

Chapter 12. Exponents and Powers

Question 1

Evaluate: (i) 5^{-4} (ii) $(-7)^{-2}$.

Solution:

$$(i) 5^{-4}$$

$$5^{-4} = \frac{1}{5^4} = \frac{1}{625}$$

$$(ii) (-7)^{-2}$$

$$(-7)^{-2} = \frac{1}{(-7)^2} = \frac{1}{49}$$

Question 2

Evaluate (i) $\left(\frac{2}{7}\right)^{-5}$ (ii) $(\sqrt{3})^0$.

Solution:

$$(i) \left(\frac{2}{7}\right)^{-5}$$

$$\begin{aligned} \left(\frac{2}{7}\right)^{-5} &= \frac{1}{\left(\frac{2}{7}\right)^5} = \frac{1}{\frac{2^5}{7^5}} \\ &= \frac{7^5}{2^5} = \frac{16807}{32} \end{aligned}$$

$$(ii) (\sqrt{3})^0$$

$$(\sqrt{3})^0 = 1$$

Question 3

Evaluate $8^9 \div 8^3$.

Solution:

$$\begin{aligned} 8^9 \div 8^3 &= \frac{8^9}{8^3} \\ &= 8^{9-3} = 8^6 \end{aligned}$$

Question 4

$$(i)(-7)^5 \div (-7)^2 \quad (ii)(\sqrt{3})^4 \times (\sqrt{3})^{-2}.$$

Solution:

$$\begin{aligned}(i)(-7)^5 \div (-7)^2 \\ (-7)^5 \div (-7)^2 &= \frac{(-7)^5}{(-7)^2} \\ &= (-7)^{5-2} = (-7)^3 \\ (ii)(\sqrt{3})^4 \times (\sqrt{3})^{-2} \\ (\sqrt{3})^4 \times (\sqrt{3})^{-2} &= (\sqrt{3})^{4-2} \\ &= (\sqrt{3})^2\end{aligned}$$

Question 5

Simplify $(i)(3^{-2} \times 4^{-2})^{-3} \quad (ii)\left(\frac{2}{3}\right)^5 \times \left(\frac{2}{3}\right).$

Solution:

$$\begin{aligned}(i)(3^{-2} \times 4^{-2})^{-3} \\ (3^{-2})^{(-3)} \times (4^{-2})^{-3} &= 3^6 \times 4^6 \\ (ii)\left(\frac{2}{3}\right)^5 \times \left(\frac{2}{3}\right) \\ \left(\frac{2}{3}\right)^5 \times \left(\frac{2}{3}\right)^1 &= \left(\frac{2}{3}\right)^{5+1} = \left(\frac{2}{3}\right)^6\end{aligned}$$

Question 6

Solve the following equation & find the value of m in $7^m = 343$.

Solution:

$$\begin{aligned}7^m &= 343 \\ \Rightarrow 7^m &= 7^3 \\ \Rightarrow m &= 3\end{aligned}$$

Question 7

Solve the following equation & find the value of m in $2^{m-3} = 1$.

Solution:

$$2^{m-3} = 1$$

$$2^{m-3} = 2^0$$

$$\Rightarrow m - 3 = 0$$

$$\Rightarrow m = 3$$

Question 8

Solve the following equation & find the value of m in $3^m = \frac{1}{9}$.

Solution:

$$3^m = \frac{1}{9}$$

$$3^m = \frac{1}{9}$$

$$\Rightarrow 3^m = \frac{1}{3^2} \Rightarrow 3^m = 3^{-2} \Rightarrow m = -2$$

Question 9

Find the value of $(243)^{\frac{2}{5}}$.

Solution:

$$(243)^{\frac{2}{5}} = (3^5)^{\frac{2}{5}}$$

$$= 3^{\left(5 \times \frac{2}{5}\right)} = 3^2 = 9$$

Question 10

Find the value of $(512)^{\frac{-2}{9}}$.

Solution:

$$\begin{aligned}(512)^{\frac{-2}{9}} &= (2^9)^{\frac{-2}{9}} \\ &= 2^{9 \times \frac{-2}{9}} \\ &= 2^{-2} = \frac{1}{2^2} \\ &= \frac{1}{4}\end{aligned}$$

Question 11

Find the value of $\left\{(216)^{\frac{2}{3}}\right\}^{\frac{1}{2}}$.

