Principle of Mathematical Induction

Question 1.
For all n∈N, $3n^5 + 5n^3 + 7n$ is divisible by
(a) 5
(b) 15
(c) 10
(d) 3

Answer: (b) 15
Given number = $3n^5 + 5n^2 + 7n$
Let n = 1, 2, 3, 4, ……..
$3n^5 + 5n^3 + 7n = 3 \times 1^2 + 5 \times 1^3 + 7 \times 1 = 3 + 5 + 7 = 15$
$3n^5 + 5n^3 + 7n = 3 \times 2^5 + 5 \times 2^3 + 7 \times 2 = 3 \times 32 + 5 \times 8 + 7 \times 2 = 96 + 40 + 14 = 150 = 15 \times 10$
$3n^5 + 5n^3 + 7n = 3 \times 3^5 + 5 \times 3^3 + 7 \times 3 = 3 \times 243 + 5 \times 27 + 7 \times 3 = 729 + 135 + 21 = 885 = 15 \times 59$
Since, all these numbers are divisible by 15 for n = 1, 2, 3, …..
So, the given number is divisible by 15

Question 2.
{1 – (1/2)}{1 – (1/3)}{1 – (1/4)} ……. {1 – 1/(n + 1)} =
(a) 1/(n + 1) for all n ∈ N.
(b) 1/(n + 1) for all n ∈ R
(c) n/(n + 1) for all n ∈ N.
(d) n/(n + 1) for all n ∈ R

Answer: (a) 1/(n + 1) for all n ∈ N.
Let the given statement be P(n). Then,
P(n): {1 – (1/2)}{1 – (1/3)}{1 – (1/4)} ……. {1 – 1/(n + 1)} = 1/(n + 1).
When n = 1, LHS = {1 – (1/2)} = ½ and RHS = 1/(1 + 1) = ½.
Therefore LHS = RHS.
Thus, P(1) is true.
Let P(k) be true. Then,

P(k): {1 – (1/2)}{1 – (1/3)}{1 – (1/4)} ……. [1 – {1/(k + 1)}] = 1/(k + 1)
Now, [{1 – (1/2)}{1 – (1/3)}{1 – (1/4)} ……. [1 – {1/(k + 1)}] · [1 – {1/(k + 2)}]
= [1/(k + 1)] · [{(k + 2) – 1}/(k + 2)}]
= [1/(k + 1)] · [(k + 1)/(k + 2)]
= 1/(k + 2)
Therefore p(k + 1): [{1 – (1/2)}{1 – (1/3)}{1 – (1/4)} ……. [1 – {1/(k + 1)}] = 1/(k + 2)
⇒ P(k + 1) is true, whenever P(k) is true.
Thus, P(1) is true and P(k + 1) is true, whenever P(k) is true.
Hence, by the principle of mathematical induction, P(n) is true for all n ∈ N.

Question 3.
For all n ∈ N, $3^{2n} + 7$ is divisible by
(a) non of these
(b) 3
(c) 11
(d) 8

Answer: (d) 8
Given number = 32n + 7
Let n = 1, 2, 3, 4, ……..
$3^{2n} + 7 = 3^2 + 7 = 9 + 7 = 16$
$3^{2n} + 7 = 3^4 + 7 = 81 + 7 = 88$
$3^{2n} + 7 = 3^6 + 7 = 729 + 7 = 736$
Since, all these numbers are divisible by 8 for n = 1, 2, 3, …..
So, the given number is divisible by 8

Question 4.
The sum of the series 1 + 2 + 3 + 4 + 5 + ………..n is
(a) n(n + 1)
(b) (n + 1)/2
(c) n/2
(d) n(n + 1)/2

Answer: (d) n(n + 1)/2
Given, series is series 1 + 2 + 3 + 4 + 5 + ………..n
Sum = n(n + 1)/2

Question 5.
The sum of the series $1^2 + 2^2 + 3^2 + \ldots\ldots\ldots\ldots n^2$ is

