ICSE SEMESTER 2 EXAMINATION

SPECIMEN QUESTION PAPER

MATHEMATICS

Maximum Marks: 40

Time allowed: One and a half hours

Answers to this Paper must be written on the paper provided separately.

You will not be allowed to write during the first 10 minutes.

This time is to be spent in reading the question paper.

The time given at the head of this Paper is the time allowed for writing the answers.

Attempt all questions from Section A and any three questions from Section B.

The intended marks for questions or parts of questions are given in brackets [].

SECTION A

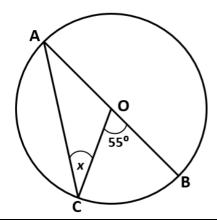
(Attempt **all** questions from this Section.)

Question 1

Choose the correct answers to the questions from the given options. (Do not copy the question, Write the correct answer only.)

[10]

- (i) The point (3,0) is invariant under reflection in:
 - (a) The origin
 - (b) x-axis
 - (c) y-axis
 - (d) both x and y axes
- (ii) In the given figure, AB is a diameter of the circle with centre 'O'. If $\angle COB = 55^{\circ}$ then the value of x is:

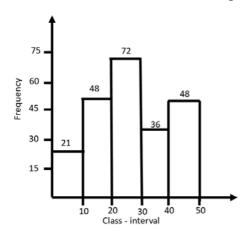


- (a) 27.5°
- (b) 55^0
- (c) 110^{0}
- (d) 125°
- (iii) If a rectangular sheet having dimensions 22 cm x 11 cm is rolled along its shorter side to form a cylinder. Then the curved surface area of the cylinder so formed is:
 - (a) 968 cm^2
 - (b) 424 cm^2
 - (c) 121 cm^2
 - (d) 242 cm^2
- (iv) If the vertices of a triangle are (1,3), (2, -4) and (-3, 1). Then the co-ordinate of its centroid is:
 - (a) (0,0)
 - (b) (0, 1)
 - (c) (1,0)
 - (d) (1, 1)
- (v) $\tan \theta \times \sqrt{1 \sin^2 \theta}$ is equal to:
 - (a) $\cos \theta$
 - (b) $\sin \theta$
 - (c) $\tan \theta$
 - (d) $\cot \theta$
- (vi) The median class for the given distribution is:

Class Interval	1 – 5	6 – 10	11–15	16 –20
Cumulative Frequency	2	6	11	18

- (a) 1-5
- (b) 6-10
- (c) 11 15
- (d) 11 20

- (vii) If the lines 7y = ax + 4 and 2y = 3 x, are parallel to each other, then the value of 'a' is:
 - (a) 1
 - (b) $\frac{-7}{2}$
 - (c) $\frac{-2}{7}$
 - (d) 14
- (viii) Volume of a cylinder is 330 cm³. The volume of the cone having same radius and height as that of the given cylinder is:
 - (a) 330 cm^3
 - (b) 165 cm^3
 - (c) 110 cm^3
 - (d) 220 cm^3
- (ix) In the given graph, the modal class is the class with frequency:



- (a) 72
- (b) 21
- (c) 48
- (d) 36
- (x) If the probability of a player winning a game is 0.56. The probability of his losing this game is:
 - (a) 0.56
 - (b) 1
 - (c) 0.44
 - (d) 0

SECTION B

(Attempt any three questions from this Section.)

Question 2

(i) Find the ratio in which the x-axis divides internally the line joining points A (6, -4) and B (-3, 8).

[2]

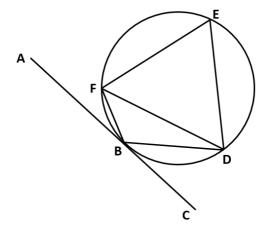
(ii) Three rotten apples are accidently mixed with twelve good ones. One apple is picked at random. What is the probability that it is a good one?

[2]

(iii) In the given figure, AC is a tangent to circle at point B. ΔEFD is an equilateral triangle and $\angle CBD = 40^{\circ}$. Find:

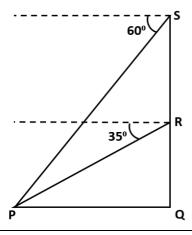
[3]

- (a) ∠**BFD**
- (b) ∠*FBD*
- (c) ∠*ABF*



(iv) A drone camera is used to shoot an object P from two different positions R and S along the same vertical line QRS. The angle of depression of the object P from these two positions are 35° and 60° respectively as shown in the diagram. If the distance of the object P from point Q is 50 metres, then find the distance between R and S correct to the nearest meter.

