12. Circle

Exercise 12.1

1. Question

Fill in the blank:

(i) The centre of the circle lies in …….. of the circle. (exterior/interior)

(ii) A point, where distance from the centre of a circle is greater than its radius lies in ………. of the circle. (exterior/interior)

(iii) The longest chord of a circle is a …….. of the circle.

(iv) An arc is a …………… when its ends are the ends of a diameter.

(v) A circle divides the plane, on which it lies, in ………….. parts.

Answer

(i) The centre of the circle lies in interior of the circle.

We know that the centre is a fixed point in a circle i.e. interior of the circle.

(ii) A point, where distance from the centre of a circle is greater than its radius lies in exterior of the circle.

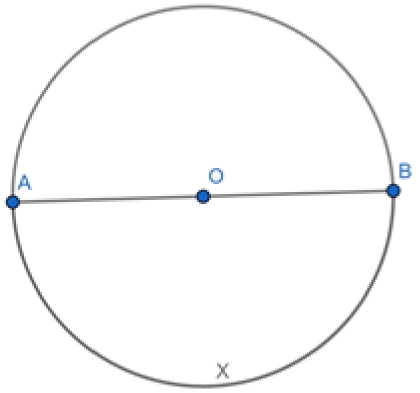
If the point's distance from centre of circle is equals to its radius, then it lies on the circle.

If the distance is greater than the radius, then it lies in exterior of the circle.

(iii) The longest chord of a circle is a diameter of the circle.

A chord that passes through the centre of the circle is the longest chord and therefore it is the diameter.

(iv) An arc is a semicircle when its ends are the ends of the diameter.



Let AB be the diameter and AXB be the arc.

Here, we can see that AXB is a semicircle.

Thus, an arc is a semicircle when its ends are the ends of a diameter.

(v) A circle divides the plane, on which it lies, in three parts.

We know that a circle divides the plane on which it lies into three parts i.e.

1. Interior

2. Circle

3. Exterior

2. Question

Write True/False. Give reason also for your answers.

(i) Line segment joining the centre to any point on the circle is a radius of the circle.

(ii) A circle has only finite number of equal chords.

(iii) If a circle is divided into three equal parts, each is a major are.

(iv) A chord of a circle, which is twice as long as its radius, is a diameter of the circle.

(v) A circle is a plane figure.

(vi) The collection of those points in a plane, which are at a fixed distance from a fixed point in the plane, is called a diameter.

(vii) The chord on which centre lies is called radius.

Answer

(i) True

We know that the constant distance is radius of a circle.

The line segment joining the centre to any point on the circle has constant distance.

∴ Line segment joining the centre to any point on the circle is a radius of the circle.

(ii) False

In a circle, infinitely many diameters can be drawn.

A diameter is the longest chord.

∴ a circle has infinitely many chords.

(iii) False

When a circle is divided into three equal parts, each part will be less than a semicircle.

(iv) True

We know that diameter is twice the radius i.e. $d = 2r$.

∴ A chord of a circle, which is twice as long as its radius is a diameter of the circle.

(v) True

A circle is drawn on a plane which divides it into three parts.

∴ A circle is a plane figure.

(vi) False

The collection of those points in a plane, which are at a fixed distance from a fixed point in the plane, is called a circle.

(vii) False

We know that if any chord passes through the centre of the circle then it is called the diameter of the circle.

Exercise 12.2

1. Question

Write True/False in the following and give the reason of your answer if possible.

(i) AB and CD are chords of measure 3 cm and 4 cm respectively of a circle by which the angles subtended at the centre are respectively 70° and 50°.

(ii) Chords of a circle whose lengths are 10 cm and 8 cm are initiated at distances 8 cm and 5 cm respectively from the centre.

(iii) Out of the two chords AB and CD of a circle each is at a distance of 4 cm from the centre. Then AB = CD.

(iv) Congruent circles with centres O and O' intersect at two points A and B. Then ∠AOB = ∠AO'B.

(v) A circle can be drawn through three collinear points.

(vi) A circle of radius 4 cm can be drawn through two points A and B of AB = 8 cm.

Answer

(i) False

We know that longer chords subtend at greater angles and smaller chords at smaller angles.

Here CD (longer chord) is subtending at 50° (smaller angle) while AB (smaller chord) is subtending at 70° (greater angle).

So, it is false.

(ii) False

We know that longer the chord, smaller the distance from centre.

Here, 10 cm chord is at a distance of 8 cm from the center, hence it is not possible that 8 cm chord is at a distance of 5 cm from the center.

(iii) True

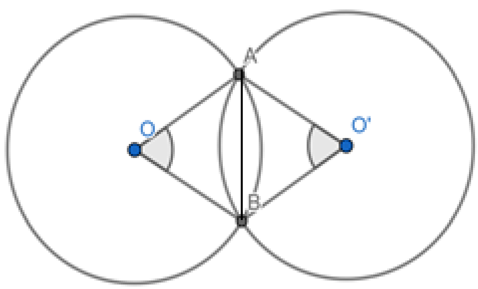
We know that if two chords are at equal distance from the centre, they are equal.

Here, chords AB and CD are at equal distance of 4 cm from the centre.

∴ AB = CD

Hence, it is true.

(iv) True



We know that equal chords of congruent circles subtend equal angles at the corresponding centres.

If we take AB as chord, and as radius of both circle are equal

⇒ ∠AOB = ∠AO'B

So, it is true.

(v) False

A circle passing through two collinear points cannot pass through the third point.

Hence, it is false.

(vi) True

Given radius = 4 cm

We know that diameter is twice the radius.

When a circle is drawn through two points A and B, diameter will be AB = 8 cm.

Taking a compass, from the centre O and OA or OB as radius, we join A to B and B to A.

So, the circle can be drawn through A and B.

∴ It is true.

2. Question

If the radius of a circle is 13 cm and length of its one chord is 10 cm, then find the distance of this chord from the centre.

Answer

We know that Length of chord, $l = 2\sqrt{r^2 - d^2}$

Where r = radius of circle, d = distance of chord from centre

Given radius = 13 cm and chord = 10 cm

Now,

$\Rightarrow\ 10 = 2\sqrt{13^2 - d^2}$

$\Rightarrow\ 10 = 2\sqrt{169 - d^2}$

Squaring on both sides,

$\Rightarrow\ 10^2 = \left(2\sqrt{169 - d^2}\right)^2$

⇒ 100 = 4 (169 – d^2)

⇒ 100/ 4 = 169 – d^2

$\Rightarrow 25 = 169 - d^2$

$\Rightarrow d^2 = 169 - 25$

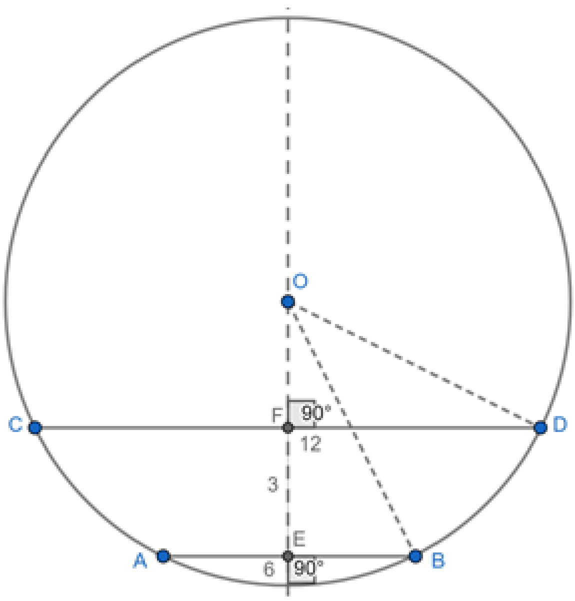
$\Rightarrow d^2 = 144$

∴ Distance of chord, d = 12 cm

3. Question

Two chords AB and CD of a circle whose lengths are 6 cm and 12 cm respectively, are parallel to each other and these lie in the same side of the centre of circle. If the distance between AB and CD be 3 cm, then find the radius of the circle.

Answer



Given chords AB = 6 cm and CD = 12 cm,

Draw OE⊥ AB intersecting CD at F,

As, AB||CD

OF⊥ CD [Corresponding angles]

Distance between AB and CD, EF= 3 cm

Now, we know that perpendicular from center to chord bisects the chord.

Then CF = FD = 1/2 CD = 6 cm

⇒ AE = EB = 1/2 AB = 3 cm

Let OF = y cm, OE = 3 + y cm and OD = OB = x cm = Radius.

Consider ΔOFD,

By Pythagoras Theorem,

$\Rightarrow OD^2 = OF^2 + FD^2$

$\Rightarrow x^2 = y^2 + 6^2$

$\Rightarrow x^2 = y^2 + 36$ … (1)

Consider ΔOEB,

By Pythagoras Theorem,

$\Rightarrow OB^2 = OE^2 + EB^2$

$\Rightarrow x^2 = (3 + y)^2 + 3^2$

$\Rightarrow x^2 = 9 + y^2 + 6y + 9$

$\Rightarrow x^2 = y^2 + 6y + 18$ … (2)

From (1) and (2),

$\Rightarrow y^2 + 36 = y^2 + 6y + 18$

$\Rightarrow 36 = 6y + 18$

$\Rightarrow 6y = 36 – 18$

$\Rightarrow 6y = 18$

$\Rightarrow y = 18/ 6$

$\therefore y = 3$

Substituting y value in (1),

$\Rightarrow x^2 = y^2 + 36$

$\Rightarrow x^2 = 3^2 + 36$

$\Rightarrow x^2 = 9 + 36$

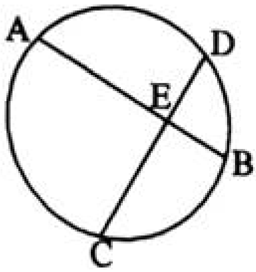
$\Rightarrow x^2 = 45$

$[45 = 3 \times 3 \times 5]$

$\therefore$ Radius $= x = 3\sqrt{5}$ cm

4. Question

In figure, two equal chords AB and CD intersect each other at E. Prove that arc DA = arc CB.



Answer

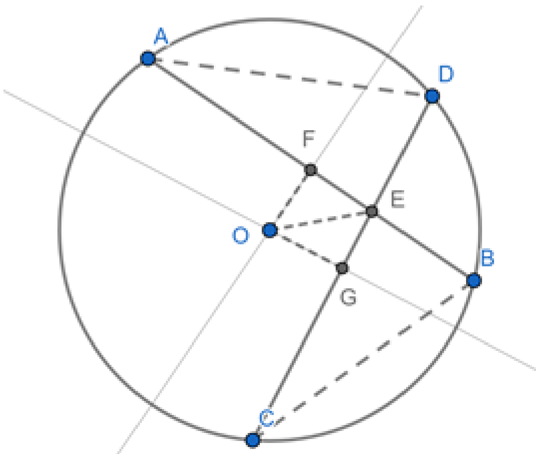
Given chord AB = chord CD intersect at E.

Construction:

Join AD and BC.

Draw OF ⊥ AB and OG ⊥ CD.

Join OE.



We know that when a perpendicular is drawn from centre to chord, it bisects them.