(a) $n(n + 1) (2n + 1)$
(b) $n(n + 1) (2n + 1)/2$
(c) $n(n + 1) (2n + 1)/3$
(d) $n(n + 1) (2n + 1)/6$

Answer: (d) $n(n + 1) (2n + 1)/6$
Given, series is $1^2 + 2^2 + 3^2 + \ldots\ldots\ldots\ldots n^2$
Sum = $n(n + 1)(2n + 1)/6$

Question 6.
For all positive integers n, the number $n(n^2 - 1)$ is divisible by:
(a) 36
(b) 24
(c) 6
(d) 16

Answer: (c) 6
Given,
number = $n(n^2 - 1)$
Let n = 1, 2, 3, 4….
$n(n^2 - 1) = 1(1 - 1) = 0$
$n(n^2 - 1) = 2(4 - 1) = 2 \times 3 = 6$
$n(n^2 - 1) = 3(9 - 1) = 3 \times 8 = 24$
$n(n^2 - 1) = 4(16 - 1) = 4 \times 15 = 60$
Since all these numbers are divisible by 6 for n = 1, 2, 3,……..
So, the given number is divisible 6

Question 7.
If n is an odd positive integer, then $a^n + b^n$ is divisible by :
(a) $a^2 + b^2$
(b) $a + b$
(c) $a - b$
(d) none of these

Answer: (b) $a + b$
Given number = $a^n + b^n$
Let n = 1, 3, 5, ……..
$a^n + b^n = a + b$
$a^n + b^n = a^3 + b^3 = (a + b) \times (a^2 + b^2 + ab)$ and so on.
Since, all these numbers are divisible by $(a + b)$ for n = 1, 3, 5,…..
So, the given number is divisible by $(a + b)$

Question 8.
$n(n + 1)(n + 5)$ is a multiple of ____ for all $n \in N$
(a) 2
(b) 3
(c) 5
(d) 7

Answer: (b) 3
Let P(n): $n(n + 1)(n + 5)$ is a multiple of 3.
For n = 1, the given expression becomes $(1 \times 2 \times 6) = 12$, which is a multiple of 3.
So, the given statement is true for n = 1, i.e. P(1) is true.
Let P(k) be true. Then,
P(k): $k(k + 1)(k + 5)$ is a multiple of 3
$\Rightarrow K(k + 1)(k + 5) = 3m$ for some natural number m, …… (i)
Now, $(k + 1)(k + 2)(k + 6) = (k + 1)(k + 2)k + 6(k + 1)(k + 2)$
$= k(k + 1)(k + 2) + 6(k + 1)(k + 2)$
$= k(k + 1)(k + 5 – 3) + 6(k + 1)(k + 2)$
$= k(k + 1)(k + 5) – 3k(k + 1) + 6(k + 1)(k + 2)$
$= k(k + 1)(k + 5) + 3(k + 1)(k + 4)$ [on simplification]
$= 3m + 3(k + 1)(k + 4)$ [using (i)]
$= 3[m + (k + 1)(k + 4)]$, which is a multiple of 3
$\Rightarrow P(k + 1)(k + 1)(k + 2)(k + 6)$ is a multiple of 3
$\Rightarrow$ P(k + 1) is true, whenever P(k) is true.
Thus, P(1) is true and P(k + 1) is true, whenever P(k) is true.
Hence, by the principle of mathematical induction, P(n) is true for all $n \in N$.

