination by substitution.

(ii) Method of elimination by equating the coefficients.

(iii) Method of cross multiplication.

Substitution Method

In this method, we first find the value of one variable (y) in terms of another variable (x) from one equation. Substitute this value of y in the second equation. Second equation becomes a linear equation in x only and it can be solved for x.

Putting the value of x in the first equation, we can find the value of y.

This method of solving a system of linear equations is known as the method of **elimination by substitution.**

'Elimination', because we get rid of y or 'eliminate' y from the second equation. 'Substitution', because we 'substitute' the value of y in the second equation.

Working rule :

Let the two equations be

$a_1x + b_1y + c_1 = 0$ ....(1)

$a_2x + b_2y + c_2 = 0$ ....(2)

Step I : Find the value of one variable, say y, in terms of the other i.e., x from any equation, say (1).

Step II : Substitute the value of y obtained in step 1 in the other equation i.e., equation (2). This equation becomes equation in one variable x only.

Step III : Solve the equation obtained in step II to get the value of x.

Step IV : Substitute the value of x from step II to the equation obtained in step I. From this equation, we get the value of y. In this way, we get the solution i.e. values of x and y.

Remark : Verification is a must to check the answer.

❖ EXAMPLES ❖

Ex.50 Solve each of the following system of equations by eliminating x (by substitution) :

(i) $x + y = 7$ (ii) $x + y = 7$

$2x - 3y = 11$ $12x + 5y = 7$

(iii) $2x - 7y = 1$ (iv) $3x - 5y = 1$

$4x + 3y = 15$ $5x + 2y = 19$

(v) $5x + 8y = 9$

$2x + 3y = 4$

Sol. (i) We have ;

$x + y = 7$ ....(1)

$2x - 3y = 11$ ....(2)

We shall eliminate x by substituting its value from one equation into the other. from equaton (1), we get ;

$x + y = 7 \Rightarrow x = 7 - y$

Substituting the value of x in equation (2), we get ;

$2 \times (7 - y) - 3y = 11$

$\Rightarrow 14 - 2y - 3y = 11$

$\Rightarrow -5y = -3$ or, $y = 3/5$.

Now, substituting the value of y in equation (1), we get;

$x + 3/5 = 7 \Rightarrow x = 32/5$.

Hence, $x = 32/5$ and $y = 3/5$.

(ii) We have,

$x + y = 7$ ....(1)

$12x + 5y = 7$ ....(2)

From equation (1), we have;

$x + y = 7 \Rightarrow x = 7 - y$

Substituting the value of y in equation (2), we get ;

$\Rightarrow 12(7 - y) + 5y = 7$

$\Rightarrow 84 - 12y + 5y = 7$

$\Rightarrow -7y = -77 \Rightarrow y = 11$

Now, Substituting the value of y in equation (1), we get ;

$x + 11 = 7 \quad \Rightarrow x = -4$

Hence, $x = -4, \quad y = 11.$

(iii) We have;

$2x - 7y = 1$ ....(1)

$4x + 3y = 15$ ....(2)

From equation (1), we get

$2x - 7y = 1 \quad \Rightarrow x = \frac{7y+1}{2}$

Substituting the value of x in equation (2), we get ;

$\Rightarrow 4 \times \frac{7y+1}{2} + 3y = 15$

$\Rightarrow \frac{28y+4}{2} + 3y = 15$

$\Rightarrow 28y + 4 + 6y = 30$

$\Rightarrow 34y = 26 \quad \Rightarrow y = \frac{26}{34} = \frac{13}{17}$

Now, substituting the value of y in equation (1), we get;

$2x - 7 \times \frac{13}{17} = 1$

$\Rightarrow 2x = 1 + \frac{91}{17} = \frac{108}{17}$

$\Rightarrow x = \frac{108}{34} = \frac{54}{17}$

Hence, $x = \frac{54}{17}, y = \frac{13}{17}$

(iv) We have ;

$3x - 5y = 1$ .... (1)

$5x + 2y = 19$ .... (2)

From equation (1), we get;

$3x - 5y = 1 \quad \Rightarrow x = \frac{5y+1}{3}$

Substituing the value of x in equation (2), we get ;

$\Rightarrow 5 \times \frac{5y+1}{3} + 2y = 19$

$\Rightarrow 25y + 5 + 6y = 57 \quad \Rightarrow 31y = 52$

Thus, $y = \frac{52}{31}$

Now, substituting the value of y in equation (1), we get ;

$3x - 5 \times \frac{52}{31} = 1$

$\Rightarrow 3x - \frac{260}{31} = 1 \quad \Rightarrow 3x = \frac{291}{31}$

$\Rightarrow x = \frac{291}{31 \times 3} = \frac{97}{31}$

Hence, $x = \frac{97}{31}$, $y = \frac{52}{31}$

(v) We have,

$5x + 8y = 9$ ....(1)

$2x + 3y = 4$ ....(2)

From equation (1), we get;

$5x + 8y = 9 \quad \Rightarrow \quad x = \frac{9-8y}{5}$

Substituting the value of x in equation (2), we get ;

$\Rightarrow 2 \times \frac{9-8y}{5} + 3y = 4$

$\Rightarrow 18 - 16y + 15y = 20$

$\Rightarrow -y = 2$ or $y = -2$

Now, substituting the value of y in equation (1), we get ;

$5x + 8(-2) = 9$

$\Rightarrow 5x = 25 \quad \Rightarrow \quad x = 5$

Hence, $x = 5$, $y = -2$.

Ex.51 Solve the following systems of equations by eliminating 'y' (by substitution) :

(i) $3x - y = 3$ (ii) $7x + 11y - 3 = 0$

$7x + 2y = 20$ $\quad 8x + y - 15 = 0$

(iii) $2x + y - 17 = 0$

$17x - 11y - 8 = 0$

Sol. (i) We have;

$3x - y = 3$ ....(1)

$7x + 2y = 20$ ....(2)

From equation (1), we get ;

$3x - y = 3 \quad \Rightarrow \quad y = 3x - 3$

Substituting the value of 'y' in equation (2), we get ;

$\Rightarrow 7x + 2 \times (3x - 3) = 20$

$\Rightarrow 7x + 6x - 6 = 20$

$\Rightarrow 13x = 26 \quad \Rightarrow \quad x = 2$

Now, substituting x = 2 in equation (1), we get;

$3 \times 2 - y = 3 \Rightarrow y = 3$

Hence, $x = 2$, $y = 3$.

(ii) We have;

$7x + 11y - 3 = 0$ .... (1)

$8x + y - 15 = 0$ .... (2)

From equation (1), we get;

$7x + 11y = 3 \quad \Rightarrow \quad y = \frac{3-7x}{11}$

Substituting the value of 'y' in equation (2), we get;

$\Rightarrow 8x + \frac{3-7x}{11} = 15$

$\Rightarrow 88x + 3 - 7x = 165$

$\Rightarrow 81x = 162 \Rightarrow \quad x = 2$

Now, substituting, x = 2 in the equation (2), we get ;

$8 \times 2 + y = 15 \quad \Rightarrow \quad y = -1$

Hence, $x = 2, \quad y = -1$.

(iii) We have,

$2x + y = 17$ ....(1)

$17x - 11y = 8$ ....(2)

From equation (1), we get;

$2x + y = 17 \quad \Rightarrow \quad y = 17 - 2x$

Substituting the value of 'y' in equation (2), we get ;

$17x - 11(17 - 2x) = 8$

$\Rightarrow 17x - 187 + 22x = 8$

$\Rightarrow 39x = 195 \Rightarrow \quad x = 5$

Now, substituting the value of 'x' in equation (1), we get ;

$2 \times 5 + y = 17 \quad \Rightarrow \quad y = 7$

Hence, $x = 5, \quad y = 7$.

Ex.52 Solve the following systems of equations,

(i) $2x - y = 11$ (ii) $-6x + 5y = 2$

$5x + 4y = 1$ $-5x + 6y = 9$

(iii) $4x + 7y = 20$ (iv) $\frac{15}{u} + \frac{2}{v} = 17$

$21x - 13y = 21$ $\frac{1}{u} + \frac{1}{v} = \frac{36}{5}$

(v) $\frac{11}{v} - \frac{7}{u} = 1$

$\frac{9}{v} - \frac{4}{u} = 6$

Sol. The given system of equation is ;

$2x - y = 11$ ....(1)

$5x + 4y = 14$ ....(2)

From equation (1), we get ;

$x = \frac{y+11}{2}$

Substituting the value of 'x' in equation (2), we get;

$5 \times \left(\frac{y+11}{2}\right) + 4y = 1$

$\Rightarrow 5y + 55 + 8y = 2$

$\Rightarrow$ 13y = – 53 $\Rightarrow$ $y = \frac{-53}{13}$

Substituting the value of 'y' in equation (1), we get ;

$$2x - \left(\frac{-53}{13}\right) = 11$$

$\Rightarrow 2x = 11 - \frac{53}{13} = \frac{143-53}{13} = \frac{90}{13}$

$\Rightarrow x = \frac{90}{2\times 13} = \frac{45}{13}$

Hence, $x = \frac{45}{13}$, $y = \frac{-53}{13}$

(ii) The given system of equation is ;

$-6x + 5y = 2$ ....(1)

$-5x + 6y = 9$ ....(2)

Multiplying the equation (1) by 6 and the equation (2) by 5, we get ;

$-36x + 30y = 12$ ....(3)

$-25x + 30y = 45$ ....(4)

Subtracting (4) form (3), we get ;

$-11x = -33 \Rightarrow x = 3$

Substituting the above value of x in equation (1), we get ;

$-6 \times 3 + 5y = 2$

$\Rightarrow 5y = 20 \Rightarrow y = 4$

Hence, x = 3, y = 4.

(iii) The given system of equation is ;

$4x + 7y = 20$ ....(1)

$21x - 13y = 21$ ....(2)

Multiplying (1) by 13 and (2) by 7, we get ;

$52x + 91y = 260$ ....(3)

$147x - 91y = 147$ ....(4)

Adding (3) and (4), we get ;

$199x = 407 \Rightarrow x = \frac{407}{199}$

Substituting the above value of x in equation (1), we get;

$$4 \times \frac{407}{199} + 7y = 20$$

$\Rightarrow 7y = 20 - \frac{1628}{199} \Rightarrow 7y = \frac{3980-1628}{199}$

$\Rightarrow y = \frac{2352}{7\times 199} = \frac{336}{199}$ Hence, $x = \frac{407}{199}$, $y = \frac{336}{199}$

(iv) The given system of equation is ;

$\frac{15}{u} + \frac{2}{v} = 17$ ....(1)

$$\frac{1}{u} + \frac{1}{v} = \frac{36}{5} \quad ....(2)$$

Considering 1/u = x, 1/v = y, the above system of linear equations can be written as :

$$15x + 2y = 17 \quad ....(3)$$

$$x + y = \frac{36}{5} \quad ....(4)$$

Multiplying (4) by 15 and (iii) by 1, we get ;

$$15x + 2y = 17 \quad ....(5)$$

$$15x + 15y = \frac{36}{5} \times 15 = 108 \quad ....(6)$$

Subtracting (6) form (5), we get;

$$-13y = -91 \Rightarrow y = 7$$

Substituting y = 7 in (4), we get ;

$$x + 7 = \frac{36}{5} \Rightarrow x = \frac{36}{5} - 7 = \frac{1}{5}$$

But, $y = \frac{1}{v} = 7 \Rightarrow v = \frac{1}{7}$

and, $x = \frac{1}{u} = \frac{1}{5} \Rightarrow u = 5$

Hence, the required solution of the given system is u = 5, v = 1/7.

(v) The given system of equation is ;

$$\frac{11}{v} - \frac{7}{u} = 1; \quad \frac{9}{v} - \frac{4}{u} = 6$$

Taking 1/v = x and 1/u = y, the above system of equations can be written as ;

$$11x - 7y = 1 \quad ....(1)$$

$$9x - 4y = 6 \quad ....(2)$$

Multiplying (1) by 4 and (2) by 7, we get,

$$44x - 28y = 4 \quad ....(3)$$

$$63x - 28y = 42 \quad ....(4)$$

Subtracting (4) from (3) we get,

$$-19x = -38 \Rightarrow x = 2$$

Substituting the above value of x in (2), we get;

$$9 \times 2 - 4y = 6 \Rightarrow -4y = -12$$

$$\Rightarrow y = 3$$

But, $x = \frac{1}{v} = 2 \Rightarrow v = \frac{1}{2}$

and, $y = \frac{1}{u} = 3 \Rightarrow u = \frac{1}{3}$

Hence, the required solution of the given system of the equation is ;

$$v = \frac{1}{2}, \quad u = \frac{1}{3}$$

Ex.53 Solve the following system of equations by the method of elimination (substitution).

$$(a + b)x + (a - b)y = a^2 + b^2$$

$(a - b)x + (a + b)y = a^2 + b^2$

Sol. The given system of equations is

$(a + b)x + (a - b)y = a^2 + b^2$ ....(1)

$(a - b)x + (a + b)y = a^2 + b^2$ ....(2)

From (2), we get $(a + b)y = a^2 + b^2 - (a - b)x$

$\Rightarrow y = \frac{a^2+b^2}{a+b} - \frac{a-b}{a+b}x$ ....(3)

Substituting $y = \frac{a^2+b^2}{a+b} - \frac{a-b}{a+b}x$ in(1), we get

$(a + b)x + (a - b)\left[\frac{a^2+b^2}{a+b} - \frac{a-b}{a+b}x\right] = a^2 + b^2$

$\Rightarrow (a + b)x + \frac{(a-b)(a^2+b^2)}{a+b} - \frac{(a-b)^2}{(a+b)}x$
$= a^2 + b^2$

$\Rightarrow (a + b)x - \left(\frac{a^2-2ab+b^2}{a+b}\right)x = a^2 + b^2 - \frac{(a-b)(a^2+b^2)}{a+b}$

$\Rightarrow (a + b)x - \left(\frac{a^2-2ab+b^2}{a+b}\right)x$
$= (a^2 + b^2)\left[1-\frac{a-b}{a+b}\right]$

$\Rightarrow \frac{(a^2+2ab+b^2)x-(a^2-2ab+b^2)x}{a+b}$
$= (a^2 + b^2)\left(\frac{a+b-a+b}{a+b}\right)$

$\Rightarrow \frac{4ab}{a+b}x = \frac{(a^2+b^2)2ab}{a+b}$

$\Rightarrow 4abx = 2b(a^2 + b^2) \Rightarrow x = \frac{a^2+b^2}{2a}$

Putting $x = \frac{a^2+b^2}{2a}$ in (3), we get

$y = \frac{a^2+b^2}{a+b} - \frac{(a-b)}{a+b}\frac{(a^2+b^2)}{2a}$

$\Rightarrow y = \frac{(a^2+b^2)}{a+b}\left[1-\frac{a-b}{2a}\right]$

$= \left(\frac{a^2+b^2}{a+b}\right)\left(\frac{2a-a+b}{2a}\right)$

$\Rightarrow y = \left(\frac{a^2+b^2}{a+b}\right)\left(\frac{a+b}{2a}\right)$

$\Rightarrow y = \frac{a^2+b^2}{2a}$

Hence, the solution is $x = \frac{a^2 + b^2}{2a}$, $y = \frac{a^2 + b^2}{2a}$

Verification : On verifying, we find that answer is correct.

Ex.54 Solve $2x + 3y = 11$ and $2x - 4y = -24$ and hence find the value of 'm' for which $y = mx + 3$.

Sol. We have,

$2x + 3y = 11$ ....(1)

$2x - 4y = -24$ ....(2)

From (1), we have $2x = 11 - 3y$

Substituting $2x = 11 - 3y$ in (2), we get

$11 - 3y - 4y = -24$

$-7y = -24 - 11$

$\Rightarrow -7y = -35 \quad \Rightarrow y = 5$

Putting $y = 5$ in (1), we get

$2x + 3 \times 5 = 11$

$2x = 11 - 15$

$\Rightarrow x = -\frac{4}{2} = -2$

Hence, $x = -2$ and $y = 5$

Again putting $x = -2$ and $y = 5$ in $y = mx + 3$, we get

$5x = m(-2) + 3$

$\Rightarrow -2m = 5 - 3$

$\Rightarrow m = \frac{2}{-2} = -1$

Method of Elimination By Equating the Coefficients

Step I : Let the two equations obtained be

$a_1x + b_1y + c_1 = 0$ ....(1)

$a_2x + b_2y + c_2 = 0$ ....(2)

Step II : Multiplying the given equation so as to make the co-efficients of the variable to be eliminated equal.

Step III : Add or subtract the equations so obtained in Step II, as the terms having the same co-efficients may be either of opposite or the same sign.

Step IV : Solve the equations in one varibale so obtained in Step III.

Step V : Substitute the value found in Step IV in any one of the given equations and then copmpute the value of the other variable.

Type I : Solving simultaneous linear equations in two variables

❖ EXAMPLES ❖

Ex.55 Solve the following system of linear equations by applying the method of elimination by equating the co-efficients :

(i) $4x - 3y = 4$ (ii) $5x - 6y = 8$

$2x + 4y = 3$ $3x + 2y = 6$

Sol. (i) We have,

$4x - 3y = 4$ ....(1)

$2x + 4y = 3$ ....(2)

Let us decide to eliminate x from the given equation. Here, the co-efficients of x are 4 and 2 respectively. We find the L.C.M. of 4 and 2 is 4. Then, make the co-efficients of x equal to 4 in the two equations.

Multiplying equation (1) with 1 and equation (2) with 2, we get ;

$4x - 3y = 4$ ....(3)

$4x + 8y = 6$ ....(4)

Subtracting equation (4) from (3), we get ;

$-11y = -2 \Rightarrow y = \frac{2}{11}$

Substituting y = 2/11 in equation (1), we get;

$4x - 3 \times \frac{2}{11} = 4$

$\Rightarrow 4x - \frac{6}{11} = 4 \quad \Rightarrow 4x = 4 + \frac{6}{11}$

$\Rightarrow 4x = \frac{50}{11} \quad \Rightarrow x = \frac{50}{44} = \frac{25}{22}$

Hence, solution of the given system of equation is :

$x = \frac{25}{22}, \quad y = \frac{2}{11}$

(ii) We have;

$5x - 6y = 8$ ....(1)

$3x + 2y = 6$ ....(2)

Let us eliminate y from the given system of equations. The co-efficients of y in the given equations are 6 and 2 respectively. The L.C.M. of 6 and 2 is 6. We have to make the both coefficients equal to 6. So, multiplying both sides of equation (1) with 1 and equation (2) with 3, we get ;

$5x - 6y = 8$ ....(3)

$9x + 6y = 18$ ....(4)

Adding equation (3) and (4), we get ;

$14x = 26 \Rightarrow x = \frac{26}{14} = \frac{13}{7}$

Putting x = 13/7 in equation (1), we get ;

$5 \times \frac{13}{7} - 6y = 8 \Rightarrow \frac{65}{7} - 6y = 8$

$\Rightarrow 6y = \frac{65}{7} - 8 = \frac{65-56}{7} = \frac{9}{7}$

$\Rightarrow y = \frac{9}{42} = \frac{3}{14}$

Hence, the solution of the system of equations is ; $x = \frac{13}{7}, \quad y = \frac{3}{14}$

Ex.56 Solve the following system of equations by using the method of elimination by equating the co-efficients.

$$\frac{x}{2} + \frac{2y}{5} + 2 = 10; \ \frac{2x}{7} - \frac{y}{2} + 1 = 9$$

Sol. The given system of equation is

$$\frac{x}{2} + \frac{2y}{5} + 2 = 10 \Rightarrow \frac{x}{2} + \frac{2y}{5} = 8 \quad ....(1)$$

$$\frac{2x}{5} - \frac{y}{2} + 1 = 9 \Rightarrow \frac{2x}{7} - \frac{y}{2} = 8 \quad ....(2)$$

The equation (1) can be expressed as :

$$\frac{5x+4y}{10} = 8 \Rightarrow 5x + 4y = 80 \quad ....(3)$$

Similarly, the equation (2) can be expressed as :

$$\frac{4x-7y}{14} = 8 \Rightarrow 4x - 7y = 112 \quad ....(4)$$

Now the new system of equations is

$$5x + 4y = 80 \quad ....(5)$$

$$4x - 7y = 112 \quad ....(6)$$

Now multiplying equation (5) by 4 and equation (6) by 5, we get ;

$$20x - 16y = 320 \quad ....(7)$$

$$20x + 35y = 560 \quad ....(8)$$

Subtracting equation (7) from (8), we get ;

$$y = \frac{-240}{51}$$

Putting $y = \frac{-240}{51}$ in equation (5), we get ;

$$5x + 4 \times \left(\frac{-240}{51}\right) = 80 \Rightarrow 5x - \frac{960}{51} = 80$$

$$\Rightarrow 5x = 80 + \frac{960}{51} = \frac{4080+960}{51} = \frac{5040}{51}$$

$$\Rightarrow x = \frac{5040}{255} = \frac{1008}{51} = \frac{336}{17} \Rightarrow x = \frac{336}{17}$$

Hence, the solution of the system of equations is, $x = \frac{336}{17}$, $y = \frac{-80}{17}$.

