

Chapter 12

Equation of Straight line

Exercise 12.1

1. Find the slope of a line whose inclination is

(i) 45°

(ii) 30°

Solution:

The slope of a line having inclination :

(i) 45°

$$\text{Slope} = \tan 45^\circ = 1$$

(ii) 30°

$$\text{Slope} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

2. Find the inclination of a line whose gradient is

(i) 1

(ii) $\sqrt{3}$

(iii) $\frac{1}{\sqrt{3}}$

Solution:

Given,

(i) $\tan \theta = 1$

$\Rightarrow \theta = 45^\circ$

(ii) $\tan \theta = \sqrt{3}$

$\Rightarrow \theta = 60^\circ$

(iii) $\tan \theta = \frac{1}{\sqrt{3}}$

$\Rightarrow \theta = 30^\circ$

3. Find the equation of a straight line parallel to x-axis which is at a distance.

(i) 2 units above it

(ii) 3 units below it.

Solution:

(i) A line which is parallel to x-axis is $y = a$

$\Rightarrow y = 2$

Hence, the equation of line parallel to x-axis which is at a distance of 2 units above it is $y - 2 = 0$.

(ii) A line which is parallel to x –axis is $y = a$

$$\Rightarrow y = -3$$

Hence, the equation of line parallel to x – axis which is at a distance of 3 units below it is $y + 3 = 0$.

4. Find the equation of a straight line parallel to y-axis which is at a distance of :

(i) 3 units to the right

(ii) 2 units to the left.

Solution:

A line which is parallel to y – axis is $x = a$.

(i) Here, $x = 3$

Hence, the equation of line parallel to y-axis is at a distance of 3 units to the right is $x - 3 = 0$.

(ii) Here, $x = -2$

Hence, the equation of line parallel to y-axis at a distance of 2 units to the left is $x + 2 = 0$.

5. Find the equation of a straight line parallel to y-axis and passing through the point (-3,5).

Solution:

The equation of the line parallel to y-axis passing through (-3, 5) is $x = -3$.

$$\Rightarrow x + 3 = 0$$

6. Find the equation of a line whose

(i) slope = 3, y-intercept = -5

(ii) slope = $\frac{-2}{7}$, y-intercept = 3

(iii) gradient = $\sqrt{3}$, y-intercept = $\frac{-4}{3}$

(iv) inclination = 30° , y-intercept = 2

Solution:

Equation of a line whose slope and y-intercept is given by ;

$y = mx + c$, where m is the slope and c is the y-intercept.

(i) Given : slope = 3, y-intercept = -5

$$\Rightarrow y = 3x + (-5)$$

Hence, the equation of line is $y = 3x - 5$.

(ii) Given : Slope = $\frac{-2}{7}$, y-intercept = 3

$$\Rightarrow y = \left(\frac{-2}{7}\right)x + 3$$

$$\Rightarrow \frac{(-2x+21)}{7}$$

$$7y = -2x + 21$$

Hence, the equation of line is $2x + 7y - 21 = 0$.

(iii) Given : gradient = $\sqrt{3}$, y-intercept = $\frac{-4}{3}$

$$\Rightarrow y = \sqrt{3}x + \left(\frac{-4}{3}\right)$$

$$\Rightarrow y = \frac{(3\sqrt{3}x - 4)}{3}$$

$$\Rightarrow 3y = 3\sqrt{3}x - 4$$

Hence, the equation of line of $3\sqrt{3}x - 3y - 4 = 0$.

(iv) Given : inclination = 30° , y-intercept = 2

$$\text{Slope} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\Rightarrow y = \left(\frac{1}{\sqrt{3}}\right)x + 2$$

$$\Rightarrow y = \frac{(x+2\sqrt{3})}{\sqrt{3}}$$

$$\Rightarrow \sqrt{3}y = x + 2\sqrt{3}$$

Hence, the equation of line is $x - \sqrt{3}y + 2\sqrt{3} = 0$.

7. Find the slope and y-intercept of the following lines :

(i) $x - 2y - 1 = 0$

(ii) $4x - 5y - 9 = 0$

(iii) $3x + 5y + 7 = 0$

(iv) $\frac{x}{3} + \frac{y}{4} = 1$

(v) $y - 3 = 0$

(vi) $x - 3 = 0$

Solution:

We know that, equation of line whose slope and y-intercept is given by :

$y = mx + c$, where m is the slope and c is the y-intercept

Using the above and converting to this, we find

(i) $x - 2y - 1 = 0$

$$2y = x - 1$$

$$\Rightarrow y = \left(\frac{1}{2}\right)x + \left(\frac{-1}{2}\right)$$

Hence, slope = $\frac{1}{2}$ and y-intercept = $\frac{-1}{2}$

(ii) $4x - 5y - 9 = 0$

$$5y = 4x - 9$$

$$\Rightarrow y = \left(\frac{4}{5}\right)x + \left(\frac{-9}{5}\right)$$

Hence, slope = $\frac{4}{5}$ and y-intercept = $\frac{-9}{5}$

$$\text{(iii)} \quad 3x + 5y + 7 = 0$$

$$5y = -3x - 7$$

$$\Rightarrow \left(\frac{-3}{5}\right)x + \left(\frac{-7}{5}\right)$$

Hence, slope = $\frac{-3}{5}$ and y-intercept = $\frac{-7}{5}$

$$\text{(iv)} \quad \frac{x}{3} + \frac{y}{4} = 1$$

$$\frac{(4x+3y)}{12} = 1$$

$$4x + 3y = 12$$

$$3y = -4x + 12$$

$$\Rightarrow y = \left(\frac{-4}{3}\right)x + 4$$

Hence, slope = $\frac{-4}{3}$ and y-intercept = 4

$$\text{(v)} \quad y - 3 = 0$$

$$y = 3$$

$$\Rightarrow y = (0)x + 3$$

Hence, slope = 0 and y-intercept = 3

$$\text{(vi)} \quad x - 3 = 0$$

Here, the slope cannot be defined as the line does not meet y-axis.

8. The equation of the line PQ is $3y - 3x + 7 = 0$

(i) Write down the slope of the line PQ.

(ii) Calculate the angle that the line PQ makes with the positive direction of x-axis.

Solution:

Given, equation of line PQ is $3y - 3x + 7 = 0$

Re – writing in form of $y = mx + c$, we have

$$3y = 3x - 7$$

$$\Rightarrow y = x + \left(\frac{-7}{3}\right)$$

Here,

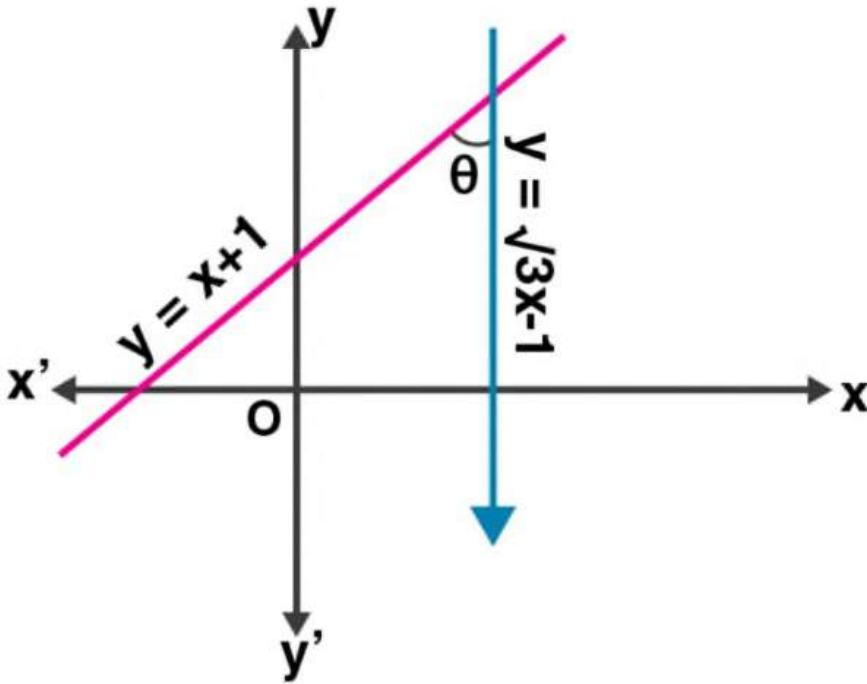
(i) Slope = 1

(ii) as $\tan \theta = 1$

$$\theta = 45^\circ$$

Hence, the angle which PQ makes with the x-axis is 45° .

9. The given figure represents the line $y = x + 1$ and $y = \sqrt{3}x - 1$. write down the angles which the lines make with the positive direction of the x-axis. Hence determine θ .



Solution:

Given line equations, $y = x + 1$ and $y = \sqrt{3}x - 1$

On comparing with $y = mx + c$,

The slope of the line : $y = x + 1$ is 1 as $m = 1$

So, $\tan \theta = 1 \Rightarrow \theta = 45^\circ$

And,

The slope of the line : $y = \sqrt{3}x - 1$ is $\sqrt{3}$ as $m = \sqrt{3}$

So, $\tan \theta = \sqrt{3} \Rightarrow \theta = 60^\circ$

Now, in triangle formed by the given two lines and x-axis

Ext. angle = sum of interior opposite angle

$$60^\circ = \theta + 45^\circ$$

$$\theta = 60^\circ - 45^\circ$$

$$\text{Thus, } \theta = 15^\circ$$

10. Find the value of p , given that the line $\frac{y}{2} = x - p$ passes through the point $(-4, 4)$

Solution :

Given, equation of line : $\frac{y}{2} = x - p$

And, it passes through the point $(-4, 4)$

Hence, it satisfies the line equation

So,

$$\frac{4}{2} = (-4) - p$$

$$2 = -4 - p$$

$$p = -4 - 2$$

$$\text{Thus, } p = -6$$

11. Given that $(a, 2a)$ lies on the line $\frac{y}{2} = 3x - 6$. Find the value of a .

Solution:

Given, equation of line : $\frac{y}{2} = 3x - 6$

and, it passes through the point $(a, 2a)$

Hence, it satisfies the line equation

So,

$$\frac{2a}{2} = 3(a) - 6$$

$$a = 3a - 6$$

$$2a = 6$$

$$\text{Thus, } a = 3$$

12. The graph of the equation $y = mx + c$ passes through the points (1,4) and (-2, -5). Determine the values of m and c.

Solution:

Given, equation of the line is $y = mx + c$

And, it passes through the points (1, 4)

So, the point will satisfy the line equation

$$\Rightarrow 4 = m \times 1 + c$$

$$4 = m + c$$

$$m + c = 4 \dots\dots (i)$$

Also, the line passes through another point (-2, -5)

So,

$$5 = m(-2) + c$$

$$5 = -2m + c$$

$$\Rightarrow 2m - c = 5 \dots (ii)$$

Now, on adding (i) and (ii) we get

$$3m = 9$$

$$\Rightarrow m = 3$$

Substituting the value of m in (i), we get

$$3 + c = 4$$

$$\Rightarrow c = 4 - 3 = 1$$

Therefore, $m = 3$, $c = 1$.

13. Find the equation of the line passing through the point (2, -5) and making an intercept of -3 on the y-axis.

Solution:

Given, a line equation passes through point (2,-5) and makes a y-intercept of -3.

We know that,

The equation of line is $y = mx + c$, where m is the slope and c is the y-intercept

So, we have

$$y = mx - 3$$

Now, this line equation will satisfy the point (2, -5)

$$-5 = m(2) - 3$$

$$-5 = 2m - 3$$

$$2m = 3 - 5 = -2$$

$$\Rightarrow m = -1.$$

Hence, the equation of the line is $y = -x + (-3) \Rightarrow x + y + 3 = 0$

14. Find the equation of a straight line passing through (-1, 2) and whose slope is $\frac{2}{5}$.

Solution:

Given, the equation of straight line passes through (-1, 2) and having slope as $\frac{2}{5}$

So, the equation of the line will be

$$y - y_1 = m(x - x_1)$$

Here, (x_1, y_1) is (-1, 2)

$$\Rightarrow y - 2 = \left(\frac{2}{5}\right)[x - (-1)]$$

$$5(y - 2) = 2(x + 1)$$

$$5y - 10 = 2x + 2$$

Thus, the line equation is $2x - 5y + 12 = 0$.