[3]

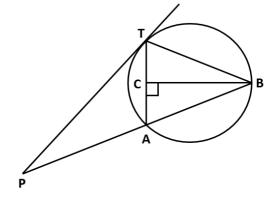


Question 3

In the given figure, PT is a tangent to the circle at T, chord BA is produced to meet the tangent at P. Perpendicular BC bisects the chord TA at C. If PA = 9cm and TB = 7cm, find the lengths of:



- AB (a)
- (b) PT



(ii) How many solid right circular cylinders of radius 2 cm and height 3 cm can be made by melting a solid right circular cylinder of diameter 12 cm and height 15 cm?

[2]

(iii) Prove that:

$$\frac{\cos^2 A}{\cos A - \sin A} + \frac{\sin A}{1 - \cot A} = \sin A + \cos A$$

(iv) Use graph paper for this question, take 2 cm = 10 marks along one axis and 2 cm = 10 students along the other axis.

The following table shows the distribution of marks in a 50 marks test in Mathematics:

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
No. of Students	6	10	13	7	4

Draw the ogive for the above distribution and hence estimate the median marks. [3]

Question 4

Find the equation of the perpendicular dropped from the point P (-1,2) onto the line joining A (1,4) and B (2,3).

[2]

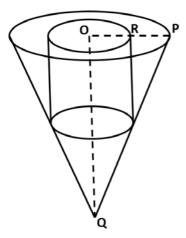
Find the mean for the following distribution:

[2]

Class Interval	20 – 40	40 – 60	60–80	80 –100
Frequency	4	7	6	3

(iii) A solid piece of wooden cone is of radius OP = 7 cm and height OQ = 12 cm. A cylinder whose radius and height equal to half of that of the cone is drilled out from this piece of wooden cone. Find the volume of the remaining piece of wood.

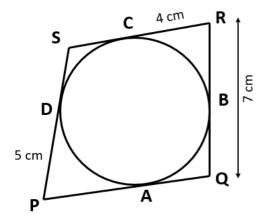
(Use,
$$\pi = \frac{22}{7}$$
)



- (iv) Use a graph sheet for this question, take 2cm = 1 unit along both x and y axis: [3]
 - (a) Plot the points A (3,2) and B (5,0). Reflect point A on the y-axis to A'. Write co-ordinates of A'.
 - (b) Reflect point B on the line AA' to B'. Write the co-ordinates of B'.
 - (c) Name the closed figure A'B'AB.

Question 5

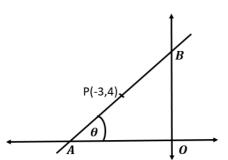
(i) In the given figure, the sides of the quadrilateral PQRS touches the circle at A,B,C and D. If RC = 4 cm, RQ = 7 cm and PD = 5cm. Find the length of PQ: [2]



(ii) Prove that: [2]

$$\frac{\sin^3\theta + \cos^3\theta}{\sin\theta + \cos\theta} = 1 - \sin\theta\cos\theta$$

(iii) In the given diagram, OA = OB, $\angle OAB = \theta$ and the line AB passes through point P (-3, 4).



Find:

(a) Slope and inclination (θ) of the line AB

(b) Equation of the line AB

(iv) Use graph paper for this question. Estimate the mode of the given distribution by plotting a histogram. [Take 2 cm = 10 marks along one axis and 2 cm = 5 students along the other axis]

 Daily wages(in ₹)
 30 - 40 40 - 50 50 - 60 60 - 70 70 - 80

 No. of Workers
 6
 12
 20
 15
 9

Question 6

(i) A box contains tokens numbered 5 to 16. A token is drawn at random. Find the probability that the token drawn bears a number divisible by:

- (a) 5
- (b) Neither by 2 nor by 3
- (ii) Point M (2, b) is the mid-point of the line segment joining points P (a, 7) and Q (6, 5). Find the values of 'a' and 'b'. [2]
- (iii) An aeroplane is flying horizontally along a straight line at a height of 3000 m from the ground at a speed of 160 m/s. Find the time it would take for the angle of elevation of the plane as seen from a particular point on the ground to change from 60° to 45°.
 Give your answer correct to the nearest second.