Ex.57 Solve the following system of linear equations by usnig the method of elimination by equating the coefficients :

$$3x + 4y = 25 ; \qquad 5x - 6y = -9$$

Sol. The given system of equations is

$$3x + 4y = 25 \quad ....(1)$$

$$5x - 6y = -9 \quad ....(2)$$

Let us eliminate y. The coefficients of y are 4 and – 6. The LCM of 4 and 6 is 12.

So, we make the coefficients of y as 12 and – 12.

Multiplying equation (1) by 3 and equation (2) by 2, we get

$9x + 12y = 75$ ....(3)

$10x - 12y = -18$ ....(4)

Adding equation (3) and equation (4), we get

$19x = 57 \Rightarrow x = 3.$

Putting x = 3 in (1), we get,

$3 \times 3 + 4y = 25$

$\Rightarrow 4y = 25 - 9 = 16 \Rightarrow y = 4$

Hence, the solution is x = 3, y = 4.

Verification : Both the equations are satisfied by x = 3 and y = 4, which shows that the solution is correct.

Ex.58 Solve the following system of equations :

$15x + 4y = 61; \quad 4x + 15y = 72$

Sol. The given system of equation is

$15x + 4y = 61$ ....(1)

$4x + 15y = 72$ ....(2)

Let us eliminate y. The coefficients of y are 4 and 15. The L.C.M. of 4 and 15 is 60. So, we make the coefficients of y as 60. Multiplying (1) by 15 and (2) by 4, we get

$225x + 60y = 915$ ....(3)

$16x + 60y = 288$ ....(4)

Substracting (4) from (3), we get

$209x = 627 \Rightarrow x = \frac{627}{209} = 3$

Putting x = 3 in (1), we get

$15 \times 3 + 4y = 61 \Rightarrow 45 + 4y = 61$

$\Rightarrow 4y = 61 - 45 = 16 \Rightarrow y = \frac{16}{4} = 4$

Hence, the solution is x = 3, y = 4.

Verification : On putting x = 3 and y = 4 in the given equations, they are satisfied. Hence, the solution is correct.

Ex.59 Solve the following system of linear equatoins by using the method of elimination by equating the coefficients

$\sqrt{3}x - \sqrt{2}y = \sqrt{3} \; ; \; \sqrt{5}x + \sqrt{3}y = \sqrt{2}$

Sol. The given equations are

$\sqrt{3}x - \sqrt{2}y = \sqrt{3}$ .... (1)

$\sqrt{5}x + \sqrt{3}y = \sqrt{2}$ .... (2)

Let us eliminate y. To make the coefficients of equal, we multiply the equation (1) by $\sqrt{3}$ and equation (2) by $\sqrt{2}$ to get

$3x - \sqrt{6}y = 3$ ....(3)

$\sqrt{10}x + \sqrt{6}y = 2$ ....(4)

Adding equation (3) and equation (4), we get

$3x + \sqrt{10}x = 5 \Rightarrow (3 + \sqrt{10})x = 5$

$\Rightarrow x = \frac{5}{3+\sqrt{10}} = \left(\frac{5}{\sqrt{10}+3}\right)\times\left(\frac{\sqrt{10}-3}{\sqrt{10}-3}\right)$

$= \frac{5(\sqrt{10}-3)}{10-9} = 5(\sqrt{10}-3)$

Putting $x = 5(\sqrt{10} - 3)$ in (1) we get

$\sqrt{3} \times 5(\sqrt{10} - 3) - \sqrt{2}y = \sqrt{3}$

$\Rightarrow 5\sqrt{30} - 15\sqrt{3} - \sqrt{2}y = \sqrt{3}$

$\Rightarrow \sqrt{2}y = 5\sqrt{30} - 15\sqrt{3} - \sqrt{3}$

$\Rightarrow \sqrt{2}y = 5\sqrt{30} - 16\sqrt{3}$

$\Rightarrow y = \frac{5\sqrt{30}}{\sqrt{2}} - \frac{16\sqrt{3}}{\sqrt{2}} = 5\sqrt{15} - 8\sqrt{6}$

Hence, the solution is $x = 5(\sqrt{10} - 3)$ and $y = 5\sqrt{15} - 8\sqrt{6}$.

Verification : After verifying, we find the solution is correct.

Ex.60 Solve for x and y :

$\frac{ax}{b} - \frac{by}{a} = a + b$; $ax - by = 2ab$

Sol. The given system of equations is

$\frac{ax}{b} - \frac{by}{a} = a + b$ ....(1)

$ax - by = 2ab$ ....(2)

Dividing (2) by a, we get

$x - \frac{by}{a} = 2b$ ....(3)

On subtracting (3) from (1), we get

$\frac{ax}{b} - x = a - b \Rightarrow x\left(\frac{a}{b}-1\right) = a - b$

$\Rightarrow x = \frac{(a-b)b}{a-b} = b \Rightarrow x = b$

On substituting the value of x in (3), we get

$b - \frac{by}{a} = 2b \Rightarrow b\left(1-\frac{y}{a}\right) = 2b$

$\Rightarrow 1 - \frac{y}{a} = 2 \Rightarrow \frac{y}{a} = 1 - 2$

$\Rightarrow \frac{y}{a} = -1 \Rightarrow y = -a$

Hence, the solution of the equations is

$x = b,\ y = -a$

Ex.61 Solve the following system of linear equations :

$2(ax - by) + (a + 4b) = 0$

$2(bx + ay) + (b - 4a) = 0$

Sol. $2ax - 2by + a + 4b = 0$ .... (1)

$2bx + 2ay + b - 4a = 0$ .... (2)

Multiplyng (1) by b and (2) by a and subtracting, we get

$2(b^2 + a^2)\, y = 4\,(a^2 + b^2) \Rightarrow y = 2$

Multiplying (1) by a and (2) by b and adding, we get

$2(a^2 + b^2)\, x + a^2 + b^2 = 0$

$\Rightarrow 2(a^2 + b^2)\, x = -(a^2 + b^2)$

$\Rightarrow x = -\frac{1}{2}$

Hence $x = -\frac{1}{2}$, and $y = 2$

Ex.62 Solve $(a - b)\, x + (a + b)\, y = a^2 - 2ab - b^2$

$(a + b)\,(x + y) = a^2 + b^2$

Sol. The given system of equation is

$(a - b)\, x + (a + b)\, y = a^2 - 2ab - b^2$ ....(1)

$(a + b)\,(x + y) = a^2 + b^2$ ....(2)

$\Rightarrow (a + b)\, x + (a + b)\, y = a^2 + b^2$ ....(3)

Subtracting equation (3) from equation (1), we get

$(a - b)\, x - (a + b)\, x = (a^2 - 2ab - b^2) - (a^2 + b^2)$

$\Rightarrow -2bx = -2ab - 2b^2$

$\Rightarrow x = \frac{-2ab}{-2b} - \frac{2b^2}{-2b} = a + b$

Putting the value of x in (1), we get

$(a - b)\,(a + b) + (a + b)\, y = a^2 - 2ab - b^2$

$\Rightarrow (a + b)\, y = a^2 - 2ab - b^2 - (a^2 - b^2)$

$\Rightarrow (a + b)\, y = -2ab \Rightarrow y = \frac{-2ab}{a+b}$

$\Rightarrow$ Hence, the solution is $x = a + b$,

$y = \frac{-2ab}{a+b}$

Verification : After verifying, we find that the solution is correct.

Type II : Solving a system of equations which is reducible to a system of simultaneous linear equations

❖ EXAMPLES ❖

Ex.63 Solve the following system of equations

$$\frac{1}{2x} - \frac{1}{y} = -1; \quad \frac{1}{x} + \frac{1}{2y} = 8$$

Sol. We have ;

$$\frac{1}{2x} - \frac{1}{y} = -1 \quad ....(1)$$

$$\frac{1}{x} + \frac{1}{2y} = 8 \quad ....(2)$$

Let us consider 1/x = u and 1/y = v.

Putting 1/x = u and 1/y = v in the above equations, we get;

$$\frac{u}{2} - v = -1 \quad ....(3)$$

$$u + \frac{v}{2} = 8 \quad ....(4)$$

Let us eliminate v from the system of equations. So, multiplying equation (3) with ½ and (4) with 1, we get ;

$$\frac{u}{4} - \frac{v}{2} = -\frac{1}{2} \quad ....(5)$$

$$u + \frac{v}{2} = 8 \quad ....(6)$$

Adding equation (5) and (6), we get ;

$$\frac{u}{4} + u = \frac{-1}{2} + 8 \Rightarrow \frac{5u}{4} = \frac{15}{2}$$

$$\Rightarrow u = \frac{15}{2} \times \frac{4}{5} \quad \Rightarrow u = 6$$

We know,

$$\frac{1}{x} = u \Rightarrow \frac{1}{x} = 6 \Rightarrow x = \frac{1}{6}$$

Putting 1/x = 6 in equation (2), we get ;

$$6 + \frac{1}{2y} = 8 \quad \Rightarrow \frac{1}{2y} = 2$$

$$\Rightarrow \frac{1}{y} = 4 \quad \Rightarrow y = \frac{1}{4}$$

Hence, the solution of the system is,

$$x = \frac{1}{6}, y = \frac{1}{4}$$

Ex.64 Solve,

$$\frac{2}{x} + \frac{1}{3y} = \frac{1}{5}\ ;\ \frac{3}{x} + \frac{2}{3y} = 2$$

and also find 'a' for which y = ax – 2.

Sol. Considering 1/x = u and 1/y = v, the given system of equations becomes

$$2u + \frac{v}{3} = \frac{1}{5} \Rightarrow \frac{6u+v}{3} = \frac{1}{5}$$

$$\Rightarrow 30u + 5v = 3 \quad ....(1)$$

$$3u + \frac{2v}{3} = 2 \Rightarrow 9u + 2v = 6 \quad ....(2)$$

Multiplying equation (1) with 2 and equation (2) with 5, we get ;

$$60u + 10v = 6 \quad ....(3)$$

$$45u + 10v = 30 \quad ....(4)$$

Subtracting equation (4) from equation (3), we get ;

$$15u = -24 \quad \Rightarrow \quad u = \frac{-24}{15} = \frac{-8}{5}$$

Putting $u = \frac{-8}{5}$ in equation (2), we get;

$$9 \times \left(\frac{-8}{5}\right) + 2v = 6 \quad \Rightarrow \quad \frac{-72}{5} + 2v = 6$$

$$\Rightarrow 2v = 6 + \frac{72}{5} = \frac{102}{5}$$

$$\Rightarrow v = \frac{102}{2\times5} = \frac{51}{5}$$

Here $\frac{1}{x} = u = -\frac{8}{5} \quad \Rightarrow \quad x = -\frac{5}{8}$

And, $\frac{1}{y} = v = \frac{51}{5} \quad \Rightarrow \quad y = \frac{5}{51}$

Putting $x = \frac{-5}{8}$ and $y = \frac{5}{51}$ in y = ax – 2, we get;

$$\frac{5}{51} = \frac{-5a}{8} - 2$$

$$\Rightarrow \frac{5a}{8} = -2 - \frac{5}{51} = \frac{-102-5}{51} = \frac{-107}{51}$$

$$\Rightarrow a = \frac{-107}{51} \times \frac{8}{5} = \frac{-856}{255} \Rightarrow a = \frac{-856}{255}$$

Ex.65 Solve,

$$\frac{2}{x+2y} + \frac{6}{2x-y} = 4$$

$$\frac{5}{2(x+2y)} + \frac{1}{3(2x-y)} = 1$$

where, $x + 2y \neq 0$ and $2x - y \neq 0$

Sol. Taking $\frac{1}{x+2y}$ = u and $\frac{1}{2x-y}$ = v, the above system of equations becomes

$2u + 6v = 4$ ....(1)

$\frac{5u}{2} + \frac{v}{3} = 1$ ....(2)

Multiplying equation (2) by 18, we have;

$45u + 6v = 18$ ....(3)

Now, subtracting equation (3) from equation (1), we get ;

$-43u = -14 \Rightarrow u = \frac{14}{43}$

Putting u = 14/43 in equation (1), we get

$2 \times \frac{14}{43} + 6v = 4$

$\Rightarrow 6v = 4 - \frac{28}{43} = \frac{172-28}{43} \Rightarrow v = \frac{144}{43}$

Now, $u = \frac{14}{43} = \frac{1}{x+2y}$

$\Rightarrow 14x + 28y = 43$ ....(4)

And, $v = \frac{144}{43} = \frac{1}{2x-y}$

$\Rightarrow 288x - 144y = 43$ ....(5)

Multiplying equation (4) by 288 and (5) by 14, the system of equations becomes

$288 \times 14x + 28y \times 288 = 43 \times 288$

$288x \times 14 - 144y \times 14 = 43 \times 4$

$\Rightarrow 4022x + 8064y = 12384$ ....(6)

$4022x - 2016y = 602$ ....(7)

Subtracting equation (7) from (6), we get;

$10080y = 11782 \Rightarrow y = 1.6$(approx)

Now, putting 1.6 in (4), we get,

$14x + 28 \times 1.6 = 63$

$\Rightarrow 14x + 44.8 = 63 \Rightarrow 14x = 18.2$

$\Rightarrow x = \frac{18.2}{14} = 1.3$ (approx)

Thus, solution of the given system of equation is x = 1.3 (approx), y = 1.6 (approx).

Ex.66 Solve,

$\frac{1}{x+y} + \frac{2}{x-y} = 2$

$\frac{2}{x+y} - \frac{1}{x-y} = 3$

where $x + y \neq 0$ and $x - y \neq 0$

Sol. Taking $\frac{1}{x+y} = u$ and $\frac{1}{x-y} = v$ the above system of equations becomes

$u + 2v = 2$ ....(1)

$2u - v = 3$ ....(2)

Multiplying equation (1) by 2, and (2) by 1, we get;

$2u + 4v = 4$ ....(3)

$2u - v = 3$ ....(4)

Subtracting equation (4) from (3), we get;

$5v = 1 \Rightarrow v = \frac{1}{5}$

Putting v = 1/5 in equation (1), we get;

$u + 2 \times \frac{1}{5} = 2 \Rightarrow u = 2 - \frac{2}{5} = \frac{8}{5}$

Here, $u = \frac{8}{5} = \frac{1}{x+y} \Rightarrow 8x + 8y = 5$ ....(5)

And, $v = \frac{1}{5} = \frac{1}{x-y} \Rightarrow x - y = 5$ ....(6)

Multiplying equation (5) with 1, and (6) with 8, we get;

$8x + 8y = 5$ ....(7)

$8x - 8y = 40$ ....(8)

Adding equation (7) and (8), we get;

$16x = 45 \Rightarrow x = \frac{45}{16}$

Now, putting the above value of x in equation (6), we get;

$\frac{45}{16} - y = 5 \Rightarrow y = \frac{45}{16} - 5 = \frac{-35}{16}$

Hence, solution of the system of the given equations is ;

$x = \frac{45}{16}, \quad y = \frac{-35}{16}$

Type-III : Equation of the form,

$ax + by = c$ and $bx + ay = d$ where $a \neq b$.

We may use the following method to solve the above type of equations.

Steps :

Step I : Let us write the equations in the form

$ax + by = c$

$bx + ay = d$

Step II : Adding or subtracting the above type of two equations, we find :

$(a + b) x + (a + b) y = c + d$

$\Rightarrow$ $x + y = \frac{c+d}{a+b}$ ....(1)

$(a - b)x - (a - b)y = c - d$

$\Rightarrow$ $x - y = \frac{c-d}{a-b}$ ....(2)

Step III : We get the values of x and y after adding or subtracting the equations (1) and (2).

❖ EXAMPLES ❖

Ex.67 Solve the following equations.

$156x + 112y = 580; \quad 112x + 156y = 492$

Sol. The given system of equation is

$156x + 112y = 580$ ....(1)

$112x + 156y = 492$ ....(2)

Adding equation (1) and (2) we get ;

$268x + 268y = 1072$

$\Rightarrow 268(x + y) = 1072$

$\Rightarrow x + y = 4$ ....(3)

Subtracting equation (2) from equation (1), we get

$44x - 44y = 88$

$x - y = 2$ ....(4)

Adding equation (3) with equation (4), we get;

$2x = 6 \quad \Rightarrow \quad x = 3$

Putting x = 3 in equation (3), we get;

$y = 1$

Thus, solution of the system of equations is

$x = 3, y = 1$

Ex.68 Solve the following system of equations.

$43x + 35y = 207; \; 35x + 43y = 183$

Sol. The given sytem of equations is ;

$43x + 35y = 207$ ....(1)

$35x + 43y = 183$ ....(2)

Adding equation (1) and (2), we get;

$78x + 78y = 390 \Rightarrow 78(x + y) = 390$

$\Rightarrow x + y = 5$ ....(3)

Subtracting equation (2) from the equation (1), we get ;

$8x - 8y = 24$

$\Rightarrow x - y = 3$ ....(4)

Adding equation (3) and (4), we get;

$2x = 8 \quad \Rightarrow \quad x = 4$

Putting x= 4 in equation (3), we get;

$4 + y = 5 \Rightarrow y = 1$

Hence, the solution of the system of equation is ; $x = 4$, $y = 1$.

Type IV : Equation of the form,

$a_1x + b_1y + c_1z = d_1$

$a_2x + b_2y + c_2z = d_2$

$a_3x + b_3y + c_3z = d_3$

We may use the following method to solve the above type of equations.

Steps :

Step I : Consider any one of the three given equations.

Step II : Find the value of one of the variable, say z, from it.

Step III : Substitute the value of z found in Step II in the other two equations to get two linear equatons in x, y.

Step IV: Taking the help of elimination method, solve the equations in x, y obtained in Step III.

Step V : Substitute the values of x, y found in Step IV and Step II to get the value of z.

❖ EXAMPLES ❖

Ex.69 Solve the following system of equations.

$x - z = 5$

$y + z = 3$

$x - y = 2$

Sol. The given system of equations to ;

$x - z = 5$ ....(1)

$y + z = 3$ ....(2)

$x - y = 2$ .....(3)

From equation (1), we have;

$z = x - 5$

Putting $z = x - 5$ in equation (2), we get ;

$y + x - 5 = 3$

$\Rightarrow x + y = 8$ ....(4)

Adding equations (3) and (4), we get;

$2x = 10 \quad \Rightarrow \quad x = 5$

Again puttting $x = 5$ in equation (1), we get;

$5 - z = 5 \Rightarrow \quad z = 0$

Hence, the solution of the given system of equation is $x = 5$, $y = 3$, $z = 0$.

Ex.70 Solve,

$$x + 2y + z = 12$$
$$2x - z = 4$$
$$x - 2y = 4$$

Sol. We have,

$$x + 2y + z = 12 \quad \text{....(1)}$$
$$2x - z = 4 \quad \text{....(2)}$$
$$x - 2y = 4 \quad \text{....(3)}$$

From equation (1), we have $z = 12 - x - 2y$.

Putting, $z = 12 - x - 2y$ in the equation (2), we get;

$$2x - (12 - x - 2y) = 4$$
$$\Rightarrow 2x - 12 + x + 2y = 4$$
$$\Rightarrow 3x + 2y = 16 \quad \text{....(4)}$$

Adding equations (3) and (4), we get;

$$4x = 20 \quad \Rightarrow \quad x = 5$$

Putting the value of $x = 5$ in equation (2), we get

$$2 \times 5 - z = 4$$
$$\Rightarrow z = 10 - 4 = 6$$

Again putting the value of $x = 5$ in equation (3), we get

$$5 - 2y = 4 \quad \Rightarrow \quad y = \frac{1}{2}$$

Hence, the solution of the given system of equations is ;

$$x = 5,\ y = \frac{1}{2}\ ,\ z = 6$$

Cross-Multiplication Method

By the method of elimination by substitution, only those equations can be solved, which have unique solution. But the method of cross multiplication discussed below is applicable in all the cases; whether the system has a unique solution, no solution or infinitely many solutions.