Solution:

$$\begin{aligned}\left\{(216)^{\frac{2}{3}}\right\}^{\frac{1}{2}} &= (216)^{\frac{2}{3} \times \frac{1}{2}} \\ &= (216)^{\frac{1}{3}} \\ &= (6^3)^{\frac{1}{3}} = 6\end{aligned}$$

Question 12

Evaluate $\frac{a^{-3}b^{-4}}{a^{-2}b^{-3}}$.

Solution:

$$\begin{aligned}\frac{a^{-3}b^{-4}}{a^{-2}b^{-3}} &= a^{-3-(-2)} \times b^{-4-(-3)} \\ &= a^{-1} \times b^{-1} = \frac{1}{a} \times \frac{1}{b} = \frac{1}{ab}\end{aligned}$$

Question 13

Evaluate $(3^2)^3 + \left(\frac{2}{3}\right)^0 + 3^5 \times \left(\frac{1}{3}\right)^4$.

Solution:

$$\begin{aligned}\left(3^2\right)^3 + \left(\frac{2}{3}\right)^0 + 3^5 \times \left(\frac{1}{3}\right)^4 &= 3^6 + 1 + 3^5 \times 3^{-4} \\ &= 3^6 + 1 + 3^{5-4} \\ &= 3^6 + 1 + 3 \\ &= 729 + 1 + 3 = 733\end{aligned}$$

Question 14

Show that $9^{\frac{3}{2}} - 3 \times 5^0 - \left(\frac{1}{81}\right)^{\frac{-1}{2}} = 15$.

Solution:

$$\begin{aligned}\text{LHS} &= 9^{\frac{3}{2}} - 3 \times 5^0 - \left(\frac{1}{81}\right)^{\frac{-1}{2}} \\ &= \left(3^2\right)^{\frac{3}{2}} - 3 \times 1 - \left(\frac{1}{9^2}\right)^{\frac{-1}{2}} \\ &= 3^3 - 3 - 9 \\ &= 27 - 12 = 15 = \text{RHS}\end{aligned}$$

Question 15

Simplify $\left(\sqrt{x^{-3}}\right)^5$.

Solution:

$$\begin{aligned}\left(\sqrt{x^{-3}}\right)^5 &= \left\{\left(x^{-3}\right)^{\frac{1}{2}}\right\}^5 \\ &= \left(x^{-3}\right)^{\frac{5}{2}} = x^{\frac{-15}{2}} = \frac{1}{x^{\frac{15}{2}}}\end{aligned}$$

Question 16

If $27^y = \frac{9}{3^y}$, find y .

Solution:

$$\begin{aligned}27^y &= \frac{9}{3^y} \\ (3^3)^y &= \frac{3^2}{3^y} = 3^{2-y} \\ (3)^{3y} &= 3^{2-y} \\ 3y &= 2 - y \\ 4y &= 2 \Rightarrow y = \frac{1}{2}\end{aligned}$$

Question 17

Simplify $a^2 \times a^3 \times a^{-5}$.

Solution:

$$\begin{aligned}a^2 \times a^3 \times a^{-5} &= a^{2+3-5} \\ &= a^{5-5} \\ &= a^0 = 1\end{aligned}$$

Question 18

Simplify: $(a^3 \times a^{-2} \times a^4)^{-2}$.

Solution:

$$\begin{aligned}(a^3 \times a^{-2} \times a^4)^{-2} &= (a^{3-2+4})^{-2} \\ &= (a^5)^{-2} \\ &= a^{-10} = \frac{1}{a^{10}}\end{aligned}$$

Question 19

Simplify: $\frac{4^{-3} \times a^{-5} \times b^{-4}}{4^{-5} \times a^{-8} \times b^3}$.

Solution:

$$\begin{aligned}\frac{4^{-3} \times a^{-5} \times b^{-4}}{4^{-5} \times a^{-8} \times b^3} &= 4^{-3-(-5)} \times a^{-5-(-8)} \times b^{-4-3} \\ &= 4^2 a^3 b^{-7} \\ &= 16 \frac{a^3}{b^7}\end{aligned}$$

Question 20

Express as a power of 3 in 729 and 343.

Solution:

(i) $729 = 3^6$

(ii) $343 = 7^3$

Question 21

Simplify $\frac{49 \times t^{-5}}{7^{-3} \times 10 \times t^{-9}}$ ($t \neq 0$).

