11. Similarity

Exercise 11.1

1. Question

Fill in the blanks:

(i) All circles are ……….

(ii) All squares are ………..

(iii) All ……… triangles are similar.

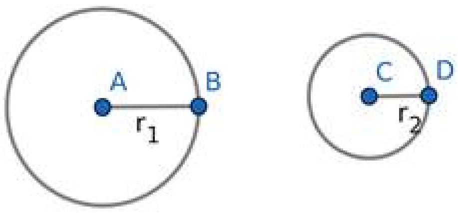
(iv) Two polygons with same number of sides are similar if

(a) …………………… (b)……………………

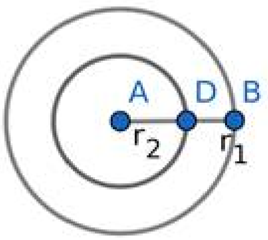
Answer

(i) All circles are <u>similar.</u>

Let there be two circles of radii r_1 and r_2.

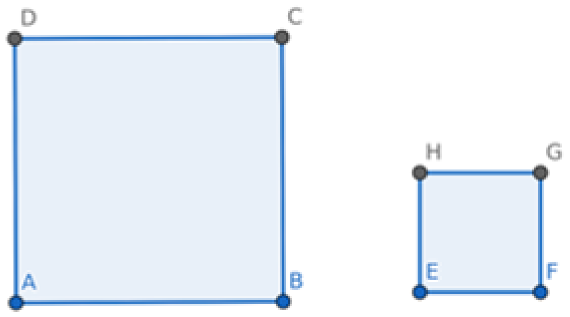


Now, shifting the centre of smaller circle to the bigger circle.



If we slowly increase the radius of smaller circle, it will coincide with bigger circle when $r_2 = r_1$. Thus, the circles are similar.

(ii) All squares are <u>similar.</u>



Let there be two squares ABCD and EFGH. When the smaller square is kept at the centre of the square ABCD, then on increasing the side of EFGH both of them will coincide. Thus, they are similar.

Two polygons are similar if their corresponding are equal. The corresponding angles of the squares are 90°. Thus, they are similar.

(iii) All equiangular triangles are similar.

Two equiangular triangles have equal corresponding angles. Thus, they are similar by AA or AAA Similarity Rule.

(iv) Two polygons with same number of sides are similar if

(a) their all the corresponding angles are equal

(b) their corresponding sides are in the same ratio

2. Question

State whether the following statement are true or false:

(i) Two congruent figures are similar.

(ii) Two similar figures are congruent.

(iii) Two polygons are similar if their corresponding sides are proportional.

(iv) Two polygons are similar if their corresponding sides are proportional and the corresponding angles are equal.

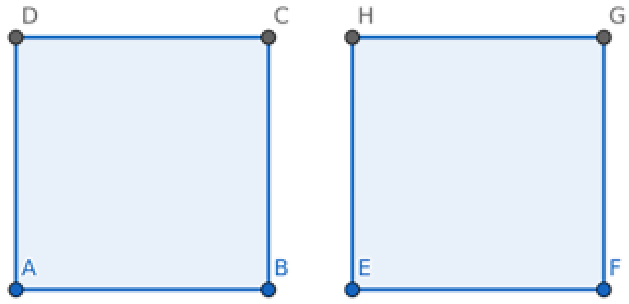
(v) Two polygons are similar if their corresponding angles are equal.

Answer

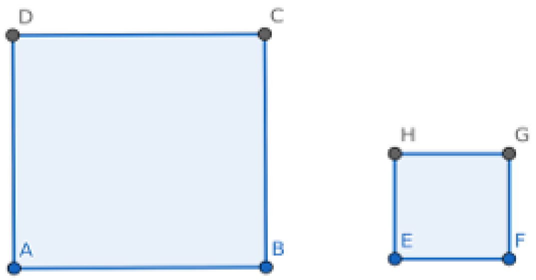
(i) True

Two figures are said to be congruent if their corresponding sides are equal and their corresponding angles are also equal.

For figures to be similar, the corresponding angles should be equal which is true in case of congruent figures. Thus, the congruent figures are also similar.



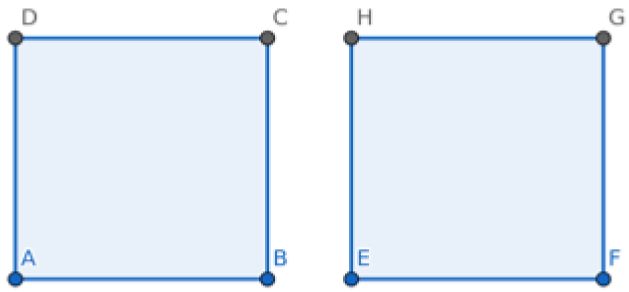
(ii) False



For figures to be similar, the corresponding angles are equal.

Two figures are said to be congruent if their corresponding sides are equal and their corresponding angles are also equal.

Thus, two similar figures are congruent only if their corresponding sides are equal.



(iii) False

Two polygons have proportional sides are similar if and only if the corresponding angles are also equal.

(iv) True

(v) True

Two polygons are similar if their corresponding sides are proportional and the corresponding angles are equal.

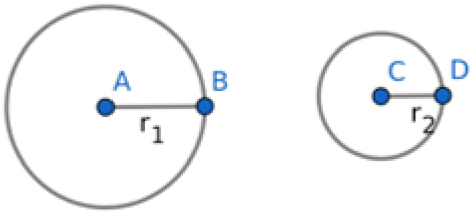
3. Question

Give two different examples of pairs of similar figures.

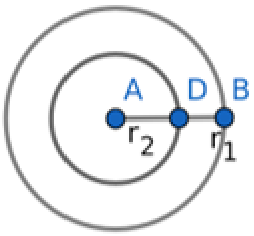
Answer

All circles are similar.

Let there be two circles of radii r_1 and r_2.

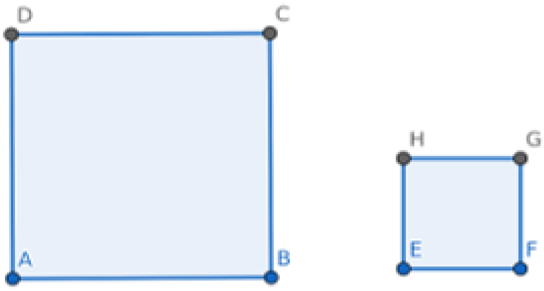


Now, shifting the centre of smaller circle to the bigger circle



If we slowly increase the radius of smaller circle, it will coincide with bigger circle when $r_2 = r_1$. Thus, the circles are similar.

All squares are similar.



Let there be two squares ABCD and EFGH. When the smaller square is kept at the centre of the square ABCD, then on increasing the side of EFGH both of them will coincide. Thus, they are similar.

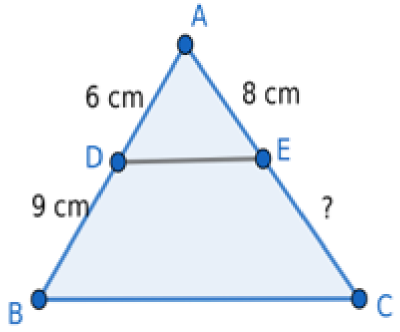
Exercise 11.2

1 A. Question

Point D and E lie on the sides AB and AC respectively of ΔABC such that DE|| BC. Then,

If AD = 6 cm, DB = 9 cm and AE = 8 cm then find the value of AC.

Answer



In ΔABC,

∵DE|| BC

$\therefore \frac{AD}{AB} = \frac{AE}{AC}$|(By Thales Theorem)

$$\frac{AD}{AB} = \frac{6}{15} = \frac{2}{5}$$

$$\frac{AE}{AC} = \frac{2}{5}$$

$$\Rightarrow AC = \frac{5}{2} \times AE$$

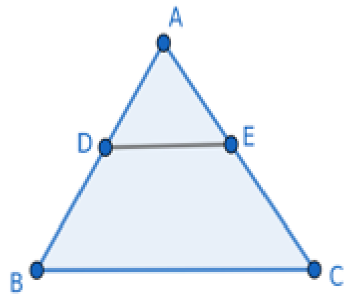
$$\Rightarrow AC = \frac{5}{2} \times 8 = 20 \text{ cm}$$

1 B. Question

Point D and E lie on the sides AB and AC respectively of ΔABC such that DE|| BC. Then,

If $\frac{AD}{DB} = \frac{4}{13}$ and AC = 20.4 cm then find the value of EC.