Let us solve the following system of equations

$$a_1x + b_1y + c_1 = 0 \quad \text{....(1)}$$
$$a_2x + b_2y + c_2 = 0 \quad \text{....(2)}$$

Multiplying equation (1) by b_2 and equation (2) by b_1, we get

$$a_1b_2x + b_1b_2y + b_2c_1 = 0 \quad \text{....(3)}$$
$$a_2b_1x + b_1b_2y + b_1c_2 = 0 \quad \text{....(4)}$$

Subtracting equation (4) from equation (3), we get

$$(a_1b_2 - a_2b_1)\,x + (b_2c_1 - b_1c_2) = 0$$

$$\Rightarrow x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1} \qquad \begin{bmatrix} a_1b_2 - a_2b_1 \neq 0 \\ \dfrac{a_1}{a_2} \neq \dfrac{b_1}{b_2} \end{bmatrix}$$

Similarly, $y = \dfrac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$

These values of x and y can also be written as

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{-y}{a_1c_2 - a_2c_1} = \frac{1}{a_1b_2 - a_2b_1}$$

❖ EXAMPLES ❖

Ex.71 Solve the following system of equations by cross-multiplication method.

$2x + 3y + 8 = 0$

$4x + 5y + 14 = 0$

Sol. The given system of equations is

$2x + 3y + 8 = 0$

$4x + 5y + 14 = 0$

By cross-multiplication, we get

$$\frac{x}{\begin{matrix}3 & 8\\ 5 & 14\end{matrix}} = \frac{-y}{\begin{matrix}2 & 8\\ 4 & 14\end{matrix}} = \frac{1}{\begin{matrix}2 & 3\\ 4 & 5\end{matrix}}$$

$$\Rightarrow \frac{x}{3\times14-5\times8} = \frac{-y}{2\times14-4\times8} = \frac{1}{2\times5-4\times3}$$

$$\Rightarrow \frac{x}{42-40} = \frac{-y}{28-32} = \frac{1}{10-12}$$

$$\Rightarrow \frac{x}{2} = \frac{-y}{-4} = \frac{1}{-2}$$

$$\Rightarrow \frac{x}{2} = -\frac{1}{2} \Rightarrow x = -1$$

and $\frac{-y}{-4} = -\frac{1}{2} \Rightarrow y = -2.$

Hence, the solution is $x = -1$, $y = -2$

We can verify the solution.

Ex.72 Solve the followning system of equations by the method of cross-multiplication.

$2x - 6y + 10 = 0$

$3x - 7y + 13 = 0$

Sol. The given system of equations is

$2x - 6y + 10 = 0$ ....(1)

$3x - 7y + 13 = 0$ ....(2)

By cross-multiplication, we have

$$\frac{x}{\begin{matrix}-6 & 10\\ -7 & 13\end{matrix}} = \frac{-y}{\begin{matrix}2 & 10\\ 3 & 13\end{matrix}} = \frac{1}{\begin{matrix}2 & -6\\ 3 & -7\end{matrix}}$$

$$\Rightarrow \frac{x}{-6\times13-(-7)\times10} = \frac{-y}{2\times13-3\times10}$$

$$= \frac{1}{2\times(-7)-3\times(-6)}$$

$$\Rightarrow \frac{x}{-78+70} = \frac{-y}{26-30} = \frac{1}{-14+18}$$

$$\Rightarrow \frac{x}{-8} = \frac{-y}{-4} = \frac{1}{4}$$

$$\Rightarrow \frac{x}{-8} = \frac{1}{4} \Rightarrow \quad x = -2$$

$$\Rightarrow \frac{-y}{-4} = \frac{1}{4} \Rightarrow \quad y = 1$$

Hence, the solution is $x = -2$, $y = 1$

Ex.73 Solve the following system of equations by the method of cross-multiplication.

$11x + 15y = -23$; $7x - 2y = 20$

Sol. The given system of equations is

$11x + 15y + 23 = 0$

$7x - 2y - 20 = 0$

Now, by cross-multiplication method, we have

$$\frac{x}{\begin{matrix}15 & 23\\ -2 & -20\end{matrix}} = \frac{-y}{\begin{matrix}11 & 23\\ 7 & -20\end{matrix}} = \frac{1}{\begin{matrix}11 & 15\\ 7 & -2\end{matrix}}$$

$$\Rightarrow \frac{x}{15\times(-20)-(-2)\times 23} = \frac{-y}{11\times(-20)-7\times 23}$$

$$= \frac{1}{11\times(-2)-7\times 15}$$

$$\Rightarrow \frac{x}{-300+46} = \frac{-y}{-220-161} = \frac{1}{-22-105}$$

$$\Rightarrow \frac{x}{-254} = \frac{-y}{-381} = \frac{1}{-127}$$

$$\Rightarrow \frac{x}{-254} = \frac{1}{-127} \Rightarrow x = 2$$

and $\frac{-y}{-381} = \frac{1}{-127} \Rightarrow y = -3$

Hence, $x = 2$, $y = -3$ is the required solution.

The students can verify the solution.

Ex.74 Solve the following system of equations by cross-multiplication method.

$ax + by = a - b$; $bx - ay = a + b$

Sol. Rewriting the given system of equations, we get

$ax + by - (a - b) = 0$

$bx - ay - (a + b) = 0$

By cross-multiplication method, we have

$$\frac{x}{\begin{matrix}b & -(a-b)\\ -a & -(a+b)\end{matrix}} = \frac{-y}{\begin{matrix}a & -(a-b)\\ b & -(a+b)\end{matrix}} = \frac{1}{\begin{matrix}a & b\\ b & -a\end{matrix}}$$

$\Rightarrow \frac{x}{b\times\{-(a+b)\}-(-a)\times\{-(a-b)\}} = \frac{-y}{-a(a+b)+b(a-b)} = \frac{1}{-a^2-b^2}$

$\Rightarrow \frac{x}{-ab-b^2-a^2+ab} = \frac{-y}{-a^2-ab+ab-b^2} = \frac{1}{-(a^2+b^2)}$

$\Rightarrow \frac{x}{-(a^2+b^2)} = \frac{-y}{-(a^2+b^2)} = \frac{1}{-(a^2+b^2)}$

$\Rightarrow \frac{x}{-(a^2+b^2)} = \frac{1}{-(a^2+b^2)} \Rightarrow x = 1$

and $\frac{-y}{-(a^2+b^2)} = \frac{1}{-(a^2+b^2)} \Rightarrow y = -1$

Hence, the solution is $x = 1$, $y = -1$.

Verification of the solution shows that the answer is correct.

Ex.75 Solve the following system of equations by cross-multiplication method.

$x + y = a - b;\ ax - by = a^2 + b^2$

Sol. The given system of equations can be rewritten as :

$x + y - (a - b) = 0$

$ax - by - (a^2 + b^2) = 0$

By cross-multiplication method, we have

$\frac{x}{\begin{matrix}1 & -(a-b) \\ -b & -(a^2+b^2)\end{matrix}} = \frac{-y}{\begin{matrix}1 & -(a-b) \\ a & -(a^2+b^2)\end{matrix}} = \frac{1}{\begin{matrix}1 & 1 \\ a & -b\end{matrix}}$

$\Rightarrow \frac{x}{-(a^2+b^2)-(-b)\times\{-(a-b)\}} = \frac{-y}{-(a^2+b^2)-a\times\{-(a-b)\}} = \frac{1}{-b-a}$

$\Rightarrow \frac{x}{-(a^2+b^2)-b(a-b)} = \frac{-y}{-(a^2+b^2)+a(a-b)} = \frac{1}{-(b+a)}$

$\Rightarrow \frac{x}{-a^2-b^2-ab+b^2} = \frac{-y}{-a^2-b^2+a^2-ab} = \frac{1}{-(a+b)}$

$\Rightarrow \frac{x}{-a(a+b)} = \frac{-y}{-b(a+b)} = \frac{1}{-(a+b)}$

$\Rightarrow \frac{x}{-a(a+b)} = \frac{1}{-(a+b)} \Rightarrow x = a$

and $\frac{-y}{-b(a+b)} = \frac{1}{-(a+b)} \Rightarrow y = -b$

Hence, the solution is $x = a$, $y = -b$.

Ex.76 Solve the followig system of equations by the method of cross-multiplication :

$\frac{x}{a} + \frac{y}{b} = a + b$

$\frac{x}{a^2} + \frac{y}{b^2} = 2$

Sol. The given system of equations is rewritten as :

$\frac{x}{a} + \frac{y}{b} - (a + b) = 0$ ....(1)

$$\frac{x}{a^2} + \frac{y}{b^2} - 2 \quad = 0 \qquad ....(2)$$

Multiplying equation (1) by ab, we get

$$bx + ay - ab\,(a + b) = 0 \quad .... \ (3)$$

Multiplying equation (2) by a^2 b^2, we get

$$b^2x + a^2y - 2a^2b^2 = 0 \quad .... \ (4)$$

By cross multiplication method, we have

$$\frac{x}{\begin{matrix} a & -ab(a+b) \\ a^2 & -2a^2b^2 \end{matrix}} = \frac{-y}{\begin{matrix} b & -ab(a+b) \\ b^2 & -2a^2b^2 \end{matrix}} = \frac{1}{\begin{matrix} b & a \\ b^2 & a^2 \end{matrix}}$$

$$\Rightarrow \frac{x}{-2a^3b^2 + a^3b(a+b)} = \frac{-y}{-2a^2b^3 + ab^3(a+b)} = \frac{1}{a^2b - ab^2}$$

$$\Rightarrow \frac{x}{-2a^3b^2 + a^4b + a^3b^2} = \frac{-y}{-2a^2b^3 + a^2b^3 + ab^4} = \frac{1}{ab(a-b)}$$

$$\Rightarrow \frac{x}{a^4b - a^3b^2} = \frac{-y}{ab^4 - a^2b^3} = \frac{1}{ab(a-b)}$$

$$\Rightarrow \frac{x}{a^3b(a-b)} = \frac{y}{ab^3(a-b)} = \frac{1}{ab(a-b)}$$

$$\Rightarrow \frac{x}{a^3b(a-b)} = \frac{1}{ab(a-b)}$$

$$\Rightarrow x = \frac{a^3b(a-b)}{ab(a-b)} = a^2$$

And $\dfrac{y}{ab^3(a-b)} = \dfrac{1}{ab(a-b)}$

$$\Rightarrow \frac{ab^3(a-b)}{ab(a-b)} = b^2$$

Hence, the solution $x = a^2$, $y = b^2$

Ex.77 Solve the following system of equations by cross-multiplication method -

$$ax + by = 1; \quad bx + ay = \frac{(a+b)^2}{a^2+b^2} - 1$$

Sol. The given system of equations can be written as.

$$ax + by - 1 = 0 \qquad ....(1)$$

$$bx + ay = \frac{(a+b)^2}{a^2+b^2} - 1$$

$$\Rightarrow bx + ay = \frac{a^2 + 2ab + b^2 - a^2 - b^2}{a^2+b^2}$$

$$\Rightarrow bx + ay = \frac{2ab}{a^2+b^2}$$

$$\Rightarrow bx + ay - \frac{2ab}{a^2+b^2} = 0 \qquad ....(2)$$

Rewritting the equations (1) and (2), we have

$$ax + by - 1 = 0$$

$$bx + ay - \frac{2ab}{a^2+b^2} = 0$$

Now, by cross-multiplication method, we have

$$\frac{x}{\begin{array}{cc} b & -1 \\ a & -\frac{2ab}{a^2+b^2} \end{array}} = \frac{-y}{\begin{array}{cc} a & -1 \\ b & -\frac{2ab}{a^2+b^2} \end{array}} = \frac{1}{\begin{array}{cc} a & b \\ b & a \end{array}}$$

$$\Rightarrow \frac{x}{b \times \left(\frac{-2ab}{a^2+b^2}\right) - a \times (-1)} = \frac{-y}{a \times \left(\frac{-2ab}{a^2+b^2}\right) - b \times (-1)} = \frac{1}{a \times a - b \times b} \Rightarrow \frac{x}{-\frac{2ab^2}{a^2+b^2} + a} = \frac{-y}{\frac{-2a^2b}{a^2+b^2} + b} = \frac{1}{a^2 - b^2}$$

$$\Rightarrow \frac{x}{\frac{-2ab^2 + a^3 + ab^2}{a^2+b^2}} = \frac{-y}{\frac{-2a^2b + a^2b + b^3}{a^2+b^2}} = \frac{1}{a^2-b^2}$$

$$\Rightarrow \frac{x}{\frac{a(a^2-b^2)}{a^2+b^2}} = \frac{-y}{\frac{b(b^2-a^2)}{a^2+b^2}} = \frac{1}{a^2-b^2}$$

$$\Rightarrow \frac{x}{\frac{a(a^2-b^2)}{a^2+b^2}} = \frac{1}{a^2-b^2} \Rightarrow x = \frac{a}{a^2+b^2}$$

and $\dfrac{-y}{\frac{b(b^2-a^2)}{a^2+b^2}} = \dfrac{1}{a^2-b^2} \Rightarrow y = \dfrac{b}{a^2+b^2}$

Hence, the solution is $x = \dfrac{a}{a^2+b^2}$, $y = \dfrac{b}{a^2+b^2}$

Ex.78 Solve the following system of equations in x and y by cross-multiplication method

$$(a - b)x + (a + b)y = a^2 - 2ab - b^2$$

$$(a + b)(x + y) = a^2 + b^2$$

Sol. The given system of equations can be rewritten as :

$$(a - b)x + (a + b)y - (a^2 - 2ab - b^2) = 0$$

$$(a + b)x + (a + b)y - (a^2 + b^2) = 0$$

By cross-multiplication method, we have

$$\frac{x}{\begin{array}{cc} (a+b) & -(a^2-2ab-b^2) \\ (a+b) & -(a^2+b^2) \end{array}} = \frac{-y}{\begin{array}{cc} (a-b) & -(a^2-2ab-b^2) \\ (a+b) & -(a^2+b^2) \end{array}}$$

$$= \frac{1}{\begin{matrix}(a-b) & \searrow\!\!\!\nearrow & (a+b) \\ (a+b) & & (a+b)\end{matrix}}$$

$$\Rightarrow \frac{x}{(a+b)\times\{-(a^2+b^2)\}-(a+b)\times\{-(a^2-2ab-b^2)\}}$$

$$= \frac{-y}{(a-b)\times\{-(a^2+b^2)\}-(a+b)\times\{-(a^2-2ab-b^2)\}}$$

$$= \frac{1}{(a-b)\times(a+b)-(a+b)\times(a+b)}$$

$$\Rightarrow \frac{x}{-(a+b)(a^2+b^2)+(a+b)(a^2-2ab-b^2)}$$

$$= \frac{-y}{-(a-b)(a^2+b^2)+(a+b)(a^2-2ab-b^2)}$$

$$= \frac{1}{(a-b)(a+b)-(a+b)^2}$$

$$\Rightarrow \frac{x}{(a+b)[-(a^2+b^2)+(a+b)(a^2-2ab-b^2)]}$$

$$= \frac{-y}{(a+b)(a^2-2ab-b^2)-(a-b)(a^2+b^2)}$$

$$= \frac{1}{(a+b)(a-b-a-b)}$$

$$\Rightarrow \frac{x}{(a+b)(-2ab-2b^2)}$$

$$= \frac{-y}{a^3-a^2b-3ab^2-b^3-a^3-ab^2+a^2b+b^3}$$

$$= \frac{1}{(a+b)(-2b)}$$

$$\Rightarrow \frac{x}{-(a+b)(2a+2b)b} = \frac{-y}{-4ab^2} = \frac{1}{-2b(a+b)}$$

$$\Rightarrow \frac{x}{-2(a+b)(a+b)b} = \frac{1}{-2b(a+b)}$$

$\Rightarrow$ $x = a + b$

and $\frac{-y}{-4ab^2} = \frac{1}{-2b(a+b)} \Rightarrow y = -\frac{2ab}{a+b}$

Hence, the solution of the given system of equations is $x = a + b$, $y = -\frac{2ab}{a+b}$.

Ex.79 Solve the following system of equations by cross-multiplications method.

$^2 - ab + b^2$

$a(x + y) - b(x - y) = a^2 + ab + b^2$

Sol. The given system of equations can be rewritten as

$ax + bx + ay - by - (a^2 - ab + b^2) = 0$

$\Rightarrow (a + b)x + (a - b)y - (a^2 - ab + b^2) = 0$....(1)

And $ax - bx + ay + by - (a^2 + ab + b^2) = 0$

$\Rightarrow (a - b)x + (a + b)y - (a^2 + ab + b^2) = 0$....(2)

Now, by cross-multiplication method, we have

$$\frac{x}{\begin{array}{l}(a-b)\\(a+b)\end{array}\times\begin{array}{l}-(a^2-ab+b^2)\\-(a^2+ab+b^2)\end{array}}=\frac{-y}{\begin{array}{l}(a+b)\\(a-b)\end{array}\times\begin{array}{l}-(a^2-ab+b^2)\\-(a^2+ab+b^2)\end{array}}$$

$$=\frac{1}{\begin{array}{l}(a+b)\\(a-b)\end{array}\times\begin{array}{l}(a-b)\\(a+b)\end{array}}$$

$$\Rightarrow \frac{x}{(a-b)\times\{-(a^2+ab+b^2)\}-(a+b)\times\{-(a^2-ab+b^2)\}}$$

$$=\frac{-y}{(a+b)\times\{-(a^2+ab+b^2)\}-(a-b)\times\{-(a^2-ab+b^2)\}}$$

$$=\frac{1}{(a+b)\times(a+b)-(a-b)(a-b)}$$

$$\Rightarrow \frac{x}{-(a-b)(a^2+ab+b^2)+(a+b)(a^2-ab+b^2)}$$

$$=\frac{-y}{-(a+b)(a^2+ab+b^2)+(a-b)(a^2-ab+b^2)}$$

$$=\frac{1}{(a+b)^2-(a-b)^2}$$

$$\Rightarrow \frac{x}{-(a^3-b^3)+(a^3+b^2)}$$

$$=\frac{-y}{-a^3-2a^2b-2ab^2-b^3+a^3-2a^2b+2ab^2-b^3}$$

$$=\frac{1}{a^2+2ab+b^2-a^2+2ab-b^2}$$

$$\Rightarrow \frac{x}{2b^3}=\frac{-y}{-4a^2b-2b^3}=\frac{1}{4ab}$$

$$\Rightarrow \frac{x}{2b^3}=\frac{-y}{-2b(2a^2+b^2)}=\frac{1}{4ab}$$

$$\Rightarrow \frac{x}{2b^3}=\frac{1}{4ab} \Rightarrow x=\frac{b^2}{2a}$$

And $\frac{-y}{-2b(2a^2+b^2)}=\frac{1}{4ab} \Rightarrow y=\frac{2a^2+b^2}{2a}$

Hence, the solution is $x=\frac{b^2}{2a}$, $y=\frac{2a^2+b^2}{2a}$

Ex.80 Solve the following system of equations by the method of cross-multiplication.

$$\frac{a}{x}-\frac{b}{y}=0\ ;\ \frac{ab^2}{x}+\frac{a^2b}{y}=a^2+b^2\ ;$$

where $x \neq 0,\ y \neq 0$

Sol. The given system of equations is

$$\frac{a}{x}-\frac{b}{y}=0 \qquad \text{....(1)}$$

$$\frac{ab^2}{x}+\frac{a^2b}{y}-(a^2+b^2)=0 \qquad \text{....(2)}$$

Putting $\frac{a}{x} = u$ and $\frac{b}{y} = v$ in equatinos (1) and (2) the system of equations reduces to

$$u - v + 0 = 0$$

$$b^2u + a^2v - (a^2 + b^2) = 0$$

By the method of cross-multiplication, we have

$$\frac{u}{\begin{matrix} -1 & \times & 0 \\ a^2 & & -(a^2+b^2) \end{matrix}} = \frac{-v}{\begin{matrix} 1 & \times & 0 \\ b^2 & & -(a^2+b^2) \end{matrix}} = \frac{1}{\begin{matrix} 1 & \times & -1 \\ b^2 & & a^2 \end{matrix}}$$

$$\Rightarrow \frac{u}{a^2 + b^2 - a^2 \times 0} = \frac{-v}{-(a^2 + b^2) - b^2 \times 0} = \frac{1}{a^2 - (-b^2)}$$

$$\Rightarrow \frac{u}{a^2 + b^2} = \frac{-v}{-(a^2 + b^2)} = \frac{1}{a^2 + b^2}$$

$$\Rightarrow \frac{u}{a^2 + b^2} = \frac{1}{a^2 + b^2} \Rightarrow u = 1$$

and $\frac{-v}{-(a^2 + b^2)} = \frac{1}{a^2 + b^2} \Rightarrow v = 1$

and $u = \frac{a}{x} = 1 \qquad \Rightarrow x = a$

$v = \frac{b}{y} = 1 \qquad \Rightarrow y = b$

Hence, the solution of the given system of equations is $x = a$, $y = b$.

