Short Answer Questions – II (PYQ)

Q. 1. Write any two important points of similarities and differences each between Coulomb's law for the electrostatic field and Biot-Savart's law for the magnetic field.
[CBSE (F) 2015]

Ans. Similarities:

Both electrostatic field and magnetic field:

Follows the principle of superposition.

Depends inversely on the square of distance from source to the point of interest.

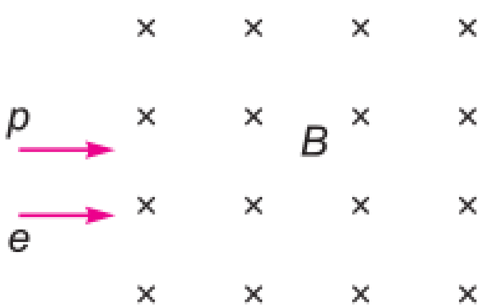
Differences:

(i) Electrostatic field is produced by a scalar source (q) and the magnetic field is produced by a vector (I $\vec{dl}$).

(ii) Electrostatic field is along the displacement vector between source and point of interest; while magnetic field is perpendicular to the plane, containing the displacement vector and vector source.

(iii) Electrostatic field is angle independent, while magnetic field is angle dependent between source vector and displacement vector.

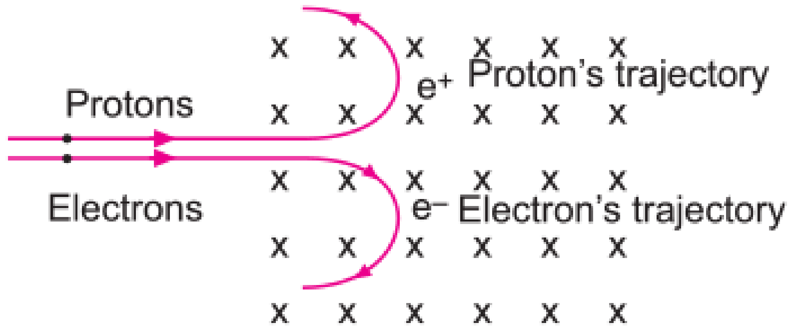
Q. 2. An electron and a proton enter a region of uniform magnetic field B with uniform speed v in a perpendicular direction (fig.).



(i) Show the trajectories followed by two particles.

(ii) What is the ratio of the radii of the circular paths of electron to proton?
[CBSE (F) 2010]

Ans. (i) Trajectories are shown in figure.



(ii)

As $r = \frac{mv}{qB} \rightarrow r \propto m$

Ratio of radii of electron path and proton path.

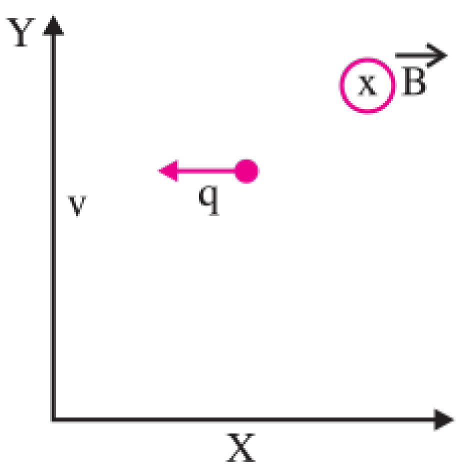
$$\frac{r_e}{r_p} = \frac{m_e}{m_p}$$

As mass of proton $m_p \approx 1840 \times$ mass of electron (m_e)

$$\therefore \frac{m_e}{m_p} \approx \frac{1}{1840} \quad \text{and} \quad \frac{r_e}{r_p} = \frac{1}{1840}$$

Q. 3. Answer the following questions

(i) A point charge q moving with speed v enters a uniform magnetic field B that is acting into the plane of the paper as shown. What is the path followed by the charge q and in which plane does it move?



(ii) How does the path followed by the charge get affected if its velocity has a component parallel to $\vec{B}$?

(iii) If an electric fieldis also applied such that the particle continues moving along the original straight line path, what should be the magnitude and direction of the electric field [CBSE (F) 2016]

Ans. (i) The force experienced by the charge particle is given by $\vec{F} = q\,(\vec{V} \times \vec{B})$

When $\vec{V}$ is perpendicular to $\vec{B}$ the force on the charge particle acts as the centripetal force and makes it move along a circular path. Path followed by charge is anticlockwise in X-Y plane. The point charge moves in the plane perpendicular to both $\vec{V}$ and $\vec{B}$.

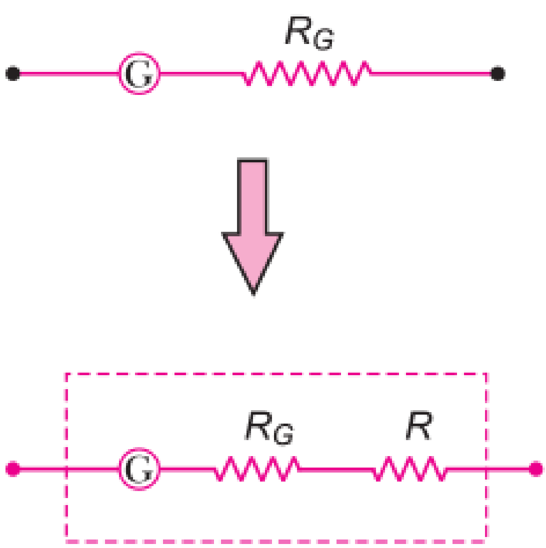
Q. 4. How is a galvanometer converted into a voltmeter and an ammeter? Draw the relevant diagrams and find the resistance of the arrangement in each case. Take resistance of galvanometer as G. [CBSE East 2016]

Ans. A galvanometer is converted into a voltmeter by connecting a high resistance 'R' in series with it.

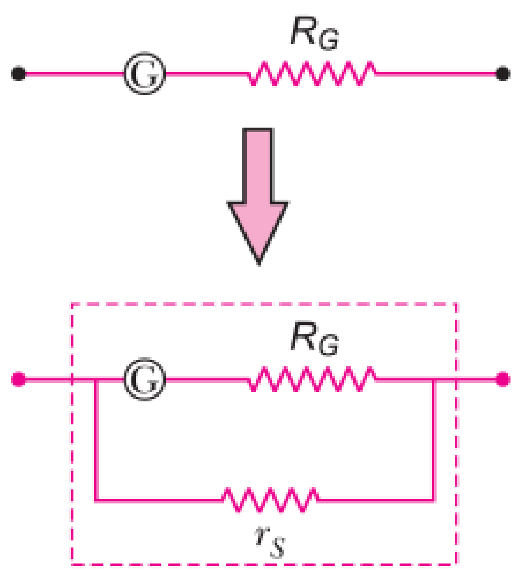
A galvanometer is converted into an ammeter by connecting a small resistance (called shunt) in parallel with it.

Resistance of voltmeter, $R_V = G + R$

Resistance of Ammeter, $R_A = \dfrac{Gr_s}{G+r_s}$



Voltmeter

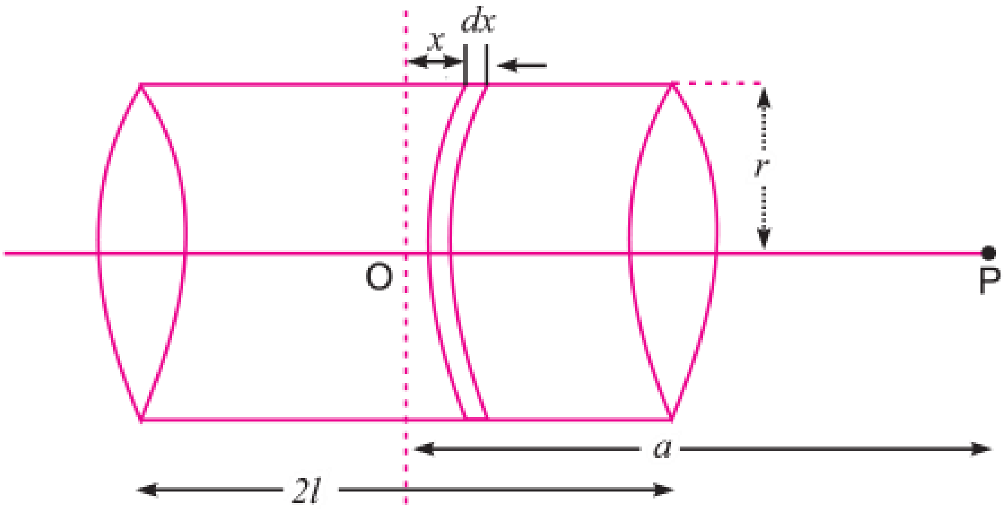


Ammeter

Q. 5. Derive an expression for the axial magnetic field of a finite solenoid of length 2l and radius r carrying current I. Under what condition does the field become equivalent to that produced by a bar magnet? [CBSE South 2016]

Ans. Consider a solenoid of length 2l, radius r and carrying a current I and having n turns per unit length.

Consider a point P at a distance a from the centre O of solenoid. Consider an element of solenoid of length dx at a distance x from its centre. This element is a circular current loop having (ndx) turns. The magnetic field at axial point P due to this current loop is



$$dB = \frac{\mu_0 (ndx) Ir^2}{2\{r^2 + (a-x)^2\}^{3/2}}$$

The total magnetic field due to entire solenoid is

$$\therefore \quad B = \int_{-l}^{+l} \frac{\mu_0 (ndx) Ir^2}{2\{r^2 + (a-x)^2\}^{3/2}}$$

For $a >> l$ and $a >> r$, we have $\{r^2 + (a-x)^2\}^{3/2} = a^3$

$$\therefore \quad B = \frac{\mu_0 nIr^2}{2a^3} \int_{-l}^{+l} dx = \frac{\mu_0 nIr^2 (2l)}{2a^3}$$

The magnetic moment of solenoid $m\ (= NIA) = (n2l)I \,.\, \pi r^2$

$$\therefore \quad B = \frac{\mu_0}{4\pi} \frac{2m}{a^3}$$

This is also the far axial magnetic field of a bar magnet. Hence, the magnetic field, due to current carrying solenoid along its axial line is similar to that of a bar magnet for far off axial points.