Answer



Given,

$$\frac{AD}{DB} = \frac{4}{13}$$

In ΔABC,

∵DE|| BC

$\therefore \frac{AD}{DB} = \frac{AE}{EC}$ |(By Thales Theorem)

$\Rightarrow \frac{AE}{EC} = \frac{4}{13}$

$\Rightarrow EC = \frac{13}{4} \times AE$

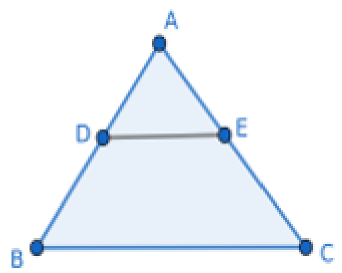
$\Rightarrow EC = \frac{13}{4} \times 8 = 26 \text{ cm}$

1 C. Question

Point D and E lie on the sides AB and AC respectively of ΔABC such that DE|| BC. Then,

If $\frac{AD}{DB} = \frac{4}{7}$ and AE = 6.3 cm then find the value of AC.

Answer



Given,

$\frac{AD}{DB} = \frac{7}{4}$

In ΔABC,

∵DE|| BC

$\therefore \frac{AD}{DB} = \frac{AE}{EC}$ |(By Thales Theorem)

$\Rightarrow \frac{AE}{EC} = \frac{7}{4}$

$\Rightarrow EC = \frac{4}{7} \times AE$

$$\Rightarrow \ EC = \frac{4}{7} \times 6.3 = 3.6 \text{ cm}$$

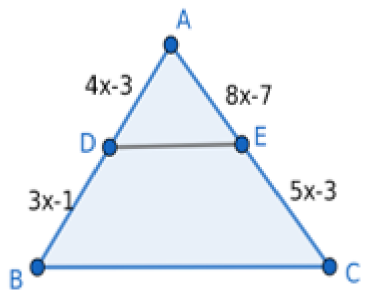
AC = AE + EC = 6.3 + 3.6 = 9.9 cm

1 D. Question

Point D and E lie on the sides AB and AC respectively of ΔABC such that DE|| BC. Then,

If AD = 4x – 3, AE = 8x – 7, BD = 3x – 1 and CE = 5x – 3, then find the value of x.

Answer



Given,

AD = 4x – 3

BD = 3x – 1

AE = 8x – 7

CE = 5x – 3

In ΔABC,

∵DE|| BC

$\therefore \frac{AD}{DB} = \frac{AE}{EC}$ |(By Thales Theorem)

$$\frac{AD}{BD} = \frac{4x-3}{3x-1}$$

$$\frac{AE}{EC} = \frac{8x-7}{5x-3}$$

$$\Rightarrow \ \frac{4x-3}{3x-1} = \frac{8x-7}{5x-3}$$

$$\Rightarrow \ (4x-3)(5x-3) = (8x-7)(3x-1)$$

$\Rightarrow 20x^2 - 27x + 9 = 24x^2 - 29x + 7$

$\Rightarrow 4x^2 - 2x - 2 = 0$

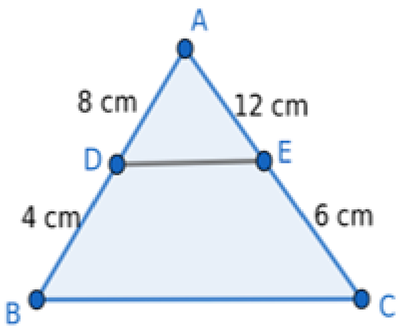
$\Rightarrow x = 1, -1/2$

2 A. Question

Two points D and E lie on sides AB and AC respectively of ΔABC. Give the information that DE|| BC is not true through the values given in the following questions:

AB = 12 cm, AD = 8 cm, AE = 12 cm and AC = 18 cm.

Answer



Suppose DE||BC,

In ΔADE & ΔABC,

∠A = ∠A |common angle

∠ADE = ∠ABC |corresponding angles

ΔADE~ΔABC by AA Similarity Rule

By Thales Theorem,

$\frac{AD}{AB} = \frac{AE}{AC}$ should be true for DE||BC.

Here, $\frac{AD}{AB} = \frac{8}{12} = \frac{2}{3}$

$$\frac{AE}{AC} = \frac{12}{18} = \frac{2}{3}$$

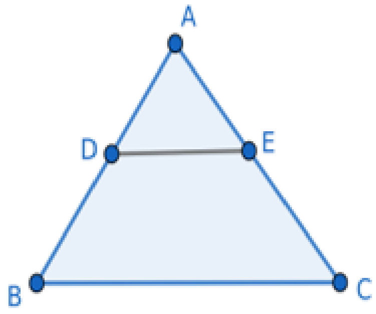
Hence, DE||BC is true.

2 B. Question

Two points D and E lie on sides AB and AC respectively of ΔABC. Give the information that DE|| BC is not true through the values given in the following questions:

AB = 5.6 cm, AD = 1.4 cm, AC = 9.0 cm and AE = 1.8 cm.

Answer



Here, $\frac{AD}{AB} = \frac{1.4}{5.6} = \frac{1}{4}$

$$\frac{AE}{AC} = \frac{1.8}{9.0} = \frac{1}{5}$$

$$\frac{AD}{AB} \neq \frac{AE}{AC}$$

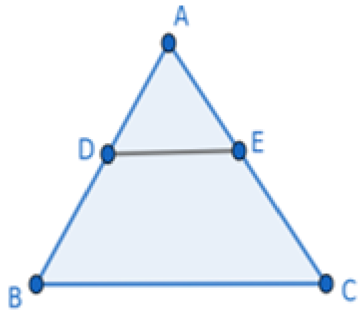
Hence, DE||BC is false.

2 C. Question

Two points D and E lie on sides AB and AC respectively of ΔABC. Give the information that DE|| BC is not true through the values given in the following questions:

AD = 10.5 cm, BD = 4.5 cm, AC = 4.8 cm and AE = 2.8 cm.

Answer



Here, $\frac{AD}{AB} = \frac{10.5}{10.5 + 4.5} = \frac{10.5}{15} = 7:10$

$$\frac{AE}{AC} = \frac{2.8}{4.8} = \frac{7}{12}$$

$$\frac{AD}{AB} \neq \frac{AE}{AC}$$

Hence, DE||BC is false.

2 D. Question

Two points D and E lie on sides AB and AC respectively of ΔABC. Give the information that DE|| BC is not true through the values given in the following questions:

AD = 5.7 cm, BD = 9.5 cm, AE = 3.3 cm and EC = 5.5 cm.

Answer

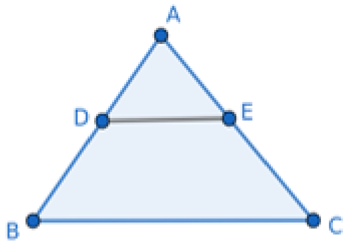
Given,

AD = 5.7 cm

BD = 9.5 cm

AE = 3.3 cm

EC = 5.5 cm.



AB = AD + BD = 5.7 + 9.5 = 15.2 cm

Here, $\frac{AD}{AB} = \frac{5.7}{5.7 + 9.5} = \frac{5.7}{15.2} = 3:8$

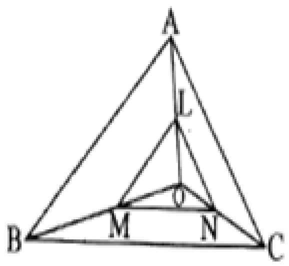
$\frac{AE}{AC} = \frac{3.3}{3.3 + 5.5} = \frac{3.3}{8.8} = 3:8$

$\frac{AD}{AB} = \frac{AE}{AC}$

Hence, DE||BC is true.

3. Question

In the given figure points L, M and N respectively lie on OA, OB and OC such that LM || AB and MN || BC. Then, show that LN || AC.