⇒ AF = FB = $\frac{AB}{2}$ and CG = GD = $\frac{CD}{2}$

Given AB = CD,

⇒ AF = FB = CG = GD ... (1)

Consider ΔOFE and ΔOGE,

We know that equal chords of a circle are equidistant from each other.

⇒ OF = OG

⇒ OE = OE [Common side]

⇒ ∠OFE = ∠OGE = 90° [Construction]

By RHS congruence rule,

⇒ ΔOFE ≅ ΔOGE

By CPCT,

⇒ FE = GE … (2)

So,

⇒ AE = AF + FE, CE = CG + GE

And EB = FB – FE, ED = GD – GE

From (1) and (2),

⇒ AE = CE … (3)

And EB = ED … (4)

Consider ΔAED and ΔCEB,

⇒ AE = CE [From (3)]

⇒ EB = ED [From (4)]

⇒ ∠AED = ∠CEB [Vertically opposite angles are equal]

By SAS congruence rule,

⇒ ΔAED ≅ ΔCEB

By CPCT,

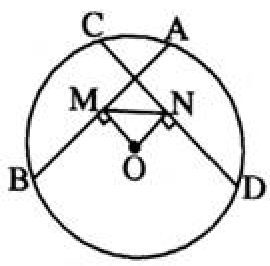
⇒ AD = CB

∴ Arc AD = Arc CB

Hence proved

5. Question

In figure, AB and CD are equal chords of a circle. O is the centre of the circle of OM ⊥ AB and ON ⊥ CD. Then prove that ∠OMN = ∠ONM.



Answer

Given AB and CD are equal chords of a circle with centre O.

Also OM ⊥ AB and ON ⊥ CD.

We have to prove that ∠OMN = ∠ONM.

Proof:

We know that equal chords of a circle are equidistant from the centre.

⇒ OM = ON … (1)

In ΔOMN,

From (1),

⇒ OM = ON

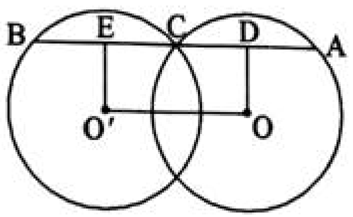
We know that angles opposite to equal sides of a triangle are equal.

∴ ∠OMN = ∠ONM

Hence proved.

6. Question

In figure, O and O’ are the centre of the given circle. AB || OO’. Prove that AB = 2 OO’.



Answer

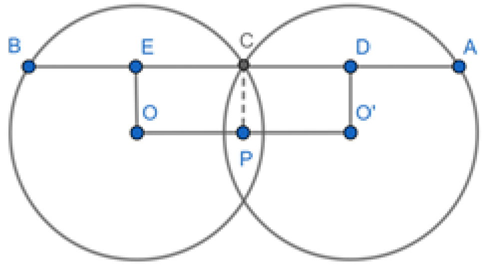
Given O and O’ are the centres of the given circles.

Also AB || OO’

We have to prove that AB = 2OO’.

Construction:

Draw perpendicular CP from point C on OO’.



Proof:

We know that perpendicular from the centre of a circle to a chord bisects the chord.

⇒ BE = EC and CD = DA

Since AB || OO' and O'E || PC || OD, [since all are perpendiculars on line AB]

⇒ EC = O'P and CD = PO

⇒ OO' = OP + PO'

= CD + EC

= 1/2 BC + 1/2 AC

= 1/2 (BC + AC)

= 1/2 AB

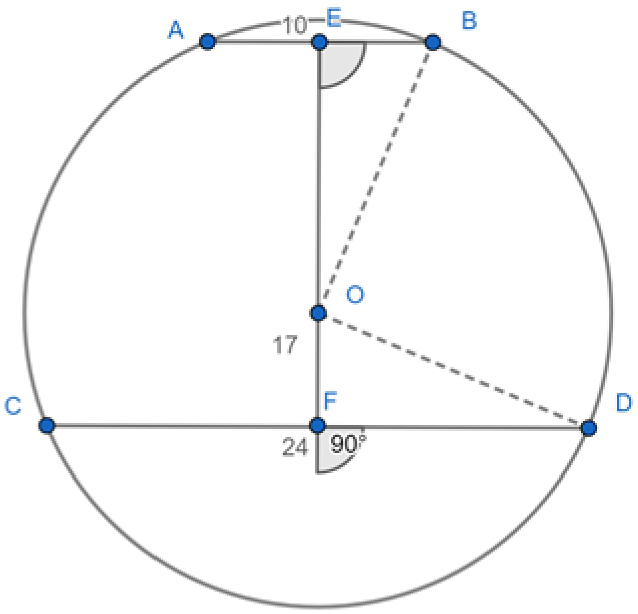
∴ 2OO' = AB

Hence proved.

7. Question

AB and CD are two chords of a circle such that AB = 10 cm, CD = 24 cm and AB || CD. The distance between AB and CD is 17 cm. Find the radius of the circle.

Answer



Given chords AB = 10 cm, CD = 24 cm

Distance between AB and CD = 17 cm

From the figure,

OE ⊥ AB and OF ⊥CD.

Now, we know perpendicular from the center to the chord bisects the chord.

⇒ AE = EB = 1/2 AB = 5 cm

$\Rightarrow$ CF = FD = 1/2 CD = 12 cm

Let OF = y cm, OE = 17 - y cm and OB = OD = x cm = Radius.

Consider ΔOEB,

By Pythagoras Theorem,

$\Rightarrow OB^2 = OE^2 + EB^2$

$\Rightarrow x^2 = (17 - y)^2 + 5^2$

$\Rightarrow x^2 = 289 + y^2 - 34y + 25$

$\Rightarrow x^2 = y^2 - 34y + 314$ ... (1)

Consider ΔOFD,

By Pythagoras Theorem,

$\Rightarrow OD^2 = OF^2 + FD^2$

$\Rightarrow x^2 = y^2 + 12^2$

$\Rightarrow x^2 = y^2 + 144$ ... (2)

From (1) and (2),

$\Rightarrow y^2 - 34y + 314 = y^2 + 144$

$\Rightarrow -34y + 314 = 144$

$\Rightarrow 34y = 314 - 144$

$\Rightarrow 34y = 170$

$\Rightarrow y = 170/ 34$

$\therefore y = 5$

Substituting y value in (2),

$\Rightarrow x^2 = 5^2 + 144$

$\Rightarrow x^2 = 25 + 144$

$\Rightarrow x^2 = 169$

$\therefore$ Radius = x = 13 cm

8. Question

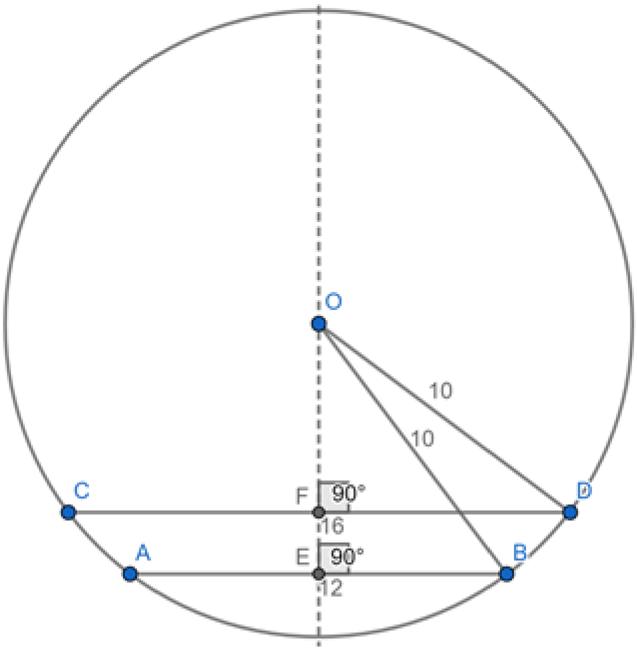
In a circle of radius 10 cm, the lengths of two parallel chords are 12 cm and 16 cm respectively. Find the distance between AB and CD. If chords (a) are on the same side of the centre (b) are on the opposite sides of the centre.

Answer

Given radius of circle = 10 cm

Chords AB = 12 cm and CD = 16 cm

(a)



Draw OE, OF⊥ AB, CD.

We know that when a perpendicular is drawn from centre to chord, it bisects them.

Then CF = FD = 1/2 CD = 8 cm

⇒ AE = EB = 1/2 AB = 6 cm

Consider ΔOFD,

By Pythagoras Theorem,

⇒ $OD^2 = OF^2 + FD^2$

⇒ $10^2 = x^2 + 8^2$

⇒$100^2 = x^2 + 64$

⇒$x^2 = 100 – 64$

⇒$x^2 = 36$

∴ OF = x = 6 cm

Consider ΔOEB,

By Pythagoras Theorem,

$\Rightarrow OB^2 = OE^2 + EB^2$

$\Rightarrow 10^2 = y^2 + 6^2$

$\Rightarrow 100 = y^2 + 36$

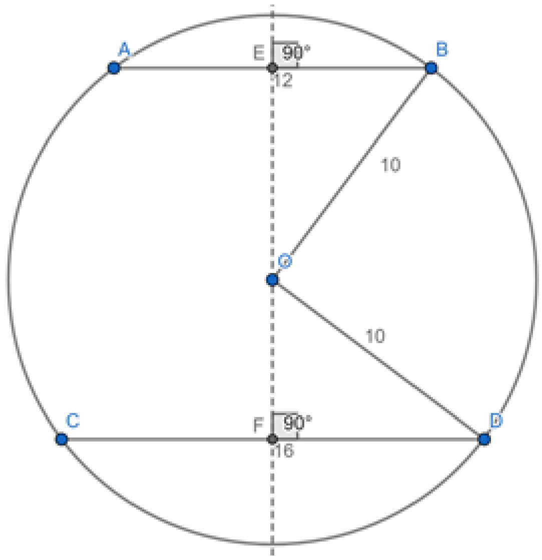
$\Rightarrow y^2 = 100 - 36$

$\Rightarrow y^2 = 64$

$\therefore OE = y = 8$ cm

$\therefore$ Distance between chords AB and CD = EF = OE − OF = y − x = 8 − 6 = 2 cm

(b)



Draw OE, OF ⊥ AB, CD.

We know that when a perpendicular is drawn from centre to chord, it bisects them.

Then CF = FD = 1/2 CD = 8 cm

⇒ AE = EB = 1/2 AB = 6 cm

Consider ΔOFD,

By Pythagoras Theorem,

$\Rightarrow OD^2 = OF^2 + FD^2$

$\Rightarrow 10^2 = x^2 + 8^2$

$\Rightarrow 100^2 = x^2 + 64$

$\Rightarrow x^2 = 100 - 64$

$\Rightarrow x^2 = 36$

$\therefore$ OF = x = 6 cm

Consider ΔOEB,

By Pythagoras Theorem,

$\Rightarrow OB^2 = OE^2 + EB^2$

$\Rightarrow 10^2 = y^2 + 6^2$

$\Rightarrow 100 = y^2 + 36$

$\Rightarrow y^2 = 100 - 36$

$\Rightarrow y^2 = 64$

$\therefore$ OE = y = 8 cm

$\therefore$ Distance between chords AB and CD is

$\Rightarrow$ EF = OE + OF

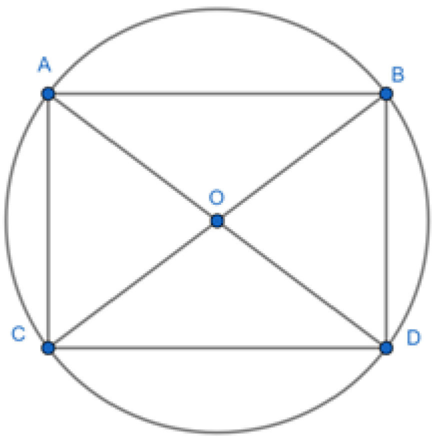
= 8 + 6

= 14 cm

9. Question

The vertices of quadrilateral ABCD lie on a circle such that AB = CD. Then prove that AC = BD.