The system of equations is given by

$a_1x + b_1y + c_1 = 0$ ....(1)

$a_2x + b_2y + c_2 = 0$ ....(2)

(a) It is consistent with unique solution, if

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

It shows that lines represented by equations (1) and (2) are not parallel.

(b) It is consistent with infinitely many solutions, if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

It shows that lines represented by equation (1) and (2) are coincident.

(c) It is inconsistent, if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

It shows that lines represented by equation (1) and (2) are parallel and non–coincident.

❖ EXAMPLES ❖

Ex.81 Show that the following system of equations has unique solution

$2x - 3y = 6; \; x + y = 1.$

Sol. The given system of equation can be written as

$2x - 3y - 6 = 0$

$x + y - 1 = 0$

The given equations are of the form

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Where, $a_1 = 2, \; b_1 = -3, \; c_1 = -6$

and $a_2 = 1, \; b_2 = 1, \; c_2 = -1$

$\frac{a_1}{a_2} = \frac{2}{1} = 2, \; \frac{b_1}{b_2} = \frac{-3}{1} = 3$

$\frac{c_1}{c_2} = \frac{-6}{-1} = 6$

Clearly, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

So, the given system of equations has a unique solution. i.e., it is consistent.

Ex.82 Show that the following system of equations has unique solution :

$x - 2y = 2 \; ; \; 4x - 2y = 5$

Sol. The given system of equations can be written as

$x - 2y - 2 = 0$

$4x - 2y - 5 = 0$

The given equations are of the form

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Where, $a_1 = 1, \; b_1 = -2, \; c_1 = -2$

and $a_2 = 4, \; b_2 = -2, \; c_2 = -5$

$\frac{a_1}{a_2} = \frac{1}{4}, \; \frac{b_1}{b_2} = \frac{-2}{-2} = 1, \; \frac{c_1}{c_2} = \frac{-2}{-5} = \frac{2}{5}$

Clearly, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

So, the given system of equations has a unique solution i.e. It is consistent.

Ex.83 For what value of k the following system of equations has a unique solution :

$x - ky = 2 \; ; \; 3x + 2y = -5$

Sol. The given system of equation can be written as

$x - ky - 2 = 0$

$3x + 2y + 5 = 0$

The given system of equations is of the form

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

where, $a_1 = 1$, $b_1 = -k$, $c_1 = -2$

and $a_2 = 3$, $b_2 = 2$, $c_2 = 5$

Clearly, for unique solution $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\Rightarrow \frac{1}{3} \neq \frac{-k}{2} \Rightarrow k \neq \frac{-2}{3}$

Ex.84 Show that the following system has infinitely many solutions.

$x = 3y + 3$; $9y = 3x - 9$

Sol. The given system of equations can be written as

$x - 3y - 3 = 0$

$3x - 9y - 9 = 0$

The given equations are of the form

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Where, $a_1 = 1$, $b_1 = -3$, $c_1 = -3$

and $a_2 = 3$, $b_2 = -9$, $c_2 = -9$

$\frac{a_1}{a_2} = \frac{1}{3}$, $\frac{b_1}{b_2} = \frac{-3}{-9} = \frac{1}{3}$, $\frac{c_1}{c_2} = \frac{-3}{-9} = \frac{1}{3}$

Clearly, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ so the given system of equations has infinitely many solutions.

Ex.85 Show that the following system has infinitely many solutions :

$2y = 4x - 6$; $2x = y + 3$

Sol. The given system of equations can be written as

$4x - 2y - 6 = 0$

$2x - y - 3 = 0$

The given equations are of the form

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Where, $a_1 = 4$, $b_1 = -2$, $c_1 = -6$

and $a_2 = 2$, $b_2 = -1$, $c_2 = -3$

$\frac{a_1}{a_2} = \frac{4}{2} = 2$, $\frac{b_1}{b_2} = \frac{-2}{-1} = 2$, $\frac{c_1}{c_2} = \frac{-6}{-3} = 2$

Clearly, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, so the given system of equations has infinitely many solutions.

Ex.86 Find the value of k for which the following system of equations has infinitely many solutions.

$(k - 1)x + 3y = 7$; $(k + 1)x + 6y = (5k - 1)$

Sol. The given system of equations can be written as

$(k - 1)x + 3y - 7 = 0$

$(k + 1)x + 6y - (5k - 1) = 0$

Here $a_1 = (k - 1)$, $b_1 = 3$, $c_1 = -7$

and $a_2 = (k + 1)$, $b_2 = 6$, $c_2 = -(5k - 1)$

For the system of equations to have infinite number of solutions.

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \frac{k-1}{k+1} = \frac{3}{6} = \frac{-7}{-(5k-1)}$$

$$\Rightarrow \frac{k-1}{k+1} = \frac{1}{2} = \frac{7}{5k-1}$$

Taking I and II

$$\frac{k-1}{k+1} = \frac{1}{2}$$

$\Rightarrow 2k - 2 = k + 1 \quad \Rightarrow \quad k = 3$

Taking II and III

$$\frac{1}{2} = \frac{7}{5k-1} \qquad \Rightarrow \quad 5k - 1 = 14$$

$\Rightarrow 5k = 15 \qquad \Rightarrow \quad k = 3$

Hence, $k = 3$.

Ex.87 For what values of a and b, the following system of equations have an infinite number of solutions:

$2x + 3y = 7$; $(a - b)x + (a + b)y = 3a + b - 2$

Sol. The given system of linear equations can be written as

$$2x + 3y - 7 = 0$$

$$(a - b)x + (a + b)y - (3a + b - 2) = 0$$

The above system of equations is of the form

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0,$$

where $a_1 = 2$, $b_1 = 3$, $c_1 = -7$

$a_2 = (a - b)$, $b_2 = (a + b)$, $c_2 = -(3a + b - 2)$

For the given system of equations to have an infinite number of solutions

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Here, $\frac{a_1}{a_2} = \frac{2}{a-b}$, $\frac{b_1}{b_2} = \frac{3}{a+b}$ and

$$\frac{c_1}{c_2} = \frac{-7}{-(3a+b-2)} = \frac{7}{3a+b-2}$$

$$\Rightarrow \frac{2}{a-b} = \frac{3}{a+b} = \frac{7}{3a+b-2}$$

$$\Rightarrow \frac{2}{a-b} = \frac{3}{a+b} \text{ and } \frac{3}{a+b} = \frac{7}{3a+b-2}$$

$\Rightarrow 2a + 2b = 3a - 3b$ and $9a + 3b - 6 = 7a + 7b$

$\Rightarrow$ 2a – 3a = –3b – 2b and 9a – 7a = 7b – 3b + 6

$\Rightarrow$ –a = –5b and 2a = 4b + 6

$\Rightarrow$ a = 5b .... (3) and a = 2b + 3 .... (4)

Solving (3) and (4) we get

$$5b = 2b + 3 \Rightarrow b = 1$$

Substituting b = 1 in (3), we get a = 5 × 1 = 5

Thus, a = 5 and b = 1

Hence, the given system of equations has infinite number of solutions when

$$a = 5,\ b = 1$$

Ex.88 Show that the following system of equations is inconsistent.

$$2x + 7y = 11;\quad 5x + \frac{35}{2}y = 25$$

Sol. The given system of equations can be written as

$$2x + 7y - 11 = 0$$

$$5x + \frac{35}{2}y - 25 = 0$$

The given equations are of the form

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

Where, $a_1 = 2$, $b_1 = 7$, $c_1 = -11$

and $a_2 = 5$, $b_2 = \frac{35}{2}$, $c_2 = -25$

$$\frac{a_1}{a_2} = \frac{2}{5},\ \frac{b_1}{b_2} = \frac{7}{\frac{35}{2}} = \frac{2}{5},\ \frac{c_1}{c_2} = \frac{-11}{-25} = \frac{11}{25}$$

Clearly, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

So, the given system of equations has no solution, i.e. it is inconsistent. **Proved**.

Ex.89 Show that the following system of equations has no solution :

$$2x + 4y = 10;\quad 3x + 6y = 12$$

Sol. The given system of equations can be written as

$$2x + 4y - 10 = 0$$

$$3x + 6y - 12 = 0$$

The given equations are of the form

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

The given equations are of the form

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

Where $a_1 = 2$, $b_1 = 4$, $c_1 = -10$

and $a_2 = 3, \quad b_2 = 6, \quad c_2 = -12$

$\frac{a_1}{a_2} = \frac{2}{3}, \quad \frac{b_1}{b_2} = \frac{4}{6} = \frac{2}{3}, \quad \frac{c_1}{c_2} = \frac{-10}{-12} = \frac{5}{6}$

Clearly, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

So, the given system of equations has no solution, i.e., it is inconsistent. **Proved**.

Ex.90 For what values of k will the following system of liner equations has no solution.

$3x + y = 1; \quad (2k - 1)x + (k - 1)y = 2k + 1$

Sol. The given system of equations may be written as

$$3x + y - 1 = 0$$

$$(2k - 1)x + (k - 1)y - (2k + 1) = 0$$

The above system of equations is of the form

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

where $a_1 = 3, \quad b_1 = 1, \quad c_1 = -1$

and $a_2 = (2k - 1), b_2 = (k - 1), c_2 = -(2k+1)$

$\frac{a_1}{a_2} = \frac{3}{2k-1}, \quad \frac{b_1}{b_2} = \frac{1}{k-1}, \quad \frac{c_1}{c_2} = \frac{-1}{-(2k+1)}$

Clearly, for no solution $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\Rightarrow \frac{3}{2k-1} = \frac{1}{k-1}$

$\Rightarrow 3k - 3 = 2k - 1 \Rightarrow k = 2$

and $\frac{1}{k-1} \neq \frac{-1}{-(2k+1)}$

$\Rightarrow 2k + 1 \neq k - 1 \quad \Rightarrow k \neq -2$

and $\frac{3}{2k-1} \neq \frac{1}{2k+1}$

$\Rightarrow 6k + 3 \neq 2k - 1 \Rightarrow 4k \neq -4 \Rightarrow k \neq -1$

Hence the given system of linear equations has no solution, when

$k = 2$ and $k \neq -2$ and $k \neq -1$.

Ex.91 Determine the value of k for each of the following given system of equations having unique/consistent solution.

(i) $2x + 3y - 5 = 0; \quad kx - 6y = 8$

(ii) $2x + ky = 1; \quad 5x - 7y - 5 = 0$

Sol. (i) The given system of equations may be written as

$$2x + 3y - 5 = 0$$

$$kx - 6y - 8 = 0$$

Here, $a_1 = 2, b_1 = 3, c_1 = 5,$

$a_2 = k, b_2 = -6, c_2 = -8$

As the given equations have unique solution,

we get,

$$\frac{a_1}{a_2} = \frac{2}{k} \text{ and } \frac{b_1}{b_2} = \frac{3}{-6} = \frac{-1}{2}$$

Here $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\Rightarrow \frac{2}{k} \neq \frac{-1}{2} \quad \Rightarrow k \neq -4$

Thus the given system of equations have a unique solution for all real values of k except –4.

(ii) The given system of equations may be written as

$$2x + ky - 1 = 0$$
$$5x - 7y - 5 = 0$$

Here, $a_1 = 2, b_1 = k, c_1 = -1,$

$a_2 = 5, b_2 = -7, c_2 = -5$

We have $\frac{a_1}{a_2} = \frac{2}{5}$ and $\frac{b_1}{b_2} = \frac{k}{-7} = \frac{-k}{7}$

Here $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow \frac{2}{5} \neq \frac{-k}{7}$

It satisfies the condition that the system of given solutions has a unique solution.

So, $\frac{2}{5} \neq \frac{-k}{7}$

$\Rightarrow k \neq \frac{-14}{5}$

Thus, the given system of equations has a unique solution for all real values of k except $\frac{-14}{5}$.

Ex.92 Determine the value of k for each of the following given system of equations having unique/ consistent solution.

(i) $x - ky - 2 = 0;\ 3x + 2y + 5 = 0$

(ii) $2x - 3y - 1 = 0;\ kx + 5y - 7 = 0$

Sol. (i) We have,

$$x - ky - 2 = 0$$
$$3x + 2y + 5 = 0$$

Here, $a_1 = 1, b_1 = -k, c_1 = -2,$

$a_2 = 3, \ b_2 = 2, c_2 = 5$

Since, the given system of equations has a unique solution, we have

$\Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ or $\frac{1}{3} \neq \frac{-k}{2}$

$\Rightarrow k \neq \frac{-2}{3}$

Thus, the given system of equations has a solution for all values of k except $\frac{-2}{3}$

(ii) We have

$$2x - 3y - 1 = 0$$
$$kx + 5y - 7 = 0$$

Here, $a_1 = 2, b_1 = -3, c_1 = -1,$

$a_2 = k, b_2 = 5, c_2 = -7$

Since, the given system of equations has a unique solution, we get

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$$\Rightarrow \frac{2}{k} \neq \frac{-3}{5} \qquad \Rightarrow k \neq \frac{-10}{3}$$

Thus, the given system of equation has a unique solution for all value of k except $\frac{-10}{3}$.

Ex.93 Find the value of k for each of the following systems of equations having infinitely many solutions.

(i) $2x + 3y = k; (k - 1)x + (k + 2)y = 3k$

(ii) $2x + 3y = 2; (k + 2)x + (2k + 1)y = 2(k - 1)$

Sol. (i) We have

$$2x + 3y - k = 0$$
$$(k - 1)x + (k + 2)y - 3k = 0$$

Here $a_1 = 2, b_1 = 3, c_1 = -k,$

$a_2 = k - 1, b_2 = k + 2, c_2 = -3k$

Since, the given system of equations has infinitely many solutions, we get

$$\frac{a_1}{a_2} = \frac{2}{k-1}, \quad \frac{b_1}{b_2} = \frac{3}{k+2}, \quad \frac{c_1}{c_2} = \frac{-k}{-3k}$$

and $\frac{a_1}{a_1} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$$\Rightarrow \frac{2}{k-1} = \frac{3}{k+2} = \frac{1}{3}$$

$$\Rightarrow \frac{2}{k-1} = \frac{3}{k+2} \quad \text{or} \quad \frac{3}{k+2} = \frac{1}{3}$$

$$\Rightarrow 2k + 4 = 3k - 3 \quad \text{or} \quad k + 2 = 9$$

$$\Rightarrow 3k - 2k = 4 + 3 \quad \text{or} \quad k = 7$$

$$\Rightarrow k = 7 \quad \text{or} \quad k = 7$$

$$\Rightarrow k = 7$$

It shows that the given system of equations has infinitely many solutions at $k = 7$

(ii) We have

$$2x + 3y - 2 = 0$$
$$(k + 2)x + (2k + 1)y - 2(k - 1) = 0$$

Here, $a_1 = 2, b_1 = 3, c_1 = -2,$

$a_2 = k + 2,\ b_2 = 2k + 1,\ c_2 = -2\ (k - 1)$

Since, the given system of equations has infinitely many solutions, we get

$$\frac{a_1}{a_2} = \frac{2}{k+2},\ \frac{b_1}{b_2} = \frac{3}{3k+1},\ \frac{c_1}{c_2} = \frac{-2}{-2(k-1)}$$

and $\frac{a_1}{a_1} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \frac{2}{k+2} = \frac{3}{2k+1} = \frac{1}{k-1}$

$\Rightarrow \frac{2}{k+2} = \frac{3}{2k+1}$ or $\frac{3}{2k+1} = \frac{1}{k-1}$

$\Rightarrow 4k + 2 = 3k + 6$ or $3k - 3 = 2k + 1$

$\Rightarrow 4k - 3k = 6 - 2$ or $3k - 2k = 1 + 3$

$\Rightarrow k = 4$ or $k = 4$

$\Rightarrow k = 4$

Ex.94 Determine the values of k for the following system of equations having no solution.

$x + 2y = 0;\ 2x + ky = 5$

Sol. The given system of equations may be written as

$x + 2y = 0$

$2x + ky - 5 = 0$

Here, $a_1 = 1, b_1 = 2, c_1 = 0, a_2 = 2, b_2 = k, c_2 = -5$

As the given system of equations has no solution, we get

$$\frac{a_1}{a_2} = \frac{1}{2} = \frac{b_1}{b_2} = \frac{2}{k} \neq \frac{c_1}{c_2} = \frac{0}{-5}$$

We must write

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$\Rightarrow \frac{1}{2} = \frac{2}{k} \Rightarrow k = 4$

Here, for this value of k, we get $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Ex.95 Find the value of k of the following system of equations having infinitely many solutions.

$2x - 3y = 7;\ (k + 2)\ x - (2k + 1)\ y = 3(2k - 1)$

Sol. A given system of equations has infinitely many solutions, if

$$\frac{a_1}{a_1} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

So, we get

$\Rightarrow \frac{2}{k+2} = \frac{-3}{-(2k+1)} = \frac{7}{3(2k-1)}$

$\Rightarrow \frac{2}{k+2} = \frac{3}{2k+1}$ or $\frac{3}{2k+1} = \frac{7}{6k-3}$

$\Rightarrow 4k + 2 = 3k + 6$ or $18k - 9 = 14k + 7$

$\Rightarrow k = 4 \quad$ or $\quad k = 4$

$\Rightarrow k = 4$

Thus, the given system of equations has infinitely many solutions at k = 4.

Ex.96 Determine the values of a and b so that the following given system of linear equations has infinitely many solutions.

$2x - (2a + 5)y = 5; \ (2b + 1)x - 9y = 15$

Sol. We have

$2x - (2a + 5)y - 5 = 0$

$(2b + 1)x - 9y - 15 = 0$

Hence, $a_1 = 2, \ b_1 = -(2a + 5), \ c_1 = -5,$

$a_2 = 2b + 1, \ b_2 = -9, \ c_2 = -15$:

The given system of equations has infinitely many solutions, if

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ such that

$\frac{2}{2b+1} = \frac{-(2a+5)}{-9} = \frac{-5}{-15}$

$\Rightarrow \frac{2}{2b+1} = \frac{2a+5}{9} = \frac{1}{3}$

$\Rightarrow \frac{2}{2b+1} = \frac{1}{3}$ and $\frac{2a+5}{9} = \frac{1}{3}$

$\Rightarrow 2b + 1 = 6$ or $6a + 15 = 9$

$\Rightarrow b = \frac{5}{2}$ and $a = -1$

Thus, the given system of equations has infinitely many solutions at $a = -1, b = \frac{5}{2}$.

Ex.97 Find the value of c if the following system of equation has no solution.

$cx + 3y = 3; \ 12x + cy = 5$

Sol. We have

$cx + 3y - 3 = 0$

$12x + cy - 6 = 0$

The given system of equations has no solution, if

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ such that $\frac{c}{12} = \frac{3}{c} \neq \frac{-3}{-6}$

$\Rightarrow \frac{c}{12} = \frac{3}{c}$ and $\frac{3}{c} \neq \frac{1}{2}$

$\Rightarrow c^2 = 36$

$\Rightarrow c = \pm 6$

Thus, the given system of equation has no solution at $c = \pm 6$.

Ex.98 For what value of p, the system of equations will have no solution ?