Q. 6. A cyclotron's oscillator frequency is 10 MHz. What should be the operating magnetic field for accelerating protons? If the radius of its 'dees' is 60 cm, calculate the kinetic energy (in MeV) of the proton beam produced by the accelerator.
[CBSE Ajmer 2015]

Ans. The oscillator frequency should be same as proton cyclotron frequency, then

Magnetic field,

$$B = \frac{2\pi \text{ mv}}{q}$$

$$= \frac{2 \times 3.14 \times 1.67 \times 10^{-27} \times 10^{7}}{1.6 \times 10^{-19}} = 0.66\,T$$

$$v = r\omega = r \times 2\pi\nu$$

$$= 0.6 \times 2 \times 3.14 \times 10^{7} = 3.78 \times 10^{7} \text{m/s}$$

$$\text{So, Kinetic energy, } \text{KE} = \frac{1}{2}\text{mv}^2$$

$$= \frac{1}{2} \times 1.67 \times 10^{-27} \times (3.78 \times 10^{7})^2 J$$

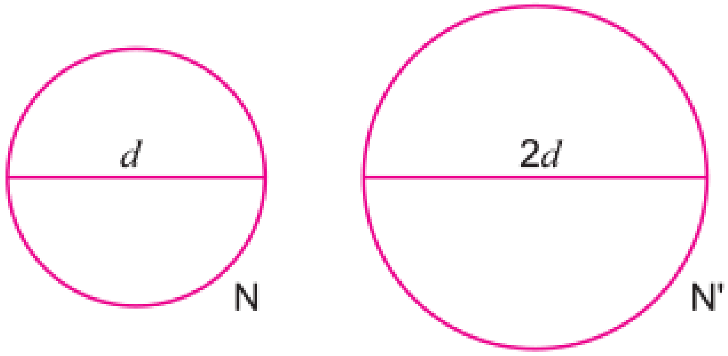
$$= \frac{1}{2} \times \frac{1.67 \times 10^{-27} \times 14.3 \times 10^{14}}{1.6 \times 10^{-19} \times 10^{6}} \text{MeV} = 7.4 \text{ MeV}$$

Q. 7. A circular coil of 'N' turns and diameter 'd' carries a current 'I'. It is unwound and rewound to make another coil of diameter '2d', current 'I' remaining the same. Calculate the ratio of the magnetic moments of the new coil and the original coil.
[CBSE (AI) 2012]

Ans. We know,

Magnetic moment (m) = NIA

Where N = Number of turns



Then, length of wire remains same

$$\text{Thus, } N \times \left[2\pi\left(\frac{d}{2}\right)\right] = N'\left[2\pi\left(\frac{2d}{2}\right)\right]$$

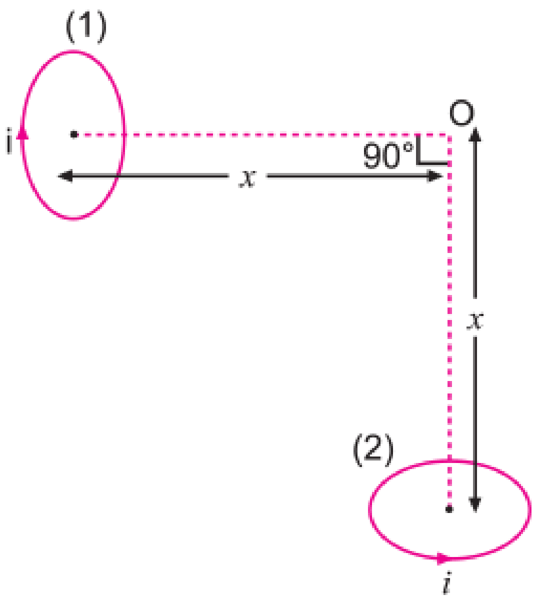
$$\Rightarrow \quad N' = \frac{N}{2}$$

$$\text{Now, } m_A = \text{NIA}_A = \text{NI}\,(\pi r_A^2) = \frac{1}{4}\text{NI}\,\pi d^2$$

$$\text{Similarly } m_B = N'I\,A_B = \frac{\text{NI}}{2}\left(\pi r_B^2\right) = \frac{1}{2}(\text{NI}\,\pi d^2)$$

$$\frac{m_B}{m_A} = \frac{\frac{1}{2}}{\frac{1}{4}} = \frac{2}{1} \quad \Rightarrow \quad \frac{m_B}{m_A} = \frac{2}{1}$$

Q. 8. Two small identical circular loops, marked (1) and (2), carrying equal currents, are placed with the geometrical axes perpendicular to each other as shown in the figure. Find the magnitude and direction of the net magnetic field produced at the point O. [CBSE (F) 2013, 2014]



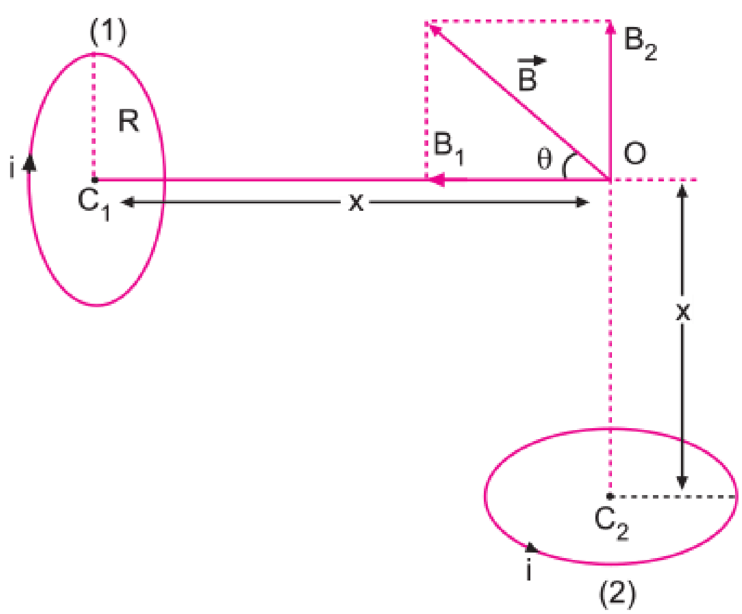
Ans. Magnetic field due to coil 1 at point O

$$\overrightarrow{B_1} = \frac{\mu_0 \text{ iR}^2}{2(R^2+x^2)^{3/2}} \text{ along } \overrightarrow{\text{OC}_1}$$

Magnetic field due to coil 2 at point O

$$\overrightarrow{B_2} = \frac{\mu_0 \text{ iR}^2}{2(R^2+x^2)^{3/2}} \text{ along } \overrightarrow{C_2O}$$

Both $\overrightarrow{B_1}$ and $\overrightarrow{B_2}$ are mutually perpendicular, so the net magnetic field at O is



$$B = \sqrt{B_1^2 + B_2^2} = \sqrt{2}B_1 \,(\text{as } \; B_1 = B_2)$$

$$= \sqrt{2}\frac{\mu_0 \, \text{iR}^2}{2(R^2+x^2)^{3/2}}$$

As $R << x$

$$B = \frac{\sqrt{2}\mu_0 \, \text{iR}^2}{2.x^3} = \frac{\mu_0}{4\pi} . \; \frac{2\sqrt{2}.\mu_0 i \,(\pi R^2)}{x^3}$$

$$= \frac{\mu_0}{4\pi} \; \frac{2\sqrt{2}.\mu_0 i \, A}{x^3}$$

where $A = \pi R^2$ is area of loop.

$$\tan \, \theta = \frac{B_2}{B_1} \quad \Rightarrow \quad \tan \, \theta = 1 \qquad (\because B_2 = B_1)$$

$$\Rightarrow \qquad \theta = \frac{\pi}{4}$$

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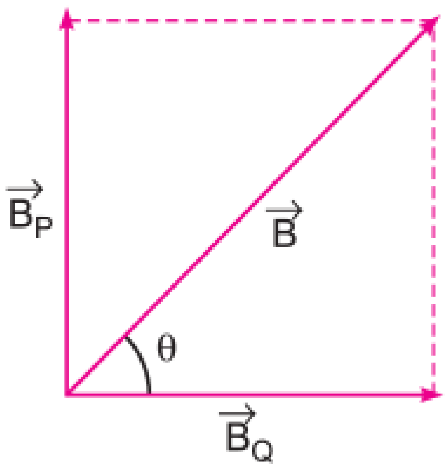
$$\Rightarrow \qquad \theta = \frac{\pi}{4}$$

$\therefore \vec{B}$ is directed at an angle $\frac{\pi}{4}$ with the direction of magnetic field $\overrightarrow{B_1}$.

Q. 9. Two identical coils P and Q each of radius R are lying in perpendicular planes such that they have a common centre. Find the magnitude and direction of magnetic field at the common centre of the two coils, if they carry currents equal to I and $-\sqrt{3}$ I respectively. [CBSE (F) 2016] [HOTS]

Ans. Given that two identical coils are lying in perpendicular planes and having common centre. P and Q carry current I and $\sqrt{3}$ I respectively.