Answer



Given AB = CD.

Construction:

Join AD and BC.

Consider ΔABC and ΔBCD,

⇒ AB = CD [Given]

⇒ ∠ABC = ∠BCD [Vertically opposite angles are equal]

⇒ BC = BC [Common side]

By SAS congruence rule,

⇒ ΔABC ≅ ΔBCD

By CPCT,

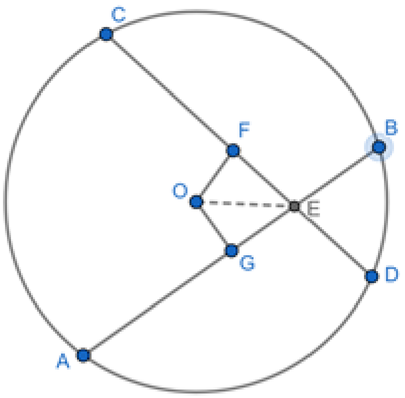
⇒ AC = BD

Hence proved.

10. Question

If two equal chords of a circle intersect each other then prove that the ordered points of one chord are respectively equal to the corresponding points of the record chord.

Answer



Let AB and CD be two equal chords of a circle which are intersecting at a point E.

Construction:

Draw OF and OG perpendiculars on the chords

Join OE.

Consider ΔOFE and ΔOGE,

⇒ OF = OG [Equal chords]

⇒∠OFE = ∠OGE [Each 90°]

⇒ OE = OE [Common]

By RHS congruence rule,

⇒ ΔOFE ≅ ΔOGE

By CPCT,

⇒ FE = GE … (1)

Given AB = CD … (2)

⇒ 1/2 AB = 1/2 CD

⇒ AG = CF … (3)

Adding equations (1) and (3),

⇒ AG + GE = CF + FE

⇒ AE = CE … (4)

Subtracting equation (4) from (2),

⇒ AB – AE = CD - CE

⇒ BE = DE … (5)

From (4) and (5),

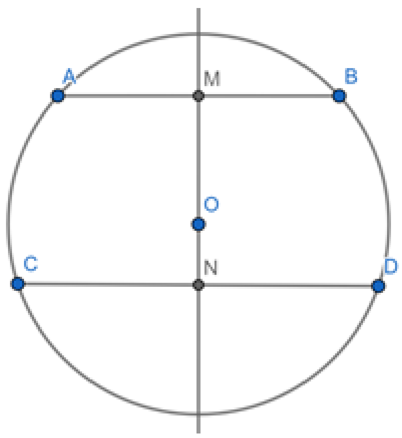
We can see that two parts of one chord are equal to two parts of another chord.

Hence proved

11. Question

Prove that the line segment joining the mid-points of two parallel chords of a circle passes through the centre of the circle.

Answer



Let AB and CD be the two parallel chords of a circle such that M and N are the mid-points of AB and CD respectively.

Since the perpendicular bisector of the chord passes through the centre,

⇒ ON ⊥ CD and OM ⊥ AB

Since AB || CD, NOM is a straight line.

Hence the line joining the midpoints of two parallel chords of a circle passes through the centre of the circle.

Exercise 12.3

1. Question

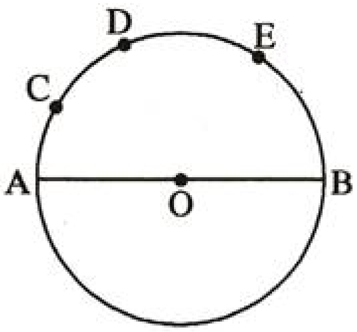
Write True/False for each and write the reason also for your answer.

(i) The angles subtended at any two points lying of the circle by every chord are equal.

(ii) In figure, AB is a diameter of a circle and C is any point on the circle. Then $AC^2 + BC^2 = AB^2$.

(iii) In figure 12.48, if ∠ADE = 120°, then ∠EAB = 30°.

(iv) In figure 12.48, ∠CAD = ∠CED.



Answer

(i) False

We know that if two points lie on the same segment, then they are equal.

Hence, it is false.

(ii) False

⇒ ∠C = 90°

∴ By Pythagoras Theorem,

⇒ $AB^2 = AC^2 + BC^2$

(iii) True

Join AD, DE, EB and DB.

∠ADB = 90°

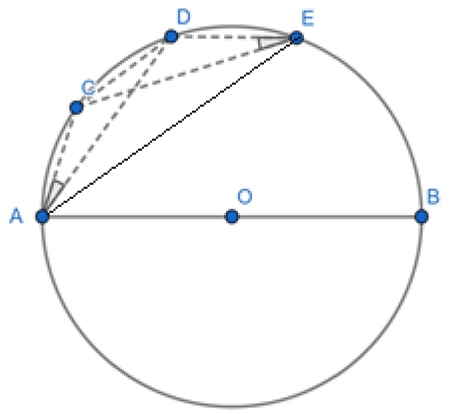
And ∠BDE = 120° - 90° = 30°

We know that angles that lie on the same segment are equal.

∴ ∠BDE = ∠EAB = 30°

(iv) True

Join AC, CE, CD, DE, AE and AD.



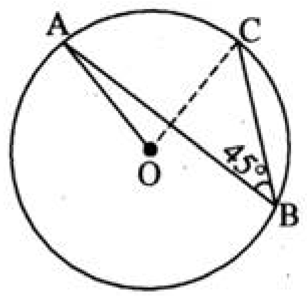
ACDE is minor segment of the circle.

We know that angles in the same segment are equal.

∴ ∠CAD = ∠CED

2. Question

In figure, ∠ABC = 45°. Then prove that OA ⊥ OC.



Answer

We know that angle subtended by an arc at the centre is double the angle subtended at any point on the remaining part of the circle.

⇒∠AOC = 2∠ABC

Given ∠ABC = 45°

⇒∠AOC = 2(45°)

⇒∠AOC = 90°

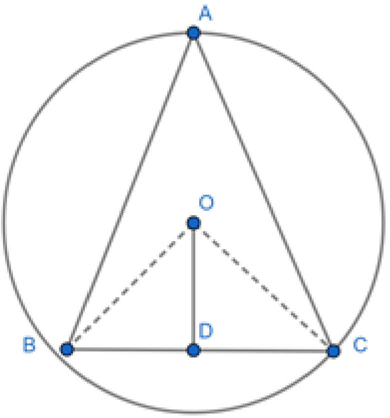
Since ∠AOC = 90°, OA ⊥ OC.

Hence proved.

3. Question

O is the circumcircle of triangle ABC and D is the mid-point of the base BC. Prove that ∠BOD = ∠A.

Answer



Given in ΔABC, O is the circum circle and D is the mid-point of the base BC.

We have to prove that ∠BOD = ∠A.

Construction:

Join OB and OC

Proof:

Consider ΔOBD and ΔOCD,

⇒ OB = OC [Radii of same circle]

⇒ BD = DC [D is mid-point of BC]

⇒ OD = OD [Common side]

By SSS congruence rule,

ΔOBD ≅ ΔOCD

By CPCT,

⇒ ∠BOD = ∠COD

∴ ∠BOD = 1/2 ∠BOC … (1)

Arc BC subtends ∠BOC at the centre and ∠BAC at point A in the remaining part of the circle.

∴ ∠BAC = 1/2 ∠BOC … (2)

From (1) and (2),

⇒∠BOD = ∠BAC

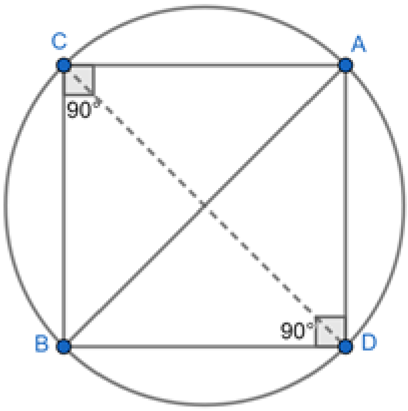
∴ ∠BOD = ∠A

Hence proved

4. Question

On a common hypotenuse AB two right angled triangles ACB and ADB are drawn such that they lie on the opposite sides. Prove that ∠BAC = ∠BDC.

Answer



Given ACB and ADB are two right angled triangles having common hypotenuse AB.

We have to prove that ∠BAC = ∠BDC.

Construction: Join CD.

Proof:

⇒∠C + ∠D = 90° + 90° = 180°

We know that if opposite angles of a quadrilateral are supplementary, then it is a cyclic quadrilateral.

∴ ADBC is a cyclic quadrilateral.

We know that if two angles are of the same arc, then they are equal.

Here, ∠BAC and ∠BDC are made by the same arc BC.

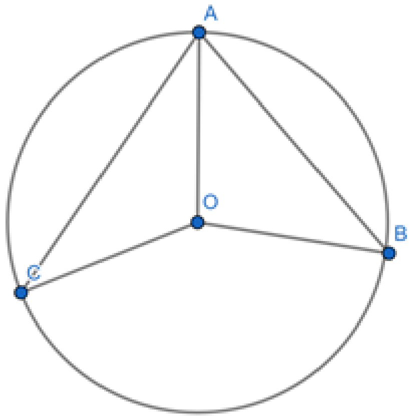
∴ ∠BAC = ∠BDC

Hence proved

5. Question

Two chords AB and AC of a circle subtend angles 90° and 150° respectively on its centre. Find ∠BAC if AB and AC lie on opposite side of the centre.

Answer



Given AB subtends at an angle 90° and AC subtends at 150°.

We know that sum of all angles at a point = 360°

$\Rightarrow \angle AOC + \angle AOB + \angle COB = 360°$

$\Rightarrow 150° + 90° + \angle COB = 360°$

$\Rightarrow 240° + \angle COB = 360°$

$\Rightarrow \angle COB = 360° - 240°$

$\therefore \angle COB = 120°$

We know that the angle subtended by an arc of a circle at the centre is twice the angle subtended by it at any point.

$\Rightarrow \angle COB = 2\angle CAB$

$\Rightarrow \angle CAB = 1/2 \angle COB$

$= 1/2\ (120°)$

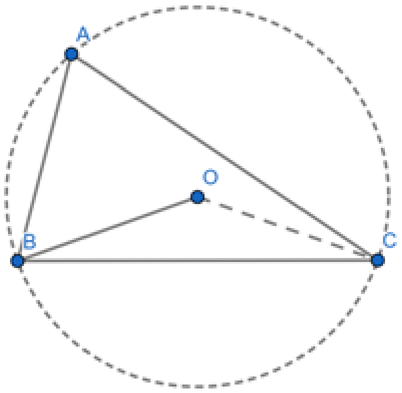
$= 60°$

$\therefore \angle BAC = 60°$

6. Question

The circumcentre of a triangle ABC is O. Prove that $\angle OBC + \angle BAC = 90°$.

