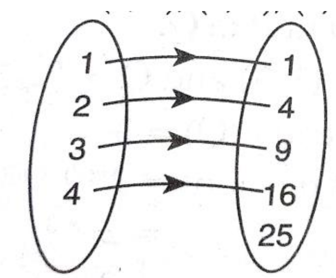
Long Answer Type Question – I

Q. 1. Let A = {1, 2, 3, 4}, B = {1, 4, 9, 16, 25} and R be a Relation defined from A to B as,
R = {(x, y) : ∈ A, y ∈ B and y = x^2}

(i) Depict this relation using arrow diagram.
(ii) Find domain of R.
(iii) Find range of R.
(iv) Write co-domain of R. [DDE – 2017]

Sol. Given, A = {1, 2, 3, 4} and B = {1, 4, 9, 16, 25} and
R = {(x, y): ∈ A, y ∈ B and y = x^2
(i) Relation R = {(1, 1), (2, 4), (3, 9), (4, 16)}



(ii) Domain of R = {1, 2, 3, 4}
(iii) Range of = {1, 4,9, 16}
(iv) Co-domain of R = {1, 4, 9, 16, 25}

Q. 2. If A = {2, 4, 6, 9} B = {4, 6, 18, 27, 54} and a relation R from A to B is defined by R = {(a, b): a ∈ A, b ∈ B a is factor of b and a < b}, then find in roster from, Also find its domain and range.

Sol. Given, A = {2, 4, 6, 9} and B = {4, 6, 18, 27, 54} and R = {(a, b): a ∈ A, b ∈ B, a is a factor of b and a < b}

Roster form
R = {(2, 4), (2, 6), (2, 18), (2, 54), (6, 18), (6, 54), (9, 18), (9, 27), (9, 54)}
Domain of R = {2, 6, 9}
Range of R = {4, 6, 18, 27, 54}

Q. 3. Let A = {2, 3, 4, 5, 6, 7, 8, 9}. Let R be the relation on A Defined by {(x, y); x, y ∈ A, x is a multiple of y and $x \neq$ y [KVS 2016, Mumbai]

(a) find the relation.
(b) find the domain of R.
(c) find the range of R.
(d) find the inverse relation. [DDE – 2017]

Sol. (i) R = {(4, 2) (6, 2) (8, 2) (6, 3) (9, 3) (8, 4)}
(ii) Domain of R = {4, 6, 8, 9}
(iii) Range of R = {2, 3, 4}

(iv) $R^{-1} = \frac{1}{R}$ {(2, 4) (2, 6) (2, 8) (3, 6) (3, 9) (4, 8)}

Q. 4. Define a relation R on the set N of natural numbers by R = {(x, y): y = 2x – 1; x, $y \in$ N, $x \leq 5$}. Depict this relationship using roster form, Write down the domain and range. [KVS 2016, Agra]

Sol. Given y = 2x – 1 and $x \leq 5$
for x = 1, y = 2 × 1 – 1 = 1
x = 2, y = 2 × 2 – 1 = 3
x = 3, y = 2 × 3 – 1 = 5
x = 4, y = 2 × 4 – 1 = 7
x = 5, y = 2 × 5 – 1 = 9
Relation R = {(1, 1), (2, 3), (3, 5), (4, 7), (5, 9)}
∴ Domain = {1, 2, 3, 4, 5}
∴ Range = {1, 3, 5, 7, 9}

Q. 5. Let A = {1, 2, 3}, B = {3, 4 and C = {4, 5, 6}. Find
(i) A × (B ∩ C) (ii) (A × C) ∩ (A × C) [KVS 2014 – 15]

Sol. A {1, 2, 3}, B = {3, 4}, C = {4, 5, 6}, B ∩ C = {4}
(i) A × (B ∩ C)
= {1, 2, 3} × {4}
= {(1, 4), (2, 4), 3, 4)}

(ii) (A × B) ∩ (A × C)
= {(1, 2, 3} × {3, 4)} ∩ ({1, 2, 3} × {4, 5, 6})
= {(1, 3), (1, 4), (2, 3), (2, 4), (3, 3), (3, 4)}
∩ {(1, 4), 1, 5), (1, 6), (2, 4), (2, 5),
(2, 6), (3, 4), (3, 5), (3, 6)}

