2

Multiple-Choice Questions: Type 2

*Each question in this chapter has **four** options (a, b, c and d). **One or more** options may be correct.*

2.1 General Physics

1. Consider (i) a vernier callipers in which each 1 cm on the main scale is divided into 8 equal divisions and (ii) a screw gauge with 100 divisions on its circular scale. In the vernier callipers 5 divisions of the vernier scale coincides with 4 divisions on the main scale and in the screw gauge, one complete rotation of the circular scale moves it by two divisions on the linear scale. Now, which of the following statements is/are correct?

(a) If the pitch of the screw gauge is twice the least count of the vernier callipers, the least count of the screw gauge is 0.01 mm.

(b) If the pitch of the screw gauge is twice the least count of the vernier callipers, the least count of the screw gauge is 0.005 mm.

(c) If the least count of the linear scale of the screw gauge is twice the least count of the vernier callipers, the least count of the screw gauge is 0.01 mm.

(d) If the least count of the linear scale of the screw gauge is twice the least count of the vernier callipers, the least count of the screw gauge is 0.005 mm.

2. A student uses a simple pendulum of exactly 1-m length to determine g, the acceleration due to gravity. He uses a stopwatch with the least count of 1 second for this and records 40 seconds for 20 oscillations. For this observation, which of the following statements is/are true?

(a) The error ΔT in measuring T, the time period, is 0.05 second.
(b) The error ΔT in measuring T, the time period, is 1 second.
(c) The percentage error in the determination of g is 5.
(d) The percentage error in the determination of g is 2.5.

3. In an experiment to determine the acceleration due to gravity g, the formula used for the time period of a periodic motion is $T = 2\pi\sqrt{\frac{7(R-r)}{5g}}\cdot$ The values of R and r are measured to be (60 ± 1) mm and (10 ± 1) mm respectively. In five successive measurements, the time period is found to be 0.52 s, 0.56 s, 0.54 s, 0.59 s and 0.57 s. The least count of the watch used for measurement of time period is 0.01 s. Which of the following statements is/are true?

(a) The error in the measurement of r is 10%.
(b) The error in the measurement of T is 3.57%.
(c) The error in the measurement of T is 2%.
(d) The error in the measurement of g is 11%.

4. Planck constant h, speed of light c and gravitational constant G are used to form a unit of length L and a unit of mass M. Then the correct option(s) is/are

(a) $M \propto \sqrt{c}$ (b) $M \propto \sqrt{G}$
(c) $L \propto \sqrt{h}$ (d) $L \propto \sqrt{G}$

5. The length-scale (l) depends on the permittivity (ε) of the dielectric material, Boltzmann constant (k_B), the absolute temperature (T), the number per unit volume (n) of a certain charged particle, and the charge (q) carried by each of the particles. Which of the following expressions for l is/are dimensionally correct?

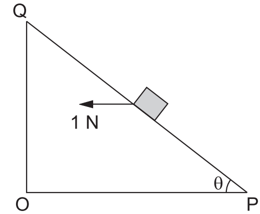
(a) $l = \sqrt{\frac{nq^2}{\varepsilon k_B T}}$ (b) $l = \sqrt{\frac{\varepsilon k_B T}{nq^2}}$

(c) $l = \sqrt{\frac{q^2}{\varepsilon n^{2/3} k_B T}}$ (d) $l = \sqrt{\frac{q^2}{\varepsilon n^{1/3} k_B T}}$

6. In terms of potential difference V, electric current I, permittivity ε_0, permeability μ_0 and speed of light c, the dimensionally correct equation(s) is/are

(a) $\mu_0 I^2 = \varepsilon_0 V^2$ (b) $\varepsilon_0 I = \mu_0 V$
(c) $I = \varepsilon_0 c V$ (d) $\mu_0 c I = \varepsilon_0 V$

7. A small block of mass 0.1 kg lies on a fixed inclined plane PQ which makes an angle θ with the horizontal. A horizontal force of 1 N is applied on this block through its centre of mass as shown in the figure. The block remains stationary if (take $g = 10$ m s^{-2})

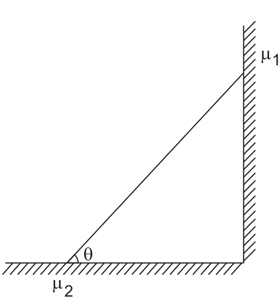


(a) $\theta = 45°$
(b) $\theta > 45°$ and a frictional force acts on the block towards P
(c) $\theta > 45°$ and a frictional force acts on the block towards Q
(d) $\theta < 45°$ and a frictional force acts on the block towards Q

8. A flat plate is moving normal to its plane through a gas under the action of a constant force F. The gas is kept at a very low pressure. The speed v of the plate is much less than the average speed u of the gas molecules. Which of the following options is/are true?

(a) The resistive force experienced by the plate is proportional to v.
(b) The pressure difference between the leading and trailing faces of the plate is proportional to uv.
(c) The plate will continue to move with constant nonzero acceleration at all times.
(d) At a later time, the external force F balances the resistive force.

9. In the figure, a ladder of mass m is shown leaning against a wall. It is in static equilibrium, making an angle θ with the horizontal floor. The coefficient of friction between the wall and the ladder is μ_1 and that between the floor and the ladder is μ_2. The normal reaction of the wall on the ladder is $\mathcal{N}_1$ and that of the floor is $\mathcal{N}_2$. If the ladder is about to slip then



(a) $\mu_1 = 0, \mu_2 \neq 0$ and $\mathcal{N}_2 \tan\theta = \dfrac{mg}{2}$

(b) $\mu_1 \neq 0, \mu_2 = 0$ and $\mathcal{N}_1 \tan\theta = \dfrac{mg}{2}$

(c) $\mu_1 \neq 0, \mu_2 = 0$ and $\mathcal{N}_2 = \dfrac{mg}{1+\mu_1\mu_2}$

(d) $\mu_1 = 0, \mu_2 \neq 0$ and $\mathcal{N}_1 \tan\theta = \dfrac{mg}{2}$

10. A particle of mass m is attached to one end of a massless spring of force constant k, lying on a frictionless horizontal plane. The other end of the spring is fixed. The particle starts moving horizontally from its equilibrium position at time $t = 0$ with an initial velocity u_0. When the speed of the particle is $0.5u_0$, it collides elastically with a rigid wall. After this collision,

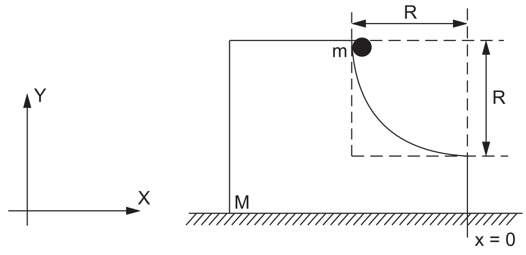
(a) the speed of the particle when it returns to its equilibrium position is u_0

(b) the time at which the particle passes through the equilibrium position for the first time is $t = \pi\sqrt{\dfrac{m}{k}}$

(c) the time at which the maximum compression in the spring occurs is $t = \dfrac{4\pi}{3}\sqrt{\dfrac{m}{k}}$

(d) the time at which the particle passes through the equilibrium position for the second time is $t = \dfrac{5\pi}{3}\sqrt{\dfrac{m}{k}}$

11. A block of mass M has a circular cut with a frictionless surface as shown. The block rests on a horizontal frictionless surface of a fixed table. Initially the right edge of the block is at $x = 0$, in a coordinate system fixed to the table. A point mass m is released from rest at the topmost point of the path as shown and it slides down. When the mass loses contact with the block, its position is at x and the velocity is v. At that instant, which of the following options is/are correct?



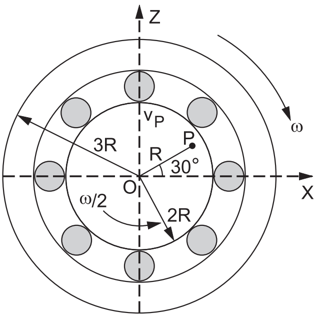
(a) The x-component of displacement of the centre of mass of the block M is $-\dfrac{mR}{M+m}$.

(b) The position of the point mass is $x = -\sqrt{2}\dfrac{mR}{M+m}$.

(c) The velocity of the point mass m is $v = \sqrt{\dfrac{2gR}{1+\dfrac{m}{M}}}$.

(d) The velocity of the block M is $v = -\dfrac{m}{M}\sqrt{2gR}$.

12. The figure shows a system consisting of (i) a ring of outer radius $3R$ rolling clockwise without slipping on a horizontal surface with angular speed ω and (ii) an inner disc of radius $2R$ rotating anticlockwise with angular speed $\dfrac{\omega}{2}$. The ring and the disc are separated by frictionless ball bearings. The system is in the xz-plane. The point P on the inner disc is at a distance R from the origin, where OP makes an angle of 30° with this horizontal. Then with respect to this horizontal surface



(a) the point O has linear velocity $3R\omega\hat{i}$

(b) the point P has linear velocity $\left(\dfrac{11}{4}R\omega\hat{i} + \dfrac{\sqrt{3}}{4}R\omega\hat{k}\right)$

(c) the point P has linear velocity $\left(\dfrac{13}{4}R\omega\hat{i} - \dfrac{\sqrt{3}}{4}R\omega\hat{k}\right)$

(d) the point P has linear velocity $\left(\left(3 - \dfrac{\sqrt{3}}{4}\right)R\omega\hat{i} + \dfrac{1}{4}R\omega\hat{k}\right)$

13. The position vector $\vec{r}$ of a particle of mass m is given by the equation $\vec{r}(t) = \alpha t^3\hat{i} + \beta t^2\hat{j}$, when $\alpha = \dfrac{10}{3}\text{ m s}^{-3}$, $\beta = 5\text{ m s}^{-2}$ and $m = 0.1$ kg. At $t = 1$ s, which of the following statements is/are true about the particle?

(a) The velocity $\vec{v}$ is given by $\vec{v} = (10\hat{i} + 10\hat{j})\text{ m s}^{-1}$.

(b) The angular momentum $\vec{L}$ with respect to the origin is given by $\vec{L} = -\frac{5}{3}\hat{k}$ N m s.

(c) The force $\vec{F}$ is given by $\vec{F} = (\hat{i} + 2\hat{j})$ N.

(d) The torque $\vec{\tau}$ with respect to the origin is given by $\vec{\tau} = -\left(\frac{20}{3}\right)\hat{k}$ N m.

14. Two solid cylinders P and Q of the same mass and same radius start rolling down a fixed inclined plane from the same height at the same time. The cylinder P has most of its mass concentrated near its surface, while the cylinder Q has most of its mass concentrated near the axis. Which of the following statements is/are correct?

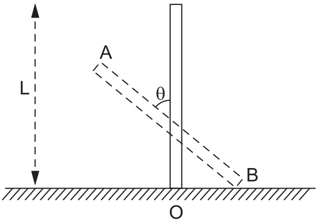
(a) Both the cylinders P and Q reach the ground at the same time.

(b) The cylinder P has larger linear acceleration than the cylinder Q.

(c) Both the cylinders reach the ground with the same translational kinetic energy.

(d) The cylinder Q reaches the ground with large angular speed.

15. A rigid uniform bar AB of length L is slipping from its vertical position on a frictionless floor (as shown in the figure). At some instant of time, the angle made by the bar with the vertical is θ. Which of the following statements about the motion is/are correct?



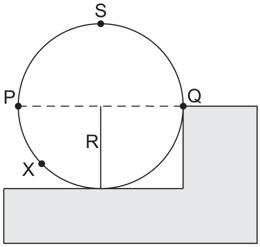
(a) The trajectory of the point A is a parabola.

(b) The instantaneous torque about the point in contact with the floor is proportional to sin θ.

(c) The mid-point of the bar will fall vertically downwards.

(d) When the bar makes an angle θ with the vertical, the displacement of its mid-point from the initial position is proportional to (1 – cos θ).

16. A wheel of radius R and mass M is placed at the bottom of a fixed step of height R as shown in the figure. A constant force is continuously applied on the surface of the wheel so that it just climbs the step without slipping. Consider the torque τ about an axis normal to the plane of the paper passing through the point Q. Which of the following options is/are correct?



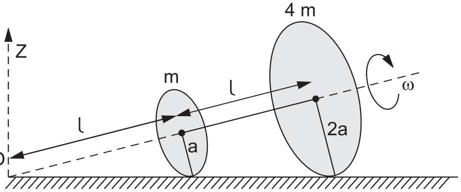
(a) If the force is applied tangentially at point X then $\tau \neq 0$ but the wheel never climbs the step.

(b) If the force is applied normal to the circumference at point P then τ is zero.

(c) If the force is applied normal to the circumference at point X then τ is constant.

(d) If the force is applied at point P tangentially then τ decreases continuously as the wheel climbs.

17. Two thin circular discs of mass m and $4m$, having radii of a and $2a$ respectively, are tightly fixed by a massless rigid rod of length $l = \sqrt{24}\, a$ through their centres. This assembly is laid on a firm and flat surface, and set rolling without stopping on the surface so that the angular speed about the axis of the rod is ω. The angular momentum of the entire assembly about the point O is $\vec{L}$ (see the figure). Which of the following statements is/are true?



(a) The magnitude of the angular momentum of the assembly about its centre of mass is $17ma^2\omega/2$.

(b) The magnitude of the z-component of $\vec{L}$ is $55ma^2\omega$.

(c) The magnitude of angular momentum of centre of mass of the assembly about the point O is $81ma^2\omega$.

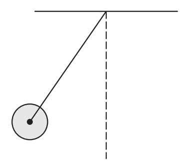
(d) The centre of mass of the assembly rotates about the z-axis with an angular speed of $\omega/5$.

18. Two spherical planets P and Q have the same uniform density ρ, masses M_P and M_Q and surface areas A and $4A$ respectively. A spherical planet R also has uniform density ρ and its mass is $(M_P + M_Q)$. The escape velocities from the planets P, Q and R are v_P, v_Q and v_R respectively. Then

(a) $v_Q > v_R > v_P$ (b) $v_R > v_Q > v_P$

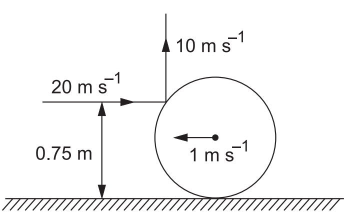
(c) $\frac{v_R}{v_P} = 3$ (d) $\frac{v_P}{v_Q} = \frac{1}{2}$

19. A metal rod of length L and mass m is pivoted at one end. A thin disc of mass M and radius R ($< L$) is attached at its centre to the free end of the rod. Consider the two ways the disc is attached. Case A: the disc is not free to rotate about its centre. Case B: the disc is free to rotate about its centre. The rod–disc system performs SHM in a vertical plane after being released from the same displaced position. Which of the following statements is/are true?



(a) Restoring torque in Case A = restoring torque in Case B
(b) Restoring torque in Case A < restoring torque in Case B
(c) Angular frequency in Case A > angular frequency in Case B
(d) Angular frequency in Case A < angular frequency in Case B

20. A thin ring of mass 2 kg and radius 0.5 m is rolling without slipping on a horizontal plane with velocity 1 m s^{-1}. A small ball of mass 0.1 kg moving with velocity 20 m s^{-1} in the opposite direction hits the ring at a height of 0.75 m and goes vertically up with a velocity of 10 m s^{-1}. Immediately after the collision



(a) the ring has pure rotation about the stationary CM
(b) the ring comes to a complete stop
(c) friction between the ring and the ground is to the left
(d) there is no friction between the ring and the ground

21. Two spherical bodies, each of mass M, are kept with their centres at a distance $2L$ apart. A particle of mass m is projected from the mid-point of the line joining their centres, perpendicular to the line. The gravitational constant is G. Which of the following statements is/are true?

(a) The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is $4\sqrt{\dfrac{GM}{L}}$.

(b) The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is $2\sqrt{\dfrac{GM}{L}}$.

(c) The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is $\sqrt{\dfrac{2GM}{L}}$.

(d) The total mechanical energy of the mass m remains constant.

22. A spherical body of radius R consists of a fluid of constant density and is in equilibrium under its own gravity. If $p(r)$ is the pressure at $r(r < R)$ then the correct option(s) is/are

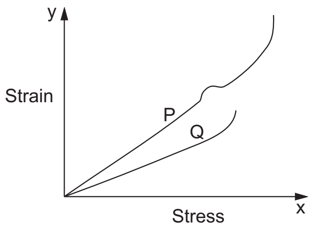
(a) $p(r = 0) = 0$

(b) $\dfrac{p\left(r = \dfrac{3R}{4}\right)}{p\left(r = \dfrac{2R}{3}\right)} = \dfrac{63}{80}$

(c) $\dfrac{p\left(r = \dfrac{3R}{5}\right)}{p\left(r = \dfrac{2R}{5}\right)} = \dfrac{16}{21}$

(d) $\dfrac{p\left(r = \dfrac{R}{2}\right)}{p\left(r = \dfrac{R}{3}\right)} = \dfrac{20}{27}$

23. While plotting stress versus strain curves for two materials P and Q, a student by mistake puts strain on the y-axis and stress on the x-axis as shown in the figure. Which of the following statements is/are correct?



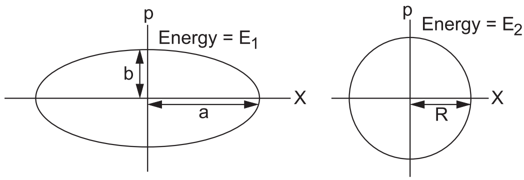
(a) P has more tensile strength than Q.

(b) P is more ductile than Q.

(c) P is more brittle than Q.

(d) The Young modulus of P is more than that of Q.

24. Two independent harmonic oscillators of equal mass are oscillating about the origin with angular frequencies ω_1 and ω_2 and have total energies E_1 and E_2 respectively. The variations of their momenta p with positions x are shown in the figure.



If $\frac{a}{b} = n^2$ and $\frac{a}{R} = n$ then the correct equation(s) is/are

(a) $E_1\omega_1 = E_2\omega_2$ (b) $\frac{\omega_2}{\omega_1} = n^2$

(c) $\omega_1\omega_2 = n^2$ (d) $\frac{E_1}{\omega_1} = \frac{E_2}{\omega_2}$

25. A block of mass M is connected by a massless spring with stiffness constant K to a rigid wall and moves without friction on a horizontal surface. The block oscillates with a small amplitude A about an equilibrium position x_0. Consider two cases: (i) when the block is at x_0, and (ii) when the block is at $x = x_0 + A$. In both the cases, a particle of mass m ($< M$) is gently placed on the block after which they stick to each other. Which of the following statements is/are true about the motion after the mass m is placed on the mass M?

(a) The amplitude of oscillation in the first case changes by a factor of $\sqrt{\frac{M}{M+m}}$, whereas in the second case it remains unchanged.
(b) The final time period of oscillation in both the cases is same.
(c) The total energy decreases in both the cases.
(d) The instantaneous speed at x_0 of the combined masses decreases in both the cases.

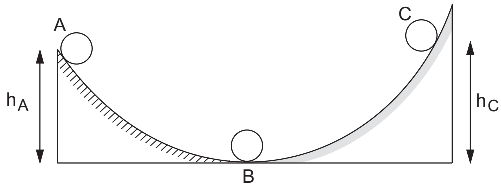
26. A solid cylinder of mass m and radius r is rolling on a rough inclined plane of inclination θ. The coefficient of friction between the cylinder and the incline is μ. Then

(a) frictional force is always $\mu mg\cos\theta$
(b) friction is a dissipative force

(c) by decreasing θ, frictional force decreases

(d) friction opposes translation and supports rotation

27. A ball moves on a fixed track as shown in the figure. From A to B, the ball rolls without slipping. Surface BC is frictionless. K_A, K_B and K_C are kinetic energies of the ball at A, B and C respectively. Then



(a) $h_A > h_C; K_B > K_C$ (b) $h_A > h_C; K_C > K_A$

(c) $h_A = h_C; K_B = K_C$ (d) $h_A < h_C; K_B > K_C$

28. If the resultant of all the external forces acting on a system of particles is zero then from an inertial frame, one can surely say that

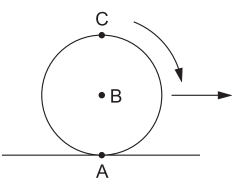
(a) the linear momentum of the system does not change in time

(b) the kinetic energy of the system does not change in time

(c) the angular momentum of the system does not change in time

(d) the potential energy of the system does not change in time

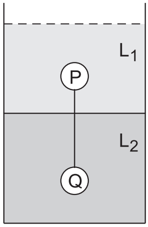
29. A sphere is rolling without slipping on a fixed horizontal plane surface. In the figure, A is the point of contact, B is the centre of the sphere and C is its topmost point. Then,



(a) $\vec{v}_C - \vec{v}_A = 2(\vec{v}_B - \vec{v}_C)$ (b) $\vec{v}_C - \vec{v}_B = \vec{v}_B - \vec{v}_A$

(c) $|\vec{v}_C - \vec{v}_A| = 2|\vec{v}_B - \vec{v}_C|$ (d) $|\vec{v}_C - \vec{v}_A| = 4|\vec{v}_B|$

30. Two spheres P and Q of equal radii have densities ρ_1 and ρ_2 respectively. The spheres are connected by a massless string and placed in liquids L_1 and L_2 of densities σ_1 and σ_2 and viscosities η_1 and η_2 respectively. They float in equilibrium with sphere P in L_1 and sphere Q in L_2, the string being taut (see the figure). If the sphere P alone in L_2 has terminal velocity $\vec{v}_P$ and the sphere Q alone in L_1 has terminal velocity $\vec{v}_Q$ then



(a) $\dfrac{|\vec{v}_P|}{|\vec{v}_Q|} = \dfrac{\eta_1}{\eta_2}$ (b) $\dfrac{|\vec{v}_P|}{|\vec{v}_Q|} = \dfrac{\eta_2}{\eta_1}$

(c) $\vec{v}_P \cdot \vec{v}_Q > 0$ (d) $\vec{v}_P \cdot \vec{v}_Q < 0$

31. A solid sphere of radius R and density ρ is attached to one end of a massless spring of force constant k. The other end of the spring is connected to another solid sphere of radius R and density 3ρ. The complete arrangement is placed in a liquid of density 2ρ and is allowed to reach equilibrium. Now, which of the following statements is/are correct?