Now, magnetic field at the centre of P due to its current, I



$$\overrightarrow{B_P} = \frac{\mu_0 I}{2R}$$

And, magnetic field at centre of Q, due to its current $\sqrt{3}\ I$

$$\overrightarrow{B_Q} = \frac{\mu_0 \sqrt{3} I}{2R}$$

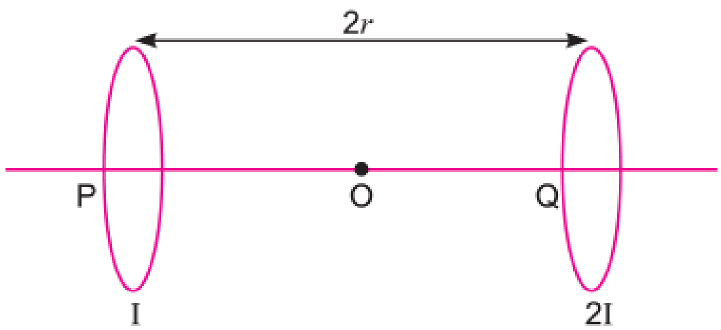
$$\therefore\ \overrightarrow{\overrightarrow{B_{\text{net}}}} = \overrightarrow{B_P} + \overrightarrow{B_Q} = \frac{\mu_0 I}{2R} + \frac{\mu_0 \sqrt{3} I}{2R}$$

$$\therefore\ \left|\overrightarrow{\overrightarrow{B_{\text{net}}}}\right| = \sqrt{\left(\frac{\mu_0 I}{2R}\right)^2 + \left(\frac{\mu_0 \sqrt{3} I}{2R}\right)^2} = \frac{\mu_0 I}{2R} \times 2 = \frac{\mu_0 I}{R}$$

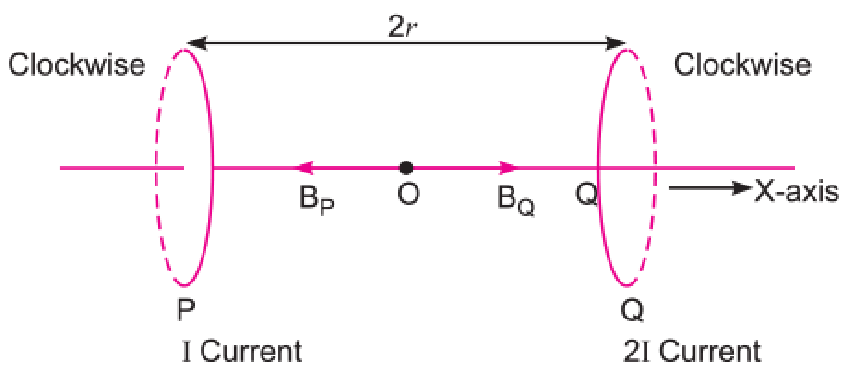
$$\therefore\ \tan\,\theta = \frac{|\overrightarrow{B_P}|}{|\overrightarrow{B_Q}|} = \left(\frac{\frac{\mu_0 I}{2R}}{\frac{\mu_0 \sqrt{3} I}{2R}}\right) = \frac{1}{\sqrt{3}}$$

$$\Rightarrow\quad \theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30^\circ$$

Q. 10. Two identical circular loops, P and Q, each of radius r and carrying currents I and 2I respectively are lying in parallel planes such that they have a common axis. The direction of current in both the loops is clockwise as seen from O which is equidistant from the both loops. Find the magnitude of the net magnetic field at point O.
[CBSE (AI) 2012] [HOTS]



Ans.



$$|\overrightarrow{B_P}| = \frac{\mu_0 r^2 I}{2(r^2+r^2)^{3/2}} = \frac{\mu_0 I}{4\sqrt{2}r} \text{ Pointing towards } P$$

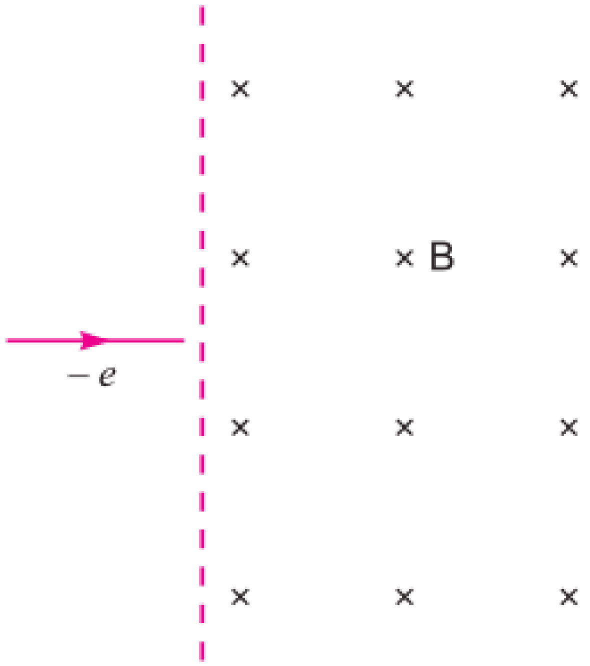
$$|\overrightarrow{B_Q}| = \frac{\mu_0 (2I) r^2}{2(r^2+r^2)^{3/2}} = \frac{\mu_0 2I}{4\sqrt{2}r} \text{ Pointing towards } Q$$

$$|\overrightarrow{B}| = |\overrightarrow{B_Q}| - |\overrightarrow{B_P}| = \frac{\mu_0 I}{4\sqrt{2}r}$$

So, magnetic field at point O has a magnitude $\frac{\mu_0 I}{4\sqrt{2}r}$.

Q. 11. Answer the following questions

(i) An electron moving horizontally with a velocity of 4 × 10^4 m/s enters a region of uniform magnetic field of 10^{-5} T acting vertically upward as shown in the figure. Draw its trajectory and find out the time it takes to come out of the region of magnetic field.



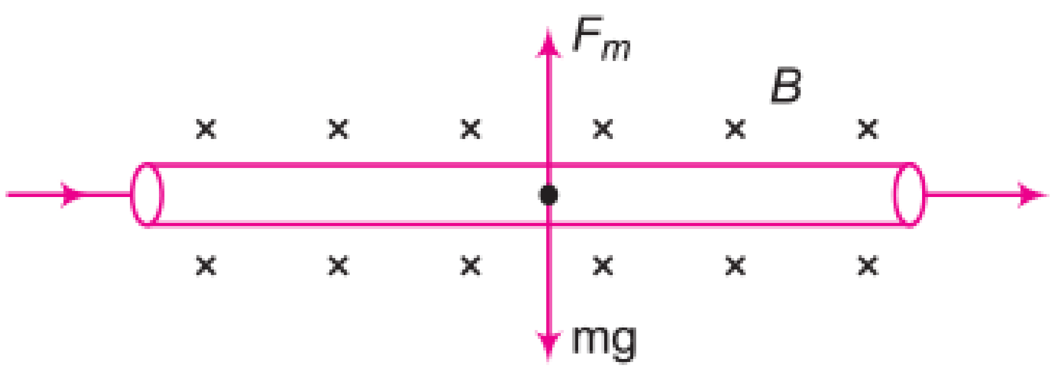
(ii) A straight wire of mass 200 g and length 1.5 m carries a current of 2A. It is suspended in midair by a uniform magnetic field B. What is the magnitude of the magnetic field? [CBSE (F) 2015] [HOTS]

Ans. If Ampere's force acts in upward direction and balances the weight, that is,

F_m = mg

$$B = \frac{\text{mg}}{\text{Il}} = \frac{0.2 \times 10}{2 \times 1.5} = \frac{2}{3} = 0.67$$

$$BIl = mg$$

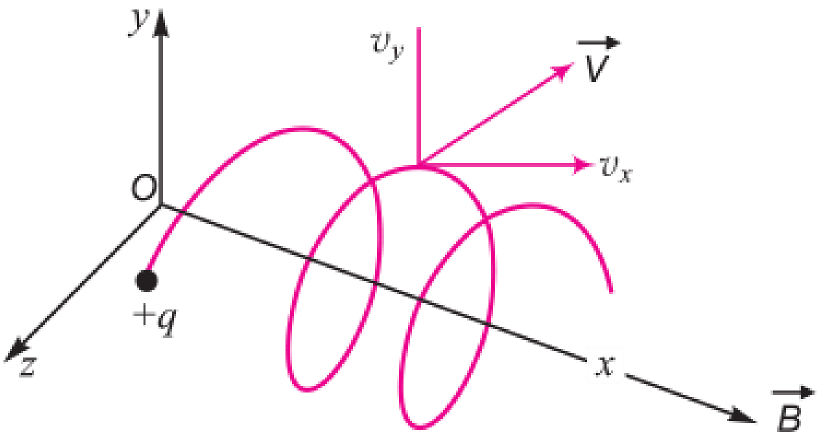


$$B = \frac{\text{mg}}{\text{Il}} = \frac{0.2 \times 10}{2 \times 1.5} = \frac{2}{3} = 0.67$$

Q. 12. A uniform magnetic field $\vec{B}$ is set up along the positive x-axis. A particle of charge 'q' and mass 'm' moving with a velocity $\vec{V}$ enters the field at the origin in X-Y plane such that it has velocity components both along and perpendicular to the magnetic field $\vec{B}$. Trace, giving reason, the trajectory followed by the particle. Find out the expression for the distance moved by the particle along the magnetic field in one rotation. [CBSE Allahabad 2015] [HOTS]

Ans. If component v_x of the velocity vector is along the magnetic field, and remain constant, the charge particle will follow a helical trajectory; as shown in fig.

If the velocity component v_y is perpendicular to the magnetic field B, the magnetic force acts like a centripetal force $qv_y B$.