$px - (p - 3) = -3y; \ py = p - 12x$

Sol. The given system of equations may be written as

$px + 3y - (p - 3) = 0$

$12x + py - p = 0$

Here, $a_1 = p, b_1 = 3, c_1 = -(p - 3),$

$a_2 = 12, b_2 = p, c_2 = -p$

The given system of equations will have no solution, if $\frac{a_1}{a_1} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

For it we get, $\frac{p}{12} = \frac{3}{p}$ and $\frac{3}{p} \neq \frac{p-3}{p}$

$\Rightarrow p_2 = 36 \quad \Rightarrow p = \pm 6$

When p = 6 , $\frac{3}{p} = \frac{3}{6} = \frac{1}{2}$ and $\frac{p-3}{p} = \frac{6-3}{6} = \frac{1}{2}$

So, $\frac{3}{p} = \frac{p-3}{p} = \frac{1}{2}$. Thus, p = 6 does not satisfy the equation $\frac{3}{p} \neq \frac{p-3}{p}$

When p = –6 , $\frac{3}{p} = \frac{3}{-6} = \frac{-1}{2}$

and $\frac{p-3}{p} = \frac{-6-3}{-6} = \frac{-9}{-6} = \frac{3}{2}$

So, $\frac{3}{p} \neq \frac{p-3}{p}$

Thus, p = –6 satisfy the equation $\frac{3}{p} \neq \frac{p-3}{p}$.

Thus, the given system of equations will have no solution, if p = – 6

Ex.99 Find the value of k for the following system of equations has no solution.

$(3k + 1)\, x + 3y = 2\ ;\ (k^2 + 1)\, x - 5 = -(k - 2)y$

Sol. The given system of equations may be written as

$(3k + 1)\, x + 3y - 2 = 0$

$(k^2 + 1)\, x + (k - 2)\, y - 5 = 0$

Here, $a_1 = 3k + 1, b_1 = 3, c_1 = -2,$

$a_2 = k^2 + 1, b_2 = k - 2, c_2 = -5$

Since the given system of equations has no solution therefore, we can write ;

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\Rightarrow \frac{3k+1}{k^2+1} = \frac{3}{k-2} \neq \frac{-2}{-5}$$

$$\Rightarrow \frac{3k+1}{k^2+1} = \frac{3}{k-2} \text{ and } \frac{3}{k-2} \neq \frac{2}{5}$$

So, $\frac{3k+1}{k^2+1} = \frac{3}{k-2}$

$\Rightarrow 3k^2 - 6k + k - 2 = 3k^2 + 3$

$\Rightarrow -5k = 5 \Rightarrow k = -1$

Putting k = – 1 in the equation $\frac{3}{k-2} \neq \frac{2}{5}$, we get

$\frac{3}{-1-2} = -1 \neq \frac{2}{5}$ Thus, k = –1 satisfy $\frac{3}{k-2} \neq \frac{2}{5}$

Thus, the given system of equation has no solution at k = –1

Ex.100 Determine the values of a and b so that the following system of equations has infinite number of solutions.

$3x + 4y - 12 = 0$

$2(a-b)y - (5a-1) = -(a+b)x$

Sol. The given system of equations may be written as

$3x + 4y - 12 = 0$

$(a+b)x + 2(a-b)y - (5a-1) = 0$

Here, $a_1 = 3, b_1 = 4, c_1 = -12,$

$a_2 = a + b, b_2 = 2(a-b), c_2 = -(5a-1)$

Since, the given system of equations has infinite number of solutions therefore, we get

$\frac{a_1}{a_1} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \frac{3}{a+b} = \frac{4}{2(a-b)} = \frac{-12}{-(5a-1)}$

$\Rightarrow \frac{3}{a+b} = \frac{4}{2(a-b)}$ and $\frac{4}{2(a-b)} = \frac{12}{(5a-1)}$

$\Rightarrow 6a - 6b = 4a + 4b$ and $20a - 4 = 24a - 24b$

$\Rightarrow 6a - 4a - 6b - 4b = 0$ and $20a - 24a + 24b = 4$

$\Rightarrow 2a - 10b = 0$ and $24b - 4a = 4$

$\Rightarrow a - 5b = 0$ and $6b - a = 1$

Adding the above two equations, we get

$-5b + 6b = 1 \Rightarrow b = 1$

Putting b = 1 in the equation 6b – a = 1, we get

$6 \times 1 - a = 1 \Rightarrow 6 - a = 1 \Rightarrow a = 5$

Thus, the given system of equations has infinitely many solutions at a = 5, b = 1.

Homogeneous Equations

The system of equations

$a_1x + b_1y = 0$

$a_2x + b_2y = 0$

called homogeneous equations has only solution x = 0, y = 0, when $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

(i) when $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$,

The system of equations has only one solution, and the system is consistent.

(ii) When $\frac{a_1}{a_2} = \frac{b_1}{b_2}$

The system of equations has infinitely many solutions and the system is consistent.

Ex.101 Find the value of k for which the system of equations

$$4x + 5y = 0; \; kx + 10y = 0$$

has infinitely many solutions.

Sol. The given system is of the form

$$a_1x + b_1y = 0$$

$$a_2x + b_2y = 0$$

$a_1 = 4$, $b_1 = 5$ and $a_2 = k$, $b_2 = 10$

If $\frac{a_1}{a_2} = \frac{b_1}{b_2}$, the system has infinitely many solutions.

$\Rightarrow \frac{4}{k} = \frac{5}{10} \quad \Rightarrow k = 8$

Word Problems on simultaneous Linear Equation

<u>Problems Based on Articles</u>

❖ EXAMPLES ❖

Ex.102 The coach of a cricket team buys 7 bats and 6 balls for Rs. 3800. Later, he buys 3 bats and 5 balls for Rs. 1750. Find the cost of each bat and each ball.

Sol. Let the cost of one bat be Rs. x and cost of one ball be Rs. y. Then

$7x + 6y = 3800$ ....(1)

$3x + 5y = 1750$ ....(2)

From (1) $y = \frac{3800 - 7x}{6}$

Putting $y = \frac{3800 - 7x}{6}$ in (2), we get

$3x + 5\left(\frac{3800 - 7x}{6}\right) = 1750$ ....(3)

Multiplying (3) by 6, we get

$18x + 5(3800 - 7x) = 10500$

$\Rightarrow 18x + 19000 - 35x = 10500$

$\Rightarrow -17x = 10500 - 19000$

$\Rightarrow -17x = -8500 \quad \Rightarrow x = 500$

Putting $x = 500$ in (1), we get

$7(500) + 6y = 3800$

$\Rightarrow 3500 + 6y = 3800$

$\Rightarrow 6y = 3800 - 3500$

$\Rightarrow$ $6y = 300$ $\Rightarrow$ $y = 50$

Hence, the cost of one bat = Rs. 500

and the cost of one ball = Rs. 50

Ex.103 Meena went to a bank to withdraw Rs. 2000. She asked the cashier to give Rs. 50 and Rs. 100 notes only. Meena got 25 notes in all. Find how many notes of Rs. 50 and Rs. 100 she received ?

Sol. Let the number of notes of Rs. 50 be x,

and the number of notes of Rs. 100 be y,

Then according to the question,

$$x + y = 25 \quad ....(1)$$

$$50x + 100y = 2000 \quad ....(2)$$

Multiplying (1) by 50, we get

$$50x + 50y = 1250 \quad ....(3)$$

Subtracting (3) from (2), we have

$$50y = 750 \Rightarrow y = 15$$

Putting y = 15 in (1), we get

$$x + 15 = 25 \Rightarrow x = 25 - 15 = 10$$

Hence, the number of notes of Rs. 50 was 10 and that of Rs. 100 was 15.

Ex.104 Yash scored 40 marks in a test, receiving 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer, then Yash would have scored 50 marks. How many questions were there in the test ?

Sol. Let the number of correct answers of Yash be x and number of wrong answers be y. Then according to question :

Case I. He gets 40 marks if 3 marks are given for correct answer and 1 mark is deducted for incorrect answers.

$$3x - y = 40 \quad ....(1)$$

Case II. He gets 50 marks if 4 marks are given for correct answer and 2 marks are deducted for incorrect answers.

$$4x - 2y = 50 \quad ....(2)$$

Multiplying (1) by 2, we get

$$6x - 2y = 80 \quad ....(3)$$

Subtracting (2) from (3), we get

$$2x = 30 \Rightarrow x = \frac{30}{2} = 15$$

Putting x = 15 in (1); we get

$$3 \times 15 - y = 40$$

$$\Rightarrow 45 - y = 40 \Rightarrow y = 5$$

Total number of questions = number of correct answers + number of incorrect answers.

= 15 + 5 = 20

Problems Based on Numbers

Ex.105 What number must be added to each of the numbers, 5, 9, 17, 27 to make the numbers in proportion ?

Sol. Four numbers are in proportion if

First × Fourth = Second × Third.

Let x be added to each of the given numbers to make the numbers in proportion. Then,

$(5 + x)(27 + x) = (9 + x)(17 + x)$

$\Rightarrow 135 + 32x + x^2 = 153 + 26x + x^2$

$\Rightarrow 32x - 26x = 153 - 135$

$\Rightarrow 6x = 18 \quad \Rightarrow x = 3$

Ex.106 The average score of boys in an examination of a school is 71 and that of girls is 73. The average score of the school in the examination is 71.8. Find the ratio of the number of boys to the number of girls that appeared in the examination.

Sol. Let the number of boys = x

Average score of boys = 71

Total score of boys = 71x

Let the number of girls = y

Average score of girls = 73

Total score of girls = 73 y

According to the question,

$$\text{Average score} = \frac{\text{Total average}}{\text{Total number of students}}$$

$$\Rightarrow 71.8 = \frac{71x + 73y}{x + y}$$

$\Rightarrow 71.8x + 71.8y = 71x + 73y \Rightarrow 0.8x = 1.2y$

$$\Rightarrow \frac{x}{y} = \frac{1.2}{0.8} = \frac{3}{2}$$

Hence, the ratio of the number of boys to the number of girls = 3 : 2.

Ex.107 The difference between two numbers is 26 and one number is three times the other. Find them.

Sol. Let the numbers be x and y.

Difference of two numbers is 26.

i.e., $x - y = 26$ ....(1)

One number is three times the other.

i.e., $x = 3y$ ....(2)

Putting x = 3y in (1), we get

$3y - y = 26$

$\Rightarrow$ 2y = 26 $\Rightarrow$ y = 13

Putting y = 13 in (2), we get

x = 3 × 13 = 39

Hence, the numbers are x = 39 and y = 13.

Problems Based on Ages

Ex.108 Father's age is three times the sum of ages of his two children. After 5 years his age will be twice the sum of ages of two children. Find the age of father.

Sol. Let the age of father = x years.

And the sum of the ages of his two children = y years

According to the question

Father's age = 3 × (sum of the ages of his two children)

$\Rightarrow$ x = 3y ....(1)

After 5 years

Father's age = (x + 5) years

sum of the ages of his two childrens

= y + 5 + 5 = y + 10

[Age of his each children increases by 5 years]

According to the question,

After 5 years

Father's age = 2 × (sum of ages of his two children)

$\Rightarrow$ x + 5 = 2 × (y + 10)

$\Rightarrow$ x + 5 = 2y + 20

$\Rightarrow$ x – 2y = 15 ....(2)

Putting x = 3y from (1) in (2), we get

3y – 2y = 15

$\Rightarrow$ y = 15 years

And x = 3y $\Rightarrow$ x = 3 × 15 = 45

$\Rightarrow$ x = 45 years.

Hence, father's age = 45 years

Ex.109 Five years hence, the age of Jacob will be three times that of his son. Five years ago, Jacob's age was seven times that of his son. What are their present ages.

Sol. Let the present age of Jacob and his son be x and y respectively.

Case I. After five years age of Jacob = (x + 5),

After five years the age of his son = (y + 5).

According to question

x + 5 = 3(y + 5)

$\Rightarrow$ x – 3y = 10 ....(1)

Case II. Five years ago Jacob's age = x – 5, and his son's age = y – 5. Then, according to question,

$x - 5 = 7(y - 5)$

$\Rightarrow x = 7y - 30$ ....(2)

Putting x = 7y – 30 from (2) in (1), we get

$7y - 30 - 3y = 10$

$\Rightarrow 4y = 40 \quad \Rightarrow y = 10$

Putting y = 10 in (1), we get

$x - 3 \times 10 = 10$

$\Rightarrow x = 10 + 30 \quad \Rightarrow x = 40$

Hence, age of Jacob is 40 years, and age of his son is 10 years.

Problems Based on two digit numbers

Ex.110 The sum of a two digit number and the number obtained by reversing the order of its digits is 99. If the digits differ by 3, find the number.

Sol. Let the unit's place digit be x and the ten's place digit be y.

∴ Original number = x + 10 y

The number obtained by reversing the digits = 10x + y

According to the question,

Original number + Reversed number = 99

$\Rightarrow (x + 10y) + (10x + y) = 99$

$\Rightarrow 11x + 11y = 99$

$\Rightarrow x + y = 9$

$\Rightarrow x = 9 - y$ ....(1)

Given the difference of the digit = 3

$\Rightarrow x - y = 3$ ....(2)

On putting the value of x = 9 – y from equation (1) in equation (2), we get

$(9 - y) - y = 3 \quad \Rightarrow 9 - 2y = 3$

$\Rightarrow 2y = 6 \quad \Rightarrow y = 3$

Substituting the the value of y = 3 in equation (1), we get

$x = 9 - y = 9 - 3 = 6$

Hence, the number is $x + 10y = 6 + 10 \times 3 = 36$.

Ex.111 The sum of a two-digit number and the number obtained y reversing the order of its digits is 165. If the digits differ by 3, find the number.

Sol. Let unit's place digit = x

And ten's place digit = y

∴ Original number = x + 10y

The number obtained by reversing the digits = 10 x + y

According to first condition.

The original number + Reversed number = 165

$\Rightarrow$ $x + 10y + 10x + y = 165$

$\Rightarrow$ $11x + 11y = 165$

$\Rightarrow$ $x + y = \frac{165}{11} = 15$

$\Rightarrow$ $x = 15 - y$ ....(1)

According to second condition.

The difference of the digits = 3

$\Rightarrow$ $x - y = 3$ ....(2)

Substituting $x = 15 - y$ from equation (1) in equation (2), we get

$(15 - y) - y = 3$

$\Rightarrow$ $15 - 2y = 3$

$\Rightarrow$ $2y = 12$ $\Rightarrow$ $y = 6$

Putting $y = 6$ in equation (1), we have

$x = 15 - 6$ $\Rightarrow$ $x = 9$

Hence, the original number = $x + 10y$

$= 9 + 10 \times 6 = 69$

Ex.112 The sum of the digits of a two-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the number. Find the number.

Sol. Let the ten's and the unit's digits in the number be x and y, respectively. So, the number may be written as $10x + y$.

When the digits are reversed, x becomes the unit's digit and y becomes the ten's digit.

The number can be written as $10y + x$.

According to the given condition,

$x + y = 9$ ....(1)

We are also given that nine times the number i.e., $9(10x + y)$ is twice the numbers obtained by reversing the order of the number i.e. $2(10y + x)$.

$\therefore$ $9(10x + y) = 2(10y + x)$

$\Rightarrow$ $90x + 9y = 20y + 2x$

$\Rightarrow$ $90x - 2x + 9y - 20y = 0$

$\Rightarrow$ $88x - 11y = 0$ $\Rightarrow$ $8x - y = 0$ ....(2)

Adding (1) and (2), we get

$9x = 9$ $\Rightarrow$ $x = 1$

Putting $x = 1$ in (1), we get

$y = 9 - 1 = 8$

Thus, the number is $10 \times 1 + 8 = 10 + 8 = 18$

Problems Based on Fraction

Ex.113 The sum of the numerator and denominator of a fraction is 4 more than twice the numerator. If the numerator and denominator are increased by 3, they are in the ratio 2 : 3. Determine the fraction.

Sol. Let Numerator = x and Denominator = y

$\therefore$ Fraction = $\frac{x}{y}$

According to the first condition,

Numerator + denominator = 2 × numerator + 4

$\Rightarrow$ x + y = 2x + 4

$\Rightarrow$ y = x + 4 ....(1)

According to the second condition,

$$\frac{\text{Increased numerator by 3}}{\text{Increased deno min ator by 3}} = \frac{2}{3}$$

$\Rightarrow \frac{x+3}{y+3} = \frac{2}{3}$

$\Rightarrow$ 3x + 9 = 2y + 6

$\Rightarrow$ 3x − 2y + 3 = 0 ....(2)

Substituting the value of y form equation (1) into equation (2), we get

3x − 2(x + 4) + 3 = 0

$\Rightarrow$ 3x − 2x − 8 + 3 = 0 $\Rightarrow$ x = 5

On putting x = 5 in equation (1), we get

y = 5 + 4 $\Rightarrow$ y = 9

Hence, the fraction $= \frac{x}{y} = \frac{5}{9}$

Ex.114 The sum of the numerator and denominator of a fraction is 3 less than twice the denominator. If the numerator and denominator are decreased by 1, the numerator becomes half the denominator. Determine the fraction.

Sol. Let Numerator = x and Denominator = y,

Then, fraction = $\frac{x}{y}$

According to the first condition,

Numerator + denominator = twice of the denominator – 3

$\Rightarrow$ x + y = 2y − 3

$\Rightarrow$ 2y − y = 3 + x

$\Rightarrow$ y = 3 + x ....(1)

According to the second condition,

Decreased numerator by 1 = $\frac{1}{2}$ (decreased denominator)

(x − 1) = $\frac{1}{2}$ (y − 1)

$\Rightarrow$ 2(x − 1) = y − 1

$\Rightarrow$ 2x − y = −1 + 2

$\Rightarrow$ 2x − y = 1 ....(2)

Substituting y = 3 + x in equation (2), we have

$2x - (3 + x) = 1$

$\Rightarrow 2x - x = 1 + 3 \quad \Rightarrow x = 4$

On putting x = 4 in equation (1), we get

$y = 3 + 4 \Rightarrow y = 7$

Hence, the fraction $= \frac{x}{y} = \frac{4}{7}$

Ex.115 A fraction becomes $\frac{9}{11}$, if 2 is added to both the numerator and the denominator. If 3 is added to both the numerator and the denominator it becomes $\frac{5}{6}$. Find the fraction.

Sol. Let the numerator be x and denominator be y. Then, according to the question,

Case 1 : $\frac{x+2}{y+2} = \frac{9}{11}$

$\Rightarrow 11(x + 2) = 9(y + 2)$

$\Rightarrow 11x + 22 = 9y + 18$

$\Rightarrow 11x - 9y = -4$ ....(1)

Case 2 : $\frac{x+3}{y+3} = \frac{5}{6}$

$\Rightarrow 6(x + 3) = 5(y + 3)$

$\Rightarrow 6x + 18 = 5y + 15$

$\Rightarrow 6x - 5y = -3$ ....(2)

$\Rightarrow x = \frac{5y-3}{6}$

Putting $x = \frac{5y-3}{6}$ in (1), we get

$11\left(\frac{5y-3}{6}\right) - 9y = -4$ ....(3)

Multiplying (3) by 6, we get

$11(5y - 3) - 54y = -24$

$55y - 33 - 54y = -24$

$y = 33 - 24 = 9$

Putting y = 9 in (1), we get

$11x - 9 \times 9 = -4$

$11x = -4 + 81 = 77 \Rightarrow x = 7$

Hence, the required fraction is $\frac{7}{9}$.

Ex.116 A fraction becomes $\frac{4}{5}$ if 1 is added to each of the numerator and denominator. However, if we subtract 5 from each, the fraction becomes $\frac{1}{2}$. Find the fraction.

Sol. Let the required fraction be $\frac{x}{y}$ where x be the numerator and y be the denominator.

First Case :

According to the question,

$$\frac{x+1}{y+1} = \frac{4}{5}$$

$\Rightarrow 5x + 5 = 4y + 4$

$\Rightarrow 5x - 4y = -1$

Second Case : 5 is subtracted from x and y

So, $\frac{x-5}{y-5} = \frac{1}{2}$

$\Rightarrow 2x - 10 = y - 5$

$\Rightarrow 2x - y = 5$ ....(2)

Multiplying equation (2) by 4 and equation (1) by 1, we get

$5x - 4y = -1$ ....(3)

$8x - 4y = 20$ ....(4)

Subtracting (4) from (3), we get

$-3x = -21 \quad \Rightarrow x = 7$

Substituting the value of x in (2) we get

$2 \times 7 - y = 5 \Rightarrow y = 9$

So, $\frac{x}{y} = \frac{7}{9}$

Hence, the required fraction is $\frac{7}{9}$.