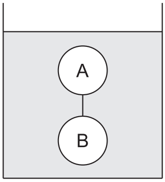
(a) The net elongation of the spring is $\dfrac{4\pi R^3 \rho g}{3k}$.

(b) The net elongation of the spring is $\dfrac{8\pi R^3 \rho g}{3k}$.

(c) The lighter sphere is partly submerged.

(d) The lighter sphere is completely submerged.

32. Two solid spheres A and B of equal volumes but of different densities d_A and d_B are connected by a string. They are fully immersed in a fluid of density d_F. They get arranged into an equilibrium as shown in the figure with a tension in the string. The arrangement is possible only if



(a) $d_A < d_F$ (b) $d_B > d_F$

(c) $d_A > d_F$ (d) $d_A + d_B = 2d_F$

2.2 Heat and Thermodynamics

1. A container of fixed volume has a mixture of one mole of hydrogen and one mole of helium in equilibrium at temperature T. Assuming the gases to be ideal, which of the following statements is/are correct?

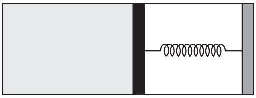
(a) The average energy per mole of the gas mixture is $2RT$.

(b) The ratio of the speed of sound in the gas mixture to that in the helium gas is $\sqrt{\frac{6}{5}}$.

(c) The ratio of the rms speed of helium atoms to that of the hydrogen molecules is $\frac{1}{2}$.

(d) The ratio of the rms speed of helium atoms to that of the hydrogen molecules is $\frac{1}{\sqrt{2}}$.

2. An ideal monatomic gas is confined in a horizontal cylinder by a spring-loaded piston (as shown in the figure). Initially the gas is at temperature T_1, pressure p_1 and volume V_1 and the spring is in its relaxed state. The gas is then heated very slowly to temperature T_2, pressure p_2 and volume V_2. During this process the piston moves out by a distance x. Now, which of the following statements is/are correct? (Ignore the friction between the piston and the cylinder.)



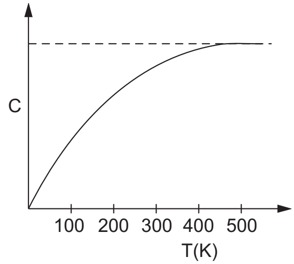
(a) If $V_2 = 2V_1$ and $T_2 = 3T_1$ then the energy stored in the spring is $\frac{1}{4}p_1V_1$.

(b) If $V_2 = 2V_1$ and $T_2 = 3T_1$ then the change in internal energy is $3p_1V_1$.

(c) If $V_2 = 3V_1$ and $T_2 = 4T_1$ then the work done by the gas is $\frac{7}{3}p_1V_1$.

(d) If $V_2 = 3V_1$ and $T_2 = 4T_1$ then the heat supplied to the gas is $\frac{17}{6}p_1V_1$.

3. The variation of specific heat capacity (C) of a solid as a function of temperature (T) is shown in the figure. The temperature is increased continuously from 0 to 500 K at a constant rate. Ignoring any volume change, find which of the following statements is/are correct to a reasonable approximation.

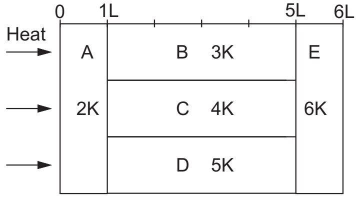


(a) The rate at which heat is absorbed in the range 0–100 K varies linearly with temperature.
(b) Heat absorbed in increasing the temperature from 0 to 100 K is less than the heat required for increasing the temperature from 400 to 500 K.
(c) There is no change in the rate of heat absorption in the range 400–500 K.
(d) The rate of heat absorption increases in the range 200–300 K.

4. A human body has a surface area of approximately 1 m^2. The normal body temperature is 10 K above the surrounding room temperature T_0. Take the room temperature T_0 to be 300 K. For T_0 = 300 K, the value of σT_0 = 460 W m^{-2} (where σ is the Stefan–Boltzmann constant). Which of the following options is/are correct?

(a) The amount of energy radiated by the body in 1 second is close to 60 J.
(b) If the surrounding temperature reduces by a small amount $\Delta T_0 << T_0$ then to maintain the same body temperature the same (living) human being needs to radiate $\Delta W = 4\sigma T_0^3 \Delta T_0$ more energy per unit time.
(c) Reducing the exposed surface area of the body (e.g., by curling up) allows humans to maintain the same body temperature by reducing the energy lost by radiation.
(d) If the body temperature rises significantly then the peak in the spectrum of electromagnetic radiation emitted by the body would shift to longer wavelengths.

5. A composite block is made of slabs A, B, C, D and E of different thermal conductivities (given in terms of a constant K) and sizes (given in terms of length L) as shown in the figure. All slabs are of the same width. Heat Q flows only from left to right through the blocks. Then in the steady state,



(a) heat flow through slabs A and E are same

(b) heat flow through slab E is maximum

(c) temperature difference across slab E is smallest

(d) heat flow through C = heat flow through B + heat flow through D

6. In a dark room with ambient temperature T_0, a black body is kept at a temperature T. Keeping the temperature of the black body constant (at T), sunrays are allowed to fall on the black body through a hole in the roof of the dark room. Assuming that there is no change in the ambient temperature of the room, which of the following statements is/are true?

(a) The quantity of radiation absorbed by the black body in unit time will increase.

(b) Since emissivity = absorptivity, the quantity of radiation emitted by the black body in unit time will increase.

(c) The black body radiates more energy in unit time in the visible spectrum.

(d) The reflected energy in unit time by the black body remains the same.

7. C_V and C_p denote the molar heat capacities of a gas at constant volume and constant pressure respectively. Then,

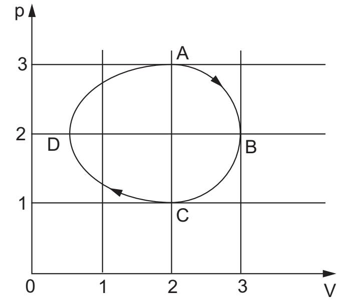
(a) $C_p - C_V$ is larger for a diatomic ideal gas than for a monatomic ideal gas

(b) $C_p + C_V$ is larger for a diatomic ideal gas than for a monatomic ideal gas

(c) C_p / C_V is larger for a diatomic ideal gas than for a monatomic ideal gas

(d) $C_p \cdot C_V$ is larger for a diatomic ideal gas than for a monatomic ideal gas

8. The figure shows the p-V plot of an ideal gas taken through a cycle ABCDA. The part ABC is a semicircle and CDA is half of an ellipse. Then,



(a) the process during the path $A \rightarrow B$ is isothermal

(b) heat flows out of the gas during the path $B \rightarrow C \rightarrow D$

(c) work done during the path $A \rightarrow B \rightarrow C$ is zero

(d) positive work is done by the gas in the cycle ABCDA

2.3 Sound Waves

1. A horizontally stretched string fixed at two ends is vibrating in its fifth harmonic according to the equation

$$y(x, t) = (0.01\text{ m}) \sin[(62.8\text{ m}^{-1})x] \cos[(628\text{ s}^{-1})t].$$

Assuming $\pi = 3.14$, which of the following statements is/are correct?

(a) The number of nodes is 5.

(b) The length of the string is 0.25 m.

(c) The maximum displacement of the mid-point of the string from its equilibrium position is 0.01 m.

(d) The fundamental frequency is 100 Hz.

2. Two loudspeakers M and N are located 20 m apart and emit sound at frequencies 118 Hz and 121 Hz respectively. A car is initially at a point P, 1800 m away from the mid-point Q of the line MN and moves towards Q constantly at 60 km h^{-1} along the perpendicular bisector of MN. It crosses Q and eventually reaches the point R, 1800 m away from Q. Let $\nu(t)$ represent the beat frequency measured by a person sitting in the car at time t. Let ν_P, ν_Q and ν_R be the beat frequencies measured at locations P, Q and R respectively.

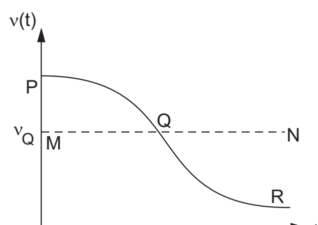


Fig. A

The speed of sound in air is 330 m s^{-1}. Which of the following statements is/are true regarding the sound heard by the person?

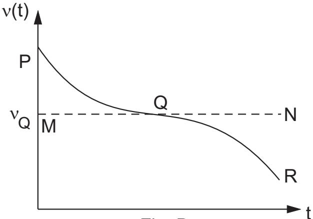


Fig. B

(a) Figure A represents schematically the variation of the beat frequency with time.

(b) Figure B represents schematically the variation of beat frequency with time.

(c) The rate of change of beat frequency is maximum when the car passes through Q.

(d) $\nu_P + \nu_B = 2\nu_Q$.

3. Two vehicles, each moving with speed u on the same horizontal straight road, are approaching each other. Wind blows along the road with velocity w. One of these vehicles blows a whistle of frequency f_1. An observer in the other vehicle hears the frequency of the whistle to be f_2. The speed of sound in still air is v. Identify the correct statement(s).

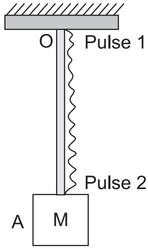
(a) If the wind blows from the observer to the source then $f_2 > f_1$.

(b) If the wind blows from the source to the observer then $f_2 > f_1$.

(c) If the wind blows from the observer to the source then $f_2 < f_1$.

(d) If the wind blows from the source to the observer then $f_2 < f_1$.

4. A block M hangs vertically from the bottom end of a uniform rope of constant mass per unit length. The top end of the rope is attached to a fixed rigid support at O. A transverse wave pulse (Pulse 1) of wavelength λ_O is produced at point O on the rope. The pulse takes time T_{OA} to reach point A. If the wave pulse of wavelength λ_O is produced at point A (Pulse 2) without disturbing the position of M, it takes time T_{AO} to reach the point O. Which of the following options is/are correct?



(a) The time $T_{AO} = T_{OA}$.

(b) The velocities of the two pulses (Pulse 1 and Pulse 2) are the same at the mid-point of the rope.

(c) The wavelength of Pulse 1 becomes longer when it reaches point A.

(d) The velocity of any pulse along the rope is independent of its frequency and wavelength.

5. One end of a taut string of length 3 m along the x-axis is fixed at $x = 0$. The speed of the waves in the string is 100 m s^{-1}. The other end of the string is vibrating in the y-direction so that stationary waves are set up in the string. The possible waveform(s) of these stationary waves is/are

(a) $y(t) = A\sin\frac{\pi x}{6}\cos\frac{50\pi t}{3}$ (b) $y(t) = A\sin\frac{\pi x}{3}\cos\frac{100\pi t}{3}$

(c) $y(t) = A\sin\frac{5\pi x}{6}\cos\frac{250\pi t}{3}$ (d) $y(t) = A\sin\frac{5\pi x}{2}\cos 250\pi t$

6. A student is performing an experiment using a resonance column and a tuning fork of frequency 244 s^{-1}. He is told that the air in the tube has been replaced by another gas (assume that the column remains filled with the gas). If the minimum height at which resonance occurs is (0.350 ± 0.005) m, the gas in the tube is (useful information: $\sqrt{167\,\text{RT}} = 640\,\text{J}^{1/2}\,\text{mol}^{-1/2}$; $\sqrt{140\,\text{RT}} = 590\,\text{J}^{1/2}\,\text{mol}^{-1/2}$, the molar mass M in grams is given in each option; take the value of $\sqrt{\frac{10}{M}}$ for each gas as given there)

(a) neon $\left(M = 20, \sqrt{\frac{10}{20}} = \frac{7}{20}\right)$ (b) nitrogen $\left(M = 28, \sqrt{\frac{10}{28}} = \frac{3}{5}\right)$

(c) oxygen $\left(M = 32, \sqrt{\frac{10}{32}} = \frac{9}{16}\right)$ (d) argon $\left(M = 36, \sqrt{\frac{10}{36}} = \frac{17}{32}\right)$

7. A person blows into the open end of a long pipe. As a result, a high-pressure pulse of air travels down the pipe. When this pulse reaches the other end of the pipe

(a) a high-pressure pulse starts travelling up the pipe, if the other end of the pipe is open

(b) a low-pressure pulse starts travelling up the pipe, if the other end of the pipe is open

(c) a low-pressure pulse starts travelling up the pipe, if the other end of the pipe is closed

(d) a high-pressure pulse starts travelling up the pipe, if the other end of the pipe is closed

8. Function $X = A\sin^2 \alpha x + B\cos^2 \alpha x + C\sin \alpha x \cos \alpha x$ represents SHM

(a) for any value of A, B and C (except $C = 0$)

(b) if $A = -B$, $C = 2B$, amplitude = $|B\sqrt{2}|$

(c) if $A = B$, $C = 0$

(d) if $A = B$, $C = 2B$, amplitude = $|B|$

9. A student performed the experiment to measure the speed of sound in air using resonance air-column method. Two resonances in the air column were obtained by lowering the water level. The resonance with the shorter air column is the first resonance and that with the longer air column is the second resonance. Then,

(a) the intensity of the sound heard at the first resonance was more than that at the second resonance

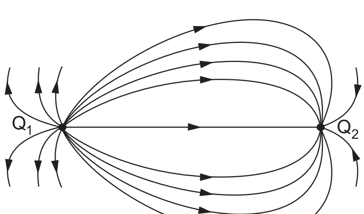
(b) the prongs of the tuning fork were kept in a horizontal plane above the resonance tube

(c) the amplitude of vibration of the ends of the prongs is typically around 1 cm

(d) the length of the air column at the first resonance was somewhat shorter than 1/4th of the wavelength of the sound in air

2.4 Electrostatics

1. A few electric field lines for a system of two charges Q_1 and Q_2 fixed at two different points on the x-axis are shown in the figure. These lines suggest that



(a) $|Q_1| > |Q_2|$ (b) $|Q_1| < |Q_2|$

(c) at a finite distance to the left of Q_1 the electric field is zero

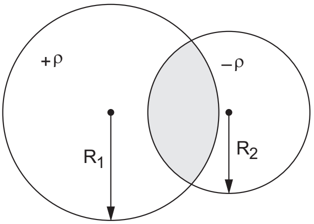
(d) at a finite distance to the right of Q_2 the electric field is zero

2. A spherical metal shell A of radius R_A and a solid metal sphere B of radius R_B ($< R_A$) are kept far apart and each is given charge $+Q$. If they are connected by a thin metal wire then

(a) $E_A^{\text{inside}} = 0$ (b) $Q_A > Q_B$

(c) $\dfrac{\sigma_A}{\sigma_B} = \dfrac{R_B}{R_A}$ (d) $E_A^{\text{on surface}} < E_B^{\text{on surface}}$

3. Two nonconducting spheres of radii R_1 and R_2 and carrying uniform volume charge densities $+\rho$ and $-\rho$ respectively are placed such that they partially overlap, as shown in the figure. At all points in the overlapping region,



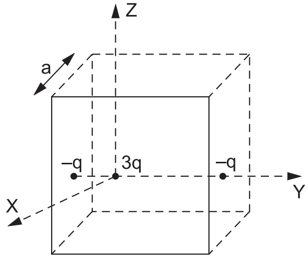
(a) the electrostatic field is zero

(b) the electrostatic potential is constant

(c) the electrostatic field is constant in magnitude

(d) the electrostatic field has the same direction

4. A cubical region of side a has its centre at the origin. It encloses three fixed point charges: $-q$ at $\left(0, -\frac{a}{4}, 0\right)$, $+3q$ at $(0, 0, 0)$ and $-q$ at $\left(0, +\frac{a}{4}, 0\right)$. Choose the correct option(s).



(a) The net electric flux crossing the plane $x = +\frac{a}{2}$ is equal to the net electric flux crossing the plane $x = -\frac{a}{2}$.

(b) The net electric flux crossing the plane $y = +\frac{a}{2}$ is more than the net electric flux crossing the plane $y = -\frac{a}{2}$.

(c) The net electric flux crossing the entire region is $\frac{q}{\varepsilon_0}$.

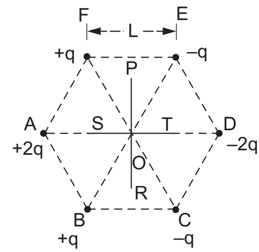
(d) The net electric flux crossing the plane $z = +\frac{a}{2}$ is equal to the net electric flux crossing the plane $x = +\frac{a}{2}$.

5. Two nonconducting solid spheres of radii R and $2R$, having uniform volume charge densities ρ_1 and ρ_2 respectively touch each other. The net electric field at a distance $2R$ from the centre of the smaller sphere, along the line joining the centres of the spheres is zero. The ratio ρ_1/ρ_2 can be

(a) -4 (b) $-\frac{32}{25}$

(c) $\frac{32}{25}$ (d) 4

6. Six point charges are kept at the vertices of a regular hexagon of side L and centre O, as shown in the figure. Given that $K = \frac{1}{4\pi\varepsilon_0}\frac{q}{L^2}$, which of the following statements is/are correct?



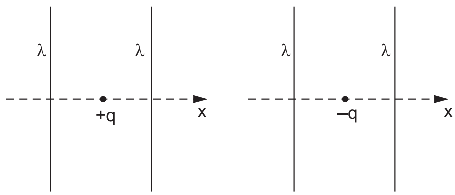
(a) The electric field at O is $6K$ along OD.

(b) The potential at O is zero.

(c) The potential at all points on the line PR is the same.

(d) The potential at all points on the line ST is the same.

7. The figures below depict two situations, in each of which two infinitely long static line charges of constant positive line charge density λ are kept parallel to each other. In their resulting electric fields, point charges $+q$ and $-q$ are kept in equilibrium between the lines. The point charges are confined to move in the x-direction only. If they are given a small displacement about their equilibrium positions then which of the following statements is/are correct?



(a) Both charges execute simple harmonic motion.

(b) Both charges will continue to move in the directions of their displacements.

(c) Charge $+q$ executes SHM while charge $-q$ continues to move in the direction of its displacement.

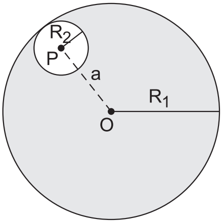
(d) Charge $-q$ executes SHM while charge $+q$ continues to move in the direction of its displacement.

8. Let $E_1(r)$, $E_2(r)$ and $E_3(r)$ be the respective electric fields at a distance r from a point charge Q, an infinitely long wire with a constant linear charge density λ, and an infinite plane with a uniform surface charge density σ. If $E_1(r_0) = E_2(r_0) = E_3(r_0)$ at a given distance r_0 then

(a) $Q = 4\sigma\pi r_0^2$ (b) $r_0 = \dfrac{\lambda}{2\pi\sigma}$

(c) $E_1\left(\dfrac{r_0}{2}\right) = 2E_2\left(\dfrac{r_0}{2}\right)$ (d) $E_2\left(\dfrac{r_0}{2}\right) = 4E_3\left(\dfrac{r_0}{2}\right)$

9. Consider a uniform spherical charge distribution of radius R_1, centred at the origin O. In this distribution, an eccentric spherical cavity of radius R_2, centred at P with distance OP = $a = R_1 - R_2$ (see the figure) is made. If the electric field inside the cavity at position $\vec{r}$ is $\vec{E}(\vec{r})$ then which of the following statements is/are correct?



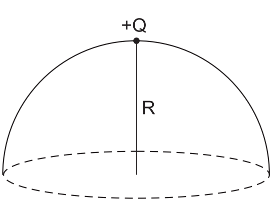
(a) $\vec{E}$ is uniform, its magnitude is independent of R_2 but its direction depends on $\vec{r}$.

(b) $\vec{E}$ is uniform, its magnitude depends on R_2 but its direction depends on $\vec{r}$.

(c) $\vec{E}$ is uniform, its magnitude is independent of a but its direction depends on $\vec{a}$.

(d) $\vec{E}$ is uniform and both its magnitude and direction depends on $\vec{a}$.

10. A point charge $+Q$ is placed just outside an imaginary hemispherical surface of radius R as shown in the figure. Which of the following statements is/are correct?



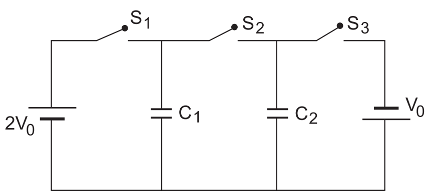
(a) The electric flux passing through the curved surface of the hemisphere is $-\frac{Q}{2\varepsilon_0}\left(1-\frac{1}{\sqrt{2}}\right)$.

(b) The component of electric field normal to the flat surface is constant over the surface.

(c) The total flux through the curved and the flat surfaces is $\frac{Q}{\varepsilon_0}$.

(d) The circumference of the flat surface is all equipotential.