= {(1, 4), (2, 4), (3, 4)}

Q. 6. Let $f(x) = \begin{cases} x^2, when\ 0 \leq x \leq 2. \\ 2x, when\ 2 \leq x \leq 5. \end{cases}$

$$g(x) = \begin{cases} x^2, when\ 0 \leq x \leq 3. \\ 2x, when\ 3 \leq x \leq 5. \end{cases}$$

Show that f is a function while g is not a function. [DDE – 2017]

Sol. The relation f is defined as

$$f\ (x) = \begin{cases} x^2, when\ 0 \leq x \leq 3 \\ 2x, when\ 3 \leq x \leq 5 \end{cases}$$

It is observed that for

$$0 \le x \le 2, f(x) = x^2$$

And $$2 \le x \le 5, f(x) = 2x$$

Also, at x = 2, f (x) = 2^2 = 4 or $f(x)$ = 2 × 2 = 4
i,e at x = 2, f (x) = 4.

Therefore, for 0≤ x ≤ 5, the image of f (x) are unique
Thus, the given relation 'f' is a function.

The relation g is defined as $g(x) = \begin{cases} x^2, when\ 0 \le x \le 3 \\ 2x, when\ 3 \le x \le 5 \end{cases}$

It can be observed that for x = 3 $g(x)$ = 3^2 = 9
and g (x) = 2 × 3 = 6
Hence, element 3 of the domain of relation 'g'
corresponds to two different images i.e., 9 and 6.
Hence, this relation is not a function.

Q. 7. Find the domain and range of $f(x)$ = $|2x - 3| - 3$ [DDE – 2017]

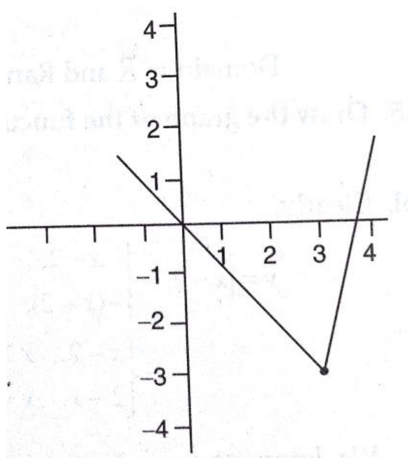
Sol. Given, $f(x) = |2x - 3| - 3$

The domain of the expression in all real number expect where the expression is undefined. In this case, there is not real number that makes the expression undefined.

∴ Domain of f = $(-\infty, \infty)$ = R

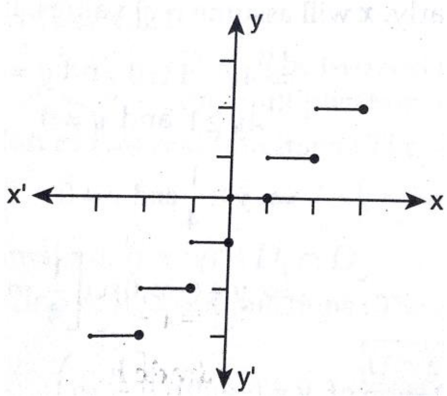
The absolute value of expression has a 'V' shape. The range of a positive absolute value expression starts at its vertex and extends to infinity.

Range of f = $[-3, \infty]$ or $\{y/y \ge -3\}$



Q. 8. Draw the graph of the greatest Integer Function [DDE – 2017]

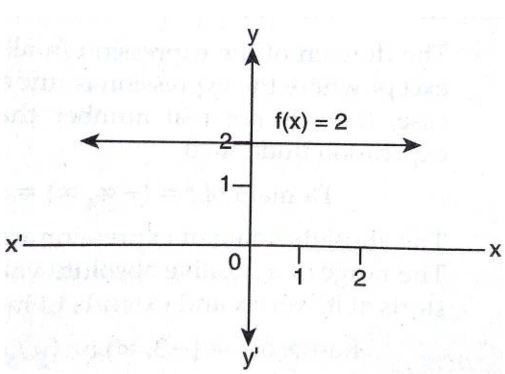
Sol. The greatest integer function is denoted by y = $[x]$. For all real number, x, the greatest integer function returns the largest integer less than or equal to x.



Value of x	$f(x)$ = $[x]$
. .	. .
$-3 \leq x < -2$	–3
$-2 \leq x < -1$	–2
$-1 \leq x < -0$	–1
$0 \leq x < 1$	0
$1 \leq x < 2$	1
$2 \leq x < 3$	2
$3 \leq x < 4$	3
. .	. .