15. Find the equation of a straight line whose inclination is 60° and which passes through the point (0, -3).

Solution:

Given,

Inclination of a straight line is 60°

So, the slope $= \tan 60^\circ = \sqrt{3} = m$

And, the equation of line passes through the point $(0, -3) = (x_1, y_1)$

Hence, the equation of line is given by

$$y - y_1 = m(x - x_1)$$

$$y + 3 = \sqrt{3}(x - 0)$$

$$y + 3 = \sqrt{3}x$$

$$\sqrt{3}x - y - 3 = 0$$

Thus, the line equation is $\sqrt{3}x - y - 3 = 0$

16. Find the gradient of a line passing through the following pairs of points.

(i) (0, -2), (3, 4)

(ii) (3, -7), (-1, 8)

Solution :

Gradient of a line (m) = $\frac{y_2 - y_1}{x_2 - x_1}$

(i) (0, -2), (3, 4)

$$m = \frac{(4+2)}{(3-0)} = \frac{6}{3} = 2$$

Hence, gradient = 2

(ii) (3, -7), (-1,8)

$$m = \frac{8+7}{-1-3} = \frac{15}{-4}$$

Hence, gradient = $\frac{-15}{4}$.

17. The coordinates of two points E and F are (0,4) and (3,7) respectively. Find :

(i) The gradient of EF

(ii) The equation of EF

(iii) The coordinates of the point where the line EF intersects the x-axis.

Solution:

Given, co-ordinates of points E and F are (0,4) and (3,7) respectively

(i) The gradient of EF

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(7-4)}{(3-0)} = \frac{3}{3}$$

$$\Rightarrow m = 1$$

(ii) Equation of line EF is given by,

$$y - y_1 = m(x - x_1)$$

$$y - 7 = 1(x - 3)$$

$$y - 7 = x - 3$$

$$x - y + 7 - 3 = 0$$

Hence, the equation of line EF is $x - y + 4 = 0$.

(iii) It's seen that the co-ordinates of point of intersection of EF and the x-axis will be $y = 0$

So, substituting the value $y = 0$ in the above equation

$$x - y + 4 = 0$$

$$x - 0 + 4 = 0$$

$$x = -4$$

Hence, the co-ordinates are (-4,0).

18. Find the intercepts made by the line $2x - 3y + 12 = 0$ on the co-ordinate axis.

Solution:

Given line equation is $2x - 3y + 12 = 0$

On putting $y = 0$, we will get the intercept made on x-axis.

$$2x - 3y + 12 = 0$$

$$2x - 3 \times 0 + 12 = 0.$$

$$2x - 0 + 12 = 0$$

$$2x = -12$$

$$\Rightarrow x = -6$$

Now, on putting $x = 0$, we get the intercepts made on y-axis

$$2x - 3y + 12 = 0$$

$$2 \times 0 - 3y + 12 = 0$$

$$-3y = -12$$

$$\Rightarrow y = 4$$

Hence, the x – intercept and y-intercept of the given line is -6 and 4 respectively.

19. Find the equation of the line passing through the points P(5,1) and Q (1, -1). Hence, show that the points P, Q and R (11, 4) are collinear.

Solution:

Give, two points P(5,1) and G(1,-1)

$$\text{Slope of the line (m)} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-1-1}{1-5}$$

$$= \frac{-2}{-4}$$

$$= \frac{1}{2}$$

So, the equation of the line is

$$y - y_1 = m (x - x_1)$$

$$y - 1 = \frac{1}{2}(x - 5)$$

$$2y - 2 = x - 5$$

$$x - 2y - 3 = 0$$

Now, if point R (11,4) is collinear to points P and Q then, R (11, 4) should satisfy the line equation

On substituting, we have

$$11 - 2(4) - 3 = 11 - 8 - 3 = 0$$

As point R satisfy the line equation.

Hence, P, Q and R are collinear.

20. Find the value of 'a' for which the following points A (a, 3), B (2,1) and C (5,a) are collinear. Hence find the equation of the line.

Solution:

Given,

Points A (a,3), B (2,1) and C (5,a) are collinear.

So, slope of AB = slope of BC

$$\frac{1-3}{2-a} = \frac{a-1}{5-2}$$

$$\frac{-2}{2-a} = \frac{a-1}{3}$$

$$\Rightarrow -6 = (a - 1)(2 - a) \text{ [On cross multiplying]}$$

$$-6 = 2a - 2 - a^2 + a$$

$$-6 = 3a - a^2 - 2$$

$$= a^2 - 3a - 4 = 0$$

$$= a^2 - 4a + a - 4 = 0$$

$$a(a - 4) + (a - 4) = 0$$

$$(a + 1)(a - 4) = 0$$

$$a = -1 \text{ or } 4$$

As a = -1 doesn't satisfy the equation

$$\Rightarrow a = 4$$

Now,

$$\text{Slopw of BC} = \frac{(a-1)}{(5-2)} = \frac{(4-1)}{3} = \frac{3}{3} = 1 = m$$

So, the equation of BC is

$$(y - 1) = 1(x - 2)$$

$$x - y = -1 + 2$$

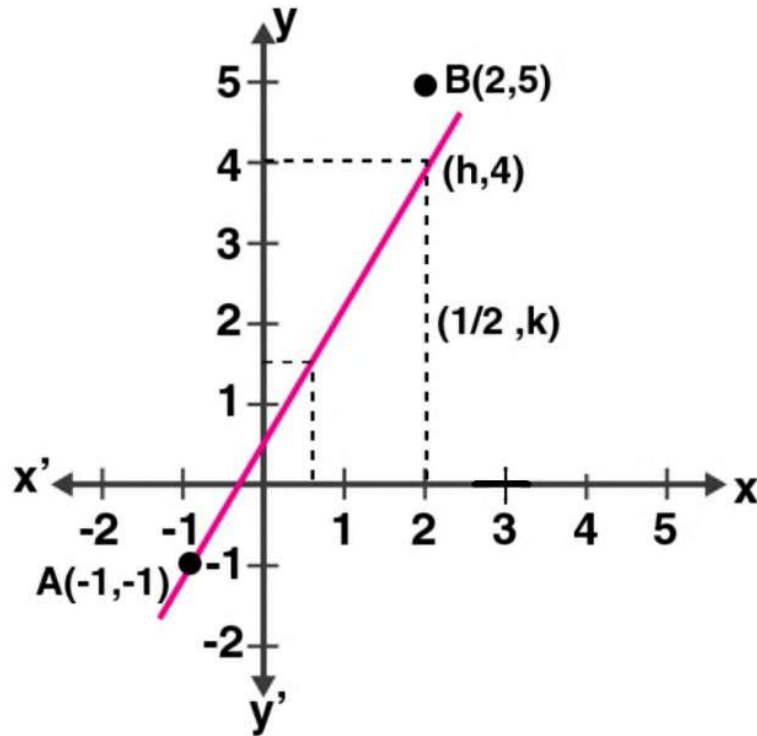
Thus, the equation of BC is $x - y = 1$.

21. Use a graph paper for this equation. The graph of a linear equation in x and y, passees through A (-1, -1) and B (2,5). From your graph, find the values of h and k, if the line passes through (h, 4) and $(\frac{1}{2}, k)$.

Solution:

Given,

Points (h,4) and $(\frac{1}{2}, k)$ lie on the line passing through A(-1, -1) and B (2,5)



From the graph, its clearly seen that

$$h = \frac{3}{2} \text{ and}$$

$$k = 2$$

22. ABCD is a parallelogram where A(x,y), B(5,8), C(4,7) and D(2, -4). Find

(i) The coordinates of A

(ii) The equation of the diagonal BD.

Solution:

(i) Given,

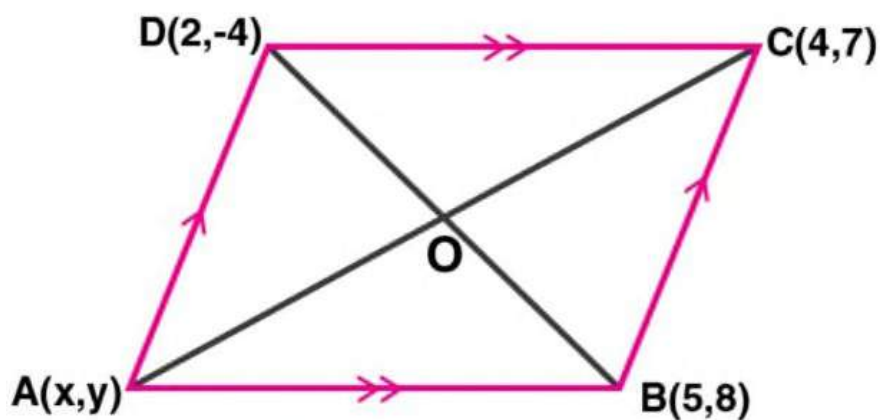
ABCD is a parallelogram where A(x,y), B(5,8), C(4,7) and D(2, -4)

O is the point of intersection of the diagonals of the parallelogram

So, the co-ordinates of O = $\left(\frac{[5+2]}{2}, \frac{[8-4]}{2}\right) = (3.5, 2)$

Now, for the line AC we have

$$3.5 = \frac{(x+4)}{2} \text{ and } 2 = \frac{(y+7)}{2}$$



$$7 = x + 4 \text{ and } 4 = y + 7$$

$$x = 7 - 4 \text{ and } y = 4 - 7$$

$$x = 3 \text{ and } y = -3$$

Thus, the co-ordinates of A are (3, -3).

(ii) Equation of diagonal BD is given by

$$y - 8 = \frac{(-4-8)}{(2-5)} \times (x - 5)$$

$$y - 8 = \left(\frac{-12}{-3}\right) \times (x - 5)$$

$$y - 8 = 4(x - 5)$$

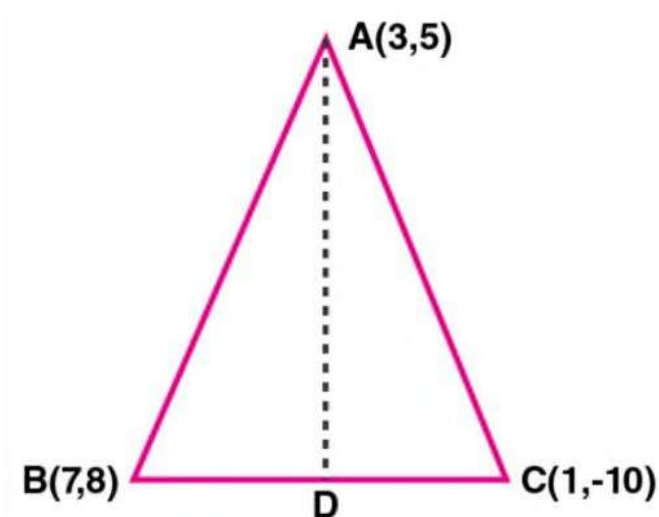
$$y - 8 = 4x - 20$$

$$4x - y - 20 + 8 = 0$$

Hence, the equation of the diagonal is $4x - y - 12 = 0$.

23. In $\triangle ABC$, $A(3, 5)$, $B(7, 8)$ and $C(1, -10)$. Find the equation of the median through A.

Solution:



Given,

ΔABC and their vertices A(3,5), B(7,8) and C(1, -10).

And, AD is median

So, D is mid-point of BC

Hence, the co-ordinates of D is $\left(\frac{[7+1]}{2}, \frac{[8-10]}{2}\right) = (4, -1)$

Now,

Slope of AD, $m = \frac{y_2 - y_1}{x_2 - x_1}$

$$m = \frac{(5+1)}{(3-4)}$$

$$= \frac{6}{-1}$$

$$= -6$$

Thus, the equation of AD is given by

$$y - y_1 = m(x - x_1)$$

$$y + 1 = -6(x - 4)$$

$$y + 1 = -6x + 24$$

$$\Rightarrow 6x + y - 23 = 0$$

24. Find the equation of a line passing through the point (-2, 3) and having x – intercept 4 units.

Solution:

Given, point (-2, 3) and the x-intercept of the line passing through that point is 4 units.

