APPLICATION OF TRIGONOMETRY

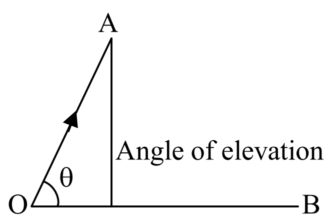
3
CHAPTER

CONTENTS

- Angle of Elevation
- Angle of Depression

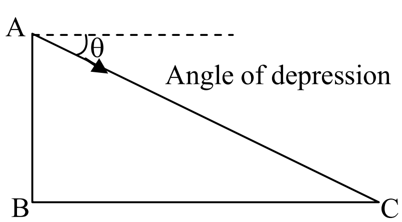
➢ ANGLE OF ELEVATION

The angle of elevation of the point viewed is the angle formed by the line of sight with the horizontal when the point being viewed is above the horizontal level, i.e. the case when we raise our head to look at the object. (see fig.)



➢ ANGLE OF DEPRESSION

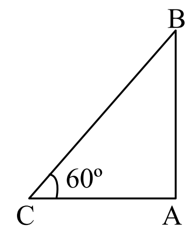
The angle of depression of a point on the object being viewed is the angle formed by the line of sight with the horizontal when the point is below the horizontal level, i.e. the case when we lower our head to look at the point being viewed. (See fig.)



❖ EXAMPLES ❖

Ex.1 The shadow of a building is 20 m long when the angle of elevation of the sun is 60º. Find the height of the building.

Sol. Let AB be the building and AC be its shadow.



Then, AC = 20 m and $\angle ACB = 60^\circ$.

Let AB = h.

Then, $\dfrac{AB}{AC} = \tan 60^\circ = \sqrt{3}$

$\Rightarrow \dfrac{h}{20} = \sqrt{3}$

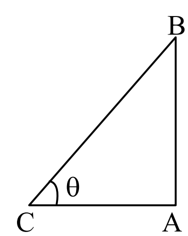
$\therefore$ h = $(20 \times \sqrt{3})$m = (20×1.732) m

= 34.64 m.

Ex.2 If a vertical pole 6m high has a shadow of length $2\sqrt{3}$ metres, find the angle of elevation of the sun.

Sol. Let AB be the vertical pole and AC be its shadow.

Let the angle of elevation be θ. Then,



AB = 6 m, AC = $2\sqrt{3}$ m

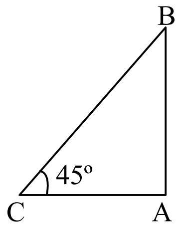
and $\angle ACB = \theta$.

Now, $\tan\theta = \dfrac{AB}{AC} = \dfrac{6}{2\sqrt{3}} = \sqrt{3} = \tan 60^\circ$.

$\therefore$ $\theta = 60^\circ$.

Ex.3 A ladder against a vertical wall makes an angle of 45º with the ground. The foot of the ladder is 3m from the wall. Find the length of the ladder.

Sol. Let AB be the wall and CB, the ladder.



Then, AC = 3m and $\angle ACB = 45^\circ$

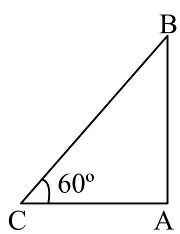
Now, $\frac{CB}{AC} = \sec 45^\circ = \sqrt{2} \Rightarrow \frac{CB}{3} = \sqrt{2}$

$\therefore$ Length of the ladder = CB = $3\sqrt{2}$

= (3 × 1.41) m = 4.23 m

Ex.4 A balloon is connected to a meteorological station by a cable of length 200 m, inclined at 60º to the horizontal. Find the height of the balloon from the ground. Assume that there is no slack in the cable.

Sol. Let B be the balloon and AB be the vertical height. Let C be the meteorological station and CB be the cable.



Then, BC = 200 m and $\angle ACB = 60^\circ$

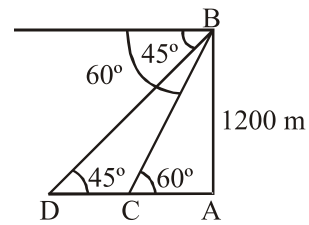
Then, $\frac{AB}{BC} = \sin 60^\circ = \frac{\sqrt{3}}{2}$

$\Rightarrow \frac{AB}{200} = \frac{\sqrt{3}}{2}$

$\Rightarrow AB = \left(\frac{200 \times \sqrt{3}}{2}\right)$ m = 173.2 m.

Ex.5 The pilot of a helicopter, at an altitude of 1200m finds that the two ships are sailing towards it in the same direction. The angle of depression of the ships as observed from the helicopter are 60º and 45º respectively. Find the distance between the two ships.

Sol. Let B the position of the helicopter and let C, D be the ships. Let AB be the vertical height.



Then, AB = 1200 m,

$\angle ACB = 60^\circ$ and $\angle ADB = 45^\circ$.

Then, $\frac{AD}{AB} = \cot 45^\circ = 1$

$\Rightarrow \frac{AD}{1200} = 1 \Rightarrow AD = 1200$ m

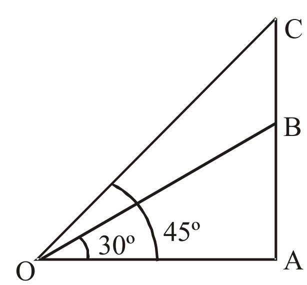
And, $\frac{AC}{AB} = \cot 60^\circ = \frac{1}{\sqrt{3}}$

$\Rightarrow \frac{AC}{1200} = \frac{1}{\sqrt{3}}$

$\Rightarrow AC = \frac{1200}{\sqrt{3}} = 400\sqrt{3}$ m.

Ex.6 A vertical tower stands on a horizontal plane and is surmounted by a flagstaff of height 7m. At a point on the plane, the angle of elevation of the bottom of the flagstaff is 30º and that of the top of the flagstaff is 45º. Find the height of the tower.

Sol. Let AB be the tower and BC be the flagstaff.



Then, BC = 7 m. Let AB = h.

Let O be the point of observation.

Then, $\angle AOB = 30^\circ$ and $\angle AOC = 45^\circ$.

Now, $\frac{OA}{AC} = \cot 45^\circ = 1$

$\Rightarrow$ OA = AC = h + 7.