Problem on Fixed Charges & Running Charges

Ex.117 A Taxi charges consist of fixed charges and the remaining depending upon the distance travelled in kilometers. If a persons travels 10 km, he pays Rs. 68 and for travelling 15 km, he pays Rs. 98. Express the above statements with the help of simultaneous equations and hence, find the fixed charges and the rate per km.

Sol. Let fixed charges of taxi = Rs. x.

And running charges of taxi = Rs. y per km.

According to the question,

Expenses of travelling 10 km = Rs. 68.

$\therefore x + 10y = 68$ ....(1)

Again expenses of travelling 15 km = Rs. 98.

$\therefore x + 15y = 98$ ....(2)

Subtracting equation (1) from equation (2), we get

$5y = 30 \quad \Rightarrow y = 6$

On putting y = 6 in equation (1), we have

$x + 10 \times 6 = 68$

$\Rightarrow$ x = 68 – 60 $\Rightarrow$ x = 8

Hence, fixed charges of taxi = x = Rs. 8 and running charges per km = y = Rs. 6.

Ex.118 A lending library has a fixed charge for the first three days and an addition charge for each day thereafter. Sarika paid Rs. 27 for a book kept for seven days. While Susy paid Rs. 21 for the book the kept for five days. Find the fixed charge and the charge for each extra day.

Sol. Let fixed charge be Rs. x.

and the charge for each extra day be Rs. y.

According to the question

Case I. Sarika paid Rs. 27 for 7 days i.e. 4 extra days.

$\therefore$ x + 4y = 27 ....(1)

Susy paid Rs. 21 for 5 days i.e. 2 extra days

$\therefore$ x + 2y = 21 ....(2)

Subtracting (2) from (1), we get

2y = 6 $\Rightarrow$ y = 3

Putting y = 3 in (1), we get

x + 4 × 3 = 27

$\Rightarrow$ x = 27 – 12 = 15

Hence, the fixed charge is Rs. 15 and the charge for each extra days is Rs. 3.

Ex.119 The taxi charges in a city consist of a fixed charge together with the charge for the distance covered. For a distance of 10 km, the charge paid is Rs. 105 and for a journey of 15 km, the charge paid is 155. What are the fixed charges and the charges per kilometer ? How much does a person have to pay for travelling a distance of 25 km ?

Sol. Let fixed charges of taxi = Rs. x

And running charges of taxi = Rs. y per km.

According to the question,

Express of travelling 10 km = Rs. 105

$\therefore$ x + 10y = 105 ....(1)

Again expenses of travelling 15 km = Rs. 155

$\therefore$ x + 15y = 155 ....(2)

$\Rightarrow$ x = 155 – 15y

Putting x = 155 – 15y in (1), we get

155 – 15y + 10y = 105

$\Rightarrow$ 155 – 5y = 105

$\Rightarrow$ –5y = 105 – 155

$\Rightarrow$ –5y = – 50 $\Rightarrow$ y = 10

Putting y = 10 in (2), we get

x + 15 × 10 = 155

$\Rightarrow$ x + 150 = 155 $\Rightarrow$ x = 155 – 150 = 5

Hence, fixed charges of taxi = x = Rs. 5 and running charges per km = y = Rs. 10 A person should pay for travelling 25 km = 5 + 25 × 10

= 5 + 250 = Rs. 255

Problems Based on Speed & Time

Ex.120 Places A and B are 100 km apart on the highway. One car stars from A and another from B at the same time. If the cars travel in the same direction at a different speed, they meet in 5 hours. If they travel towards each other, they meet in 1 hour. What are the speed of the two cars ?

Sol. Let the speed of the first car, starting from A = x km/hr.

And the speed of second car, starting from B = y km/hr.

Distance travelled by first car in 5 hours = AC = 5x

Distance travelled by second car in 5 hours = BC = 5y

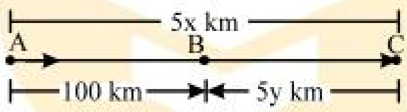
According to the question,

Let they meet at C, when moving in the same direction.

AC= AB + BC

5x = 100 + 5y

⇒ x = 20 + y ....(1)



Distance = Speed × Time

When moving in the opposite direction, let they meet at D

Distance travelled by first car in 1 hour = AD = x. Distance travelled by second car in 1 hour = BD = y

AD + BD = AB

⇒ x + y = 100 ....(2)

Substituting x = 20 + y from equation (1) in equation (2), we have

(20 + y) + y = 100

⇒ 20 + 2y = 100

⇒ 2y = 100 – 20 = 80 ⇒y = 40 km/hour

On putting y = 40 in equation (1), we get

x = 20 + 40 = 60 km/hour

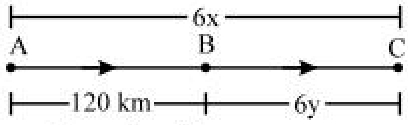
Hence, the speed of first car = 60 km/hour

and the speed of the second car = 40 km/hour.

Ex.121 Two places A and B are 120 km apart from each other on a highway. One car starts from A and another from B at the same time. If they move in the same direction, they meet in 6 hours and if they move in opposite directions, they meet in 1 hour and 12 minutes. Find the speed of the cars.

Sol. Let the speed of car starting from

A = x km/hr.



And the speed of car starting from B = y km/h.

While moving in the same-direction let they meet at C.

Distance travelled by first car in 6 hours = AC = 6x.

Distance travelled by second car in 6 hours = BC = 6y.

According to the first condition.

$AC = AB + BC$

$\Rightarrow 6x = 120 + 6y$ ($\because$ distance = Speed × Time)

$\Rightarrow x = 20 + y$ ....(1)

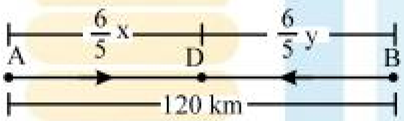
According to the second condition,

Distance travelled by first car in $\frac{6}{5}$ hours $= AD = \frac{6}{5}x$

Distance travelled by second car in $\frac{6}{5}$ hours $= BD = \frac{6}{5}y$

While moving in the opposite direction let they meet at D.

$AD + DB = AB$



$\Rightarrow \frac{6}{5}x + \frac{6}{5}y = 120$

[1 hour 12 minutes = $\frac{6}{5}$ hours]

$\Rightarrow x + y = 120 \times \frac{5}{6}$

$\Rightarrow x + y = 100$ ....(2)

Substituting x = 20 + y from equation (1) in equation (2), we get

$(20 + y) + y = 100$

$\Rightarrow 2y = 80 \Rightarrow y = 40$ km/hour

Putting y = 40 in equation (1), we have $x = 20 + 40 = 60$ km/hour

Hence, the speed of first car = 60 km/hour.

And the speed of second car = 40 km/hour.

Ex.122 A plane left 30 minutes later than the scheduled time and in order to reach the destination 1500 km away in time, it has to increase the speed by 250 km/hr from the usual speed. Find its usual speed.

Sol. Let the usual speed of plane = x km/hr.

The increased speed of the plane = y km/hr.

$\Rightarrow$ y = (x + 250) km/hour. ....(1)

Distance = 1500 km.

According to the question,

(Scheduled time) – (time in increasing the speed) = 30 minutes.

$$\frac{1500}{x} - \frac{1500}{y} = \frac{1}{2} \quad ....(2)$$

$$\frac{1500}{x} - \frac{1500}{x+250} = \frac{1}{2} \quad \left[\text{Time} = \frac{\text{Distance}}{\text{Speed}}\right]$$

$$\Rightarrow \frac{1500x + 375000 - 1500x}{x(x+250)} = \frac{1}{2}$$

$\Rightarrow$ x(x + 250) = 750000

$\Rightarrow$ $x^2 + 250x - 750000 = 0$

$\Rightarrow$ $x^2 + 1000x - 750x - 750000 = 0$

$\Rightarrow$ (x – 750) (x + 1000) = 0

$\Rightarrow$ x = 750 or x = – 1000

But speed can never be – ve

Hence, Usual speed = 750 km/hr.

<u>Problems Based on Boat & Stream</u>

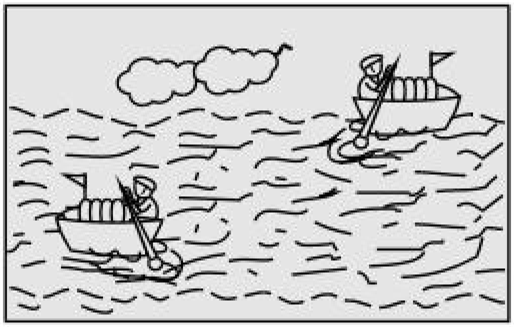
Ex.123 A boat goes 16 km upstream and 24 km downstream in 6 hours. It can go 12 km upstream and 36 km downstream in the same time. Find the speed of the boat in still water and the speed of the stream.

Sol. Let the speed of stream = y km/hr ;

speed of boat in still water = x km/hr.

And the speed of boat in upstream = (x – y) km/hr.

The speed of boat in downstream = (x + y) km/hr.



According to the question,

Time taken in going 16 km upstream + time taken in going 24 km downstream = 6 hours.

$$\Rightarrow \frac{16}{x-y} + \frac{24}{x+y} = 6 \qquad ....(1)$$

$$\left[\text{Time} = \frac{\text{Distance}}{\text{Speed}}\right]$$

Again, according to the question,

Time taken in going 12 km upstream + time taken in going 36 km downstream = 6 hours.

$$\Rightarrow \frac{12}{x-y} + \frac{36}{x+y} = 6 \qquad ....(2)$$

Let $\frac{1}{x-y} = p$, $\frac{1}{x+y} = q$

Equation (1) becomes $16p + 24q = 6$ ....(3)

Equation (2) becomes $12p + 36q = 6$ ....(4)

Multiplying equation (3) by 3 and equation (4) by 4, we get

$48p + 72q = 18$ ....(5)

$48p + 144q = 24$ ....(6)

Subtracting equation (5) from equation (6), we get

$$72q = 6 \Rightarrow q = \frac{6}{72} = \frac{1}{12}$$

Putting the value of q in equation (3), we get

$$16p + 24\left(\frac{1}{12}\right) = 6 \Rightarrow 16p + 2 = 6$$

$\Rightarrow 16p = 6 - 2 = 4 \quad \Rightarrow p = 1/4$

$$\therefore \frac{1}{x-y} = \frac{1}{4} \text{ and } \frac{1}{x+y} = \frac{1}{12}$$

$\Rightarrow x - y = 4$ ....(7)

And, $x + y = 12$ ....(8)

By adding $2x = 16 \Rightarrow x = 8$

Putting $x = 8$ in equation (7), we get

$8 - y = 4 \Rightarrow y = 8 - 4 = 4$

Hence, speed of boat in still water = 8 km/hr.

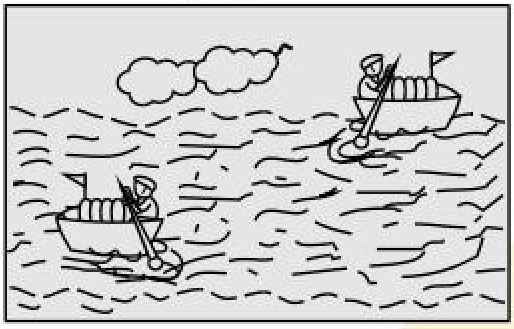
and speed of stream = 4 km/hr.

Ex.124 A boat goes 30 km upstream and 44 km downstream in 10 hours. In 13 hours, it can go 40 km up stream and 55 km down stream. Determine the speed of the stream and that of the boat.

Sol. Let the speed of the boat in still water be x km/hr and speed of the stream be y km/hr. Then the speed of the boat downstream = (x + y) km/hr, and the speed of the boat upstream = (x – y) km/hr. Also time = distance/speed.

In the first case, when the boat goes 30 km upstream, let the time taken be t_1. Then

$$t_1 = \frac{30}{(x-y)}$$



Let t_2 be the time taken by the boat to go 44 km downstream. Then $t_2 = \frac{44}{(x+y)}$. The total time taken, $t_1 + t_2$, is 10 hours Therefore, we get the equation

$$\frac{30}{(x-y)} + \frac{44}{(x+y)} = 10 \qquad \text{....(1)}$$

In the second case in 13 hours it can go 40 km upstream and 55 km downstream. We get the equation

$$\frac{40}{x-y} + \frac{55}{x+y} = 13 \qquad \text{....(2)}$$

Let $\frac{1}{(x-y)} = u$ and $\frac{1}{(x+y)} = v$ ....(3)

On substituting these values in equations (1) and (2), we get the linear pair

$30u + 44v = 10$ ....(4)

$40u + 55v = 13$ ....(5)

Multiplying equation (3) by 4 and equation (5) by 3, we get

$120u + 176v = 40$

$120u + 165v = 39$

On subtracting the two equations, we get

$11v = 1$, i.e., $v = \frac{1}{11}$

Substituting the value of v in equation (4), we get

$30u + 4 = 10$

$\Rightarrow 30u = 6 \qquad \Rightarrow u = \frac{1}{5}$

On putting these values of u and v in equation (3), we get

$$\frac{1}{(x-y)} = \frac{1}{5} \text{ and } \frac{1}{(x+y)} = \frac{1}{11}$$

i.e.,$(x - y) = 5$ and $(x + y) = 11$

Adding these equations, we get

i.e.,$2x = 16$ i.e.,$x = 8$

Subtracting the equations, we get

$2y = 6$ i.e.,$y = 3$

Hence, the speed of the boat in still water is 8 km/hr and the speed of the stream is 3 km/hr.

Ex.125 A sailor goes 8km downstream in 40 minutes and returns in 1 hour. Determine the speed of the sailor in still water and speed of the current.

Sol. We know that

$$40 \text{ minutes} = \frac{40}{60} \text{ hr} = \frac{2}{3} \text{ hr}$$

Let the speed of the sailor in still water be x km./hr and the speed of the current be y km/hr.

We know that speed $= \frac{\text{distance}}{\text{time}}$

$$\text{time} = \frac{\text{distance}}{\text{speed}}$$

speed of upstream = $(x - y)$ km/hr

and speed of downstream = $(x + y)$ km/hr

For the first case, we get

$$\frac{2}{3} = \frac{8}{x+y} \quad \left\{\text{time} = \frac{\text{distanc}}{\text{speed}}\right\}$$

$\Rightarrow 2x + 2y = 24$

$\Rightarrow x + y = 12$ ....(1)

For the second case, we get

$$1 = \frac{8}{x-y} \quad \left\{\text{time} = \frac{\text{distance}}{\text{speed}}\right\}$$

$\Rightarrow x - y = 8$ ....(2)

Adding equations (1) and (2), we get

$2x = 20 \Rightarrow x = 10$ km/hr

Substituting $x = 10$ in equation (1), we get

$10 + y = 12 \Rightarrow y = 2$ km/hr

Hence, speed of the sailor in still water and speed of the current are 10 km/hr and 2km/hr respectively.

Ex.126 A person rows downstream 20 km in 2 hours and upstream 4 km in 2 hours. Find man's speed of rowing in still water and the speed of the current.

Sol. Let man's speed of rowing in still water and the speed of the current be x km/hr and y km/

hr respectively.

Then, the upstream speed = (x – y) km/hr

and the downstream speed = (x + y) km/hr

we know that

$$\text{time} = \frac{\text{distance}}{\text{speed}}$$

First case :

$\Rightarrow 2 = \frac{20}{x+y} \quad \Rightarrow 2(x + y) = 20$

$\Rightarrow x + y = 10$ ....(1)

2nd Case :

$2 = \frac{4}{x-y} \quad \Rightarrow 2(x - y) = 4$

$\Rightarrow x - y = 2$ ....(2)

Adding (1) and (2), we get

$\Rightarrow 2x = 12 \quad \Rightarrow x = 6$

Substituting the value of x in equation (1), we get

$6 + y = 10 \quad \Rightarrow y = 4$

Hence, man's speed of rowing in still water and the speed of the current are 6 km/hr and 4 km/hr respectively.

Problems Based on Area

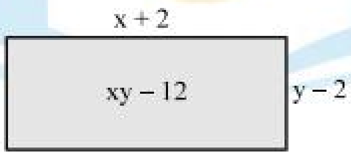
Ex.127 If in a rectangle, the length is increased and breadth reduced each by 2 metres, the area is reduced by 28 sq. metres. If the length is reduced by 1 metre and breadth increased by 2 metres, the area increases by 33 sq. metres. Find the length and breadth of the rectangle.

Sol. Let length of the rectangle = x metres

And breadth of the rectangle = y metres

Area = length × breadth = xy sq. metres

Case 1: As per the question



Increased length = x + 2

Reduced breadth = y – 2

Reduced area = (x + 2) (y – 2)

Reduction in area = 28

Original Area – Reduced area = 28

$xy - [(x + 2)(y - 2)] = 28$

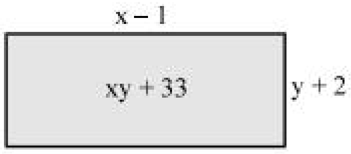
$\Rightarrow xy - [xy - 2x + 2y - 4] = 28$

$\Rightarrow xy - xy + 2x - 2y + 4 = 28$

$\Rightarrow 2x - 2y = 28 - 4 = 24$

$\Rightarrow$ $x - y = 12$ ....(1)

Case 2 :



Reduced length = $x - 1$

$\Rightarrow$ Increased breadth = $y + 2$

$\Rightarrow$ Increased area = $(x - 1)(y + 2)$

Increase in area = 33

$\therefore$ Increased area – original area = 33

$\Rightarrow$ $(x - 1)(y + 2) - xy = 33$

$\Rightarrow$ $xy + 2x - y - 2 - xy = 33$

$\Rightarrow$ $2x - y = 33 + 2 = 35$

$\Rightarrow$ $2x - y = 35$ ....(2)

Subtracting equation (1) from equation (2), we get

$x = 23$

Substituting the value of x in equation (1), we get

$2x - y = 12$

$\Rightarrow$ $y = 23 - 12 = 11$

$\Rightarrow$ Length = 23 metres.

$\Rightarrow$ Breadth = 11 metres.

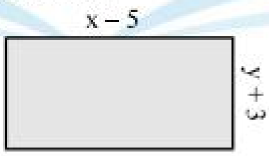
Ex.128 The area of a rectangle gets reduced by 9 square units if its length is reduced by 5 units and breadth is increased by 3 units. If we increase the length by 3 units and the breadth by 2 units, the area increases by 67 square units. Find the dimensions of the rectangle.

Sol. Let the length of rectangle be x units

and the breadth of the rectangle be y units

Area of the rectangle = xy

Case 1 : According to the first condition,



Reduced length = $x - 5$

Increased breadth = $y + 3$

Reduced area = $(x - 5)(y + 3)$

Reduction in area = 9

Original area – Reduced area = 9

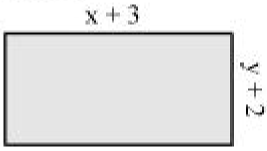
$xy - [(x - 5)(y + 3)] = 9$

$\Rightarrow$ $xy - [xy + 3x - 5y - 15] = 9$

$\Rightarrow$ xy – xy – 3x + 5y + 15 = 9

$\Rightarrow$ 3x – 5y = 6 ....(1)

Case 2. According to the second condition,



Increased length = x + 3

Increased breadth = y + 2

Increased area = (x + 3) (y + 2)

Increase in area = 67

Increased area – Original area = 67

$\Rightarrow$ (x + 3) (y + 2) – xy = 67

$\Rightarrow$ xy + 2x + 3y + 6 – xy = 67

2x + 3y = 61 ....(2)

On solving (1) and (2), we get

x = 17 units and and y = 9 units

Hence, length of rectangle = 17 units,

and breadth of rectangle = 9 units.

<u>Problems Based on Geometry</u>

Ex.129 The larger of two supplementary angles exceeds the smaller by 18 degrees. Find them.

Sol. Let the angles be x and y. Then according to the question.

x + y = 180 ....(1)

and x = y + 18 ....(2)

Putting x = y + 18 from (2) in (1), we get

y + 18 + y = 180

2y = 180 – 18

$\Rightarrow$ 2y = 162 $\Rightarrow$ y = 81

Putting y = 81 in (2), we get

x = 81 + 18 = 99

Hence, angles are x = 99º and y = 81º.

Ex.130 In a ΔABC, $\angle C = 3\angle B = 2(\angle A + \angle B)$. Find the three angles.