So, the co-ordinates of the point where the line meets the x-axis is (4,0)

Now, slope of the line passing through the points (-2,3) and (4,0)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{(0-3)}{(4+2)} = \frac{-3}{6} = \frac{-1}{2}$$

Hence, the equation of the line will be

$$y - y_1 = m (x - x_1)$$

$$y - 0 = \frac{-1}{2}(x - 4)$$

$$2y = -x + 4$$

$$\Rightarrow x + 2y = 4$$

25. Find the equation of the line whose x-intercept is 6 and y-intercept is -4.

Solution:

Given, x-intercept of a line is 6

So,

The line will pass through the point (6,0)

Also given, the y-intercept of the line is $-4 \Rightarrow c = -4$

So, the line will pass through the point (0, -4)

Now,

$$\text{Slope, } m = m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{(-4 - 0)}{(0 - 6)} = \frac{-4}{-6} = \frac{2}{3}$$

Thus, the equation of the line is given by

$$y = mx + c$$

$$y = \left(\frac{2}{3}\right)x + (-4)$$

$$3y = 2x - 12$$

$$\Rightarrow 2x - 3y - 12 = 0$$

26. Write down the equation of the line whose gradient is $\frac{3}{2}$ and which passes through P where P divides the line segment joining A (-2, 6) and B(3, -4) in the ratio 2 : 3.

Solution:

Given, P divides the line segment joining the points A (-2,6) and B (3, -4) in the ratio 2 : 3.

So, the co-ordinates of P will be

$$x = \frac{(m_1x_2+m_2x_1)}{(m_1+m_2)}$$

$$= \frac{(2 \times 3 + 3 \times (-2))}{(2+3)}$$

$$= \frac{(6-6)}{5}$$

$$= \frac{0}{5} = 0$$

$$y = \frac{(m_1y_2+m_2y_1)}{(m_1+m_2)}$$

$$= \frac{(2 \times (-4) + 3 \times (6))}{(2+3)}$$

$$= \frac{(-8+18)}{5}$$

$$= \frac{10}{5}$$

$$= 2$$

Hence, the co-ordinates of P are (0,2)

Now, the slope (m) of the line passing through (0,2) is $\frac{3}{2}$

Thus, the equation will be

$$y - y_1 = m(x - x_1)$$

$$y - 2 = \frac{3}{2}(x - 0)$$

$$2y - 4 = 3x$$

$$\Rightarrow 3x - 2y + 4 = 0$$

27. Find the equation of the line passing through the point (1,4) and intersecting the line $x - 2y - 11 = 0$ on the y-axis.

Solution:

Given, line $x - 2y - 11 = 0$ passes through y- axis and point (1,4)

So, putting $x = 0$ in the line equation we get the y-intercept

$$0 - 2y - 11 = 0$$

$$y = \frac{-11}{2}$$

The co-ordinates are $\left(0, \frac{-11}{2}\right)$

Now, the slope of the line joining the points (1,4) and $\left(0, \frac{-11}{2}\right)$ is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{\left(\frac{-11}{2} - 4\right)}{(0 - 1)}$$

$$= \frac{19}{2}$$

Thus, the line equation will be

$$y - y_1 = m (x - x_1)$$

$$y + \frac{11}{2} = \frac{19}{2} (x - 0)$$

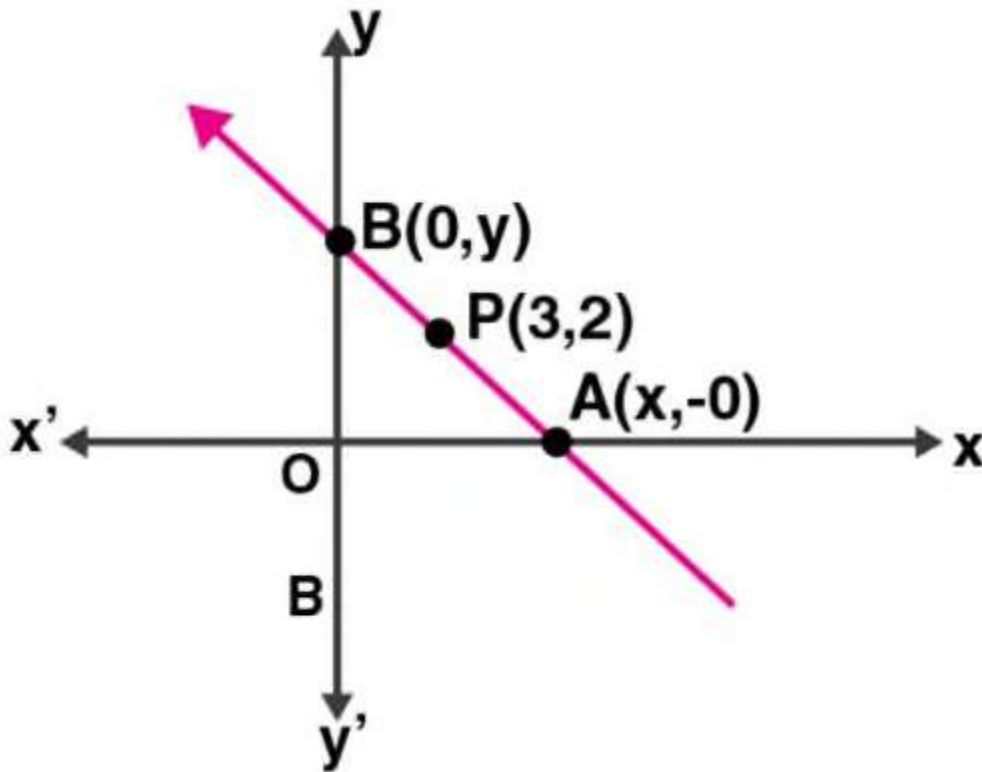
$$2y + 11 = 19x$$

$$\Rightarrow 19x - 2y - 11 = 0$$

28. Find the equation of the straight line containing the point (3,2) and making positive equal intercepts on axes.

Solution:

Let the line containing the point P (3,2) pass through x-axis at A(x,0) and y-axis at B (0,y)



Given $OA = OB$

Thus, $x = y$

Now, the slope of the line (m) $= \frac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{0-y}{x-0}$$

$$= \frac{-y}{x}$$

$$= -1$$

Hence, the equation of the line will be

$$y - y_1 = m(x - x_1)$$

$$y - 2 = -1(x - 3)$$

$$y - 2 = -x + 3$$

$$\Rightarrow x + y - 5 = 0$$

29. Three vertices of a parallelogram ABCD taken in order are A(3, 6), B(5, 10) and C(3, 2) find :

(i) The coordinates of the fourth vertex D.

(ii) length of diagonal BD.

(iii) equation of side AB of the parallelogram ABCD.

Solution:

Given, the three vertices of a parallelogram ABCD taken in order are A(3, 6), B(5, 10) and C(3, 2)

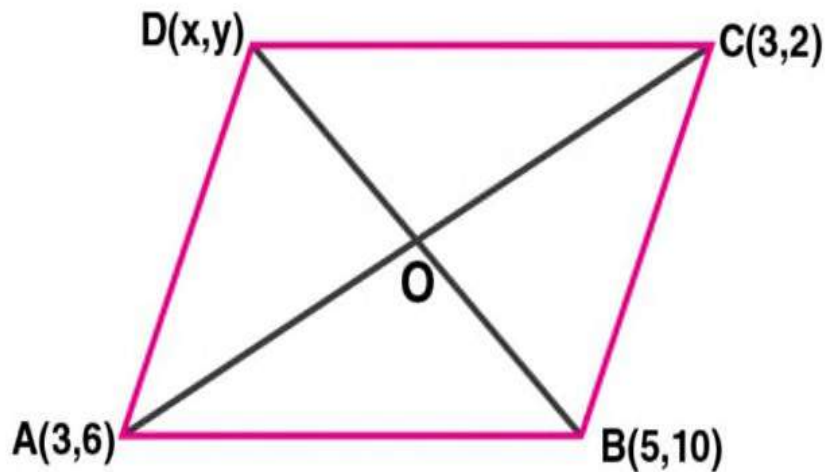
(i) We know that the diagonals of a parallelogram bisect each other.

Let (x, y) be the co-ordinates of D

Hence, we have

$$\text{Mid-point of diagonal AC} = \left(\frac{(3+3)}{2}, \frac{(6+2)}{2} \right) = (3,4)$$

$$\text{Mid-point of diagonal BD} = \left(\frac{(5+x)}{2}, \frac{(10+y)}{2} \right)$$



And, these two should be the same

On equating we get,

$$\frac{(5+x)}{2} = 3 \quad \text{and} \quad \frac{(10+y)}{2} = 4$$

$$5 + x = 6 \quad \text{and} \quad 10 + y = 8$$

$$x = 1 \quad \text{and} \quad y = -2$$

Thus, the co-ordinates of D = (1, -2)

(ii) Length of diagonal BD

$$= \sqrt{(1 - 5)^2 + (-2 - 10)^2} = \sqrt{(4)^2 + (-12)^2}$$

$$= \sqrt{16 + 144}$$

$$= \sqrt{160} \text{ units}$$

(iii) Equation of the side joining A (3,6) and D(1, -2) is given by

$$\frac{x-3}{3-1} = \frac{y-6}{6+2}$$

$$\Rightarrow \frac{x-3}{2} = \frac{y-6}{8}$$

$$4(x - 3) = y - 6$$

$$4x - 12 = y - 6$$

$$4x - y = 6$$

Thus, the equation of the side joining A(3,6) and D(1, -2) is $4x - y = 6$.

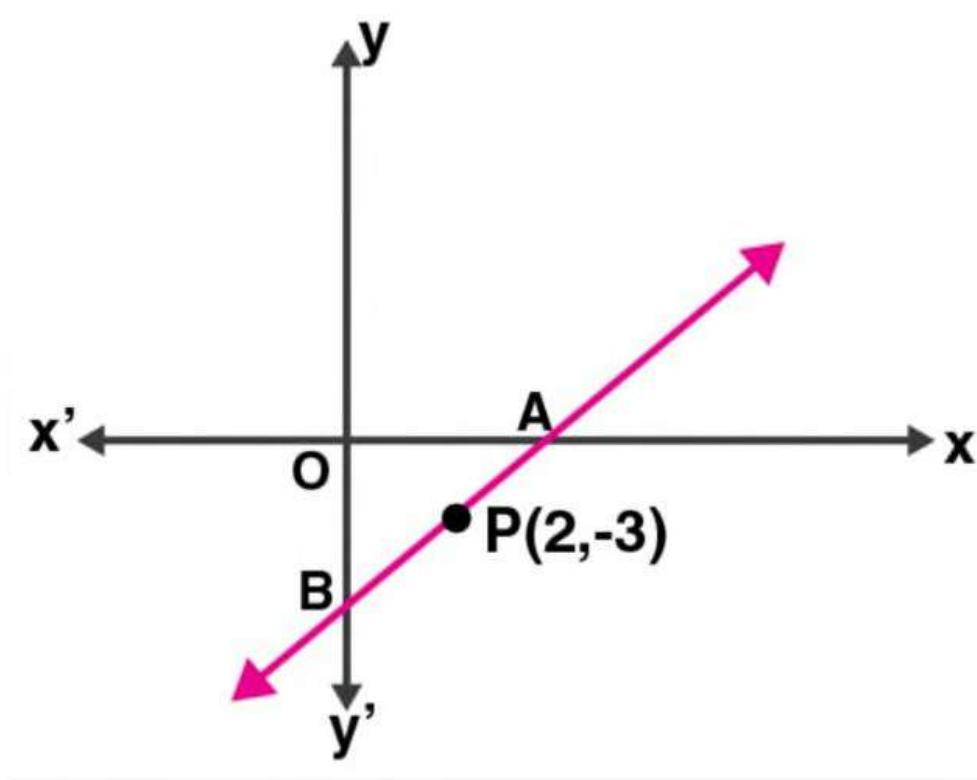
30. A and B are two points on the x-axis and y-axis respectively. P (2, -3) is the mid point of AB. Find the

(i) the co-ordinates of A and B.

(ii) the slope of the line AB.

(iii) the equation of the line AB.

