Chapter—4

MULTIPLICATION & DIVISION OF ALGEBRAIC EXPRESSIONS

You are already acquainted with addition and subtraction of algebraic expressions. In the process of addition & subtraction integer numbers are added & subtracted respectively and the algebriac remains the same. Similarly in class VII you have studied – On multiplying any two algebraic expressions, their constants are multiplied with the constants and the variables are multiplied with the variables.

Activity 1.

In the table given below, algebraic expressions as the two variables and their products are shown. Some blank spaces are given in the table. Fill in the blanks with correct values.

Table 4.1

S.No	First Expressions	Second Expressions	1st Expressions x 2nd Expressions	2nd Expressions x 1st Expressions	Product
01	-3	a	-3.a	a.(-3)	-3a
02	x	5	x.5	5.x	5x
03	2a	3a	2a.3a	3a.2a	$6a^2$
04	7x	-4y	--------------	--------------	-------------
05	-5xy	2x	--------------	--------------	-------------
06	4a2	------------	--------------	--------------	$-12a^2b$
07	$-7a^2b^2$	8ab	--------------	--------------	-------------

Here we can see in table the product value remains the same even if the place of the expression are exchanged. From this which rule about multiplication does this observation satisfy?

Let us look at a few more examples :

1. $3x \cdot 5x = (3 \cdot 5)x \cdot x = 15x^2$

2. $(-4x)6y = (-4 \times 6)\,x.y = -24xy$

3. $(-ab)5b^2 = (-1 \times 5)\,ab.b^2 = -5a.b.b^2 = -5ab^3$

Thus, we can see that when the base is the same, the exponents get added according to the rule of exponential numbers.

While adding a algebriac expression you have seen that the factors get added to each other. For example : $x + x = (1+1)x = 2x$ (here is the multiple factor of x)

Similarly 2x is obtained by adding x to himself twice.

Thus $\quad x + x + x \quad = 3x$

$\quad\quad x + x + x + x = 4x$

So the multiple factor of x becomes the numbers that is the number of times it is added and hence in 2x, 2 is the product factor and x is the variable.

Therefore, for different values of 'x' the value of 2x will be different.

If $x = 3,$ than $2x = 2.(3) = 06$

If $x = -5,$ than $2x = x.(-5) = -10$

If $x = 0,$ than $2x = 2.(0) = 0$

If $x = \frac{3}{8},$ than $2x\frac{3}{8} = \frac{3}{4}$

Fill in the blanks in the table below :-

x		$2x$
1	→	2
3	→	-----
2	→	-----
8	→	-----
7	→	-----

some time while calculating it goes wrong If $x = 5,$ thus $2x$ will not be 25, $2x \neq 25$ but 2x=2×5=10 is correct.

One day the teacher asked Neeraj in the class,

Teacher : "What is your age?"

Neraj : I am 13 years old.

Teacher : What will be your age after 2 years ?

Neraj : After 2 years my age will be 13 + 2 = 15 years.

Teacher : How old are you, Jeetendra ?

Jeetendra : I am nearly 12 years old.

Teacher : What will be your age after 2 years ?

Jeetendra : After 2 years, I will be 12+2 = 14 years old.

Teacher : If a person is x year old, then after 2 years. How old would he be ?

Manisha answered that after 2 years, the person would be $(x+2)$ years old.

If we keep the value of x different, then the value of $(x+2)$ will also be different.

If $x = 3$, then $x + 2 = 3 + 2 = 05$ years

If $x = 8$, then $x + 2 = 8 + 2 = 10$ years

If $x = 5$, then $x + 2 = 5 + 2 = 07$ years

Fill in the blanks in the table below -

$2 + x = x + 2$

1	→	3
7	→	-----
12	→	-----
20	→	-----
31	→	-----

Thus, we can see that $2x$ represent twice of x, while $2 + x$ means a value that is 2 more than x.

If $x = 0$, then $2x = 2 \times 0 = 0$

If $x = 0$, then $2 + x = 2 + 0 = 2$

Therefore $2x \neq 2 + x$

Multiplying monomial expression with polynomial expression.

In class VII, we have learnt how to multiply -

Any monomial algebraic expression to binomial algebraic expression.