Answer

In OAB & OLM

LM||AB

$\Rightarrow \frac{OL}{AL} = \frac{OM}{BM}$ ...(1) |By Basic Proportionality Theorem(BPT)

In ΔOMN & ΔOBC,

MN||BC

$\Rightarrow \frac{OM}{BM} = \frac{ON}{NC}$ ...(2) |By BPT

From (1)&(2)

$\frac{OM}{BM} = \frac{ON}{NC}$ ...(3)

In ΔOLN & ΔOAC

$$\frac{OM}{BM} = \frac{ON}{NC}$$

⇒ LN||AC by Converse of BPT

Hence, proved.

4. Question

In ΔABC points D and E are situated on sides AB and AC respectively such that BD = CE. If ∠B = ∠C then show that DE || BC.

Answer

In ΔABC & ΔADE

∠B = ∠C |Given

⇒ AC = AB |sides opposite to equal angles are equal

⇒ AB = AC ...(1)

Given,

BD = CE

Subtracting BD from AB and CE from AC,

⇒ AB – BD = AC – CE

⇒ AD = AE |...(2)

In ΔABC & ΔADE,

From (1) and (2)

$$\frac{AD}{AB} = \frac{AE}{AC}$$

∠B = ∠C

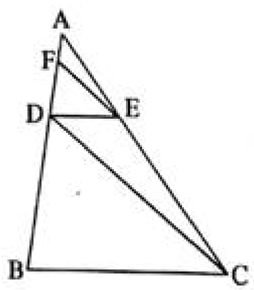
⇒ ABC~ADE by SAS Similarity Rule

⇒ ∠ADE = ∠ABC

⇒ DE||BC as all corresponding angles are equal.

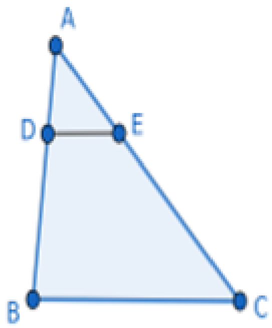
5. Question

In figure, if DE || BC and CD || EF then prove that $AD^2 = AB \times AF$.



Answer

In ΔADE & ΔABC,



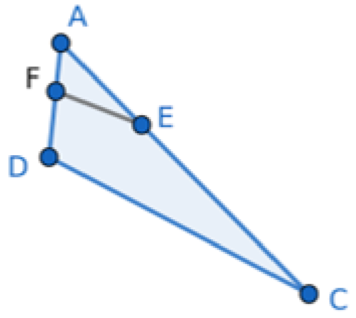
∠A = ∠A |common angle

∠ADE = ∠ABC |corresponding angles

ΔADE~ ΔABC by AA Similarity Rule

$$\frac{AD}{AB} = \frac{AE}{AC} \ldots(1)$$

In ΔAFE & ΔADC,



∠A = ∠A |common angle

∠AFE = ∠ADC |corresponding angles

ΔAFE~ ΔADC by AA Similarity Rule

$\frac{AF}{AD} = \frac{AE}{AC}$...(2)

From (1) & (2),

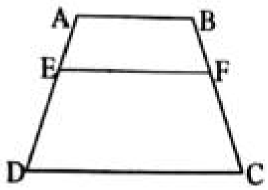
$\frac{AD}{AB} = \frac{AF}{AD}$

⇒ $AD^2 = AB \times AF$

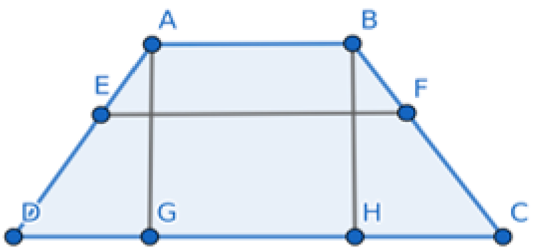
Hence, proved.

6. Question

In figure, if EF || DC || AB then prove that $\frac{AE}{ED} = \frac{BF}{FC}$.

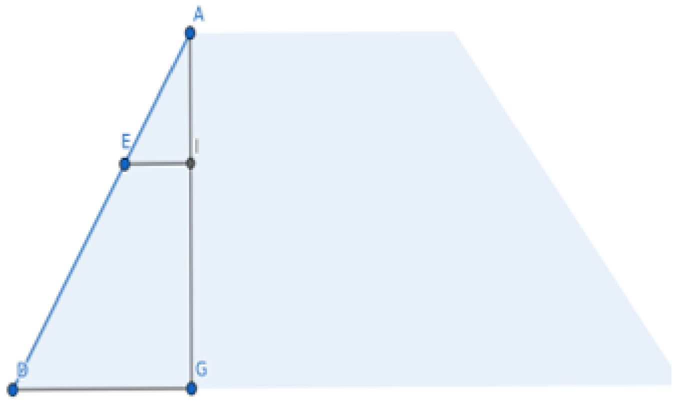


Answer



Let us drop a perpendicular AG and BH to CD cutting EG at I and J and CD.

In ΔADG & ΔAEI,



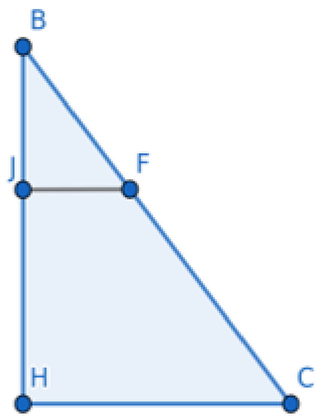
∠AGD = ∠AIE |Right Angle

∠AEI = ∠ADG |corr. ∠s

ΔADG~ΔAEI by AA Similarity Rule

$$\Rightarrow \frac{AE}{ED} = \frac{AI}{IG}$$

In ΔBJF & ΔBHC,



∠BJF = ∠BHC |Right angle

∠BFJ = ∠BCH |corr. ∠s

ΔBJF~ΔBHC by AA Similarity Rule

$$\Rightarrow \frac{BJ}{BH} = \frac{BF}{FC}$$

In rectangle ABHG & ABJI,

AI = BJ ...(a) |opposite sides of rectangle are equal

AG = BH ...(b) |opp. sides of rectangle

From eqn. (b) – (a)

AG – AI = BH – BJ

⇒ GI = HJ

$$\Rightarrow \frac{AI}{IG} = \frac{BJ}{BH}$$

$$\Rightarrow \frac{BF}{FC} = \frac{BJ}{BH} = \frac{AI}{IG} \ldots(2)$$

From (1) & (2),

$$\frac{AE}{ED} = \frac{BF}{FC}$$

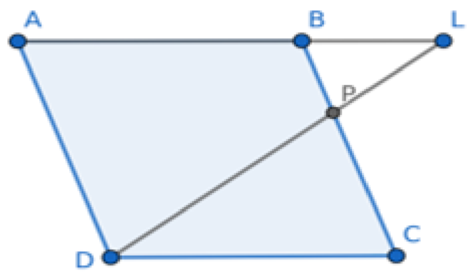
Hence, proved.

7. Question

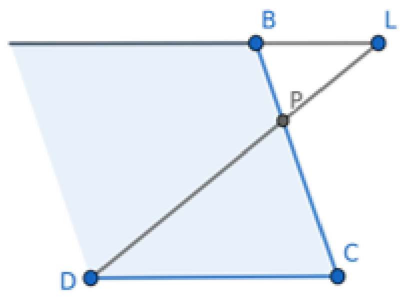
ABCD is a parallelogram on whose side BC a point P lies. It DP and AB are produced ahead then they meet at L. Then prove that

(i) $\frac{DP}{PL} = \frac{DC}{BL}$ (ii) $\frac{DL}{DP} = \frac{AL}{DC}$

Answer

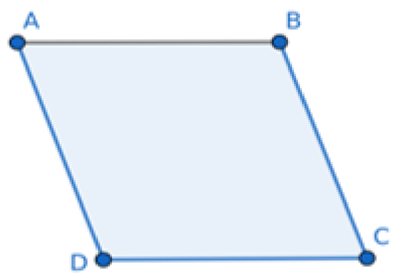


(i) In ΔDPC & ΔBPL,



∠DPC = ∠BPL |vertically opposite ∠s

In ||gm ABCD,



DC||AB or DC||AL,

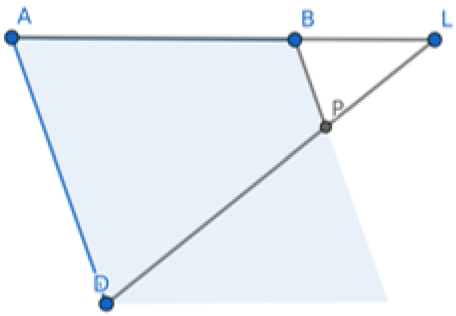
⇒ ∠DCP = ∠LBP

ΔDPC~ΔBPL by AA Similarity Rule

$$\frac{DP}{PL} = \frac{DC}{BL}$$

Hence,proved.

