Sample Paper-04 SUMMATIVE ASSESSMENT –II MATHEMATICS Class – X

Time allowed: 3 hours

Maximum Marks: 90

General Instructions:

- a) All questions are compulsory.
- b) The question paper consists of 31 questions divided into four sections A, B, C and D.
- c) Section A contains 4 questions of 1 mark each which are multiple choice questions, Section B contains 6 questions of 2 marks each, Section C contains 10 questions of 3 marks each and Section D contains 11 questions of 4 marks each.
- d) Use of calculator is not permitted.

Section A

- 1. A bag contains 3 red balls and 5 black balls. A ball is drawn at random from the bag. What is the probability that the ball drawn is red?
- 2. $\triangle OAB$ is a rectangle whose three vertices are A (0, 3), O (0, 0) and B (5, 0). Find the length of its diagonal.
- 3. In an AP, if d = -4, n = 7, $a_n = 4$ then find a.
- 4. In given figure, ABCD is a cyclic quadrilateral. If $ZBAC = 50^{\circ}$ and $ZDBC = 60^{\circ}$ then find ZBCD.



Section **B**

5. Find the area of the shaded region in figure, if ABCD is a square of side 14 cm and APD and BPC are semicircles.



6. A solid metallic sphere of radius 12 cm is melted and recast into a number of small cones, each of radius 4 cm and height 3 cm. Find the number of cones so formed.

- 7. Water is flowing at the rate of 15 km per hour through a pipe of diameter 14 cm into a rectangular tank which is 50 m long and 44 m wide. Find the time in which the level of water in the tank will rise by 21 m.
- 8. Show that x = -3 is a solution of the equation $x^2 + 6x + 9 = 0$.
- 9. How many terms are there in A.P? 18,15 $\frac{1}{2}$,13,.....,-47
- 10. In figure, BOA is a diameter of the circle and the tangent at a point P meets BA extended at T. If \angle PBO = 35°, then find \angle PTA.



Section C

- 11. If A(-3,2), B(a,b) and C(-1,4) are the vertices of a isosceles triangle, show that a+b=1, if AB = BC.
- 12. Find the value of P if the point A(0,2) is equidistant from (3, p) and (p,3).
- 13. A copper wire when bent in the form of a square encloses an area of 121 cm². If the same wire is bent into the form of a circle, then find the area of the circle. $\left(\text{Use } \pi = \frac{22}{7}\right)$
- 14. The circumference of a circular plot is 220 m. A 15 m wide concrete track runs around outside the plot. Find the area of the track. $\left(\text{Use } \pi = \frac{22}{7}\right)$
- 15. A hemispherical bowl of internal radius 9 cm is full of liquid. The liquid is to be filled into cylindrical shaped small bottles each of diameter 3 cm and height 4 cm. How many bottles are needed to empty the bowl?
- 16. Solve the quadratic equation by quadratic formula:

 $\frac{1}{2}x^2 - \sqrt{11}x + 1 = 0$

- 17. Find the sum of AP in $-5+(-8)+(-11)+\dots+(-230)$
- 18. In figure, ABC is a right angled triangle with AB = 6 cm and AC = 8 cm. A circle with centre 0 has been inscribed inside the triangle. Calculate the value of r, the radius of the inscribed circle.



19. An aeroplane when flying at a height of 3125 m form the ground passes vertically below another aeroplane at an instant when the angle of elevation of the two planes from the same

point on the ground are 30° and 60° respectively. Find the distance between the two aeroplanes at that instant.

20. A bag contains 6 red balls and some blue balls. If the probability of drawing a blue ball from the bag is twice that of a red, find the number of blue balls in the bag.

Section D

21. Construct $a \triangle ABC$ in which BC = 6.5 cm, AB = 4.5 cm and $\angle ACB = 60^{\circ}$. Construct another triangle similar to $\triangle ABC$ such that each side of new triangle is $\frac{4}{5}$ of the corresponding sides of

 Δ ABC.