Let us revise the multiplication of monomial expression with binomial expressions.

Once again by activity.

Activity - 2

In the table given below the product of monomial expressions, with binomial expressions is shown. Below are given some blank spaces fill these up.

Table 4.2

S.No.	Monomial Expression	Binomial Expression	Monomial×binomial Expression	Product
1.	x	a + b	x(a + b)	ax + bx
2.	−4y	3a + b	----------	----------
3.	xy	7 + 8x	$xy(7 + 8x)$	----------
4.	$2t^2$	$3r^2 - 55$	----------	----------
5.	$\frac{1}{2}m$	$m^3 + \frac{3}{2}n$	----------	----------
6	4a	$5x - \frac{1}{2}y$	----------	----------

Similarly we can multiply any monomial expression by polynomial expression.

Or a (b + c + d) = ab + ac + ad

(b + c + d) a = ba + ca + da

Similarly a (b + c + d + e) = ab + ac + ad + ae

Or (b + c + d + e) a = ba + ca + da + ea

Example 1. 2a (a + 2b + 5c) = 2a· a + 2a · 2b + 2a · 5c

= $2a^2$ + 4 ab + 10ac

Example 2. (2q + r + 3s - t) p = 2q · p + r · p + 3s · p + t p

= 2 pq + pr + 3 ps + pt

Example 3. $(xy + 2y^2z + x^2)\ yz^2$ = $xy \cdot yz^2 + 2y^2z \cdot yz^2 + x^2 \cdot yz^2$

= $xy^2z^2 + 2\ y^3z^3 + x^2\ yz^2$

Activity 3

Fill in the blanks -

Table 4.3

S.No.	Multiplicatoin of algebraic expression	Process of multiply	Product
1.	(2a +b + c) 5d	2a × 5d + b × 5d + c × 5d	10ad + 5 bd + 5 cd
2.	$7a^2$ (b + 2d - t)	....	....
3.	..... ($x^2 + xy + z$)	p × x^2 + p × xy + p × z	p x^2 + ...
4.	–5 m (....+....+ b)	....	$-5m^2 - 10mn - 5mb$
5.	$7p^2m\ (m + n^3 + p)$	....	....

Let us think of multiplication of two binomial expression -

Multiplication of two binomial expressions

Multiplication of two binomial expression is equal to the sum of product of two monomial with polynomial expression.

$$(a+b)(c+d) = a(c+d)+b(c+d)$$

$$=(ac+ad)+(bc+bd)$$

$$=ac+ad+bc+bd$$

we can also solve this in the following way

$$(a+b)(c+d)$$
$$=(a+b)c+(a+b)d$$
$$=ac+bc+ad+bd$$

In this process the product using the distribution property of multiplication over addition used twice.

Example - 4

Multiply $(5x+3y)$ and $(4x+5y)$ to each other.

Solution :

$$(5x+3y)(4x+5y)=5x(4x+5y)+3y(4x+5y)$$

[using $(a+b)(c+d)=a(c+d)+b(c+d)$]

$$=5x.4x+5x.5y+3y.4x+3y.5y$$

[using $a(b+c) = ab+ac$]

$$=20x^2+25xy+12yx+15y^2$$

$$=20x^2+37xy+15y^2$$

This can also be solved in the following manner :

$$(5x+3y)(4x+5y)=5x+3y.4x+(5x+3y).5y$$

[using $(a+b)(c+d)=(a+b)c+(a+b)d$]

$$=5x.4x+3y.4x+5x.5y+3y.5y$$

[using $(a+b).c=ac+bc$]

$$=20x^2+12yx+25xy+15y^2$$

$$=20x^2+37xy+15y^2$$

Example - 5

Multiply $\left(3s^2+2t\right)$ and $\left(2r^2+5t\right)$

Solution :

$$\left(3s^2+2t\right)\left(2r^2+5t\right)=3s^2.\left(2r^2+5t\right)+2t.\left(2r^2+5t\right)$$