(ii)In ΔPLB & ΔDLA,



∠L = ∠L |common angle

In ||gm ABCD, AD||BC or AD||BP,

⇒ ∠LPB = ∠LDC |corresponding angles

ΔPLB~ΔDLA by AA Similarity Rule

$\frac{PL}{DL} = \frac{BL}{AL}$ ...(1)

$$\frac{DL}{PL} = \frac{AL}{BL}$$

Subtracting 1 from both sides of the above equation,

$$\frac{DL}{PL} - 1 = \frac{AL}{BL} - 1$$

$$\frac{DL - PL}{PL} = \frac{AL - BL}{BL}$$

$\frac{DP}{PL} = \frac{AB}{BL} = \frac{DC}{BL}$ ...(2)

Multiplying (1) & (2),

$$\frac{DP}{DL} = \frac{DC}{AL}$$

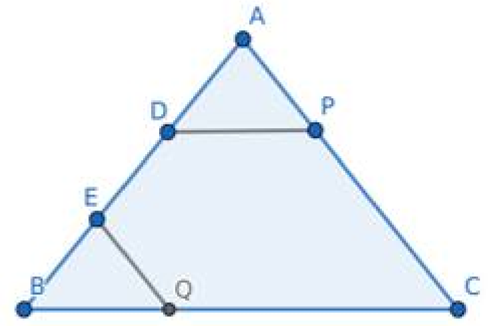
Or, $\frac{DL}{DP} = \frac{AL}{DC}$

Hence, proved.

8. Question

On side AB of ΔABC two points D and E lie such that AD = BE. If DP || BC and EQ || AC then prove that PQ || AB.

Answer



In ΔABC,

EQ||AC

By Basic Proportionality Theorem,

$$\frac{BQ}{CQ} = \frac{BE}{AE}$$

AD = BE |Given

AE = AD + DE = BE + ED = BD

$$\frac{BQ}{QC} = \frac{AD}{BD} \text{ ...(1)}$$

In ΔABC,

DP||BC

By Basic Proportionality Theorem,

$$\frac{AD}{BD} = \frac{AP}{PC} \text{ ...(2)}$$

From (1) & (2),

$$\frac{BQ}{QC} = \frac{AP}{PC}$$

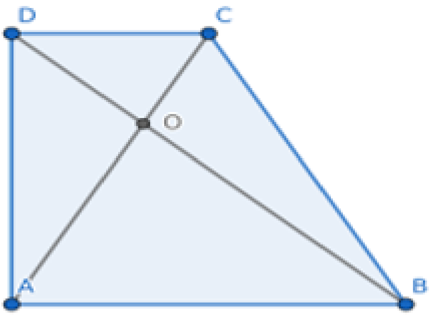
By Converse of BPT, PQ||AB.

Hence, proved.

9. Question

ABCD is a trapezium in which AB || DC and its diagonals intersect at O. Show that $\frac{AO}{BO} = \frac{CO}{DO}$.

Answer



In ΔAOB & ΔCOD,

∠AOB = ∠COD

∠ABO = ∠ODC

∠OAB = ∠OCD

ΔAOB~ΔCOD by AAA Similarity Rule

$\Rightarrow \frac{AO}{CO} = \frac{BO}{DO}$

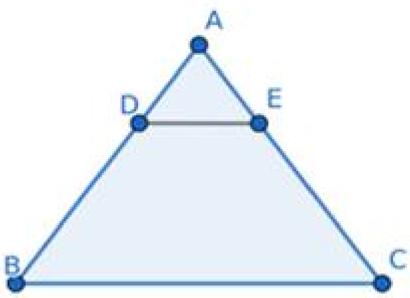
$\Rightarrow \frac{AO}{BO} = \frac{CO}{DO}$

Hence,proved.

10. Question

If D and E are points lying on sides AB and AC respectively of ΔABC such that BD = CE. Then prove that ΔABC is an isosceles triangle.

Answer



In ΔADE & ΔABC,

∠ADE = ∠ABC

∠A = ∠A

ΔADE~ΔABC by AA Similarity Rule

$$\frac{AD}{BD} = \frac{AE}{CE}$$

BD = CE

⇒ AD = AE

Now,

AD + BD = AE + CE

AB = AC

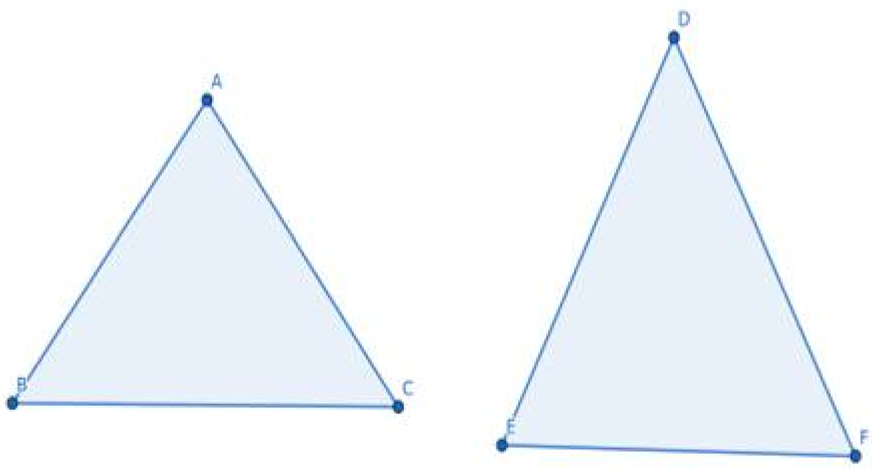
Thus, the triangle ABC is isosceles.

Exercise 11.3

1. Question

In two triangles ABC and PQR $\frac{AB}{PQ} = \frac{BC}{QR}$. Name two angles of two triangles which must be equal so that these triangles may be similar. Give reason also for your answer.

Answer



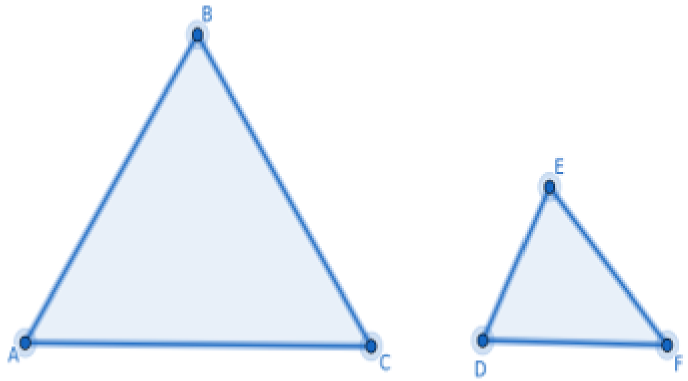
As per SAS rule of similarity, the angle between sides AB & BC of ΔABC and PQ & QR of ΔPQR should be equal.

∠ABC = ∠PQR

2. Question

In triangles ABC and DEF, if ∠A = ∠D, ∠B = ∠F then is ΔABC ~ ΔDEF? Give reason for your answer.

Answer



Thus,

ΔABC is not similar to ΔDEF as the order should be ΔDFE to be similar as given conditions.

3. Question

If ΔABC ~ ΔFDE then can you say that $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$? Write your answer with reason.

Answer

For similar triangles, the ratio of corresponding sides are equal. The order of sides should be such that the similar sides are written at the same position of naming.