Question 9.
For any natural number n, $7^n – 2^n$ is divisible by
(a) 3
(b) 4
(c) 5
(d) 7

Answer: (c) 5
Given, $7^n – 2^n$
Let n = 1
$7^n – 2^n = 7^1 – 2^1 = 7 – 2 = 5$
which is divisible by 5
Let n = 2
$7^n – 2^n = 72 – 22 = 49 – 4 = 45$
which is divisible by 5
Let n = 3

$7^n – 2^n = 7^3 – 2^3 = 343 – 8 = 335$
which is divisible by 5
Hence, for any natural number n, $7^n – 2^n$ is divisible by 5

Question 10.
The sum of the series $1^3 + 2^3 + 3^3 + \ldots\ldots n^3$ is
(a) $\{(n + 1)/2\}^2$
(b) $\{n/2\}^2$
(c) $n(n + 1)/2$
(d) $\{n(n + 1)/2\}^2$

Answer: (d) $\{n(n + 1)/2\}^2$
Given, series is $1^3 + 2^3 + 3^3 + \ldots\ldots n^3$
Sum = $\{n(n + 1)/2\}^2$

Question 11.
$(1^2 + 2^2 + \ldots\ldots + n^2)$ _____ for all values of $n \in N$
(a) $= n^3/3$
(b) $< n^3/3$
(c) $> n^3/3$
(d) None of these

Answer: (c) $> n^3/3$
Let P(n): $(1^2 + 2^2 + \ldots.. + n^2) > n^3/3$.
When = 1, LHS = $1^2 = 1$ and RHS = $1^3/3 = 1/3$.
Since $1 > 1/3$, it follows that P(1) is true.
Let P(k) be true. Then,
P(k): $(1^2 + 2^2 + \ldots.. + k^2) > k^3/3$ …. (i)
Now,
$1^2 + 2^2 + \ldots.. + k^2$
$+ (k + 1)^2$
$= \{1^2 + 2^2 + \ldots.. + k^2 + (k + 1)^2$
$> k^3/3 + (k + 1)^3$ [using (i)]
$= 1/3 \cdot (k^3 + 3 + (k + 1)^2) = 1/3 \cdot \{k^2 + 3k^2 + 6k + 3\}$
$= 1/3[k^3 + 1 + 3k(k + 1) + (3k + 2)]$
$= 1/3 \cdot [(k + 1)^3 + (3k + 2)]$
$> 1/3(k + 1)^3$
P(k + 1):
$1^2 + 2^2 + \ldots.. + k^2 + (k + 1)^2$
$> 1/3 \cdot (k + 1)^3$
P(k + 1) is true, whenever P(k) is true.

Thus P(1) is true and P(k + 1) is true whenever p(k) is true.
Hence, by the principle of mathematical induction, P(n) is true for all $n \in N$.

Question 12.
$\{1/(3 \cdot 5)\} + \{1/(5 \cdot 7)\} + \{1/(7 \cdot 9)\} + \ldots\ldots. + 1/\{(2n + 1)(2n + 3)\} =$
(a) $n/(2n + 3)$
(b) $n/\{2(2n + 3)\}$
(c) $n/\{3(2n + 3)\}$
(d) $n/\{4(2n + 3)\}$

Answer: (c) $n/\{3(2n + 3)\}$
Let the given statement be P(n). Then,
P(n): $\{1/(3 \cdot 5) + 1/(5 \cdot 7) + 1/(7 \cdot 9) + \ldots\ldots. + 1/\{(2n + 1)(2n + 3)\} = n/\{3(2n + 3)$.
Putting n = 1 in the given statement, we get
and LHS = $1/(3 \cdot 5) = 1/15$ and RHS = $1/\{3(2 \times 1 + 3)\} = 1/15$.
LHS = RHS
Thus, P(1) is true.
Let P(k) be true. Then,
P(k): $\{1/(3 \cdot 5) + 1/(5 \cdot 7) + 1/(7 \cdot 9) + \ldots\ldots.. + 1/\{(2k + 1)(2k + 3)\} = k/\{3(2k + 3)\}$ ….. (i)
Now, $1/(3 \cdot 5) + 1/(5 \cdot 7) + \ldots\ldots. + 1/[(2k + 1)(2k + 3)] + 1/[\{2(k + 1) + 1\}2(k + 1) + 3$
$= \{1/(3 \cdot 5) + 1/(5 \cdot 7) + \ldots\ldots. + [1/(2k + 1)(2k + 3)]\} + 1/\{(2k + 3)(2k + 5)\}$
$= k/[3(2k + 3)] + 1/[2k + 3)(2k + 5)]$ [using (i)]
$= \{k(2k + 5) + 3\}/\{3(2k + 3)(2k + 5)\}$
$= (2k^2 + 5k + 3)/[3(2k + 3)(2k + 5)]$
$= \{(k + 1)(2k + 3)\}/\{3(2k + 3)(2k + 5)\}$
$= (k + 1)/\{3(2k + 5)\}$
$= (k + 1)/[3\{2(k + 1) + 3\}]$
= P(k + 1): $1/(3 \cdot 5) + 1/(5 \cdot 7) + \ldots\ldots.. + 1/[2k + 1)(2k + 3)] + 1/[\{2(k + 1) + 1\}\{2(k + 1) + 3\}]$
$= (k + 1)/\{3\{2(k + 1) + 3\}]$
⇒ P(k + 1) is true, whenever P(k) is true.
Thus, P(1) is true and P(k + 1) is true, whenever P(k) is true.
Hence, by the principle of mathematical induction, P(n) is true for $n \in N$.