So, $qv_yB = \frac{mv_y^2}{r} \Rightarrow v_y = \frac{qBr}{m}$

Since tangent velocity $v_y = r\omega$

$\Rightarrow r\omega = \frac{qBr}{m} \Rightarrow \omega = \frac{qB}{m}$

Time taken for one revolution, $T = \frac{2\pi}{\omega} = \frac{2\pi m}{qB}$

and the distance moved along the magnetic field in the helical path is

$x = v_x . T = v_x . \frac{2\pi m}{qB}$

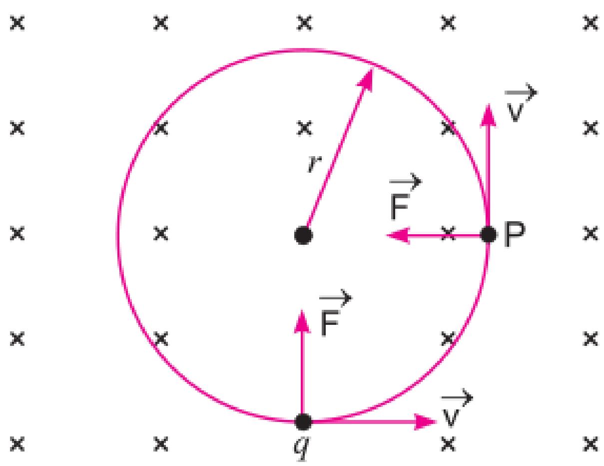
Q. 13. A particle of charge 'q' and mass 'm' is moving with velocity $\vec{V}$. It is subjected to a uniform magnetic field $\vec{B}$ directed perpendicular to its velocity. Show that it describes a circular path. Write the expression for its radius. [CBSE (F) 2012] [HOTS]

Ans. When a particle of charge 'q' of mass 'm' is directed to move perpendicular to the uniform magnetic field 'B' with velocity '$\vec{V}$'

The force on the charge

$$\vec{F} = q(\vec{v} \times \vec{B})$$

This magnetic force acts always perpendicular to the velocity of charged particle. Hence magnitude of velocity remains constant but direction changes continuously. Consequently the path of the charged particle in a perpendicular magnetic field becomes circular. The magnetic force (qvB) provides the necessary centripetal force to move along a circular path. Then,



$$qvB = \frac{mv^2}{r} \quad \Rightarrow \quad r = \frac{mv}{qB}$$

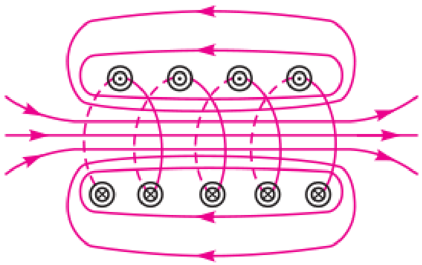
Here r = radius of the circular path followed by the charge.

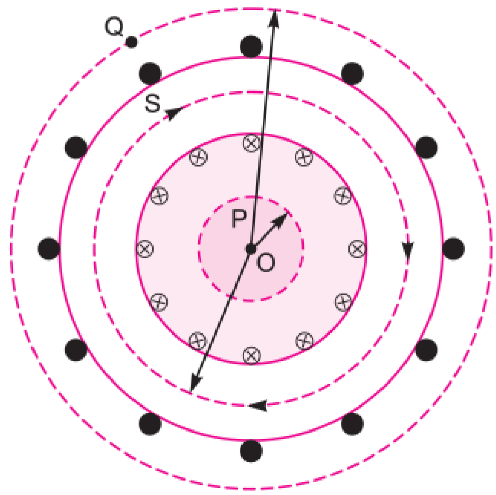
Q. 14. Answer the following questions

(i) In what respect is a toroid different from a solenoid? Draw and compare the pattern of the magnetic field lines in the two cases. [CBSE (AI) 2011]

(ii) How is the magnetic field inside a given solenoid made strong? [CBSE (AI) 2011]

Ans. (i) A toroid is a solenoid bent into the form of a closed ring. The magnetic field lines of solenoid are straight lines parallel to the axis inside the solenoid.





(ii) The magnetic field lines of toroid are circular having common centre.

Inside a given solenoid, the magnetic field may be made strong by (i) passing large current and (ii) using laminated coil of soft iron.

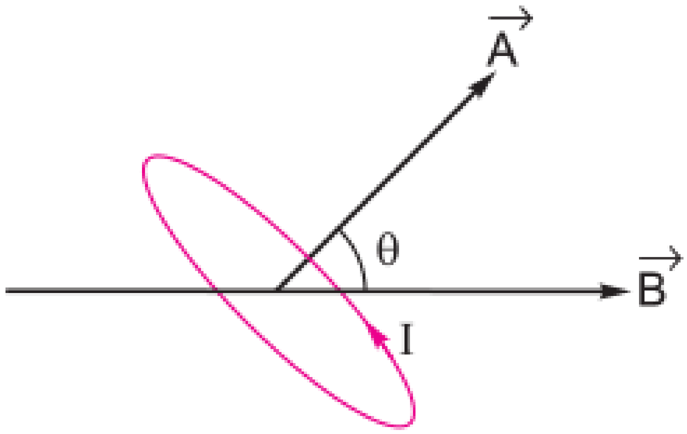
Q. 15. Answer the following questions.

(i) (a) A circular loop of area $\vec{B}$, carrying a current I is placed in a uniform magnetic field $\vec{B}$ Write the expression for the torque $\vec{t}$ acting on it in a vector form.

(b) If the loop is free to turn, what would be its orientation of stable equilibrium? Show that in this orientation, the flux of net field (external field + the field produced by the loop) is maximum.

(ii) Find out the expression for the magnetic field due to a long solenoid carrying a current I and having n number of turns per unit length.

Ans. (i)



i. Torque acting on the current loop $\vec{\tau} = \vec{m} \times \vec{B} = I(\vec{A} \times \vec{B})$

ii. If magnetic moment $\vec{m} = I\vec{A}$ is in the direction of external magnetic field *i.e.*, θ=0°.

Magnetic flux $\varphi_B = (\overrightarrow{B^{\text{ext}}} + B_C) \cdot \vec{A}$

$$\varphi_{\max} = \left[\left|\overrightarrow{B^{\text{ext}}}\right| + \frac{\mu_0 I}{2r}\right]|A| \cos 0^\circ$$

where r is radius of the loop.

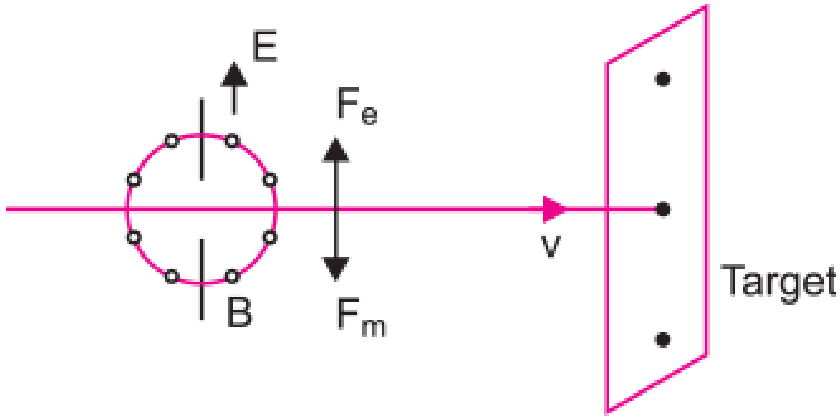
Q. 16. A beam of protons passes undeflected with a horizontal velocity v, through a region of electric and magnetic fields, mutually perpendicular to each other and normal to the direction of the beam. If the magnitudes of the electric and magnetic fields are 50 kV/m and 100 mT respectively; calculate the [CBSE (AI) 2008] [HOTS]

(i) Velocity of the beam.

(ii) Force with which it strikes a target on the screen, if the proton beam current is equal to 0.80 mA.

Ans. (i) For a beam of charged particles to pass undeflected crossed electric and magnetic fields, the condition is that electric and magnetic forces on the beam must be equal and opposite i.e.,

$$\text{eE} = \text{evB} \quad \Rightarrow \quad v = \frac{E}{B}$$



Given, $E = 50 \text{kV/m} = 50 \times 10^3 \text{ V/m}$

$B = 100 \text{ mT} = 100 \times 10^{-3} \text{ T}$

$$\therefore \quad v = \frac{50 \times 10^3}{100 \times 10^{-3}} = 5 \times 10^5 \text{ ms}^{-1}$$

Q. 17. Answer the following questions.

(i) Obtain the expression for the cyclotron frequency.

(ii) A deuteron and a proton are accelerated by the cyclotron. Can both be accelerated with the same oscillator frequency? Give reason to justify your answer.
[CBSE Delhi 2017]

Ans. (i) Suppose the positive charge ion with charge q moves in a dee with a velocity v, then

$$\frac{mv^2}{r} = qvB \qquad \Rightarrow r = \frac{mv}{qB}$$

$$\text{Frequency of revolution } \nu = \frac{1}{\text{Time Period } (T)} = \frac{v}{2\pi r} \qquad \left(\because \quad T = \frac{2\pi r}{v}\right)$$

$$\nu = \frac{qB}{2\pi m}$$

(ii) No.

The mass of the two particles, i.e., deuteron and proton, is different. Since cyclotron frequency depends inversely on the mass, they cannot be accelerated by the same oscillator frequency.

Q. 18. Answer the following questions

(i) Write the expression for the force $\vec{F}$ acting on a particle of mass m and charge q moving with velocity $\vec{V}$ in a magnetic field $\vec{B}$ Under what conditions will it move in (a) a circular path and (b) a helical path? [CBSE Delhi 2017]

(ii) Show that the kinetic energy of the particle moving in magnetic field remains constant. [CBSE Delhi 2017]

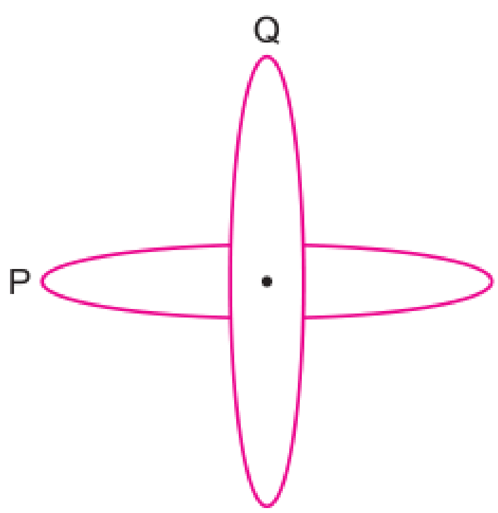
Ans. (i) i. $\vec{F} = q(\vec{v} \times \vec{B})$

(a) When velocity of charged particle and magnetic field are perpendicular to each other charged particle will move in circular path.