[using $(a+b)(c+d)=a(c+d)+b(c+d)$]

$$=3s^2.2r^2+3s^25t+2t.2r^2+2t.st$$

[using $a(b+c)=ab+ac$]

$$=6s^2r^2+15s^2t+4tr^2+10t^2$$

Example - 6

Multiply $(5x+3y)$ to $(x+y)$ and verify the product for $x=3,\ y=-2$

Solution : $(5x+3y)(x+y)\ =5x(x+y)+3y(x+y)$

$$=5x.x+5x.y+3y.x\quad 3y.y$$

$$=5x^2+5xy+3xy+3y^2$$

$$(5x+3y)(x+y)\ =5x^2+8xy+3y^2$$

Verification

L.H.S. $=(5x+3y)(x+y)$

$=\{5(3)+3(-2)\}(3-2)$ (when $x=3,\ y=-2$)

$=(15-6)(1)$

$=9\times1=9$

R.H.S. $=\ 5x^2+8xy+3y^2$

$=5(3)^2+8(3)(-2)+3(-2)^2$

$=5(9)-48+3(4)$

$=45-48+12$

$=45-48+12$

$=57-48=9$

It is clear that

L.H.S. = R.H.S.

Therfore the product is correct.

Activity -4

With the process of multiplication fill in the blanks in the table given below -

Table 4.4

Multiplication of two algebraic expressions	Process of Multiplication		Product obtained
	Using the distributive law	Using the distributive law the second time	
1. (a + b) (c + d)	a (c + d) + b (c + d) or (a+b)c + (a+b) d	ac + ad +bc +bd or ac + bc+ad+bd	ac+ad+bc+bd or ac+bc+ad+bd
(a) $(4x+5y)(2x+3y)$	$4x(2x+3y)+5y(2x+3y)$	$4x\times2x+4x\times3y+5y\times2x+5y\times3y$	$8x^2+22xy+15y^2$
(b) $(5x^2+2s)(2t+5)$	....	....	....
(c) $(2r^2+5s^3)(r^2+t^3)$	....	....	....
2. (a + b) (c – d)	a (c – d) + b (c – d)	ac – ad +bc – bd	ac–ad+bc–bd
(a) (b+2c) (3b – c)	....	....	....
(b) $(5x+3y)(2y^2-z)$	....	....	....
3. (a – b) (c + d)	a (c + d) – b (c + d)	ac + ad – bc – bd	ac+ad–bc–bd
(a) $(2x-3y)(3x+z)$	....	....	....
(b) $(5p-2q)(3x+4s)$	....	....	....
4. (a – b) (c – d)	a (c – d) – b (c – d)	ac – ad – bc+bd	ac–ad–bc+bd
(a) $(2s-3p)(4x-5t)$	....	....	....
(b) $(x^2+xy)(y^2-z)(y^2-z)$	....	....	....

Exercise 4.1

Q. 1. Multiply the given expressions to each other

(i) $(2x+7)(3x+2)$ (ii) $(3x-5)(2x+9)$ (iii) $(7x-6)(15x-2)$

(iv) $\left(\frac{1}{2}x+5y\right)\left(3x-\frac{6}{5}y\right)$ (v) $(x+5y)(7x-y)$

Q. 2. Find the values

(i) $(x+y)(2y+3x)+(3x+y)(y+2x)$ (ii) $\left(2x+\frac{1}{2}\right)\left(\frac{3x}{2}-\frac{1}{4}\right)$

(iii) $(x^2+y^2)(3x-5y)$ (iv) $(a+b)(a+b)$

Q. 3. Multiply $(x+y)$ and $(3x+4y)$ to each other & verify the product for the following values :

(i) $x=2, \quad y=-1$, (ii) $x=1, \quad y=0$

DIVISION OF ALGEBRAIC EXPRESSIONS

You know how to multiply a whole number and another whole number & also how to divide them. Let us see some examples ?