Question 13.
If n is an odd positive integer, then $a^n + b^n$ is divisible by :
(a) $a^2 + b^2$
(b) $a + b$
(c) $a - b$
(d) none of these

Answer: (b) a + b
Given number = $a^n + b^n$
Let n = 1, 3, 5, ……..
$a^n + b^n = a + b$
$a^n + b^n = a^3 + b^3 = (a + b) \times (a^2 + b^2 + ab)$ and so on.
Since, all these numbers are divisible by (a + b) for n = 1, 3, 5,…..
So, the given number is divisible by (a + b)

Question 14.
$(2 \cdot 7^N + 3 \cdot 5^N - 5)$ is divisible by ……….. for all N ∈ N
(a) 6
(b) 12
(c) 18
(d) 24

Answer: (d) 24
Let P(n): $(2 \cdot 7^n + 3 \cdot 5^n - 5)$ is divisible by 24.
For n = 1, the given expression becomes $(2 \cdot 7^1 + 3 \cdot 5^1 - 5) = 24$, which is clearly divisible by 24.
So, the given statement is true for n = 1, i.e., P(1) is true.
Let P(k) be true. Then,
P(k): $(2 \cdot 7^n + 3 \cdot 5^n - 5)$ is divisible by 24.
⇒ $(2 \cdot 7^n + 3 \cdot 5^n - 5) = 24m$, for m = N
Now, $(2 \cdot 7^n + 3 \cdot 5^n - 5)$
$= (2 \cdot 7^k \cdot 7 + 3 \cdot 5^k \cdot 5 - 5)$
$= 7(2 \cdot 7^k + 3 \cdot 5^k - 5) = 6 \cdot 5^k + 30$
$= (7 \times 24m) - 6(5^k - 5)$
$= (24 \times 7m) - 6 \times 4p$, where $(5^k - 5) = 5(5^{k-1} - 1) = 4p$
[Since $(5^{k-1} - 1)$ is divisible by (5 – 1)]
$= 24 \times (7m - p)$
= 24r, where r = (7m – p) ∈ N
⇒ P (k + 1): $(2 \cdot 7^k + 13 \cdot 5^k + 1 - 5)$ is divisible by 24.
⇒ P(k + 1) is true, whenever P(k) is true.
Thus, P(1) is true and P(k + 1) is true, whenever P(k) is true.
Hence, by the principle of mathematical induction, P(n) is true for all n ∈ N.