(b) When velocity is neither parallel nor perpendicular to the magnetic field charged particle will move in a helical path.

(ii) The force, experienced by the charged particle is perpendicular to the instantaneous velocity, $\vec{V}$ at all instants. Hence the magnetic force cannot bring any change in the speed of the charged particle. Since speed remain constant, the kinetic energy also remains constant.

Q. 19. Two identical loops P and Q each of radius 5 cm are lying in perpendicular planes such that they have a common centre as shown in the figure. Find the magnitude and direction of the net magnetic field at the common centre of the two coils, if they carry currents equal to 3A and 4A respectively. [CBSE (AI) 2017]



Ans.

Magnetic field at the centre of a circular coil = $\frac{\mu_0 I}{2R}$

Magnetic field due to coil P,

$$B_P = \frac{\mu_0 \times 3}{2 \times 5 \times 10^{-2}} T = 12\pi \times 10^{-6} T$$

Magnetic field due to coil Q,

$$B_Q = \frac{\mu_0 \times 4}{2 \times 5 \times 10^{-2}} T = 16\pi \times 10^{-6} T$$

Net magnetic field, $B = \sqrt{B_P^2 + B_Q^2}$

$$= \sqrt{(12\pi \times 10^{-6})^2 + (16\pi \times 10^{-6})^2}$$

$$= \pi\sqrt{144 + 256} \times 10^{-6} T = 20\pi \times 10^{-6} T$$

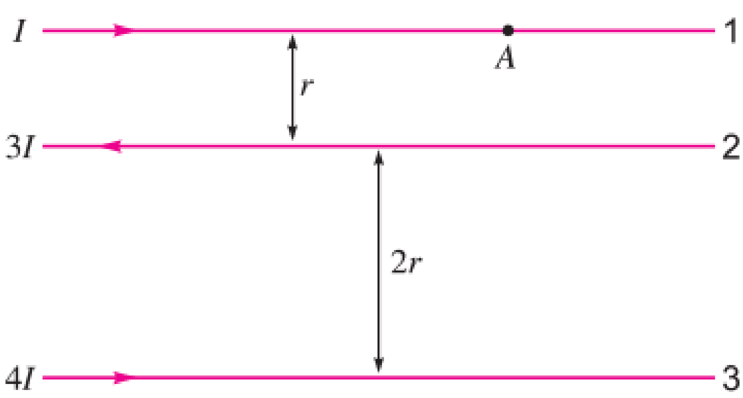
Let the field make an angle θ with the magnetic field due to Q.

$$\tan\theta = \frac{12\pi \times 10^{-6}}{16\pi \times 10^{-6}} = \frac{3}{4} \qquad \Rightarrow \theta = \tan^{-1} = \frac{3}{4}$$

Q. 20. The figure shows three infinitely long straight parallel current carrying conductors. Find the

(i) Magnitude and direction of the net magnetic field at point A lying on conductor 1,

(ii) Magnetic force on conductor 2.



Ans. (i)

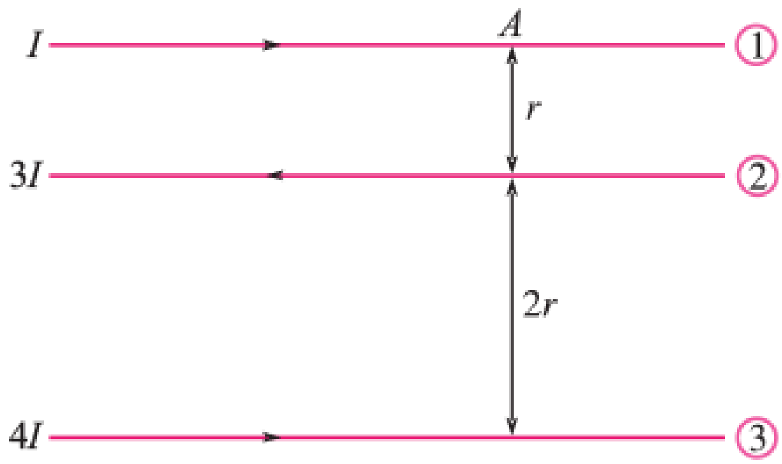
$$B_2 = \frac{\mu_0}{4\pi}\frac{2(3I)}{r} = \frac{\mu_0(6I)}{4\pi r} \text{ into the plane of the } paper.$$

$$B_3 = \frac{\mu_0}{4\pi}\frac{2(4I)}{3r} = \frac{\mu_0}{4\pi}\left(\frac{8I}{3r}\right) \text{ out of the plane of the paper.}$$

$B_A = B_2 - B_3$ into the paper.

$$= \frac{\mu_0}{4\pi}\frac{10I}{3r} \text{ into the } paper.$$

(ii)



Magnetic force per unit length on wire (2)

$$F = \frac{\mu_0}{2\pi r}.3I^2 - \frac{\mu_0 12I^2}{2\pi(2r)}$$

$$= \frac{3}{2}\frac{\mu_0 I^2}{\pi r} - 3\frac{\mu_0 I^2}{\pi r} = -\frac{3}{2}\frac{\mu_0 I^2}{\pi r}$$

Hence, $F = \frac{3}{2}\frac{\mu_0 I^2}{\pi r}$ in the direction of wire 1.

Q. 21. Answer the following questions.

(i) State the condition under which a charged particle moving with velocity v goes undeflected in a magnetic field B.

(ii) An electron, after being accelerated through a potential difference of 10^4 V, enter a uniform magnetic field of 0.04T, perpendicular to its direction of motion. Calculate the radius of curvature of its trajectory. [CBSE (AI) 2017]

Ans. (i)

Force in magnetic field on a charged particle

$$\vec{F} = q(\vec{v} \times \vec{B}) \quad \Rightarrow \; F \;\mathrm{qvB}\;\sin\;\theta$$

If $F = 0$,

$$\Rightarrow \quad 0 = qvB \sin \theta$$

$$\Rightarrow \sin \theta = 0 \qquad \theta = \pm\, n\pi$$

So, magnetic field will be parallel or antiparallel to the velocity of charged particle.

(ii)

For a charged particle moving in a constant magnetic field and $\vec{v} \perp \vec{B}$

$$\frac{\mathrm{mv}^2}{r} = \mathrm{qvB} \quad \Rightarrow \quad r = \frac{\mathrm{mv}}{\mathrm{qB}} = \frac{P}{\mathrm{qB}} \qquad ...(i)$$

If e is accelerated through a potential difference of 10^4 V, then

K. E of electron = eV

$$\Rightarrow \quad \frac{P^2}{2m} = \mathrm{eV} \quad \Rightarrow \; P = \sqrt{2\,\mathrm{meV}} \quad ...(ii)$$

From (i) & (ii)

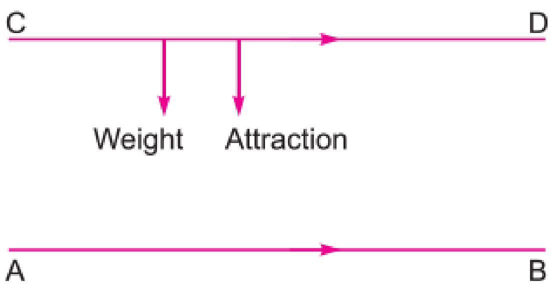
$$\Rightarrow \quad r = \frac{\sqrt{2\,\mathrm{meV}}}{\mathrm{qB}}$$

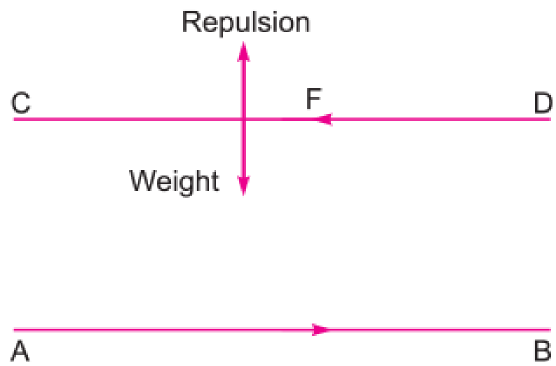
$$= \frac{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times 10^4}}{1.6 \times 10^{-19} \times 0.04}$$

$$= \frac{5.39 \times 10^{-23}}{6.4 \times 10^{-21}} m = 8.4 \times 10^{-3} m$$

Q. 22. A wire AB is carrying a steady current of 12 A and is lying on the table. Another wire CD carrying 5A is held directly above AB at a height of 1 mm. Find the mass per unit length of the wire CD so that it remains suspended at its position when left free. Give the direction of the current flowing in CD with respect to that in AB. [Take the value of g = 10 ms^{-2}] [CBSE (AI) 2013]

Ans. (i) Current carrying conductors repel each other, if current flows in the opposite direction.





(ii) Current carrying conductors attract each other if current flows in the same direction.

If wire CD remain suspended above AB then

$$F_{repulsion} = \text{Weight}$$

$$\frac{\mu_0 I_1 I_2 l}{2\pi r} = mg$$

where r = Separation between the wires

$$\frac{m}{l} = \frac{\mu_0 I_1 I_2}{2\pi rg}$$

$$= \frac{2 \times 10^{-7} \times 12 \times 5}{1 \times 10^{-3} \times 10}$$

$$1.2 \times 10^{-3} \text{ kg / m}$$

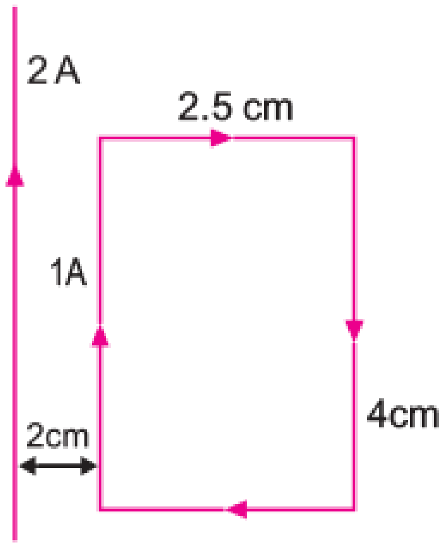
Current in CD should be in opposite direction to that in AB.