Solution:

$$\begin{aligned}\frac{49 \times t^{-5}}{7^{-3} \times 10 \times t^{-9}} &= \frac{7^2 \times t^{-5}}{7^{-3} \times 10 \times t^{-9}} = \frac{7^{2-(-3)} t^{-5-(-9)}}{10} \\ &= \frac{7^5 t^4}{10}\end{aligned}$$

Question 22

Express the following numbers in usual form (i) 5.08×10^{-6} (ii) 7.9×10^4 .

Solution:

$$(i) 5.08 \times 10^{-6}$$

$$5.08 \times 10^{-6} = \frac{5.08}{1000000} = 0.00000508$$

$$(ii) 7.9 \times 10^4$$

$$7.9 \times 10^4 = 7.9 \times 10000 = 79000$$

Question 23

Express the number appearing in the following statements in standard form.

Solution:

(i) Thickness of a chart paper is 0.05 mm.

(ii) Charge of an electron is 0.000,000,000,000,000,00012 coulomb.

Answer:

(i) Thickness of a chart paper is 0.05 mm.

Standard form of thickness of chart paper = 5×10^{-2} mm

(ii) Charge of an electron is 0.000,000,000,000,000,00012 coulomb.

Charge of electron = 1.2×10^{-19}

Question 24

Express the following number in standard form.

Solution:

$$(i) 6080000000000000$$

$$= 6.08 \times 10^{15}$$

$$(ii) 798600000000$$

$$= 7.986 \times 10^{10}$$

Question 25

simplify: $\left(\frac{256}{81}\right)^{\frac{5}{4}}$.

Solution:

$$\begin{aligned}(i) \left(\frac{256}{81}\right)^{\frac{5}{4}} \\ \left(\frac{256}{81}\right)^{\frac{5}{4}} &= \left(\frac{4^4}{3^4}\right)^{\frac{5}{4}} \\ &= \left\{\left(\frac{4}{3}\right)^4\right\}^{\frac{5}{4}} = \left(\frac{4}{3}\right)^{4 \times \frac{5}{4}} \\ &= \left(\frac{4}{3}\right)^5 = \frac{4^5}{3^5} = \frac{1024}{243}\end{aligned}$$

Question 26

Simplify $(\sqrt{4})^{-3}$.

Solution:

$$\begin{aligned}(\sqrt{4})^{-3} &= \left(\left(4^{\frac{1}{2}}\right)^{-3}\right) \\ &= 4^{\frac{-3}{2}} = \frac{1}{4^{\frac{3}{2}}} \\ &= \frac{1}{\left(4^3\right)^{\frac{1}{2}}} = \frac{1}{(64)^{\frac{1}{2}}} \\ &= \frac{1}{\left(8^2\right)^{\frac{1}{2}}} = \frac{1}{8}\end{aligned}$$

Question 27

Simplify $(\sqrt{5})^{-3}(\sqrt{2})^{-3}$.

Solution:

$$\begin{aligned}(\sqrt{5})^{-3} \times (\sqrt{2})^{-3} &= \left(5^{\frac{1}{2}}\right)^{-3} \times \left(2^{\frac{1}{2}}\right)^{-3} \\&= 5^{\frac{-3}{2}} \times 2^{\frac{-3}{2}} \\&= (5 \times 2)^{\frac{-3}{2}} \\&= \left(10^{\frac{-3}{2}}\right) \\&= \frac{1}{(10^3)^{\frac{1}{2}}} = \frac{1}{(1000)^{\frac{1}{2}}} \\&= \frac{10^{\frac{1}{2}}}{100}\end{aligned}$$

Question 28

In a library, there are 6 books each of thickness 30mm and 6 paper sheets each of thickness 0.018mm. What is the total thickness?

Solution:

Thickness of one book = 30mm

Thickness of 6 books = $30 \times 6 = 18\text{mm}$ ——(1)

Thickness of one paper = 0.018

Thickness of 6 papers = $6 \times 0.018 = 0.108$ ——(2)

+ (2) $18 + 0.108 = 18.108\text{ mm}$

Total thickness = 18.108 mm

Question 29

Solve $2^{2x+2} = 4^{2x-1}$.

Solution:

$$(i) 2^{2x+2} = 4^{2x-1}$$

$$\begin{aligned} 2^{2x+2} &= 4^{2x-1} = (2^2)^{2x-1} \\ &= 2^{4x-2} \end{aligned}$$

Equating the powers on both sides,

$$2x + 2 = 4x - 2$$

$$4 = 2x \Rightarrow x = 2$$