Question 15.
For all n∈N, $5^{2n} - 1$ is divisible by
(a) 26
(b) 24

(c) 11
(d) 25

Answer: (b) 24
Given number = $5^{2n} - 1$
Let n = 1, 2, 3, 4, ……..
$5^{2n} - 1 = 5^2 - 1 = 25 - 1 = 24$
$5^{2n} - 1 = 5^4 - 1 = 625 - 1 = 624 = 24 \times 26$
$5^{2n} - 1 = 5^6 - 1 = 15625 - 1 = 15624 = 651 \times 24$
Since, all these numbers are divisible by 24 for n = 1, 2, 3, …..
So, the given number is divisible by 24

Question 16.
$1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \ldots.. + n(n + 1) =$
(a) n(n + 1)(n + 2)
(b) {n(n + 1)(n + 2)}/2
(c) {n(n + 1)(n + 2)}/3
(d) {n(n + 1)(n + 2)}/4

Answer: (c) {n(n + 1)(n + 2)}/3
Let the given statement be P(n). Then,
P(n): $1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \ldots.. + n(n + 1) = (1/3)\{n(n + 1)(n + 2)\}$
Thus, the given statement is true for n = 1, i.e., P(1) is true.
Let P(k) be true. Then,
P(k): $1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \ldots.. + k(k + 1) = (1/3)\{k(k + 1)(k + 2)\}$.
Now, $1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \ldots + k(k + 1) + (k + 1)(k + 2)$
$= (1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \ldots\ldots. + k(k + 1)) + (k + 1)(k + 2)$
$= (1/3)\, k(k + 1)(k + 2) + (k + 1)(k + 2)$ [using (i)]
$= (1/3)\, [k(k + 1)(k + 2) + 3(k + 1)(k + 2)$
$= (1/3)\{(k + 1)(k + 2)(k + 3)\}$
$\Rightarrow$ P(k + 1): $1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \ldots\ldots + (k + 1)(k + 2)$
$= (1/3)\{k + 1)(k + 2)(k + 3)\}$
$\Rightarrow$ P(k + 1) is true, whenever P(k) is true.
Thus, P(1) is true and P(k + 1)is true, whenever P(k) is true.
Hence, by the principle of mathematical induction, P(n) is true for all values of $\in$ N.

Question 17.
$1/(1 \cdot 2 \cdot 3) + 1/(2 \cdot 3 \cdot 4) + \ldots\ldots.. + 1/\{n(n + 1)(n + 2)\} =$
(a) {n(n + 3)}/{4(n + 1)(n + 2)}
(b) (n + 3)/{4(n + 1)(n + 2)}

(c) $n/\{4(n + 1)(n + 2)\}$
(d) None of these

Answer: (a) $\{n(n + 3)\}/\{4(n + 1)(n + 2)\}$
Let P (n): $1/(1 \cdot 2 \cdot 3) + 1/(2 \cdot 3 \cdot 4) + \ldots\ldots. + 1/\{n(n + 1)(n + 2)\} = \{n(n + 3)\}/\{4(n + 1)(n + 2)\}$
Putting $n = 1$ in the given statement, we get
LHS $= 1/(1 \cdot 2 \cdot 3) = 1/6$ and RHS $= \{1 \times (1 + 3)\}/[4 \times (1 + 1)(1 + 2)] = (1 \times 4)/(4 \times 2 \times 3) = 1/6$.
Therefore LHS = RHS.
Thus, the given statement is true for n = 1, i.e., P(1) is true.
Let P(k) be true. Then,
P(k): $1/(1 \cdot 2 \cdot 3) + 1/(2 \cdot 3 \cdot 4) + \ldots\ldots\ldots + 1/\{k(k + 1) (k + 2)\} = \{k(k + 3)\}/\{4(k + 1) (k + 2)\}$.
…….(i)
Now, $1/(1 \cdot 2 \cdot 3) + 1/(2 \cdot 3 \cdot 4) + \ldots\ldots\ldots\ldots.. + 1/\{k(k + 1) (k + 2)\} + 1/\{(k + 1) (k + 2) (k + 3)\}$
$= [1/(1 \cdot 2 \cdot 3) + 1/(2 \cdot 3 \cdot 4) + \ldots\ldots\ldots\ldots + 1/\{ k(k + 1) (k + 2\}] + 1/\{(k + 1)(k + 2) (k + 3)\}$
$= [\{k(k + 3)\}/\{4(k + 1)(k + 2)\} + 1/\{(k + 1)(k + 2)(k + 3)\}]$ [using(i)]
$= \{k(k + 3)^2 + 4\}/\{4(k + 1)(k + 2) (k + 3)\}$
$= (k^3 + 6k^2 + 9k + 4)/\{4(k + 1) (k + 2) (k + 3)\}$
$= \{(k + 1) (k + 1) (k + 4)\}/\{4 (k + 1) (k + 2) (k + 3)\}$
$= \{(k + 1) (k + 4)\}/\{4(k + 2) (k + 3)$
$\Rightarrow$ P(k + 1): $1/(1 \cdot 2 \cdot 3) + 1/(2 \cdot 3 \cdot 4) + \ldots\ldots\ldots\ldots.. + 1/\{(k + 1) (k + 2) (k + 3)\}$
$= \{(k + 1) (k + 2)\}/\{4(k + 2) (k + 3)\}$
$\Rightarrow$ P(k + 1) is true, whenever P(k) is true.
Thus, P(1) is true and P(k + 1) is true, whenever P(k) is true.
Hence, by the principle of mathematical induction, P(n) is true for all $n \in N$.