1. If $6 \times 8 = 48$, then $48 \div 8 = 6$ and $48 \div 6 = 8$
2. If $-15 \times 3 - 45$ then $-45 \div -15 = 3$ and $-45 \div 3 = -15$
3. If $m \times n = mn$ then $mn \div m = n$ and $mn \div n = n$

Activity - 5

Fill in the blanks in the following table -

Table 4.5

S.No	1st Number x 2nd Number	Product of the two numbers	Exhibiting the process of Division	
			1st Expressions	2nd Expressions
01	$3x \times 4y$	$12xy$	$12xy \div 3x = 4y$	$12xy \div 4y = 3x$
02	$2x(-7x)$	$-14x^2$	--------------	--------------
03	$m \times 4n$	$4mn$	--------------	--------------
04	$18a^2 \times 2b^2$	--------------	--------------	-------------
05	$13p^2 \times 7pq$	$91p^3q$	--------------	--------------

Thus we note that on multiplying $3x$ by $4y$ we get $12xy$ and on dividing $12xy$ by $3x$, we get $4y$ and if $12xy$ is divided by $4y$, we would get $3x$. Hence multiplication and division process are opposite to each other.

Division of a monomial expression by another monomial expression -

Let us know how to divide a monomial expression by another monomial expression.

Example - 7

Divide $18x^2y$ by $6xy$

Solution :

Here $18x^2y \div 6xy = \dfrac{18x^2y}{6xy}$

$$= \frac{18}{6} \times \frac{x^2}{x} \times \frac{y}{y} = 3 \times \frac{x \times \cancel{x}}{\cancel{x}} \times \frac{\cancel{y}}{\cancel{y}}$$

$$= 3x$$

Example - 8

Divide $-35mn^2p$ by $7np$

Solution :

$$-35mn^2p \div 7np$$

$$= \frac{-35mn^2p}{7np}$$

$$= \frac{-35}{7} \times \frac{m}{1} \times \frac{n^2}{n} \times \frac{p}{p}$$

$$= -5 \times m \times \frac{n \times \cancel{n}}{\cancel{n}} \times \frac{\cancel{p}}{\cancel{p}}$$

$$= -5mn$$

So, you have observed that the process of division is taken up in the following steps.

1. If the sign of the divisor and the dividend are same, the sign of the quotient is positive.
2. If the sign of the divisor and the dividend are different the quotient will have a negative sign.
3. The multiple factor of the dividend is divided by the multiple factor of the division.
4. To find the value of the exponent of any variable in the quotient the exponential law $a^m \div a^n = a^{m-n}$ is used. Let us take the following example :

Example - 9

Divide $-25a^3b^2c$ by $-5ab^2c$

Solution :

Hence $\left(-25a^3b^2c\right) \div \left(-5ab^2c\right)$

$$= \frac{-25a^3b^2c}{-5ab^2c}$$

$$= \frac{-25}{-5} \times \frac{a^3}{a} \times \frac{b^2}{b^2} \times \frac{c}{c}$$

$$= 5 \times a^{3-1} \times b^{2-2} \times c^{1-1} \qquad \left(\because a^m \div a^n = a^{m-n}\right)$$

$$= 5a^2b^0c^0 \qquad \text{(since } b^0 = 1,\ c^0 = 1\text{)}$$

$$= 5a^2$$

Division of a polynomial expression by a monomial expression -

You have known how to divide a monomial expression by another monomial expression. Now let us see the division of polynomial expression by a monomial expression.

Example - 10

Divide $16m^2 + 4mn - 12mn^2$ by $2m$

Solution :

$$16m^2 + 4mn - 12mn^2 \div 2m$$

or

$$\frac{16m^2 + 4mn - 12mn^2}{2m}$$

$$= \frac{16m^2}{2m} + \frac{4mn}{2m} - \frac{12mn^2}{2m}$$

$$= 8m^{2-1} + 2m^{1-1}n - 6m^{1-1}n^2$$

$$= 8m + 2n - 6n^2$$

Here the polynomial has been changed in to monomial expression to continue the process of division.

Exerciese 4.2

1. Find the values.

(i) $(18x^2y^2) \div (-6xy)$ (ii) $(-15x^3y^2z) \div (-5x^2yz)$

(iii) $(-x^5y^7) \div (-x^4y^5)$ (iv) $32a^4b^2c \div (-8abc)$

(v) $(28a^4b^6c^8) \div (-7a^2b^4c^6)$

2. Divide

(i) $2x^4 - 6x^3 + 4x^2$ by $2x^2$ (ii) $5a^4b^3 - 10a^3b^2 - 15a^2b^2$ by $5a^2b^2$

(iii) $27a^4 - 36a^2$ by $-9a$ (iv) $x^4 + 2x^3 + 2x^2$ by $4x^2$

(v) $a^2 + ab + ac$ by a

Division of a polynomial by binomial -

You now know how to divide a polynomial by a monomial expression. Now let us see this.