In the given question, the ratio cannot be written as the order of sides is not same. The order should be

$$\frac{AB}{FD} = \frac{BC}{DE} = \frac{AC}{EF}$$

4. Question

If two sides and one angle of a triangle and respectively proportional and equal to two sides and one angle of another triangle then the two triangles are similar. Is this statement true? Write answer with reason.

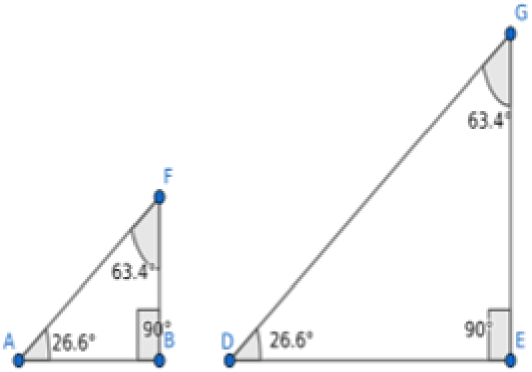
Answer

The triangles may not be similar. The triangles should have corresponding sides proportional and the angle between them to be similar by SAS Similarity Rule.

5. Question

What do you mean by equiangular triangles? What mutual relation can these hold?

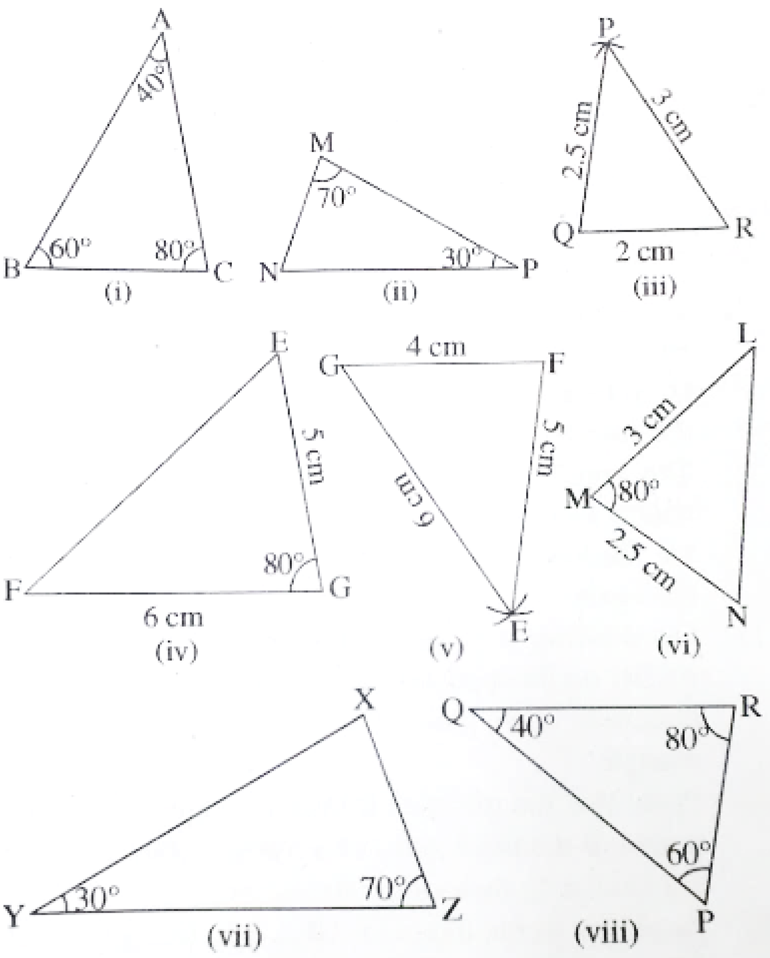
Answer



If the corresponding angles in two triangles are equal, then they are said to be equiangular. The equiangular triangles are always similar by AAA Similarity Rule.

6. Question

Select the pairs of similar triangles from the figures of triangles given below and write them in symbolic language of their being similar.



Answer

In ΔABC & $\Delta Q'P'R$,

$\angle A = \angle Q'$

$\angle B = \angle P'$

$\angle C = \angle R'$

Thus, $\Delta ABC \sim \Delta Q'P'R'$ |AAA Similarity Rule

In ΔMNP & ΔZXY,

$\angle M = \angle Z$

$\angle P = \angle Y$

Thus, $\Delta MNP \sim \Delta ZXY$ |AA Similarity Rule

In ΔPQR & ΔEFG,

$$\frac{PQ}{EF} = \frac{QR}{FG} = \frac{PR}{EG} = \frac{1}{2}$$

Thus, the corresponding sides are proportional.

Thus, $\Delta PQR \sim \Delta EFG$ |SSS Similarity Rule

In $\Delta E'G'F'$ & ΔNML,

$\angle E'G'F' = \angle NML$

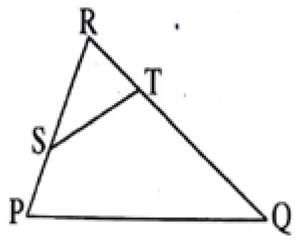
$$\frac{E'G'}{G'F'} = \frac{NM}{ML}$$

Thus, the corresponding sides are proportional with the angle between them equal.

Thus, $\Delta E'G'F' \sim \Delta NML$ |SAS Similarity Rule

7. Question

Figure $\Delta PRQ \sim \Delta TRS$. Then state which angles must be mutually equal in this pair of similar triangles.

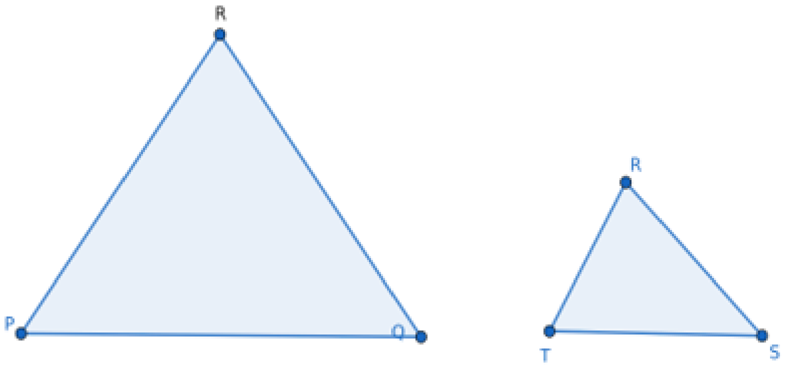


Answer

Given,

$\Delta PRQ \sim \Delta TRS$

$$\Rightarrow \frac{PR}{TR} = \frac{PQ}{ST} = \frac{QR}{SR}$$



The mutually equal angles are

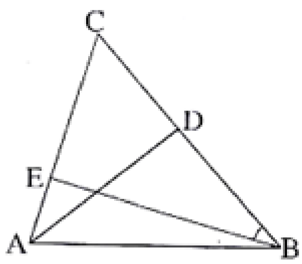
∠PRQ = ∠TRS

∠RPQ = ∠RTS

∠RQP = ∠RST

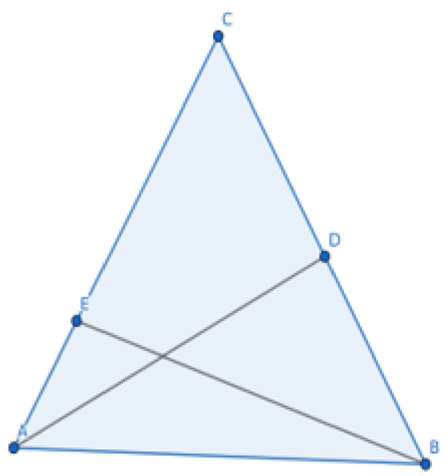
8. Question

You are to select two triangles in the figure which are mutually similar if ∠CBE is = ∠CAD.