11. In the circuit shown in the figure, there are two parallel plate capacitors, each of capacitance C. The switch S_1 is pressed first to fully charge the capacitor C_1 and then released. The switch S_2 is then pressed to charge the capacitor C_2. After a pause, S_2 is released and then S_3 is pressed. After some time



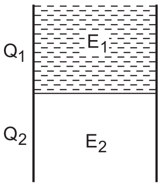
(a) the charge on the upper plate of C_1 is $2CV_0$

(b) the charge on the upper plate of C_1 is CV_0

(c) the charge on the upper plate of C_1 is 0

(d) the charge on the upper plate of C_2 is $-CV_0$

12. A parallel plate capacitor has a dielectric slab of dielectric constant K between the plates that covers $\frac{1}{3}$rd of the area of its plates as shown in the figure. The total capacitance of the capacitor is C while that of the portion with dielectric in between is C_1. When the capacitor is charged, the plate area covered by the dielectric gets charge Q_1 and the rest of the area gets charge Q_2. The electric field in the dielectric is E_1 and that on the other portion is E_2. Choose the correct option(s), ignoring the edge effects.



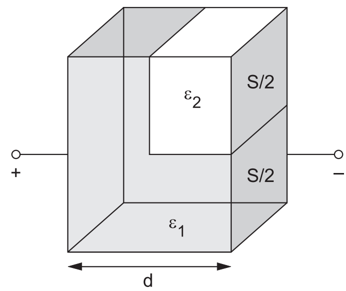
(a) $\frac{E_1}{E_2}=1$

(b) $\frac{E_1}{E_2}=\frac{1}{K}$

(c) $\frac{Q_1}{Q_2}=\frac{3}{K}$

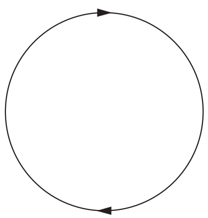
(d) $\frac{C}{C_1}=\frac{2+K}{K}$

13. A parallel plate capacitor having plates of area A and plate separation d, has capacitance C_1 in air. When two dielectrics of different relative permittivities ($\varepsilon_1 = 2$ and $\varepsilon_2 = 4$) are introduced between the two plates as shown in the figure, the capacitance becomes C_2. The ratio C_2/C_1 is



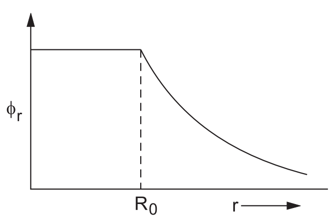
(a) $\frac{6}{5}$ (b) $\frac{5}{3}$ (c) $\frac{7}{5}$ (d) $\frac{7}{3}$

14. In the given diagram, a field-line of a particular force-field is shown. It can never represent



(a) an electrostatic field
(b) a magnetostatic field
(c) a gravitational field of a mass at rest
(d) an induced electric field

15. The electrostatic potential (ϕ_r) of a spherical symmetric system kept at origin is shown in the adjacent figure, and given as



$$\phi_r = \frac{q}{4\pi\varepsilon_0 r} \cdots (r \geq R_0),$$

$$\phi_r = \frac{q}{4\pi\varepsilon_0 R_0} \cdots (r \leq R_0).$$

Which of the following options is/are correct?

(a) For spherical region $r \leq R_0$, the total electrostatic energy stored is zero.
(b) Within $r = 2R_0$, the total charge is q.
(c) There will be no charge anywhere except at $r = R_0$.
(d) The electric field is discontinuous at $r = R_0$.

16. Under the influence of the coulomb field of charge $+Q$, a charge $-q$ is moving around it in an elliptical orbit. Find out the correct statement(s).

(a) The angular momentum of the charge $-q$ is constant

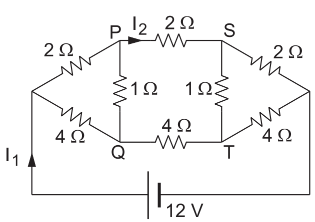
(b) The linear momentum of the charge $-q$ is constant
(c) The angular velocity of the charge $-q$ is constant
(d) The linear speed of the charge $-q$ is constant

2.5 Current Electricity and Magnetism

1. An incandescent bulb has a thin filament of tungsten that is heated to a high temperature by passing an electric current. The hot filament emits black-body radiation. The filament is observed to break up at random locations after a sufficiently long time of operation due to nonuniform evaporation of tungsten from the filament. If the bulb is powered at a constant voltage, which of the following statements is/are true?

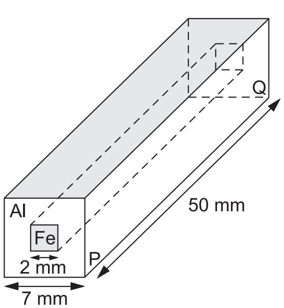
(a) The temperature distribution over the filament is uniform.
(b) The resistance over small sections of the filament decreases with time.
(c) The filament emits more light at higher band frequencies before it breaks up.
(d) The filament consumes less electrical power towards the end of the life of the bulb.

2. For the resistance network shown in the figure, choose the correct option(s).



(a) The current through PQ is zero.
(b) $I_1 = 3A$.
(c) The potential at S is less than that at Q.
(d) $I_2 = 2A$.

3. In an aluminium (Al) bar of square cross section, a square hole is drilled and is filled with iron (Fe) as shown in the figure. The electrical resistivities of Al and Fe are 2.7×10^{-8} Ω m and 1.0×10^{-7} Ω m respectively. The electrical resistance between the two faces P and Q of the composite bar is



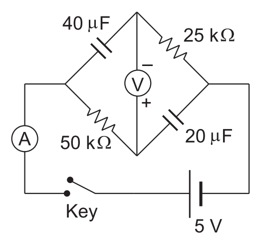
(a) $\frac{2475}{64}\mu\,\Omega$ (b) $\frac{1875}{64}\mu\,\Omega$

(c) $\frac{1875}{49}\mu\,\Omega$ (d) $\frac{2475}{132}\mu\,\Omega$

4. The heater of an electric kettle is made of a wire of length L and diameter d. It takes 4 minutes to raise the temperature of 0.5 kg of water by 40 K. This heater is replaced by a new heater having two wires of the same material, each of length L and diameter $2d$. The way these wires are connected is given in the options. How much time in minutes will it take to raise the temperature of the same amount of water by 40 K?

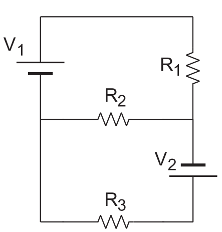
(a) 4, if the wires are in parallel
(b) 2, if the wires are in series
(c) 1, if the wires are in series
(d) 0.5, if the wires are in parallel

5. In the given circuit, the key is pressed at time t = 0. Which of the following statements is/are true?



(a) The voltmeter displays –5 V as soon as the key is pressed, and displays +5 V after a long time.
(b) The voltmeter will display 0 V at time $t = \ln 2$ s.
(c) The current in the ammeter becomes $1/e$ of the initial value after 1 s.
(d) The current in the ammeter become zero after a long time.

6. Two ideal batteries of emf V_1 and V_2 and three resistors R_1, R_2 and R_3 are connected as shown in the figure. The current in resistance R_2 would be zero if



(a) $V_1 = V_2$ and $R_1 = R_2 = R_3$
(b) $V_1 = V_2$ and $R_1 = 2R_2 = R_3$
(c) $V_1 = 2V_2$ and $2R_1 = 2R_2 = R_3$
(d) $2V_1 = V_2$ and $2R_1 = R_2 = R_3$

7. Consider two identical galvanometers and two identical resistors with resistance R. If the internal resistance of the galvanometers $R_g < R/2$, which of the following statements about any one of the galvanometers is/are true?

(a) The maximum voltage range is obtained when all the components are connected in series.

(b) The maximum voltage range is obtained when the two resistors and one galvanometer are connected in series, and the second galvanometer is connected in parallel to the first galvanometer.

(c) The maximum current range is obtained when all the components are connected in parallel.

(d) The maximum current range is obtained when the two galvanometers are connected in series and the combination is connected in parallel with both the resistors.

8. A steady current I flows along an infinitely long hollow cylindrical conductor of radius R. This cylinder is placed coaxially inside an infinitely long solenoid of radius $2R$. The solenoid has n turns per unit length and carries a steady current I. Consider a point P at a distance r from the common axis. Which of the following statements is/are correct?

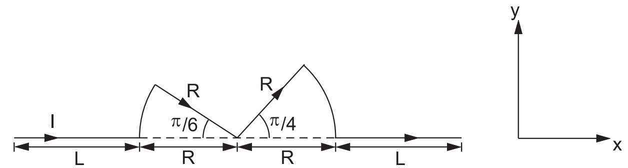
(a) In the region $0 < r < R$, the magnetic field is nonzero.

(b) In the region $R < r < 2R$, the magnetic field is along the common axis.

(c) In the region $R < r < 2R$, the magnetic field is tangential to the circle of radius r, centred on the axis.

(d) In the region $r > 2R$, the magnetic field is nonzero.

9. A conductor (shown in the figure) carrying a constant current I is kept in the xy-plane in a uniform magnetic field $\vec{B}$. If $\vec{F}$ is the magnitude of the total magnetic force acting on the conductor, the correct statement(s) is/are



(a) If $\vec{B}$ is along $\hat{z}$, $F \propto (L + R)$ (b) If $\vec{B}$ is along $\hat{x}$, $F = 0$

(c) If $\vec{B}$ is along $\hat{y}$, $F \propto (L + R)$ (d) If $\vec{B}$ is along $\hat{z}$, $F = 0$

10. Consider the motion of a positive point charge in a region where there are simultaneous uniform electric and magnetic fields $\vec{E} = E_0\hat{j}$ and $\vec{B} = B_0\hat{j}$. At time $t = 0$, this charge has velocity $\vec{v}$ in the xy-plane, making an angle θ with the x-axis. Which of the following options is/are correct for time $t > 0$?

(a) If $\theta = 0$, the charge moves in a circular path in the xz-plane.

(b) If $\theta = 0$, the charge undergoes helical motion with a constant pitch along the y-axis.

(c) If $\theta = 10°$, the charge undergoes helical motion with its pitch increasing with time.

(d) If $\theta = 90°$, the charge undergoes linear but accelerated motion along the y-axis.

11. An electron and a proton are moving on straight parallel paths with the same velocity. They enter a semi-infinite region of uniform magnetic field perpendicular to the velocity. Which of the following statements is/are true?

(a) They will never come out of this magnetic field region.

(b) They will come out travelling along parallel paths.

(c) They will come out at the same time.

(d) They will come out at different times.

12. A particle of mass M and positive charge Q, moving with a constant velocity $\vec{u_1} = (4\hat{i})$ m s^{-1}, enters a region of uniform static magnetic field normal to the xy-plane. The region of the magnetic field extends from $x = 0$ to $x = L$ for all values of y. After passing through the region, the particle emerges into the other side after 10 ms with a velocity $\vec{u_2} = 2(\sqrt{3}\,\hat{i} + \hat{j})$ m s^{-1}. Which of the following statements is/are correct?

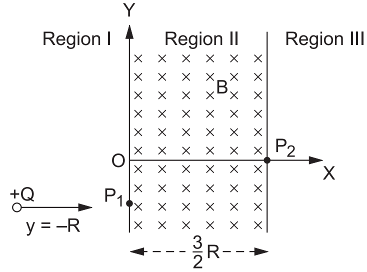
(a) The direction of the magnetic field is along the $-z$ axis.

(b) The direction of the magnetic field is along the $+z$ axis.

(c) The magnitude of the magnetic field is $\dfrac{50\pi M}{3Q}$ units.

(d) The magnitude of the magnetic field is $\dfrac{100\pi M}{3Q}$ units.

13. A uniform magnetic field B exists in the region between $x = 0$ and $x = \dfrac{3R}{2}$ (Region II in the figure), pointing normally into the plane of the paper. A particle with charge $+Q$ and momentum p directed along the x-axis enters Region II from Region I at a point P_1 ($y = -R$). Which of the following options is/are correct?



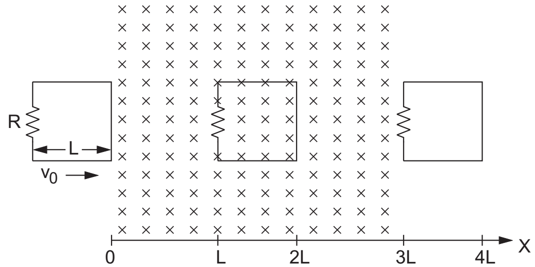
(a) For $B = \dfrac{8}{13}\dfrac{p}{QR}$, the particle will enter Region III through the point P_2 on the x-axis.

(b) For $B > \dfrac{2}{3}\dfrac{p}{QR}$, the particle will re-enter Region I.

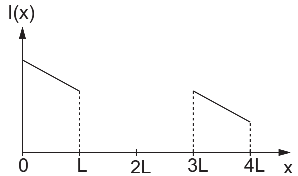
(c) For a fixed magnetic field B, particles carrying the same charge Q and moving with the same velocity v, the distance between the point P_1 and the point of re-entry into Region I is inversely proportional to the mass of the particle.

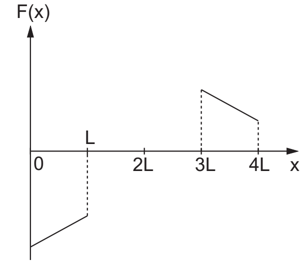
(d) When the particle re-enters Region I through the longest possible path in Region II, the magnitude of the change in its linear momentum between the point P_1 and the farthest point from the y-axis is $\dfrac{p}{\sqrt{2}}$.

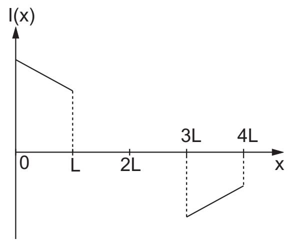
14.

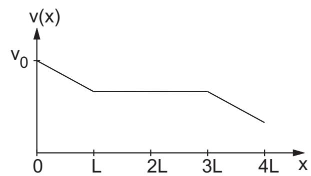


A rigid square-shaped wire loop of side L and resistance R is moving along the x-axis with a constant velocity v_0 in the plane of the paper. At $t = 0$, the right edge of the loop enters the region of length $3L$ where there is a uniform magnetic field B_0 directed into the plane of the paper, as shown in the figure. For a sufficiently large v_0, the loop eventually crosses the region. Let x be the location of the right edge of the loop. Let $v(x)$, $I(x)$ and $F(x)$ represent the velocity of the loop, current in the loop and force on the loop respectively, as a function of x. Counterclockwise current is taken as positive. Which of the following schematic plots is/are correct? (Ignore gravity.)

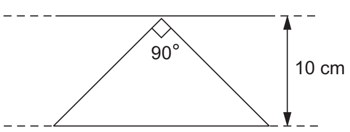
(a) 

(b) 

(c) 

(d) 

15. A conducting loop in the shape of a right-angled isosceles triangle of height 10 cm is kept such that the 90° vertex is very close to an infinitely long conducting wire as shown in the figure. The wire is electrically insulated from the loop. The hypotenuse of the triangle is parallel to the wire. The current in the triangular loop is in counterclockwise direction and increased at a constant rate of 10 A s^{-1}. Which of the following statements is/are true?



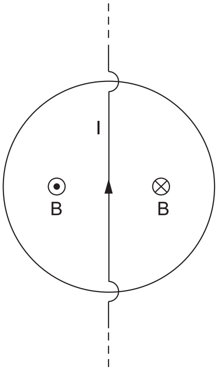
(a) The induced current in the wire is in the opposite direction to the current along the hypotenuse.

(b) There is a repulsive force between the wire and the loop.

(c) The magnitude of the induced emf in the wire is $\left(\frac{\mu_0}{\pi}\right)$ volt.

(d) If the loop is rotated at a constant angular speed about the wire, an additional emf of $\left(\frac{\mu_0}{\pi}\right)$ volt is induced in the wire.

16. A current-carrying, infinitely long wire is kept along the diameter of a circular wire loop, without touching it. Which of the statements is/are correct?



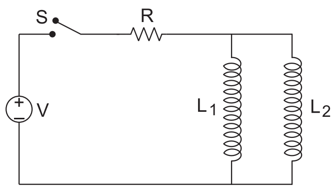
(a) The emf induced in the loop is zero if the current is constant.

(b) The emf induced in the loop is finite if the current is constant.

(c) The emf induced in the loop is zero if the current decreases at a steady state.

(d) The emf induced in the loop is infinite if the current decreases at a steady state.

17. A source of constant voltage V is connected to a resistance R and two ideal inductors L_1 and L_2 through a switch S as shown. There is no mutual inductance between the two inductors. The switch is initially open. At $t = 0$, the switch is closed and current begins to flow. Which of the following options is/are correct?



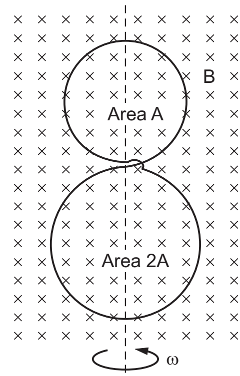
(a) At $t = 0$, the current through the resistance R is $\frac{V}{R}$.

(b) After a long time, the current through L_2 will be $\frac{V}{R}\frac{L_1}{L_1+L_2}$.

(c) After a long time, the current through L_1 will be $\frac{V}{R}\frac{L_2}{L_1+L_2}$.

(d) The ratio of the current through L_1 and L_2 is fixed at all times ($t > 0$).

18. A circular insulated copper wire loop is twisted to form two loops of areas A and $2A$ as shown in the figure. At the point of crossing, the wires remain electrically insulated from each other. The entire loop lies in the plane (of the paper). A uniform magnetic field $\vec{B}$ points towards the plane of the paper. At $t = 0$, the loop starts rotating about the common diameter as axis with a constant angular velocity ω in the magnetic field. Which of the following options is/are correct?



(a) The rate of change of the magnetic flux is maximum when the plane of the loops is perpendicular to the plane of the paper.

(b) The net emf induced due to both the loops is proportional to cos ωt.

(c) The emf induced in the loop is proportional to the sum of the areas of the two loops.

(d) The amplitude of the maximum net emf induced due to both the loops is equal to the amplitude of the maximum emf induced in the smaller loop alone.

19. The instantaneous voltages at three terminals marked X, Y and Z are given by

$$V_X = V_0 \sin \omega t,$$

$$V_Y = V_0 \sin\left(\omega t + \frac{2\pi}{3}\right) \text{ and}$$

$$V_Z = V_0 \sin\left(\omega t + \frac{4\pi}{3}\right).$$

An ideal voltmeter is configured to read the rms value of the potential difference between its terminals. It is connected between the points X and Y and then between Y and Z. The readings of the voltmeter will be

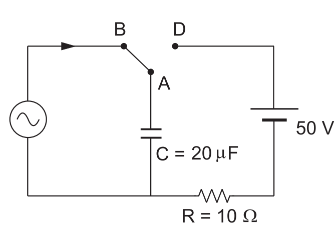
(a) $V_{YZ}^{rms} = V_0\sqrt{\frac{3}{2}}$

(b) $V_{XY}^{rms} = V_0$

(c) $V_{YZ}^{rms} = V_0\sqrt{\frac{1}{2}}$

(d) independent of the choice of the two terminals

20. At time $t = 0$, terminal A in the circuit shown in the figure is connected to B by a key and an alternating current $I(t) = I_0 \cos \omega t$ (where I_0 = 1 A and ω = 500 rad s^{-1}) starts flowing in it with the initial direction as shown



in the figure. At $t = \frac{7\pi}{6\omega}$, the key is shifted from B to D. Now onwards, only A and D are connected. A total charge Q flows from the battery to charge the capacitor fully. If $C = 20\ \mu F$, $R = 10\ \Omega$ and battery is ideal with emf of 50 V, identify the correct statement(s).

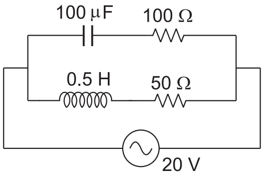
(a) The magnitude of maximum charge on the capacitor before $t = \frac{7\pi}{\omega}$ is 1×10^{-3} C.

(b) The current in the left part of the circuit just before $t = \frac{7\pi}{6\omega}$ is clockwise.

(c) Immediately after A is connected to D, the current in R is 10 A.

(d) $Q = 2 \times 10^{-3}$ C.

21. In the given circuit, the AC source has $\omega = 100$ rad s^{-1}. Considering the inductor and the capacitor to be ideal, which of the following statements is/are correct?



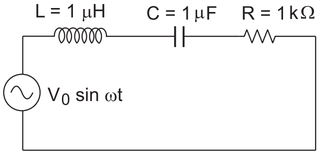
(a) The current through the circuit is 0.3 A.

(b) The current through the circuit is $0.3\sqrt{2}$ A.

(c) The voltage across 100 Ω resistor is $10\sqrt{2}$ V.

(d) The voltage across 50 Ω resistor is 10 V.

22. In the circuit shown, $L = 1\ \mu H$, $C = 1\ \mu F$ and $R = 1\ k\Omega$. They are connected in series with an AC source $V = V_0 \sin \omega t$ as shown. Which of the following options is/are correct?



(a) The frequency at which the current will be in phase with the voltage is independent of R.

(b) At $\omega \to 0$, the current flowing through the circuit becomes nearly zero.

(c) At $\omega >> 10^6$ rad s^{-1}, the circuit behaves like a capacitor.

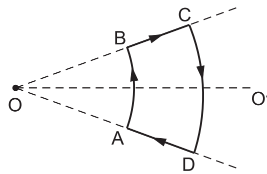
(d) The current will be in phase with the voltage if $\omega = 10^4$ rad s^{-1}.

23. A series RC circuit is connected to an AC voltage source. Consider two cases: (A) when C is without a dielectric medium, and (B) when C is filled with a medium of dielectric constant 4. The current I_R through the resistor and the voltage V_C across the capacitor are compared in the two cases. Which of the following is/are true?