Question 18.
For any natural number n, $7^n - 2^n$ is divisible by
(a) 3
(b) 4
(c) 5
(d) 7

Answer: (c) 5
Given, $7^n - 2^n$
Let n = 1
$7^n - 2^n = 7^1 - 2^1 = 7 - 2 = 5$
which is divisible by 5
Let n = 2
$7^n - 2^n = 7^2 - 2^2 = 49 - 4 = 45$
which is divisible by 5
Let n = 3
$7^n - 2^n = 7^3 - 2^3 = 343 - 8 = 335$

which is divisible by 5
Hence, for any natural number n, $7^n - 2^n$ is divisible by 5

Question 19.
The sum of n terms of the series $1^2 + 3^2 + 5^2 + \dots\dots\dots$ is
(a) $n(4n^2 - 1)/3$
(b) $n^2(2n^2 + 1)/6$
(c) none of these.
(d) $n^2(n^2 + 1)/3$

Answer: (a) $n(4n^2 - 1)/3$
Let $S = 1^2 + 3^2 + 5^2 + \dots\dots\dots (2n - 1)^2$
$\Rightarrow S = \{1^2 + 2^2 + 3^2 + 4^2 \dots\dots\dots (2n - 1)^2 + (2n)^2\} - \{2^2 + 4^2 + 6^2 + \dots\dots\dots + (2n)^2\}$
$\Rightarrow S = \{2n \times (2n + 1) \times (4n + 1)\}/6 - \{4n \times (n + 1) \times (2n + 1)\}/6$
$\Rightarrow S = n(4n^2 - 1)/3$

Question 20.
For all $n \in N$, $3n^5 + 5n^3 + 7n$ is divisible by:
(a) 5
(b) 15
(c) 10
(d) 3

Answer: (b) 15
Given number $= 3n^5 + 5n^3 + 7n$
Let n = 1, 2, 3, 4, ……..
$3n^5 + 5n^3 + 7n = 3 \times 1^2 + 5 \times 1^3 + 7 \times 1 = 3 + 5 + 7 = 15$
$3n^5 + 5n^3 + 7n = 3 \times 2^5 + 5 \times 2^3 + 7 \times 2 = 3 \times 32 + 5 \times 8 + 7 \times 2 = 96 + 40 + 14 = 150 = 15 \times 10$
$3n^5 + 5n^3 + 7n = 3 \times 3^5 + 5 \times 3^3 + 7 \times 3 = 3 \times 243 + 5 \times 27 + 7 \times 3 = 729 + 135 + 21 = 885 = 15 \times 59$
Since, all these numbers are divisible by 15 for n = 1, 2, 3, …..
So, the given number is divisible by 15