(iv) Given that the mean of the following frequency distribution is 30, find the missing frequency 'f' [3]

Class Interval	0 – 10	10 – 20	20–30	30 –40	40 – 50	50 – 60
Frequency	4	6	10	f	6	4

[3]

[3]

[2]



Section-A

Answer 1.

(i) (b) x-axis

Explanation:

A point lying on the x-axis is invariant under reflection in x-axis. Hence, (3, 0) is invariant under reflection in x-axis.

(ii) (a) 27.5°

Explanation:

$$\angle AOC = 180^{\circ} - 55^{\circ} = 125^{\circ}$$

[linear pair]

In ΔAOC,

$$\angle A + \angle O + \angle C = 180^{\circ}$$

$$x + 125^{\circ} + x = 180^{\circ}$$
$$x = \frac{55^{\circ}}{2}$$

$$\begin{bmatrix} \because AO = OC(radii) \\ \therefore \angle A = \angle C \end{bmatrix}$$

 \Rightarrow

$$x = 27.5^{\circ}$$

(iii) (d) 242 cm²

Explanation:

Given,

$$h = 22$$

Shorter side of the rectangular sheet = circumference of the circle.

 \Rightarrow

$$11 = 2\pi r$$

 \Rightarrow

$$r = \frac{11}{2 \times \frac{22}{7}} = \frac{7}{4} \text{cm}$$

 \therefore Curved surface area = $2\pi rh$

$$= 2 \times \frac{22}{7} \times \frac{7}{4} \times 22$$

$$= 242 \text{ cm}^2$$

Explanation:

Centroid =
$$\left[\left(\frac{x_1 + x_2 + x_3}{3} \right), \left(\frac{y_1 + y_2 + y_3}{3} \right) \right].$$

Centroid =
$$\left[\frac{(1+2-3)}{3}, \frac{(3-4+1)}{3}\right]$$

= $(0, 0)$

n = 18

(v) (b)
$$\sin \theta$$

Explanation:

$$\tan \times \sqrt{1 - \sin^2 \theta} = \tan \theta \times \cos \theta$$

$$= \frac{\sin \theta}{\cos \theta} \times \cos \theta$$

$$= \sin \theta$$

$$= \sin \theta$$
(: \cdot \cos^2 \theta + \sin^2 \theta = 1).

•:•

Explanation:

 $\frac{n}{2} = \frac{18}{2}$ *:*. = 9

∴ 9 belongs to class 11 - 15. Hence, Median class is 11 - 15

(vii) (b)
$$-\frac{7}{2}$$

Explanation:

$$7y = ax + 4$$

$$y = \frac{a}{7}x + \frac{4}{7} \qquad \dots(1)$$

 $m_1 = \frac{a}{7}$ *:*.

2y = 3 - xand

 $y = -\frac{1}{2}x + \frac{3}{2}$ \Rightarrow

 $m_2 = -\frac{1}{2}$ *:*.

Since, lines are parallel

 $m_1 = m_2$

 $\frac{a}{7} = -\frac{1}{2}$ \Rightarrow

 $a = -\frac{7}{2}$ \Rightarrow

(viii) (c) 110 cm³

Explanation:

Given,

Volume of cylinder = $\pi r^2 h$

$$= 330 \text{ cm}^3$$

Since, both have same radius and height

$$\therefore \qquad \text{Volume of cone} = \frac{1}{3} \pi r^2 h$$

$$= \frac{1}{3} \times 330 \text{ cm}^3 \qquad (\because \pi r^2 h = 330 \text{ cm}^3)$$

$$= 110 \text{ cm}^3$$

(ix) (a) 72

Explanation:

Here, we can only locate a class with the maximum frequency, called the modal class.

(x) (c) 0.44

Explanation:

Let *p* be the probability of a player winning a game and q be the probability of a player losing a game.

∴
$$p+q=1$$
 (∴ total probability is 1)
⇒ $q=1-p$
⇒ $q=1-0.56$
 $=0.44$

Section-B

Answer 2

(i) Let the ratio be k:1.

$$(x, y) = \left(\frac{k \times x_2 + 1 \times x_1}{k + 1}, \frac{k \times y_2 + 1 \times y_1}{k + 1}\right)$$

$$y = \frac{(k \times y_2 + 1 \times y_1)}{(k + 1)}$$

$$0 = \frac{(8 \times k - 4)}{(k + 1)} \qquad (\because \text{ point lies on } x\text{-axis})$$

$$\Rightarrow \qquad \qquad 8k - 4 = 0$$

$$\Rightarrow \qquad k : 1 = 1 : 2$$

 \therefore 1:2 is the required ratio.