Answer



Given O is the circum centre of ΔABC.

Construction:

Join OC

We have to prove that ∠OBC + ∠BAC = 90°.

Proof:

Consider ΔOBC,

⇒ OB = OC [radii of same circle]

We know that angles opposite to equal sides are equal

⇒ ∠OBC = ∠OCB … (1)

We know that angle at the center is twice the angle at the circumference.

⇒∠BOC = 2∠BAC … (2)

We know that sum of angles of a triangle is 180°.

⇒∠OBC + ∠OCB + ∠BOC = 180°

⇒ 2∠OBC + ∠BOC = 180° [From (1)]

⇒ 2∠OBC + 2∠BAC = 180°

⇒∠OBC + ∠BAC = 180°/ 2

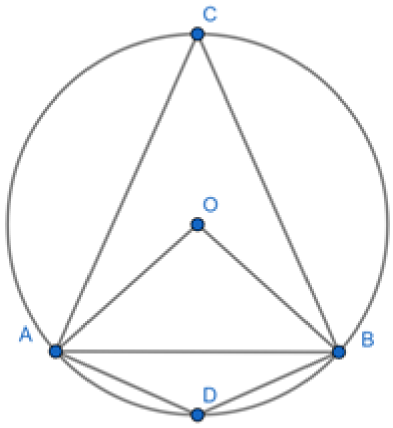
∴ ∠OBC + ∠BAC = 90°

Hence proved

7. Question

A chord of a circle is equal to its radius. Find the angle subtended by this chord on any point in the major segment.

Answer



Given a chord of a circle is equal to its radius.

In ΔOAB,

⇒ AB = OA = OB.

∴ ΔOAB is an equilateral triangle.

Each angle of an equilateral triangle is 60°.

∴ ∠AOB = 60°

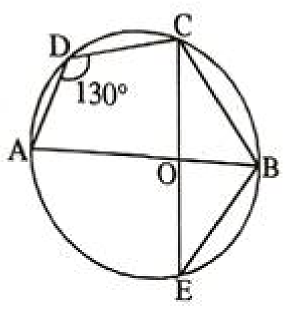
We know that angle subtended at the centre of a circle by an arc is double the angle subtended by it on any point on the remaining part of the circle.

⇒∠ACB = 1/2 ∠AOB = 1/2 (60°) = 30°

∴ Angle subtended by this chord at a point on the major arc is 30°.

8. Question

In figure, ∠ADC = 130° and chord BC = chord BE. Find ∠CBE.



Answer

Given ∠ADC = 130° and chord BC = chord BE.

Consider the points A, B, C and D which form a cyclic quadrilateral.

We know that in a cyclic quadrilateral the opposite angles are supplementary.

In cyclic quadrilateral ADCB,

⇒∠ADC + ∠OBC = 180°

$\Rightarrow 130° + \angle OBC = 180°$

$\Rightarrow \angle OBC = 180° - 130° = 50°$

Consider ΔBOC and ΔBOE,

⇒ BC = BE [given]

⇒ OC = OE [radii of same circle]

⇒ OB = OB [common side]

By SSS congruence rule,

$\Rightarrow \Delta BOC \cong \Delta BOE$

By CPCT,

$\Rightarrow \angle OBC = \angle OBE = 50°$

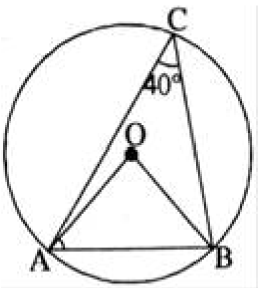
$\Rightarrow \angle CBE = \angle CBO + \angle EBO$

$= 50° + 50°$

$\therefore \angle CBE = 100°$

9. Question

In figure, $\angle ACB = 40°$. Find $\angle OAB$.



Answer

Given $\angle ACB = 40°$

We know that a segment subtends an angle to the circle is half the angle that subtends to the circle.

i.e. $\angle AOB = 2\angle ACB$

$\Rightarrow \angle AOB = 2(40°)$

$\Rightarrow \angle AOB = 80°$ … (1)

In ΔAOB,

⇒ OA = OB [Radii of same circle]

We know that angles opposite to equal sides are equal.

∴ ∠OBA = ∠OAB

We know that sum of all angles in a triangle is 180°.

⇒∠AOB + ∠OBA + ∠OAB = 180°

⇒ 80° + ∠OAB + ∠OAB = 180°

⇒ 2∠OAB = 180° - 80°

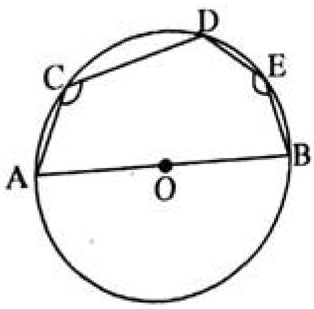
⇒ 2∠OAB = 100°

⇒∠OAB = 100°/ 2

∴ ∠OAB = 50°

10. Question

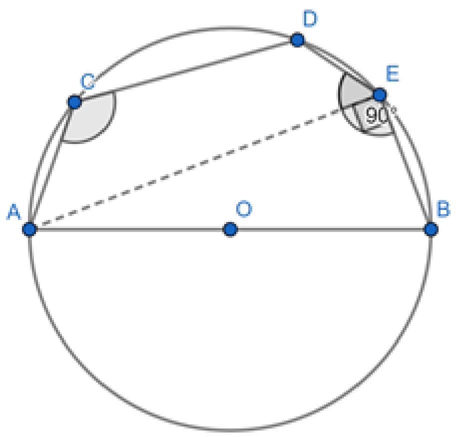
In figure AOB is a diameter if the circle and C, D and E are any three points of the semicircle. Find the value of ∠ACD + ∠BED.



Answer

ACDE is a cyclic quadrilateral because A, C, D and E are four points on circle.

Construction: Join AE



We know that in a cyclic quadrilateral the opposite angles are supplementary.

⇒∠ACD + ∠AED = 180° ... (1)

Given AOB is a diameter.

We know that diameter subtends a right angle to the circle.

$\Rightarrow\angle AEB = 90°$

On adding equations (1) and (2),

$\Rightarrow\angle ACD + \angle AED + \angle AEB = 180° + 90° = 270°$

$\therefore \angle ACD + \angle BED = 270°$

Exercise 12.4

1. Question

One angle of a cyclic quadrilateral is given. Find the opposite angle.

(i) 70° (ii) 135°

(iii) $112\frac{1}{2}^{\circ}$ (iv) $\frac{3}{5}$ right angle

(iv) 165°

Answer

We know that the pair of opposite angles of a cyclic quadrilateral is supplementary i.e. sum is 180°.

Let the opposite angle of the cyclic quadrilateral be x°.

(i) Given angle = 70°

$\Rightarrow$ 70° + x° = 180°

$\Rightarrow$x° = 180° - 70°

$\therefore$ x° = 110°

(ii)Given angle = 135°

$\Rightarrow$ 135° + x° = 180°

$\Rightarrow$x° = 180° - 135°

$\therefore$ x° = 45°

(iii)Given angle = 1121/2°

$\Rightarrow$ 1121/2° + x° = 180°

$\Rightarrow$x° = 180° - 1121/2°

$\therefore$ x° = 671/2°

(iv)Given angle = $\frac{3}{5}$ right angle = $\frac{3}{5} \times 90° = 54°$

⇒ 54° + x° = 180°

⇒x° = 180° - 54°

∴ x° = 126°

(v) Given angle = 165°

⇒ 165° + x° = 180°

⇒x° = 180° - 165°

∴ x° = 15°

2. Question

Find the opposite angle of a cyclic quadrilateral of one angel of these in

(i) $\frac{2}{7}$ of other

(ii) $\frac{11}{4}$ of other

Answer

We know that the pair of opposite angles of a cyclic quadrilateral is supplementary i.e. sum is 180°.

Let the angle be x°.

(i) $\frac{2}{7}$of other

$\Rightarrow \frac{2}{7}x° + x° = 180°$

$\Rightarrow \frac{9}{7}x° = 180°$

$\Rightarrow x° = 180° \times \frac{7}{9}$

⇒ x° = 140°

And $\frac{2}{7}x° = \frac{2}{7}(140°) = 40°$

∴ Angles are 140° and 40°.

(ii) $\frac{11}{4}$of other

$\Rightarrow \frac{11}{4}x^\circ + x^\circ = 180^\circ$

$\Rightarrow \frac{15}{4}x^\circ = 180^\circ$

$\Rightarrow x^\circ = 180^\circ \times \frac{4}{15}$

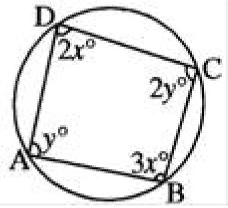
$\Rightarrow x^\circ = 48^\circ$

And $\frac{11}{4}x^\circ = \frac{11}{4}(48^\circ) = 132^\circ$

∴ Angles are 48° and 132°.

3. Question

In figure, find all the four angles of the cyclic quadrilateral ABCD.



Answer

We know that the pair of opposite angles of a cyclic quadrilateral is supplementary i.e. sum is 180°.

Firstly, $2x^\circ + 3x^\circ = 180^\circ$

$\Rightarrow 5x^\circ = 180^\circ$

$\Rightarrow x^\circ = 180^\circ / 5$

∴ $x^\circ = 36^\circ$

Also, $y^\circ + 2y^\circ = 180^\circ$

$\Rightarrow 3y^\circ = 180^\circ$

$\Rightarrow y^\circ = 180^\circ / 3$

∴ $y^\circ = 60^\circ$

Now,

∴ $\angle A = y^\circ = 60^\circ$

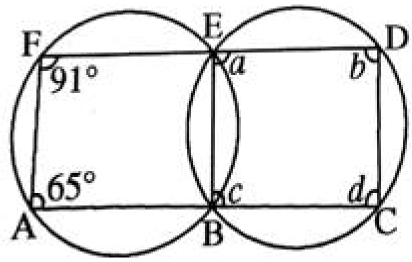
∴ $\angle B = 3x^\circ = 3(36^\circ) = 108^\circ$

∴ $\angle C = 2y^\circ = 2(60^\circ) = 120^\circ$

∴ $\angle D = 2x^\circ = 2(36^\circ) = 72^\circ$

4. Question

In figure, some angles have been marked with a, b, c and d. Find the measures of these angles.



Answer

We know that in a cyclic quadrilateral, the exterior angle formed by producing one side of a quadrilateral is equal to its interior opposite angle.

∴ a = 65° and c = 91°

We know that the pair of opposite angles of a cyclic quadrilateral is supplementary i.e. sum is 180°.