- 22. At the foot of a mountain the elevation of its summit is 45° . After ascending 1000 m towards the mountain up a slope of 30° inclination, the elevation is found to be 60° . Find the height of the mountain.
- 23. From a pack of 52 playing cards, jacks, queens, kings and aces of red colour are removed. From the remaining a card is drawn at random. Find the probability that the card drawn is
 - (i) a black queen,(ii) a red card,
 - (iii) a black jack,
 - (iv) a honorable card
- 24. If the point A (2, 4) is equidistant from P (3, 8) and Q (- 10, y), find the values of y. Also find distance PQ.
- 25. A solid is in the form of a right circular cylinder with hemispherical ends. The total height of the solid is 28 cm. Find the total surface area of the solid. $\left(\text{Use } \pi = \frac{22}{7}\right)$



- 26. A bucket is in the form of a frustum of a cone with capacity 12305.8 cm³ of water. The radii of the top and bottom circular ends are 20 cm and 12 cm respectively. Find the total height of the bucket and the area of the metal sheet used in its making. (Use $\pi = 3.14$)
- 27. Two circles touch internally. The sum of their areas is 116π cm² and the distance between their centres is 6 cm. Find the radii of the circles.
- 28. A trader bought a number of articles for Rs.900. Five articles were found damaged. He sold each of the remaining articles at Rs.80 in the whole transaction. Find the number of articles he bought.

29. Ram asks the labour to dig a well up to a depth of 10 m. Labour charges `150 for first meter and

`50 for each subsequent meters. As labour was uneducated, he claims `550 for the whole work. Read the above passage and answer the following questions:

- (i) What should be the actual amount to be paid to the labour?
- (ii) What value of Ram is depicted in the question, if he pays `600 to the labour?
- 30. Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that \angle PTQ = 2 \angle OPQ.



31. Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact.

Using the above result, find the length of PQ, if a tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that OQ = 12 cm.

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(Solutions)

SECTION-A

 There are 3 + 5 = 8 balls in a bag. Out of these 8 balls, one can be chosen in 8 ways.
 ∴ Total number of elementary events = 8 Since the bag contains 3 red balls, therefore, one red ball can be drawn in 3 ways. Favourable number of elementary events = 3

Hence P (getting a red ball) = $\frac{3}{8}$

2. Length of diagonal =
$$AB = \sqrt{(5-0)^2 + (0-3)^2}$$

$$=\sqrt{25+9}=\sqrt{34}$$

3. We know, $a_n = a + (n-1)d$

Putting the values given, we get

$$\Rightarrow 4 = a + (7-1)(-4)$$

a = 4 + 24
a = 28

4. Here $ZBDC = ZBAC = 50^{\circ}$ (angles in same segment are equal)

In ABCD, we have $ZBCD = 180^{\circ} - (ZBDC + ZDBC)$ $= 180^{\circ} - (50^{\circ} + 60^{\circ})$ $= 70^{\circ}$

5. Area of shaded region= Area of square ABCD – (Area of semicircle APD + Area of semicircle BPC)

$$= 14 \times 14 - \left[\frac{1}{2} \times \frac{22}{7} \left(\frac{14}{2}\right)^2 + \frac{1}{2} \times \frac{22}{7} \left(\frac{14}{2}\right)^2\right]$$
$$= 196 - \frac{22}{7} \times 7 \times 7$$
$$= 196 - 154 = 42 \text{ cm}^2$$

6. Let *n* cones be formed. Then,

 $n \times$ Volume of one cone = Volume of sphere

$$\Rightarrow \qquad n.\frac{1}{3}\pi r^2 h = \frac{4}{3}\pi r_1^3 \qquad \Rightarrow \qquad n.r^2 h = 4r_1^3$$
$$\Rightarrow \qquad n \times (4)^2 (3) = 4 \times (12)^3 \Rightarrow \qquad n = \frac{4 \times 12 \times 12 \times 12}{4 \times 4 \times 3}$$
$$\Rightarrow \qquad n = 144$$

7. Volume of water that flows out through the pipe in 1 hour.

$$= \frac{22}{7} \times \left(\frac{14}{2}\right)^2 \times 15 \times 1000 \times 100 \,\mathrm{cm}^3$$

Volume of tank = $50 \times 44 \times 21 \times 100 \times 100 \text{ cm}^3$

$$\therefore \qquad \text{Required time} = \frac{50 \times 44 \times 21 \times 100 \times 100}{\frac{22}{7} \times \left(\frac{14}{2}\right)^2 \times 15 \times 1000 \times 100} = 2 \text{ hours}$$

8. $x^2 + 6x + 9 = 0$

$$\Rightarrow (x)^2 + 2.3.x + (3)^2$$

$$\Rightarrow (x+3)^2 \Rightarrow x = -3, -3$$

Hence x = -3 is a solution.