(a) $I_R^A > I_R^B$ (b) $I_R^A < I_R^B$

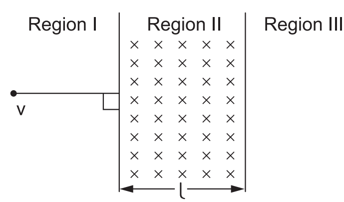
(c) $V_C^A > V_C^B$ (d) $V_C^A < V_C^B$

24. An infinitely long current-carrying wire passes through point O and is perpendicular to the plane containing a current-carrying loop ABCD as shown in the figure. Choose the correct option(s).



(a) The net force on the loop is zero.
(b) The net torque on the loop is zero.
(c) As seen from O, the loop rotates clockwise.
(d) As seen from O, the loop rotates anticlockwise.

25. A particle of mass m and charge q, moving with a velocity v enters Region II normal to the boundary as shown in the figure. Region II has a uniform magnetic field B perpendicular to the plane of the paper. The length of Region II is l. Choose the correct choice(s).



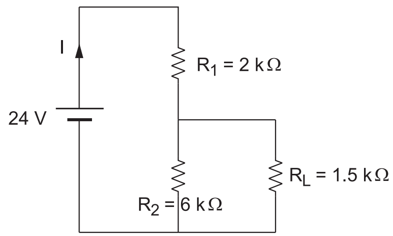
(a) The particle enters Region III only if its velocity $v > \frac{qlB}{m}$.

(b) The particle enters Region III only if its velocity $v < \frac{qlB}{m}$.

(c) Path length of the particle in Region II is maximum when velocity $v = \frac{qlB}{m}$.

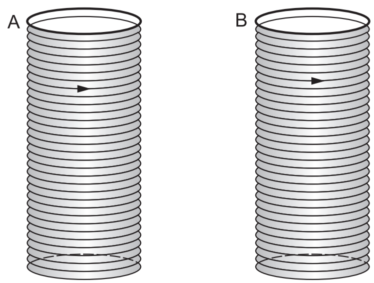
(d) Time spent in Region II is same for any velocity v as long as the particle returns to Region I.

26. For the circuit shown in the figure,



(a) the current I through the battery is 7.5 mA
(b) the potential difference across R_L is 18 V
(c) the ratio of powers dissipated in R_1 and R_2 is 3
(d) if R_1 and R_2 are interchanged, the magnitude of power dissipated in R_L will decrease by a factor of 9.

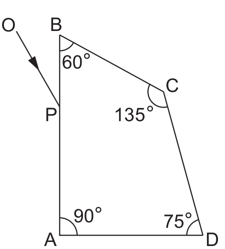
27. Two metallic rings A and B, identical in shape and size but having different resistivities ρ_A and ρ_B, are kept on top of two identical solenoids as shown in the figure. When current I is switched on in both the solenoids in identical manner, the rings A and B jump to heights h_A and h_B respectively, with $h_A > h_B$. The possible relation(s) between their resistivities and their masses m_A and m_B is/are



(a) $\rho_A > \rho_B$ and $m_A = m_B$ (b) $\rho_A < \rho_B$ and $m_A = m_B$
(c) $\rho_A > \rho_B$ and $m_A > m_B$ (d) $\rho_A < \rho_B$ and $m_A < m_B$

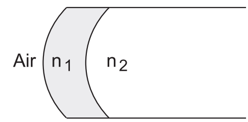
2.6 Ray Optics and Wave Optics

1. A ray OP of monochromatic light is incident on the face AB of prism ABCD near vertex B at an incident angle of 60° (see figure). If the refractive index of the material of the prism is $\sqrt{3}$, which of the following is/are correct?



(a) The ray gets totally internally reflected at face CD.
(b) The ray comes out through face AD.
(c) The angle between the incident ray and the emergent ray is 90°.
(d) The angle between the incident ray and the emergent ray is 120°.

2. A transparent thin film of uniform thickness and refractive index $n_1 = 1.4$ is coated on the convex spherical surface of radius R at one end of a long solid glass cylinder of refractive index $n_2 = 1.5$, as shown in the figure. Rays of light parallel to the axis of the cylinder traversing through the film from air to glass get focused at a distance f_1 from the film, while rays of light traversing from glass to air get focused at a distance f_2 from the film. Then

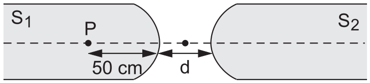


(a) $|f_1| = 3R$ (b) $|f_1| = 2.8R$
(c) $|f_2| = 2R$ (d) $|f_2| = 1.4R$

3. For an isosceles prism of angle A and refractive index μ, it is found that the angle of minimum deviation $\delta_m = A$. Which of the following options is/are correct?

(a) At minimum deviation, the incident angle i_1 and the refracting angle r_1 at the first refracting surface are related by $r_1 = \dfrac{i_1}{2}$.
(b) For the prism, the refractive index μ and the angle of prism A are related as $A = \dfrac{1}{2}\cos^{-1}\left(\dfrac{\mu}{2}\right)$.
(c) For the prism, the emergent ray at the second surface will be tangential to the surface when the angle of incidence at the first surface is $i_1 = \sin^{-1}\left[\sin A\sqrt{4\cos^2\dfrac{A}{2} - 1} - \cos A\right]$.
(d) For the angle of incidence $i_1 = A$, the ray inside the prism is parallel to the base of the prism.

4. Two identical glass rods S_1 and S_2 (refractive index = 1.5) have one convex end of radius of curvature 10 cm. They are placed with curved surfaces at a distance d as shown in the figure with their axes (shown by the dashed line) aligned. When a point source of light P is placed inside the rod S_1 on its axis at a distance of 50 cm from the curved face, the light rays emanating from it are found to be parallel to the axis inside S_2. The distance d is

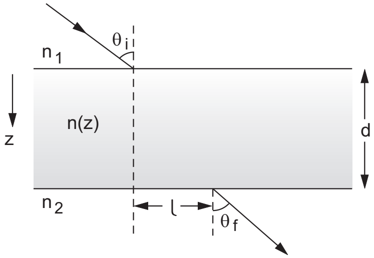


(a) 60 cm (b) 70 cm
(c) 80 cm (d) 90 cm

5. A plano-convex lens is made of a material of refractive index n. When a small object is placed 3 cm away in front of the curved face of the lens, an image double the size of the object is produced. Due to reflection from the convex surface of the lens, another faint image is observed at a distance of 10 cm away from the lens. Which of the following statements is/are true?

(a) The refractive index of the lens is 2.5.
(b) The radius of curvature of the convex surface is 45 cm.
(c) The faint image is erect and real.
(d) The focal length of the lens is 20 cm.

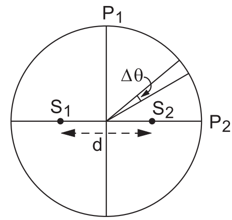
6. A transparent slab of thickness d has a refractive index $n(z)$ that increases with z. Here z is the vertical distance measured from the top. The slab is placed between two media with uniform refractive indices n_1 and n_2 ($n_2 > n_1$) as shown in the figure. A ray of light is incident with angle θ_i from medium 1 and emerges into medium 2 with refraction angle θ_f with lateral displacement l. Which of the following statements is/are correct?



(a) $n_1 \sin\theta_i = n_2 \sin\theta_f$. (b) $n_1 \sin\theta_i = (n_2 - n_1)\sin\theta_f$.

(c) l is independent of n_2. (d) l is dependent on $n(z)$.

7. Two coherent point sources S_1 and S_2 of wavelength $\lambda = 600$ nm are placed symmetrically on either side of the centre of a circle as shown. The sources are separated by a distance $d = 1.8$ mm. This arrangement produces interference fringes visible as alternate bright and dark spots on the circumference of the circle. The angular separation between two consecutive bright spots is $\Delta\theta$. Which of the following options is/are correct?



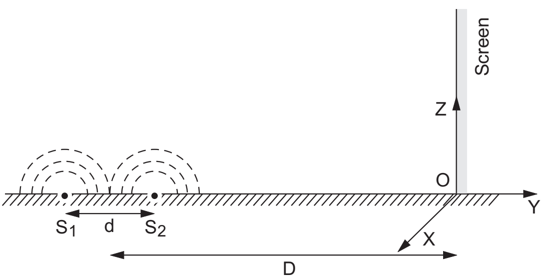
(a) The total number of fringes produced between P_1 and P_2 in the first quadrant is close to 3000.

(b) A dark spot will be formed at the point P_2.

(c) At P_2, the order of the fringe will be maximum.

(d) The angular separation between two consecutive bright spots decreases as we move from P_1 to P_2 along the first quadrant.

8. While conducting the Young's double slit experiment, a student replaced the two slits with a large opaque plate in the xy-plane containing two small holes S_1 and S_2 that act as two coherent point sources emitting light of wavelength 600 nm. The student mistakenly placed the screen parallel to the xz-plane (for $z > 0$) at a distance $D = 3$ m from the mid-point of S_1S_2 as shown schematically in the figure. The distance between the sources is $d = 0.6003$ mm. The origin O is at the intersection of the screen and the line joining S_1S_2. Which of the following is/are true of the intensity pattern on the screen?



(a) Hyperbolic bright and dark bands with foci symmetrically placed about O in the x-direction

(b) Semicircular bright and dark bands centred at point O

(c) Straight bright and dark bands parallel to the x-axis

(d) The region very close to the point O will be dark

9. Using the expression $2d \sin \theta = \lambda$, one calculates the value of d by measuring the corresponding angles θ in the range from 0 to 90°. The wavelength λ is exactly known and the error in θ is constant for all values of θ. As θ increases from 0,

(a) the absolute error in d remains constant
(b) the absolute error in d increases
(c) the fractional error in d remains constant
(d) the fractional error in d decreases

10. A light source which emits two wavelengths $\lambda_1 = 400$ nm and $\lambda_2 = 600$ nm is used in a Young's double slit experiment. If the recorded fringe widths for λ_1 and λ_2 are β_1 and β_2, and the number of fringes for them within a distance y on one side of the central maximum are m_1 and m_2 respectively then

(a) $\beta_2 > \beta_1$
(b) $m_1 > m_2$
(c) from the central maximum, the 3rd maximum of λ_2 overlaps with 5th minimum of λ_1
(d) the angular separation of the fringes for λ_1 is greater than λ_2

11. In a Young's double slit experiment, the separation between the two slits is d and the wavelength of the light is λ. The intensity of light falling on slit 1 is four times the intensity of light falling on slit 2. Choose the correct choice(s).

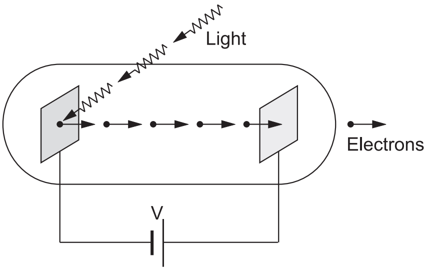
(a) If $d = \lambda$, the screen will contain only one maximum.
(b) If $\lambda < d < 2\lambda$, at least one more maximum (besides the central maximum) will be observed on the screen.
(c) If the intensity of light falling on slit 1 is reduced so that it becomes equal to that of slit 2, the intensities of the observed dark and bright fringes will increase.
(d) If the intensity of light falling on slit 2 is increased so that it becomes equal to that of slit 1, the intensities of the observed dark and bright fringes will increase.

12. A student performed the experiment to determine the focal length of a concave mirror by u–v method using an optical bench of length 1.5 m. The focal length of the mirror used is 24 cm. The maximum error in the location of the image can be 0.2 cm. The 5 sets of (u, v) values recorded by the student (in cm) are: (42, 56), (48, 48), (60, 40), (66, 33), (78, 39). The data set(s) that cannot come from the experiment and is/are incorrectly recorded, is/are

(a) (42, 56) (b) (48, 48)
(c) (66, 33) (d) (78, 39)

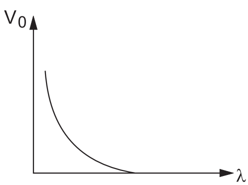
2.7 Modern Physics

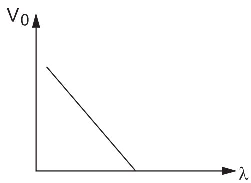
1. Light of wavelength λ_{Ph} falls on a cathode plate inside a vacuum tube as shown in the figure. The work function of the cathode surface is ϕ and the anode is a wire mesh of a conducting material kept at a distance d from the cathode. A potential difference V is maintained between the electrodes. If the minimum de Broglie wavelength of the electrons passing through the anode is λ_e, which of the following statements is/are true?

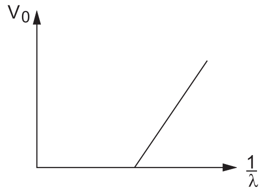


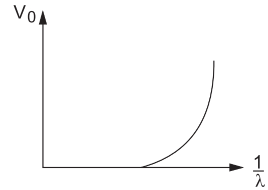
(a) For a large potential difference ($V >> \phi/e$), λ_e is approximately halved if V is made four times.
(b) λ_e increases at the same rate as λ_{Ph} does for $\lambda_{Ph} < hc < \phi$.
(c) λ_e decreases with increase in ϕ and λ_{Ph}.
(d) λ_e is approximately halved if d is doubled.

2. For photoelectric effect with incident photons of wavelength λ, the stopping potential is V_0. Identify the correct variations of V_0 with λ and $1/\lambda$.

(a) 

(b) 

(c) 

(d) 

3. The highly excited states ($n >> 1$) for hydrogen-like atoms (also called Rydberg states) with nuclear charge Ze are defined by their principal quantum number n. Which of the following statements is/are true?

(a) The relative change in the radii of two consecutive orbitals does not depend on Z.

(b) The relative change in the radii of two consecutive orbitals varies as $1/n$.

(c) The relative change in the energy of two consecutive orbitals varies as $1/n^3$.

(d) The relative change in the angular momentum of two consecutive orbitals varies as $1/n$.

4. The radius of the orbit of an electron in a hydrogen-like atom is $4.5a_0$, where a_0 is the Bohr radius. Its orbital angular momentum is $3h/2\pi$. It is given that h is Planck constant and R is Rydberg constant. The possible wavelength(s) when the atom de-excites, is/are:

(a) $\dfrac{9}{32R}$ (b) $\dfrac{9}{16R}$

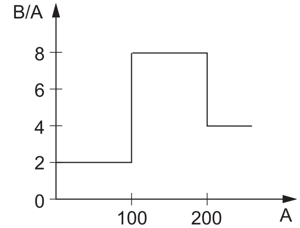
(c) $\dfrac{9}{5R}$ (d) $\dfrac{4}{3R}$

5. A fission reaction is given by ${}^{236}_{92}\mathrm{U} \rightarrow {}^{140}_{54}\mathrm{Xe} + {}^{94}_{38}\mathrm{Sr} + \mathrm{X} + \mathrm{Y}$, where X and Y are two particles. Considering ${}^{236}_{92}\mathrm{U}$ to be at rest, the kinetic energies of the products are denoted by K_{Xe}, K_{Sr}, K_X (2 MeV) and K_Y (2 MeV) respectively. Let the binding energies per nucleon of ${}^{236}_{92}\mathrm{U}$, ${}^{140}_{54}\mathrm{Xe}$ and ${}^{94}_{38}\mathrm{Sr}$ be 7.5 MeV, 8.5 MeV and 8.5 MeV respectively. Considering the different conservation laws, the correct options is/are

(a) $\mathrm{X} = n$, $\mathrm{Y} = n$, $K_{Sr} = 129$ MeV, $K_{Xe} = 86$ MeV

(b) X = p, Y = e^-, K_{Sr} = 129 MeV, K_{Xe} = 86 MeV
(c) X = p, Y = n, K_{Sr} = 129 MeV, K_{Xe} = 86 MeV
(d) X = n, Y = n, K_{Sr} = 86 MeV, K_{Xe} = 129 MeV

6. Assume that the nuclear binding energy per nucleon (B/A) versus mass number (A) is as shown in the figure. Use this plot to choose the correct choice(s) given below.



(a) Fusion of two nuclei with mass numbers lying in the range of $1 < A < 50$ will release energy.
(b) Fusion of two nuclei with mass numbers lying in the range of $51 < A < 100$ will release energy.
(c) Fission of a nucleus lying in the mass range of $100 < A < 200$ will release energy when broken into two equal fragments.
(d) Fission of a nucleus lying in the mass range of $200 < A < 260$ will release energy when broken into two equal fragments.

Answers

2.1 General Physics

1. b, c	**2.** a, c	**3.** a, b, d	**4.** a, c, d	**5.** b, d	**6.** a, c
7. a, c	**8.** a, b, d	**9.** c, d	**10.** a, d	**11.** a, c	**12.** a, b
13. a, b, d	**14.** d	**15.** c, d	**16.** b, c	**17.** a, d	**18.** b, d
19. a, d	**20.** c	**21.** b, d	**22.** b, c	**23.** a, b	**24.** b, d
25. a, b, d	**26.** c, d	**27.** a, b, d	**28.** a	**29.** b, c	**30.** a, d
31. a, d	**32.** a, b, d				

2.2 Heat and Thermodynamics

1. a, b, d	**2.** a, b, c	**3.** a, b, c, d	**4.** a, b, c	**5.** a, b, c, d	**6.** a, b, c, d
7. b, d	**8.** b, d				

2.3 Sound Waves

1. b, c	**2.** a, c, d	**3.** a, b	**4.** a, b, d	**5.** a, c, d	**6.** d
7. b, d	**8.** a, b, d	**9.** a, d			

2.4 Electrostatics

1. a, d	**2.** a, b, c, d	**3.** c, d	**4.** a, c, d	**5.** b, d	**6.** a, b, c
7. c	**8.** c	**9.** d	**10.** a, d	**11.** b, d	**12.** a, d
13. d	**14.** a, c	**15.** a, b, c, d	**16.** a		

2.5 Current Electricity and Magnetism

1. c, d	**2.** a, b, c, d	**3.** b	**4.** b, d	**5.** a, b, c, d	**6.** a, b, d
7. b, c	**8.** a, d	**9.** a, b, c	**10.** c, d	**11.** b, d	**12.** a, c
13. a, b	**14.** c, d	**15.** b, c	**16.** a, c	**17.** b, c, d	**18.** a, d
19. a, d	**20.** c, d	**21.** a, c	**22.** a, b	**23.** b, c	**24.** a, c
25. a, c, d	**26.** a, c, d	**27.** b, d			

2.6 Ray Optics and Wave Optics

1. a, b, c	**2.** a, c	**3.** a, c, d	**4.** b	**5.** a, d	**6.** a, c, d
7. a, c	**8.** b, d	**9.** d	**10.** a, b, c	**11.** a, b	**12.** c, d

2.7 Modern Physics

1. a	**2.** a, c	**3.** a, b, d	**4.** a, c	**5.** a	**6.** b, d

Hints and Solutions

2.1 General Physics

1. For the vernier callipers, 1 ms = $\frac{1}{8}$ cm. 5 vs = 4 ms

$\Rightarrow$ 1 vs = $\frac{4}{5}$ ms = $\frac{4}{5} \times \frac{1}{8}$ cm = $\frac{1}{10}$ cm.

$\therefore$ LC of the vernier callipers = 1 ms – 1 vs = $\frac{1}{40}$ cm = 0.025 cm.

Pitch of the screw gauge = 2 × 0.025 cm = 0.05 cm.

LC of the screw gauge = $\frac{0.05}{100}$ cm = 0.005 mm.

LC of the linear scale of the screw gauge = 0.05 cm.
Pitch = 0.05 × 2 cm = 0.1 cm.

$\therefore$ LC of the screw gauge = $\frac{0.1}{100}$ cm = 0.01 mm.

Correct options are (b) and (c).

2. Error in measuring $T = \frac{2\text{ s}}{40} = 0.05\text{ s} = \Delta T$.

$$T \propto \frac{1}{\sqrt{g}} \Rightarrow \frac{\Delta g}{g} = \frac{2\Delta T}{T} = \frac{1}{20} \Rightarrow \frac{\Delta g}{g} \times 100\% = 5\%.$$

3. Measured value of r is (10 + 1) mm.

Thus % error in $r = \frac{\Delta r}{r} \times 100 = \frac{1}{10} \times 100 = 10\%$.

$(T)_{\text{av}} = \frac{1}{5}\Sigma T_{\text{i}} = \frac{2.78\text{ s}}{5} = 0.56\text{ s}$.

Mean absolute error = $\frac{1}{5}\Sigma|T - T_{\text{av}}| = 0.02$ S.

$\therefore$ % error in $T = \frac{0.02}{0.56} \times 100\% = 3.57\%$.

Now, $g \propto \frac{R-r}{T^2}$, so $\frac{dg}{g} = 2\frac{dT}{T} + \frac{dR+dr}{R-r}$

$$\Rightarrow \frac{dg}{g} = 2(3.57\%) + \frac{1+1}{60-10} \times 100\% = 7.14\% + 4\% = 11.14\% \approx 11\%.$$

4. $[h] = ML^2T^{-1}$, $[c] = LT^{-1}$, $G = M^{-1}L^3T^{-2}$

$$\Rightarrow M \propto \sqrt{\frac{hc}{G}}, L \propto \sqrt{\frac{hG}{c^3}}.$$

5. Energy $\equiv FL \equiv k_B T$, where F = force, L = length, k_B = Boltzmann constant and T = temperature.

Permittivity $\varepsilon \equiv \frac{Q^2}{FL^2}$, where Q = charge and mole $(n) \equiv L^{-3}$.

With these expressions, correct options are (b) and (d).

6. Magnetic energy in the inductor $= \frac{1}{2}LI^2 = \frac{1}{2}\frac{\mu_0 N^2 A}{l}I^2$.

Electrical energy with the capacitor $= \frac{1}{2}CV^2 = \frac{1}{2}\frac{\varepsilon_0 A}{d}V^2$.