And, $\frac{OA}{AB} = \cot 30^\circ = \sqrt{3}$

$\Rightarrow \frac{OA}{h} = \sqrt{3} \Rightarrow OA = h\sqrt{3}$

$\therefore \quad h + 7 = h\sqrt{3}$

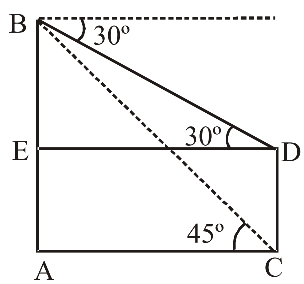
$\Rightarrow h = \frac{7}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{7(\sqrt{3}+1)}{2} = 9.562$ m

Ex.7 From the top of a building 30 m high, the top and bottom of a tower are observed to have angles of depression 30º and 45º respectively. The height of the tower is :

(a) $15(1+\sqrt{3})$ m (b) $30(\sqrt{3}-1)$ m

(c) $30\left(1+\frac{1}{\sqrt{3}}\right)$m (d) $30\left(1-\frac{1}{\sqrt{3}}\right)$m

Sol. Let AB be the building and CD be the tower.



Then, AB = 30 m. Let DC = x.

Draw DE ⊥ AB. Then AE = CD = x.

$\therefore \quad BE = (30 - x)$ m.

Now, $\frac{AC}{AB} = \cot 45^\circ = 1$

$\Rightarrow \frac{AC}{30} = 1 \Rightarrow AC = 30$ m.

$\therefore \quad DE = AC = 30$ m.

$\frac{BE}{DE} = \tan 30^\circ = \frac{1}{\sqrt{3}} \Rightarrow \frac{BE}{30} = \frac{1}{\sqrt{3}}$

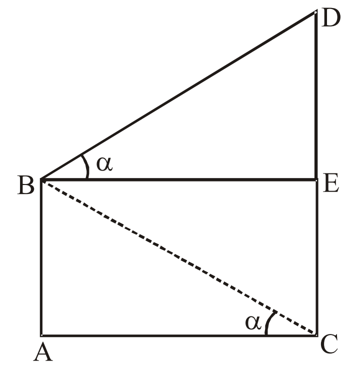
$\Rightarrow BE = \frac{30}{\sqrt{3}}$.

$\therefore \quad CD = AE = AB - BE = \left(30 - \frac{30}{\sqrt{3}}\right)$

$= 30\left(1-\frac{1}{\sqrt{3}}\right)$m

Ex.8 From the top of a cliff 25 m high the angle of elevation of a tower is found to be equal to the angle of depression of the foot of the tower. Find the height of the tower.

Sol. Let AB be the cliff and CD be the tower.



Then, AB = 25 m. From B draw BE ⊥ CD.

Let ∠EBD = ∠ACB = α.

Now, $\frac{DE}{BE} = \tan\alpha$ and $\frac{AB}{AC} = \tan\alpha$

$\therefore \frac{DE}{BE} = \frac{AB}{AC}$. So, DE = AB

[Θ BE = AC]

$\therefore$ CD = CE + DE = AB + AB = 2AB = 50m

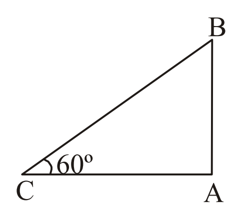
Ex.9 The altitude of the sun at any instant is 60º. The height of the vertical pole that will cast a shadow of 30 m is

(A) $30\sqrt{3}$ m (B) 15 m

(C) $\frac{30}{\sqrt{3}}$m (D) $15\sqrt{2}$ m

Sol. Let AB be the pole and AC be its shadow.

Then, θ = 60º and AC = 30 m.



$\therefore \frac{AB}{AC} = \tan 60^\circ$

$\Rightarrow \frac{AB}{30} = \sqrt{3} \Rightarrow AB = 30\sqrt{3}$ m

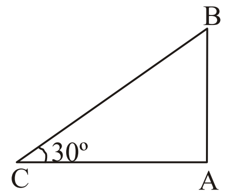
Ex.10 When the sun is 30º above the horizontal, the length of shadow cast by a building 50m high is-

(A) $\frac{50}{\sqrt{3}}$ m (B) $50\sqrt{3}$ m

(C) 25 m (D) $25\sqrt{3}$ m

Sol. Let AB be the building and AC be its shadow.

Then, AB = 50 m and θ = 30º.



$$\therefore \quad \frac{AC}{AB} = \cot 30^\circ = \sqrt{3}$$

$$\Rightarrow \frac{AC}{50} = \sqrt{3}$$

$$\Rightarrow \quad AC = 50\sqrt{3} \text{ cm.}$$

Ex.11 If the elevation of the sun changed from 30º to 60º, then the difference between the lengths of shadows of a pole 15 m high, made at these two positions, is–

(A) 7.5 m (B) 15 m

(C) $10\sqrt{3}$ m (D) $\frac{15}{\sqrt{3}}$ m

Sol. When AB = 15m, θ = 30º, then $\frac{AC}{AB}$ = tan30º

$$\Rightarrow \quad AC = \frac{15}{\sqrt{3}} \text{ m.}$$

When AB = 15m, θ = 60º, then $\frac{AC}{AB}$ = tan60º

$$\Rightarrow \quad AC = 15\sqrt{3} \text{ m}$$

∴ Diff. in lengths of shadows

$$= \left(15\sqrt{3} - \frac{15}{\sqrt{3}}\right)$$

$$= \frac{30}{\sqrt{3}} = 10\sqrt{3} \text{ m.}$$

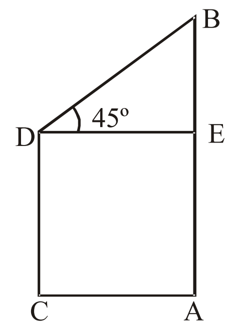
Ex.12 The heights of two poles are 80 m and 62.5 m. If the line joining their tops makes an angle of 45º with the horizontal, then the distance between the poles, is -

(A) 17.5 m (B) 56.4 m

(C) 12.33 m (D) 44 m

Sol. Let AB and CD be the poles such that

AB = 80 m and CD = 62.5 m.



Draw DE ⊥ AB. Then,

$\angle$EDB = 45º

Now, BE = AB – AE = AB – CD = 17.5

$$\frac{DE}{BE} = \cot 45^\circ = 1$$

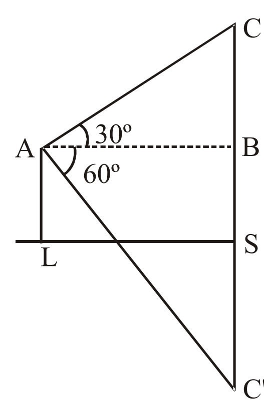
$$\Rightarrow \quad DE = BE = 17.5 \text{ m.}$$

Ex.13 If the angle of elevation of cloud from a point 200 m above a lake is 30º and the angle of depression of its reflection in the lake is 60º, then the height of the cloud above the lake, is

(A) 200 m (B) 500 m

(C) 30 m (D) None of these

Sol. Let C be the cloud and C′ be its reflection in the lake.