9.
$$a = 18$$
, $d = \frac{31}{2} - \frac{18}{1} = \frac{-5}{2}$
 $a_n = -47$
 $a_n = a + (n-1)d$
 $-47 = 18 + (n-1)\left(\frac{-5}{2}\right)$
 $\Rightarrow -47 - 18 = \frac{-5}{2}n + \frac{5}{2}$
 $\Rightarrow n = 27$
10. $\angle BPA = 90^{\circ}$ and $\angle PBA = 35^{\circ}$
 $\therefore \angle PAB = 180^{\circ} - (90^{\circ} + 35^{\circ}) = 55^{\circ}$
 $\therefore \qquad \bigcirc OA = OP$
 $\therefore \qquad \angle OAP = \angle OPA = 55^{\circ}$
 $\angle OPT = 90^{\circ}$
 $\Rightarrow \qquad \angle OPA = \angle APT = 90^{\circ}$
 $\Rightarrow \qquad \angle APT = 35^{\circ}$
 $\therefore \qquad \angle PTA = 55^{\circ} - 35^{\circ} = 20^{\circ}$

11. AB = BC (Given)

$$\Rightarrow AB^{2} = BC^{2}$$

$$\Rightarrow (a+3)^{2} + (b-2)^{2} = (-1-a)^{2} + (4-b)^{2}$$

$$\Rightarrow a^{2} + 9 + 6a + b^{2} + 4 - 4b = 1 + a^{2} + 2a + 16 + b^{2} - 8b$$

$$\Rightarrow 4a + 4b = 4$$

$$\Rightarrow a + b = 1$$
12. Let $B(3, p)$ and $C(p, 3)$
AB = AC (Given)
$$\Rightarrow AB^{2} = AC^{2}$$

$$\Rightarrow (0-3)2 + (2-p)^{2} = (p-0)^{2} + (3-2)^{2}$$

$$\Rightarrow 9 + 4 + p^{2} - 4p = p^{2} + 1$$

$$\Rightarrow -4p = -12$$

$$\Rightarrow p = 3$$
13. Side of square = $\sqrt{121} = 11 \text{ cm}$

$$\therefore \quad \text{Circumference of circle} = 44 \text{ cm}$$

$$\Rightarrow 2\pi r = 44$$

$$\Rightarrow r = \frac{44 \times 7}{2 \times 22} = 7 \text{ cm}$$

$$\therefore \quad \text{Area of the circle} = \pi r^{2}$$

$$= \frac{22}{7} \times 7 \times 7 = 154 \text{ cm}^{2}$$
14. $\therefore 2\pi r = 220$

$$\Rightarrow r = 35 \text{ m}$$
Width of track = 15 m

$$\therefore \quad \text{Area of circular track} = \pi R^{2} - \pi r^{2}$$

$$= \pi (50)^{2} - \pi (35)^{2}$$

$$= \pi (2500 - 1225)$$

$$= \frac{22}{7} \times 1275$$

$$= \frac{28050}{7} \text{ m}^{2}$$
15. Volume of hemispherical bowl = $\frac{2}{3}\pi r^{3}$

$$= \frac{2}{3}\pi (9)^{3} \text{ cm}^{3}$$

Volume of one bottle = $\pi r^2 h$

$$= \pi \left(\frac{3}{2}\right)^{2} (4)$$

$$\therefore \qquad \text{Number of bottles required} = \frac{\text{Volume of hemisphere}}{\text{Volume of 1 bottle}}$$

$$= \frac{\frac{2}{3} \pi (9)^{3}}{\pi \left(\frac{3}{2}\right)^{2} (4)} = 54$$
16. Given, $a = \frac{1}{2}, b = -\sqrt{11}, c = 1$