$\Rightarrow \frac{\mu_0 AI^2}{l}$ and $\frac{\varepsilon_0 AV^2}{d}$ have the same dimension.

$\therefore$ $\mu_0 I^2$ and $\varepsilon_0 V^2$ have the same dimension.

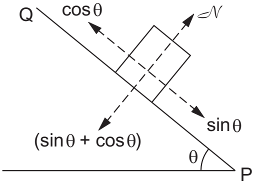
For a capacitor, $\quad Q = CV$

or $\quad \frac{Q}{t} = \frac{CV}{t} = \frac{\varepsilon_0 A}{d} \cdot \frac{V}{t}$.

or $\quad I = \varepsilon_0 \frac{A}{td} V$.

$\because$ $\frac{A}{td}$ has the dimension of speed,

$\therefore$ $I = \varepsilon_0 CV$.



7. $f = 0$, if $\sin\theta = \cos\theta \Rightarrow \theta = 45°$.

f directed towards Q, $\sin\theta > \cos\theta$

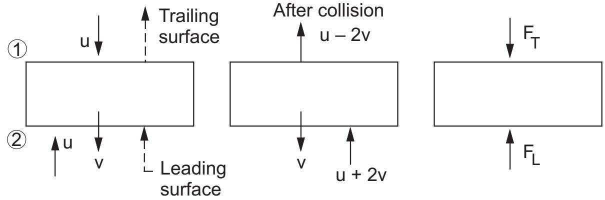
$\Rightarrow Q > 45°$.

f directed towards P,

$\sin\theta < \cos\theta \Rightarrow \theta < 45°$.

Hence, options (a) and (c) are correct.

8.



The change in momentum at leading surface,

$$\Delta P = 2m(u+v).$$

$$F_L = \frac{\Delta P}{\Delta t} = \frac{2m(u+v)}{(d/u)}$$

(assuming that the particles return after a distance d, when $\Delta t = d/u$).

Similarly, for the trailing surface:

$$\Rightarrow \quad F_L = \frac{2m(u+v)u}{d}.$$

Similarly, for the trailing surface:

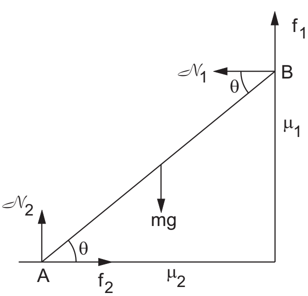
$$F_T = \frac{2m(u-v)u}{d}.$$

$$\text{Pressure difference} = \frac{2m(u+v)u}{Ad} - \frac{2m(u-v)u}{Ad}$$

$$\Rightarrow \quad \Delta P = \frac{2muv}{Ad}.$$

9. Since the ladder is about to slip, both f_1 and f_2 will be limiting.

$$\Rightarrow \quad f_1 = \mu_1\mathcal{N}_1 \text{ and } f_2 = \mu_2\mathcal{N}_2.$$



For translational equilibrium

$$\mathcal{N}_1 = \mu_2\mathcal{N}_2, \text{ and}$$

$$\mathcal{N}_2 + \mu_1\mathcal{N}_1 = mg.$$

Solving, $$\mathcal{N}_1 = \frac{\mu_2 mg}{1+\mu_1\mu_2} \quad \text{...(1)}$$

and $$\mathcal{N}_2 = \frac{mg}{1+\mu_1\mu_2}. \quad \text{...(2)}$$

Taking torque about A,

$$mg\frac{l}{2}\cos\theta = \mathcal{N}_1 l\sin\theta + \mu_1\mathcal{N}_1 l\cos\theta$$

or $$\frac{mg}{2}\cos\theta = \frac{\mu_2 mg}{1+\mu_1\mu_2}\sin\theta + \mu_1\mathcal{N}_1\sin\theta. \quad \text{...(3)}$$

Taking $\mu_1 = 0$ and $\mu_2 \neq 0$, we get $\mathcal{N}_1 = \mu_2\, mg$, hence

from equation (3), $\frac{mg}{2}\cos\theta = \mathcal{N}_2$ mg sin θ $= \mathcal{N}_1$ sin θ

$$\Rightarrow \quad \mathcal{N}_1 \tan\theta = \frac{mg}{2}.$$

Taking, $\mu_1 \neq 0$, $\mu_2 \neq 0$, from (2) we get

$$\mathcal{N}_2 = \frac{mg}{1+\mu_1\mu_2}.$$

Taking torque about B, we can conclude that options (a) and (b) are incorrect.

10. Initially, at $t = 0$, $v = u_0$. Thus $v = u_0 \cos \omega t$. At the time of collision,

$$\frac{u_0}{2} = u_0 \cos \omega t_1 \Rightarrow t_1 = \frac{\pi}{3\omega}.$$

After collision the motion is reversed and at time $t_2 = 2t_1 = \frac{2\pi}{3\omega}$, it acquires the same speed u_0.

If t_3 be the time at which the particle passes through the equilibrium position for the second time then

$$t_3 = \frac{T}{2} + 2t_1 = \frac{\pi}{\omega} + \frac{2\pi}{3\omega} = \frac{5\pi}{3\omega} = \frac{5\pi}{3}\sqrt{\frac{m}{k}}.$$

Hence, the correct options are (a) and (d).

11. Conserving linear momentum, $mv = MV$

$$\Rightarrow \qquad V = \frac{mv}{M}.$$

Conserving energy, $\frac{1}{2}mv^2 + \frac{1}{2}M\left(\frac{mv}{M}\right)^2 = mgR$

$$\Rightarrow \qquad v = \sqrt{\frac{2gR}{1+\frac{m}{M}}}.$$

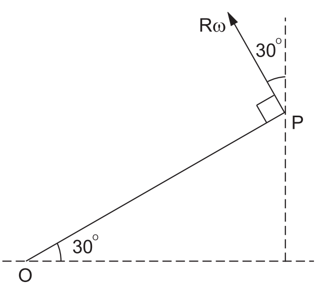
In absence of external force, $\Delta x_{cm} = 0$,

$$m\Delta x_m = -M\Delta x_M,\ m(R-x) = Mx$$

$$\Rightarrow \qquad x = \frac{mR}{(M+m)} \text{ (leftward)}.$$

12. $\vec{V}_0 - (3R)\omega\hat{i} = 0.$

$\therefore \ \vec{V}_0 = 3R\omega\hat{i}.$



Now, $\vec{V}_{P,O} = -\frac{R\omega}{4}\hat{i} + \frac{R\omega\sqrt{3}}{4}\hat{j}$

$$\Rightarrow \qquad \vec{V}_P = \vec{V}_{P,O} + \vec{V}_0$$

$$= -\frac{R\omega}{4}\hat{i} + \frac{R\omega\sqrt{3}}{4}\hat{j} + 3R\omega\hat{i}$$

$$= \frac{11}{4}R\omega\hat{i} + R\omega\frac{\sqrt{3}}{4}\hat{j}.$$

Hence, the correct options are (a) and (b).

13.
$$\vec{r}(t) = \alpha t^3 \hat{i} + \beta t^2 \hat{j}$$
$$\vec{v}(t) = \frac{d\vec{r}}{dt} = 3\alpha t^2 \hat{i} + 2\beta t\hat{j} = 10t^2 \hat{i} + 10t\hat{j}$$
$$\vec{a} = (20t\hat{i} + 10\hat{j}) \text{ m s}^{-2}.$$

At $t = 1\text{s}, \vec{v} = 10(\hat{i} + \hat{j}) \text{ m s}^{-1}$.

Angular momentum
$$\vec{L}_0 = \vec{r} \times m\vec{v}.$$

At $t = 1 \text{ s}, \vec{r} = \left(\frac{10}{3}\hat{i} + 5\hat{j}\right) \text{m}.$

$$\therefore \quad \vec{L}_0 = \left(\frac{10}{3}\hat{i} + 5\hat{j}\right) \text{m } (0.1 \text{ kg}) \times (10\hat{i} + 10\hat{j}) \text{ m s}^{-1}$$
$$= \left(-\frac{5}{3}\hat{k}\right) \text{N m s}.$$
$$\vec{F} = m\vec{a} = (0.1 \text{ kg}) \left(20\hat{i} + 10\hat{j}\right) \text{m s}^{-2}$$
$$= (2\hat{i} + \hat{j}) \text{ N}.$$

Torque about the origin,
$$\vec{\tau}_0 = \vec{r} \times \vec{F} = \left(\frac{10}{3}\hat{i} + 5\hat{j}\right) \text{m} \times (2\hat{i} + \hat{j}) \text{ N} = \left(-\frac{20}{3} \text{ N m}\right) \hat{k}.$$

Hence, the correct options are (a), (b) and (d).

14. Acceleration of a body rolling down the plane,
$$a = \frac{mg \sin\theta}{m + \frac{I}{R^2}}.$$

For P, $$a_P = \frac{mg \sin\theta}{m + \frac{mR^2}{R^2}} = \frac{g\sin\theta}{2}.$$

For Q, $$a_Q = \frac{mg\sin\theta}{m} = g\sin\theta, \text{ since } I_Q \approx 0.$$

$$\omega_P = \frac{\sqrt{2 \cdot \frac{g}{2} \cdot l}}{R} \text{ and } \omega_Q = \frac{\sqrt{2gl}}{R}.$$

$$\therefore \quad \omega_Q > \omega_P.$$

Hence, the correct option is (d).

15. In the absence of external horizontal forces on the bar, its CM will fall vertically downwards.

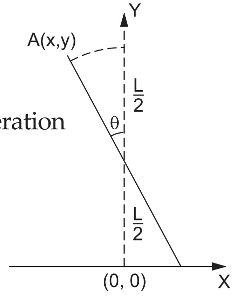
For translational motion:

$$mg - N = ma_{CM}. \quad \text{...(i)}$$

For rotational motion:

$$N \cdot \frac{L}{2}\sin\theta = I_{CM}\alpha \quad \text{...(ii)}$$

and $$a_{CM} = \alpha \cdot \frac{L}{2}\sin\theta. \quad \text{...(iii)}$$



Solving equations (i), (ii) and (iii), the angular acceleration

$$\alpha = \frac{mg\frac{L}{2}\sin\theta}{I_{CM} + \frac{mL^2}{4}\sin^2\theta}.$$

Torque about the point of contact $= I\alpha = \frac{mL^2}{3}\alpha$.

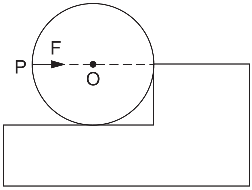
Displacement of the mid-point $= \frac{L}{2}(1 - \cos\theta)$.

For the point A, $x = -\frac{L}{2}\sin\theta$ and $y = L\cos\theta$

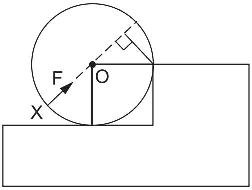
$$\Rightarrow \quad \frac{y^2}{L^2} + \frac{4x^2}{L^2} = 1, \text{ which represents an ellipse.}$$

Hence, the correct options are (c) and (d).

16. When the force is applied normal to the circumference at point P, the line of action always passes through O, so the torque is always zero.

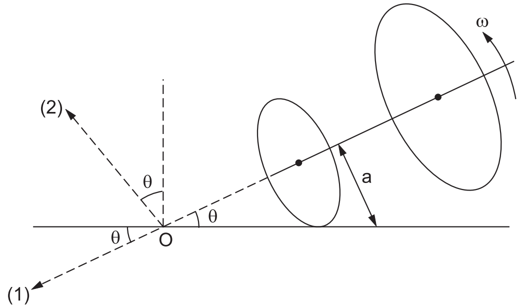


When the force is applied normal to the circumference at point X, the force and its perpendicular distance from the line of action remain constant, so torque τ = constant.



Hence, the correct options are (b) and (c).

17.



For angular momentum about the CM, we find L about the axis denoted by (1):

$$L_1 = \frac{ma^2}{2}\omega + \frac{4m(2a)^2}{2}\omega = \frac{17ma^2}{2}\omega.$$

$$V_{CM} = \frac{1}{5m}(m\cdot\omega a + 4m\cdot\omega 2a) = \frac{9\omega a}{5},$$

$$l_{CM} = \frac{1}{5m}(ml + 4m\cdot 2l) = \frac{9}{5}l.$$

Angular velocity (ω) about the axis (2): $\dfrac{V_{CM}}{l_{CM}} = \dfrac{\omega a}{l}$.

Component of ω along the z-axis,

$$\omega\cos\theta = \frac{\omega a}{l}\frac{l}{5a} = \frac{\omega}{5}.$$

Hence, the correct options are (a) and (d).

18. By simple calculation, if mass of P is M, and its radius is R then

for Q: mass = $8M$ and radius = $2R$,

for R: mass = $9M$ and radius = $9^{1/3}R$.

Since escape velocity $v_e = \sqrt{2GM/R}$,

$$(v_e)_P = \sqrt{2G\frac{M}{R}},\ (v_e)_Q = \sqrt{2G.\frac{8M}{2R}} = 2(v_e)_P, \text{ and}$$

$$(v_e)_R = \sqrt{2G\times\frac{9M}{9^{1/3}R}} = 9^{1/3}(v_e)_P.$$

$\Rightarrow \quad (v_e)_R > (v_e)_Q > (v_e)_P.$

Now, $(v_e)_Q/(v_e)_P = 2.$

Hence, the correct options are (b) and (d).

19. In case A:

$$mg\frac{L}{2}\sin\theta + MgL\sin\theta = \left(\frac{1}{3}mL^2 + \frac{1}{2}MR^2 + ML^2\right)\alpha_A.$$

In case B:

$$mg\frac{L}{2}\sin\theta + MgL\sin\theta = \left(\frac{1}{3}mL^2 + ML^2\right)\alpha_B.$$

$\Rightarrow \quad \tau_A = \tau_B;\ \omega_A < \omega_B.$

Hence, the correct options are (a) and (d).

20. During collision, friction is impulsive and soon after collision the ring has clockwise angular velocity, so friction acts towards left.

Hence, the correct option is (c).

21. $\frac{1}{2}mv^2 + \left(-\frac{2GMm}{L}\right) = 0$

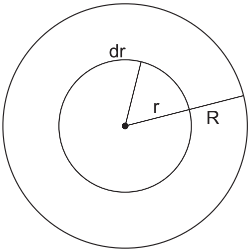
$\Rightarrow \quad v = 2\sqrt{\frac{GM}{L}}.$

Gravitational field is conservative, in which total mechanical energy (KE + PE) remains conserved. Hence, the kinetic energy imparted to the mass m is gradually reduced and gets converted into its potential energy, so that at every point of its flight the total mechanical energy remains constant.

Hence, the correct options are (b) and (d).

22. Gravitational field E at a distance r from the centre of the solid sphere:

$E = G \cdot \frac{4}{3}\pi r^3 \rho / r^2 = \frac{4}{3}\pi G \rho r.$



$\therefore$ force on the concentric shell of radius r and thickness dr is

$dF = E \cdot 4\pi r^2 dr\rho$

$\Rightarrow$ pressure $dp = \frac{dF}{\Delta A} = \frac{dF}{4\pi r^2} = E\rho\, dr.$

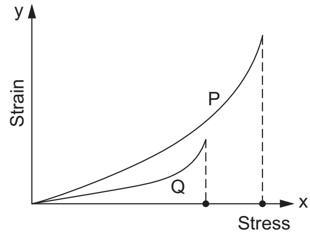
Integrating, $-p = -\int_0^p dp = \int_R^r \left(\frac{4}{3}\pi G\rho r\right)(\rho dr) = K\int_R^r r dr$

$\Rightarrow \quad p = \frac{K}{2}(R^2 - r^2) = A\left(1 - \frac{r^2}{R^2}\right).$

Let us calculate p for different values of $\frac{r}{R}$:

$\frac{r}{R}$	p
0	A
1/2	$3A/4$
1/3	$8A/9$
2/3	$5A/9$
2/5	$21A/25$
3/5	$16A/25$
3/4	$7A/16$

With these values, options (b) and (c) are correct.



23. From the stress–strain graph, breaking stress of P is more than that of Q.

So, P is more ductile then Q.

Strain $= \frac{1}{Y} \cdot$ stress, so $Y_P < Y_Q$.

In addition, P has greater tensile strength.

Hence, the correct options are (a) and (b).

24. First oscillator:

$$b = ma\omega_1.$$

$$\therefore \quad \frac{a}{b} = \frac{1}{m\omega_1} = n^2.$$

$$E_1 = \frac{1}{2}m\omega_1^2 a^2.$$

Second oscillator:

$$\frac{1}{m\omega_2} = 1.$$

$$\therefore \quad \frac{\omega_2}{\omega_1} = n^2.$$

$$E_2 = \frac{1}{2}m\omega_2^2 R^2.$$

$$\frac{E_1}{E_2} = \left(\frac{\omega_1}{\omega_2}\right)^2 \cdot n^2 = \left(\frac{\omega_1}{\omega_2}\right)^2 \frac{\omega_2}{\omega_1} = \frac{\omega_1}{\omega_2}.$$

$$\Rightarrow \quad \frac{E_1}{\omega_1} = \frac{E_2}{\omega_2}.$$

Hence, the correct options are (b) and (d).

25. $\omega_{v_1} = \sqrt{\frac{K}{M}}, \quad \omega' = \sqrt{\frac{K}{M+m}}.$

Case (i): When m is placed at mean position—

Let v' = velocity of system $(M + m)$ just after placing m.

Conserving linear momentum, $Mv_0 = (M + m)v'$

$$\Rightarrow \quad M\omega A = (M+m)\,\omega' A'$$

or $\frac{A'}{A} = \left(\frac{M}{M+m}\right)\frac{\omega}{\omega'} = \sqrt{\frac{M}{M+m}}.$

Case (ii): When m is placed at the extreme position—

v_M before placing is zero $\Rightarrow v_{M+m} = 0$ after placing m

$\Rightarrow$ extreme position and mean position remain unchanged, so $A = A'$.

$T' = \frac{2\pi}{\omega'}$, which is same in both the cases.

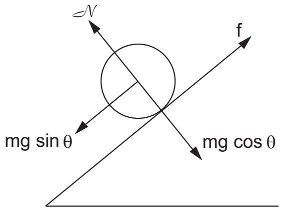
Energy decreases in case (i) but not in case (ii).

Velocity at the mean position $= \omega' A'$, which decreases in both the cases.

Hence, the correct options are (a), (b) and (d).

26. Friction acts upwards, so it supports rotation and opposes translation.

Acceleration of a rolling body,



$$a_{CM} = \frac{g\sin\theta}{1+K^2/R^2} = \frac{2}{3}g\sin\theta.$$

Frictional force,

$$f = \frac{mg\sin\theta}{1+R^2/K^2} = \frac{1}{3}mg\sin\theta.$$

$\Rightarrow$ f decreases with decrease in θ.

For pure rolling, there is no energy dissipation due to absence of slipping.

Hence, the correct options are (c) and (d).

27. Since mechanical energy is conserved, so

$$E_A = E_B = E_C.$$

$$\Rightarrow \quad K_A + mgh_A = K_B = K_C + mgh_C.$$

$$\therefore \quad K_B > K_A \; ; \quad K_B > K_C.$$

Now, $E_A - E_C = 0$, so $mg(h_A - h_C) = (K_C - K_A)$.

For, $h_A > h_C; K_C > K_A$.

For kinetic energy K_B, only the translational parts gets converted into potential energy at point C.

28. $$\Sigma\vec{F}_{ext} = \frac{d\vec{p}}{dt}.$$

When $\Sigma\vec{F}_{ext} = 0\cdot$, linear momentum $\vec{p}$ is conserved.

Hence, the correct option is (a).

29. $$\vec{v}_C = \vec{v}_{CM} + \vec{r\omega} = 2\vec{v}_{CM}\,;$$

$$\vec{v}_B = \vec{v}_{CM} \text{ and } \vec{v}_A = 0.$$

$$\Rightarrow \quad \vec{v}_C - \vec{v}_B = \vec{v}_{CM}\,;\ \vec{v}_B - \vec{v}_A = \vec{v}_{CM}$$

$$\vec{v}_C - \vec{v}_A = 2\vec{v}_{CM}\,;\ \vec{v}_B - \vec{v}_C = -\vec{v}_{CM}$$

Hence, the correct options are (b) and (c).

30. For the system to float,

$\downarrow (\rho_1 + \rho_2) V = (\sigma_1 + \sigma_2) V \uparrow \;\Rightarrow\; \rho_1 + \rho_2 = \sigma_1 + \sigma_2.$

Since the string is taut,

$$\rho_1 < \sigma_1 \text{ and } \rho_2 > \sigma_2.$$

Now, $v_P = \frac{2}{9}\frac{r^2(\sigma_2 - \rho_1)g}{\eta_2}$, and $v_Q = \frac{2}{9}\frac{r^2(\sigma_1 - \rho_2)g}{\eta_1}$.

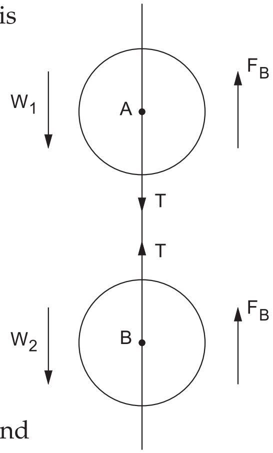
Since, $\sigma_2 - \rho_1 = -(\sigma_1 - \rho_2)$; $\left|\frac{\vec{v}_P}{\vec{v}_Q}\right| = \frac{\eta_1}{\eta_2}$

$\vec{v}_P \cdot \vec{v}_Q < 0$, as they are in opposite directions.

Hence, the correct options are (a) and (d).

