CBSE Test Paper 05

Chapter 7 System of Particles and Rotational Motion

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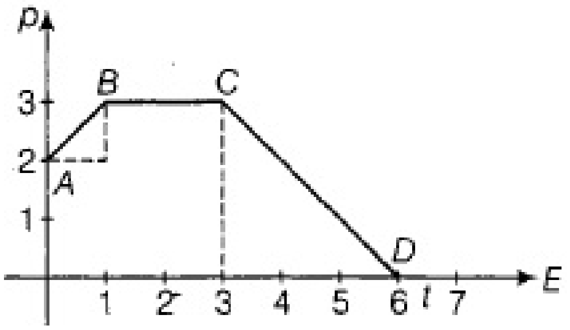
1. In pure translational motion of a rigid body **1**
   a. at any instant of time every particle of the body has the same velocity
   b. at any instant of time different particles of the body have different velocities.
   c. at any instant of time velocity is dependent on the position vector of a point on the body
   d. at different instants of time every particle of the body has the same velocity.
2. A thin circular ring of mass M and radius R is rotating about its central axis with angular velocity. Four point objects each of mass m are attached gently to the opposite ends of two perpendicular diameters, the angular velocity of the ring is given by **1**
   a. $\frac{M-4m}{M+4m} \cdot \omega$
   b. $\frac{M+4m}{M} \cdot \omega$
   c. $\frac{M}{M+m} \cdot \omega$
   d. $\frac{M}{M+4m} \cdot \omega$
3. If a gymnast sitting on a rotating stool with his arms outstretched, suddenly lowers his hands **1**
   a. the angular velocity decreases
   b. his moment of inertia decreases
   c. the angular momentum increases
   d. the angular velocity stays constant
4. Two discs of the same mass and same thickness t are made from two different materials of densities d_1 and d_2 respectively. The ratio of the moments of inertia of the two about an axis passing through the centre and perpendicular to the plane of lamina is **1**
   a. 1: $d_1 d_2$
   b. d_1/d_2 : 1
   c. $d_2 : d_1$
   d. $d_1 : d_2$
5. The vector product of two vectors a and b is a vector c such that the magnitude of c is

given by **1**

a. |a||b| cos θ
b. |a||b| tan θ
c. |a||b| cot θ
d. |a||b| sin θ

6. When a labourer cuts down a tree, he makes a cut on the side facing the direction in which he wants it to fall. Why? **1**
7. Is radius of gyration a constant quantity? **1**
8. Is centre of mass and centre of gravity body always coincide? **1**
9. Prove that the centre of mass of two particles divides the line joining the particles in the inverse ratio of their masses. **2**
10. Explain how a cat is able to land on its feet after a fall taking advantage of the principle of conservation of angular momentum? **2**
11. The figure shows momentum versus time graph for a particle moving along x-axis. In which region force on the particle is large. Why? **2**



12. If angular momentum is conserved in a system whose moment of inertia is decreased, will its rotational kinetic energy be also conserved? Explain. **3**
13. A wheel has a constant angular acceleration of 4.2 rad/s^2. During a certain 8.05 s interval, it turns through angle of 140 rad. Assuming that wheel started from rest, how long it had been in motion before the start of the 8.0 s? **3**
14. State the conditions for complete equilibrium of a body. **3**
15. i. Find the moment of inertia of a sphere about a tangent to the sphere, given the moment of inertia of the sphere about any of its diameters to be $\frac{2MR^2}{5}$, where M is the mass of the sphere and R is the radius of the sphere.
    ii. Given the moment of inertia of a disc of mass M and radius R about any of its diameters to be $\frac{MR^2}{4}$, find its moment of inertia about an axis normal to the disc and passing through a point on its edge. **5**

Answer

1. a. at any instant of time every particle of the body has the same velocity.
   Explanation: In translational motion when the body moves along a straight line or more exactly when every point of the body travels on paralell lines, thus at any instant of time every particle of the body has the same velocity.

2. d. $\frac{M}{M+4m} \cdot \omega$
   Explanation: Let ω be the angular velocity of the Ring of Mass M , thus the moment of inetia about given axis is $I_1 = MR^2$ and the four point objects are genlty placed at perpendicular diameters at opposite end, so thus the distance of each object from axis of rotation is R, so total moment of inetia of ring and four objects is $I_2 = MR^2 + 4mR^2$.
   According to law of conservation of angular momentum $I_1 \omega = I_2\omega_2$, So on solving $\omega_2 = \left(\frac{MR^2}{MR^2+4mR^2}\right)\omega = \frac{M}{M+4m} \cdot \omega$

3. b. his moment of inertia decreases
   Explanation: When gymnast lowers his hand the distance of mass from rotational axis decrease. Hence his moment of inertia decreases and angular velocity increase to conserve angular momentum.

4. c. $d_2 : d_1$
   Explanation: if V = volume of disc (At), A = area of disc = πr^2, t = thickness of disc, M = mass of disc, d = density of material of disc
   $V = \frac{M}{d}$
   $At = \frac{M}{d}$
   $\pi r^2 t = \frac{M}{d}$
   $r^2 = \frac{M}{\pi d_1 t}$
   $r^2 \propto \frac{1}{d}$
   $\left(\frac{r_1}{r_2}\right)^2 = \frac{d_2}{d_1}$

$$\frac{I_1}{I_2} = \frac{\frac{1}{2}M{r_1}^2}{\frac{1}{2}M{r_1}^2} = \left(\frac{r_1}{r_2}\right)^2 = \frac{d_2}{d_1}$$