$$\therefore \qquad x = \frac{-b \pm \sqrt{b^{2} - 4ac}}{2a}$$

$$\therefore \qquad x = \frac{-(-\sqrt{11}) \pm \sqrt{\left(-\sqrt{11}\right)^{2} - 4, \frac{1}{2}, 1}}{2\left(\frac{1}{2}\right)}$$

$$\Rightarrow \qquad x = \sqrt{11} \pm \sqrt{11 - 2} \qquad \Rightarrow \qquad x = \sqrt{11} \pm 3$$
17. $a = -5$
 $d = -8 - (5)$
 $= -8 + 5 = -3$
 $a_{n} = -230$
 $a_{n} = 4 + (n - 1)d$

$$\Rightarrow \qquad -230 = -5 + (n - 1)(-3)$$

$$\Rightarrow \qquad -230 = -5 + (n - 1)(-3)$$

$$\Rightarrow \qquad n = 76$$

$$S_{n} = \frac{\pi}{2} [2a + (n - 1)d]$$

$$S_{76} = \frac{76}{2} [2\times(-5) + (76 - 1)(-3)]$$

$$= 38[-10 + 75\times(-3)]$$

$$= 38[-10 - 225]$$

$$= 38\times(-235)$$

$$= -8930$$
18. $\because \qquad \text{Area of triangle} = \frac{1}{2} \text{ Base x Height}$

$$\therefore \qquad \text{Area of \Delta ABC = Area of $\Delta ADC + Area of \Delta BOC + Area of \Delta COA$$$



19. Let A_1 and A_2 be the positions of the two aeroplanes



$$\frac{x}{6+x} = 2 \cdot \frac{6}{6+x}$$

Hence, no. of blue balls = 12

21. **Steps of construction**:



- (a) Draw a right angled triangle ABC with given measurements.
- (b) Draw any ray BX making an acute angle with BC on the side opposite to the vertex A.
- (c) Locate 5 points B_1 , B_2 , B_3 , B_4 , B_5 on BX so that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5$.
- (d) Join B_5C and draw a line through B_4 parallel to B_5C , intersecting the extended line segment BC at C'.

(e) Draw a line through C' parallel to CA intersecting the extended line segment BA at A'. The A'BC' is the required triangle.

22. In right $\triangle OEC$,

$$\frac{CE}{1000} = \cos 30^{\circ}$$
$$\Rightarrow \frac{OE}{1000} = \frac{1}{2}$$
$$\Rightarrow OE = 500 \text{ m}$$

In right $\triangle ADO$,



In right $\triangle ABC$,

$$\frac{h}{500\sqrt{3} + \frac{h - 500}{\sqrt{3}}} = \tan 45^{\circ}$$
$$\Rightarrow \frac{\sqrt{3}h}{1500 + h - 500} = \frac{1}{1}$$
$$\Rightarrow \sqrt{3}h = 1000 + h$$
$$\Rightarrow h = \frac{100}{\sqrt{3} - 1}$$
$$\Rightarrow h = 1369.86 m$$

23. Total number of outcomes = 52

Cards removed = 2+2+2+2 = 8[2 jack, 2 queen, 2 king and 2 aces of red colour]

- :. Remaining number of cards = 52 8 = 44
- \therefore Total number of outcomes = 44
- (i) Favourable outcomes = 2 [There are 2 black queen]

:. Required probability =
$$\frac{2}{44} = \frac{1}{22}$$

(ii) Favourable outcomes = number of red cards left = 26 - 8=18

- :. Probability for a red card = $\frac{18}{44} = \frac{9}{22}$
- (iii) Favourable outcomes = Number of black jacks = 2

Required probability = $\frac{2}{44} = \frac{1}{22}$

(iv) Number of picture cards left =2+2+2 = 6 [jack, queen, King are picture cards]

$$\therefore$$
 Required probability = $\frac{6}{44} = \frac{3}{22}$

(v) Honorable cards [ace, jack, queen and king] No. of honorable cards left = 2+2+2+2=8

$$\therefore \text{ Required probability} = \frac{8}{44} = \frac{2}{11}$$

24. Given points are A(2, -4), P(3, 8) and Q(-10, y) According to the question,

PA=QA

$$\sqrt{(2-3)^{2} + (-4-8)^{2}} = \sqrt{(2+10)^{2} + (-4-y)^{2}}$$
$$\sqrt{(-1)^{2} + (-12)^{2}} = \sqrt{(12)^{2} + (4+y)^{2}}$$
$$\sqrt{1+144} = \sqrt{144+16+y^{2}+8y}$$
$$\sqrt{145} = \sqrt{160+y^{2}+8y}$$
$$145 = 160 + y^{2} + 8y$$