Let CS = C′S = x.

Now, $\frac{BC}{AB} = \tan 30^o = \frac{1}{\sqrt{3}}$

$\Rightarrow \ x - 200 = \frac{AB}{\sqrt{3}}$

Also, $\frac{BC'}{AB} = \tan 60^o = \sqrt{3}$

$\Rightarrow \ x + 200 = (AB)\sqrt{3}$.

$\therefore \ \sqrt{3}(x - 200) = \frac{x + 200}{\sqrt{3}}$ or x = 400.

$\therefore$ CS = 400 m.

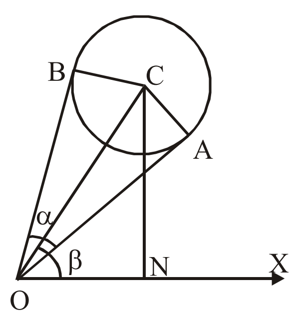
Ex.14 A balloon of radius γ makes an angle α at the eye of an observer and the angle of elevation of its centre is β. The height of its centre from the ground level is given by :

(A) $\gamma \cos\frac{\beta}{2}\sec\alpha$ (B) $\gamma\cos\beta\sec\frac{\alpha}{2}$

(C) $\gamma\sin\frac{\beta}{2}\operatorname{cosec}\alpha$ (D) $\gamma\sin\beta\operatorname{cosec}\frac{\alpha}{2}$

Sol. Let C be the centre of the balloon and O be the position of the observer at the horizontal line OX. Let OA and OB be the tangents to the balloon so that $\angle AOB = \alpha$, $\angle XOC = \beta$ and CA = CB = γ.

Clearly, right angled triangles OAC and OBC are congruent.



$\therefore \ \angle AOC = \angle BOC = \frac{\alpha}{2}$.

Let CN $\perp$ OX.

Now, $\frac{OC}{CA} = \operatorname{cosec}\frac{\alpha}{2}$

$\Rightarrow \ OC = \gamma \operatorname{cosec}\frac{\alpha}{2}$ ....(i)

Also, $\frac{CN}{OC} = \sin\beta$

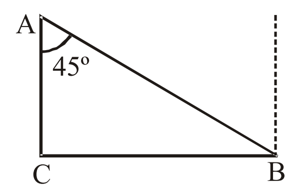
$\Rightarrow \ CN = OC\sin\beta = \gamma\operatorname{cosec}\frac{\alpha}{2}\sin\beta$ [Using(i)]

Ex.15 The banks of a river are parallel. A swimmer starts from a point on one of the banks and swims in a straight line inclined to the bank at 45º and reaches the opposite bank at a point 20 m from the point opposite to the starting point. The breadth of the river is -

(A) 20 m (B) 28.28 m

(C) 14.14 m (D) 40 m

Sol. Let A be the starting point and B, the end point of the swimmer. Then AB = 20 m and $\angle BAC = 45^o$.



Now, $\frac{BC}{AB} = \sin 45^o = \frac{1}{\sqrt{2}}$

$\Rightarrow \ \frac{BC}{20} = \frac{1}{\sqrt{2}}$

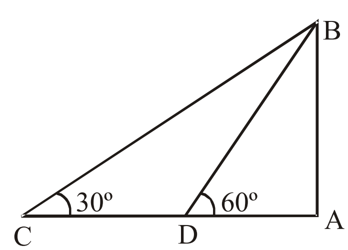
$\Rightarrow \ BC = \frac{20\times\sqrt{2}}{2} = 14.14$ m.

Ex.16 A man on a cliff observes a fishing trawler at an angle of depression of 30º which is approaching the shore to the point immediately beneath the observer with a uniform speed. 6 minutes later, the angle of depression of the trawler is found to be 60º. The time taken by the trawler to reach the shore is -

(A) $3\sqrt{3}$ min (B) $\sqrt{3}$ min

(C) 1.5 min (D) 3 min

Sol. Let AB be the cliff and C and D be the two positions of the fishing trawler.



Then, $\angle ACB = 30^o$ and $\angle ADB = 60^o$

Let AB = h.

Now, $\frac{AD}{AB} = \cot 60^o = \frac{1}{\sqrt{3}}$

$\Rightarrow$ $AD = \frac{h}{\sqrt{3}}$.

And, $\frac{AC}{AB} = \cot 30^o = \sqrt{3}$

$\Rightarrow AC = \sqrt{3}\,h$

$CD = AC - AD = \left(\sqrt{3}\,h - \frac{h}{\sqrt{3}}\right) = \frac{2h}{\sqrt{3}}$

Let u m/min be the uniform speed of the trawler.

Distance covered in 6 min = 6u metres.

$\therefore$ CD = 6u $\Rightarrow$ $\frac{2h}{\sqrt{3}} = 6u$ $\Rightarrow$ $h = 3\sqrt{3}\,u$

Now, $AD = \frac{h}{\sqrt{3}} = \frac{3\sqrt{3}\,u}{\sqrt{3}} = 3u$.

Time taken by trawler to reach A

$= \frac{\text{distance AD}}{\text{speed}}$ $\Rightarrow$ $A = \frac{3u}{u} = 3$ min

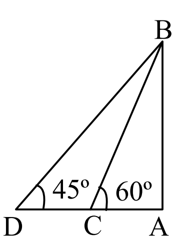
Ex.17 A boat is being rowed away from a cliff 150m high. At the top of the cliff the angle of depression of the boat changes from 60º to 45º in 2 minutes. The speed of the boat is –

(A) 2 km/hr (B) 1.9 km/hr

(C) 2.4 km/hr (D) 3 km/hr

Sol. Let AB be the cliff and C and D be the two positions of the ship. Then, AB = 150 m,

$\angle ACB = 60^o$ and $\angle ADB = 45^o$.



Now, $\frac{AD}{AB} = \cot 45^o = 1$

$\Rightarrow$ $\frac{AD}{150} = 1$ $\Rightarrow$ AD = 150 m.

$\frac{AC}{AB} = \cot 60^o = \frac{1}{\sqrt{3}}$ $\Rightarrow$ $\frac{AC}{150} = \frac{1}{\sqrt{3}}$

$\Rightarrow$ $AC = \frac{150}{\sqrt{3}} = 50\sqrt{3} = 86.6$ m.