Firstly, a + d = 180°

⇒ 65° + d = 180°

⇒ d = 180° - 65°

∴ d = 115°

Similarly, b + c = 180°

⇒ b + 91° = 180°

⇒ b = 180° - 91°

∴ b = 89°

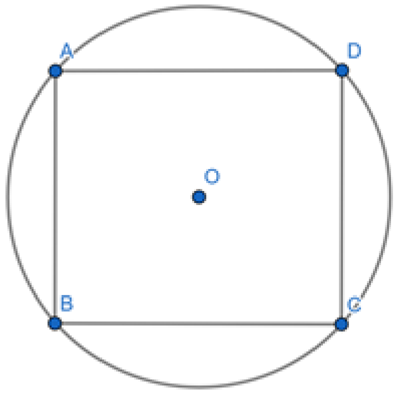
∴ a = 65°, b = 89°, c = 91° and d = 115°

5. Question

If in a cyclic quadrilateral ABCD, AD || BC, then prove that ∠A = ∠D.

Answer

Given ABCD is a cyclic quadrilateral and AD || BC.



We have to prove that ∠A = ∠D.

Proof:

Since AD || BC and CD is a traversal.

So, ∠BCD + ∠ADC = 180° … (1)

But ABCD is a cyclic quadrilateral.

⇒∠BCD + ∠BAD = 180° … (2)

From (1) and (2),

⇒∠ADC = ∠BAD

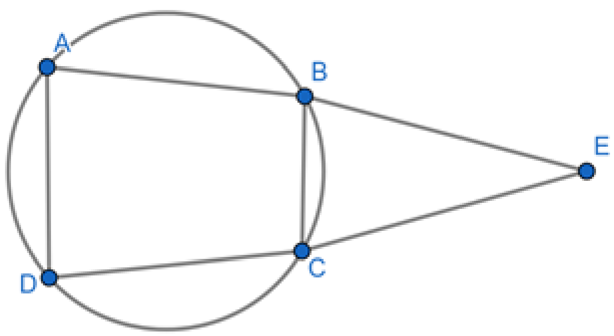
∴ ∠D = ∠A

Hence proved.

6. Question

ABCD is a cyclic quadrilateral. AB and DC when produced meet at E. Prove that ΔEBC and ΔEDA are similar.

Answer



Given that ABCD is a cyclic quadrilateral and AB and DC are produced to meet at E.

We have to prove that ΔEBC and ΔEDA are similar.

Proof:

We know that in a cyclic quadrilateral, opposite angles are supplementary.

$\Rightarrow \angle ABC + \angle ADC = 180°$ … (1)

Also, $\angle ABC + \angle EBC = 180°$ … (2) [linear pair]

From (1) and (2),

$\Rightarrow \angle ABC + \angle ADC = \angle ABC + \angle EBC$

$\Rightarrow \angle ADC = \angle EBC$ … (3)

Similarly,

$\Rightarrow \angle BAD + \angle BCD = 180°$ … (4)

Also, $\angle BCD + \angle BCE = 180°$ … (5) [linear pair]

From (4) and (5),

$\Rightarrow \angle BAD + \angle BCD = \angle BCD + \angle BCE$

$\Rightarrow \angle BAD = \angle BCE$ … (6)

And $\angle BEC = \angle AED$ … (7)

From (3), (6) and (7),

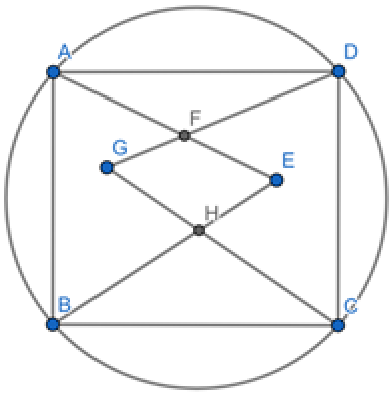
∴ ΔEBC and ΔEDA are similar.

Hence proved

7. Question

Prove that the quadrilateral formed by the bisectors of the angles of a cyclic quadrilateral is also a cyclic quadrilateral.

Answer



Given that ABCD is a cyclic quadrilateral.

We have to prove that EFGH is also a cyclic quadrilateral.

Proof:

Let

⇒ 1/2∠A = x; 1/2∠B = w; 1/2∠C = z and 1/2∠D = y

We know that opposite angles of a cyclic quadrilateral are supplementary.

⇒∠A + ∠C = 180° and ∠B + ∠D = 180°

⇒ 1/2 (A + C) = 90° and 1/2 (B + D) = 180°

⇒∠x + ∠z = 90° and ∠y + ∠w = 90° … (1)

In ΔAFD and ΔBHC,

⇒∠x + ∠y + ∠AFD = 180°

⇒∠AFD = 180° - (∠x + ∠y) … (2)

And ∠z + ∠w + ∠BHC = 180°

⇒∠BHC = 180° - (∠z + ∠w) … (3)

Adding (2) and (3),

⇒∠AGD + ∠BHC = 360° - (∠x + ∠y + ∠z + ∠w)

From (1),

⇒∠AGD + ∠BHC = 360° - 180°

∴ ∠AGD + ∠BHC = 180°

⇒∠FGH + ∠HEF = 180° [Vertically opposite angles]

We know that opposite angles of a cyclic quadrilateral are supplementary.

∴ EFGH is also a cyclic quadrilateral.

Hence proved

Miscellaneous Exercise 12

1. Question

The length of the chord situated at a distance of 6 cm from the centre of a circle of radius 10 cm is—

A. 16 cm

B. 8 cm

C. 4 cm

D. 5 cm

Answer

We know that Length of chord, $l = 2\sqrt{r^2 - d^2}$

Where r = radius of circle, d = distance of chord from centre

$\Rightarrow\ l = 2\sqrt{10^2 - 6^2}$

$\Rightarrow\ l = 2\sqrt{100 - 36}$

$\Rightarrow\ l = 2\sqrt{64}$

⇒ l = 2 (8)

∴ Length of chord, l = 16 cm

2. Question

In a circle of radius 13 cm a chord of length 24 cm has been drawn. The distance of the chord from the centre of the circle is—

A. 12 cm

B. 5 cm

C. 6.5 cm

D. 12 cm

Answer

We know that Length of chord, $l = 2\sqrt{r^2 - d^2}$

Where r = radius of circle, d = distance of chord from centre

$\Rightarrow 24 = 2\sqrt{13^2 - d^2}$

$\Rightarrow 24 = 2\sqrt{169 - d^2}$

Squaring on both sides,

$\Rightarrow 24^2 = \left(2\sqrt{169 - d^2}\right)^2$

⇒ 576 = 4 (169 – d^2)

⇒576/ 4 = 169 – d^2

⇒ 144 = 169 – d^2

⇒ d^2 = 169 – 144

⇒ d^2 = 25

∴ Distance of chord, d = 5 cm

3. Question

The degree measure of minor arc is—

A. less than 180°

B. greater than 180°

C. 360°

D. 270°

Answer

A minor arc is an arc less than a semicircle.

Angle of a semicircle is 180°.

∴ Degree measure of minor arc is less than 180°.

4. Question

The degree measure of major arc is—

A. less than 180°

B. greater than 180°

C. 360°

D. 90°

Answer

A major arc is an arc greater than a semicircle.

Angle of a semicircle is 180°.

∴ Degree measure of major arc is greater than 180°.

5. Question

The chords lying at equal distances from the centre in a circle are—

A. double

B. triple

C. half

D. equal of each other

Answer

From the centre to a point on the circle, it is called a radius.

Radius of a circle has constant distance.

So, the chords lying at equal distances from the centre in a circle are equal to each other.

6. Question

The degree measure of an arc of a circle is 180°, that arc is—

A. major arc

B. minor arc

C. circle

D. semicircle

Answer

The angle of a semicircle is 180°.

∴ The arc is a semicircle if its degree measure is 180°.

7. Question

The number of circles passing through three non-collinear points is—

A. one

B. two

C. zero

D. infinitely

Answer

We know that there is one and only one circle passing through three non-collinear points.

8. Question

If in any circle arc AB = arc BA, then the arc is—

A. major arc

B. minor arc

C. semicircle

D. circle

Answer

A circle can be divided into two equal parts i.e. semicircles.

So, if arc AB = arc BA, then arc is a semicircle.

9. Question

If a diameter of a circle bisects each of the two chords then the chords will be —

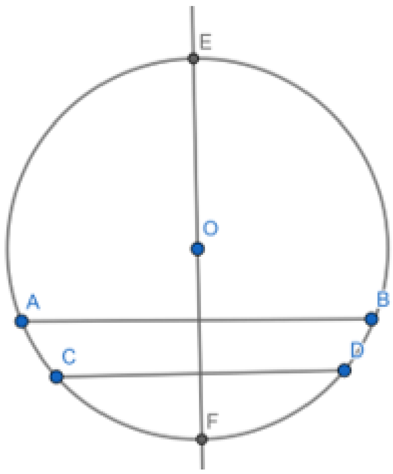
A. parallel

B. perpendicular

C. intersecting

D. none of the above

Answer



AB and CD are two chords.

Let EF be the diameter of the circle.

We know that if a radius bisects a chord, then it is perpendicular to the chord.

⇒ EF ⊥ CD and EF ⊥ AB

∴ AB || CD

∴ If a diameter bisects each of the chords, then the chords will be parallel.

10. Question

If in congruent circles two arcs are equal, then their corresponding chords will be—

A. parallel

B. equal

C. perpendicular

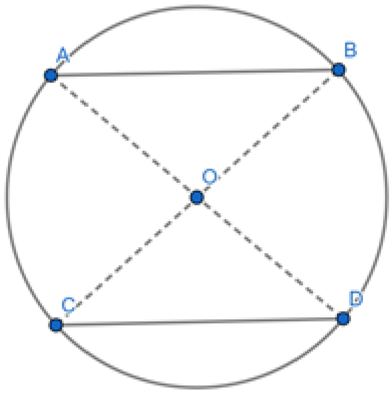
D. intersecting

Answer

Let us take a circle with O as centre and radius r in which arc AB ≅ arc CD.

Construction:

Join OA, OB, OC and OD.



In ΔAOB and ΔCOD,

⇒ OA = OC [radii of same circle]

⇒OB = OD [radii of same circle]

⇒∠AOB = ∠COD [measure (arc AB) = measure (arc CD)]

By SAS congruency,

⇒ ΔAOB ≅ ΔCOD

By CPCT,

⇒ AB = CD

∴ In congruent circles, if two arcs are equal, then their corresponding chords will be equal.

11. Question

AD is a diameter of a circle and AB is a chord. If AD = 34 cm, AB = 30 cm, then the distance of AB from the centre of the circle is—

A. 17 cm

B. 15 cm

C. 4 cm

D. 8 cm

Answer

We know that Length of chord, $l = 2\sqrt{r^2 - d^2}$

Where r = radius of circle, d = distance of chord from centre

Given diameter AD = 34 cm and chord AB = 30 cm

⇒ Radius = 34/ 2 = 17 cm

Now,

$\Rightarrow 30 = 2\sqrt{17^2 - d^2}$

$\Rightarrow 30 = 2\sqrt{289 - d^2}$

Squaring on both sides,

$\Rightarrow 30^2 = \left(2\sqrt{289 - d^2}\right)^2$

⇒ 900 = 4 (289 – d^2)

⇒900/ 4 = 289 – d^2

⇒225 = 289 – d^2

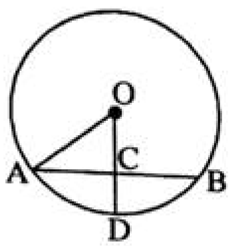
⇒ d^2 = 289 – 225

⇒ d^2 = 64

∴ Distance of chord, d = 8 cm

12. Question

In figure, if OA = 5 cm, AB = 8 cm and OD is perpendicular to chord AB, then CD is equal to—



A. 2 cm

B. 3 cm

C. 4 cm

D. 5 cm

Answer

Given OA = 5 cm, AB = 8 cm and OD ⊥ AB.