31. The buoyant force on each of the two spheres is

$$F_B = \frac{4}{3}\pi R^3 2\rho g.$$



The weights of the spheres are

$$W_1 = \frac{4}{3}\pi R^3 \rho g \quad \text{and} \quad W_2 = \frac{4}{3}\pi R^3 3\rho g.$$

Let T = tension in the spring.

For equilibrium of A:

$$\frac{4}{3}\pi R^3 2\rho g = \frac{4}{3}\pi R^3 \rho g + T$$

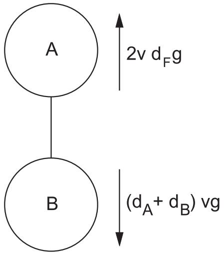
$$\Rightarrow \quad T = \frac{4}{3}\pi R^3 \rho g = k\Delta l \Rightarrow \Delta l = \frac{4\pi R^3 \rho g}{3k}.$$

For the whole system, T is the internal force, and

$W_1 + W_2 = 2F_B$ is true when both the spheres are completely submerged.

Hence, the correct options are (a) and (d).

32. For equilibrium, forces acting on the system are shown in the adjoining figure.



Now, it is clear from the figure that

$$d_A < d_F, d_B > d_F$$

and $d_A + d_B = 2d_F$.

Hence, the correct options are (a), (b) and (d).

2.2 Heat and Thermodynamics

1. Total internal energy of the system,

$$U = U_1 + U_2 = (nC_VT)_1 + (nC_VT)_2 = \frac{5}{2}RT + \frac{3}{2}RT = 4RT.$$

Average energy per mole $= \frac{4RT}{2} = 2RT$.

$\because$ the mixture has two moles with $U = 4RT$,

$\therefore$ $2(C_V)_{\text{mix}}\, T = 4RT \;\Rightarrow\; (C_V)_{\text{mix}} = 2R$.

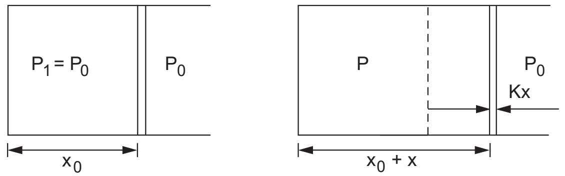
The speed of sound, c, in a gas $= \sqrt{\dfrac{\gamma RT}{M}}$.

$$\Rightarrow \frac{c_{\text{mix}}}{c_{\text{He}}} = \sqrt{\left(\frac{\gamma_{\text{mix}}}{\gamma_{\text{He}}}\right)\left(\frac{M_{\text{He}}}{M_{\text{max}}}\right)} = \sqrt{\frac{3}{2}\times\frac{3}{5}\times\frac{4}{3}} = \sqrt{\frac{6}{5}}.$$

$$\text{Now, } \frac{(v_{\text{rms}})_{\text{He}}}{(v_{\text{rms}})_{H_2}} = \sqrt{\frac{M_{H_2}}{M_{\text{He}}}} = \frac{1}{\sqrt{2}}.$$

Hence, the correct options are (a), (b) and (d).

2.



$$p = p_1 + \frac{Kx}{A}.$$

$$p_2 = \frac{3}{2}p_1 \;\Rightarrow\; x = \frac{V_1}{A}.$$

$$\frac{3}{2}p_1 = p_1 + \frac{Kx}{A} \;\text{ or }\; Kx = \frac{p_1 A}{2}.$$

Energy stored in the spring $= \dfrac{1}{2}Kx^2 = \dfrac{1}{2}(Kx)x = \dfrac{1}{4}p_1V_1$.

$$\text{Now, } \Delta U = \frac{f}{2}(p_2V_2 - p_1V_1) = 3p_1V_1.$$

In option (c):

$$\because \quad p_2 = \frac{4}{3}p_1,\; Kx = \frac{p_1}{3}A, \text{ so } x = \frac{2V_1}{A}.$$

$$\text{Now, } W_{\text{gas}} = -(W_{\text{atm}} + W_{\text{spring}}) = p_1Ax + \frac{1}{2}Kx^2$$

$$= \left[p_1A\cdot\frac{2V_1}{A} + \frac{1}{2}\cdot\frac{p_1A}{3}\cdot\frac{2V_1}{A}\right] = \frac{7}{3}p_1V_1,$$

Heat supplied to the system,

$$\Delta Q = W + \Delta U = \frac{7}{3}p_1V_1 + \frac{3}{2}(p_2V_2 - p_1V_1) = \frac{41}{6}p_1V_1.$$

which is incorrect.

Hence, the correct options are (a), (b) and (c).

3. The graph between 0 and 100 K is linear, so statement (a) is correct.
The area under the given curve is proportional to the heat absorbed, so statement (b) is correct.
The given graph is parallel to T axis and has constant C, so statement (c) is correct.
The value of C increases with temperature in the range 200–300 K, so statement (d) is correct.

4. Net amount of heat radiated per second

$$\begin{aligned} Q &= \sigma A(T^4 - T_0^4) = \sigma A[(T_0 + \Delta T_0)^4 - T_0^4] \\ &= \sigma A\, T_0^4\left[\left(1 + \frac{\Delta T_0}{T_0}\right)^4 - 1\right] = \sigma A T_0^4\left[\left(1 + 4\frac{\Delta T_0}{T_0}\right) - 1\right] \\ &= \sigma A \cdot 4\frac{\Delta T_0}{T_0} \cdot T_0^4 = A(\sigma T_0^4)\left(\frac{4\Delta T_0}{T_0}\right) \\ &= (1\ \text{m}^2)\ (460\ \text{W m}^{-2})\left(\frac{4\times 10}{300}\right) = 61.3\ \text{J} \approx 60\ \text{J}. \end{aligned}$$

In the first case, $Q_1 = \sigma[(T_0 + \Delta T)^4 - T_0^4]$.

In the second case, $Q_2 = \sigma[(T_0 + \Delta T)^4 - (T_0 - \Delta T_0)^4]$

$\Rightarrow$ more heat energy required to be radiated per second to maintain the body temperature,

$$\begin{aligned} Q_2 - Q_1 &= \sigma[T_0^4 - (T_0 - \Delta T_0)^4] \quad [\because \Delta T << T_0] \\ &= \sigma T_0^4\left[1 - \left(1 - \frac{\Delta T_0}{T_0}\right)^4\right] \approx 4\sigma T_0^3 \Delta T_0. \end{aligned}$$

Net heat radiated by a human body in 1 s,

$$Q = \sigma A(T^4 - T_0^4).$$

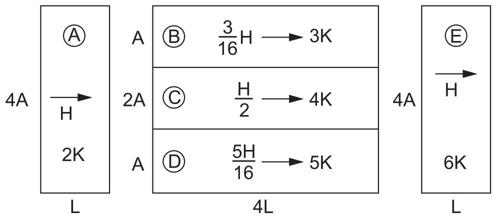
If the exposed area is reduced, the rate of heat loss Q is also reduced, so that the body temperature is maintained the same.

From Wien's law, $\lambda_m T$ = constant.

Thus the increase in body temperature corresponds to the shift of the peak of wavelength λ_m of the spectrum of electromagnetic radiation towards *smaller* wavelength side.

Hence, the correct options are (a), (b) and (c).

5.



The given system comprises three components shown separately in the figure, whose thermal resistances are

$$R_A = \frac{L}{8KA}, R_B = \frac{4L}{3KA},$$

$$R_C = \frac{4L}{8KA}, R_D = \frac{4L}{5KA}$$

and $$R_E = \frac{L}{24KA}.$$

Since the three are in series, thermal current through A and E will be the same.

Temperature difference: $\Delta\theta_A = HR_A = \dfrac{HL}{8KA}$,

$$\Delta\theta_B = \frac{3H}{16}R_B = \frac{HL}{4KA},\ \Delta\theta_C = \frac{HR_C}{2} = \frac{HL}{4KA},$$

$$\Delta\theta_D = \frac{5H}{16}R_D = \frac{HL}{4KA} \text{ and } \Delta\theta_E = HR_E = \frac{HL}{24KA}.$$

$\Rightarrow \Delta\theta_E$ is the smallest.

Now, $$I_C = \frac{\Delta\theta_C}{R_C} = \frac{H}{2}, I_B = \frac{\Delta\theta_B}{R_B} = \frac{3}{16}H$$

and $$I_D = \frac{\Delta\theta_D}{R_D} = \frac{5H}{16}$$

$\Rightarrow$ $$I_C = I_B + I_D.$$

Hence, the correct options are (a), (c) and (d).

6. Sunrays falling on the black body leads to more absorption of radiation. Since the temperature remains constant, it emits more radiation.

Sunlight is white light, which comprises the visible spectrum (red to violet), hence it radiates more energy in the visible spectrum.

Hence, the correct options are (b), (c) and (d).

7.

Gas	C_V	C_P	$C_P - C_V$	$C_P + C_V$	$\frac{C_p}{C_V}$	$C_P \times C_V$
Monatomic	$\frac{3}{2}R$	$\frac{5}{2}R$	R	$4R$	$\frac{5}{3}$	$\frac{15}{4}R^2$
Diatomic	$\frac{5}{2}R$	$\frac{7}{2}R$	R	$6R$	$\frac{7}{5}$	$\frac{35}{4}R^2$

So, options (b) and (d) are true.

8. The process A → B is a part of the semicircle, so it cannot be isothermal. For a process to be isothermal, the path should be a rectangular hyperbola.

In the process B → C → D, the gas undergoes compression. The work done by the gas is negative ($\Delta W < 0$), hence heat is expelled (flows out).

The work done during the process A → B → C is the area enclosed by the semicircle ($\int p\mathrm{d}V$), which is not zero.

For the clockwise process A → B → C → D → A, the work done by the gas is positive.

Hence, the correct options are (b) and (d).

2.3 Sound Waves

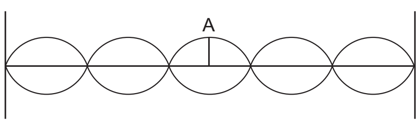
1. Comparing the given equation with the equation of a standing wave,

$$y(x, t) = A\sin\frac{2\pi}{\lambda}x\cos(\omega t),$$

we have, $A = 0.01$ m.

$$\frac{2\pi}{\lambda} = 62.8 \text{ m}^{-1}$$

$$= 20\,\pi \text{ m}^{-1}$$



or $$\frac{\lambda}{2} = \frac{1}{20}\text{ m} = 5 \text{ cm}.$$

∴ length of the string, $L = \frac{5\lambda}{2} = 25 \text{ cm} = 0.25 \text{ m}$.

Angular frequency of this mode, $\omega = 628 \text{ s}^{-1}$, $2\pi\nu = 200\pi \text{ s}^{-1}$.

∴ fundamental frequency $= \frac{\nu}{5} = \frac{100}{5}\text{Hz} = 20 \text{ Hz}$.

Hence, the correct options are (b) and (c).

2. Beat frequency:

(i) When the observer is at P,

$$\nu_P = (121 - 118)\frac{(V + V_0 \cos\alpha)}{V}\text{Hz} = 3\left(\frac{V + V_0\cos\alpha}{V}\right)\text{Hz.}$$

(ii) When the observer is at Q,

$$\nu_Q = 121 - 118 = 3 \text{ Hz.}$$

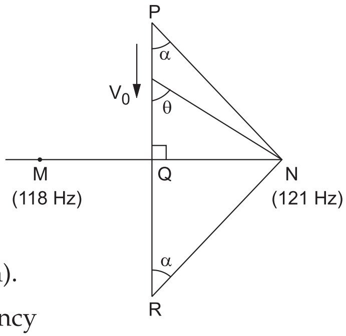
(iii) When the observer is at R,

$$\nu_R = (121 - 118)\frac{(V - V_0 \cos\alpha)}{V}\text{Hz} = 3\left(\frac{V - V_0\cos\alpha}{V}\right)\text{Hz}$$

$$\Rightarrow \nu_P + \nu_R = 2\nu_Q.$$

At any position (θ),

the beat frequency



$$\nu(\theta) = 3\left(\frac{V + V_0\cos\theta}{V}\right)\text{Hz}$$

$$\Rightarrow \quad \frac{d\nu}{d\theta} = -3\frac{V_0}{V}\sin\theta.$$

At Q, $\sin\theta = \sin 90° = 1$ (maximum).

$\therefore$ the rate of change of beat frequency with time is maximum at Q.

Hence, the correct options are (a), (c) and (d).

3. Expressions for apparent frequency are

$f_2 = \left(\frac{v + w + u}{v + w - u}\right)f_1$, for wind blowing from the source to the observer.

$f_2 = \left(\frac{v - w + u}{v - w - u}\right)f_1$, for wind blowing from the observer to the source.

$\Rightarrow$ $f_2 > f_1$ in both the cases.

Hence, the correct options are (a) and (b).

4. $v = \sqrt{\frac{F}{\mu}}\cdot$ Speed is same at any point for both the pulses,

hence $T_{AO} = T_{OA}$.

$\because$ $v = f\lambda$, so $\lambda \propto v$. At A, tension F decreases, so v decreases

$\Rightarrow \lambda$ decreases, so option (c) is incorrect.

v depends on F and μ and is independent of f and λ.

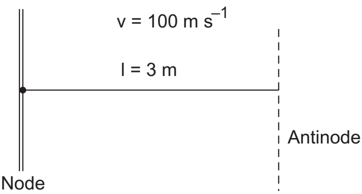
Hence, the correct options are (a), (b) and (d).

5. For possible modes of vibration in standing waves,

$$l = (2n+1)\frac{\lambda}{4} \Rightarrow \lambda = \frac{12}{(2n+1)} \text{ m.}$$

$$k = \frac{2\pi}{\lambda} = \frac{(2n+1)\pi}{6} \text{ and}$$

$$\omega = vk = \frac{(2n+1)50\pi}{3}.$$



For $n = 0$, $k = \frac{\pi}{6}$ and $\omega = \frac{50\pi}{3}$.

For $n = 2$, $k = \frac{5\pi}{6}$ and $\omega = \frac{250\pi}{3}$.

For $n = 7$, $k = \frac{5\pi}{2}$ and $\omega = 250\pi$.

Hence, the correct options are (a), (c) and (d).

6. $$v = \sqrt{\frac{\gamma p}{\rho}} = \sqrt{\frac{\gamma RT}{M}}, \text{ also } \nu = \frac{v}{\lambda} = \frac{v}{4l}$$

$$\Rightarrow l = \frac{1}{4\nu}\sqrt{\frac{\gamma RT}{M}}.$$

From the above equation the values of l for gases mentioned in the options (a), (b), (c) and (d) are found to be 0.459 m, 0.363 m, 0.340 m and 0.348 m respectively.

As $l = (0.350 \pm 0.005)$ m, the correct option is (d).

7. When reflection of a sound wave occurs at a rigid boundary (like the closed end of a pipe), the particles at the boundary are unable to vibrate. The reflected wave thus generated interferes with the incident wave to produce zero displacement (or node). At this displacement node exists the pressure antinode. Thus reflected pressure wave has the same phase as the incident wave and a high pressure compression pulse gets reflected as a compression pulse.

Similarly, for reflection from the open end of the pipe, the particles vibrate with increased amplitude (displacement antinode) and pressure remains at the average value (pressure node). The reflected pressure wave interferes destructively with the incident wave so that a phase change of π occurs from the open end.

Hence, a high pressure compression pulse reflects as a low pressure rarefaction pulse.

Thus, the correct options are (b) and (d).

8. $X = A\sin^2\alpha x + B\cos^2\alpha x + C\sin\alpha x\cos\alpha x$

$= \frac{A}{2}(1-\cos 2\alpha x) + \frac{B}{2}(1+\cos 2\alpha x) + \frac{C}{2}\sin 2\alpha x$

$= \frac{1}{2}(A+B) - \frac{1}{2}(A\cos 2\alpha x - B\cos 2\alpha x) + \frac{C}{2}\sin 2\alpha x.$

For $A = -B$, $C = 2B$, $X = B\cos 2\alpha x + B\sin 2\alpha x$.

This represents SHM with amplitude $\sqrt{2}B$.

For $A = B$, $C = 0$, we get

$X = A$. The motion is not SHM.

For $A = B$, $C = 2B$,

$X = B + B\sin 2\alpha x$.

This represents SHM with displaced origin and amplitude = B.

For any value of A, B and C (except $C = 0$),

$$X = \frac{1}{2}(A+B) + \frac{1}{2}(B-A)\cos 2\alpha x + \frac{C}{2}\sin 2\alpha x.$$

This represents SHM.

Hence, the correct options are (a), (b) and (d).

9. The first resonance is more intense than the second resonance.

The prongs are kept vertically (not horizontally) to produce longitudinal vibration in the air column.

For the first resonance, $l_1 + E = \frac{\lambda}{4}$, where E = end correction. So l_1 is slightly shorter than $\frac{\lambda}{4}$.

Hence, the correct options are (a) and (d).

2.4 Electrostatics

1. Q_1 is positive, Q_2 is negative. Also $|Q_1| > |Q_2|$.

Lines of force start from a positive charge and end on a negative charge, and are denser near a larger charge.

Hence, the correct options are (a) and (d).

2. $k\frac{Q_A}{R_A} = k\frac{Q_B}{R_B} = k\frac{2Q}{R_A+R_B}$ = common potential V

$$\Rightarrow \frac{Q_A}{Q_B} = \frac{R_A}{R_B} > 1.$$

$$\therefore \; Q_A > Q_B.$$

$$\frac{\sigma_A R_A^2}{\sigma_B R_B^2} = \frac{R_A}{R_B} \Rightarrow \frac{\sigma_A}{\sigma_B} = \frac{R_B}{R_A}.$$

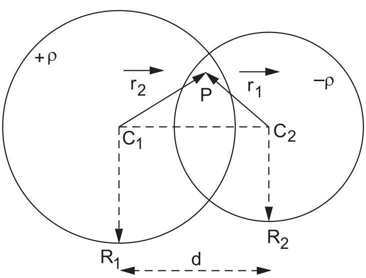
The field in the cavity of the metallic shell is zero.

On the surface, $\dfrac{E_A}{E_B} = \dfrac{Q_A}{Q_B}\dfrac{R_B^2}{R_A^2} = \dfrac{R_A}{R_B}\cdot\dfrac{R_B^2}{R_A^2} = \dfrac{R_B}{R_A} < 1$

$$\Rightarrow \qquad E_A < E_B.$$

Hence, the correct options are (a), (b), (c) and (d).

3. In $\triangle PC_1C_2$, $\vec{r}_2 = \vec{d} + \vec{r}_1$.



The electrostatic field in the overlapped region at any point P is

$$\vec{E} = \frac{k\frac{4}{3}\pi r_2^3 \rho}{r_2^2}\hat{r}_2 + \frac{k\frac{4}{3}\pi r_1^3(-\rho)}{r_1^2}\hat{r}_1$$

$$= k\cdot\frac{4}{3}\pi\rho(r_2\hat{r}_2 - r_1\hat{r}_1)$$

$$= k\frac{4}{3}\pi\rho(\vec{r}_2 - \vec{r}_1)$$

$$= k\frac{4}{3}\pi\rho\vec{d} = \frac{1}{4\pi\varepsilon_0}\frac{4\pi}{3}\rho\vec{d}$$

$$= \frac{\rho\vec{d}}{3\varepsilon_0} = \text{constant and parallel.}$$

Hence, the correct options are (c) and (d).

4. The position of all the charges are symmetric about the planes $x = \pm\dfrac{a}{2}$, so the net electric flux through them will be the same.

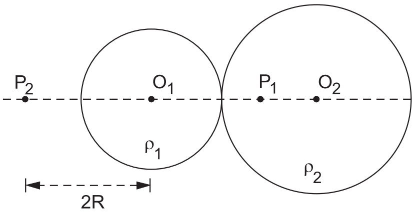
The same is true for the planes $y = \pm\dfrac{a}{2}$.

Hence $\phi = \dfrac{1}{\varepsilon_0}\Sigma q_{\text{inside}} = \dfrac{3q - q - q}{\varepsilon_0} = \dfrac{q}{\varepsilon_0}$.

By symmetry, flux through $z = +\dfrac{a}{2}$ is equal to flux through $x = +\dfrac{a}{2}$.

Hence, the correct options are (a), (c) and (d).

5. At point P_1,



$$\frac{1}{4\pi\varepsilon_0}\frac{\rho_1\left(\frac{4}{3}\pi R^3\right)}{4R^2} = \frac{\rho_2 R}{3\varepsilon_0}$$

$$\Rightarrow \quad \frac{\rho_1}{\rho_2} = 4.$$

At point P_2,

$$\frac{\rho_1\left(\frac{4}{3}\pi R^3\right)}{(2R)^2} + \frac{\rho_2\left(\frac{4}{3}\pi\times 8R^3\right)}{(5R)^2} = 0$$

$$\Rightarrow \quad \frac{\rho_1}{\rho_2} = -\frac{32}{25}.$$

Hence, the correct options are (b) and (d).

6. Field at O is $6K$, along OD.

Line PR is the perpendicular bisector of all the dipoles, so $V_0 = 0$.

Hence, the correct options are (a), (b) and (c).

7. The displacement of $+q$ towards $+x$ direction produces a net restoring force towards $-x$ direction and vice versa.

Restoring force,

$$F = q\left[\frac{2K\lambda}{d-x} - \frac{2K\lambda}{d+x}\right] \approx \frac{4K\lambda q}{d^2}x$$

$\Rightarrow$ motion is SHM with

$$T = 2\pi\sqrt{\frac{md^2}{4K\lambda q}}.$$

In the second case, $-q$ will move along the direction of displacement.

Hence, the correct option is (c).