Sol. $\angle C = 2(\angle A + \angle B)$ ....(1)

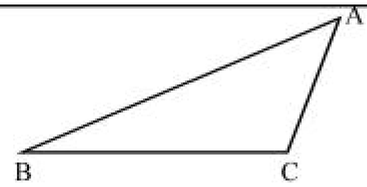
(given)

Adding $2\angle C$ on both sides of (1), we get

$\angle C + 2\angle C = 2(\angle A + \angle B) + 2\angle C$

$\Rightarrow 3\angle C = 2(\angle A + \angle B + \angle C)$

$\Rightarrow \angle C = \frac{2}{3} \times 180^\circ = 120^\circ$



Again $\angle C = 3\angle B$ (given)

$120^\circ = 3\angle B \Rightarrow \angle B = \frac{120^\circ}{3} = 40^\circ$

But $\angle A + \angle B + \angle C = 180^\circ$

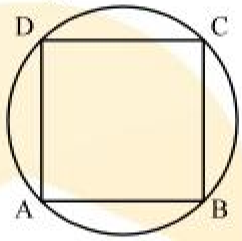
$\angle A + 40^\circ + 120^\circ = 180^\circ$

$\Rightarrow \angle A = 180^\circ - 40^\circ - 120^\circ = 20^\circ$

$\angle A = 20^\circ, \angle B = 40^\circ, \angle C = 120^\circ$

Ex.131 Find a cyclic quadrilateral ABCD, $\angle A = (2x + 4)^\circ$, $\angle B = (y + 3)^\circ$, $\angle C = (2y + 10)^\circ$ and $\angle D = (4x - 5)^\circ$. Find the four angles.

Sol. $\angle A = (2x + 4)^\circ$, and $\angle C = (2y + 10)$;



But $\angle A + \angle C = 180^\circ$ (Cyclic quadrilateral)

$\Rightarrow (2x + 4)^\circ + (2y + 10)^\circ = 180^\circ$

$\Rightarrow 2x + 2y = 166^\circ$

Also $\Rightarrow \angle B = (y + 3)^\circ, \angle D = (4x - 5)^\circ$

But $\angle B + \angle D = 180^\circ$ (Cyclic quadrilateral)

$\Rightarrow (y + 3)^\circ + (4x - 5)^\circ = 180^\circ$

$\Rightarrow 4x + y = 182^\circ$

On solving (1) and (2), we get $x = 33^\circ, y = 50^\circ$

$\angle A = (2x + 4)^\circ = (66 + 4)^\circ = 70^\circ$

$\angle B = (y + 3)^\circ = (50 + 3)^\circ = 53^\circ$

$\angle C = (2y + 10)^\circ = (100 + 10)^\circ = 110^\circ$,

$\angle D = (4x - 5)^\circ = (4 \times 33 - 5)^\circ = 127^\circ$

$\angle A = 70^\circ, \angle B = 53^\circ, \angle C = 110^\circ, \angle D = 127^\circ$

Ex.132 The area of a rectangle remains the same if the length is decreased by 7 dm and breadth is increased by 5 dm. The area remains unchanged if its length is increased by 7 dm and and breadth decreased by 3 dm. Find the dimensions of the rectangle.

Sol. Let the length and breadth of a rectangle be x and y units respectively. So, area = (xy) sq. units.

First Case : Length is decreased by 7 dm and breadth is increased by 5 dm.

According to the question,

$xy = (x - 7)(y + 5)$

$\Rightarrow$ xy = xy + 5x – 7y – 35

$\Rightarrow$ 5x – 7y – 35 = 0 .... (1)

Second Case : Length is increased by 7 dm and breadth is decreased by 3 dm.

Here, area also remains same

so, we get

xy = (x + 7) (y – 3) = xy – 3x + 7y – 21

$\Rightarrow$ 3x – 7y + 21 = 0 ....(2)

So, the system of equations becomes

$\Rightarrow$ 5x – 7y – 35 = 0 ....(3)

3x – 7y + 21 = 0 ....(4)

Subtracting equation (4) from (3), we get

2x – 56 = 0

$\Rightarrow$ 2x = 56 $\Rightarrow$ x = 28 dm

Substituting x = 28 in equation (3), we get

5 × 28 – 7y = 35

$\Rightarrow$ 7y = 105 $\Rightarrow$ y = 15 dm

Hence, length and breadth of the rectangle are 28 and 15 dm respectively.

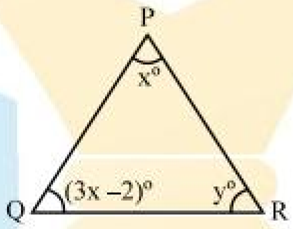
Ex.133 In a triangle PQR, $\angle P = x^\circ$, $\angle Q = (3x - 2)^\circ$, $\angle R = y^\circ$, $\angle R - \angle Q = 9^\circ$, Determine the three angles.

Sol. It is given that

$\angle P = x^\circ$, $\angle Q = (3x - 2)^\circ$,

$\angle R = y^\circ$ and

$\angle R - \angle Q = 9^\circ$



We know that the sum of three angles in a triangle is 180°.

So, $\angle P + \angle Q + \angle R$ = x + 3x – 2 + y = 180

$\Rightarrow$ 4x + y = 182 ....(1)

It is also given that

$\angle R - \angle Q = 9^\circ$

or y – (3x – 2) = 9

$\Rightarrow$ y – 3x + 2 = 9

$\Rightarrow$ 3x – y = –7 ....(2)

Adding equation (1) with (2), we get

7x = 175 $\Rightarrow$ x = 25

Substituting x = 25 in equation (2), we get

3 × 25 – y = – 7

$\Rightarrow$ y = 75 + 7 = 82

Thus, P = x° = 25°

Q = (3x – 2)° = (3 × 25 – 2)° = 73°

and R = y = 82°

Important Points To Be Remembered

1. An equation of the form $ax + by + c = 0$ is linear in two variables x and y. For all a and b are the coefficients of x and y respectively such that $a, b \in R$ and $a \neq 0$, $b \neq 0$

2. The graph of a linear equation in two variables is a straight line

3. A linear equation in two variables has infinitely many solutions

4. Slope of the line $ax + by + c = 0$ is $- a/b$

5. Equation of x-axis is $y = 0$ and equation of y-axis is $x = 0$

6. The graph of the line $x = a$ is parallel to y-axis

7. The graph of the line $y = b$ is parallel to x-axis.

8. Every point on the graph of a linear equation in two variables is a solution of the equation.

9. A pair of linear equations in two variables x and y can be represented algebraically as follows :

   $a_1x + b_1y + c_1 = 0$

   $a_2x + b_2y + c_2 = 0$

   where a_1, a_2, b_1, b_2, c_1, c_2 are real number such that $a_1^2 + b_1^2 \neq 0$, $a_2^2 + b_2^2 \neq 0$.

10. Graphically or geometrically a pair of linear equations

    $a_1x + b_1y + c_1 = 0$

    $a_2x + b_2y + c_2 = 0$

    in two variables represents a pair of straight lines which are

    (i) intersecting, if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

    (ii) parallel, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

    (iii) coincident , if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

11. A pair of linear equations in two variables can be solved by the :

    (i) Graphical method

    (ii) Algebraic method

12. To solve a pair of linear equations in two variables by Graphical method, we first draw the lines represented by them.

    (i) If the pair of lines intersect at a point, then we say that the pair is consistent and the coordinates of the point provide us the unique solution.

    (ii) If the pair of lines are parallel, then the pair has no solution and is called inconsistent pair of equations.

    (iii) If the pair of lines are coincident, then it has infinitely many solutions each point on the line being of solution. In this case, we say that the pair of linear equations is consistent with infinitely many solutions.

13. To solve a pair of linear equation in two variables algebraically, we have following methods :

    (i) Substitution method

    (ii) Elimination method

    (iii) Cross-multiplication method

14. If $a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

is a pair of linear equation in two variable x and y such that :

(i) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, then the pair of linear equations is consistant with a unique solution.

(ii) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, then the pair of linear equations is inconsistent.

(iii) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, then the pair of linear equations is consistent with infinitely many solutions.

EXERCISE # 1

Very short answer type Questions

In each of the following verify whether the given value of the x is a solution or not :

Q.1 $\frac{x}{3}+\frac{x}{4}=8,\ x=12$

Q.2 $(4x+7)-2=3x+1,\ x=-4$

Q.3 $\frac{5x+4}{4}-\frac{3x-2}{2}=5,\ x=\frac{1}{2}$

Q.4 $2x-4+1=3x-6,\ x=3$

Q.5 Solve : $\frac{6}{x}+11=\frac{3}{x}+12$

Q.6 If $2x-8=8$, then find the value of x^2+x-70.

Q.7 For each of the following, state the quadrant in which the point lies.

(i) (3, 3) (ii) (–3, 2) (iii) (2, –4)

(iv) (–1, –2) (v) (–5, –5) (vi) (5, 3).

Q.8 Draw the graph of y = x. Show that point (4, 4) is on the graph.

Q.9 Express x in terms of y, given that 3x + 4y = 6. Check whether the point (3, 2) is on the given line.

Q.10 Draw the graph of y = – 2x. Show that the point (2, –5) is not on the graph.

Short answer type Questions

Q.11 Indicate the quadrants in which the following points lie and plot them on a graph paper.

(i) (–2, 0) (ii) (0, 1) (iii) (–2, –3)

Q.12 Draw the graph of (i) x = 3 (ii) y = –2.

Q.13 Find the value of k, if line represented by the equation 2x – ky = 9 passes through the point (–1, –1)

Q.14 Express x in terms of y, it is being given that 7x – 3y = 15. Check if the line represented by the given equation intersects the y-axis at y = – 5.

Q.15 Draw the graph of 6 – 1.5x = 0.

Q.16 The following observed values of x and y are thought to fulfil the law y = ax + b. Find the values of a and b.

x	1	2	–3	0	5
y	12	19	–16	5	–30

Q.17 Show that the points A (1, 2), B (–1, –16), C(0, –7) are on the graph y = 9x – 7.

Q.18 Find the point of intersection of the line represented by the equation $7x + y = -2$ with x-axis. Check whether the point (2, 1) is a solution set of the given equation.

Q.19 Express y in terms of x, given that $2x - 5y = 7$. Check whether the point (–3, –2) is on the given line.

Q.20 Verify whether $x = 2, y = 1$ and $x = 1$ and $y = 2$ are the solutions of the linear equation $2x + y = 5$. Find two more solutions.

Q.21 Draw the graph of the equation $4x - 5y = 20$ and check whether the points (3, 1) and and (5, 0) lie on the graph.

Q.22 Draw the graph of the equation $3x + 4y = 14$ and check whether $x = 1$ and $y = 2$ is a solution or not.

Q.23 Draw the graph of the equation $2y + x = 7$ and determine from the graph whether $x = 3$ and $y = 2$ is a solution.

Q.24 Draw the graph of the equation $3x + 7y = 10$. Find whether $x = 1$ and $y = 1$ is the solution of these equation.

Q.25 Draw the graph of the following pair of linear equations.

(i) $3x + y - 5 = 0$
$2x - y - 5 = 0$

(ii) $2x + y = 8;$
$3x - 2y = 12$

(iii) $2x - y - 5 = 0$
$x - y - 3 = 0$

(iv) $2x - \quad y - 4 = 0$
$x + y + 1 = 0$

(v) $2x + y - 11 = 0$
$x - y - 1 = 0$

Q.26 Solve graphically the following system of linear equations.

(i) $2x - 3y + 13 = 0$
$3x - 2y + 12 = 0$

(ii) $x + y = 3$
$3x - 2y = 4$

Q.27 Solve graphically the following system of equations. Also, shade the region bounded by these lines and x-axis.

(i) $2x + 3y = -5$
$3x - 2y = 12$

(ii) $3x + 2y = 9$
$2x + y = 5$

Q.28 Solve the following system of equations graphically and shade the region bounded by these lines and y-axis.

(i) $3x + y - 11 = 0$
$x - y - 1 = 0$

(ii) $x + 2y - 7 = 0$
$2x - y - 4 = 0$

(iii) $x - y = 1$
$2x + y = 8$

Q.29 Solve the following system of equations graphically. Also, find out the points, where these lines meet the x-axis.

$x - 2y = 1$ $\qquad$ $2x + y = 7$

Q.30 Solve the following system of equations graphically. Also, find out the points, where these lines meet the y-axis.

(i) $x + 2y - 7 = 0$ (ii) $2x + y = 8$

$2x - y + 1 = 0$ $x + 1 = 2y$

(iii) $2x + 3y = 12$

$2y - 1 = x$

Q.31 Draw the graphs of the following systems of equations, state whether it is consistent (dependent), consistent (independent) or inconsistent :

(i) $x + y = 7$ (ii) $2x + 4y = 7$

$2x - 3y = 9$ $3x + 6y = 10$

(iii) $2x + 3y - 12 = 0$ (iv) $3x - 5y + 4 = 0$

$2x + 3y - 6 = 0$ $9x = 15y - 12$

(v) $x + 3y = 1$ (vi) $x + 4y = 7$

$2x + 6y = 2$ $2x - y = 5$

Q.32 Solve the following pair of linear equations by the substitution method :

(i) $7x - 15y = 2$ (ii) $2x + 3y = 9$

$x + 2y = 3$ $4x + 6y = 18$

(iii) $x + 2y = 5$ (iv) $0.2x + 0.3y = 1.3$

$2x + 3y = 8$ $0.4x + 0.5y = 2.3$

(v) $x + 2y = -1$ (vi) $3x - 5y + 1 = 0$

$2x - 3y = 12$ $x - y + 1 = 0$

Q.33 Solve the following equations by the method of elimination by equating the coefficients.

(i) $12x + 5y = 17$; $7x - y = 6$

(ii) $17x + 12y = -2$; $15x + 8y = 6$

(iii) $23x + 17y = 6$; $39x - 19y = 58$

(iv) $43x - 37y = 31$; $13x + 23y = -59$

(v) $0.4x + 3y = 1.2$, $7x - 2y = \frac{17}{6}$

(vi) $(a + 2b)\,x + (2a - b)\,y = 2$,

$(a - 2b)\,x + (2a + b)\,y = 3$

(vii) $a(x + y) + b(x - y) = a^2 - ab + b^2$,

$a(x + y) - b(x - y) = a^2 + ab + b^2$

Q.34 Solve the following system of equations by cross-multiplication method :

(i) $3x - 4y = 7$ (ii) $3x - 5y = 1$

$5x + 2y = 3$ $7x + 2y = 16$

(iii) $2x + 3y = 8$ (iv) $3x - 4y = 1$

$3x + 2y = 7$ $4x - 3y = 6$

(v) $3x - 4y = 10$
$4x + 3y = 5$

(vi) $2x - 6y + 10 = 0$
$3x - 9y + 15 = 0$

(vii) $\frac{2}{x-1} + \frac{3}{y+1} = 2$

$\frac{3}{x-1} + \frac{2}{y+1} = \frac{13}{6}$, $x \neq 1$, $y \neq -1$

(viii) $\frac{5}{x+y} - \frac{2}{x-y} = -1$

$\frac{15}{x+y} + \frac{7}{x-y} = 10$; $x + y \neq 0$, $x - y \neq 0$

Q.35 For what value of k will the following system of equations have a unique solution.

(i) $2x + ky = 1$ and $3x - 5y = 7$

(ii) $x - 2y = 3$ and $3x + ky = 1$

(iii) $2x + 5y = 7$ and $3x - ky = 5$

Q.36 For what value of k will the following system of equations have infinitely many solutions.

(i) $7x - y = 5$ and $21x - 3y = k$

(ii) $5x + 2y = k$ and $10x + 4y = 3$

(iii) $kx + 4y = k - 4$ and $16x + ky = k$

Q.37 Find the conditions so that the following systems of equations have infinitely many solutions.

(i) $3x - (a + 1)y = 2b - 1$ and $5x + (1 - 2a)y = 3b$, find a and b.

(ii) $2x + 3y = 7$ and $(p + q)x + (2p - q)y = 3(p + q + 1)$, find p and q.

(iii) $2x - (2a + 5)y = 5$ and $(2b + 1)x - 9y = 15$, find a and b.

Q.38 Show that the following systems of equation are inconsistent.

(i) $x - 2y = 6$
$3x - 6y = 0$

(ii) $2y - x = 9$
$6y - 3x = 21$

(iii) $2x - y = 9$
$4x - 2y = 15$

Q.39 For what value of k the following systems of equations have no solution.

(i) $8x + 5y = 9$ and $kx + 10y = 8$

(ii) $x - 4y = 6$ and $3x + ky = 5$

(iii) $kx - 5y = 2$ and $6x + 2y = 7$

(iv) $4x + 6y = 11$ and $2x + ky = 7$

(v) $2x + ky = 11$ and $5x - 7y = 5$

Q.40 Solve the following pair of linear equations

(i) $\frac{1}{2x} - \frac{1}{y} = -1.$

$\frac{1}{x} + \frac{1}{2y} = 8$, $x \neq 0$, $y \neq 0$

(ii) $\frac{2}{x} + \frac{2}{3y} = \frac{1}{6}$, $\frac{3}{x} + \frac{2}{y} = 0$; $x \neq 0$ $y \neq 0$ and hence, find a for which $y = ax - 4$.

(iii) $\frac{1}{7x} + \frac{1}{6y} = 3$,

$\frac{1}{2x} - \frac{1}{3y} = 5$; $x \neq 0$ $y \neq 0$

(iv) $\frac{m}{x} - \frac{n}{y} = a$,

$px - qy = 0$; $x \neq 0$ $y \neq 0$

(v) $\frac{2}{y} + \frac{3}{x} = \frac{7}{xy}$,

$\frac{1}{y} + \frac{9}{x} = \frac{11}{xy}$; $x \neq 0$, $y \neq 0$

(vi) $\frac{xy}{x+y} = \frac{6}{5}$,

$\frac{xy}{y-x} = 6$; $xy \neq 0$, $y \neq 0$

(vii) $x + y = 5xy$

$3x + 2y = 13\,xy$

Q.41 Solve the following pair of linear equations.

(i) $3(a + 3b) = 11\,ab$,

$3(2a + b) = 7ab$

(ii) $5x + \frac{4}{y} = 9$,

$7x - \frac{2}{y} = 5$; $y \neq 0$

(iii) $\frac{3}{x} + 4y = 7$,

$\frac{-2}{x} + 7y = \frac{19}{3}$; $x \neq 0$

(iv) $\frac{5}{x+1} - \frac{2}{y-1} = \frac{1}{2}$

$\frac{10}{(x+1)} + \frac{2}{(y-1)} = \frac{5}{2}$, $x \neq -1$, $y \neq 1$

(v) $\frac{6}{x+y} = \frac{7}{x-y} + 3$,

$\frac{1}{2(x+y)} = \frac{1}{3(x-y)}$, $x + y \neq 0$ $x - y \neq 0$

(vi) $ax + by = c$,

$bx + ay = 1 + c$

(vii) $ax + by = 1$,

$bx + ay = \frac{(a+b)^2}{a^2+b^2} - 1$

(viii) $\frac{148}{x} + \frac{231}{y} = \frac{527}{xy}$;

$$\frac{231}{x} + \frac{148}{y} = \frac{610}{xy};\ x \neq 0,\ \ y \neq 0$$

Q.42 2 tables and 3 chairs together cost Rs. 2000 whereas 3 tables and 2 chairs together cost Rs. 2500. Find the total cost of 1 table and 5 chairs.

Q.43 3 bags and 4 pens together cost Rs. 257 whereas 4 bags and 3 pens together cost Rs. 324. Find the total cost of 1 bag and 10 pens.

Q.44 Reena has pens and pencils which together are 40 in number. If she has 5 more pencils and 5 less pens, then number of pencils would become 4 times the number of pens. Find the original number of pens and pencils.

Q.45 5 pens and 6 pencils together cost Rs. 9.00, and 3 pens and 2 pencils cost Rs. 5.00. Find the cost of 1 pen 1 pencil.

Q.46 Two numbers differ by 2 and their product is 360. Find the numbers.

Q.47 Two numbers differ by 4 and their product is 192. Find the numbers.

Q.48 Five years hence, father's age will be three times the age of his son. Five years ago, father was seven times as old as his son Find their present ages.

Q.49 The age of father is 4 times the age of his son. 5 years hence, the age of father will be three times the age of his son. Find their present ages.