Answer

Given,



∠CBE = ∠CAD

In ΔCAD & ΔCBE,

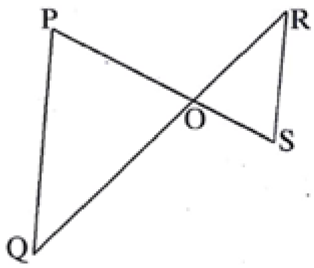
∠ACD = ∠BCE |Common Angle

∠CBE = ∠CAD |Given

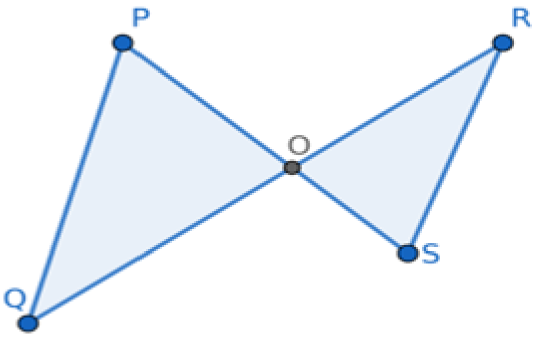
⇒ ΔCAD~ΔCBE by AA Similarity Rule

9. Question

In figure PQ and RS are parallel. Then prove that ΔPOQ ~ ΔSOR.



Answer



Given,

PQ||RS

In ΔPOQ & ΔSOR,

⇒ ∠PQO = ∠ORS |Alternate angles

⇒ ∠QPO = ∠OSR |Alternate angles

⇒ ∠POQ = ∠ROS |Vertically opposite angles

Thus, ΔPOQ~ΔSOR by AAA Similarity Rule

10. Question

A give of height 90 cm is walking away from the base of a lamp – post at a speed of 1.2 m/s. If the lamp is at a height of 3.6 m above the ground, find the length of her shadow after 4 seconds.

Answer

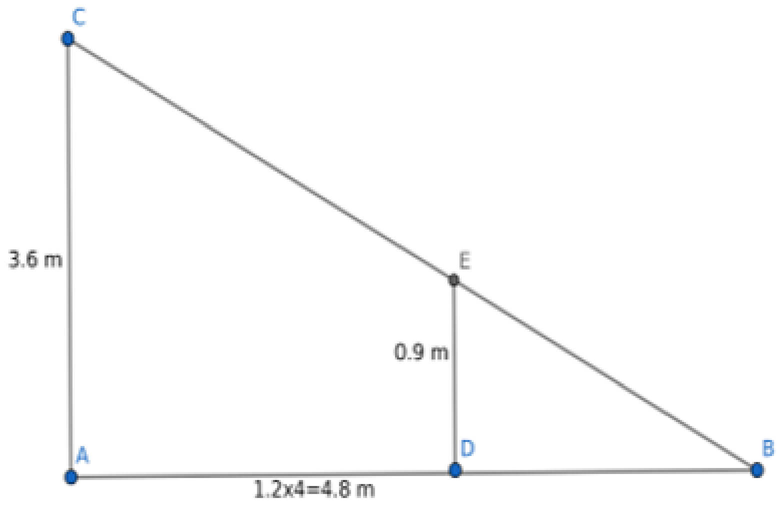
The speed of give = 1.2 ms^{-1}

Time taken = 4 seconds

Distance = speed × time taken

In 4 seconds, the distance walked is 1.2×4 = 4.8 m.

As both the lamp post and the give are perpendicular to the ground, they make the following similar triangles.



In ΔBDE & ΔBAC,

∠B = ∠B |common angle

DE||AC

∠BDE = ∠BAC

ΔBDE~ΔBAC by AA Similarity Rule

$$\frac{BD}{AB} = \frac{DE}{AC}$$

$$\frac{x}{x + 4.8} = \frac{0.9}{3.6} = \frac{1}{4}$$

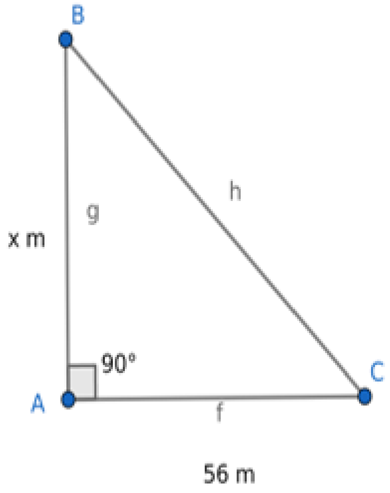
4x = x + 4.8

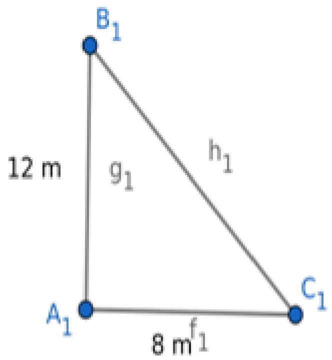
⇒ x = 1.6 m

11. Question

The length of the shadow of a vertical pillar of length 12 m is 8 m. At the same time the length of the shadow of a tower is 56 m. Find the height of the tower.

Answer





Given,

length of the pillar = A_1B_1 = 12 m

length of the shadow of pillar = A_1C_1 = 8 m

length of the shadow of the tower = AC = 56 m

Let the length of the tower = AB be x m.

As seen from the figure,

ΔABC is similar to $\Delta A_1B_1C_1$

$$\Rightarrow \frac{AB}{AC} = \frac{A_1B_1}{A_1C_1}$$

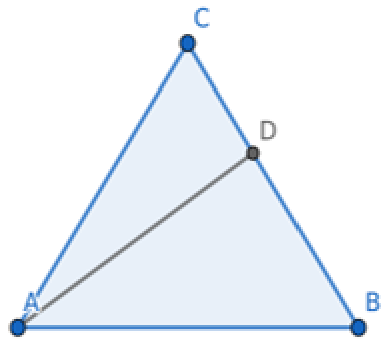
$$\Rightarrow AB = \frac{A_1B_1 \times AC}{A_1C_1}$$

$$\Rightarrow AB = \frac{12 \times 56}{8} = 84\text{ m}$$

12. Question

On drawing a perpendicular from vertex A of a ΔABC on its opposite side BD AD^2 = BD × DC is obtained. Then prove that ABC is a right angled triangle.

Answer



Given,

$AD^2 = BD \times DC$

$\Rightarrow \frac{AD}{BD} = \frac{DC}{AD}$ |(1)

$\angle ADB = 90°$

In ΔADC & ΔADB,

$$\frac{AD}{BD} = \frac{DC}{AD}$$

$\angle ADB = \angle CDA = 90°$

ΔADC~ΔADB by SAS Similarity Rule

$\Rightarrow \angle CAD = \angle ABD$

$\Rightarrow \angle ACD = \angle BAD$ ...(1)

In ΔADC,

$\angle CAD + \angle ACD + \angle ADC = 180°$

$\Rightarrow \angle CAD + \angle ACD = 180° - 90° = 90°$

From (1),

$\Rightarrow \angle CAD + \angle BAD = 90°$

$\Rightarrow \angle BAC = 90°$

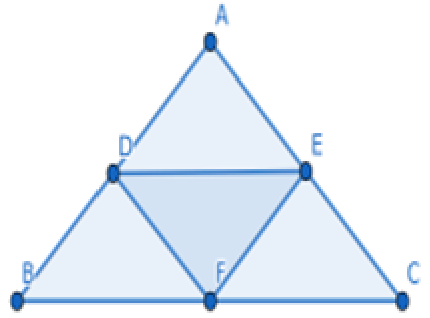
Thus, ABC is right angled triangle.

13. Question

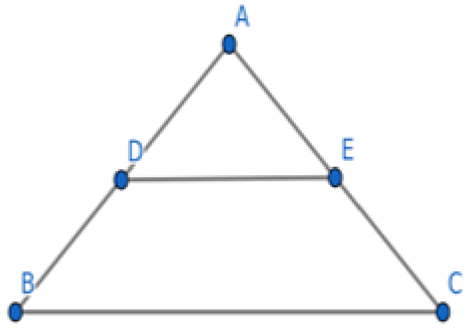
Prove that the triangles formed by joining the mid – points of the three sides of a triangle consecutively are similar to their original triangle.

Answer

Let there be a ΔABC with the mid points D,E and F of sides AB, AC and BC.



In ΔADE and ΔABC,



D is the mid point of AB and E is the mid point of AC.