Example - 11

Divide $18a^2 + 12a + 27a^3 + 8$ by $3a + 2$

Solution :

Write done the polynomial in the decreasing order of its exponents.

Example : $27a^3 + 18a^2 + 12a + 8$

Step 1- Here the frist dividend is $27a^3$. In the beginning this is divided by the first part of the divisor that is $3a$

$$\frac{27a^3}{3a} = 9a^2 \qquad 3a+2\overline{)27a^3 + 18a^2 + 12a + 8}$$

and $9a^2$ is written as the quotient.

Step 2- $9a^2$ is multipled to the complete divisor.

$$9a^2(3a+2) = 27a^3 + 18a^2$$

$$\begin{array}{r|l} & 9a^2 \\ \hline 3a+2 & 27a^3 + 18a^2 + 12a + 8 \\ & \pm 27a^3 \pm 18a^2 \end{array}$$

$27a^3 + 18a^2$ is written under the similar factors of the dividend and is subtracted. That is the sign of the lower expression is changed.

Step 3- After subtraction the remaining number is written below.

$$\begin{array}{r|l} & 9a^2 \\ \hline 3a+2 & 27a^3 + 18a^2 + 12a + 8 \\ & \underline{\pm 27a^3 \pm 18a^2} \\ & \qquad 12a + 8 \end{array}$$

Step 4- The intial part of the remaining part of the dividend $12a$ is divided by the first part of the divsor, $3a$. so, $12a \div 3a = 4$

+ 4 is to be written in the quotient and + 4 is again multipled to the whole divisor.

Therefore $(3a + 2)\times 4 = 12a + 8$

$$\begin{array}{r|l} & 9a^2 + 4 \\ \hline 3a+2 & 27a^3 + 18a^2 + 12a + 8 \\ & \underline{\pm 27a^3 \pm 18a^2} \\ & \qquad 12a + 8 \end{array}$$

Step 5- Similar factors are written under each other of the divididend 12a+8 and is subtacted.

$$\begin{array}{r|l} & 9a^2 + 4 \\ \hline 3a+2 & 27a^3 + 18a^2 + 12a + 8 \\ & \underline{\pm 27a^3 \pm 18a^2} \\ & \qquad \pm 12a + 8 \\ & \qquad \underline{\pm 12a \pm 8} \\ & \qquad\qquad 0 \end{array}$$

Step 6- Subtraction would give the remainder as zero.

$$\begin{array}{r|l} & 9a^2+4 \\ 3a+2 & 27a^3+18a^2+12a+8 \\ & 27a^3+18a^2 \\ \hline & 0 \;+\; 0 \;+12a+8 \\ & \pm 12a \pm 8 \\ \hline & 0 \quad 0 \\ \hline \end{array}$$

Step 7- Thus, desired quotient = $9a^2 + 4$

You already know that when a number is completely divided by another number and the remainder is zero, then the second number is know as the multiple factor of the first number.

Here $27a^3+18a^2+12a+8$ divided by ($3a$ + 2) gets completely divded and the remainder is zero, therefore ($3a$ +2) is a multiple factor of $27a^3+18a^2+12a+8$

Let us now take another example :

Example - 12

Divide $-12x^3-8x^2-5x+10$ *by* $(2x-3)$

Solution :

$$\begin{array}{r|l} 2x-3 & -12x^3-8x^2-5x+10 \;(-6x^2-13x-22 \\ & \mp 12x^3 \pm 18x^2 \\ \hline & -26x^2-5x+10 \\ & \mp 26x^2 \pm 39x \\ \hline & -44x+10 \\ & \mp 44x \pm 66 \\ \hline & -55 \\ \hline \end{array}$$

Here also, the divison has been done as in the earlier example, but the remander is -55, not zero, So we might say that ($2x$ - 3) is not a multiple factor of the polynomial

$-12x^3-8x^2-5x+10$.