$\therefore$ CD = AD – AC = (150 – 86.6) m = 63.4 m

Thus, distance covered in 2 min. = 63.4 m

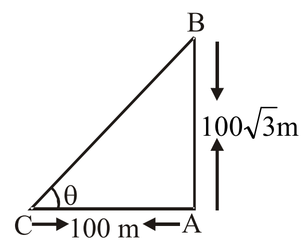
$\therefore$ Speed of the boat

$= \left(\frac{63.4}{2} \times \frac{60}{1000}\right)$ km/hr. = 1.9 km/hr.

Ex.18 A tower is $100\sqrt{3}$ metres high. Find the angle of elevation of its top from a point 100 metres away from its foot.

Sol. Let AB be the tower of height $100\sqrt{3}$ metres, and let C be a point at a distance of 100 metres from the foot of the tower.

Let θ be the angle of elevation of the top of the tower from point C.



In ΔCAB, we have

$\tan\theta = \frac{AB}{AC}$

$\Rightarrow$ $\tan\theta = \frac{100\sqrt{3}}{100} = \sqrt{3}$

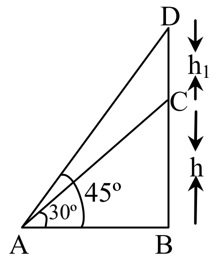
$\Rightarrow$ $\theta = 60^o$

Hence, the angle of elevation of the top of the tower from a point 100 metres away from its foot is 60º.

Ex.19 From a point on the ground 40 m away from the foot of a tower, the angle of elevation of the top of the tower is 30º. The angle of elevation of the top of a water tank (on the top of the tower) is 45º. Find the (i) height of the tower (ii) the depth of the tank.

Sol. Let BC be the tower of height h metre and CD be the water tank of height h_1 metre.

Let A be a point on the ground at a distance of 40 m away from the foot B of the tower.



In ΔABD, we have $\tan 45° = \frac{BD}{AB}$

$\Rightarrow\ 1 = \frac{h+h_1}{40} \Rightarrow h + h_1 = 40$ m ....(i)

In ΔABC, we have

$$\tan 30° = \frac{BC}{AB} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{40}$$

$$\Rightarrow h = \frac{40}{\sqrt{3}} \text{ m} = \frac{40\sqrt{3}}{3} \text{ m} = 23.1 \text{ m}$$

Substituting the value of h in (i), we get

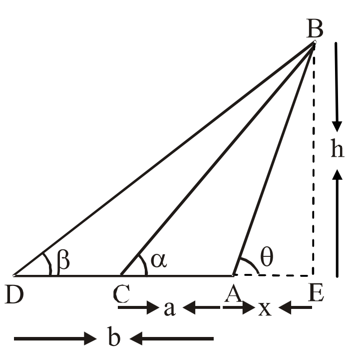
$23.1 + h_1 = 40$

$\Rightarrow h_1 = (40 - 23.1)\text{m} = 16.9$ m

Ex.20 Two stations due south of a leaning tower which leans towards the north are at distance a and b from its foot. If α, β be the elevations of the top of the tower from these stations, prove that its inclination θ to the horizontal is given by

$$\cot\theta = \frac{b\cot\alpha - a\cot\beta}{b-a}$$

Sol. Let AB be the leaning tower and let C and D be two given stations at distances a and b respectively from the foot A of the tower.



Let AE = x and BE = h

In ΔABE, we have

$$\tan\theta = \frac{BE}{AE} \Rightarrow \tan\theta = \frac{h}{x}$$

$\Rightarrow x = h\cot\theta$ ....(i)

In ΔCBE, we have

$$\tan\alpha = \frac{BE}{CE}$$

$$\Rightarrow \tan\alpha = \frac{h}{a+x}$$

$\Rightarrow a + x = h\cot\alpha$

$\Rightarrow x = h\cot\alpha - a$ ....(ii)

In Δ DBE, we have

$$\tan\beta = \frac{BE}{DE}$$

$$\Rightarrow \tan\beta = \frac{h}{b+x}$$

$\Rightarrow b + x = h\cot\beta$

$\Rightarrow x = h\cot\beta - b$ ....(iii)

From equations (i) and (ii), we have

$h\cot\theta = h\cot\alpha - a$

$\Rightarrow h(\cot\alpha - \cot\theta) = a$

$\Rightarrow h = \frac{a}{\cot\alpha - \cot\theta}$ ....(iv)

From equation (i) and (iii), we get

$h\cot\theta = h\cot\beta - b$

$\Rightarrow h(\cot\beta - \cot\theta) = b$

$$\Rightarrow h = \frac{b}{\cot\beta - \cot\theta}$$

Equating the values of h from equations (iv) and (v), we get

$$\frac{a}{\cot\alpha - \cot\theta} = \frac{b}{\cot\beta - \cot\theta}$$

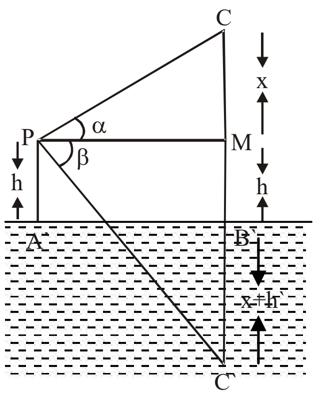
$\Rightarrow a(\cot\beta - \cot\theta) = b(\cot\alpha - \cot\theta)$

$\Rightarrow (b - a)\cot\theta = b\cot\alpha - a\cot\beta$

$$\Rightarrow \cot\theta = \frac{b\cot\alpha - a\cot\beta}{b-a}$$

Ex.21 If the angle of elevation of a cloud from a point h metres above a lake is α and the angle of depression of its reflection in the lake is β, prove that the height of the cloud is $\dfrac{h(\tan\beta + \tan\alpha)}{\tan\beta - \tan\alpha}$.

Sol. Let AB be the surface of the lake and let P be a point of observation such that AP = h metres. Let C be the position of the cloud and C′ be its reflection in the lake. Then, CB = C′B. Let PM be perpendicular from P on CB. Then, $\angle CPM = \alpha$ and $\angle MPC' = \beta$. Let CM = x.



Then, CB = CM + MB = CM + PA = x + h.