Q. 9. Draw the graph of constant function: R → R; $f(x)$ = 2 ∀ x ∈ R. Also, find its domain and range [DDE – 2017]

Sol. Given: R → R: $f(x)$ = 2 ∀ x ∈ R



Domain = R and range = {2}

Q. 10. Draw the graph of the function $|x - 2|$ [DDE = 2017]

Sol. Clearly,

$$y = |x - 2| = \begin{cases} x - 2 & x - 2 \geq 0 \\ -(x - 2), & x - 2 < 0 \end{cases}$$

$$= \begin{cases} x - 2, & x \geq 2 \\ 2 - x, & x < 2 \end{cases}$$

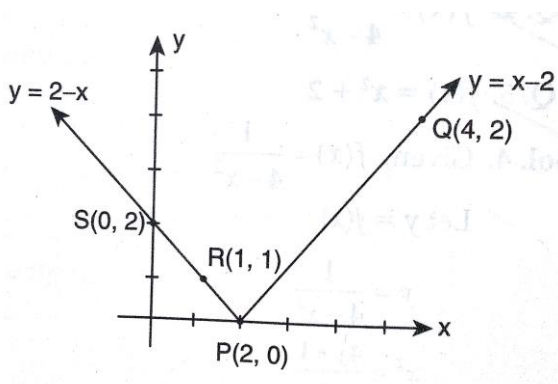
We know that, a linear equation in x and y represents a line for drawing a line, we need only two points for y = x – 2

x	2	4
y	4	2

So, plot the points P (2, 0), Q (4, 2) and join PQ to get the graph of y = x – 2 for y = 2 – x

x	1	0
y	1	2

Plot the points R (1, 1), So (0, 2) and join RS to get the graph of y = y = 2 – x



Find the domain and range of the following real functions (Questions 11 – 16)

Q. 11. $f(x) = \sqrt{x^2 + 4}$

Q. 12. $f(x) = \frac{|x-1|}{x-1}$

Q. 13. $f(x) = \frac{|x-1|}{x-1}$

Q. 14. $f(x) = \frac{x^2-9}{x-3}$

Q. 15. $f(x) = \frac{4-x}{x-4}$

Q. 16. $f(x) = 1 - |x - 3|$

Sol. 11. Given, $f(x) = \sqrt{x^2+4}$

Domain of f: We observe that $f(x)$ is defined for all x satisfying $x^2 + 4 \geq 0$

∴ Domain = The set of all real number

= R

∴ Range of:

Let y = $f(x)$

$$y = \sqrt{x^2+4}$$

$$x = \sqrt{y^2-4}$$

x is defined, $y^2 - 4 \geq 0$

$$(y+2)(y-2) \geq 0$$

∴ Range of f = $(-\infty, -2] \cup [2, \infty)$

Sol. 12. Given, $f(x) = \frac{x+1}{x-2}$

Domain: We known that $f(x)$ is defined when $x - 2 \neq 0$

i.e., $x \neq 2$

∴ The Domain is all values of x that makes the expression defined i.e.,

Domain of f = R – {2}

Range:

Let y = $f(x)$

$$\therefore\ y = \frac{x+1}{x-2}$$

$$Y(x-2) = x + 1$$

$$x(y-1) = 2y + 1$$

$$x = \frac{2y+1}{y-1}$$

∵ x is defined, when $y - 1 \neq 0$ i. e., $y \neq 1$

∴ Range of f = R – {1}

Sol. 13. Given, $f(x) = \frac{|x-1|}{x-1}$

Domain: Clearly, $f(x)$ is defined for all $x \leftarrow$ R

except $x = 1$

∴ Domain of f = R – $\left\{\vec{1}\right\}$

Range: Now, $f\ (x)$ = $\frac{x-1}{x-1} = 1$, when x > 1

And $f(x) = \frac{-(x-1)}{x-1} = -1$, when x < 1

∴ Range of f = {– 1, 1}

Sol. 14. Given, $f(x) = \frac{x^2-9}{x-3}$

Domain: Clearly, $f(x)$ is defined for all x ∈ R expect x = 3

∴ Domain of f = R – {3} = $(-\infty, 3)$,∪ $(3, \infty)$
Range: Let y = $f(x)$

$$\therefore\ y = \frac{x^2-9}{x-3} \Rightarrow y = x + 3$$

It follow from the above relation that y takes all real values except 6 when x takes values in the set R – {3}