Q.50 The sum of a two-digit number and the number formed by interchanging its digits is 110. If 10 is subtracted from the first number, the new number is 4 more than 5 times the sum of the digits in the first number. Find the first number.

Q.51 The sum of a two-digit number and the number formed by interchanging the digits is 132. If 12 is added to the number, the new number becomes 5 times the sum of the digits. Find the number.

Q.52 A two-digit number is 3 more than 4 times the sum of its digits. If 18 is added to the number, the digits are reversed. Find the number.

Q.53 A two-digit number is 4 times the sum of its digits. If 18 is added to the number, the digit are reversed. Find the number.

Q.54 The denominator of a fraction is 4 more than twice the numerator. When both the numerator and denominator are decreased by 6, then the denominator becomes 12 times the numerator. Determine the fraction.

Q.55 If 2 be added to the numerator of a fraction, it reduces to 1/2 and if 1 be subtracted from the denominator, it reduces to 1/3. Find the fraction.

Q.56 The sum of the numerator and denominator of a fraction is 18. If the denominator is increased by 2, the fraction reduces to 1/3. Find the fraction.

Q.57 The area of a rectangle gets reduced by 80 sq. units if its length is reduced by 5 units and the breadth is increased by 2 units. If we increase the length by 10 units and decrease the breadth by 5 units, the area is increased by 50 sq. units. Find the length and breadth of the rectangle.

Q.58 The length of a rectangle exceeds its width by 8 cm and the area of the rectangle is 240 sq. cm. Find the dimensions of the rectangle.

Q.59 The side of a square exceeds the side of another square by 4 cm and the sum of the area of the two squares is 400 sq. cm. Find the dimensions of the squares.

Q.60 The area of a rectangle gets reduced by 8 sq. metres, if its length is reduced by 5 metres and width is increased by 3 metres. If we increase the length by 3 metres and breadth by 2 metres, the area is increased by 74 sq. metres. Find the length and breadth of the rectangle.

Q.61 In a triangle, the sum of two angles is equal to the third. If the difference between them is 50°, find the angles.

Q.62 In ΔABC, $\angle A = y°$, $\angle B = (y - 9)°$, $\angle C = x°$. Also $\angle B - \angle C = 48°$, find the three angles.

Q.63 The largest angles of the triangle is twice the sum of the other two, the smallest is one-sixth of the largest. Find the angles in degrees.

Q.64 Find the four angles of the following cyclic quadrilateral ABCD in which

(i) $\angle A = 5x°$, $\angle B = 9x° + 2y°$, $\angle C = x° + 8y°$ and $\angle D = x° + 4y°$.

(ii) $\angle A = (2x + y)°$, $\angle B = 2(x + y)°$, $\angle C = (3x + 2y)°$, $\angle D = (4x - 2y)°$.

Long answer type Questions

Q.65 The difference between two numbers is 642. When the greater is divided by the smaller, the quotient is 8 and remainder is 19. Find the numbers.

Q.66 The ages of Ram and Mohan are in ratio 2 : 3. If sum of their ages is 65, find the difference of their ages.

Q.67 The difference between two numbers is 1365. When larger is divided by the smaller one, the quotient is 6 and remainder is 15. Find the numbers.

Q.68 The denominator of a fraction is 1 more than its numerator. If 1 is subtracted from both the numerator and denominator, the fraction becomes 1/2. Find the fraction.

Q.69 The measures of angles of a triangle in degrees are x, x + 12 and x + 27. Find the measure of angles.

Q.70 Of the three angles of a triangle the second is one-third the first and third is 26° more than the first. How many degrees are there in each angle ?

Q.71 In a factory, women are 35% of all the workers, the rest of the workers being men. The number of men exceeds that of women by 252. Find the total number of workers in the factory.

Q.72 A total of 1400 kg of potatoes were sold in three days. On the first day 100 kg less potatoes were sold than on the second day and on the third day, 3/5 of the amount sold on the first day. How many kilograms of potatoes were sold on each day ?

Q.73 The sum of a certain even number and the fourth even number after it is 68. Find the numbers.

Q.74 Solve for x

$$\frac{4x+17}{18} - \frac{13x-2}{17x-32} + \frac{x}{3} = \frac{7x}{12} - \frac{x+16}{36}$$

Q.75 The coach of a cricket team buys 3 bats and 6 balls for Rs. 3900. Later, she buys another bat and 2 balls of the same kind for Rs. 1300. Represent this situation algebraically and geometrically. [**Hint :** $3x + 6y = 3900$ and $x + 2y = 1300$]

Q.76 Gloria is walking along the path joining (–2, 3) and (2, –2) while Suresh is walking along the path joining (0, 5) and (4, 0). Represent this situation graphically.

Q.77 Shonam purchased two pencils and three erasers for Rs. 9. Gita also purchased one pencil and two erasers for Rs. 5 from the same shop. Find the cost of one pencil and one eraser.

Q.78 John purchased 2 tables and 12 chairs for Rs. 6800 for his house in Janakpuri. He also purchased 3 tables and 15 chairs for his house in Green Park for Rs. 9000. How much John paid for each table and each chair ?

Q.79 Solve the following system of equations by cross-multiplication method :

(i) $ax + by = a^2$
$bx + ay = b^2$

(ii) $\frac{2x}{a} + \frac{y}{b} = 2.$
$\frac{x}{a} - \frac{y}{b} = 4;\ a \neq 0,\ b \neq 0$

(iii) $x - y = a + b$
$ax + by = a^2 - b^2$

(iv) $\frac{x}{a} + \frac{y}{b} = 2,$
$ax - by = a^2 - b^2;\ a \neq 0,\ b \neq 0$

(v) $x + y = a + b$
$ax - by = a^2 - b^2$

Q.80 Three tables and five chairs together cost Rs. 3555, while seven tables and four chairs together cost Rs. 5788. Find the cost of a table and that of a chair.

Q.81 Fifty nine pens and forty seven pencils together cost Rs. 513, while forty seven pens and fifty nine pencils together cost Rs. 441. Find the cost of a pen and that of a pencil.

Q.82 Two numbers differ by 4 and their product is 96. Find the numbers.

Q.83 Two numbers are in the ratio of 3 : 5, If 5 is subtracted from each of the number, they become in ratio of 1 : 2. Find the numbers.

Q.84 Two numbers are in the ratio of 3 : 4. If 8 is added to each number, they become in the ratio of 4 : 5. Find the numbers.

Q.85 The sum of two numbers is 15 and sum of their reciprocals is $\frac{3}{10}$. Find the numbers.

Q.86 The sum of two numbers is 16 and the sum of their reciprocals is $\frac{1}{3}$. Find the numbers.

Q.87 Two numbers are in the ratio of 5 : 6. If 8 is subtracted from each of the numbers, they become in the ratio of 4 : 5. Find the numbers.

FILL IN THE BLANKS

Q.88 The point of intersection of two lines represented by $5x - 3y = 2$ and $2x + y = 3$ is ....

Q.89 If the point P lies on the line $y = 7$ and has its abscissa equal to -2, then its coordinates are ....

Q.90 The locus of a point whose x coordinate is always 5 is the equation ....

Q.91 The slope of the straight line $y - 4 = 0$ is ....

Q.92 The y intercept of the straight line $4y - 5x = 11$ is ....

TRUE/FALSE TYPE QUESTIONS

Q.93 The straight line $y = 2x + 5$ will not meet the x-axis.

Q.94 The slope of the graph of an increasing function can never be negative.

Q.95 The positive side of x-axis lies in the region represented by the inequation $y \leq x$.

Q.96 If $x - 1 > 0$ and $x - 5 < 0$, then in that case $1 < x < 5$.

Q.97 There is no common point to the graphs of the functions $y = 2x + 3$; $y = -2x + 3$

Q.98 The graph of $x = 2$ will never meet the y-axis.

Q.99 The graph of $y = 5$ will meet the x-axis.

Q.100 If the graph of the function $y = mx + c$ passes through origin, then 'm' must be zero.

Q.101 The slope of the straight line $5x + y = 2$ is 5.

Q.102 If x is a whole number satisfying the inequality $-2 < x < 3$, then there exist four different values for x.

MATCH THE COLUMN TYPE QUESTIONS

Q.103 Match the column

Column 1	**Column 2**
(i) $\frac{5}{2}x + 3 = \frac{21}{2}$	(A) $11\frac{1}{2}$
(ii) $\frac{x-1}{3} + \frac{x-4}{5} = 5$	(B) 3
(iii) $x + 4 = 2x$	(C) 4

Q.104 Solution of the equation

Column 1	**Column 2**
(i) $\frac{2x-3}{5} + \frac{x+3}{4} = \frac{4x+1}{7}$ is	(A) 7
(ii) $\frac{7x-1}{4} - \frac{1}{3}\left[2x - \frac{1-x}{2}\right] = \frac{19}{3}$	(B) $-\frac{41}{11}$
(iii) $\frac{4x+5}{6} - \frac{2(2x+7)}{3} = \frac{3}{2}$, is	(C) $\frac{1}{11}$

Q.105 Solution of the equation

Column 1	**Column 2**
(i) $\frac{2y-3}{5}+\frac{y-3}{4}=\frac{4y+1}{7}$	(A) $\frac{8}{5}$
(ii) $\frac{3}{x-1}+\frac{4}{x-2}=\frac{7}{x-3}$, $x \neq 1, 2, 3$ is	(B) $\frac{209}{11}$
(iii) $(x + 1)(2x + 1) = (x + 3)(2x + 3) - 14$, is	(C) 1

EXERCISE # 2

SINGLE CHOICE QUESTIONS

Q.1 If $p + q = 1$ and the ordered pair (p, q) satisfies $3x + 2y = 1$, then it also satisfies–

(A) $3x + 3y = 3$ (B) $5x + 4y = 4$

(C) $5x + 5y = 4$ (D) None of these

Q.2 The age of a father is twice that of the elder son. Ten years hence the age of the father will be three times that of the younger son. If the difference of ages of the two sons is 15 years, the age of the father is–

(A) 100 years (B) 70 years

(C) 60 years (D) 50 years

Q.3 The equation $\frac{(x+2)(x-5)}{(x-3)(x+6)} = \frac{x-2}{x+4}$ has –

(A) 3 roots (B) 2 roots

(C) 1 root (D) no root

Q.4 The cost of 9 chairs and 3 tables is Rs 306, while the cost of 8 chairs and 2 tables is Rs. 246. Then the cost of 6 chairs and 1 table is–

(A) Rs. 164 (B) Rs. 165

(C) Rs. 166 (D) Rs. 186

Q.5 If $2^x - 2^{x-1} = 4$, then x^x is equal to –

(A) 7 (B) 3

(C) 27 (D) none of these

Q.6 The value of x satisfying the equation $x^2 + p^2 = (q - x)^2$ is –

(A) $\frac{p^2 - q^2}{2}$ (B) $\frac{q^2 - p^2}{2q}$

(C) $\frac{q^2 - p^2}{2}$ (D) $\frac{p^2 - q^2}{2q}$

Q.7 The equations $3x - 5y + 2 = 0$ and $6x + 4 = 10y$ have–

(A) No solution (s)

(B) A single solution

(C) Two solutions

(D) An infinite number of solutions

Q.8 If $2x^2 + xy - 3y^2 + x + ay - 10 = (2x + 3y + b)(x - y - 2)$, the value of a and b are –

(A) 11 and 5 (B) 1 and –5

(C) –1 and –5 (D) – 11 and 5

Q.9 Given the point (–2, 5). Then the point symmetrical about the origin for the given point is –

(A) (2, –5)   (B) (2, 5)

(C) (–2, –5)   (D) none of these

Q.10 Given the point with coordinates (p, q), then the point symmetrical about the y-axis for the given point must lie on the graph of–

(A) $qx + py = 0$ (B) $px - qy = 0$

(C) $x + qy = p$ (D) none of these

Q.11 Given the point with coordinates (c, d), then the point symmetrical about the x-axis for the given point must lie on the graph of–

(A) $cx + dy = 0$ (B) $cx - dy = 0$

(C) $cy + dx = 0$ (D) none of these

Q.12 I am walking at the rate of 4 km an hour along a road and a tram car over takes me in a quarter of an hour. If I had gone to meet the car, I should have met in three minutes. Then the car was going at the rate of–

(A) 6 km an hour   (B) 8 km an hour

(C) 12 km an hour   (D) none of these

Q.13 The graphs of $x = 3$ and $y = 4$ are drawn on the same axes with the same scale. The distance of the point where they cross from the origin is–

(A) 1   (B) 5

(C) 7   (D) none of these

Q.14 The graph of the equation $4x + 3y + 12 = 0$ meets the X-axis at the point –

(A) (3, 0)   (B) (0, 4)

(C) (–3, 0)   (D) none of these

Q.15 The point (6, –7) will lie in the –

(A) II quadrant   (B) III quadrant

(C) IV quadrant   (D) none of these

Q.16 The slope of the line joining the two points (3, –2) and (5, –4) is–

(A) –1   (B) 1

(C) 0   (D) none of these

ONE OR MORE THAN ONE CORRECT CHOICE TYPE QUESTIONS

Q.17 Which of these are not the value of x in

$$8x + \frac{21}{4} = 3x + 7$$

(A) $\frac{7}{10}$ (B) $\frac{7}{20}$ (C) $\frac{7}{30}$ (D) $\frac{7}{40}$

Q.18 Which of these are not the value of x in

$$2x - (3x - 4) = 3x - 5$$

(A) $\frac{9}{5}$ (B) $\frac{9}{3}$ (C) $\frac{9}{4}$ (D) $\frac{9}{2}$

Q.19 Which of these are not the value of x in

$\frac{6}{x} + 11 = \frac{2}{x} + 9, \ x \neq 0$

(A) –2 (B) 1 (C) –1 (D) 2

Q.20 The sum of three consecutive natural numbers is 225. Then the numbers are–

(A) 74, 75, 72 (B) 74

(C) 75 (D) 76

Q.21 If we divide 64 into two parts such that one part is three times the other, then two parts are–

(A) 16 (B) 48 (C) 15 (D) 13

Q.22 Which of these are not the value of x in

$15(x - 1) + 4(x + 3) = 2(7 + x)$

(A) 1 (B) 2 (C) 3 (D) 4

Q.23 Which of these are not the value of x in

$8(x - 3) - (6 - 2x) = 2(x + 2) - 5(5 - x)$

(A) 1 (B) 2 (C) 3 (D) 4

Q.24 Solution of $\frac{x^2 + 5x + 6}{x^2 + 4x + 4} = \frac{3x + 1}{5x - 2}, x \neq -2, \frac{2}{5}$ is –

(A) –4 (B) 1 (C) 4 (D) –2

Q.25 Solution of $\frac{x}{x - 2} = 3, x \neq 2$, is

(A) 1 (B) 2 (C) 3 (D) 4

Q.26 Solution of $\frac{2}{x} - \frac{5}{3x} = \frac{1}{3}$, is–

(A) 1 (B) 2 (C) $\frac{1}{3}$ (D) 0

PASSAGE BASED QUESTIONS

Passage # 1 (Q.27 to Q. 31)

An equation of the form ax + by + c = d or ax + by = d, where a, b, c, d are real numbers, $a \neq 0, b \neq 0$ and x, y are variables, is called linear equation in two variables.

On the basis of above information answer the following question.

Q.27 The solution of the equations $x + y = 3$,

$3x - 2y = 4$ is –

(A) $x = 2, y = 1$ (B) $x = 1, y = 2$

(C) $x = -2, y = 1$ (D) $x = -2, y = -1$

Q.28 By the graphical method solution of the equations $3x - y = 2$, $9x - 3y = 6$ is–

(A) one (B) two

(C) infinitely many (D) none of these

Q.29 By graphical method the system of equations

$2x + 4y = 10$, $3x + 6y = 12$ has

(A) one solution (B) two solution

(C) three solution (D) No solution

Q.30 In which quadrant do the point $(-2, -5)$ lies?

(A) first (B) second

(C) third (D) fourth

Q.31 The solution of $3x - 2y = 5$ is–

(A) $x = 3, y = 2$ (B) $x = 3, y = -2$

(C) $x = -3, y = 2$ (D) none of these

ANSWER KEY

Q.No	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Ans.	A	D	C	B	C	B	D	D	A	A	C	A	B	C	C	A	A,C,D	A,B,D
Q.No	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34		
Ans.	B,C,D	B,C,D	A,B	B,C,D	A,B,D	A,B	C	A	A	C	D	C	A	B	C	D		

HINTS & SOLUTION

A. SINGLE CHOICE QUESTIONS :

1.[A] $p + q = 1$

$\therefore\ q = 1 - p$

ordered pair (p,q) satisfies $3x + 2y = 1$

it means if $x = p$ and $y = q$

Then , $3p + 2q = 1$

$\therefore\ p = -1$ and $q = 2$

$\therefore\ 3x + 3y = 3$ is satisfied, if $x = p$ and $y = q$

2.[D] Let the age of father be x years, the age of elder son be y years and the age of younger son be z years.

From the first condition

$x = 2y \quad .... \text{(i)}$

From the second condition

$x + 10 = 3(z + 10)$

$\therefore\ x - 3z = 20 \quad .... \text{(ii)}$

from the third condition

$y - z = 15 \quad .... \text{(iii)}$

put the value of y from equation (i) in (iii), we get

$x - 2z = 30 \quad .... \text{(iv)}$

Solving (ii) and (iv) we get $x = 50$

3. [C] $\dfrac{(x+2)(x-5)}{(x-3)(x+6)} = \dfrac{(x-2)}{(x+4)}$

$\Rightarrow (x^3 + x^2 - 22x - 40) - (x^3 + x^2 - 24x + 36) = 0$

i.e. $(x - 38) = 0$, has only one root.

4.[B] Let the cost of one chair and one table be Rs. x and Rs. y respectively.

$\therefore\ 9x + 3y = 306 \quad .... \text{(i)}$

$8x + 2y = 246 \quad .... \text{(ii)}$

Solving equation (i) and (ii), we get

x = Rs. 21 and y = Rs. 39

$\therefore$ Cost of (6 chair + 1 table)

$= 6 \times 21 + 1 \times 39 =$ Rs. 165

5.[C] $2^x - 2^{x-1} = 4$ or $2^{x-1} = 2^2$

or $x - 1 = 2 \therefore x^x = 3^3 = 27$

6.[B] $x^2 + p^2 = q^2 - 2qx + x^2$

or $2qx = q^2 - p^2$ or $x = \frac{q^2 - p^2}{2q}$

7.[D] From eq.(1) $3x - 5y + 2 = 0$ .... (i)

From eq.(2) $6x - 10y + 4 = 0$

or $3x - 5y + 2 = 0$ .... (ii)

eq. (i) and (ii) are same

Hence, it will have the infinite number of solutions.

8.[D] $2x^2 + xy - 3y^2 + x + ay - 10$

$= 2x^2 + xy - 3y^2 + (b - 4)x - (b + c)y - 2b$

Compare the coefficients of x, y and constant on both sides.

$b - 4 = 1 \Rightarrow b = 5$

$-(b + 6) = a \Rightarrow a = -11$

B. ONE OR MORE THAN ONE CORRECT CHOICE TYPE QUESTIONS:

17.[A,C,D] $8x + \frac{21}{4} = 3x + 7$

$\Rightarrow 32x + 21 = 12x + 28$

$x = \frac{7}{20}$, (A), (C), (D) are the correct answers.

18.[A,B,D] $2x - (3x - 4) = 3x - 5$

$\Rightarrow 2x - 3x + 4 = 3x - 5 \quad \Rightarrow x = \frac{9}{4}$

(A), (B), (D) are the correct answers.

19.[B,C,D] $\frac{6}{x} + 11 = \frac{2}{x} + 9$

$2x = -4 \quad \Rightarrow \quad x = -2$

(B), (C), (D) are the correct answers.

20.[B,C,D] $x + x + 1 + x + 2 = 225$

$3x + 3 = 225 \Rightarrow 3x = 222 \Rightarrow x = 74$

(B), (C), (D) are the correct answers

21.[A,B] $x + 3x = 64 \Rightarrow x = 16$

(A) and (B) are the correct answers.

C. PASSAGE BASED QUESTIONS ;

27.[A] $x + y = 3$

$3x - 2y = 4$

$\Rightarrow 5x = 10 \Rightarrow x = 2 \Rightarrow y = 1$

30.[C] The point $(-2, -5)$ lies in the third quadrant.