Q. 23. A rectangular loop of wire of size 2.5 cm × 4 cm carries a steady current of 1 A. A straight wire carrying 2 A current is kept near the loop as shown. If the loop and the wire are coplanar, find the (i) torque acting on the loop and (ii) the magnitude and direction of the force on the loop due to the current carrying wire. [CBSE Delhi 2012]

Ans.

(*i*) Torque on the loop 'τ' = $MB \sin\theta = \overrightarrow{M} \times \overrightarrow{B}$

$\tau = 0$ $\quad [\therefore \overrightarrow{M}$ and $\overrightarrow{B}$ are parallel$]$



(ii) Magnitude of force

$$\left|\overrightarrow{F}\right| = \frac{\mu_0 I_1 I_2 l}{2\pi}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

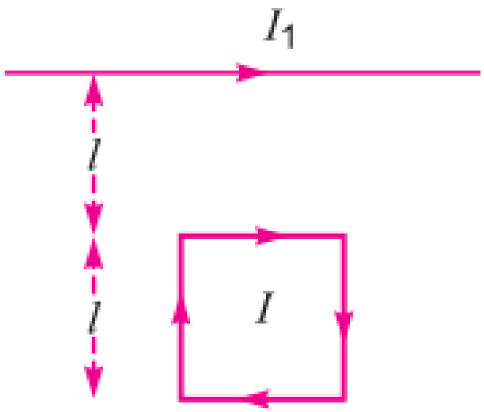
$$= 2 ? 10^{-7} ?2 ?1 ?4 ?10^{-2}\left[\frac{1}{2 ?10^{-2}} - \frac{1}{4.5 ? 10^{-2}}\right]$$

$$= 16 \times 10^{-7} \times \left[\frac{4.5-2}{2 \times 4.5}\right] = \frac{8 \times 5 \times 10^{-7}}{9} = 4.44 \times 10^{-7} N$$

Direction of force is towards conductor (attractive).

Q. 24. Write the expression for the magnetic moment ($\vec{m}$) due to a planar square loop of side 'l' carrying a steady current I in a vector form.

In the given figure this loop is placed in a horizontal plane near a long straight conductor carrying a steady current I_1 at a distance l as shown. Give reasons to explain that the loop will experience a net force but no torque. Write the expression for this force acting on the loop. [HOTS][CBSE Delhi 2010]

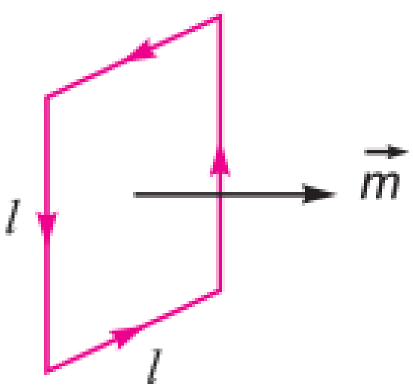


Ans. Magnetic moment due to a planar square loop of side l carrying current I is

$$\vec{m} = I\,\vec{A}$$

For square loop $A = l^2$

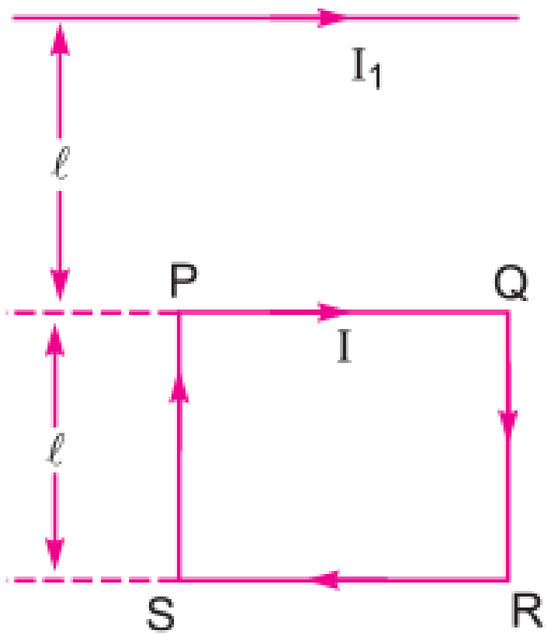
$$\therefore \qquad \vec{m} = I\,l^2\,\hat{n}$$



Where $\hat{n}$ is unit vector normal to loop.

Magnetic field due to current carrying wire at the location of loop is directed downward perpendicular to plane of loop. Since the magnetic moment is parallel to area vector hence torque is zero.

Force on QR and SP are equal and opposite, so net force on these sides is zero.



Force on side PQ, $\overrightarrow{F}_{PQ} = I\ \overrightarrow{l} \times \overrightarrow{B}_1\ l\hat{i}$

$$= \text{Il}\,\hat{i} \times \frac{u_0 I_1}{2\pi l}(-\hat{k})$$

$$= \frac{\mu_0 \text{II}_1}{2\pi}\ \hat{j}\ ;$$

Force on side *RS*, $\overrightarrow{F_{RS}} = I\ l\ (-\hat{i}) \times \frac{\mu_0 I_1}{2\pi(2l)}\ (-\hat{k})$

Net force $\overrightarrow{F} = \overrightarrow{F_{PQ}} - \overrightarrow{F_{RS}} = \frac{\mu_0 \text{II}_1}{4\pi}\ \hat{j}\ ;$

That is loop experiences a repulsive force but no torque.

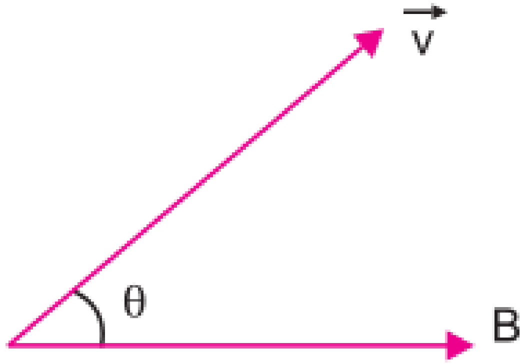
Short Answer Questions –II (OIQ)

Q. 1. Write the expression for the force on a charge moving in a magnetic field. Use this expression to define the SI unit of magnetic field.

Ans. Force on a charge (q) moving in a magnetic field B with velocity $\underset{v}{\rightarrow}$ making an angle θ (with the direction of magnetic field ($\underset{B}{\rightarrow}$) is given by

$F_m = qvB \sin \theta$

When $\theta=90° \Rightarrow \sin \theta=1$, so



$$F_m = qvB$$

$$\text{or} \quad B = \frac{F_m}{qv}$$

If $v=1$ m/s, $B = \frac{F_m}{q}$ newton/coulomb.

SI unit of magnetic field is tesla.

Thus, **1 tesla is the magnetic field in which a charged particle moving with velocity 1 m/s perpendicular to velocity experiences a force of 1 newton/coulomb.**

Q. 2. Define the term magnetic moment of a current loop. Write the expression for the magnetic moment when an electron revolves at a speed v around an orbit of radius 'r' in hydrogen atom.

Ans. Magnetic moment of a current loop: The torque on current loop is

$\tau = MB \sin \theta$, where θ is angle between magnetic moment and magnetic field.

$$\Rightarrow \quad M = \frac{\tau}{B \sin \theta}$$

If B or 1 T, $\sin \theta = 1$ or $\theta = 90°$ then $M = \tau$.

That is ***the magnetic moment of a current loop is defined as the torque acting on the loop when placed in a magnetic field of 1 T such that the loop is oriented with its area vector normal to the magnetic field.***

Also, $M = IA$

i.e., magnetic moment of a current loop is the product of current flowing in the loop and area of loop. Its direction is perpendicular to the plane of the loop and determined by using right hand thumb rule.

Magnetic moment of revolving electron,

$$M = \frac{evr}{2}$$

Q. 3. Which of the following will describe the smallest circle when projected with the same velocity perpendicular to the magnetic field B (i) α-particle and (ii) β-particle?

Ans. Radius of circular path in transverse magnetic field

$r = \frac{mv}{qB} \propto \frac{m}{q}$ for same v and B

For α-particle $\left(\frac{m}{q}\right)_\alpha = \frac{4m_p}{2e} = \frac{2m_p}{e}$ where m_p is mass of proton.

For β-particle $\left(\frac{m}{q}\right)_\beta = \frac{\frac{1}{1840}m_p}{e} = \frac{1}{1840}\left(\frac{m_p}{e}\right)$

Clearly β-particle has smallest value of $\frac{m}{q}$; so β-particle will describe the smallest circle.

Q. 4. A solenoid of length 1.0 m, radius 1 cm and total turns 1000 wound on it, carries a current of 5 A. Calculate the magnitude of the axial magnetic field inside the solenoid. If an electron was to move with a speed of 104 ms^{-1} along the axis of this current carrying solenoid, what would be the force experienced by this electron?

Ans. Magnetic field inside a solenoid,

$B = \mu_0 n I$

$n = \frac{N}{l} = \frac{1000 \ \text{turns}}{1.0 \ m} = 1000 \, \text{turns/m}$

$I = 5\text{A}$

$\therefore \quad B = (4\pi \times 10^{-7}) \times 1000 \times 5$

$= 20 \times 3.14 \times 10^{-4}$ J= $\mathbf{6.28 \times 10^{-3}}$ **T**, along the axis

Force experienced by electron

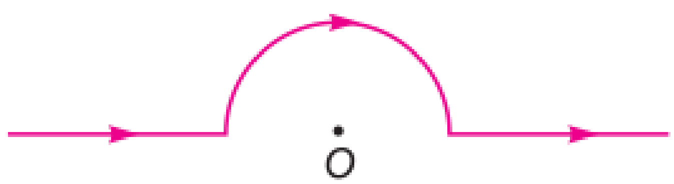
$F_m = qvB \sin\theta$

Here $q = -e, v = 10^4$ m/s,

θ = angle between $\vec{v}$ and $\vec{B} = 0$

$\therefore \quad F_m = -evB \sin 0^\circ = 0$ (zero)

Q. 5. A straight wire carrying a current of 12 A is bent into a semi-circular arc of radius 2.0 cm as shown. What is the magnetic field at O due to (i) straight segments (ii) the semi-circular arc?