By Midpoint theorem,

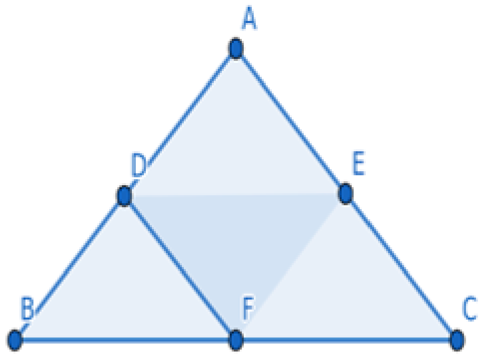
$$\frac{DE}{BC} = \frac{1}{2}$$

$$\Rightarrow \frac{DE}{2 \times BF} = \frac{1}{2}$$

$$\Rightarrow \frac{DE}{BF} = 1$$

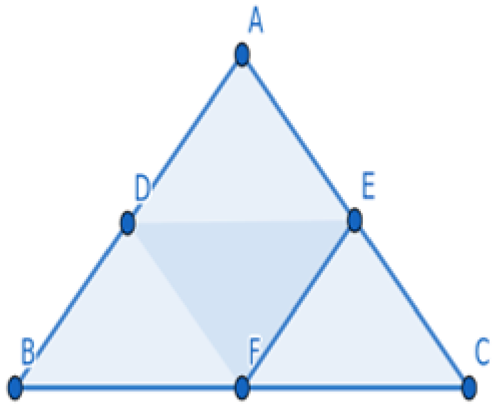
⇒ DE = BF |(1)

Similarly in ΔBFD & ΔBCA,



DF = EC = AE |(2)

Similarly in ΔCFE & ΔCBA



EF = AD = DB |(3)

In ΔADE & ΔBDF,

AD = DB |D is mid point

BF = DE |From (1)

DF = EA |From (2)

Thus, ΔADE & ΔBDF are similar to each other by SSS Similarity Rule.

⇒ ΔADE~ΔBDF

Similarly,

ΔADE~ΔEFC

ΔDBF~ΔEFC

In ΔADE & ΔDEF,

AD = EF |From (3)

DE = DE

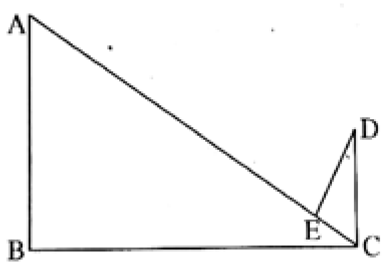
EA = DF |From (2)

⇒ ΔADE~ΔDEF

Thus, all the smaller triangles are similar to each other.

14. Question

As shown in the figure of AB ⊥ BC, DC ⊥ BC and DE ⊥ AC then prove that ΔCED ~ ΔABC.



Answer

In ΔABC and ΔCED,

DC ⊥ BC

⇒ ∠ACB + ∠DCE = 90°

Let ∠ACB = x

⇒ ∠DCE = 90° – x

In ΔABC,

∠ACB + ∠ABC + ∠BAC = 180°

⇒ x + 90° + ∠BAC = 180°

⇒ ∠BAC = 90° – x

In ΔABC and ΔCED,

∠ABC = ∠DEC = 90°

∠ACB = ∠CDE = x

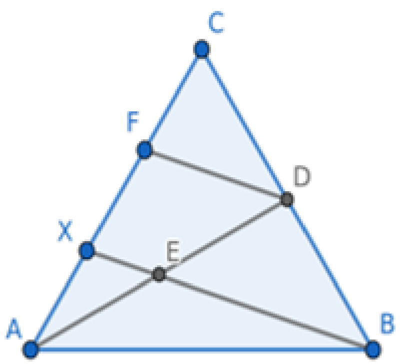
∠BAC = ∠DCE = 90° – x

Thus, the ΔABC and ΔCED are similar by AAA Similarity Rule.

15. Question

The mid point of side BC of ΔABC is D. If from B a line is drawn bisecting AD such that cutting side AD at E if cuts AC at X. Then prove that $\frac{EX}{BE} = \frac{1}{3}$.

Answer



Let a point F on AC such that DF||BX.

By Converse of Mid Point Theorem, as D is mid point of BC, F is the mid point of AC.

⇒ CF = XF

In ΔCFD & ΔCXB,

$\frac{BX}{DF} = \frac{CX}{CF} = \frac{CF + CX}{CF} = \frac{2}{1}$ |By Mid Point Theorem

⇒ BX = 2DF

In ΔAXE & ΔAFD

E is the mid point of AD

and EX | | DF

By Mid Point Theorem,

AX = XF

$$EX = \frac{DF}{2}$$

$$\frac{BE}{EX} = \frac{BX - EX}{EX} = \frac{2DF - \frac{DF}{2}}{\frac{DF}{2}} = 3:1$$

$$\Rightarrow \frac{EX}{BE} = \frac{1}{3}$$

Hence, proved.

Exercise 11.4

1. Question

Answer the following in True or False. Write the reason of your answer (if possible).

(i) The ratio of the corresponding sides of two similar triangles is 4 : 9. Then the ratio of the areas of these triangles is 4 : 9.

(ii) In two triangles respectively ΔABC and ΔDEF of $\frac{\Delta ABC}{\Delta DEF} = \frac{AB^2}{DE^2} = \frac{9}{4}$ then ΔABC ≅ ΔDEF.

(iii) The ratio of the areas of two similar triangles in proportional to the squares of their sides.

(iv) If ΔABC and ΔAXY are similar and the values of their areas are the same then XY and BC may be coincident sides.

Answer

(i) False

The ratio of areas of two similar triangles is equal to the ratio of square of their corresponding sides.

So, in the given question the ratio of areas should be 16:81.

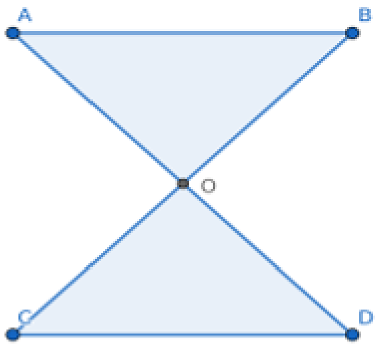
(ii) False

The ratio of areas of two similar triangles is equal to the ratio of square of their corresponding sides.

In two triangles respectively ΔABC and ΔDEF of $\frac{\text{area of ABC}}{\text{area of DEF}} = \frac{AB^2}{DE^2}$ then ΔABC~ΔDEF.

(iii) True

(iv) True



In the ΔABO & ΔOCD,

∠AOB = ∠DOC |vertically opp. angles

As AB||CD

∠ABO = ∠DCO |alternate angles

ΔABO~ΔOCD

The sides BC and XY may or may not be coincident.

2. Question

If ΔABC~ΔDEF and their areas are respectively 64 sq cm and 121 sq cm. If EF = 15.4 cm then find BC.

Answer

The ratio of areas of two similar triangles is equal to the ratio of square of their corresponding sides.

In two triangles respectively ΔABC~ΔDEF $\frac{\text{area of ABC}}{\text{area of DEF}} = \frac{BC^2}{EF^2}$ then

$$\frac{BC}{EF} = \sqrt{\frac{\text{area of ABC}}{\text{area of DEF}}}$$

$$\Rightarrow \frac{BC}{EF} = \sqrt{\frac{64}{121}} = \frac{8}{11}$$

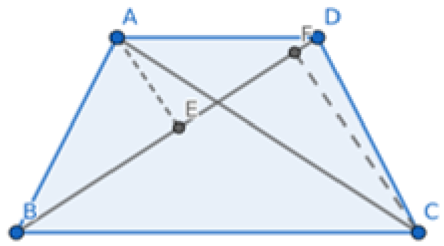
$$\Rightarrow BC = EF \times \frac{8}{11} = 15.4 \times \frac{8}{11}$$

$$\Rightarrow BC = 11.2 \text{ cm}$$

3. Question

Two triangles ABC and DBC are formed on the same base BC. If AD and BC intersect each other at O then prove that $\frac{\text{area of } \Delta ABC}{\text{area of } \Delta DBC} = \frac{AO}{DO}$.