Solution:



Given, points A and B are on x-axis and y-axis respectively

Let co-ordinates of A be $(x,0)$ and of B be $(0,y)$ and $P(2, -3)$ is the midpoint of AB

So, we have

$$2 = \frac{(x+0)}{2} \text{ and } -3 = \frac{(0+y)}{2}$$

$$x = 4 \text{ and } y = -6$$

(i) Hence, the co-ordinates of A are $(4,0)$ and of B are $(0, -6)$.

(ii) Slope of AB = $\frac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{(-6-0)}{(0-4)}$$

$$= \frac{-6}{-4}$$

$$= \frac{3}{2} = m$$

(iii) Equation of AB will be

$$y - y_1 = m(x - x_1)$$

$$y - (-3) = \frac{3}{2} (x - 2) \text{ [As P lies on it]}$$

$$y + 3 = \frac{3}{2} (x - 2)$$

$$2y + 6 = 3x - 6$$

$$3x - 2y - 12 = 0$$

31. Find the equations of the diagonals of a rectangle whose sides are $x = -1$, $x = 2$, $y = -2$ and $y = 6$.

Solution:

Given,

The equations of sides of a rectangle are

$$x_1 = -1, x_2 = 2, y_1 = -2 \text{ and } y_2 = 6.$$

The equations of sides of a rectangle are

$$x_1 = -1, x_2 = 2, y_1 = -2, y_2 = 6.$$

These lines form a rectangle when they intersect at A, B, C, D respectively

Now,

The co-ordinates of A, B, C and D will be (-1, -2), (2, -2), (2,6) and (-1, 6) respectively.

And, AC and BD are its diagonals.

Slope of the diagonal AC

$$= \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{(6 - (-2))}{(2 - (-1))}$$

$$= \frac{8}{3} = m$$

So, the equation of AC will be

$$y - y_1 = m(x - x_1)$$

$$y + 2 = \frac{8}{3}(x + 1)$$

$$3y + 6 = 8x + 8$$

$$\Rightarrow 8x - 3y + 2 = 0$$

32. Find the equation of a straight line passing through the origin and through the point of intersection of the line $5x + 1y - 3$ and $2x - 3y = 7$

Solution:

Given line equations,

$$5x + 7y = 3 \dots\dots\dots (i)$$

$$2x - 3y = 7 \dots\dots (ii)$$

Now, performing multiplication of (i) by 3 and (ii) by 7, we get

$$15x + 21y = 9$$

$$14x - 21y = 49$$

On adding we get,

$$29x = 58$$

$$x = \frac{58}{29} = 2$$

Substituting the value of x in (i), we get

$$5(2) + 7y = 3$$

$$10 + 7y = 3$$

$$7y = 3 - 10$$

$$y = \frac{-7}{7} = -1$$

Hence, the point of intersection of lines is (2, -1)

Now, the slope of the line joining the points (2, -1) and (0, 0) will be

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{(0 - (-1))}{(0 - 2)}$$

$$= \frac{-1}{2}$$

Equation of the line is given by :

$$y - y_1 = m(x - x_1)$$

$$y - 0 = \frac{-1}{2}(x - 0)$$

$$2y = -x$$

Thus the required line equation is $x + 2y = 0$.

33. Point A (3,-2) on reflection in the x-axis is mapped as 'A' and point B on reflection in the y-axis is mapped onto B' (-4, 3).

(i) Write down the co-ordinates of A' and B.

(ii) Find the slope of the line A'B, hence find its inclination.

Solution:

Given,

‘A’ is the image of A(3, -2) on reflection in the x-axis.

(i) The co-ordinates of A’ will be (3,2).

Again B; (-4, 3) is the image of A’, when reflected in the y-axis

Hence, the co-ordinates of B will be (4, 3)

(ii) Slope of the line joining the points A’ (3,2) and B (4,3) will be

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{(2-3)}{(3-4)}$$

$$= \frac{-1}{-1}$$

$$= 1$$

So, $\tan\theta = 45^\circ$

Thus, the angle of inclination is 45° .

Exercise 12.2

1. State which one of the following is true : The straight line $y = 3x - 5$ and $2y = 4x + 7$ are

- (i) Parallel
- (ii) perpendicular
- (iii) neither parallel nor perpendicular

Solution:

Given straight lines : $y = 3x - 5$ and $2y = 4x + 7 \Rightarrow y = 2x + \frac{7}{2}$

And, their slopes are 3 and 2

The product of slopes is $3 \times 2 = 6$.

Hence, as the slopes of both lines are neither equal nor their product is -1 the given pair of straight lines are neither parallel nor perpendicular.

2. If $6x + 5y - 7 = 0$ and $2px + 5y + 1 = 0$ are parallel lines, find the value of p.

Solution:

For two lines to be parallel, their slopes must be same.

Given line equations,

$$6x + 5y - 7 = 0 \text{ and } 2px + 5y + 1 = 0$$

$$6x + 5y - 7 = 0$$

$$2px + 5y + 1 = 0$$

In equation $6x + 5y - 7 = 0$,

$$5y = -6x + 7$$

$$y = \left(\frac{-6}{5}\right)x + \frac{7}{5}$$

So, the slope of the line (m_1) = $\frac{-6}{5}$

Again, in equation $2px + 5y + 1 = 0$

$$5y = -2px - 1$$

$$y = \left(\frac{-2p}{5}\right)x - \frac{1}{5}$$

So, the slope of the line (m_2) = $\left(\frac{-2p}{5}\right)$

For these two lines to be parallel

$$m_1 = m_2$$

$$\frac{-6}{5} = \frac{-2p}{5}$$

$$p = \left(\frac{-6}{5}\right) \times \left(\frac{-5}{2}\right)$$

Thus, $p = 3$

3. Line $2x - by + 5 = 0$ and $ax + 3y = 2$ are parallel. Find the relation connecting a and b.

Solution:

Given lines are : $2x - by + 5 = 0$

and $ax + 3y = 2$

If two lines to be parallel then their slopes must be equal.

In equation $2x - by + 5 = 0$

$$by = 2x + 5$$

$$y = \left(\frac{2}{b}\right)x + \frac{5}{b}$$

So, the slope of the line $(m_1) = \frac{2}{b}$

And in equation $ax + 3y = 2$.

$$3y = -ax + 2$$

$$y = \left(\frac{-a}{3}\right)x + \frac{2}{3}$$

So, the slope of the line $(m_2) = \left(\frac{-a}{3}\right)$

As the lines are parallel

$$m_1 = m_2$$

$$\frac{2}{b} = \frac{-a}{3}$$

$$6 = -ab$$

Hence, the relation connecting a and b is $ab + 6 = 0$

4. Given that the line $\frac{y}{2} = x - p$ and the line $ax + 5 = 3y$ are parallel, find the value of a.

Solution:

Given,

Line equation : $\frac{y}{2} = x - p$

$$\Rightarrow y = 2x - 2p$$

Here, the slope of the line is 2.

And, another line equation : $ax + 5 = 3y$

$$\Rightarrow 3y = ax + 5$$

$$y = \left(\frac{a}{3}\right)x + \frac{5}{3}$$

Hence, the slope of the line is $\frac{a}{3}$

As the line are parallel, their slopes must be equal

$$\Rightarrow 2 = \frac{a}{3}$$

$$a = 6$$

Thus, the value of a is 6.

5. If the lines $y = 3x + 7$ and $2y + px = 3$ are perpendicular to each other, find the value of p .

Solution:

If two lines are perpendicular, then the product of their slopes is -1

Now, slope of the line $y = 3x + 7$ is $m_1 = 3$

And,

The slope of the line : $2y + px = 3$

$$2y = -px + 3$$

$$y = \left(\frac{-p}{2}\right)x + 3$$

$$m_2 = \frac{-p}{2}$$

As the lines are perpendicular,

$$\Rightarrow m_1 \times m_2 = -1$$

$$= 3 \times \left(\frac{-p}{2}\right) = -1$$

$$p = \frac{2}{3}$$

Thus, the value of p is $\frac{2}{3}$.

6. If the straight lines $kx - 5y + 4 = 0$ and $4x - 2y + 5 = 0$ are perpendicular to each other. Find the value of k.

Solution:

Given,

In equation, $kx - 5y + 4 = 0$

$$\Rightarrow 5y = kx + 4$$

$$y = \left(\frac{k}{5}\right)x + \frac{4}{5}$$

So, the slope (m_1) = $\frac{k}{5}$

And, in equation $4x - 2y + 5 = 0$

$$\Rightarrow 2y = 4x + 5$$

$$y = 2x + \frac{5}{2}$$

So, the slope (m_2) = 2

As the line are perpendicular to each other

$$\Rightarrow m_1 \times m_2 = -1$$

$$= \frac{k}{5} \times 2 = -1$$

$$= k = \frac{(-1 \times 5)}{2}$$

Hence, the value of k = $\frac{-5}{2}$.

7. If the lines $3x + by + 5 = 0$ and $ax - 5y + 7 = 0$ are perpendicular to each other, find the relation connection a and b.

Solution:

Given that the lines $3x + by + 5 = 0$ and $ax - 5y + 7 = 0$ are perpendicular to each other

Then the product of their slopes must be -1.

Slope of line $3x + by + 5 = 0$ is,

$$by = -3x - 5$$

$$y = \left(\frac{-3}{b}\right)x - \frac{5}{b}$$

$$\text{So, slope } (m_1) = \frac{-3}{b}$$

And,

The slope of line $ax - 5y + 7 = 0$ is

$$5y = ax + 7$$

$$y = \left(\frac{a}{5}\right)x + \frac{7}{5}$$

$$\text{So, slope } (m_2) = \frac{a}{5}$$

As the lines are perpendicular, we have

$$m_1 \times m_2 = -1$$

$$\frac{-3}{b} \times \frac{a}{5} = -1$$

$$\frac{-3a}{5b} = -1$$

$$-3a = -5b$$

$$3a = 5b$$

Hence, the relation connecting a and b is $3a = 5b$.

8. Is the line through (-2, 3) and (4,1) perpendicular to the line $3x = y + 1$?

Does the line $3x = y + 1$ bisect the join of (-2, 3) and (4, 1)

Solution:

Slope of the line passing through the points (-2, 3) and (4, 1) is given by

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{(1-3)}{(4+2)}$$

$$= \frac{-2}{6}$$

$$= \frac{-1}{3}$$

And, the slope of the line : $3x = y + 1$

$$y = 3x - 1$$

$$\text{Slope } (m_2) = 3$$

Now,

$$m_1 \times m_2 = \frac{-1}{3} \times 3 = -1$$

Thus, the lines are perpendicular to each other as the product of their slopes is -1.

Now,

Co-ordinates of the mid-point of the line joining the points (-2, 3) and (4, 1) is

$$\left(\frac{[-2+4]}{2}, \frac{[3+1]}{2} \right) = (1, 2)$$

Now, if the line $3x = y + 1$ passes through the mid-point then it will satisfy the equation

$$3(1) = (2) + 1$$

$$3 = 3$$

Hence, the line $3x = y + 1$ bisects the line joining the points (-2, 3) and (4, 1).

9. The line through A(-2, 3) and B(4,b) is perpendicular to the line $2x - 4y = 5$. Find the value of b.

Solution:

The slope of the line passing through A(-2, 3) and B(4,b) will be $m_1 = \frac{(b-3)}{(4+2)} = \frac{(b-3)}{6}$

Now, the gradient of the given line $2x - 4y = 5$ is

$$4y = 2x + 5$$

$$y = \left(\frac{2}{4}\right)x + \frac{5}{4}$$

$$y = \frac{1}{2}x + \frac{5}{4}$$

$$\text{So, } m_2 = \frac{1}{2}$$

As the line are perpendicular to each other, we have

$$m_1 \times m_2 = -1$$

$$\frac{(b-3)}{6} \times \frac{1}{2} = -1$$

$$\frac{(b-3)}{12} = -1$$

$$(b - 3) = -12$$

$$b = -12 + 3 = -9$$

Hence, the value of b is -9.

10. If the lines $3x + y = 4$, $x - ay + 7 = 0$ and $bx + 2y + 5 = 0$ form three consecutive sides of a rectangle, find the value of a and b.