Ans. Magnetic field due to a current carrying element.

$$d\overrightarrow{B} = \frac{\mu_0}{4\pi} \frac{I\overrightarrow{\delta l} \times \overrightarrow{r}}{r^3}$$

i. For straight segments $\theta = 0$ or $\pi \Rightarrow \quad \overrightarrow{\delta l} \times \overrightarrow{r} = \delta l \; r \; \sin \; 0 \; \widehat{n} = 0 \quad \therefore \; B_1 = 0$

ii. For semicircular arc $\Sigma \; \mathrm{dl} = \pi r, \theta = \frac{\pi}{2}$

$$\therefore \overrightarrow{B_2} = \frac{\mu_0}{4\pi} \frac{\Sigma I \; \overrightarrow{\delta l} \times \overrightarrow{r}}{r^3} = \frac{\mu_0}{4\pi} \frac{I \; \Sigma \; \delta l \sin\frac{\pi}{2}}{r^2} \widehat{n}$$

$$= \frac{\mu_0}{4\pi} \frac{I\pi r}{r^2} \widehat{n} = \frac{\mu_0 I}{4r},$$

directed perpendicular to plane of paper downward.

Q. 6. A semi-circular arc of radius 20 cm carries a current of 10 A. Calculate the magnitude of magnetic field at the centre of the arc.

Ans. The magnetic field due to a semi-circular arc of radius 'r' carrying current (I) at centre is given by

$$\Delta B = \frac{\mu_0}{4\pi} \frac{I \Delta l \sin 90^o}{r^2} = \frac{\mu_0}{4\pi} \frac{I\Delta}{r^2}$$

The net magnetic field due to whole length of arc l will be

$$B = \frac{\mu_0}{4\pi} \frac{I}{r^2} \sum \; \Delta l$$

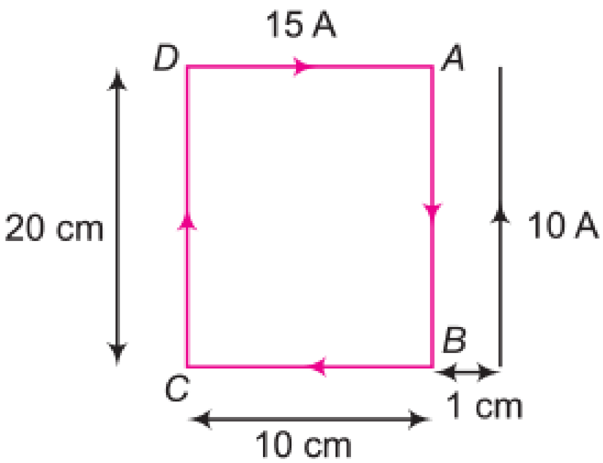
For semi-circular arc $\sum \Delta l = \pi r$

$$\therefore \quad B = \frac{\mu_0}{4\pi} \frac{I}{r^2} (\pi r) = \frac{\mu_0 I}{4r}$$

Given $I = 10$ A, $r = 20$m $= 0.20$ m

$$\therefore \quad B = \frac{4\pi \times 10^{-7} \times 10}{4 \times 0.20} = \frac{4 \times 3.14 \times 10^{-7} \times 10}{4 \times 0.20} = 1.57 \times 10^{-5} \; T$$

Q. 7. A rectangular current carrying loop is placed 1 cm away from a long straight current-carrying conductor as shown.



(i) Will the net force acting on the loop due to straight conductor be attractive or repulsive in nature? Justify your answer. Calculate the magnitude of this force.

Ans. The net force acting on the loop will be repulsive in nature.

Justification: Part AB of the loop will experience a force of repulsion whereas part CD will experience attraction. Parts BC and AD will not experience any force. Thus, the overall force will be a force of repulsion because AB is closer to the straight conductor than CD and the force between two current carrying conductors is inversely proportional to the distance between them.

$$F = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{r} l$$

Net Force

Force F_1 on AB due to current in the straight conductor

$$F_1 = \frac{\mu_0}{2\pi} \times \frac{2\times10\times15}{1\times10^{-2}} \times 20 \times 10^{-2}$$

$$= 3 \times 10^{-4}\ N \text{ towards left} \quad \left(\frac{\mu_0}{4\pi} = 10^{-7}\right)$$

Force F_2 on CD due to current in the straight conductor

$$F_2 = \frac{\mu_0}{2\pi} \times \frac{2\times10\times15}{11\times10^{-2}} \times 20 \times 10^{-2}$$

$$= 0.2725 \times 10^{-4} \text{ N towards right}$$

Hence, net force on the loop = 2.72 N towards left.

Q. 8. The magnitude F of the force between two straight parallel current carrying conductors kept at a distance d apart in air is given by

$$F = \frac{\mu_0 I_1 I_2}{2\pi d}$$

Where I_1 and I_2 are the currents flowing through the two wires.

Use this expression, and the sign convention that the:

"Force of attraction is assigned a negative sign and Force of repulsion is assigned a positive sign".

Draw graphs showing dependence of F on

(i) I_1 I_2 when d is kept constant

(ii) d when the product I_1 I_2 is maintained at a constant positive value.

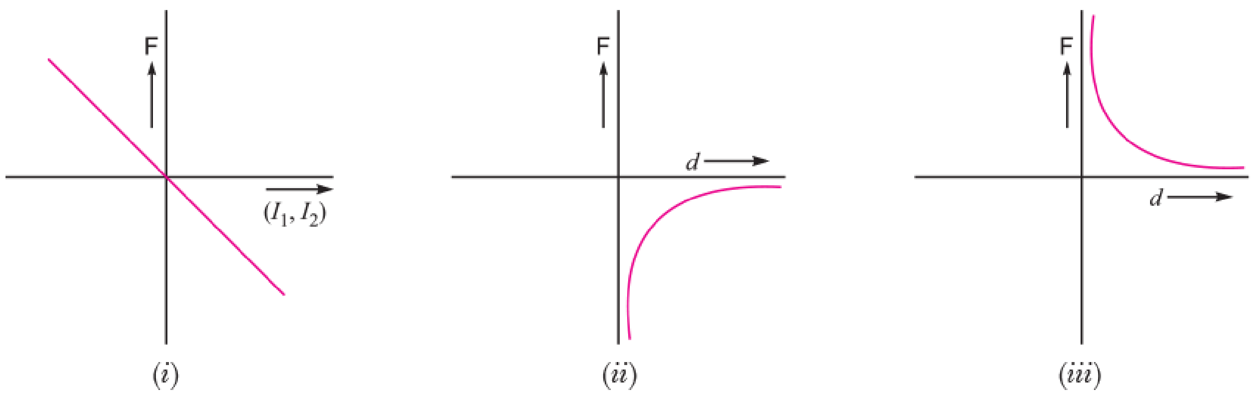
(iii) d when the product I_1 I_2 is maintained at a constant negative value. [CBSE Sample Paper] [HOTS]

Ans. We know that F is an **attractive (–ve)** force when the currents I_1 and I_2 are 'like' currents i.e. when the product I_1 I_2 is positive.

Similarly F is a **repulsive (+ve)** force when the currents I_1 and I_2 are 'unlike' currents, i.e. when the product I_1 I_2 is negative.

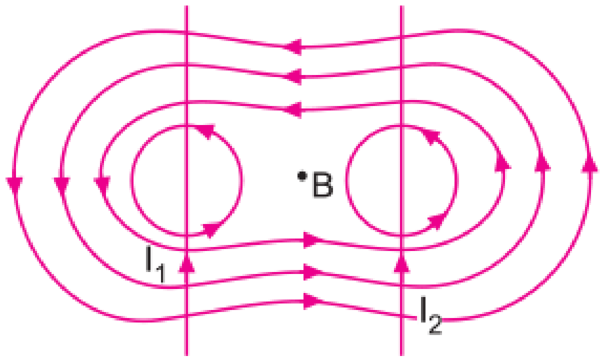
Now $F \propto (I_1 I_2)$, when d is kept constant and **$F \propto 1/d$** when $I_1 I_2$ is kept constant.

The required graphs, therefore, have the forms shown below:

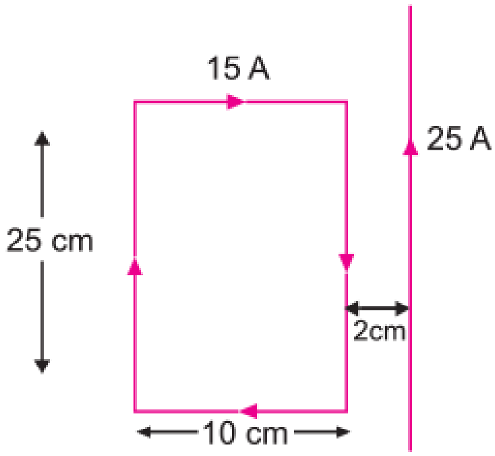


Q. 9. Answer the following questions.

(i) Draw the magnetic field lines due to two straight, long, parallel conductors carrying currents I_1 and I_2 in the same direction. Write an expression for the force acting per unit length on one conductor due to other. Is this force attractive or repulsive?