Example - 13

Divide $8q^3+2q-8q^2-1$ *by* $4q+2$

Solution :

Here the exponents of are not in descending order of q, so first the expression is written in the descending order of the exponents of q.

Thus $8q^3 - 8q^2 + 2q - 1$

$$
\begin{array}{r|l}
 & 2q^2 - 3q + 2 \\
4q+2 & 8q^3 - 8q^2 + 2q - 1 \\
 & \pm 8q^3 \pm 4q^2 \\
\hline
 & -12q^2 + 2q - 1 \\
 & \mp 12q^2 \mp 6q \\
\hline
 & 8q - 1 \\
 & \pm 8q \pm 4 \\
\hline
 & -5
\end{array}
$$

Here also the division has been done like the previous solutions. The steps of division are continued until the exponents of the algebraic variable of the remainder does not become less than exponents of the algebraic variable of the divisor.

Verification

Dividend = divisor x quotient + remainder in this question.

Dividend = $8q^3 - 8q^2 + 2q - 1$

Divisor = $4q + 2$

Quotient = $2q^2 - 3q + 2$

Remander = -5

Right hand side = divisor x quotient + remainder

= $(4q+2)(2q^2 - 3q + 2) + (-5)$

= $4q(2q^2 - 3q + 2) + 2(2q^2 - 3q + 2) - 5$

= $8q^3 - 12q^2 + 8q + 4q^2 - 6q + 4 - 5$

= $8q^3 - 8q^2 + 2q - 1$

= Left hand side

This means :

Dividend = divisor x quotient + remainder

Therefore obtained quotient = $2q^2 - 3q + 2$

remainder = -5, is correct

Exercise 4.3

Q. 1. Write the given polynomial in the decreasing order of the variables.

(i) $15x^2+3x+8x^4-4x^3-15$

(ii) $12m^5-9m^3+16-6m^2+8m$

(iii) $9m^4-16m^2-4m+16-m^3$

(iv) $4-8y^3+12y^4-6y^2$

Q. 2. Divide & say whether the divisor are multiple factors of the dividend.

(i) $x^2-11x+30$ *by* $(x-5)$

(ii) $x^2+20x+91$ *by* $(x+7)$

(iii) x^2-5x-6 *by* $(x-6)$

(iv) $x^3-5x^2-2x+24$ *by* $(x-4)$

(v) $a^2+2ab+b^2$ *by* $(a+b)$

Q. 3. Divide and prove that the divisor and the dividend are not multiple factors. Write down the quotient and the remainder for the following expressions.

(i) x^3+2x^2+3x+4 *by* $(x-1)$

(ii) $-12+3x^2-4x+x^3$ *by* $(x+5)$

(iii) $4x^4-2x^3-10x+13x-6$ *by* $(2x+3)$

(iv) $8x^3-6x^2+10x+15$ *by* $(4x+1)$

Q. 4. Divide and verify :

Dividend = Divisor x quotient + remainder.

(i) m^2-3m+7 *by* $m-2$

(ii) a^3-2a^2+a+2 *by* $a+2$

(iii) $9x^3+15x^2-5x+3$ *by* $3x+1$

(iv) $2x^3+3x^2+7x+15$ *by* x^2+4

We Have Learnt

1. Before multiplying two monomial expressions, we multiply factors first and then multiply its variables.
2. To multiply a monomial expression to a binomial expression, the monomial expression is multiplied to each term of the binomial expression and the products are added. Thus the distributive law is used.
3. While multiplying the variable exponential law is followed.
4. To multiply two binomial expressions to each other, the distribution law is used twice e.g.

$$(a+b)(c+d) = a(c+d)+b(c+d)$$

$$= ac+ad+bc+bd$$

5. While multiplying if the sign of the algebraic expression are same, then the sign of the product is also positive and when the signs are dissimilar the product is negative.
6. The process of division is continue till the exponent of the divisor does not become less than the exponent of the remainder for the algebraic expression.
7. To divide a polynomial by a monomial, it is convenient to divide each term of the polynomial by the monomial.