Solution:

Given lines are;

$$3x + y = 4 \dots (i)$$

$$x - ay + 7 = 0 \dots\dots(ii)$$

$$bx + 2y + 5 = 0 \dots(iii)$$

It's said that these lines form three consecutive sides of a rectangle.

So,

Lines (i) and (ii) must be perpendicular

Also, lines (ii) and (iii) must be perpendicular

We know that, for two perpendicular lines the product of their slopes will be -1.

Now,

Slope of line (i) is

$$3x + y = 4 \Rightarrow y = -3x + 4$$

Hence, slope (m_1) = -3

And, slope of line (ii) is

$$x - ay + 7 = 0 \Rightarrow ay = x + 7$$

$$y = \left(\frac{1}{a}\right)x + \frac{7}{a}$$

Hence, slope (m_2) = $\frac{1}{a}$

Finally, the slope of line (iii) is

$$bx + 2y + 5 = 0 \Rightarrow 2y = -bx - 5$$

$$y = \left(\frac{-b}{2}\right)x - \frac{5}{2}$$

Hence, slope (m_3) = $\frac{-b}{2}$

As lines (i), (ii) and (iii) are consecutive sides of rectangle, we have

$$= m_1 \times m_2 = -1 \text{ and } m_2 \times m_3 = -1$$

$$= (-3) \times \left(\frac{1}{a}\right) = -1 \text{ and } \left(\frac{1}{a}\right) \times \left(\frac{-b}{2}\right) = -1$$

$$= -3 = -a \text{ and } \frac{-b}{2a} = -1$$

$$= a = 3 \text{ and } b = 2a$$

$$\Rightarrow b = 2(3) = 6$$

Thus, the value of a is 3 and the value of b is 6.

11. Find the equation of a line, which has the y-intercept 4, and is parallel to the line $2x - 3y - 7 = 0$. Find the coordinates of the point where it cuts the x-axis.

Solution:

$$\text{Given line} = 2x - 3y - 7 = 0$$

Its slope is,

$$3y = 2x - 7$$

$$y = \left(\frac{2}{3}\right)x - \frac{7}{3}$$

$$\Rightarrow m = \frac{2}{3}$$

So, the equation of the line parallel to the given line will be $\frac{2}{3}$.

Also given, the y-intercept is $4 = c$

Hence, the equation of the line is given by

$$y = mx + c$$

$$y = \left(\frac{2}{3}\right)x + 4$$

$$3y = 2x + 12$$

$$2x - 3y + 12 = 0$$

Now, when this line intersects the x-axis the y co-ordinate becomes zero.

So, putting $y = 0$ in the line equation, we get

$$2x - 3(0) + 12 = 0$$

$$2x + 12 = 0$$

$$x = \frac{-12}{2} = -6$$

Hence the co-ordinates of the point where it cuts the x-axis is $(-6, 0)$.

12. Find the equation of a straight line perpendicular to the line $2x + 5y + 7 = 0$ and with y-intercept -3 units.

Solution:

Given line : $2x + 5y + 7 = 0$

So, its slope is given by

$$5y = -2x - 7$$

$$y = \left(\frac{-2}{5}\right)x - \frac{7}{5}$$

$$\Rightarrow m = \frac{-2}{5}$$

Now, let the slope of the line perpendicular to this line be m'

Then,

$$m \times m' = -1$$

$$= \left(\frac{-2}{5}\right) \times m' = -1$$

$$\Rightarrow m' = \frac{5}{2}$$

Also given, the y-intercept (c) = -3

Hence, the equation of the line is given by

$$y = m'x + c$$

$$y = \left(\frac{5}{2}\right)x + (-3)$$

$$2y = 5x - 6$$

$$5x - 2y - 6 = 0$$

13. Find the equation of a st. line perpendicular to the line $3x - 4y + 12 = 0$ and having same y-intercept as $2x - y + 5 = 0$.

Solution:

Given line : $3x - 4y + 12 = 0$

The slope of the line is given by

$$3x - 4y + 12 = 0 \Rightarrow 4y = 3x + 12$$

$$y = \left(\frac{3}{4}\right)x + 3$$

$$\text{Thus, slope } (m_1) = \frac{3}{4}$$

Now, let the slope of the line perpendicular to the given line be taken as m_2

So,

$$m_1 \times m_2 = -1$$

$$\left(\frac{3}{4}\right) \times m_2 = -1$$

$$m_2 = \frac{-4}{3}$$

And, given, the y-intercept of the line is same as $2x - y + 5 = 0$

$$\Rightarrow y = 2x + 5$$

So, the y- intercept is $5 = c$.

Hence, the equation of line is given by

$$y = m_2x + c$$

$$y = \left(\frac{-4}{3}\right)x + 5$$

$$3y = -4x + 15$$

$$4x + 3y = 15$$

14. Find the equation of the line which is parallel to which is parallel to $3x - 2y = -4$ and passes through the point $(0, 3)$.

Solution:

Given line : $3x - 2y = -4$

Slope (m_1) is given by

$$2y = 3x + 4$$

$$y = \left(\frac{3}{2}\right)x + 2$$

$$\text{So, } m_1 = \frac{3}{2}$$

Now, the slope of the line parallel to the given line will have the same slope as $\frac{3}{2} = m$

And the line passes through point $(0, 3)$

Thus, the equation of the required line is given by

$$y = mx + c$$

$$y = \left(\frac{3}{2}\right)x + 3$$

$$2y = 3x + 6$$

$$3x - 2y + 6 = 0$$

15. Find the equation of the line passing through (0, 4) and parallel to the line $3x + 5y + 15 = 0$.

Solution:

$$\text{Given line : } 3x + 5y + 15 = 0$$

$$5y = -3x - 15$$

$$y = \left(\frac{-3}{5}\right)x - 3$$

$$\text{So, slope (m)} = \left(\frac{-3}{5}\right)$$

The slope of the line parallel to the given line will be the same $\frac{-3}{5}$

And, the line passes through the point (0, 4)

Hence, equation of the line will be

$$y - y_1 = m(x - x_1)$$

$$y - 4 = \left(\frac{-3}{5}\right)(x - 0)$$

$$5y - 20 = -3x$$

$$3x + 5y - 20 = 0$$

16. The equation of line is $y = 3x - 5$, Write down the slope of this line and the intercept made by it on the y-axis. Hence or otherwise, write down the equation of a line which is parallel to the line and which passes through the point (0, 5).

Solution:

Given line : $y = 3x - 5$

Here slope (m_1) = 3

Substituting $x = 0$, we get $y = -5$

Hence, the y-intercept = -5

Now, the slope of the line parallel to the given line will be 3 and it passes through the point (0, 5).

Thus, equation of the line will be

$$y - y_1 = m(x - x_1)$$

$$y - 5 = 3(x - 0)$$

$$y = 3x + 5$$

17. Write down the equation of the line perpendicular to $3x + 8y = 12$ and passing through the point $(-1, -2)$.

Solution:

Given line ; $3x + 8y = 12$

$$8y = -3x + 12$$

$$y = \left(\frac{-3}{8}\right)x + 12$$

$$\text{So, the slope } (m_1) = \frac{-3}{8}$$

Let's consider the slope of the line perpendicular to the given line as m_2

$$\text{Then, } m_1 \times m_2 = -1$$

$$\frac{-3}{8} \times m_2 = -1$$

$$m_2 = \frac{8}{3}$$

Now,

The equation of the line perpendicular to the given line and passing through the point $(-1, -2)$ will be

$$y - y_1 = m(x - x_1)$$

$$y - (-2) = \left(\frac{8}{3}\right)(x - (-1))$$

$$y + 2 = \left(\frac{8}{3}\right)(x + 1)$$

$$3y + 6 = 8x + 8$$

$$3y = 8x + 2$$

Thus, the equation of the required line is $3y = 8x + 2$.

18. (i) The line $4x - 3y + 12 = 0$ meets the x-axis at A. Write down the co-ordinates of A.

(ii) Determine the equation of the line passing through A and perpendicular to $4x - 3y + 12 = 0$.

Solution:

Given line : $4x - 3y + 12 = 0$

(i) When this line meets the $x - axis$, its y co-ordinate becomes 0.

So, putting $y = 0$ in the given equation, we get

$$4x - 3(0) + 12 = 0$$

$$4x + 12 = 0$$

$$x = \frac{-12}{4}$$

$$x = -3$$

Hence, the line meets the x-axis at A (-3, 0).

(ii) Now, the slope of the line is given by

$$4x - 3y + 12 = 0$$

$$3y = 4x + 12$$

$$y = \left(\frac{4}{3}\right)x + 4$$

$$\Rightarrow m_1 = \frac{4}{3}$$

Let's assume the slope of the line perpendicular to the given line be m_2

$$\text{Then, } m_1 \times m_2 = -1$$

$$\frac{4}{3} \times m_2 = -1$$

$$m_2 = \frac{-3}{4}$$

Thus, the equation of the line perpendicular to the given line passing through A will be

$$y - 0 = \frac{-3}{4}(x + 3)$$

$$4y = -3(x + 3)$$

$$3x + 4y + 9 = 0$$

19. Find the equation of the line that is parallel to $2x + 5y - 7 = 0$ and passes through the mid-point of the line segment joining the points (2, 7) and (-4,1).

Solution:

$$\text{Given line : } 2x + 5y - 7 = 0$$

$$5y = -2x + 7$$

$$y = \left(\frac{-2}{5}\right)x + \frac{7}{5}$$

So, the slope is $\frac{-2}{5}$

Hence, the slope of the line that is parallel to the given line will be the same, $m = \frac{-2}{5}$

Now, the mid-point of the line segment joining points (2, 7) and (-4, 1) is

$$\left(\frac{(2-4)}{2}, \frac{(7+1)}{2}\right) = (-1, 4)$$

Thus, the equation of the line will be

$$y - y_1 = m(x - x_1)$$

$$y - 4 = \left(\frac{-2}{5}\right)(x + 1)$$

$$5y - 20 = -2x - 2$$

$$2x + 5y = 18$$

20. Find the equation of the line that is perpendicular to $3x + 2y - 8 = 0$ and passes through the mid-point of the line segment joining the points $(5, -2)$ and $(2, 2)$.

Solution:

Given line : $3x + 2y - 8 = 0$

$$2y = -3x + 8$$

$$y = \left(\frac{-3}{2}\right)x + 4$$

$$\text{Here, slope } (m_1) = \frac{-3}{2}$$

Now, the co-ordinates of the mid-point of the line segment joining the points $(5, -2)$ and $(2, 2)$ will be

$$\left(\frac{(5+2)}{2}, \frac{(-2+2)}{2}\right) = \left(\frac{7}{2}, 0\right)$$

Let's consider the slope of the line perpendicular to the given line be m_2

Then,

$$m_1 \times m_2 = -1$$

$$\left(\frac{-3}{2}\right) \times m_2 = -1$$

$$m_2 = \frac{2}{3}$$

So, the equation of the line with slope m_2 and passing through $\left(\frac{7}{2}, 0\right)$ will be

$$y - 0 = \left(\frac{2}{3}\right)\left(x - \frac{7}{2}\right)$$

$$3y = 2x - 7$$

$$2x - 3y - 7 = 0$$

Thus, the required line equation is $2x - 3y - 7 = 0$.

21. Find the equation of a straight line passing through the intersection of $2x + 5y - 4 = 0$ with x - axis and parallel to the line $3x - 7y + 8 = 0$.

Solution:

Let's assume the point of

Let's assume the point of intersection of the line $2x + 5y - 4 = 0$ and x -axis be $(x, 0)$

Now, substituting the value $y = 0$ in the line equation, we have

$$2x + 5(0) - 4 = 0$$

$$2x - 4 = 0$$

$$x = 4/2 = 2$$

Hence, the co-ordinates of the point of intersection is $(2, 0)$

Also given, line equation: $3x - 7y + 8 = 0$

$$7y = 3x + 8$$

$$y = \left(\frac{3}{7}\right)x + \frac{8}{7}$$

So, the slope $(m) = \frac{3}{7}$

We know that the slope of any line parallel to the given line will be the same.