In ΔCPM, we have

$$\tan\alpha = \frac{CM}{PM}$$

$$\Rightarrow \tan\alpha = \frac{x}{AB} \qquad [\because PM = AB]$$

$$\Rightarrow AB = x\cot\alpha \qquad ....(i)$$

In Δ PMC′, we have

$$\tan\beta = \frac{C'M}{PM}$$

$$\Rightarrow \tan\beta = \frac{x+2h}{AB}$$

$$[\because C'M = C'B + BM = x + h + h]$$

$$\Rightarrow AB = (x + 2h)\cot\beta \qquad ....(ii)$$

From (i) and (ii), we have

$$x\cot\alpha = (x + 2h)\cot\beta$$

$$\Rightarrow x(\cot\alpha - \cot\beta) = 2h\cot\beta$$

$$\Rightarrow x\left(\frac{1}{\tan\alpha} - \frac{1}{\tan\beta}\right) = \frac{2h}{\tan\beta}$$

$$\Rightarrow x\left(\frac{\tan\beta - \tan\alpha}{\tan\alpha\tan\beta}\right) = \frac{2h}{\tan\beta}$$

$$\Rightarrow x = \frac{2h\tan\alpha}{\tan\beta - \tan\alpha}$$

Hence,

Height of the cloud = x + h

$$= \frac{2h\tan\alpha}{\tan\beta - \tan\alpha} + h$$

$$= \frac{2h\tan\alpha + h\tan\beta - h\tan\alpha}{\tan\beta - \tan\alpha}$$

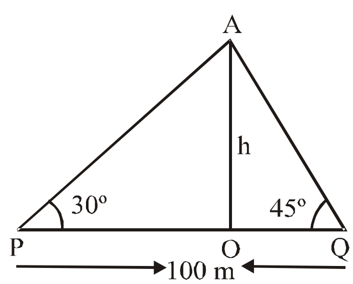
$$= \frac{h(\tan\alpha + \tan\beta)}{\tan\beta - \tan\alpha}$$

Ex.22 There is a small island in the middle of a 100 m wide river and a tall tree stands on the island. P and Q are points directly opposite to each other on two banks, and in line with the tree. If the angles of elevation of the top of the tree from P and Q are respectively 30º and 45º, find the height of the tree.

Sol. Let OA be the tree of height h metre.

In triangles POA and QOA, we have

$$\tan 30^\circ = \frac{OA}{OP} \text{ and } \tan 45^\circ = \frac{OA}{OQ}$$



$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{OP} \text{ and } 1 = \frac{h}{OQ}$$

$$\Rightarrow OP = \sqrt{3}\,h \text{ and } OQ = h$$

$$\Rightarrow OP + OQ = \sqrt{3}\,h + h \Rightarrow PQ = (\sqrt{3} + 1)h$$

$$\Rightarrow 100 = (\sqrt{3} + 1)h \qquad [\because PQ = 100 \text{ m}]$$

$$\Rightarrow h = \frac{100}{\sqrt{3}+1}\text{ m} \Rightarrow h = \frac{100(\sqrt{3}-1)}{2}\text{ m}$$

$$\Rightarrow h = 50(1.732 - 1)\text{ m} = 36.6 \text{ m}$$

Hence, the height of the tree is 36.6 m

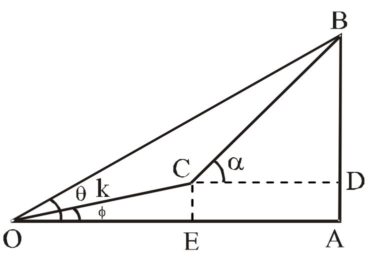
Ex.23 The angle of elevation of a cliff from a fixed point is θ. After going up a distance of k metres towards the top of cliff at an angle of ϕ, it is found that the angle of elevation is α. Show that the height of the cliff is

$$\frac{k(\cos\phi - \sin\phi\cot\alpha)}{\cot\theta - \cot\alpha} \text{ metres}$$

Sol. Let AB be the cliff and O be the fixed point such that the angle of elevation of the cliff from O is θ i.e. $\angle AOB = \theta$. Let $\angle AOC = \phi$ and OC = k metres. From C draw CD and CE perpendiculars on AB and OA respectively.

Then, $\angle DCB = \alpha$.

Let h be the height of the cliff AB.



In ΔOCE, we have

$$\sin\phi = \frac{CE}{OC}$$

$$\Rightarrow \sin\phi = \frac{CE}{k}$$

$\Rightarrow$ CE = k sin ϕ ...(i) [$\because$ CE = AD]

$\Rightarrow$ AD = k sin ϕ

And, $\cos\phi = \dfrac{OE}{OC}$

$\Rightarrow \cos\phi = \dfrac{OE}{k}$

$\Rightarrow$ OE = k cos ϕ ....(ii)

In Δ OAB, we have

$$\tan\theta = \frac{AB}{OA}$$

$$\Rightarrow \tan\theta = \frac{h}{OA}$$

$\Rightarrow$ OA = h cot θ ....(iii)

$\therefore$ CD = EA = OA – OE

= h cot θ – k cos ϕ .....(iv)

[Using eqs.(ii) and (iii)]

and, BD = AB – AD = AB – CE

= (h – k sin ϕ) ....(v)

[Using equation (i)]

In ΔBCD, we have

$$\tan\alpha = \frac{BD}{CD} \Rightarrow \tan\alpha = \frac{h - k\sin\phi}{h\cot\theta - k\cos\phi}$$

[Using equations (iv) and (v)]

$$\Rightarrow \frac{1}{\cot\alpha} = \frac{h - k\sin\phi}{h\cot\theta - k\cos\phi}$$

$\Rightarrow$ h cot α – k sin ϕ cot α = h cot θ – k cos ϕ

$\Rightarrow$ h(cot θ – cot α) = k(cos ϕ – sin ϕ cot α)

$$\Rightarrow h = \frac{k(\cos\phi - \sin\phi\cot\alpha)}{\cot\theta - \cot\alpha}$$

Ex.24 At the foot of a mountain the elevation of its summit is 45º; after ascending 1000 m towards the mountain up a slope of 30º inclination is found to be 60º. Find the height of the mountain.

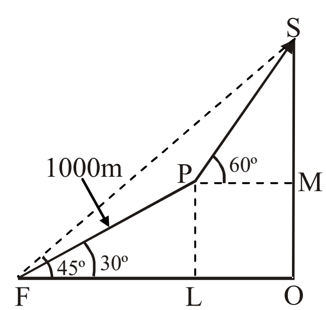
Sol. Let F be the foot and S be the summit of the mountain FOS. Then, $\angle OFS$ = 45º and therefore $\angle OSF$ = 45º. Consequently,

OF = OS = h km(say).

Let FP = 1000 m = 1 km be the slope so that $\angle OFP$ = 30º. Draw PM $\perp$ OS and PL $\perp$ OF.

Join PS. It is given that $\angle MPS$ = 60º.