On squaring both sides, we get

$$y^2 + 8y + 160 - 145 = 0$$

 $y^2 + 8y + 15 = 0$
 $y^2 + 5y + 3y + 15 = 0$
 $\Rightarrow y(y+5) + 3(y+5) = 0$
 $\Rightarrow (y+5)(y+3) = 0$
And $y + 5 = 0 \Rightarrow y = -5$
 $\therefore y + 3 = 0 \Rightarrow y = -3$
 $y = -3, -5$
Now, $PQ = \sqrt{(-10-3)^2 + (y-8)^2}$
For $y = -3$ PQ $= \sqrt{(-13)^2 + (-3-8)^2}$
 $= \sqrt{169 + 121} = \sqrt{290}$ units
And for $y = -5$ PQ $= \sqrt{(-13)^2 + (-5-8)^2}$
 $= \sqrt{169 + 169} = \sqrt{338}$

Hence, values of y are -3 and -5, $PQ = \sqrt{290}$ and $\sqrt{338}$

- 25. Total surface area of the solid
 - = Curved surface area of cylinder + 2 x Curved surface area of the hemisphere

$$= 2\pi rh + 2 \times 2\pi r^{2}$$

$$= 2\pi r (h + 2r)$$

$$= 2 \times \frac{22}{7} \times 14 [58 - (14 + 14) + 2 \times 14]$$

$$= 88 [58 - 28 + 28]$$

$$= 5104 \text{ cm}^{2}$$

26. Volume = 12308.8 cm³
 $r_{1} = 20 \text{ cm}$
 $r_{2} = 12 \text{ cm}$
Volume (V) = $\frac{1}{3} \pi h (r_{1}^{2} + r_{2}^{2} + r_{1}r_{2})$
 $\Rightarrow 12308.8 = \frac{1}{3} \times 3.14 \times h [(20)^{2} + (12)^{2} + 20 \times 12]$
 $\Rightarrow 12308.8 = \frac{3.14}{3} h [400 + 144 + 240]$
 $\Rightarrow 12308.8 = h \times 1.05 \times 784$
 $\Rightarrow h = 15 \text{ cm}$
And $l = \sqrt{h^{2} + (r_{1} - r_{2})^{2}} = \sqrt{(15)^{2} + (20 - 12)^{2}} = \sqrt{225 - 64} = 17 \text{ cm}$

Area of metal sheet used = $\pi l(r_1 + r_2) + \pi r_2^2$

 $= 3.14 \times 17(20+12) + 3.14 \times (12)^{2}$ $= 3.14 \times 17 \times 32 + 3.14 \times 144$ $= 2160.32 \text{ cm}^{2}$

27. Let r_1 and r_2 be the radius of two circles

According to question, *r*²

$$\pi r_1^2 + \pi r_2^2 = 116\pi$$

$$\Rightarrow r_1^2 + r_2^2 = 116....(i)$$

$$r_2 - r_1 = 6 \text{ (Given)}$$

$$\Rightarrow r_2 = 6 + r_1$$

Put the value of r_2 in eq. ... (i)

$$r_1^2 + (6 + r_1)^2 = 116$$

$$\Rightarrow r_1^2 + 36 + r_1^2 + 12r_1 = 116$$

$$\Rightarrow 2r_1^2 + 12r_1 - 80 = 0$$

$$\Rightarrow r_1^2 + 6r_1 - 40 = 0$$

$$\Rightarrow r_1^2 + 6r_1 - 4r_1 - 40 = 0$$

$$\Rightarrow r_1(r_1 + 10) - 4(r_1 + 10) = 0$$

$$\Rightarrow (r_1 + 10)(r_1 - 4) = 0$$

$$\Rightarrow r_1 = -10 \text{ (Neglect) or } r_1 = 4 \text{ cm}$$

when $r_1 = 4 \text{ cm}$

$$r_2 = 6 + r_1$$

$$= 6 + 4$$

$$r_2 = 10 \text{ cm}$$

28. Let he buys x articles.

C.P. of x articles = Rs. 900

 $\therefore \qquad \text{C.P. of 1 article} = \text{Rs.} \frac{900}{x}$

No. of articles damaged = 5

 \therefore No. of articles not damaged = (x-5)

:.