So, the equation of the line having slope $\frac{3}{7}$ and passing through the point (2, 0) will be

$$y - 0 = \left(\frac{3}{7} \right) (x - 2)$$

$$7y = 3x - 6$$

$$3x - 7y - 6 = 0$$

Thus, the required line equation is $3x - 7y = 0$.

22. The equation of a line is $3x + 4y - 7 = 0$. Find (i) the line. (ii) the equation of a line perpendicular to the given line and passing through the intersection of the lines $x - y + 2 = 0$ and $3x + y - 10 = 0$.

Solution:

Given line equation : $3x + 4y - 7 = 0$

(i) Slope of the line is given by,

$$4y = -3x + 7$$

$$y = \left(\frac{-3}{4} \right) x + 7$$

Hence, slope (m_1) = $\frac{-3}{4}$

(ii) Let the slope of the perpendicular to the given line be m_2

Then, $m_1 \times m_2 = -1$

$$\left(\frac{-3}{4}\right) \times m_2 = -1$$

$$m_2 = \frac{4}{3}$$

Now, to find the point of intersection of

$$x - y + 2 = 0 \dots\dots\dots(i)$$

$$3x + y - 10 = 0 \dots\dots(ii)$$

On adding (i) and (ii), we get

$$4x - 8 = 0$$

$$4x = 8$$

$$x = \frac{8}{4} = 2$$

Putting $x = 2$ in (i), we get

$$2 - y + 2 = 0$$

$$y = 4$$

Hence, the point of intersection of the lines is (2, 4).

The equation of the line having slope m_2 and passing through (2,4) will be

$$y - 4 = \left(\frac{4}{3}\right)(x - 2)$$

$$3y - 12 = 4x - 8$$

$$4x - 3y + 4 = 0$$

Thus, the required line equation is $4x - 3y + 4 = 0$.

23. Find the equation of the line perpendicular from the point (1, -2) on the line $4x - 3y - 5 = 0$. Also find the co-ordinates of the foot of perpendicular.

Solution:

Given line equation ; $4x - 3y - 5 = 0$

$$3y = 4x - 5$$

$$y = \left(\frac{4}{3}\right)x - 5$$

$$\text{Slope of the line } (m_1) = \frac{4}{3}$$

Let the slope of the line perpendicular to the given line be m_2

$$\text{Then, } m_1 \times m_2 = -1$$

$$\left(\frac{4}{3}\right) \times m_2 = -1$$

$$m_2 = \frac{-3}{4}$$

Now, the equation of the line having slope m_2 and passing through the point (1, -2) will be

$$y + 2 = \left(\frac{-3}{4}\right)(x - 1)$$

$$4y + 8 = -3x + 3$$

$$3x + 4y + 5 = 0$$

Next, for finding the co-ordinates of the foot of the perpendicular which is the point of intersection of the lines

$$4x - 3y - 5 = 0 \dots(1) \text{ and}$$

$$3x + 4y + 5 = 0 \dots(2)$$

On multiplying (1) by 4 and (2) by 3, we get

$$16x - 12y - 20 = 0$$

$$9x + 12y + 15 = 0$$

Adding we get,

$$25x - 5 = 0$$

$$x = \frac{5}{25}$$

$$x = \frac{1}{5}$$

Putting the value of x in (1), we have

$$4\left(\frac{1}{5}\right) - 3y - 5 = 0$$

$$\frac{4}{5} - 3y - 5 = 0$$

$$3y = \frac{4}{5} - 4$$

$$= \frac{(4-25)}{5}$$

$$= 3y = \frac{-21}{5}$$

$$y = \frac{-7}{5}$$

Thus, the co-ordinates are $\left(\frac{1}{5}, \frac{-7}{5}\right)$.

24. Prove that the line through (0,0) and (2, 3) is parallel to the line through (2, -2) and (6, 4).

Solution:

Let the slope of the line through (0, 0) and (2, 3) be m_1

$$\text{So, } m_1 = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

$$= \frac{(3-0)}{(2-0)}$$

$$= \frac{3}{2}$$

And, let the slope of the line through (2, -2) and (6, 4) be m_2

$$\text{So, } m_2 = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

$$= \frac{(4+2)}{(6-2)}$$

$$= \frac{6}{4}$$

$$= \frac{3}{2}$$

It's clearly seen that the slopes $m_1 = m_2$

Thus, the lines are parallel to each other.

25. Prove that the line through (-2, 6) and (4,8) is perpendicular to the line through (8,12) and (4, 24).

Solution:

Let the slope of the line through points (-2, 6) and (4, 8) be m_1

$$\text{So, } m_1 = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

$$= \frac{(8-6)}{(4+2)}$$

$$= \frac{2}{6}$$

$$= \frac{1}{3}$$

And, let the slope of the line through (8, 12) and (4, 24) be m_2

$$\text{So, } m_2 = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

$$= \frac{(24-12)}{(4-8)}$$

$$= \frac{12}{-4}$$

$$= -3$$

Now, product of slopes is

$$m_1 \times m_2 = \frac{1}{3} \times (-3) = -1$$

Thus, the lines are perpendicular to each other.

26. Show that the triangle formed by the points A (1,3), B(3,-1) and C (-5, -5) is a right-angles triangle by using slopes.

Solution:

Given, points A(1, 3), B (3, -1) and C(-5, -5) form a triangle

Now,

$$\text{Slope of the line AB} = m_1 = \frac{(-1-3)}{(3-1)} = \frac{-4}{2} = -2$$

And,

$$\text{Slope of the line BC} = m_2 = \frac{(-5+1)}{(-5-3)}$$

$$= \frac{-4}{-8}$$

$$= \frac{1}{2}$$

Hence,

$$m_1 \times m_2 = (-2) \times \left(\frac{1}{2}\right) = -1$$

So, the lines AB and BC are perpendicular to each other.

Therefore, ΔABC is a right-angled triangle.

26. Show that the triangle formed by the points A(1,3), B (3, -1) and C(-5, -5) is a right angled triangle by using slopes.

Solution:

Given, points A(1,3), B(3, -1) and C(-5, -5) form a triangle

Now,

$$\text{Slope of the line AB} = m_1 = \frac{(-1-3)}{(3-1)} = \frac{-4}{2} = -2$$

Hence,

$$m_1 \times m_2 = (-2) \times \left(\frac{1}{2}\right) = -1$$

So, the lines AB and BC are perpendicular to each other.

Therefore, ΔABC is a right-angled triangle.

27. Find the equation of the line through the point (-1, 3) and parallel to the line joining the points (0, -2) and (4, 5).

Solution:

Slope of the line joining the points (0, -2) and (4, 5) is

$$m = \frac{(5+2)}{(4-0)}$$

$$= \frac{7}{4}$$

Now, the slope of the line parallel to it and passing through (-1, 3) will be also be $\frac{7}{4}$

Hence, the equation of the line is

Hence, the equation of the line is

$$y - y_1 = m(x - x_1)$$

$$\Rightarrow y - y_1 = m(x - x_1) \Rightarrow y - 3 = \frac{7}{4}(x + 1)$$

$$\Rightarrow 4y - 12 = 7x + 7$$

$$\Rightarrow 7x - 4y + 19 = 0$$

28. Are the vertices of a triangle.

(i) Find the coordinates of the centroid G of the triangle.

(ii) Find the equation of the line through G and parallel to AC.

Solution:

Given, A (-1, 3), B (4, 2), C (3, -2)

(i) Co- ordinates of centroid G is

$$G (x,y) = \left(\frac{(x_1+x_2+x_3)}{3}, \frac{(y_1+y_2+y_3)}{3} \right)$$

$$= \left(\frac{(-1+4+3)}{3}, \frac{(3+2-2)}{3} \right)$$

$$= \left(\frac{6}{3}, \frac{3}{3} \right)$$

$$= (2,1)$$

Hence, the co-ordinates of the centroid G of the triangle is (2,1)

$$\text{(ii) Slope of AC} = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

$$= \frac{(-2 - 3)}{(3 - (-1))}$$

$$= \frac{-5}{4}$$

So, the slope of the line parallel to AC is also $\frac{-5}{4}$

Now, the equation of line through G is

$$y - 1 = \left(\frac{-5}{4}\right)(x - 2)$$

$$4y - 4 = -5x + 10$$

$$5x + 4y = 14$$

Thus, the required line equation is $5x + 4y = 14$.

29. The line through P(5, 3) intersects y-axis at Q. (i) Write the slope of the line. (ii) Write the equation of the line. (iii) Find the coordinates of Q.

Solution:

(i) Here, $\theta = 45^\circ$

So, the slope of the line $= \tan \theta$

So, the slope of the line $= \tan \theta = \tan 45^\circ = 1$

(ii) Equation of the line through P and Q is

$$y - 3 = 1(x - 5)$$

$$x - y - 2 = 0$$

(iii) Let the co-ordinates of Q be (0, y)

$$\text{Then, } m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

$$1 = \frac{(3 - y)}{(5 - 0)}$$

$$5 = 3 - y$$

$$y = 3 - 5 = -2$$

Thus, co-ordinates of Q are (0, -2).

30. In the adjoining diagram, write down (i) the co-ordinates of the points A, B and C. (ii) the equation of the line through A parallel to BC.

Solution :

From the given figure, its clearly seen that

Co- ordinates of A are (2, 3) and of B are (-1, 2) and of C are (3, 0).

Now,

$$\text{Slope of BC} = \frac{(0 - 2)}{(3 - (-1))}$$

Now,

$$= \frac{-2}{4}$$

$$= \frac{-1}{2}$$

So, the slope of the line parallel to BC is also $\frac{-1}{2}$

And, the line passes through A(2, 3)

Hence, the equation will be

$$y - 3 = \left(\frac{-1}{2}\right)(x - 2)$$

$$2y - 6 = -x + 2$$

$$x + 2y = 8$$

31. Find the equation of the line through (0, -3) and perpendicular to the line joining the points (-3, 2) and (9, 1).

Solution:

The slope of the line joining the points (-3, 2) and (9, 1) is

$$m_1 = \frac{(1-2)}{(9+3)} = \frac{-1}{12}$$

Now, let the slope of the line perpendicular to the above line be m_2

$$\text{Then, } m_1 \times m_2 = -1$$

$$= \left(\frac{-1}{12}\right) \times m_2 = -1$$

$$= m_2 = 12$$

So, the equation of the line passing through (0, -3) and having slope of m_2 will be

$$y - (-3) = 12(x - 0)$$

$$y + 3 = 12x$$

$$12x - y = 3$$

Thus, the required line equation is $12x - y = 3$.

32. The vertices of a triangle are A (10, 4), B (4, -9) and C(-2, -1). Find the equation of the altitude through A. The opposite side is called altitude.

Solution:

Given, vertices of a triangle are A(10, 4), B(4, -9) and C(-2, -1)

Now,

$$\text{Slope of line BC } (m_1) = \frac{(-1+9)}{(-2-4)} = \frac{8}{-6} = \frac{-4}{3}$$

Let the slope of the altitude from A (10,4) to BC be m_2

$$\text{Then, } m_1 \times m_2 = -1$$

$$= \left(\frac{-4}{3}\right) \times m_2 = -1$$

$$= m_2 = \frac{3}{4}$$

So, the equation of the line will be

$$y - 4 = \frac{3}{4}(x - 10)$$

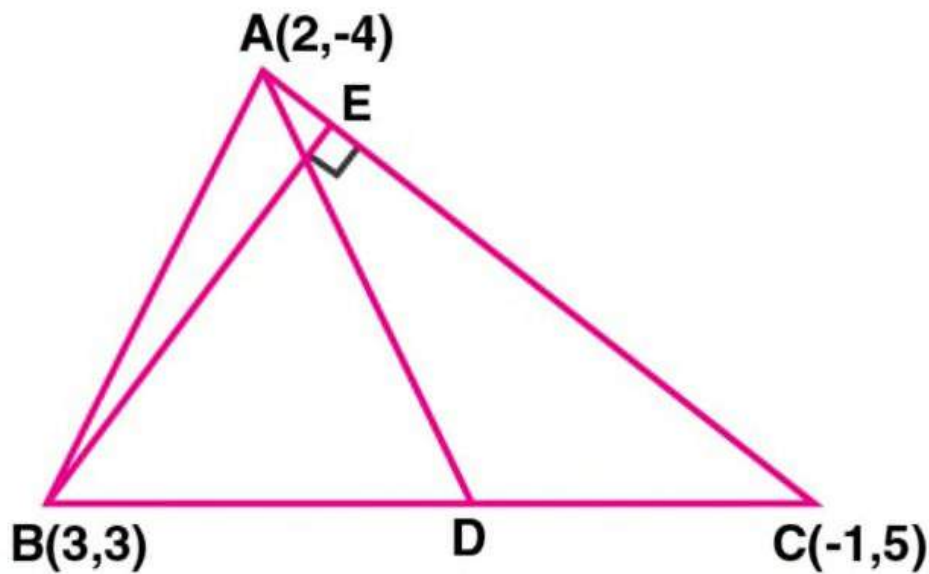
$$4y - 16 = 3x - 30$$

$$3x - 4y - 14 = 0$$

33. A(2, -4), B (3,3) and C(-1, 5) are the vertices of triangle ABC. Find the equation of :

- (i) the median of the triangle through A
- (ii) the altitude of the triangle through B.