$$\therefore 1:2 \text{ is the required ratio.}$$
(ii)

No. of good apples = 12

No. of rotten apples = 3

Total no. of apples = 15

$$P(\text{good apples}) = \frac{\text{no.of favourable outcomes}}{\text{total no.of possible outcomes}}$$

$$= \frac{12}{15} = \frac{4}{5}$$

(By alternate segment theorem)

∫60°

(b)

$$\angle$$
FBD + \angle FED = 180°

$$\angle$$
FBD + \angle FED = 180°

 \Rightarrow

(∵ ΔFED is an equilateral triangle)

 \Rightarrow

$$\angle$$
FBD = $180^{\circ} - 60^{\circ} = 120^{\circ}$

(c) Since, ABC is a straight line.

$$\angle ABF + \angle FBD + \angle CBD = 180^{\circ}$$

$$\Rightarrow$$

$$\angle ABF + 120^{\circ} + 40^{\circ} = 180^{\circ}$$

 \Rightarrow

$$\angle ABF = 180^{\circ} - 160^{\circ}$$

$$\angle ABF = 20^{\circ}$$

$$\tan 35^\circ = \frac{RQ}{PQ}$$

$$= \frac{RQ}{50}$$

 \Rightarrow

$$0.7002 \times 50 = RQ$$

 $(\because \tan 35^\circ = 0.7002)$

 \Rightarrow

$$RQ = 35.01 \text{ m}$$

and

$$\tan 60^\circ = \frac{SQ}{PQ} = \frac{SQ}{50}$$

 \Rightarrow

:.

$$1.73 \times 50 = SQ$$

$$SQ = 86.5 \text{ m}$$
$$RS = SQ - RQ$$

= 86.5 - 35.01

= 51.49 m

≈ 51 m

Ans.

35°\

₹\35°

С



$$PA = 9 cm$$

$$TB = 7 \text{ cm}$$

(a) In \triangle ABT, BC is the perpendicular bisector of AT.

$$\Delta ACB \cong \Delta BCT$$

$$AB = TB = 7 \text{ cm}$$

(b) By tangent secant theorem,

$$PT^2 = PA \times PB$$

Given,

$$PA = 9 cm$$

and

$$PB = PA + AB = 9 + 7 = 16 \text{ cm}$$

So,

$$PT^2 = 9 \times 16$$

 \Rightarrow

$$PT = \sqrt{9 \times 16}$$

 \Rightarrow

$$PT = 3 \times 4$$

 \Rightarrow

$$PT = 12 \text{ cm}$$

(ii) Volume of bigger cylinder

$$V_1 = \pi \times r^2 \times h$$

$$= \pi \times 6 \times 6 \times 15 \text{ cm}^3$$

$$\left(\because r = \frac{12}{2}\right)$$

Volume of smaller cylinder

$$V_2 = \pi \times 2 \times 2 \times 3 \text{ cm}^3$$

$$\begin{array}{l} \therefore \qquad \qquad \text{No. of cylinder} = \frac{V_1}{V_2} \\ &= \frac{\pi \times 6 \times 6 \times 15}{\pi \times 2 \times 2 \times 3} \\ &= 45 \\ \text{LHS} = \left(\frac{\cos^2 A}{\cos A - \sin A}\right) + \left(\frac{\sin A}{1 - \cot A}\right) \\ &= \left(\frac{\cos^2 A}{\cos A - \sin A}\right) + \left(\frac{\sin A \sin A}{\sin A - \cos A}\right) \quad \left[\because \cot A = \frac{\cos A}{\sin A}\right] \\ &= \left(\frac{\cos^2 A}{\cos A - \sin A}\right) + \left(\frac{\sin^2 A}{\sin A - \cos A}\right) \\ &= \left(\frac{\cos^2 A}{\cos A - \sin A}\right) - \left(\frac{\sin^2 A}{\cos A - \sin A}\right) \\ &= \left(\frac{\cos^2 A - \sin^2 A}{\cos A - \sin A}\right) \\ &= \frac{\left[(\cos A + \sin A)(\cos A - \sin A)\right]}{(\cos A - \sin A)} \\ &= (\cos A + \sin A) \end{array}$$