8. $$\frac{Q}{4\pi\varepsilon_0 r_0^2} = \frac{\lambda}{2\pi\varepsilon_0 r_0} = \frac{\sigma}{2\varepsilon_0}$$

$$\Rightarrow \quad Q = 2\pi\sigma r_0^2.$$

$$r_0 = \frac{\lambda}{\pi\sigma}.$$

$$E_1\left(\frac{r_0}{2}\right) = \frac{4E_1(r_0)}{1}.$$

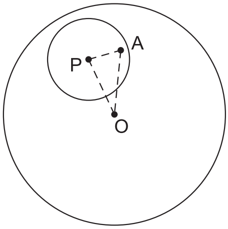
$$E_1\left(\frac{r_0}{2}\right) = 2E_2\left(\frac{r_0}{2}\right).$$

$$E_3\left(\frac{r_0}{2}\right) = E_3(r_0) = E_2(r_0).$$

Hence, the correct option is (c).

9. The electric field at A ($\overrightarrow{OA} = \vec{r}$) due to the solid sphere,

$$\vec{E}_1 = \frac{1}{4\pi\varepsilon_0}\left(\frac{4}{3}\pi r^3 \rho\right)\frac{\hat{r}}{r^2} = \frac{\rho\,\vec{r}}{3\varepsilon_0}.$$



Similarly, the field at A due to cavity,

$$\vec{E}_2 = \frac{\rho}{3\varepsilon_0}(-\overrightarrow{PA})$$

(–ve sign due to absence of charge)

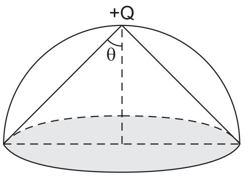
$$= \frac{\rho}{3\varepsilon_0}\overrightarrow{AP}.$$

⇒ net field at A

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{\rho}{3\varepsilon_0}(\overrightarrow{OA} + \overrightarrow{AP}) = \frac{\rho}{3\varepsilon_0}\overrightarrow{OP} = \frac{\rho}{3\varepsilon_0}\vec{a}.$$

Hence, the correct option is (d).

10. The electric flux linked with the hemisphere will be contained within the solid angle subtended by the flat surface at the position of the charge,



which is $\Omega = 2\pi(1-\cos 45°) = 2\pi\left(1-\frac{1}{\sqrt{2}}\right)$.

⇒ flux through the curved surface,

$$\psi = -\frac{Q}{\varepsilon_0}\frac{2\pi}{4\pi}\left(1-\frac{1}{\sqrt{2}}\right) = -\frac{Q}{2\varepsilon_0}\left(1-\frac{1}{\sqrt{2}}\right).$$

All points at the circumference are at the same distance ($\sqrt{2}R$) from the charge, so it is equipotential.

Hence, the correct options are (a) and (d).

11. After closing the switch S_1, C_1 is charged by $2CV_0$. When S_2 is closed, this charge is equally shared between C_1 and C_2 and both of their upper plates have charge CV_0.

When S_3 is closed, the upper plate of C_2 is charged by $-CV_0$ and the lower plate, by $+CV_0$.

Hence, the correct options are (b) and (d).

12. Since both the parts have a common potential difference (V) and the same separation d, the ratio $\frac{E_1}{E_2} = 1$ $\qquad \left[\because\ E = \frac{V}{d}\right]$.

Let C_0 = capacitance of the whole air-capacitor (without dielectric):

$$K\frac{C_0}{3} + 2\frac{C_0}{3} = C \quad \text{or} \quad C = \left(\frac{K+2}{3}\right)C_0.$$

For the upper portion, $C_1 = K\frac{C_0}{3}$.

$\therefore \qquad \frac{C}{C_1} = \frac{2+K}{K}$.

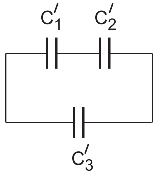
Hence, the correct options are (a) and (d).

13. The system comprises three capacitors whose capacitances are

$$C_1' = \frac{\varepsilon_1\varepsilon_0 A/2}{d/2} = 4\frac{\varepsilon_0 A}{d} = 4C_1,$$

$$C_2' = \frac{\varepsilon_2\varepsilon_0 A/2}{d/2} = \frac{2\varepsilon_0 A}{d} = 2C_1,$$

and $$C_3' = \frac{\varepsilon_1\varepsilon_0 A/2}{d} = \frac{2\varepsilon_0 A}{2d} = C_1.$$



Here C_1', C_2' are in series with C_3' in parallel with the combination.

$\therefore$ equivalent capacitance, $C_2 = \frac{C_1' \times C_2'}{C_1' + C_2'} + C_3' = \left(\frac{4}{3}+1\right)C_1 = \frac{7}{3}C_1$.

$\therefore$ ratio $\frac{C_2}{C_1} = \frac{7}{3}$.

Hence, the correct option is (d).

14. Electrostatic and gravitational static fields are both conservative and they never form closed loops.

Hence, the correct options are (a) and (c).

15. The nature of the graph showing the variation of the potential ϕ_r with distance r corresponds to that due to a uniformly charged spherical conducting shell of radius R_0.

Inside the shell ($r < R_0$), electric field $E = 0$, so the total electrostatic energy stored is zero.

Total charge q will be on the surface of the shell ($r = R_0$) and nowhere else.

Within the concentric spherical shell of radius $r = 2R_0$, total charge contained is q.

Electric field within the shell is zero and at the surface ($r = R_0$), it suddenly changes to $\frac{1}{4\pi\varepsilon_0}\frac{q}{R_0^2}$, so there is discontinuity at $r = R_0$.

Hence, the correct options are (a), (b), (c) and (d).

16. Coulomb force between Q and $-q$ is radial,

so the torque $\vec{\tau} = \vec{r} \times \vec{F} = rF\sin 180° = 0$.

$\therefore$ angular momentum $(\vec{r} \times \vec{p})$ is conserved.

The linear speed, linear momentum and angular velocity change with time.

Hence, the correct option is (a).

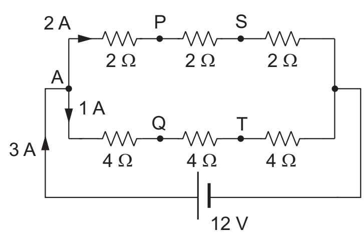
2.5 Current Electricity and Magnetism

1. At the time of breaking up of the filament, the temperature becomes higher and according to Wien's law $\lambda_{max} \propto \frac{1}{T}$, so $\nu_m \propto T$, the filament emits more light at higher band frequencies.

As the supply voltage remains constant, so consumed power $P = \frac{V^2}{R}$. Since the resistance R increases with the rise in temperature, so the filament consumes less power towards the end of the life of the bulb.

Hence, the correct options are (c) and (d).

2. By symmetry, P and Q as well as S and T are at the same potential, so resistance across them are ineffective because potential difference across them is zero. The simplified circuit will be as shown in the figure.



$$R_{eq} = \frac{6 \times 12}{18} = 4\ \Omega.$$

$$I_1 = \frac{12}{4} = 3\ \text{A}.$$

$$I_2 = \frac{12}{6+12} \times 3 = 2\ \text{A}.$$

$$V_A - V_S = (2\ \text{A})\ (4\ \Omega) = 8\ \text{V}.$$

$$V_A - V_T = (1\ \text{A})\ (8\ \Omega) = 8\ \text{V}.$$

$$V_P = V_Q \Rightarrow \text{current through PQ} = 0.$$

$$V_P = V_Q \Rightarrow V_Q > V_S.$$

$$I_2 = 3\ \text{A} - 1\ \text{A} = 2\ \text{A}.$$

Hence, the correct options are (a), (b), (c) and (d).

3. $$R_{iron} = \rho\frac{l}{A} = \frac{(1.0 \times 10^{-7}\ \Omega\ \text{m})\ (50 \times 10^{-3}\ \text{m})}{4 \times 10^{-6}\ \text{m}^2} = 1250\ \Omega.$$

$$R_{aluminium} = \rho\frac{l}{A} = \frac{(2.7 \times 10^{-8}\ \Omega\ \text{m})\ (50 \times 10^{-3}\ \text{m})}{(49-4) \times 10^{-6}\ \text{m}^2} = 30\ \Omega.$$

$$\Rightarrow \text{equivalent resistance} = \frac{R_1 \times R_2}{R_1 + R_2} = \frac{1250 \times 30}{1280} \times 10^{-6}\ \Omega = \frac{1875}{64}\ \mu\ \Omega.$$

Hence, the correct option is (b).

4. $R_1 = \rho\dfrac{L}{A} = \rho\dfrac{L}{\pi r^2}$; $R_2 = \rho\dfrac{L}{\pi 4r^2} = \dfrac{R_1}{4}$.

Initially, $H = \dfrac{V^2}{R_1}\cdot 4.$

When both the wires of resistance R_2 are in series,

$$H = \frac{V^2 t_1}{2R_2} = \frac{V^2 t_1}{2(R_1/4)} = \frac{2V^2 t_1}{R_1}.$$

$\therefore \qquad 2t_1 = 4 \text{ min}, \; t_1 = 2 \text{ min}.$

When both the wires of resistance R_2 are in parallel, the equivalent resistance

$$= \frac{R_2}{2} = \frac{R_1}{8}$$

$$\Rightarrow \qquad H = \frac{V^2}{R_1/8}t_2 = 8\frac{V^2}{R_1}t_2.$$

$\therefore \qquad 8t_2 = 4 \text{ min} \Rightarrow t_2 = 0.5 \text{ min}.$

Hence, the correct options are (b) and (d).

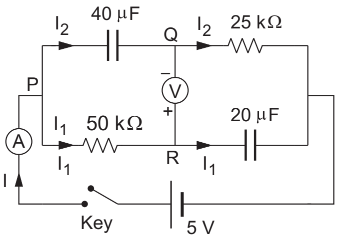
5. When the key is closed at $t = 0$, voltage across each capacitor is zero, so reading of the voltmeter is –5 V.

At $t = \infty$, the capacitors are fully charged, so the voltmeter reads +5 V.

During the transient state, $I_1 = \dfrac{5}{50}e^{-t/\tau}$ mA, $I_2 = \dfrac{5}{25}e^{-t/\tau}$ mA

and $I = I_1 + I_2$, where $\tau = RC = 1$ s.

After 1 s, current through the ammeter is $\dfrac{1}{e}$ times the initial value, and at $t = \infty$, it becomes zero.



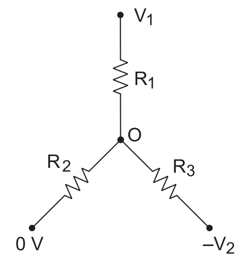
Reading of the voltmeter at any instant is

$$\begin{aligned} V_Q - V_R &= (V_P - V_R) - (V_P - V_Q) \\ &= \Delta V_{50\text{ k}\Omega} - \Delta V_{40\ \mu\text{F}} \\ &= 5e^{-t/\tau} - 5(1 - e^{-t/\tau}) = 5(2e^{-t/\tau} - 1). \end{aligned}$$

But $t = \ln 2$ s and $\tau = 1$ s, so $e^{-t/\tau} = e^{-\ln 2} = \dfrac{1}{2}$.

$\therefore$ voltmeter reads 0 V at $t = \ln 2$ s.

Hence, the correct options are (a), (b), (c) and (d).



6. Let V_0 = potential at O.

According to junction rule:

$$\frac{V_1 - V_0}{R_1} = \frac{V_0 - 0}{R_2} + \frac{V_0 + V_2}{R_3}$$

$$\Rightarrow \qquad V_0 = \frac{\dfrac{V_1}{R_1} - \dfrac{V_2}{R_3}}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}}.$$

Current through R_2 will be zero, if

$$V_0 = 0 \quad \text{so} \quad \frac{V_1}{V_2} = \frac{R_1}{R_3}.$$

This condition is satisfied by the options given (a), (b) and (d).

7. Consider the situation shown in Figure (i), in which all the elements are connected in series.

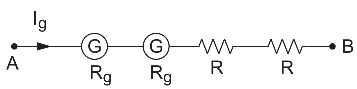


Fig. (i)

If I_g be the current through galvanometer for full-scale deflection (max range), the potential difference across AB is

$$V_1 = I_g(2R_g + 2R) = 2I_g(R + R_g).$$

Again, consider the situation shown in Figure (ii).

I_g flows through each galvanometer for full-scale deflection.

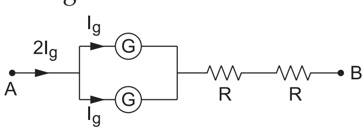


Fig. (ii)

$\therefore$ voltage range, $V_2 = V_A - V_B = I_gR_g + 2I_g \cdot 2R = I_g(R_g + 4R)$.

If $2R_g < R$ then $V_2 > V_1$.

For maximum current range, all the components are to be connected in parallel.

Hence, the correct options are (b) and (c).

8. The magnetic field is nonzero in the region $0 < r < R$ as well as in the region $r > 2R$.

Hence, the correct options are (a) and (d).

9. Effective length of the wire, $\vec{l} = 2(L + R)\hat{i}$ and magnetic force, $\vec{F} = I(\vec{l} \times \vec{B}) = 2I(L + R)\hat{i} \times \vec{B}$.

If $\vec{B} = B\hat{k}$, $\vec{F} = 2I(L+R)B\,\hat{i} \times \hat{k}$

$= 2I(L+R)B(-\hat{j})$

$\Rightarrow F \propto (L+R)$.

If $\vec{B} = B\hat{i}$, $\vec{F} = 2I(L+R)B\,(\hat{i} \times \hat{i}) = 0$.

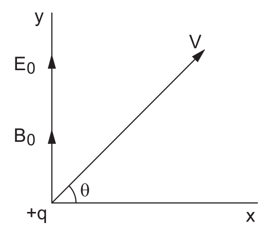
If $\vec{B} = B\hat{j}$, $\vec{F} = 2I(L+R)B\,(\hat{i} \times \hat{j}) = 2B(L \times R)\hat{k}$

$\Rightarrow F \propto (L+R)$.

If $\vec{B} = B\hat{k}$, $F \neq 0$, so option (d) is incorrect.

Hence, the correct options are (a), (b) and (c).

10. For $\theta = 0$, path of the charge is circular due to $\vec{B}$ field. In addition, it experiences force due to the electric field ($qE_0\hat{j}$); it has an acceleration along the y-axis. So the resulting path will be helical with a variable pitch.



With similar arguments, for $\theta = 10°$, the path will be helical with the pitch increasing with time.

For $\theta = 90°$, $\vec{F}_B = q(\vec{v} \times \vec{B})$ is zero but $F_{el} = qE_0$: the motion is linear, accelerated along the y-axis.

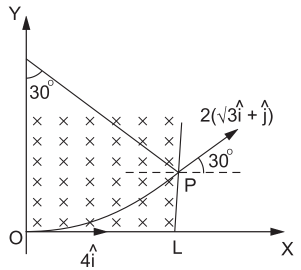
Hence, the correct options are (c) and (d).

11. $qvB = \dfrac{mv^2}{r}$, $r = \dfrac{mv}{qB}$ · So $r \propto m$.

The particle will describe semicircular paths (parallel) with radii $\propto$ mass. Time $T = \dfrac{2\pi m}{qB}$, so $T \propto m$.

Hence, the correct options are (b) and (d).

12. The magnetic field exists along the negative z-direction as shown.



The radius of the circular path $r = \dfrac{Mv}{QB}$.

The velocity at the point of emergence P is $\vec{v} = 2(\sqrt{3}\,\hat{i} + \hat{j})$ m s^{-1}.

So it makes an angle θ with the x-axis, when $\theta = \tan^{-1}\dfrac{1}{\sqrt{3}} = 30° = \dfrac{\pi}{6}$.

$$\therefore \text{ path OP} = \frac{\pi}{6}\cdot r = \frac{\pi}{6}\frac{Mv}{QB} = \frac{\pi}{6}\frac{M\,4}{QB} = \frac{2}{3}\frac{\pi M}{QB}.$$

$$\therefore \text{ time } t = 10 \text{ m s} = 10\times10^{-3}\text{ s} = \frac{\text{Path OP}}{v} = \frac{2}{3}\frac{\pi M}{QB\times 4}$$

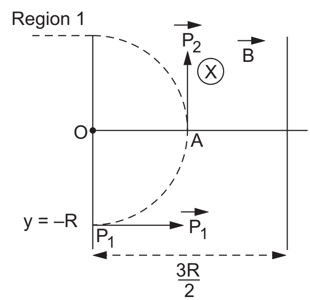
$$\Rightarrow B = \frac{50\,\pi M}{3Q}.$$

Hence, the correct options are (a) and (c).

13. Since $r = \dfrac{mv}{QB}$, the distance of re-entry into Region 1 from P_1 is $2r$

$$\Rightarrow d = \frac{2\,mv}{QB}$$

$$\Rightarrow d \propto m.$$



Given, $\quad B = \dfrac{8}{13}\dfrac{p}{QR}$,

so the radius, $r = \dfrac{p}{QB} = \dfrac{13}{8}R = 1.6R$,

which is greater than $\dfrac{3R}{2}$.

Hence the particle will enter Region 3.

For $B > \dfrac{2}{3}\dfrac{p}{QR}$, $r' < \dfrac{3}{2}R$, so the particle will re-enter Region 1.

Linear momentum has a constant magnitude in a magnetic field.

$\therefore$ at P_1, $\vec{p_i} = p\hat{i}$ and at the farthest point from the y-axis (at A), $\vec{p_f} = p\hat{j}$.

$\therefore$ the magnitude of the change in momentum,

$$|\Delta\vec{p}\,| = |\,\vec{p_f} - \vec{p_i}\,| = p|\,\hat{j}-\hat{i}\,| = \sqrt{2}\,p.$$

Hence, the correct options are (a) and (b).

14. Induced emf $0 \to L$ (region) and $3L \to 4L$ (region).

Motional emf $\mathcal{E} = -BLv$, $i = \dfrac{\mathcal{E}}{R} = \dfrac{BLv}{R}$.

Force on a current-carrying conductor in a $\vec{B}$ field,

$$F = iLB.$$

$$\text{Now, } F = -m\frac{dv}{dt} = -m\frac{dv}{dx}\cdot\frac{dx}{dt} = -mv\frac{dv}{dx} = iLB = \frac{B^2L^2v}{R}$$

$$\Rightarrow \quad dv = -\frac{B^2L^2}{mR}dx.$$

Integrating, $\int_{v_0}^{v} dv = -\frac{B^2L^2}{mR}\int_0^x dx.$

$\therefore$ velocity as a function of x,

$$v(x) - v_0 = -\frac{B^2L^2}{mR}\cdot x \Rightarrow v(x) = v_0 - \frac{B^2L^2}{mR}\cdot x.$$

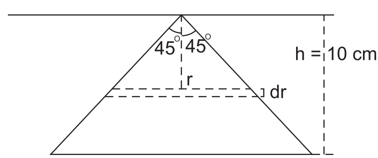
Now, current at any position (x),

$$i(x) = \frac{BL}{R}v = \frac{BL}{R}\left[v_0 - \frac{B^2L^2}{mR}x\right],$$

and the force $F(x) = LB\cdot i = \frac{L^2B^2}{R}\left[v_0 - \frac{B^2L^2}{mR}x\right].$

Considering Lenz's law with these results, the graphs represented in options (c) and (d) are correct.

15. The magnetic flux linked with the loop due to current (assumed) through the wire,



$$\phi_{l,w} = \int_0^h B\,dA$$

$$= \int_0^h \frac{\mu_0 I}{2\pi r}\cdot 2r\,dr = \frac{\mu_0 Ih}{\pi}$$

$\Rightarrow$ mutual inductance $M = \frac{\phi_{l,w}}{I} = \frac{\mu_0 h}{\pi}.$

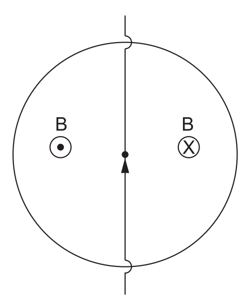
The magnitude of induced emf in the wire

$$\mathcal{E}_w = \left|-M\frac{dI}{dt}\right| = \frac{\mu_0 h}{\pi}\cdot\frac{dI}{dt} = \frac{\mu_0}{\pi}(0.1\text{ m})(10\text{ A s}^{-1}) = \left(\frac{\mu_0}{\pi}\right)\text{V}.$$

According to Lenz's law, the loop should move away, so there will be a repulsive force between the wire and the loop.

Hence, the correct options are (b) and (c).

16. The flux of the magnetic field through the wire loop is as much positive (outward) as negative (inward).



Hence, ϕ = zero and

$$\frac{d\phi}{dt} = \text{zero}.$$

Hence, the correct options are (a) and (c).

17. After a very long time, magnetic flux linked with each coil is the same, hence

$$L_1 i_1 = L_2 i_2 \qquad \left(\because -L_1 \frac{dI_1}{dt} = -L_2 \frac{dI_2}{dt}\right)$$

$$\Rightarrow i_1 = \frac{L_2}{L_1 + L_2} i = \frac{L_2}{L_1 + L_2} \frac{V}{R} \text{ and}$$

$$i_2 = \frac{L_1}{L_1 + L_2} i = \frac{L_1}{L_1 + L_2} \frac{V}{R}.$$

At $t = 0$, the inductors offer a large obstruction to the current through R. So, $i = 0$.

Now, $\frac{i_1}{i_2} = \frac{L_2}{L_1} =$ constant.

Hence, the correct options are (b), (c) and (d).

18. The net magnetic flux through the loop at time t is

$$\phi = B(2A - A) \cos \omega t = BA \cos \omega t. \qquad \text{...(i)}$$

$$\Rightarrow \left|\frac{d\phi}{dt}\right| = BA\omega \sin \omega t, \text{ which is maximum when } \omega t = \frac{\pi}{2}.$$

In the smaller loop,

$$\mathcal{E} = -\frac{d\phi}{dt} = AB\omega \sin \omega t,$$

and the net emf in the system due to both the loops (see Equation i),

$$\mathcal{E}' = -\frac{d\phi}{dt} = AB\omega \sin \omega t$$

$\Rightarrow$ amplitudes ($AB\omega$) in both the cases is the same.