$I_1 : I_2 = d_2 : d_1$

5. d. $|a||b| \sin\theta$

Explanation: As per definition of vector product :-

$$\vec{c} = \vec{a} \times \vec{b} = |\vec{a}|\left|\vec{b}\right| \sin\theta\hat{n}$$

$$|\vec{c}| = |\vec{a}|\left|\vec{b}\right| \sin\theta$$

6. Because the weight of tree exerts a torque about the point where the cut is made. This causes rotation of the tree about the cut, i.e. the direction in which the labourer wants the tree to fall.
7. No, it changes with the position of axis of rotation.
8. No, it is not necessary for CG and CM to coincide. If the body is large such that g varies from one point to another, or it is in non-uniform gravitational field, the centre of gravity is offset from centre of mass. But for small bodies, centre of mass and centre of gravity lies at their geometrical centres.
9. $\overline{r}_{\text{cm}} = \frac{m_1\bar{r}_1 + m_2\bar{r}_2}{m_1 + m_2}$

   If centre of mass is at the origin

   $\overline{r}_{cm} = 0$

   $\Rightarrow m_1\overline{r}_1 + m_2\overline{r}_2 = 0$

   $m_1\overline{r}_1 = -m_2\overline{r}_2$

   In terms of magnitude $m_1\left|\overline{r}_1\right| = m_2\left|\overline{r}_2\right|$

   $\Rightarrow \frac{m_1}{m_2} = \frac{r_2}{r_1}$
10. The cat stretches its body along with the tail when it falls to ground from a height, so that its moment of inertia becomes high. Since $I\omega = constant$, the value of angular speed ω decreases resulting in the cat be able to land on the ground gently.
11. The net force, $F_{\text{net}} = \frac{dp}{dt}$

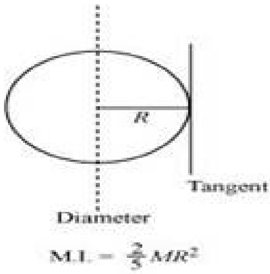
    Also, the rate of change of momentum = slope of the graph.

    From the graph, slope AB = slope CD

    And slope (BC) = slope (DE) = 0

    Therefore, the force acting on the particle is equal in regions AB and CD and in regions BC and DE (which is zero).

12. If angular momentum of a system is conserved but its moment of inertia decreases, then its rotational kinetic energy is not conserved but increases as explained below:
As angular momentum, L = $I\omega$ and rotational kinetic energy, $K_R = \frac{1}{2} I\omega^2$
$\Rightarrow K_R = \frac{L^2}{2I}$
where I is moment of inertia.
From above equation, it is clear that the rotational kinetic energy is inversely proportional to moment of inertia for a given value of angular momentum L. Thus, if moment of inertia of the system decreases, then kinetic energy of rotation increases.
13. suppose, $\omega_0 =$ initial angular speed at $t = 0$
Angle turned at the end of 8.0 s is 140°.
using $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
$\Rightarrow \omega_0 = \frac{\theta - \frac{1}{2}\alpha t^2}{t}$
$\omega_0 = \frac{140 - \frac{1}{2}(4.2)(8.0)^2}{8.0}$
ω_0 = 0.7 rad/s
Using $\omega = \omega_0 + \alpha t$ and taking $\omega = 0.$
$\Rightarrow t = \frac{\omega - \omega_0}{\alpha} = \frac{0 - \omega_0}{4.2}$
$= -\frac{0.7}{4.2} = -0.16s$
So, wheel starts from rest $0.16s$ before.
14. For complete equilibrium, condition for translational equilibrium and condition for rotational equilibrium both must be fulfilled. The conditions are :
    i. **For translational motion,** we know that $\frac{d\vec{p}}{dt} = \sum \vec{F}_{ext}$
    For equilibrium $\vec{p}$ = constant or $\frac{d\vec{p}}{dt} = 0$ or $\sum \vec{F}_{ext} = 0$
    Hence for translational equilibrium, the vector sum of all the external forces acting on the system/body must be zero. In other words we can say, that for an object to be in equilibrium, it must be experiencing no acceleration.
    ii. **For rotational motion,** we have the relation, $\frac{d\vec{L}}{dt} = \sum \tau_{ext}$
    For equilibrium, $\vec{L}$ = constant or $\frac{d\vec{L}}{dt} = 0$ or $\sum \vec{\tau}_{ext} = 0$
    Hence, the vector sum of all the external torques acting on the system must be zero for rotational equilibrium.
15. a. The moment of inertia (M.I.) of a sphere about its diameter $= \frac{2}{5} MR^2$



Given,

Moment of inertia of the sphere about its diametre = $(\frac{2}{5})mR^2$

Use, parallel axis theorem,

Moment of inertia of the sphere about tangent = $I + mR^2$

$= (\frac{2}{5})\ mR^2 + mR^2$

$= (7/5)\ mR^2$

b. Moment of inertia of disc of mass m and radius R about any of its diametre = $mR^2/4$

Moment of inertia about diametre = $I_x = I_y = (\frac{1}{4})mR^2$

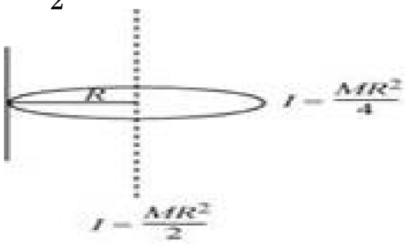
Using, perpendicular axis theorem,

$I_z = I_x + I_y$

Where I_z is moment of inertia about perpendicular axis of plane of disc.

$I_z = (\frac{1}{4})\ mR^2 + (\frac{1}{4})\ mR^2$

$= (\frac{1}{2})\ mR^2$



Moment of inertia of disc about passing through a point of its edge

Use , parallel axis theorem,

$I = I_z + mR^2$

$= (\frac{1}{2})\ mR^2 + mR^2$

$= (\frac{1}{2})\ mR^2$