In ΔFPL, We have



$$\sin 30^\circ = \frac{PL}{PF}$$

$$\Rightarrow PL = PF \sin 30^\circ = \left(1 \times \frac{1}{2}\right) \text{km.} = \frac{1}{2} \text{km.}$$

$$\therefore OM = PL = \frac{1}{2} \text{km}$$

$$\Rightarrow \text{MS} = \text{OS} - \text{OM} = \left(h - \frac{1}{2}\right) \text{km} \quad ....(i)$$

Also, $\cos 30^\circ = \frac{FL}{PF}$

$$\Rightarrow \text{FL} = \text{PF} \cos 30^\circ = \left(1 \times \frac{\sqrt{3}}{2}\right)\text{km} = \frac{\sqrt{3}}{2} \text{ km}$$

Now, $h = OS = OF = OL + LF$

$$\Rightarrow h = OL + \frac{\sqrt{3}}{2}$$

$$\Rightarrow OL = \left(h - \frac{\sqrt{3}}{2}\right) \text{km}$$

$$\Rightarrow PM = \left(h - \frac{\sqrt{3}}{2}\right) \text{km}$$

In ΔPSM, we have

$$\tan 60^\circ = \frac{SM}{PM}$$

$$\Rightarrow SM = PM. \tan 60^\circ \quad .....(ii)$$

$$\Rightarrow \left(h - \frac{1}{2}\right) = \left(h - \frac{\sqrt{3}}{2}\right)\sqrt{3}$$

[Using equations (i) and (ii)]

$$\Rightarrow h - \frac{1}{2} = h\sqrt{3} - \frac{3}{2}$$

$$\Rightarrow h(\sqrt{3} - 1) = 1$$

$$\Rightarrow h = \frac{1}{\sqrt{3} - 1}$$

$$\Rightarrow h = \frac{\sqrt{3} + 1}{(\sqrt{3} - 1)(\sqrt{3} + 1)}$$

$$= \frac{\sqrt{3} + 1}{2}$$

$$= \frac{2.732}{2} = 1.336 \text{ km}$$

Hence, the height of the mountain is 1.366 km.

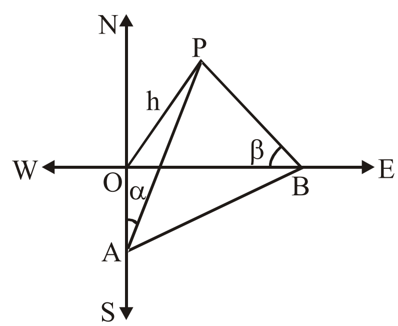
Ex.25 The angle of elevation of the top of a tower from a point A due south of the tower is α and from B due east of the tower is β. If AB = d, show that the height of the tower is $\frac{d}{\sqrt{\cot^2 \alpha + \cot^2 \beta}}$.

Sol. Let OP be the tower and let A and B be two points due south and east respectively of the tower such that $\angle OAP = \alpha$ and $\angle OBP = \beta$. Let OP = h. In ΔOAP, we have

$$\tan \alpha = \frac{h}{OA}$$

$$\Rightarrow OA = h \cot \alpha \quad ....(i)$$

In Δ OBP, we have



$$\tan \beta = \frac{h}{OB}$$

$$\Rightarrow OB = h \cot \beta. \quad ....(ii)$$

Since OAB is a right angled triangle. Therefore,

$$AB^2 = OA^2 + OB^2$$

$$\Rightarrow d^2 = h^2 \cot^2 \alpha + h^2 \cot^2 \beta$$

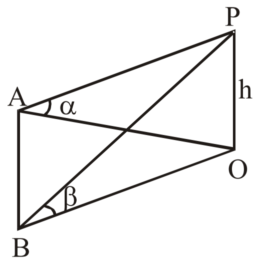
$$\Rightarrow h = \frac{d}{\sqrt{\cot^2 \alpha + \cot^2 \beta}}$$

[Using (i) and (ii)]

Ex.26 The elevation of a tower at a station A due north of it is α and at a station B due west of A is β. Prove that the height of the tower is $\frac{AB \sin \alpha \sin \beta}{\sqrt{\sin^2 \alpha - \sin^2 \beta}}$.

Sol. Let OP be the tower and let A be a point due north of the tower OP and let B be the point due west of A. Such that $\angle OAP = \alpha$ and $\angle OBP = \beta$. Let h be the height of the tower.

In right angled triangles OAP and OBP, we have



$$\tan\alpha = \frac{h}{OA} \text{ and } \tan\beta = \frac{h}{OB}$$

$\Rightarrow$ OA = h cot α and OB = h cot β.

In ΔOAB, we have

$$OB^2 = OA^2 + AB^2$$

$\Rightarrow AB^2 = OB^2 - OA^2$

$\Rightarrow AB^2 = h^2 \cot^2\beta - h^2\cot^2\alpha$

$\Rightarrow AB^2 = h^2 [\cot^2\beta - \cot^2\alpha]$

$\Rightarrow AB^2 = h^2[(\operatorname{cosec}^2\beta - 1) - (\operatorname{cosec}^2\alpha - 1)]$

$\Rightarrow AB^2 = h^2(\operatorname{cosec}^2\beta - \operatorname{cosec}^2\alpha)$

$$\Rightarrow AB^2 = h^2\left(\frac{\sin^2\alpha - \sin^2\beta}{\sin^2\alpha\sin^2\beta}\right)$$

$$\Rightarrow \quad h = \frac{AB\sin\alpha\sin\beta}{\sqrt{\sin^2\alpha - \sin^2\beta}}$$

Ex.27 An aeroplane when flying at a height of 4000m from the ground passes vertically above another aeroplane at an instant when the angles of the elevation of the two planes from the same point on the ground are 60º and 45º respectively. Find the vertical distance between the aeroplanes at that instant.

Sol. Let P and Q be the positions of two aeroplanes when Q is vertically below P and OP = 4000 m. Let the angles of elevation of P and Q at a point A on the ground be 60º and 45º respectively.

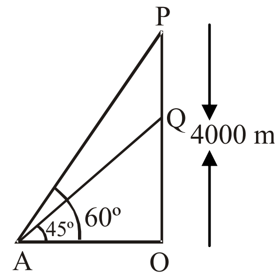
In triangles AOP and AOQ, we have

$$\tan 60^\circ = \frac{OP}{OA} \text{ and } \tan 45^\circ = \frac{OQ}{OA}$$

$$\Rightarrow \quad \sqrt{3} = \frac{4000}{OA} \text{ and } 1 = \frac{OQ}{OA}$$

$$\Rightarrow OA = \frac{4000}{\sqrt{3}} \text{ and } OQ = OA$$

$$\Rightarrow OQ = \frac{4000}{\sqrt{3}}\text{ m}$$



∴ Vertical distance between the aeroplanes

= PQ = OP – OQ

$$= \left(4000 - \frac{4000}{\sqrt{3}}\right)\text{m} = 4000\frac{(\sqrt{3}-1)}{\sqrt{3}}\text{ m}$$

= 1690.53 m

EXERCISE # 1

Q.1 The ratio of the length of a rod and its shadow is 1 : $\sqrt{3}$. Then find the angle of elevation of the sun.

