Short Answer Questions-I (PYQ)

[2 Mark]

Q.1. Evaluate: $\int \frac{(x-4)e^x}{(x-2)^3} dx$

Ans.

$$I = \int \left[\frac{(x-2)-2}{(x-2)^3}\right] e^x dx = \int \frac{e^x}{(x-2)^2} dx - 2\int \frac{e^x}{(x-2)^3} dx$$

$$= \frac{e^x}{(x-2)^2} + 2\int \frac{e^x\,dx}{(x-2)^3} - 2\int \frac{e^x\,dx}{(x-2)^3} = \frac{e^x}{(x-2)^2} + C$$

Q.2. Given $\int e^x(\tan x + 1)\sec x dx = e^x f(x) + C$

Write $f(x)$ satisfying the above.

Ans.

$$\text{Given,} \int e^x (\tan x + 1) \sec x\, dx = e^x f(x) + C$$

$$\Rightarrow \int e^x (\tan x \sec x + \sec x) dx = e^x f(x) + C$$

$$\Rightarrow \int e^x (\sec x + \tan x \sec x) dx = e^x f(x) + C$$

$$\Rightarrow e^x \sec x + C = e^x f(x) + C$$

$$\Rightarrow f(x) = \sec x$$

[Note: $[\int e^x [f(x) + f'(x)] dx = e^x . f(x) + C$, Here $f(x) = \sec x]$

Q.3. Evaluate: $\int (1-x)\sqrt{x}\, dx$

Ans.

$$\int(1-x)\sqrt{x}\,dx=\int\sqrt{x}\,dx-\int x^{1+\frac{1}{2}}\,dx=\int x^{\frac{1}{2}}\,dx-\int x^{\frac{3}{2}}\,dx$$

$$=\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}-\frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1}+C=\frac{2}{3}x^{\frac{3}{2}}-\frac{2}{5}x^{\frac{5}{2}}+C$$

Q.4. Evaluate $\int_0^1 \frac{\tan^{-1}x}{1+x^2}dx$

Ans.

Let $t=\tan^{-1}x \quad \Rightarrow \quad dt=\frac{1}{1+x^2}dx$

Also when, $x = 0$, $t = 0$ and when $x = 1 \quad \Rightarrow \; t=\frac{\pi}{4}$

$$\therefore \int_0^1 \frac{\tan^{-1}x}{1+x^2}dx=\int_0^{\frac{\pi}{4}} t\,dt$$

$$=\left[\frac{t^2}{2}\right]_0^{\pi/4}=\frac{1}{2}\left[\frac{\pi^2}{16}-0\right]=\frac{\pi^2}{32}$$

Q.5. Evaluate: $\int_0^1 \frac{dx}{\sqrt{2x+3}}dx$

Ans.

Let $I=\int_0^1 \frac{dx}{\sqrt{2x+3}}=\int_0^1 (2x+3)^{-1/2}\,dx$

$$=\left[\frac{(2x+3)^{-1/2+1}}{\left(-\frac{1}{2}+1\right)\times 2}\right]_0^1=\left[\frac{(2x+3)^{1/2}}{\frac{1}{2}\times 2}\right]_0^1=5^{1/2}-3^{1/2}=\sqrt{5}-\sqrt{3}$$

Short Answer Questions-I (OIQ)

[2 Mark]

Q.1. Evaluate : $\int \frac{\sin x}{\sin (x+a)} dx$

Ans.

Let $x + a = t \Rightarrow dx = dt$

$$\therefore \int \frac{\sin x}{\sin (x+a)} = \int \frac{\sin (t-a)}{\sin t} dt$$

$$= \int \frac{\sin t.\ \cos a - \cos t.\ \sin a}{\sin t} dt$$

$$= \int \cos a dt - \int \cot t.\ \sin a\, dt = \cos a \int dt - \sin a \int \cot t\ dt$$

$$= t.\ \cos a - \sin a.\ \log|\sin t| + C = (x + a)\ \cos a - \sin a.\ \log|\sin\ (x + a)| + C$$

$$= x \cos a + a \cos a - \sin a \log|\sin\ (x + a)| + C$$

$$= x \cos a - \sin a \log|\sin\ (x + a)| + C' \left[C' = C + a \cos a\right]$$

Q.2. Evaluate : $\int_0^{\pi/2} \log (\tan x)\, dx$

Ans.

$$I = \int_0^{\pi/2} \log (\tan x)\, dx \ \ldots (i)$$

$$= \int_0^{\pi/2} \log (\tan (\pi/2 - x))\, dx$$

$$I = \int_0^{\pi/2} \log (\cot x) dx \ldots (ii)$$

Adding (i) and (ii)

$$2I = \int_0^{\pi/2} \{\log (\tan x) + \log (\cot x)\} dx$$

$$= \int_0^{\pi/2} \log (\tan x . \cot x) dx = \int_0^{\pi/2} \log 1\ \ dx$$

$$= 0. \int_0^{\pi/2} dx = 0$$

Q.3. Evaluate : $\int_{-1}^{1} 5x^4 \sqrt{x^5+1}\, dx$

Ans.

Let $I = \int_{-1}^{1} 5x^4 \sqrt{x^5+1}\, dx$

Let $x^5 + 1 = t \quad \Rightarrow 5x^4\, dx = dt$

Also when, $x = -1 \qquad t = 0$ and when $x = 1 \qquad \Rightarrow t = 2$

$\therefore \quad I = \int_0^2 \sqrt{t}\, dt$

$= \int_0^2 t^{1/2} dt = \left[\frac{t^{1/2+1}}{1/2+1}\right]_0^2 = \frac{2}{3}(2^{3/2} - 0) = \frac{2}{3} 2\sqrt{2} = \frac{4\sqrt{2}}{3}$

Q.4. Let *f*(*x*) = *x* – [*x*], for every real number *x*, where [*x*] is the greatest integer less than or equal to *x*.

Evaluate: $\int_{-1}^{1} f(x)\, dx.$

Ans.

We have $f(x) = x - [x]$.

$\therefore \quad \int_{-1}^{1} f(x)\, dx = \int_{-1}^{0} (x - [x])\, dx + \int_{0}^{1} (x - [x])\, dx$

$= \int_{-1}^{0} (x+1) dx + \int_{0}^{1} (x - 0)\, dx$

$\because$ (When $x \in (-1, 0)$; $[x] = -1$; when $x \in (0, 1)$; $[x] = -1$)

$= \left[\frac{x^2}{2} + x\right]_{-1}^{0} + \left[\frac{x^2}{2}\right]_0^1 = \frac{1}{2} + \frac{1}{2} = 1$