We know that the perpendicular from centre of a circle to a chord bisects the chord.

⇒ AC = CB = 1/2 AB = 1/2 (8) = 4 cm

By Pythagoras Theorem,

⇒ $AO^2 = AC^2 + OC^2$

⇒ $5^2 = 4^2 + OC^2$

⇒ $25 = 16 + OC^2$

⇒ $OC^2 = 25 - 16 = 9$

⇒ OC = 3 cm

⇒ OA = OD = 5 cm [Radii of same circle]

⇒ CD = OD – OC = 5 – 3

∴ CD = 2 cm

13. Question

If AB = 12 cm, BC = 16 cm, and AB is perpendicular to line segment BC then the radius of the circle passing through A, B and C is—

A. 6 cm

B. 8 cm

C. 10 cm

D. 12 cm

Answer

We know that Length of chord, $l = 2\sqrt{r^2 - d^2}$

Where r = radius of circle, d = distance of chord from centre

Given AB = 12 cm and BC = 16 cm

Now,

$\Rightarrow 16 = 2\sqrt{r^2 - 12^2}$

$\Rightarrow 16 = 2\sqrt{r^2 - 144}$

Squaring on both sides,

$\Rightarrow 16^2 = \left(2\sqrt{r^2 - 144}\right)^2$

$\Rightarrow 256 = 4\ (r^2 – 144)$

$\Rightarrow 256/\ 4 = r^2 – 144$

$\Rightarrow 64 = r^2 – 144$

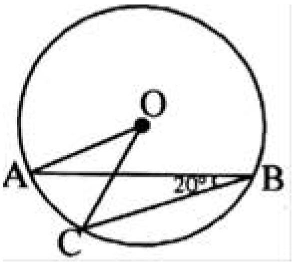
$\Rightarrow d^2 = 289 – 225$

$\Rightarrow d^2 = 64$

$\therefore$ Distance of chord, d = 8 cm

14. Question

In figure, if ∠ABC = 20°, the ∠AOC is equal to—



A. 20°

B. 40°

C. 60°

D. 10°

Answer

Given ∠ABC = 20°

We know that angle subtended at the centre by an arc is twice the angle subtended by it at the remaining part of the circle.

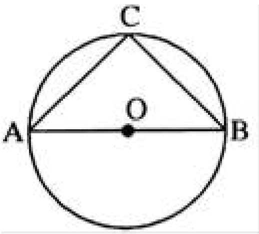
$\Rightarrow \angle AOC = 2\ \angle ABC$

$\Rightarrow \angle AOC = 2\ (20°)$

$\therefore \angle AOC = 40°$

15. Question

In figure, if AOB is a diameter of a circle and AC = BC. Then ∠CAB is equal to —



A. 30°

B. 60°

C. 90°

D. 45°

Answer

Given AOB is a diameter of a circle and AC = BC.

We know that diameter subtends a right angle to the circle.

∴ ∠BCA = 90°

We know that angles opposite to equal sides are equal.

∴ ∠ABC = ∠CAB

In ΔABC,

By angle sum property,

⇒∠CAB + ∠ABC + ∠BCA = 180°

⇒∠CAB + ∠CAB + 90° = 180°

⇒ 2 ∠CAB + 90° = 180°

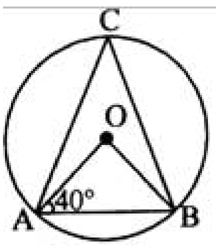
⇒ 2 ∠CAB = 180° - 90°

⇒∠CAB = 90°/ 2

∴ ∠CAB = 45°

16. Question

In figure, if ∠OAB = 40°, then ∠ACB is equal to—



A. 50°

B. 40°

C. 60°

D. 70°

Answer

Given ∠OAB = 40°

Consider ΔOAB,

⇒ OA = OB [radii of same circle]

We know that angles opposite to equal sides are equal.

⇒∠OBA = ∠OAB = 40°

By angle sum property,

⇒∠AOB + ∠OBA + ∠BAO = 180°

⇒∠AOB + 40° + 40° = 180°

⇒∠AOB = 180° - 80°

∴ ∠AOB = 100°

We know that the angle subtended by an arc at the centre is twice the angle subtended by it at the remaining part of the circle.

⇒∠AOB = 2 ∠ACB

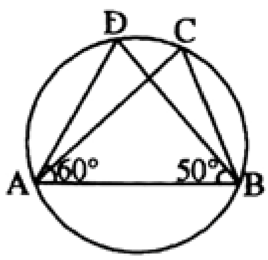
⇒ 100° = 2 ∠ACB

⇒∠ACB = 100°/ 2

∴ ∠ACB = 50°

17. Question

In figure, if ∠DAB = 60°, ∠ABD = 50°, then ∠ACB is equal to—



A. 60°

B. 50°

C. 70°

D. 80°

Answer

Given ∠DAB = 60° and ∠ABD = 50°

We know that angles in the same segment of a circle are equal.

∴ ∠ADB = ∠ACB

In ΔABD,

By angle sum property,

⇒∠ABD + ∠ADB + ∠DAB = 180°

⇒ 50° + ∠ADB + 60° = 180°

⇒ 110° + ∠ADB = 180°

⇒∠ADB = 180° - 110°

∴ ∠ADB = 70° = ∠ACB

18. Question

One side AB of a quadrilateral is a diameter of its circumscribed circle and ∠ADC = 140°. Then, ∠BAC is equal to—

A. 80°

B. 50°

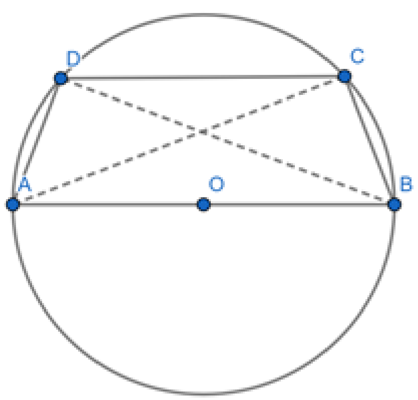
C. 40°

D. 30°

Answer

Given AB of a quadrilateral is a diameter of its circumscribed circle.

And ∠ADC = 140°



Here, O is the centre of the circle and ADCBOA is the semi circle.

∴ ∠ADB = 90°

Given that ∠ADC = 140°

⇒∠ADB + ∠BDC = 140°

⇒ 90° + ∠BDC = 140°

⇒∠BDC = 140° - 90°

∴ ∠BDC = 50°

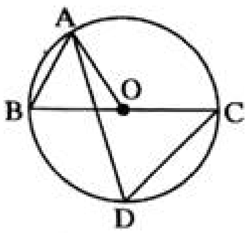
We know that angles in the same segment are equal.

⇒∠BDC = ∠BAC

∴ ∠BAC = 50°

19. Question

In figure, BC is a diameter of the circle and ∠BAO = 60° then, ∠ADC is equal to —



A. 30°

B. 45°

C. 60°

D. 120°

Answer

Given BC is a diameter of the circle and ∠BAO = 60°.

Consider ΔOAB,

⇒ OA = OB [Radii of same circle]

We know that angles opposite to equal sides are equal.

⇒∠OBA = ∠BAO = 60°

By angle sum property,

⇒∠OBA + ∠BAO + ∠AOB = 180°

⇒ 60° + 60° + ∠AOB = 180°

⇒∠AOB = 180° - 120°

∴ ∠AOB = 60°

By linear pair axiom,

⇒∠AOC = 180° - 60° = 120°

We know that angle subtended by an arc at the centre is double the angle subtended by the same arc at any point on the circle.

⇒∠AOC = 2 ∠ADC

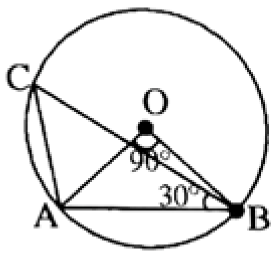
⇒ 120° = 2 ∠ADC

⇒∠ADC = 120°/ 2

∴ ∠ADC = 60°

20. Question

In figure, ∠AOB = 90° and ∠ABC = 30°. Then, ∠CAO s equal to—



A. 30°

B. 45°

C. 90°

D. 60°

Answer

Given ∠AOB = 90° and ∠ABC = 30°

We know that angles opposite to equal sides are equal.

⇒∠OAB = ∠ABO

By angle sum property,

⇒∠OAB + ∠ABO + ∠BOA = 180°

⇒ 2 ∠OAB + ∠BOA = 180°

⇒ 2 ∠OAB + 90° = 180°

⇒ 2 ∠OAB = 180° - 90° = 90°

$\Rightarrow \angle OAB = 90° / 2$

$\therefore \angle OAB = 45°$

We know that angles subtended by an arc at the centre of the circle is double the angle subtended by it to any other part of the circle.

$\therefore \angle C = 45°$

By angle sum property,

$\Rightarrow \angle ABC + \angle BCA + \angle ACB = 180°$

$\Rightarrow 30° + 45° + \angle CAB = 180°$

$\Rightarrow 75° + \angle CAB = 180°$

$\Rightarrow \angle CAB = 180° - 75°$

$\therefore \angle CAB = 105°$

Also $\angle CAB = \angle CAO + \angle OAB$

$\Rightarrow 105° = \angle CAO + 45°$

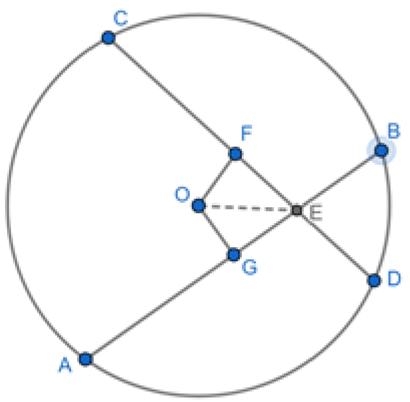
$\Rightarrow \angle CAO = 105° - 45°$

$\therefore \angle CAO = 60°$

21. Question

If two equal chords of a circle intersect each other. Then prove that two parts of one chord are respectively equal to two parts of another chord.

Answer



Let AB and CD be two equal chords of a circle which are intersecting at a point E.

Construction:

Draw perpendiculars OF and OG on the chords.

Join OE.

Consider ΔOFE and ΔOGE,

⇒ OF = OG [Equal chords]

⇒∠OFE = ∠OGE [Each 90°]

⇒ OE = OE [Common]

By RHS congruence rule,

⇒ ΔOFE ≅ ΔOGE

By CPCT,

⇒ FE = GE … (1)

Given AB = CD … (2)

⇒ 1/2 AB = 1/2 CD

⇒ AG = CF … (3)

Adding equations (1) and (3),

⇒ AG + GE = CF + FE

⇒ AE = CE … (4)

Subtracting equation (4) from (2),

⇒ AB – AE = CD - CE

⇒ BE = DE … (5)

From (4) and (5),

We can see that two parts of one chord are equal to two parts of another chord.