Hence, the correct options are (a) and (d).

19. The amplitude of the voltage across the terminals X and Y,

$$V_{XY} = \sqrt{V_0^2 + V_0^2 - 2V_0^2 \cos \frac{2\pi}{3}} = \sqrt{3}\, V_0,$$

Similarly, $V_{YZ} = V_{XZ} = \sqrt{3}\, V_0$.

The rms value $= \dfrac{\text{peak value}}{\sqrt{2}}$

$$\Rightarrow \qquad V_{XY}^{rms} = \sqrt{\frac{3}{2}}\, V_0$$

and $$V_{YZ}^{rms} = \sqrt{\frac{3}{2}}\, V_0.$$

Voltmeter reading (V^{rms}) is the same for any two terminals.
Hence, the correct options are (a) and (d).

20. In a capacitor, current leads the voltage by $\frac{\pi}{2}$, hence initially

$$V = V_0 \sin\omega t.$$

$\therefore$ charge $Q = CV = CV_0 \sin\omega t = Q_0 \sin\omega t,$

where $Q_0 = CV_0 = CI_0X_C = \frac{I_0}{\omega} = 2 \times 10^{-3}$ C.

At $t = \frac{7\pi}{6\omega}$; $\cos\omega t = -\frac{\sqrt{3}}{2}$, hence

$$I = -\frac{\sqrt{3}}{2}I_0, \text{ which is anticlockwise.}$$

Immediately after $t = \frac{7\pi}{6\omega}$, the current

$$i = \frac{V_c + 50}{R} = 10 \text{ A.}$$

Charge flow $= Q_{final} - Q_{\left(at\ \frac{7\pi}{6\omega}\right)} = 2 \times 10^{-3}$ C.

Hence, the correct options are (c) and (d).

21. $X_C = \frac{1}{\omega C} = \frac{1}{(100)(100 \times 10^{-6})} = 100\ \Omega.$

$X_L = \omega L = (100)(0.5) = 50\ \Omega.$

For the upper branch (1), $z_1 = \sqrt{R^2 + X_C^2} = 100\sqrt{2}\ \Omega.$

For the lower branch (2), $z_2 = \sqrt{R^2 + X_L^2} = 50\sqrt{2}\ \Omega.$

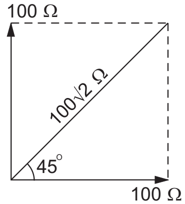
Source voltage $V = V_0 \sin\omega t = 20\sqrt{2} \sin\omega t.$

$$\therefore \quad I_1 = \frac{20\sqrt{2}}{100\sqrt{2}} \sin\left(\omega t + \frac{\pi}{4}\right),$$

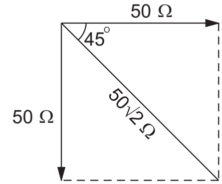
and $I_2 = \frac{20\sqrt{2}}{50\sqrt{2}} \sin\left(\omega t - \frac{\pi}{4}\right)$.

Taking the rms values,

$$\Rightarrow \quad I = \sqrt{I_1^2 + I_2^2} = \sqrt{\frac{1}{50} + \frac{4}{50}} = \sqrt{\frac{1}{10}} \approx 0.3 \text{ A.}$$



Capacitive circuit


Inductive circuit

Voltage across the resistor,

$$V_{R_{100}} = (I_1)_{rms} \times 100 = \frac{0.2}{\sqrt{2}} \times 100 = 10\sqrt{2} \text{ V}.$$

Similarly, $V_{R_{50}} = \dfrac{0.4}{\sqrt{2}} \times 50 = 10\sqrt{2}$ V.

Hence, the correct options are (a) and (c).

22. Peak current $I_0 = \dfrac{V_0}{z} = \dfrac{V_0}{\sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}}$.

At resonance,

(i) the phase difference between current and voltage is zero, and

(ii) the circuit is resistive.

$\therefore \quad X_L = X_C \Rightarrow \omega_r = \sqrt{\dfrac{1}{LC}}$

$\Rightarrow \quad \omega_r = 10^6$ rad s^{-1}.

The resonant frequency $\dfrac{1}{\sqrt{LC}}$ does not depend on R.

At ω ($>> \omega_r = 10^6$), $X_L >> X_C$, the circuit acts as inductive.

If $\omega \to 0$, $X_C = \dfrac{1}{\omega C} \to \infty$, $z \to \infty$, so $I \to 0$.

Hence, the correct options are (a) and (b).

23. $I = \dfrac{V}{Z}$, $V^2 = V_R^2 + V_C^2 = (IR)^2 + \left(\dfrac{I}{\omega C}\right)^2$ and $Z^2 = R^2 + \left(\dfrac{1}{\omega C}\right)^2$.

$\therefore$ current $I = \dfrac{V}{\sqrt{R^2 + \dfrac{1}{\omega^2 C^2}}}$ and $V_C = I\left(\dfrac{1}{\omega C}\right)$.

In case (A), C_A is less and in case (B), C_B has larger value.

$\therefore (X_C)_A > (X_C)_B \Rightarrow Z_A > Z_B \Rightarrow I_R^A < I_R^B$.

Similarly, $V_C = \frac{V}{Z} X_C = \frac{V}{\sqrt{1+R^2\omega^2C^2}}$

$\Rightarrow \; V_C^A > V_C^B$.

Hence, the correct options are (b) and (c).

24. The forces on BC and DA are equal and opposite in direction. Hence F_{net} on the loop is zero.

Torque $\vec{\tau} = \vec{m} \times \vec{B} = I\vec{A} \times \vec{B}$.

$\because \; \vec{A}$ is $\perp \vec{B}, |\vec{\tau}| = IAB \neq 0$.

As seen from O, the loop rotates clockwise.

Hence, the correct options are (a) and (c).

25. For a charge q in a magnetic field B,

$$F = qvB, \frac{mv^2}{r} = qvB, \; r = \frac{mv}{qB}.$$

For the particle to enter Region III, $r > l \Rightarrow \frac{mv}{qB} > l \;\; \Rightarrow v > \frac{qlB}{m}$.

For the path length to be maximum, $l = r$

$$\Rightarrow \; v = \frac{qBl}{m}.$$

For $r < l$, the particle describes a semicircle for which time $t = \frac{T}{2} = \frac{\pi m}{qB}$, which is independent of velocity.

Hence, the correct options are (a), (c) and (d).

26. Equivalent resistance of the circuit,

$$R \;=\; 2\text{ k}\Omega + \frac{(6\text{ k}\Omega)\,(1.5\text{ k}\Omega)}{(7.5\text{ k}\Omega)} = \frac{16}{5}\text{ k}\Omega.$$

$\therefore$ current through the battery,

$$I \;=\; \frac{V}{R} = \frac{24\text{ V}}{\frac{16}{5}\text{ k}\Omega} = 7.5\text{ mA}.$$

The potential difference across R_1 is IR_1 = (7.5 mA) (2 kΩ) = 15 V.

$\therefore$ the potential difference across the parallel combination of 6 kΩ and 1.5 kΩ will be 24 V – 15 V = 9 V.

The ratio of power dissipated in R_1 and R_2 is

$$\frac{P_1}{P_2} \;=\; \frac{I^2R_1}{V_2^2/R_2} = \frac{(7.5\text{ mA})^2 \times (2\text{ k}\Omega)}{(15\text{ V})^2/6\text{ k}\Omega} = 3:1.$$

Interchanging R_1 and R_2, equivalent resistance

$$R' = 6\ \text{k}\Omega + \frac{(2\ \text{k}\Omega)(1.5\ \text{k}\Omega)}{3.5\ \text{k}\Omega} = \frac{48}{7}\ \Omega.$$

$\therefore$ current $I' = \dfrac{24}{48/7}\ \text{A} = \dfrac{7}{2}\ \text{A}.$

The PD across R_L, $V' = 24\ \text{V} - \left(\frac{7}{2} \times 6\right)\text{V} = 3\ \text{V}.$

The power dissipated in R_L is $\dfrac{V'^2}{R_L} = \dfrac{(3\ \text{V})^2}{R_L}.$

Previous power dissipation $= \dfrac{V^2}{R_L} = \dfrac{(9\ \text{V})^2}{R_L}.$

$\therefore$ the power dissipation is decreased by a factor of 9.

Hence, the correct options are (a), (c) and (d).

27. As the induced emf $-\dfrac{d\phi}{dt}$ is same in both the rings, the strength of the induced current will depend upon the resistance of the ring.

Larger the resistivity, smaller is the current. Thus for $h_A > h_B$, the correct options are (b) and (d).

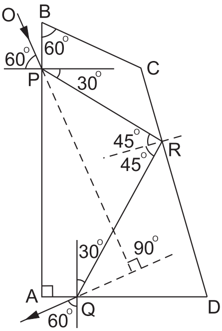
2.6 Ray Optics and Wave Optics

1. At P, $\sin 60° = \sqrt{3}\sin\gamma$. $\therefore\ \gamma = 30°$.

$\sin C = \dfrac{1}{\sqrt{3}},\quad C < 45°.$

Internal reflection occurs at R. Angle between the initial and final rays = angle of deviation = 90°.

Hence, the correct options are (a), (b) and (c).



2. For air to glass,

$$\frac{1.5}{f_1} = \frac{1.4-1}{R} + \frac{1.5-1.4}{R} \Rightarrow f_1 = 3R.$$

For glass to air,

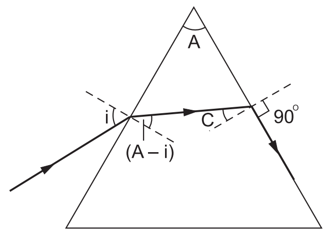
$$\frac{1}{f_2} = \frac{1.4-1.5}{-R} + \frac{1-1.4}{-R} \Rightarrow f_2 = 2R.$$

Hence, the correct options are (a) and (c).

3. $$\mu = \frac{\sin\frac{A+\delta_m}{2}}{\sin\frac{A}{2}} = \frac{\sin A}{\sin\frac{A}{2}} = 2\cos\frac{A}{2}. \qquad ...(i)$$

At minimum deviation, $i + i = A + \delta_m = 2A$

$\Rightarrow \quad i_1 = A, \; r_1 = \frac{A}{2} = \frac{i_1}{2}.$



From equation (i), $A = 2\cos^{-1}\left(\frac{\mu}{2}\right)$.

$\sin C = \frac{1}{\mu}, \; \cos C = \sqrt{1-\frac{1}{\mu^2}}.$

$$\begin{aligned}
\text{Now,}\quad \sin i &= \mu \sin(A - C) \\
&= \mu(\sin A\cos C - \cos A \sin C) \\
&= \mu\left(\sin A\sqrt{1-\frac{1}{\mu^2}} - \frac{\cos A}{\mu}\right) \\
&= \left(\sin A\sqrt{\mu^2-1} - \cos A\right)
\end{aligned}$$

$$\Rightarrow \quad \sin i = \left(\sin A\sqrt{4\cos^2\frac{A}{2}-1} - \cos A\right).$$

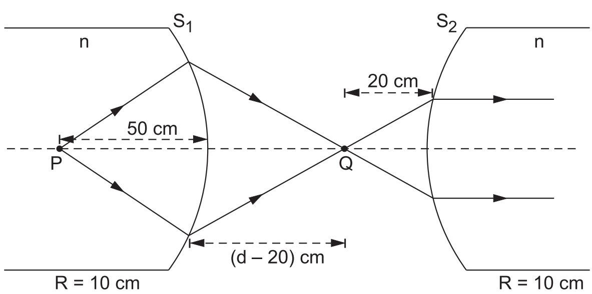
At $i_1 = A$, the ray undergoes minimum deviation through the prism and passes symmetrically so that the ray inside the prism is parallel to its base.

Hence, the correct options are (a), (c) and (d).

4. For refraction through S_1:

$$\frac{1}{v} - \frac{1.5}{-50} = \frac{1-1.5}{-10}$$

$$\Rightarrow \quad v = 50 \text{ cm (at Q)}.$$



For refraction through S_2:

$$u = -(d - 50) \text{ cm},$$
$$v = \infty \ (\because \text{ rays are parallel}).$$

$$\therefore \ \frac{1.5}{\infty} - \frac{1}{-(d-50)} = \frac{1.5-1}{+10} \Rightarrow d = 70 \text{ cm}.$$

Hence, the correct option is (b).

5. From lens formula, $\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{60} - \frac{1}{(-30)} = \frac{1}{20}$

$\Rightarrow f = 20 \text{ cm} = \frac{R}{n-1}.$

For reflection from the convex surface (using mirror formula),

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v} = \frac{1}{(-30)} + \frac{1}{10} = \frac{1}{15}.$$

$$\therefore \quad R = 2f = 30 \text{ cm}.$$

But $\quad n - 1 = \frac{R}{20} = 1.5. \ \therefore \ n = 2.5.$

The faint image is erect and virtual.

Hence, the correct options are (a) and (d).

6. From Snell's law,

$n_1 \sin\theta_i = n_2 \sin\theta_2 = n_2 \sin\theta_f$.

The deviation of ray in the slab will depend on n(z). Hence, l will depend on n(z) and is independent of n_2.

Hence, the correct options are (a), (c) and (d).

7. The path difference at any angular position P(θ) is

$$\Delta = d\cos\theta.$$

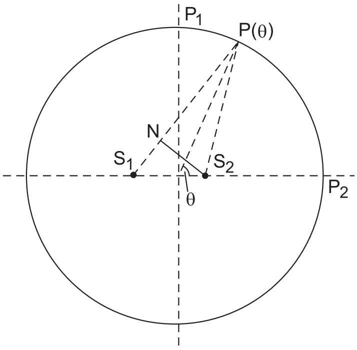
Path difference at P_1:

$$\Delta = d\cos 90° = 0 \text{ (max)}.$$

Path difference at P_2:

$$\Delta = d\cos 0° = d = 1.8 \text{ mm}$$

$$\Rightarrow \quad \frac{\Delta}{\lambda} = \frac{1.8 \text{ mm}}{600 \text{ nm}} = 3000.$$



$\therefore$ 3000th-order bright fringe will form at P_2, and the total number of fringes between P_1 and P_2 is close to 3000.

Angular separation between two consecutive fringes

$$= \left|\frac{d\theta}{dn}\right| = \frac{\lambda}{d\sin\theta}.$$

As we move from P_1 to P_2, the angular width increases.

Hence, the correct options are (a) and (c).

8. Since the line joining S_1 and S_2 is perpendicular to the screen, the fringe pattern will consist of semicircles.

At O, phase difference

$$= \frac{2\pi}{\lambda}(S_1O - S_2O) = 2\pi\frac{(0.6003\times10^{-3})\ \text{m}}{(600\times10^{-9})\ \text{m}} = 2001\,\pi.$$

Thus the region very close to O is dark.

Hence, the correct options are (b) and (d).

9. $d = \dfrac{\lambda}{2\sin\theta}$, $\ln d = \ln\left(\dfrac{\lambda}{2}\right) - \ln(\sin\theta)$.

Differentiating, $\dfrac{\Delta d}{d} = 0 - \dfrac{\cos\theta\,\Delta\theta}{\sin\theta}$.

$$\therefore\ \left(\frac{\Delta d}{d}\right)_{\max} = \cot\theta\cdot\Delta\theta \quad \text{or} \quad (\Delta d)_{\max} = d\cot\theta\cdot\Delta\theta$$

$$= \frac{\lambda}{2\sin\theta}\cdot\cot\theta\,\Delta\theta = \frac{\lambda}{2}\frac{\cos\theta}{\sin^2\theta}\Delta\theta.$$

As θ increases, $\cot\theta$ decreases and $\dfrac{\cos\theta}{\sin^2\theta}$ also decreases.

Hence fractional error in d decreases.

Hence, the correct option is (d).

10. Fringe width $\beta = \dfrac{D\lambda}{d}$. $\because \lambda_2 > \lambda_1 \Rightarrow \beta_2 > \beta_1$.

Number of fringes in a given distance $= m = \dfrac{y}{\beta} \Rightarrow m_2 < m_1$.

$$\text{3rd maximum of } \lambda_2 = 3\beta_2 = \frac{3D\lambda_2}{d} = \frac{1800D}{d},$$

and 5th minimum of $\lambda_1 = \dfrac{9\lambda_1 D}{2d} = \dfrac{1800D}{d}$.

Angular separation $\theta = \dfrac{\beta}{D} = \dfrac{\lambda}{d}$.

Hence, the correct options are (a), (b) and (c).

11. For at least one maxima, $\sin\theta = \dfrac{\lambda}{d}$.

If $d = \lambda$, $\sin\theta = 1$, $\theta = \frac{\pi}{2}$. So $y \to \infty$.

If $\lambda < d < 2\lambda$, $\sin\theta$ exists and y is finite.

$$I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}.$$
$$I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2}.$$

Initially, $I_{max} = 9I$ and $I_{min} = I$.

If $I_1 = I_2$ then $I'_{max} = 4I$, $I'_{min} = 0$.

Thus intensities of both the dark and bright fringes will decrease.

Hence, the correct options are (a) and (b).

12. Using mirror formula $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$, we get:

For (a): (42, 56); f = –24 cm

For (b): (48, 48); f = –24 cm

For (c): (66, 33); f = –22 cm

For (d): (78, 39); f = –26 cm

Thus options (c) and (d) are incorrectly recorded.

2.7 Modern Physics

1. KE_{max} of an electron just after emission,

$$KE_i = \frac{hc}{\lambda_{Ph}} - \phi,$$

and on reaching the anode,

$$KE_f = \left(\frac{hc}{\lambda_{Ph}} - \phi\right) + eV.$$

For $V >> \frac{\phi}{e}$, $KE_f \approx eV$, and $\lambda_e = \frac{h}{\sqrt{2meV}} \Rightarrow \lambda_e \propto \frac{1}{\sqrt{V}}$.

If ϕ and λ_{Ph} increase, KE_f decreases and λ_e increases. So option (c) is incorrect.

$\because$ $$\lambda_e = \frac{h}{\sqrt{2m\,(KE_f)}},$$

$\therefore$ $$\frac{d\lambda_e}{dt} \neq \frac{d\lambda_{Ph}}{dt}$$ ... option (b) is incorrect.

Hence, the correct option is (a).

2. $\frac{hc}{\lambda} = eV_0 + \phi$ or $V_0 = \frac{hc}{e}\left(\frac{1}{\lambda}\right) - \frac{\phi}{e}$.

The plot of V_0 vs $\frac{1}{\lambda}$ is linear with negative intercept on the V_0 axis.

The plot of V_0 vs λ is nonlinear.

Hence, the correct options are (a) and (c).

3. Orbital radius, $r = r_0 \frac{n^2}{Z} \Rightarrow \frac{\Delta r}{r} = \frac{2\Delta n}{n}$.

For $\Delta n = 1, \frac{\Delta r}{r} \propto \frac{1}{n}$.

Angular momentum, $L = \frac{nh}{2\pi}, \frac{\Delta L}{L} = \frac{\Delta n}{n}$.

$\therefore \quad \frac{\Delta L}{L} \propto \frac{1}{n}$.

Total energy, $E_n = -\frac{13.6\, Z^2}{n^2}$.

$\therefore \quad \frac{\Delta E_n}{E_n} = -\frac{2\Delta n}{n} \Rightarrow \frac{\Delta E}{E} \propto \frac{1}{n}$. ... option (c) incorrect.

Hence, the correct options are (a), (b) and (d).

4. Given: $4.5a_0 = a_0 \frac{n^2}{Z}$ and $\frac{nh}{2\pi} = \frac{3h}{2\pi}$.

Solving, $n = 3$ and $Z = 2$.

Possible wavelengths correspond to the transitions $3 \rightarrow 2$, $2 \rightarrow 1$ and $3 \rightarrow 1$ and are given by

$$\frac{1}{\lambda_1} = RZ^2\left(\frac{1}{1^2} - \frac{1}{3^2}\right) \Rightarrow \lambda_1 = \frac{9}{32R}$$

$$\frac{1}{\lambda_2} = RZ^2\left(\frac{1}{1^2} - \frac{1}{2^2}\right) \Rightarrow \lambda_2 = \frac{1}{3R}$$

$$\frac{1}{\lambda_3} = RZ^2\left(\frac{1}{2^2} - \frac{1}{3^2}\right) \Rightarrow \lambda_3 = \frac{9}{5R}$$

Hence, the correct options are (a) and (c).

5. $^{236}_{92}\mathrm{U} \rightarrow {}^{140}_{54}\mathrm{Xe} + {}^{94}_{38}\mathrm{Sr} + {}^{1}_{0}\mathrm{X} + {}^{1}_{0}\mathrm{Y}$,

where X = Y = n (neutron).

Q-value of reaction,

$$\begin{aligned} Q &= 236 \times 7.5 - (140 + 94) \times 8.5 \\ &= 1170 - 1989 = 219 \text{ MeV}. \end{aligned}$$

In options (a) and (d), the energy and charge conservation laws are followed, so

$$\begin{aligned} Q &= K_{Xe} + K_{Sr} + K_X + K_Y \\ &= 129 + 86 + 4 = 219. \end{aligned}$$

In (d), $p_{Xe} > p_{Sr} + p_X + p_Y$,

so, the conservation of linear momentum will not hold and only option (a) will hold.

6. Energy is released during nuclear fusion and fission when the final binding energy per nucleon is greater than the initial binding energy so that the nucleus gets more stable. This will occur in fusion [option (b)] and fission [option (d)].

❑