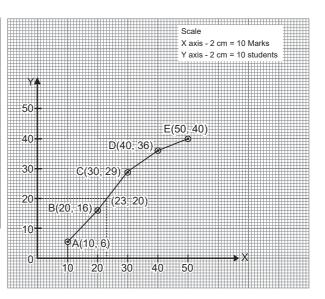
= RHS

(iv)

Class interval	Frequency	Cumulative Frequency
0-10	6	6
10-20	10	16
20-30	13	29
30-40	7	36
40-50	4	40

Points are

:. A (10, 6), B(20, 16), C(30, 29), D(40, 36), E(50, 40)



Hence Proved

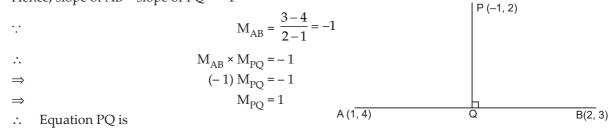
Median =
$$\frac{n}{2}$$
th term
= $\frac{40}{2}$ th term
= 20^{th} term
= 23 marks

Answer 4

(i) Given

$$AB \perp PQ$$

Hence, slope of AB \times slope of PQ = -1



Equation 1 Q is
$$(y - y_1) = M_{PQ} (x - x_1)$$

$$y - 2 = 1 [x - (-1)]$$

$$y - 2 = x + 1$$

$$y = x + 3$$

$$y - x = 3$$

(ii)

 \Rightarrow

 \Rightarrow

 \Rightarrow

Class Interval	Class Marks (x _i)	$f_{(i)}$	$f_{\underline{i}}x_i$
20-40	30	4	120
40-60	50	7	350
60-80	70	6	420
80-100	90	3	270
		$\Sigma f_i = 20$	$\Sigma f_i x_i = 1160$

Mean =
$$\overline{x} = \frac{\sum f_i x_i}{\sum f_i}$$

 $\overline{x} = \frac{1160}{20}$
= 58

(iii) Volume of cone =
$$\frac{1}{3}\pi R^2H$$

= $\frac{1}{3} \times \frac{22}{7} \times (7)^2 \times 12$
= 88×7
= 616 cm^3
Volume of cylinder = $\pi r^2 h$

$$= \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \left(\frac{12}{2}\right)$$

(∵ Radius and height of the cylinder are half of that of cone)

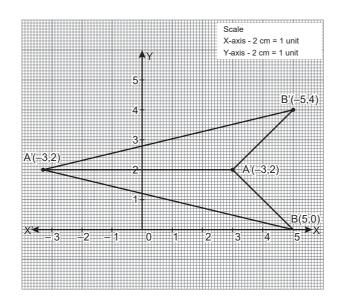
$$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 6$$
$$= 231 \text{ cm}^3$$

: Volume of Remaining solid = Volume of cone – Volume of cylinder

$$=616-231$$

$$= 385 \text{ cm}^3$$

(iv)



- (a) A'(-3, 2)
- (b) Since we know that,

The reflection of point (a, b) with respect to line y = k is a point (a, 2k - b)

Here,
$$k = 2$$
, $a = 5$, $b = 0$

$$\therefore \quad B' \equiv (5, 2 \times 2 - 0)$$

$$\therefore$$
 B' \equiv (5, 4)

(c) A'B'AB is an arrowhead

Answer 5

(i) PD = PA = 5 cm ...(1)

[Tangents from exterior point are equal in length].

QA = QB = q ...(2)

[Tangents from exterior point are equal in length].

RC = RB =
$$r = 4$$
 cm ...(3)

[Tangents from exterior point are equal in length].

SD = SC = s ...(4)

[Tangents from exterior point are equal in length].

RQ = RB + BQ (given)

RQ = RB + BQ (given)

7 cm = $4 + QB$ [from (3)]

 \Rightarrow QB = 3 cm ...(5)

 \therefore QA = QB = 3 cm ...(6) [from (2) and (5)]

 \therefore PQ = PA + AQ

= $5 + 3 = 8$ cm [from (1) and (6)]

(ii) LHS = $\frac{\sin^3 \theta + \cos^3 \theta}{\sin \theta + \cos \theta}$

= $\frac{(\sin \theta + \cos \theta)(\sin^2 \theta - \sin \theta \cos \theta + \cos^2 \theta)}{(\sin \theta + \cos \theta)}$ [$\because a^3 + b^3 = (a + b) (a^2 - ab + b^2)$]