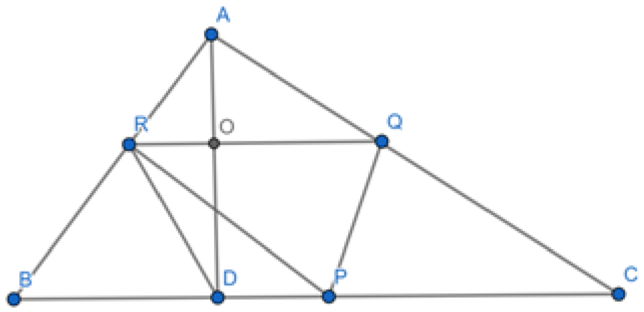
Hence proved.

22. Question

If P, Q and R are respectively the mid-points of sides BC, CA and AB respectively of a triangle and AD is the perpendicular from vertex A to BC, then prove that the points P, Q, R and D are cyclic.

Answer

Given that in ΔABC, P, Q and R are the mid-points of sides BC, CA and AB. Also AD ⊥ BC.



We have to prove that P, Q, R and D are concyclic.

Proof:

In ΔABC, R and Q are mid-points of AB and CA respectively.

By mid-point theorem, RQ || BC.

Also, PQ || AB and PR || CA

Consider quadrilateral BPQR,

⇒ BP ||RQ and PQ || BR

∴ BPQR quadrilateral is a parallelogram.

Similarly, ARPQ quadrilateral is a parallelogram.

We know that opposite angles of a parallelogram are equal.

⇒∠A = ∠RPQ

PR || AC and PC is the traversal,

⇒∠BPR = ∠C

⇒∠DPQ = ∠DPR + ∠RPQ = ∠A + ∠C … (1)

RQ || BC and BR is the traversal,

⇒∠ARO = ∠B … (2)

In ΔABD, R is the mid-point of AB and OR || BD.

∴ O is the mid-point of AD.

⇒ OA = OD

In ΔAOR and ΔDOR,

⇒ OA = OD

⇒∠AOR = ∠DOR = 90°

⇒ OR = OR [Common]

By SAS congruence rule,

⇒ ΔAOR ≅ Δ DRO

⇒∠ARO = ∠DRO [CPCT]

⇒∠DRO = ∠B [From (2)]

In quadrilateral PRQD,

⇒∠DRO + ∠DPQ = ∠B + (∠A + ∠C) = ∠A + ∠B + ∠C [From (1)]

Since ∠A + ∠B + ∠C = 180°,

⇒∠DRO + ∠DPQ = 180°

Hence, quadrilateral PRQD is a cyclic quadrilateral.

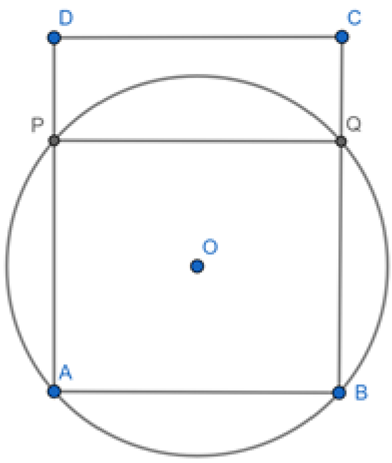
∴ Points P, Q, R and D are concyclic.

Hence proved.

23. Question

ABCD is a parallelogram. A circle is drawn through A and B such that it intersects AD at P and BC at Q. Prove that P, Q, C and D are cyclic.

Answer



Given ABCD is a parallelogram.

A circle through points A and B is drawn such that it intersects the AD at P and BC at Q.

We have to prove that points P, Q, C and D are cyclic.

Proof:

Since the circle passes through points A, B, P and Q, ABPQ is a cyclic quadrilateral.

We know that opposite angles in a cyclic quadrilateral are supplementary.

⇒∠A + ∠PQB = 180°

⇒∠CQP + ∠PQB = 180°

∴ ∠A = ∠CQP

Now, AB and CD are parallel lines and AD is the traversal.

We know that angles on the same side of traversal are supplementary.

⇒∠A + ∠D = 180°

⇒∠CQP + ∠D = 180°

Thus, in PQCD quadrilateral, opposite angles are supplementary.

Hence, quadrilateral PQCD is a cyclic quadrilateral.

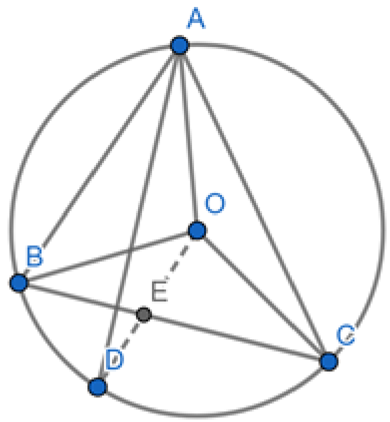
∴ P, Q, C and D are concyclic.

Hence proved.

24. Question

Prove that if the bisector of an angle of a triangle and the perpendicular bisector of its opposite side intersect, then they intersect at the circumcircle of that triangle.

Answer



Here, O is the circumcentre of ΔABC.

Let the bisector AD of ∠A and perpendicular bisector OD of BC intersect at D.

[here perpendicular bisector passes through center because circumcenter of any triangle lies on perpendicular bisector of any of its side]

Proof:

We know that angle subtended by an arc at the centre is twice the angle subtended by the arc at the point of the alternate segment of the circle.

⇒∠BOC = 2∠A

⇒ OB = OC [Radii of same circle]

∴ ΔBOC is an isosceles triangle.

Since OD is the perpendicular bisector of BC,

$$\Rightarrow \angle BOE = \angle COE = \frac{\angle BOC}{2} = \angle A$$

We know that any three points arc always concyclic.

∴ A, C and D are concyclic.

As, AD is angle bisector,

$$\angle BAD = \angle CAD = \frac{\angle A}{2}$$

⇒ Arc CD subtends $\angle CAD = \frac{\angle A}{2}$ at point A and $\angle COE = \angle A$ at point O

∴ O is the centre of the circle passing through A, C and D.

Thus, the circle passing through A, C and D is the circum circle of ΔABC.

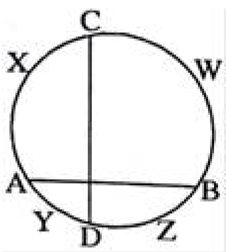
⇒ D passes through circumcircle of ΔABC

∴ They intersect the circum circle of ΔABC.

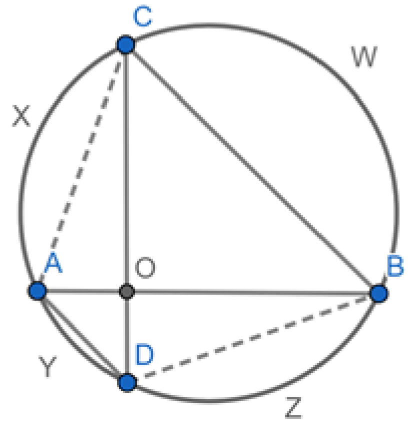
Hence proved.

25. Question

If two chords AB and CD of a circle AYDZBWCX intersect at right angles (see figure), then prove that arc CXA + arc DBZ = arc AYD + arc BWC = a semicircle.



Answer



Given AB and CD are two chords of a circle AYDZBWCX such that AB ⊥ CD.

Let the chords intersect at O.

We have to prove that arc CXA + arc DBZ = arc AYD + arc CWB = semicircle.

Construction:

Join AD, DB, AC and BC.

Proof:

Angle subtended by chord AC is ∠CBA and angle subtended by chord BD is ∠BCD.

⇒∠CBA + ∠BCD = 180° - ∠COB

= 180° - 90°

= 90°

Since angle subtended in a semicircle is 90°,

⇒ arc CXA + arc DBZ = semicircle

Similarly arc AYD + arc CWB = semicircle

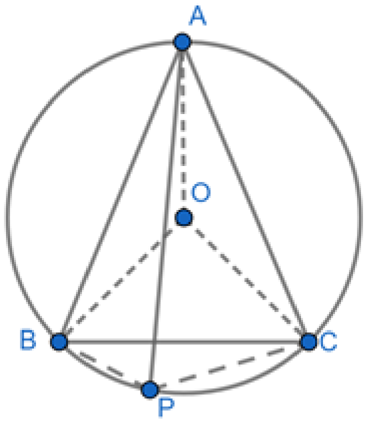
∴ arc CXA + arc DBZ = arc AYD + arc CWB = semicircle

Hence proved.

26. Question

If an equilateral triangle ABC is inscribed in a circle and P is any point lying on minor arc BC, which is not coincident with B or C, then prove that PA is the bisector of angle BPC.

Answer



Given ΔABC is an equilateral triangle inscribed in a circle with centre O.

P is a point lying on minor arc BC.

We have to prove that PA is bisector of ∠BPC.

Construction:

Join OA, OB, OC, BP and PC.

Proof:

Since ΔABC is equilateral,

⇒ AB = BC = CA

We know that equal chords subtend equal angles at centre.

⇒∠AOB = ∠AOC = ∠BOC

Consider ∠AOB = ∠AOC ... (1)

∠AOB and ∠APB are angles subtended by an arc AB at centre and at remaining part of the circle by same arc.

⇒∠APB = 1/2 ∠AOB ... (2)

⇒∠APC = 1/2 ∠AOC ... (3)

From (1), (2) and (3),

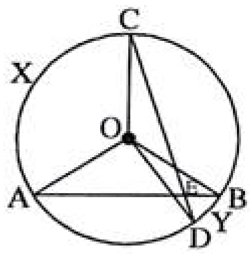
∠APB = ∠APC

∴ PA is angle bisector of ∠BPC.

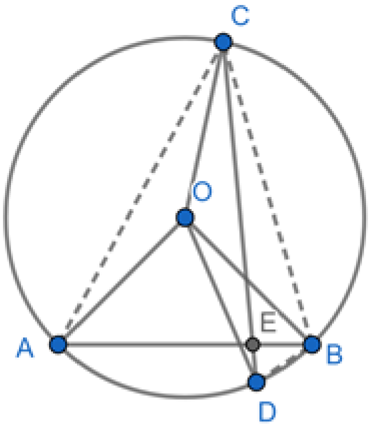
Hence proved.

27. Question

In figure, AB and CD are two chords of a circle which intersect at E. Prove that $\angle AEC = \frac{1}{2}$ (angle subtended by arc CXA at centre + angle subtended by arc DYB at centre).



Answer



Given AB and CD are two chords of the circle with centre O which intersects at E.

We have to prove that ∠AEC = 1/2 (angle subtended by arc CXA at centre + angle subtended by arc DYB at centre).

Construction:

Join AC, BC and BD.

Proof:

AC is a chord.

We know that angle subtended at center is double the angle subtended at circumference.

⇒∠AOC = 2∠ABC … (1)

⇒∠DOB = 2∠DCB … (2)

Adding (1) and (2),

⇒∠AOC + ∠DOB = 2(∠ABC + ∠DCB) … (3)

Consider ΔCEB,

We know that the sum of two opposite interior angles is the exterior angle.