Q.2 Find the angle of elevation of the moon when the length of the shadow of a pole is equal to its height.

Q.3 If the length of shadow of a pole on a level ground is twice the length of that pole, then find the angle of elevation of the sun.

Q.4 The angle of elevation of a tower from a distance 100 m from its foot is 30º. Find the height of the tower.

Q.5 In a rectangle, if the angle between a diagonal and a side is 30º and the length of diagonal is 6 cm, then find the area of the rectangle.

Q.6 The angles of elevation of an aeroplane flying vertically above the ground as observed from two consecutive stones 1 km apart are 45º and 60º. Find the height of the aeroplane above the ground in km.

Q.7 On the level ground, the angle of elevation of a tower is 30º. On moving 20 m nearer, the angle of elevation is 60º. Then find the height of the tower.

Q.8 The length of a string between a kite and a point on the ground is 90 m. The string makes an angle of 60º with the level ground. If there is no slack in the string. Find the height of the kite.

Q.9 A, B, C are three collinear points on the ground such that B lies between A and C and AB = 10 m. If the angles of elevation of the top of a vertical tower at C are respectively 30º and 60º as seen from A and B, then find the height of the tower.

Q.10 If the angles of elevation of a tower from two points distant a and b (a > b) from its foot and in the same straight line from it are 30º and 60º, then find the height of the tower.

Q.11 A vertical tower stands on a horizontal plane and is surmounted by a vertical flagstaff of height h. At a point on the plane, the angle of elevation of the bottom of the flagstaff is α and that of the top of the flagstaff is β. Find the height of the tower.

Q.12 A straight tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle of 30º with the ground. The distance from the foot of the tree to the point where the top touches the ground is 10 metres. Find the height of the tree.

Q.13 From the top of a light house, the angles of depression of two ships on the opposite sides of it are observed to be α and β. If the height of the light house be h metres and the line joining the ships passes through the foot of the light house, find the distance between the ships.

ANSWER KEY

1. 30º **2.** 45º **3.** 90º **4.** $\dfrac{100}{\sqrt{3}}$ m **5.** $9\sqrt{3}$ cm^2 **6.** $\dfrac{3+\sqrt{3}}{2}$

7. $10\sqrt{3}$ m **8.** $45\sqrt{3}$ m **9.** $5\sqrt{3}$ m **10.** $\sqrt{ab}$ **11.** $\dfrac{h\tan\beta}{\tan\beta-\tan\alpha}$

12. $10\sqrt{3}$ m **13.** $\dfrac{h(\tan\alpha+\tan\beta)}{\tan\alpha\tan\beta}$

EXERCISE # 2

Q.1 The angle of elevation of a jet plane from a point A on the ground is 60º. After a flight of 15 seconds, the angle of elevation changes to 30º. If the jet plane is flying at a constant height of $1500\sqrt{3}$ m, find the speed of the jet plane.

Q.2 Determine the height of a mountain if the elevation of its top at an unknown distance from the base is 45º and at a distance 10 km further off from the mountain, along the same line, the angle of elevation is 30º.

(Use tan 30º = 0.5774).

Q.3 The angles of elevation of the top of a rock from the top and the foot of a 100 m high tower are 30º and 45º respectively. Find the height of the rock.

Q.4 A man on the deck of a ship is 16m above water level. He observes that the angle of elevation of the top of a cliff is 45º and the angle of depression of the base is 30º. Calculate the distance of the cliff from the ship and the height of the cliff.

Q.5 From the top of a cliff 50 m high, the angles of depression of the top and bottom of a tower are observed to be 30º and 45º respectively. Find the height of the tower.

Q.6 An aeroplane, when 3000m high, passes vertically above another aeroplane at an instant when the angles of elevation of the two aeroplanes from the same point on the ground are 60º and 45º respectively. Find the vertical distance between the aeroplanes at that instant.

Q.7 The angle of elevation of the top of a hill at the foot of a tower is 60º and the angle of elevation of the top of the tower from the foot of the hill is 30º. If the tower is 50m high, what is the height of the hill ?

Q.8 A pole 5 m high is fixed on the top of a tower. The angle of elevation of the top of the pole observed from a point A on the ground is 60º and the angle of depression of the point A from the top of the tower is 45º. Find the height of the tower.

Q.9 An electric pole is 10 metres high. If its shadow is $10\sqrt{3}$ metres in length, find the elevation of the sun.

Q.10 The angle of elevation of the top of a tower, from a point on the ground and at a distance of 30 m from its foot, is 30°. Find the height of the tower.

Q.11 From a point P on the level ground, the angle of elevation of the top of a tower is 30°. If the tower is 100 m high, how far is P from the foot of tower? Take $\sqrt{3}$ = 1.732

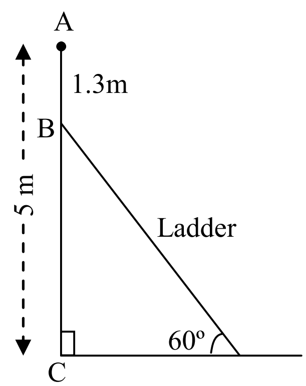
Q.12 A kite is flying at a height of 60 metres from the level ground, attached to a string inclined at 60° to the horizontal. Find the length of the string.

Q.13 A circus artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. Find the height of the pole, if the angle made by the rope with the ground level is 30°.

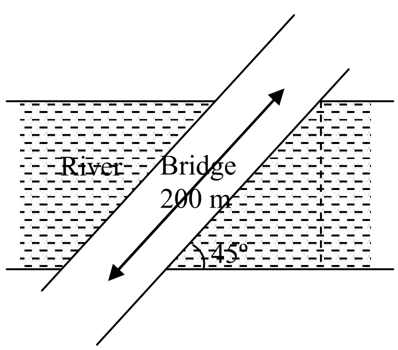
Q.14 If the length of a shadow cast by a pole be $\sqrt{3}$ times the length of the pole, find the angle of elevation of the sun.

Q.15 A river is 60 m wide. A tree of unknown height is on one bank. The angle of elevation of the top of the tree from the point exactly opposite to the foot of the tree, on the other bank, is 30°. Find the height of the tree.