Solution:



Given A(2, -4), B(3, 3) and C(-1, 5) are the vertices of triangle ABC

(i) D is the mid-point of BC

So, the co-ordinates of D will be

$$\left(\frac{(3-1)}{2}, \frac{(3+5)}{2} \right) = \left(\frac{2}{2}, \frac{8}{2} \right) = (1, 4)$$

Now,

The slope of AC (m_1) = $\frac{(5+4)}{(-1-2)} = \frac{9}{-3} = -3$

Let the slope of BE be m_2

Then, $m_1 \times m_2 = -1$

$$= -3 \times m_2 = -1$$

$$= m_2 = \frac{1}{3}$$

So, the equation of BE will be

$$y - 3 = \frac{1}{3}(x - 3)$$

$$3y - 9 = x - 3$$

$$x - 3y + 6 = 0$$

Thus, the required line equation is $x - 3y + 6 = 0$.

34. Find the equation of the right bisector of the line segment joining the points (1, 2) and (5, -6).

Solution:

The slope of the line joining the points (1, 2) and (5, -6) is

$$m_1 = \frac{(-6-2)}{(5-1)} = \frac{-8}{4} = -2$$

Now, if m_2 is the slope of the right bisector of the above line

Then,

$$m_1 \times m_2 = -1$$

$$-2 \times m_2 = -1$$

$$m_2 = \frac{1}{2}$$

The mid-point of the line segment joining (1,2) and (5, -6) will be

$$\left(\frac{(1+5)}{2}, \frac{(2-6)}{2} \right) = \left(\frac{6}{2}, \frac{-4}{2} \right) = (3, -2)$$

So, equation of the line is

$$y + 2 = \frac{1}{2}(x - 3)$$

$$2y + 4 = x - 3$$

$$x - 2y - 7 = 0$$

Thus, the equation of the required right bisector is $x - 2y - 7 = 0$.

35. Points A and B have coordinates (7, -3) and (1, 9) respectively. Find

(i) the slope of AB.

(ii) The equation of the perpendicular bisector of the line segment AB.

Solution:

Given, co-ordinates of points A are (7,-3) and of B are (1,9)

$$(i) \text{ The slope of AB}(m) = \frac{(9+3)}{(1-7)} = \frac{12}{-6} = -2$$

(ii) Let PQ be the perpendicular bisector of AB intersecting it at M

Now, the co-ordinates of M will be the mid-point of AB

Co-ordinates of M will be

$$= \frac{(7+1)}{2}, \frac{(-3+9)}{2} = \frac{8}{2}, \frac{6}{2}$$

$$= (4, 3)$$

$$\text{The slope of line PQ will be} = \frac{-1}{m} = \frac{-1}{(-2)} = \frac{1}{2}$$

Thus, the equation of PQ is

$$y - 3 = \frac{1}{2} (x - 4)$$

$$2y - 6 = x - 4$$

$$x - 2y + 2 = 0$$

(iii) As point (-2, p) lies on the above line

The point will satisfy the line equation

$$-2 - 2p + 2 = 0$$

$$- 2p = 0$$

$$p = 0$$

Thus, the value of p is 0.

36. The points B(1, 3) and D(6,8) are two opposite vertices of a square ABCD. Find the equation of the diagonal AC.

Solution:

Given, points B(1, 3) and D (6,8) are two opposite vertices of a square ABCD

Slope of BD is given by

$$m_1 = \frac{(8-3)}{(6-1)} = \frac{5}{5} = 1$$

We know that, the diagonal AC is a perpendicular bisector of diagonal BD

So, the slope of AC (m_2) will be

$$m_1 \times m_2 = -1$$

$$1 \times m_2 = -1$$

$$m_2 = -1$$

And, the co-ordinates of mid-point of BD and AC will be

$$\left(\frac{(1+6)}{2}, \frac{(3+8)}{2}\right) = \left(\frac{7}{2}, \frac{11}{2}\right)$$

So, the equation of AC is

$$y - \frac{11}{2} = -1 \left(x - \frac{7}{2}\right)$$

$$2y - 11 = -2x - 7$$

$$2x + 2y - 7 - 11 = 0$$

$$\Rightarrow 2x + 2y - 18 = 0$$

Thus, the equation of diagonal AC is $x + y - 9 = 0$.

37. ABCD is a rhombus. The co-ordinates of A and C are (3,6) and (-1, 2) respectively. Write down the equation of BD.

Solution :

Given, ABCD is a rhombus and co-ordinates of A are (3, 6) and of C are (-1, 2)

$$\text{Slope of AC } (m_1) = \frac{(2-6)}{(-1-3)} = \frac{-4}{-4} = 1$$

We know that, the diagonals of a rhombus bisect each other at right angles.

So, the diagonal BD is perpendicular to diagonal AC

Let the slope of BD be m_2

$$\text{Then, } m_1 \times m_2 = -1$$

$$m_2 = \frac{-1}{(m_1)}$$

$$= \frac{-1}{(1)}$$

$$= -1$$

Now, the co-ordinates of the mid-point of AC is given by

$$\left(\frac{(3-1)}{2}, \frac{(6+2)}{2} \right) = \left(\frac{2}{2}, \frac{8}{2} \right) = (1, 4)$$

So, the equation of BD will be

$$y - 4 = -1(x - 1)$$

$$y - 4 = -x + 1$$

$$x + y = 5$$

Thus, the equation of BD is $x + y = 5$.

38. Find the equation of the line passing through the intersection of the lines $4x + 3y = 1$ and $5x + 4y = 2$ and

(i) parallel to the line $x + 2y - 5 = 0$

(ii) perpendicular to the x-axis.

Solution:

Given, line equations:

$$4x + 3y = 1 \dots(1)$$

$$5x + 4y = 2 \dots(2)$$

On solving the above equation to find the point of intersection, we have

Multiplying (1) by 4 and (2) by 3

$$16x + 12y = 4$$

$$15x + 12y = 6$$

On subtracting, we get

$$x = -2$$

Putting the value of x in (1), we have

$$4(-2) + 3y = 1$$

$$-8 + 3y = 1$$

$$3y = 1 + 8 = 9$$

$$y = \frac{9}{3} = 3$$

Hence, the point of intersection is $(-2, 3)$.

(i) Given line, $x + 2y - 5 = 0$

$$2y = -x + 5$$

$$y = \left(\frac{-1}{2}\right)x + \frac{5}{2}$$

$$\text{Slope (m)} = \frac{-1}{2}$$

A line parallel to this line will have the same slope $m = \frac{-1}{2}$

So, the equation of line having slope m and passing through $(-2, 3)$ will be

$$y - 3 = \left(\frac{-1}{2}\right)(x + 2)$$

$$2y - 6 = -x - 2$$

$$x + 2y = 4$$

(ii) As any line perpendicular to x -axis will be parallel to y -axis.

So, the equation of line will be

$$x = -2 \Rightarrow x + 2 = 0$$

39. (i) Write down the co-ordinates of the point P that divides the line joining A(-4, 1) and B(17, 10) in the ratio 1 : 2.

(ii) Calculate the distance OP where O is the origin

(iii) In what ratio does the y-axis divide the line AB ?

Solution:

(i) Given, co-ordinates of the line joining A (-4,1) and B(17, 10) and point P divides the line segment in the ratio 1 : 2.

Let the co-ordinates of P be (x, y)

Then,

$$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2} = \frac{1 \times 17 + 2 \times -4}{1 + 2}$$

$$= \frac{17 - 8}{3} = \frac{9}{3} = 3$$

$$y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2} = \frac{1 \times 10 + 2 \times 1}{1 + 2}$$

$$= \frac{10 + 2}{3}$$

$$= \frac{12}{3} = 4$$

Thus, Co-ordinates of P will be (3, 4)

(ii) O is the origin

So, Distance between O and P

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\begin{aligned}
&= \sqrt{(0-3)^2 + (0-4)^2} \\
&= \sqrt{(-3)^2 + (-4)^2} \\
&= \sqrt{9+16} = \sqrt{25} \\
&= 5 \text{ units.}
\end{aligned}$$

(iii) Let y-axis divides AB in the ratio of $m_1 : m_2$

Then,

$$x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}$$

$$\Rightarrow 0 = \frac{m_1 \times 17 + m_2 \times (-4)}{m_1 + m_2}$$

$$\Rightarrow 17 m_1 - 4 m_2 = 0$$

$$\Rightarrow 17 m_1 = 4 m_2$$

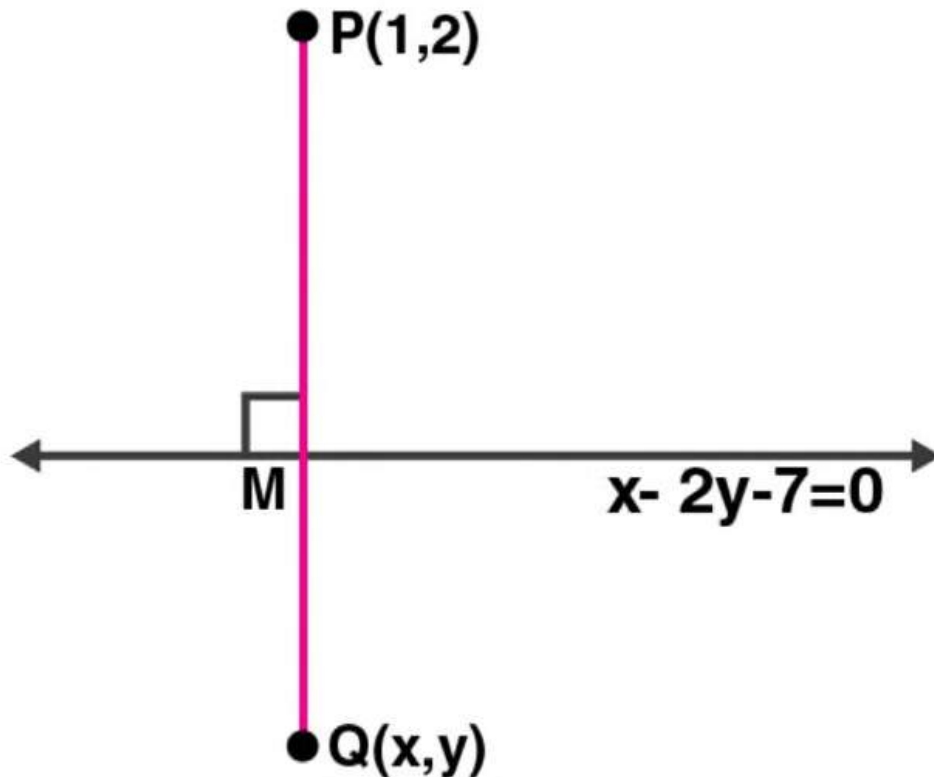
$$\frac{m_1}{m_2} = \frac{4}{17}$$

$$\Rightarrow m_1 : m_2 = 4 : 17$$

40. Find the image of the point (1, 2) in the line $x - 2y - 7 = 0$.

Solution:

Given line equation : $x - 2y - 7 = 0$ (i)



Draw a perpendicular from point P (1, 2) on the line

Let P' be the image of P and let its co-ordinates be (x, y)

The slope of the given line is given as,

$$2y = x - 7$$

$$y = \left(\frac{1}{2}\right)x - 7$$

$$\text{Slope } (m_1) = \frac{1}{2}$$

Let the slope of line segment PP' be m_2

As PP' is perpendicular to the given line, product of slopes : $m_1 \times m_2 = -1$.

$$\text{So, } \frac{1}{2} \times m_2 = -1$$

$$m_2 = -2$$

So, the equation of the line perpendicular to the given line and passing through P(1,2) is

$$y - 2 = (-2)(x - 1)$$

$$y - 2 = -2x + 2$$

$$2x + y - 4 = 0 \quad \dots(\text{ii})$$

Let the intersection point of lines (i) and (ii) be taken as M.