⇒∠AEC = ∠ECB + ∠CBE

⇒∠AEC = ∠DCB + ∠ABC … (4)

From (3) and (4),

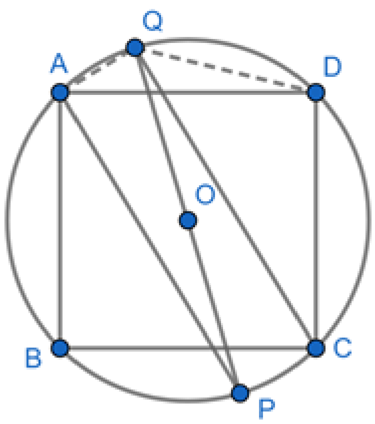
∴ ∠AOC + ∠DOB = 2 (∠AEC)

Hence proved.

28. Question

If the bisectors of the opposite angles of a cyclic quadrilateral ABCD intersects the circumscribed circle of this quadrilateral at points P and Q, then prove that PQ is a diameter of this circle.

Answer



Given ABCD is a cyclic quadrilateral. AP and CQ are bisectors of ∠A and ∠C respectively.

We have to prove that PQ is the diameter of the circle.

Construction:

Join AF and FD

Proof:

We know that in a cyclic quadrilateral, opposite angles are supplementary.

⇒∠A + ∠C = 180°

⇒ 1/2 ∠A + 1/2 ∠C = 90°

⇒∠EAD + ∠DCF = 90° … (1)

We know that angles in the same segment are equal.

⇒∠DCF = ∠DAF … (2)

From (1) and (2),

⇒∠EAD + ∠DAF = 90°

$\Rightarrow \angle EAF = 90°$

$\angle EAF$ is the angle in a semicircle.

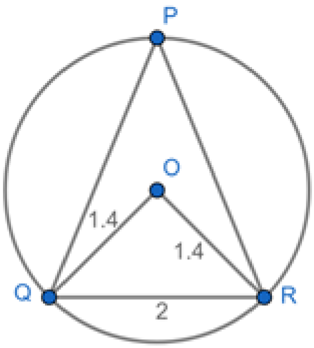
$\therefore$ EF is the diameter of the circle.

Hence proved.

29. Question

The radius of a circle is $\sqrt{2}$ cm. This circle is divided into two segments by a chord of length 2 cm. Prove that the angle subtended by this chord at any point of the major segment is 45°.

Answer



Given radius of circle OQ = OR = $\sqrt{2}$ cm

Length of chord, QR = 2 cm

In ΔOQR,

$\Rightarrow OQ^2 + OR^2 = (\sqrt{2})^2 + (\sqrt{2})^2$

$= 2 + 2$

$= 4$

$= (QR)^2$

$\Rightarrow OQ^2 + OR^2 = QR^2$

$\therefore \Delta OQR$ is a right angled triangle at $\angle QOR$.

We know that angle at the centre is double the angle on the remaining part of the circle.

$\Rightarrow \angle QOR = 2\angle QPR$

$\Rightarrow 90° = 2\angle QPR$

$\therefore \angle QPR = 45°$

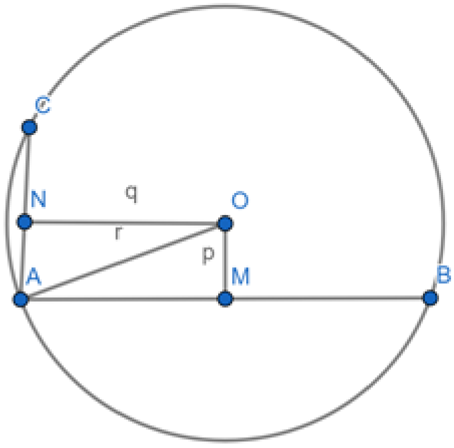
∴ The angle subtended by the chord at the point in major segment is 45°.

Hence proved

30. Question

AB and AC are two chords of a circle of radius r such that AB = 2AC. If p and q are respectively the distance of AB and AC from the centre, then prove that $4q^2 = p^2 + 3r^2$.

Answer



Given AB and AC are two chords of a circle of radius r such that AB = 2AC.

P and q are perpendicular distances of AB and AC from centre O.

⇒ OM = p and ON = q

We have to prove that $4q^2 = p^2 + 3r^2$

Proof:

We know that perpendicular from centre to chord intersect at mid-point of the chord.

Consider ΔONA,

By Pythagoras Theorem,

$\Rightarrow ON^2 + NA^2 = OA^2$

$\Rightarrow q^2 + NA^2 = r^2$

$\Rightarrow NA^2 = r^2 - q^2$ … (1)

Consider ΔOMA,

By Pythagoras Theorem,

$\Rightarrow OM^2 + AM^2 = OA^2$

$\Rightarrow p^2 + AM^2 = r^2$

$\Rightarrow AM^2 = r^2 - p^2$ ... (2)

$\Rightarrow AM = \frac{AB}{2} = \frac{2AC}{2} = AC = 2NA$

From (1) and (2),

$\Rightarrow r^2 - p^2 = AM^2 = (2NA)^2 = 4NA^2$

$\Rightarrow r^2 - p^2 = 4\ (r^2 - q^2)$

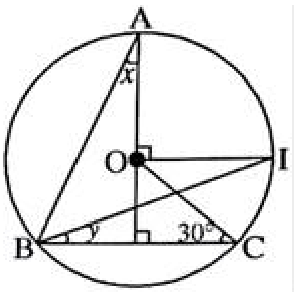
$\Rightarrow r^2 - p^2 = 4r^2 - 4q^2$

$\therefore 4q^2 = 3r^2 + p^2$

Hence proved.

31. Question

In figure, O is the centre of a circle and ∠BCO = 30°. Find x and y.

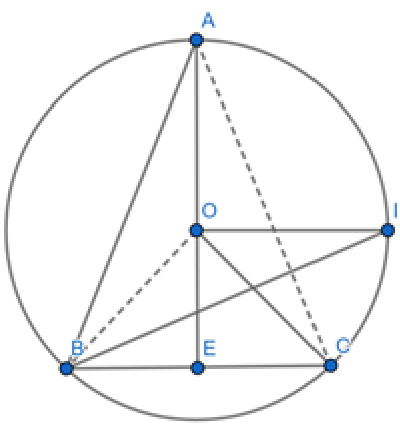


Answer

Given O is the centre of the circle and ∠BCO = 30°.

Construction:

Join AC and OB.



In ΔOBC,

$\Rightarrow$ OC = OB [Radii of circle]

$\Rightarrow \angle OBC = \angle OCB = 30°$

From angle sum property,

$\Rightarrow \angle OBC + \angle OCB + \angle BOC = 180°$

$\Rightarrow 30° + 30° + \angle BOC = 180°$

$\Rightarrow \angle BOC = 180° - 60°$

$\therefore \angle BOC = 120°$

We know that the central angle subtended by two points on a circle is twice the inscribed angle subtended by those points.

$\Rightarrow \angle BOC = 2\angle BAC$

$\Rightarrow 2\angle BAC = 120°$

$\therefore \angle BAC = 60°$

$\Rightarrow \angle AEB = 90°$

So, $OE \perp BC$

We know that a line form center to any chord is perpendicular then that line also bisects chord.

$\Rightarrow CE = BE \dots (1)$

Consider ΔABE and ΔACE,

$\Rightarrow CE = BE$ [From (1)]

$\Rightarrow \angle AEB = \angle AEC = 90°$

$\Rightarrow AE = AE$ [Common side]

By RHL rule,

$\Rightarrow \Delta ABE \cong \Delta ACE$

By CPCT,

$\angle BAE = \angle CAE$

$\Rightarrow \angle BAC = \angle BAE + \angle CAE$

$\Rightarrow x + x = 60°$

$\Rightarrow 2x = 60°$

$\therefore x = 30°$

And $\angle COI = \angle OCB = 30°$ [Alternate interior angles]

We know that the central angle subtended by two points on a circle is twice the inscribed angle subtended by those points.

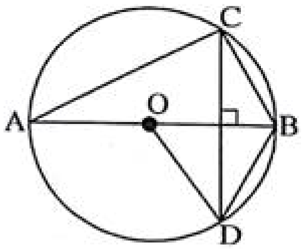
$\Rightarrow \angle COI = 2\angle CBI$

$\Rightarrow 2y = 30°$

$\therefore y = 15°$

32. Question

In figure, O is the centre of a circle, BD = OD and CD ⊥ AB. Find ∠CAB.



Answer

Given O is the centre of the circle, BD = OD and CD ⊥ AB.

⇒ OD = OB [Radii of circle]

⇒In ΔOBD,

⇒ OD = BD = OB

∴ ΔOBD is an equilateral triangle.

Each interior angle of an equilateral triangle is 60°.

$\Rightarrow \angle OBD = \angle ODB = \angle BOD = 60°$

Also given CD ⊥ AB,

We know that in equilateral triangle altitude drawn from any vertex bisects the vertex angle and also bisects the opposite side.

$\Rightarrow \angle EDB = \angle EDO = 30°$ … (1) and OE = BE

We know that if a perpendicular is drawn from center to any chord, it bisects the chord.

⇒ CE = DE … (2)

Consider ΔBED and ΔBEC,

⇒ CE = DE [From equation (2)]

$\Rightarrow \angle BED = \angle BEC = 90°$ [CD ⊥ AB]

⇒ BE = BE [Common side]

By SAS rule,

⇒ $\Delta BED \cong \Delta BEC$

By CPCT,

⇒$\angle BDE = \angle BCE$

From (1),

⇒$\angle BDE = \angle BCE = 30°$

In ΔBCE,

From angle sum property,

⇒$\angle BCE + \angle BEC + \angle CBE = 180°$

⇒ $30° + 90° + \angle CBE = 180°$

⇒$\angle CBE = 180° - 120°$

∴ $\angle CBE = 60°$

We know that the angle inscribed in a semicircle is always a right angle.

In ΔABC,

From angle sum property,

⇒$\angle CBA + \angle ACB + \angle CAB = 180°$

⇒ $60° + 90° + \angle CAB = 180°$ [$\angle CBA = \angle CBE$]

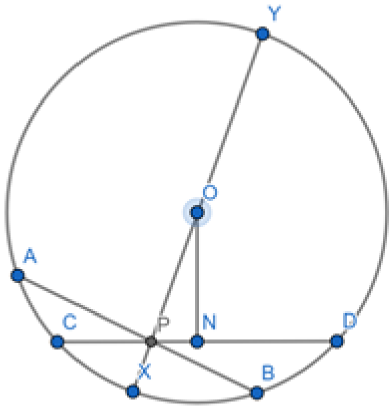
⇒$\angle CAB = 180° - 150°$

∴ $\angle CAB = 30°$

33. Question

Prove that of all the chords passing through a point inside the circle, the smallest chord is one which is perpendicular to the diameter passing through that point.

Answer



Let P be the given point inside a circle with centre O.

Draw the chord AB which is perpendicular to the diameter XY through P.

Draw ON ⊥ CD from O.

Then ΔONP is a right angled triangle.

⇒Its hypotenuse OP is larger than ON.

We know that the chord nearer to the centre is larger than the chord which is farther to the centre.

⇒ CD > AB

∴ AB is the smallest of all chords passing through P.