∴ Range of f = R – {6}

Sol. 15. Given, $f\ (x)$ = $\frac{4-x}{x-4}$

Domain: Clearly, $f\ (x)$ is defined for all x ∈ R expect x = 4

∴ Domain of f = R – {4 } = $(-\infty, 4)$,∪ $(4, \infty)$

Range: Let y = $f(x)$

$$\Rightarrow y = \frac{4 - x}{x - 4}$$

$$\Rightarrow y = \frac{-(x - 4)}{x - 4} = -1$$

∴ Range of f = {– 1}

Sol. 16. Given, $f(x)$ = 1 – $|x\ -\ 3|$
Domain: We observe that $f\ (x)$ is defined for all x ∈ R
∴ Domain of f = R

Range: Now,

$$0 \leq |x - 3| < \infty \forall \in R$$

$$\Rightarrow -\infty\ < -|x - 3| \leq 0\ \forall\ x \in R$$

$$\Rightarrow\ -\infty\ <\ -|\forall - 3|\ \leq 1\ \forall\ x\ \in\ R$$

$$\Rightarrow -\infty < f(x) \le 1 \,\forall\, x \in R$$

Hence, Range of f = $(-\infty, 1)$

Q. 17. Determine a quadratic function (f) is defined by f (x) = ax^2 + bx + c. If f (0) = 6, f(2) = 1, f(– 3) = 6. [DDE – 2017]

Sol. Given, f (x) = ax^2 + bx + c.

At x = 0, f(0) = 6 (given)

∴ a × 0 + b × 0 + c = 6

∴ c = 6 (i)

At x = 2, f(2) = 1 (given)

∴ a(2)^2 + b(2) + c = 1

⇒ 4a + 2b + 6 = 1 (using (i))

⇒ 4a + 2b = – 5 (ii)

At x = – 3, f(–3) = 6

∴ a(– 3)^2 + b(– 3) + c = 6

⇒ 9a – 3b + 6 = 6 (using (i))

⇒ 9a – 3b = 0 (iii)

On solving eqs. (ii) And (iii), we get

$$a = -\frac{1}{2} \text{ and } a = -\frac{3}{2}$$

∴ Required quadratic function $(f) = \frac{-1}{2}x^2 + \frac{-3}{2}x + 6$

Q. 18. $\boldsymbol{f(x) = \begin{cases} 1 + 2x & x < 0 \\ 3 + 5x, & x \ge 0 \end{cases}}$

Also, find its range.

Sol. Given, $f(x) = \begin{cases} 1 + 2x & x < 0 \\ 3 + 5x, & x \ge 0 \end{cases}$

Here, $f(x)$ = 1 + 2x, x <0, this given

f (– 4) = 1 + 2(– 4) = – 7

f(– 3) = – 1 + 2(–3) = – 5

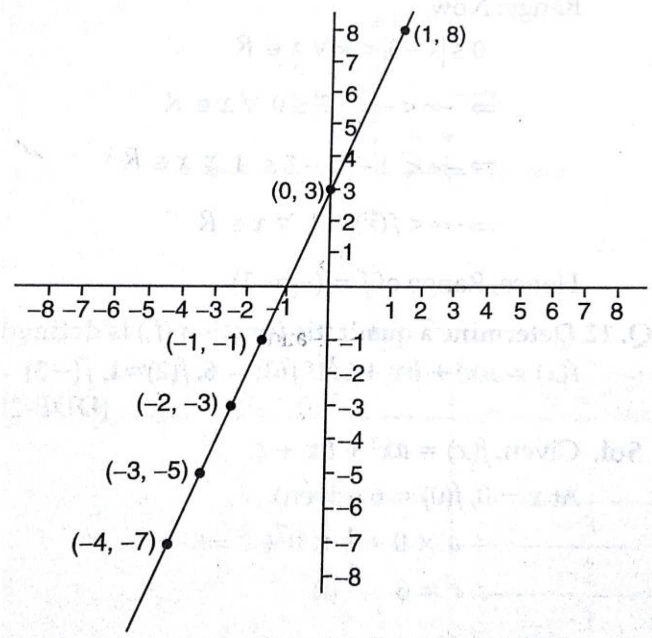
f(–2) = 1 + 2(– 2) = – 3

f(–1) = 1 + 2(–1) = – 1

and $f(x)$ = 3 + 5x, $x \ge 0$

$f(0) = 3 + 5(0) = 3$
$f(1) = 3 + 5(1) = 8$
$f(2) = 3 + 5(2) = 13$
$f(3) = 3 + 5(3) = 18$
$f(4) = 3 + 5(4) = 23$

Now the graph of f is as shown in following figure.