= $\sin^2 \theta - \sin \theta \cos \theta + \cos^2 \theta$

= $1 - \sin \theta \cos \theta$ [$\because \sin^2 \theta + \cos^2 \theta = 1$]

= RHS

$$OA = OB$$

Given, Isosceles
$$\Delta$$
 so,

$$\angle BAO = \angle ABO = x$$
 (let)

$$\angle$$
BAO + \angle ABO + \angle AOB = 180°

(:: sum of three angles of a triangle is 180°)

$$\Rightarrow$$

$$\chi + \chi + 90^{\circ} = 180^{\circ}$$

$$\Rightarrow$$

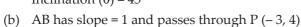
$$2x = 90^{\circ}$$

$$x = 45^{\circ}$$

$$\angle BAO = 45^{\circ} = \theta$$

(a) Slope of AB = $\tan 45^\circ = 1$

Inclination (θ) = 45°



$$\Rightarrow$$

$$y - y_1 = m(x - x_1)$$

$$\Rightarrow$$

$$y - 4 = 1 (x + 3)$$

$$\rightarrow$$

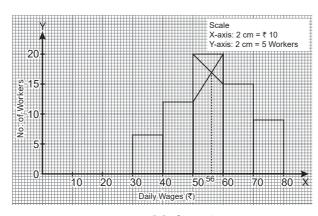
$$y = x + 3 + 4$$

$$= \chi +$$

$$\Rightarrow$$

$$x - y + 7 = 0$$





Mode = 56

Answer 6

- (i) Total no. of tokens = 12
 - (a) Favourable outcomes = {5, 10, 15}

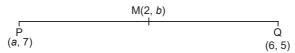
P (No. divisible by 5) = $\frac{\text{No. of favourable outcomes}}{\text{Total no. of possible outcomes}}$

$$=\frac{3}{12}=\frac{1}{4}$$

Favourable outcomes = $\{5, 7, 11, 13\}$

P(neither divisible by 2 nor by 3) =
$$\frac{4}{12}$$

$$=\frac{1}{3}$$



Mid point formula =
$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$${\rm P}=(a,\,7)=(x_1,\,y_1)$$

$$Q = (6, 5) = (x_2 y_2)$$
∴
$$(2, b) = \left(\frac{a+6}{2}, \frac{7+5}{7+5}\right)$$
∴
$$2 = \frac{a+6}{2} \text{ and } b = \frac{12}{2}$$
⇒
$$4 = a+6 \text{ and } b = 6$$
⇒
$$a = -2$$
∴
$$a = -2, b = 6$$
(iii) Given,
$$AC = ED = 3,000 \text{ m}$$
Speed = 160 m/s

In rt ΔACB
$$AC = ED = 3,000 \text{ m}$$

$$AC = ED = 3,000 \text{ m}$$

$$AC = ED = 3,000 \text{ m}$$

$$AC = ED = 3000 \text{ m}$$
∴
$$BC = \frac{3000}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{3000\sqrt{3}}{3}$$
⇒
$$BC = 1000\sqrt{3} \text{ m}$$
In rt ΔEDB
$$AD = 1000\sqrt{3} \text{ m}$$
In rt ΔEDB
$$AD = 3000 \text{ m}$$
∴
$$AE = 0000 \text{ m}$$
∴
$$AE = 0.000 \text{ m}$$
∴
$$AE$$

(iv)	Class interval	Frequency (f)	x	d = x - A	$t = \frac{d}{i}$	ft
	0 – 10	4	5	- 30	-3	- 12
	10 – 20	6	15	- 20	-2	- 12
	20-30	10	25	- 10	-1	-10
	30–40	f	35=A	0	0	0
	40–50	6	45	10	1	6
	50–60	4	55	20	2	8

Step deviation method,

 \Rightarrow

and
$$\Sigma f = 30 + f, A = 35, \text{ Class width } (i) = 10$$

$$\Sigma f = -34 + 14$$

$$= -20$$

$$\therefore \qquad \text{Mean} = A + \frac{\Sigma f t}{\Sigma f} \times i$$

$$\Rightarrow \qquad 30 = 35 + \frac{(-20)}{30 + f} \times 10$$

$$\Rightarrow \qquad -5 = \frac{-200}{30 + f}$$

$$\Rightarrow \qquad 30 + f = \frac{200}{5}$$

$$\Rightarrow 30 + f = 40$$

$$\Rightarrow f = 40 - 30$$

$$f = 10$$