Q.16 An electrician has to repair an electric fault on a pole of height 5 m. He needs to reach a point 1.3 m below the top of the pole to undertake the repair work (as shown in the adjoining figure). What should be the length of the ladder that he should use which, when inclined at an angle of 60° to the horizontal, would enable him to reach the required position? Also, how far from the foot of the pole should he place the foot of the ladder? Take $\sqrt{3}$ = 1.73



Q.17 A bridge across a river makes an angle of 45° with the river bank. If the length of the bridge across the river is 200 metres, what is the breadth of the river?



Q.18 The upper part of a tree broken by wind, falls to the ground without being detached. The top of the broken part touches the ground at an angle of 30° at a point 8 m from the foot of the tree. Calculate (i) the height at which the tree is broken. (ii) the original height of the tree.

Q.19 The horizontal distance between two towers is 140 m. The angle of elevation of the top of the first tower when seen from the top of the second tower is 30°. If the height of the second tower is 60 m, find the height of the first tower.

Q.20 A girl, 1.6 m tall, is 20 m away from a tower and observes that the angle of elevation of the top of the tower is 60°. Find the height of the tower. Take $\sqrt{3}$ = 1.73

Q.21 An observer 1.5 m tall is 28.5 m away from a chimney. The angle of elevation of the top of the chimney from his eye is 45°. What is the height of the chimney?

Q.22 From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower.

Q.23 From a point P on the ground the angle of elevation of the top of a 10 m tall building is 30°. A flag is hoisted at the top of the building and the angle of elevation of the top of the flagstaff from P is 45°. Find the length of the flagstaff and the distance of the building from the point P. (Take $\sqrt{3}$ = 1.732)

Q.24 A statue, 1.6 m tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45°. Find the height of the pedestal.

Q.25 From a point P on the ground, the angle of elevation of the top of a 10 m tall building and a helicopter, hovering over the top of the building, are 30° and 60° respectively. Find the height of the helicopter above the ground.

Q.26 The angle of elevation of the top of a building from the foot of a tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 50 m high, find the height of the building.

Q.27 A T.V. tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60°. From another point 20 m away from this point on the line joining this point to the foot of tower, the angle of elevation of the top of the tower is 30°. Find the height of the tower and the width of the canal.

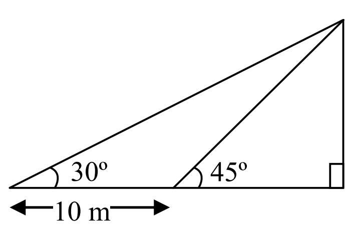
Q.28 The angle of elevation of the top of a tower from a point A (on the ground) is 30°. On walking 50 m towards the tower, the angle of elevation is found to be 60°. Calculate: (i) the height of the tower (correct to one decimal place), (ii) the distance of the tower from A.

Q.29 As observed from the top of a 75 m high lighthouse from the sea-level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships.

Q.30 From the top of a cliff 150 m high, the angles of depression of two boats are 60° and 30°. Find the distance between the boats, if the boats are (i) on the same side of cliff. (ii) on the opposite sides of the cliff.

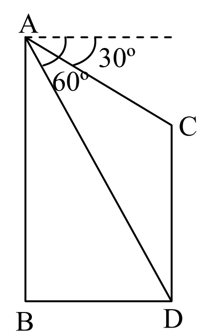
Q.31 The shadow of a tower standing on a level ground is found to be 40 m longer when the sun's altitude is 30° than when it is 60°. Find the height of the tower.

Q.32 The shadow of a vertical tower on level ground increases by 10 m, when the altitude of the sun changes from 45° to 30°. Using the given figure, find the height of the tower correct to $\frac{1}{10}$ of a metre.



Q.33 Two pillars of equal height stand on either side of a roadway which is 80 m wide. At a point in the road between pillars, the elevations of the pillars are 60° and 30°. Find the height of the pillars and the position of the point.

Q.34 In the adjoining figure, from the top of a building AB, 60 metres high, the angles of depression of the top and the bottom of a vertical lamp post CD are observed to be 30° and 60° respectively. Find (i) the horizontal distance between AB and CD. (ii) the height of the lamp post CD.



Q.35 The angles of depression of the top and the bottom of an 8 m tall building from the top of a multi-storied building are 30° and 45° respectively. Find the height of the multi-storied building and the distance between the two buildings.

Q.36 The angle of elevation of the top of an unfinished tower at a point distant 120 m from its base is 45°. How much higher must the tower be raised so that its angle of elevation at the same point may be 60°?

Q.37 From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower.

Q.38 A window in a building is at a height of 10m from the ground. The angle of depression of a point P on the ground from the window is 30°. The angle of elevation of the top of the building from the point P is 60°. Find the height of the building.

Q.39 The angle of elevation of the top of a tower from a point on the same level as the foot of the tower is 30º. On advancing 150 meters towards the foot of the tower, the angle of elevation becomes 60º. Find the height of the tower.

Q.40 A boy standing on a horizontal plane finds a bird flying at a distance of 100 m from him at an elevation of 30º. A girl standing on the roof of 20 metre high building finds the angle of elevation of the same bird to be 45º. Both the boy and the girl are on opposite sides of the bird. Find the distance of the bird from the girl.

ANSWER KEY

1. 720 km/hr	**2.** 13.66 km	**3.** 236.6 m	**4.** 27.712m ; 43.712 m	**5.** 21.13 m
6. 1268 m	**7.** 150 m	**8.** 6.83 m	**9.** 30º	**10.** $10\sqrt{3}$
11. 173.2 m	**12.** $40\sqrt{3}$	**13.** 10 m	**14.** 30º	**15.** $20\sqrt{3}$ m
16. 4.28 m ; 2.14 m	**17.** 141.4 m	**18.**(i) $\frac{8}{3}\sqrt{3}$ m ; (ii) $8\sqrt{3}$ m		**19.** 140.83 m
20. 36.2 m	**21.** 30 m	**22.** $20(\sqrt{3}-1)$ m	**23.** 7.32 m ; 17.32 m	**24.** $0.8(\sqrt{3}+1)$ m
25. 30 m	**26.** $16\frac{2}{3}$ m	**27.** $10\sqrt{3}$ m ; 10 m	**28.** (i) 43.3 m ; (ii) 75 m	**29.** $50\sqrt{3}$ m
30. (i) 173.2 m ; (ii) 346.4 m		**31.** $20\sqrt{3}$ m	**32.** 13.7 m	

33. $20\sqrt{3}$ m, 20 m from the pillar whose angle of elevation is 60º

34. (i) 34.64 m ; (ii) 40 m		**35.** $4(3+\sqrt{3})$ m ; $4(3+\sqrt{3})$		**36.** 87.84 m
37. $7\ (\sqrt{3}+1)$ m	**38.** 30 m	**39.** $75\sqrt{3}$ m	**40.** $30\sqrt{2}$ m	