(ii) Figure shows a rectangular current-carrying loop placed 2 cm away from a long, straight, current-carrying conductor. What is the direction and magnitude of the net force acting on the loop? [HOTS]



Ans. (i) The magnetic field lies due to two current carrying parallel wires are shown in figure. The force between parallel wires $\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi r} N/m$

(ii) We know that parallel currents attract and opposite currents repel and F∝1/r. As wire of loop carrying opposite current is nearer, so the net force acting on the loop is **repulsive.**

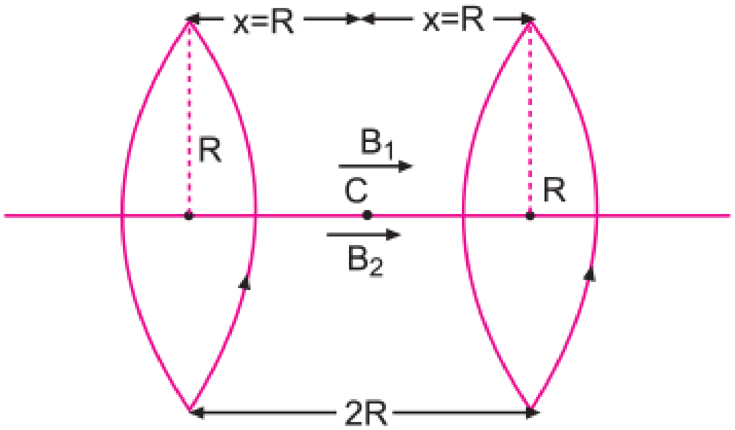
Q. 10. Two parallel coaxial circular coils of equal radius 'R' and equal number of turns 'N', carry equal currents 'I' in the same direction and are separated by a distance '2R'. Find the magnitude and direction of the net magnetic field produced at the mid-point of the line joining their centres. [HOTS]

Ans. Magnetic field due to a circular coil of radius 'R' at a distance x from centre is

$$B = \frac{\mu_0 \text{ NIR}^2}{2(R^2+x^2)^{3/2}}$$

Here $x = R$

$$\therefore \quad B = \frac{\mu_0 \text{ NIR}^2}{2(R^2+R^2)^{3/2}} = \frac{\mu_0 \text{ NIR}^2}{2.2\sqrt{2}R^3} = \frac{\mu_0 \text{ NI}}{4\sqrt{2}R}$$



Total magnetic field at centre C due to both coils

$$B = B_1 + B_2 = 2 \times \frac{\mu_0 \text{ NI}}{4\sqrt{2}R} = \frac{\mu_0 \text{ NI}}{2\sqrt{2}R}$$

Q. 11. A rectangular loop of sides 25 cm and 10 cm carrying a current of 15 A is placed with its longer side parallel to a long straight conductor 2.0 cm apart carrying a current of 25 A (fig). What is the net force on the loop? [HOTS]

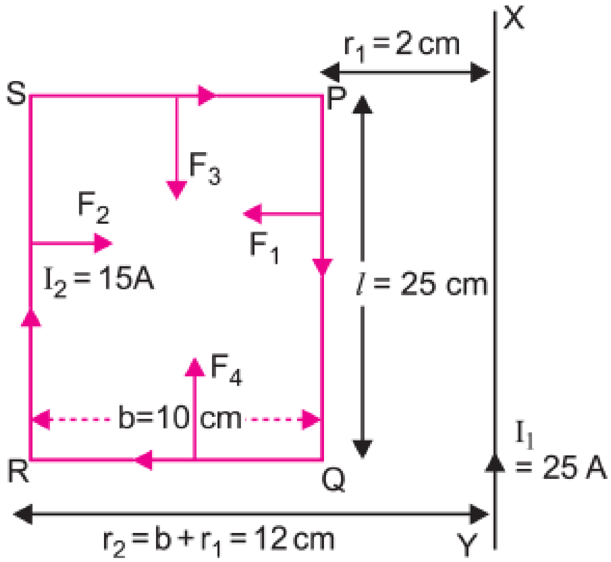
Ans. Rectangular loop PQRS is placed near a long straight wire as shown.

l = PQ = RS = 25 cm = 0.25 m

b = QR = PS =10 cm = 0.10 m

r_1 = 2 cm = 0.02 m,

r_2 = 12 cm = 0.12 m



The currents in PQ and XY are antiparallel, so PQ is repelled away from wire XY. This repulsive force is

$$F_1 = \frac{\mu_0 I_1 I_2}{2\pi r_1} l$$

$$= \frac{4\pi \times 10^{-7} \times 25 \times 15 \times 0.25}{2\pi \times 0.02} = 9.375 \times 10^{-4}\ N$$

The currents in XY and RS are in the same direction, so wire RS is attracted towards wire XY. This attractive force is

$$F_2 = \frac{\mu_0 I_1 I_2 l}{2\pi r_2} = \frac{4\pi \times 10^{-7} \times 25 \times 15 \times 0.25}{2\pi \times 0.12} = 1.563 \times 10^{-4}\ N$$

The currents in PS and QR are equal and opposite. By symmetry they exert equal and opposite forces (F_3 and F_4) and hence net force on these sides is zero.

∴ Net force on rectangular loop

$F = F_1 - F_2$ (repulsive)

$= 9.375 \times 10^{-4} - 1.563 \times 10_{-4}$

$= 7.812 \times 10^{-4}$ N (repulsive)

The net force is directed away from long wire XY.

Q. 12. To increase the current sensitivity of a moving coil galvanometer by 50%, its resistance is increased so that the new resistance becomes twice its initial resistance. By what factor does its voltage sensitivity change? [HOTS]

Ans.

Current sensitivity, $S_C = \frac{\theta}{I} = \frac{NAB}{C}$

Voltage sensitivity, $S_V = \frac{\theta}{V} = \frac{\theta}{IR} = \frac{S_C}{R}$

When current sensitivity is increased by 50%, the resistance is made twice.

$\therefore$ New current sensitivity $S_C' = S_C + \frac{50}{100} S_C = 1.5 S_C$

New resistance $R' = 2R$

$\therefore$ New Voltage sensitivity, $S_V' = \frac{S_C'}{R'} = \frac{1.5\, S_C}{2R} = 0.75\, S_V$

Clearly, $S_V' < S_V$, *i.e,* voltage sensitivity decreases

% decrease in voltage sensitivity $= \frac{S_V - S_V'}{S_V} \times 100\%$

$= \frac{S_V - 0.75 S_V}{S_V} \times 100\% = 25\%$

Q. 13. Write the expression for the force, $\vec{F}$ acting on a charged particle of charge 'q', moving with a velocity $\vec{V}$ in the presence of both electric field $\vec{E}$ and magnetic field $\vec{B}$. Obtain the condition under which the particle moves undeflected through the fields.

Ans.

Electric force on particle, $\overrightarrow{F_e} = q\overrightarrow{E}$

Magnetic force on particle, = $\overrightarrow{F_m} = q(\vec{v} + \overrightarrow{B})$

Total force, $\overrightarrow{F} = \overrightarrow{F_e} + \overrightarrow{F_m}$

$\overrightarrow{F} = q(\overrightarrow{E} + \vec{v} \times \overrightarrow{B})$

If a charge particle enter's perpendicular to both the electric and magnetic fields then it may happen that the electric and magnetic forces cancel each other and so particle will pass undeflected.

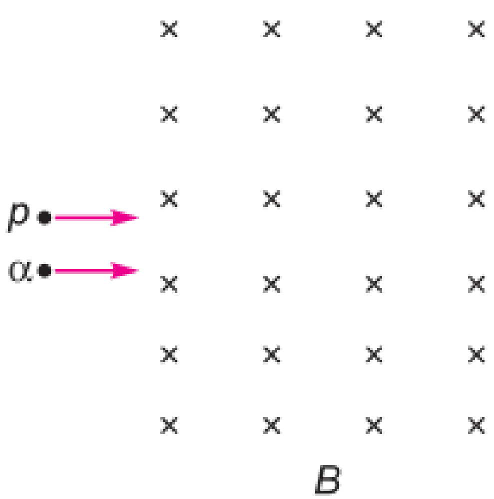
In such a case, $\overrightarrow{F} = 0$

$$\Rightarrow \quad q(\overrightarrow{E} + \vec{v} \times \overrightarrow{B}) = 0 \Rightarrow \quad \overrightarrow{E} = -(\vec{v} \times \overrightarrow{B})$$

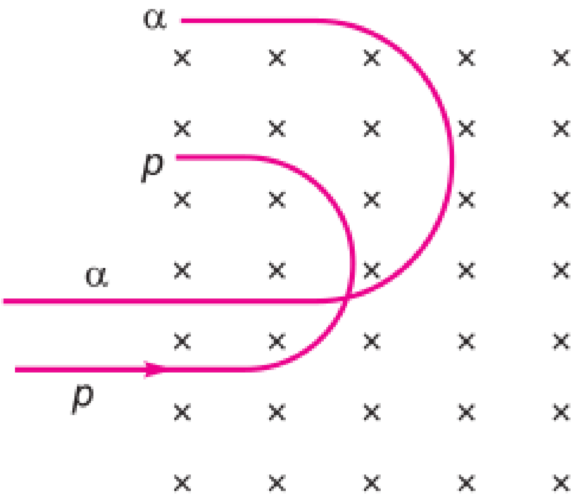
$$\Rightarrow \quad \overrightarrow{E} = \overrightarrow{B} \times \vec{v} \quad \Rightarrow \quad \overrightarrow{E} = \mathrm{Bv}\sin\theta = \mathrm{Bv} \quad (\text{when } \theta = 90^\circ)$$

$$\Rightarrow \quad v = \frac{E}{B} \text{ (when } v, E \text{ and } B \text{ are mutually perpendicular)}$$

Q. 14. An α-particle and a proton moving with the same speed enter the same magnetic field region at right angles to the direction of the field. Show the trajectories followed by the two particles in the region of the magnetic field. Find the ratio of the radii of the circular paths which the two particles may describe.



Ans. Radius of charged particle in magnetic field



$$r = \frac{\mathrm{mv}}{\mathrm{qB}}$$

$$r \propto \frac{m}{q} \text{ for same } v \text{ and } B.$$

$$\frac{r_p}{r_\alpha} = \frac{(m/q)_p}{(m/q)_\alpha}$$

$$= \frac{(m_p/e)}{((4m_p)/2e)} = \frac{1}{2}$$