Solving both the line equations, we have

Multiplying (ii) by 2 and adding with (i)

$$x - 2y - 7 = 0$$

$$4x + 2y - 8 = 0$$

$$5x - 15 = 0$$

$$x = \frac{15}{5} = 3$$

Putting value of x in (i), we get

$$3 - 2y - 7 = 0$$

$$2y = -4$$

$$y = \frac{-4}{2} = -2$$

So, the co-ordinates of M are (3, -2)

Hence, its seen that M should be the mid-point of the line segment PP'

$$(3, -2) = \left(\frac{(x+1)}{2}, \frac{(y+2)}{2} \right)$$

$$\frac{(x+1)}{2} = 3$$

$$x + 1 = 6$$

$$x = 6 - 1 = 5$$

And,

$$\frac{(y+2)}{2} = -2$$

$$y + 2 = -4$$

$$y = -4 - 2 = -6$$

Therefore, the co-ordinates of P' are (5, -6).

41. If the line $x - 4y - 6 = 0$ is the perpendicular bisector of the line segment PQ and the co-ordinates of P are (1,3), find the co-ordinates of Q.

Solution:

Given, line equation : $x - 4y - 6 = 0 \dots (i)$

Given, line equation: $x - 4y - 6 = 0 \dots (i)$

Co-ordinates of P are (1, 3)

Let the co-ordinates of Q be (x , y)

Now, the slope of the given line is

$$4y = x - 6$$

$$y = \left(\frac{1}{4} \right) x - \frac{6}{4}$$

$$\text{Slope (m)} = \frac{1}{4}$$

So, the slope of PQ will be $\left(\frac{-1}{m}\right)$

[As the product of slopes of perpendicular lines is -1]

$$\text{Slope of PQ} = \frac{-1}{\left(\frac{1}{4}\right)} = -4$$

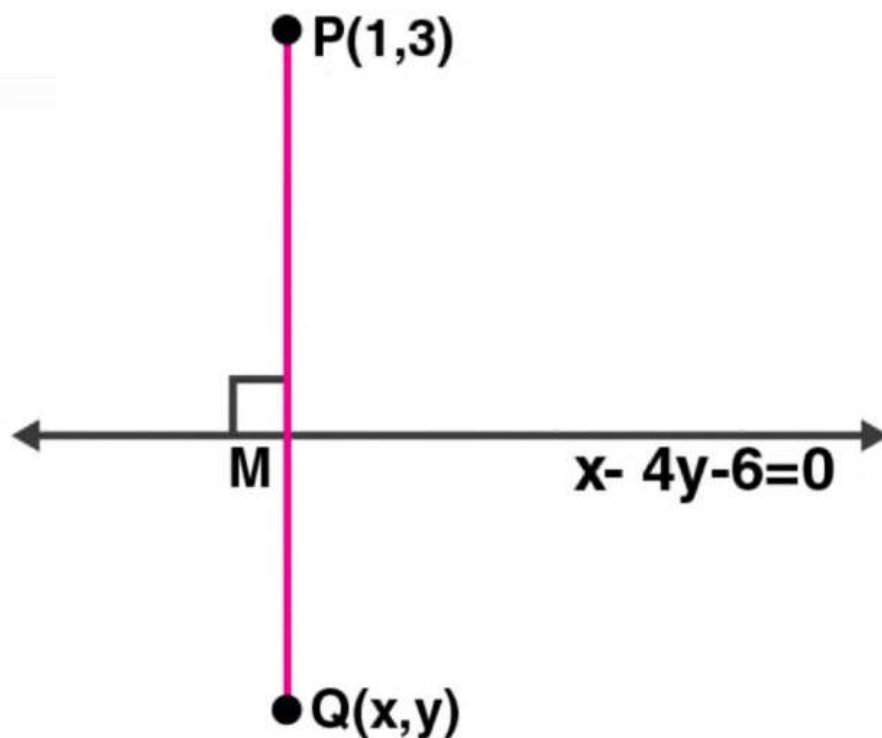
Now, the equation of line PQ will be

$$y - 3 = (-4)(x - 1)$$

$$y - 3 = -4x + 4$$

$$4x + y = 7 \text{(ii)}$$

On solving equation (i) and (ii), we get the coordinates of M



Multiplying (ii) by 4 and adding with (i), we get

$$x - 4y - 6 = 0$$

$$16x + 4y = 28$$

Multiplying (ii) by 4 and adding with (i), we get

$$x - 4y - 6 = 0$$

$$16x + 4y = 28$$

$$17x = 34$$

$$x = \frac{34}{17} = 2$$

Putting the value of x in (i)

$$2 - 4y - 6 = 0$$

$$-4 - 4y = 0$$

$$4y = -4$$

$$y = -1$$

So, the co-ordinates of M are (2, -1)

But, M is the mid-point of line segment PQ

$$(2, -1) = \frac{(x+1)}{2}, \frac{(y+3)}{2}$$

$$\frac{(x+1)}{2} = 2$$

$$x + 1 = 4$$

$$x = 3$$

And,

$$\frac{(y+3)}{2} = -1$$

$$(y + 3) = -2$$

$$y = -5$$

Thus, the co-ordinates of Q are (3, -5).

42. OABC is a square, O is the origin and the points A and B are (3, 0) and (p, q). If OABC lies in the first quadrant, find the value of p and q. Also write down the equation of AB and BC.

Solution:

Given, OABC is a square

Co- ordinates of A and B are (3, 0) and (p, q) respectively.

By distance formula, we have

$$\begin{aligned} OA &= \sqrt{(3 - 0)^2 + (0 - 0)^2} \\ &= \sqrt{(3)^2 + (0)^2} \\ &= \sqrt{9 + 0} = \sqrt{9} = 3 \end{aligned}$$

$$\begin{aligned} AB &= \sqrt{(3 - p)^2 + (0 - q)^2} \\ &= \sqrt{(3 - p)^2 + q^2} \end{aligned}$$

As OA = AB (sides of a square)

$$\Rightarrow \sqrt{(3 - p)^2 + q^2} = 3$$

$(3 - p)^2 + q^2 = 9$ (squaring both sides)

$$9 + p^2 - 6p + q^2 = 9$$

$$p^2 + q^2 - 6p = 0 \quad \dots\dots (i)$$

$$\text{Now, } OB = \sqrt{(p-0)^2 + (q-0)^2} = \sqrt{p^2 + q^2}$$

$$\text{But } OB^2 = OA^2 + AB^2$$

$$\Rightarrow \left(\sqrt{p^2 + q^2} \right)^2 = 3^2 + \left(\sqrt{(3-p)^2 + q^2} \right)^2$$

$$p^2 + q^2 = 9 + (3-p)^2 + q^2$$

$$p^2 + q^2 = 9 + 9 + p^2 - 6p + q^2$$

$$(p^2 + q^2) = 18 + (p^2 - 6p + q^2) \quad [\text{Using (i)}]$$

$$6p = 18 \Rightarrow p = \frac{18}{6} = 3$$

Substituting the value of p in (i)

$$(3)^2 + q^2 - 6(3) = 0 \Rightarrow 9 + q^2 - 18 = 0$$

$$q^2 - 9 = 0 \Rightarrow q^2 = 9 \Rightarrow q = 3$$

$$\therefore p = 3, q = 3$$

As, AB parallel to y-axis

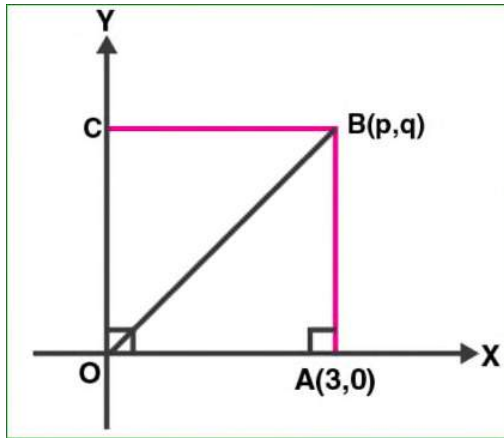
\therefore Equation AB will be $x = 3$

$$\Rightarrow x - 3 = 0$$

and equation of BC will be $y = 3$

($\because BC \parallel x$ -axis)

$$\Rightarrow y - 3 = 0$$



$$\begin{aligned}
 OA &= \sqrt{(3-0)^2 + (0-0)^2} \\
 &= \sqrt{(3)^2 + (0)^2} \\
 &= \sqrt{9+0} = \sqrt{9} = 3
 \end{aligned}$$

$$\begin{aligned}
 AB &= \sqrt{(3-p)^2 + (0-q)^2} \\
 &= \sqrt{(3-p)^2 + q^2}
 \end{aligned}$$

As $OA = AB$ (sides of a square)

$$\Rightarrow \sqrt{(3-p)^2 + q^2} = 3$$

$$(3-p)^2 + q^2 = 9 \text{ (Squaring both sides)}$$

$$9 + p^2 - 6p + q^2 = 9$$

$$p^2 + q^2 - 6p = 0 \quad \dots (i)$$

$$\text{Now, } OB = \sqrt{(p-0)^2 + (q-0)^2} = \sqrt{p^2 + q^2}$$

$$\text{But } OB^2 = OA^2 + AB^2$$

$$\Rightarrow \left(\sqrt{p^2 + q^2} \right)^2 = 3^2 + \left(\sqrt{(3-p)^2 + q^2} \right)^2$$

$$p^2 + q^2 = 9 + (3-p)^2 + q^2$$

$$p^2 + q^2 = 9 + 9 + p^2 - 6p + q^2$$

$$(p^2 + q^2) = 18 + (p^2 - 6p + q^2) \quad \text{[Using (i)]}$$

$$6p = 18 \Rightarrow p = \frac{18}{6} = 3$$

Substituting the value of p in (i)

$$(3)^2 + q^2 - 6(3) = 0 \Rightarrow 9 + q^2 - 18 = 0$$

$$q^2 - 9 = 0 \Rightarrow q^2 = 9 \Rightarrow q = 3$$

$$\therefore p = 3, q = 3$$

As, AB parallel to y-axis

\therefore Equation AB will be $x = 3$

$$\Rightarrow x - 3 = 0$$

and equation of BC will be $y = 3$

(\because BC \parallel x-axis)

$$\Rightarrow y - 3 = 0.$$

Chapter Test

1. Find the equation of a line whose inclination is 60° and y-intercept is -4.

Solution:

Given, inclination $= 60^\circ$ and y-intercept $(c) = -4$

So, slope $(m) = \tan 60^\circ = \sqrt{3}$

Hence, the equation of the line is given by

$$y = mx + c$$

$$y = \sqrt{3}x - 4$$

2. Write down the gradient and the intercept on the y – axis of the line $3y + 2x = 12$.

Solution:

Given line equation : $3y + 2x = 12$.

$$3y = -2x + 12$$

$$y = \left(\frac{-2}{3}\right)x + \frac{12}{3}$$

$$y = \left(\frac{-2}{3}\right)x + 4$$

Hence, gradient $= \frac{-2}{3}$ and the intercept on the y-axis is 4.

3. If the equation of a line is $y - \sqrt{3}x + 1$, find its inclination.

Solution:

Given line equation : $y - \sqrt{3}x + 1$

$$y = \sqrt{3}x - 1$$

Here, slope = $\sqrt{3}$

$$\Rightarrow \tan \theta = \sqrt{3}$$

$$\theta = 60^\circ$$

Hence, the inclination of the line is 60° .