Range Let y, $f(x)$, $x < 0$

$\therefore y_1 = 1 + 2x, x < 0$

$$\therefore x = \frac{y_1 - 1}{2}, x < 0$$

$\because < 0 \Rightarrow y_1 - 1 < 0 \Rightarrow y_1 < 1$

Let $y_2 = f(x), x \geq 0$

$\Rightarrow y_2 = 3 + 5x, x \geq 0$

$$\Rightarrow x = \frac{y_2 - 3}{5}, x \geq 0$$

$\because x \geq 0 \Rightarrow y_2 - 3 \geq 0 \Rightarrow y_2 \geq 3$

Therefore, range of f $(-\infty, 1), \cup [3, \infty)$

Q. 19. Draw the graph of the function

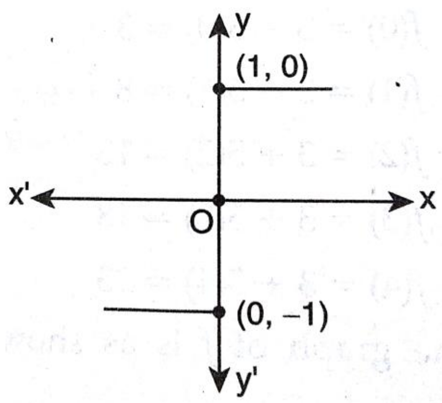
$$f(x) \begin{cases} \frac{|x|}{x} & x \neq 0 \\ 0, & x = 0 \end{cases}$$

Also, find its range. **[DDE – 2017]**

Sol. Given $f(x) \begin{cases} \frac{|x|}{x} & x \neq 0 \\ 0, & x = 0 \end{cases}$

The given function is called the signum function and can be write as

$$f(x) = \begin{cases} 1, & when\ x > 0 \\ 0, & when\ x = 0 \\ -1, & when\ x < 0 \end{cases}$$



The domain of f = R and the range of f = {– 1, 0, 1}

Find the domain of the following functions: **(Question 20 – 21)**

Q. 20. $\boldsymbol{f(x) = \frac{1}{\sqrt{x+|x|}}}$

Q. 21. $\boldsymbol{f(x) = \frac{1}{\sqrt{x-|x|}}}$

Sol. 20. Given, $f(x) = \frac{1}{\sqrt{x+|x|}}$

We know that,

$$|x| = \begin{cases} x, & when\ x \geq 0 \\ -x, & when\ x < 0 \end{cases}$$

$$x + |x| = \begin{cases} x, & when\ x \geq 0 \\ -x, & when\ x < 0 \end{cases}$$

$$x + |x| = \begin{cases} 2x, & when\ x \geq 0 \\ 0, & when\ x < 0 \end{cases}$$

Now, $f(x) = \frac{1}{\sqrt{x+|x|}}$ assume real values, if

$x + |x| > 0$
$\Rightarrow x > 0$
$\Rightarrow x \in (0, \infty)$

Hence, Domain (f) = (0, ∞)

Sol. 21. Given, $f(x) \frac{1}{\sqrt{x - |x|}}$

We know that,

$$|x| = \begin{cases} x, & when\ x \geq 0 \\ -x, & when\ x < 0 \end{cases}$$

$$x - |x| = \begin{cases} x - x & when\ x \geq 0 \\ x + x, & when\ x < 0 \end{cases}$$

$$x - |x| = \begin{cases} 0, & when\ x \geq 0 \\ 2x, & when\ x < 0 \end{cases}$$

$$= x - |x| \leq 0 \text{ For all } x$$

$\frac{1}{\sqrt{x - |x|}}$ does not take real values for any $x \in$ R 1

$\Rightarrow f(x)$ is not defined for any $x \in$ R Hence, Domain (f) = ϕ