S.P. of 1 article = Rs. $\left(\frac{900}{x} + 2\right)$ S.P. of (x-5) articles = Rs. $\left(\frac{900}{x}+2\right)(x-5)$... According to the question, $\left(\frac{900}{x}+2\right)(x-5) - 900 = 80$ $\Rightarrow \qquad 900x - 4500 + 2x^2 - 10x = 980x$ (900+2x)(x-5) = 980x \Rightarrow $2x^2 - 90x - 4500 = 0$ $\Rightarrow x^2 - 45x - 2250 = 0$ \Rightarrow $x^2 - 75x + 30x - 2250 = 0$ $x^{2}(x-75)+30(x-75)=0$ \Rightarrow \Rightarrow (x-75)(x+30) = 0x = 75, -30 \Rightarrow \Rightarrow

x = -30 is inadmissible as x is the number of articles.

Hence, No. of articles = 75

29. (i) Here, amount form an AP.

(ii) Economy, Saving

First term, a = Labour charge for first meter = `150

Since Labour charge increasing by `50 for each subsequent meters.

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∴ d = 50
Total depth = 10 m
∴ Labour charge for 10 m = a + (n-1)d
= 150 + (10 - 1) x 50 = 150 + 9 x 50
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= 150 + 450 = 600
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Hence `600 should be paid to the labours.

(ii) If Ram pays `600 to the labour, then it shows his honesty and sincerety.

30. : Tangent segments from an external point to a circle are equal in length.

 \therefore TP = TQ

And $\angle TQP = \angle TPQ$(i)[Angles opposite to equal sides of a \triangle are equal] \therefore Tangent is perpendicular to the radius through the point of contact.

[Angle sum property of a Δ]

[From eq. (ii)]

$$\angle OPT = 90^{\circ}$$
(ii)

$$\Rightarrow \qquad \angle OPQ + \angle TPQ = 90^{\circ}$$

$$\Rightarrow \angle TPQ = 90^{\circ} - \angle OPQ$$
(iii)

In ∆TPQ,

:.

 $\angle PTQ + \angle TPQ + \angle TQP = 180^{\circ}$

 $\Rightarrow \qquad \angle PTQ + 2 \angle TPQ = 180^{\circ} \qquad \dots \dots \dots (iv)$

- $\Rightarrow \qquad \angle PTQ + 2(90^{\circ} \angle OPQ) = 180^{\circ} \qquad [From eq. (iii)]$
- $\Rightarrow \qquad \angle PTQ = 2 \angle OPQ$
- 31. **First part**: <u>Given</u> : A circle with centre O and radius *r* and a tangent AB at a point P. <u>To Prove</u> : $OP \perp AB$

Construction: Take any point Q, other than P on the tangent AB. Join OQ. Suppose OQ meets the circle at R. 0 <u>Proof</u>: Clearly OP = OQ [Radii] OQ = OR + RQNow, \Rightarrow 0Q> 0R 0Q> 0P [OP = OQ] \Rightarrow 0P < 0Q \Rightarrow Thus, OP is shorter than any segment joining O to any point of AB. So, OP is perpendicular to AB. Hence, OT = OT'..... (Radii of the same circle)(Common) OP = OPand(RHS congruency) $\Delta OTP \cong \Delta OT'P$ *.*.. Hence, $OP \perp AB$ Second part: Using the above, we get, $\angle OPQ = 90^{\circ}$ $OP^2 + PQ^2 = OQ^2$ [By Pythagoras theorem] $12^2 = 5^2 + PQ^2$ \Rightarrow

 $PQ = \sqrt{119}$ cm

 \Rightarrow