Answer



$$\text{Area of } \Delta ABC = \frac{1}{2} \times BC \times AF$$

$$\text{Area of } \Delta DBC = \frac{1}{2} \times BC \times DG$$

$$\frac{\text{area of } \Delta ABC}{\text{area of } \Delta DBC} = \frac{\frac{1}{2}\times BC\times AF}{\frac{1}{2}\times BC\times DG} = \frac{AF}{DG} \ldots(1)$$

In ΔAOF & ΔDOG,

∠AOF = ∠DOG |vertically opp. angles

∠AFO = ∠DGO |both right angles

ΔAOF~ΔDOF by AA Similarity Rule

$$\frac{AF}{DG} = \frac{AO}{DO} \ldots(2)$$

From (1) & (2),

$$\frac{\text{area of } \Delta ABC}{\text{area of } \Delta DBC} = \frac{AO}{DO}$$

Hence, proved.

4 A. Question

Find the solutions of the following questions:

In ΔABC DE || BC and AD : DB = 2 : 3 then find the ratio of the areas of ΔADE and ΔABC.

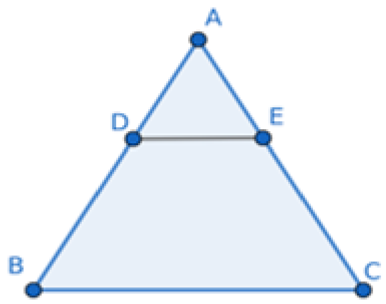
Answer

$$\text{Let } \frac{AD}{DB} = \frac{2}{3} = \frac{2k}{3k} \text{(say)}$$

Let AD and DB be 2k and 3k.

AB = AB + DB = 2k + 3k = 5k

$$\Rightarrow \frac{AD}{AB} = \frac{2k}{5k} = \frac{2}{5}$$



In ΔADE & ΔABC,

DE||BC

⇒ ∠ADE = ∠ABC

⇒ ∠AED = ∠ACB |alternate angles

ΔADE~ΔABC

$$\frac{\text{area of } \Delta ADE}{\text{area of } \Delta ABC} = \left(\frac{AD}{AB}\right)^2 = \left(\frac{2}{5}\right)^2 = \frac{4}{25}$$

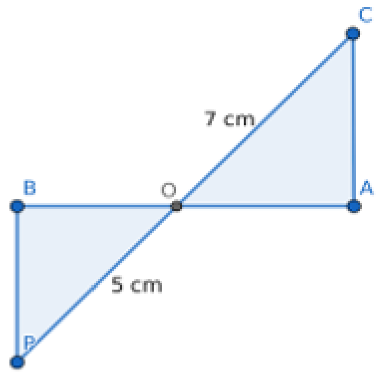
$$\frac{\text{area of } \Delta ADE}{\text{area of } \Delta ABC} = 4:25$$

4 B. Question

Find the solutions of the following questions:

PB and QA are perpendicular at points B and A of line segment AB. If P and Q lie on opposite sides of AB and on joining P and Q it intersects AB at O and PO = 5 cm, QO = 7 cm, area of ΔPOB = 150 cm^2 then find the area of ΔQOA.

Answer



In ΔPOB & ΔQOA

∠PBO = ∠QAO = 90°

∠POB = ∠QOA |vertically opposite angles

ΔPBO~ΔQOA by AA Similarity Rule

$$\frac{\text{area of } \Delta POB}{\text{area of } \Delta QOA} = \left(\frac{PO}{QO}\right)^2 = \left(\frac{5}{7}\right)^2 = \frac{25}{49}$$

$$\text{area of } \Delta QOA = \text{area of } \Delta POB \times \frac{49}{25}$$

$$\text{area of } \Delta QOA = 150 \times \frac{49}{25} = 294 \text{ cm}^2$$

4 C. Question

Find the solutions of the following questions:

Find the value of x in terms of a, b and c in the figure.

Answer

In ΔBCD & ΔACE

∠C = ∠C |common angle

∠CBD = ∠CAE |given

⇒ ΔBCD~ΔACE

$$\Rightarrow \frac{BD}{AE} = \frac{BC}{AC}$$

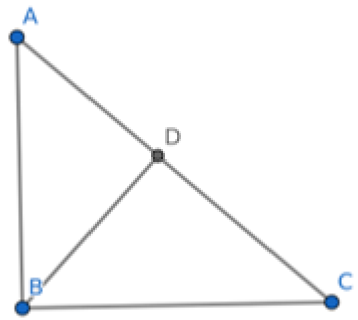
$$\Rightarrow BD = AE \times \frac{BC}{AC}$$

$$\Rightarrow BD = a \times \frac{c}{b + c} = \frac{ac}{b + c}$$

5. Question

In ΔABC if ∠B = 90° and BD perpendicular to hypotenuse AC then prove that ΔADB ~ ΔBDC.

Answer



Let ∠ABD = x

⇒ ∠BAD = 90 – x

∠DBC = 90 – x

⇒ DCB = 90 – (90 – x) = x

In ΔADB&ΔBDC,

∠ADB = ∠BDC = 90°

∠ABD = ∠DCB = x

∠BAD = ∠DBC = 90 – x

⇒ ΔADB~ΔBDC by AAA Similarity Rule

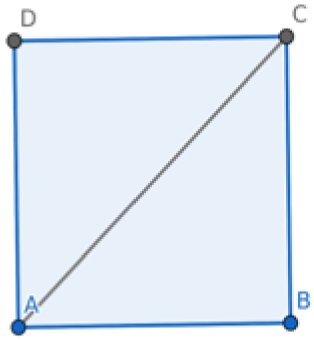
Hence, proved.

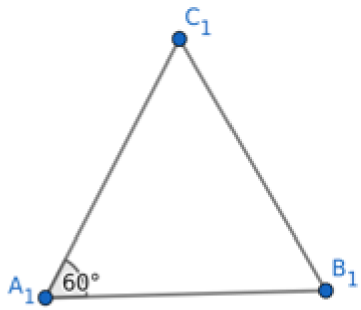
6. Question

Prove that area of an equilateral triangle formed an one side of a square is half of the area of the equilateral triangle formed on one diagonal of that square itself.

Answer

Let there be a square ABCD with diagonal AC of side ‘a’ .





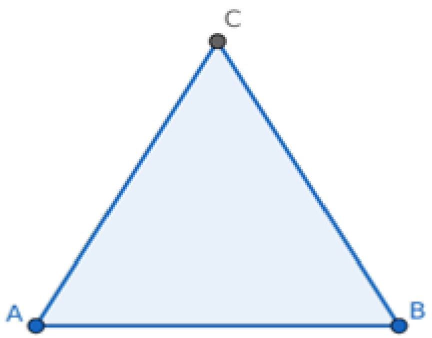
For equilateral triangle drawn on one side of the square,

In $\Delta B_1C_1E_1$,

Side = a

$$\text{Area(A1)} = \frac{\sqrt{3}a^2}{2}$$

For the equilateral triangle formed on one diagonal of that square,



In ΔABC,

side = $\sqrt{2}a$

$$\text{Area(A2)} = \frac{\sqrt{3}(\sqrt{2}a)^2}{2} = 2 \times \frac{\sqrt{3}a^2}{2} = 2 \times A_1$$

$$\Rightarrow A_1 = \frac{A_1}{2}$$

Thus, the area of an equilateral triangle formed an one side of a square is half of the area of the equilateral triangle formed on one diagonal of that square itself.

Hence, proved.

Miscellaneous Exercise 11

1. Question

If figure DE || BC. If AD = 4 cm, DB = 6 cm and AE = 5 cm, then the value of EC will be:

A. 6.5 cm

B. 7.0 cm

C. 7.5 cm

D. 8.0 cm

Answer

DE||BC

By Basic Proportionality Theorem(BPT),

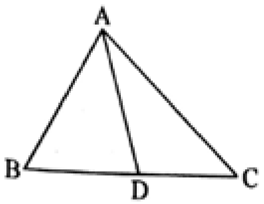
$$\frac{AD}{DB} = \frac{AE}{EC}$$

Putting the values,

$$EC = \frac{AE}{AD} \times DB = \frac{5}{4} \times 6 = 7.5 \text{ cm}$$

2. Question

In figure AD is the bisector of ∠A. If AB = 6 cm, BD = 8 cm, DC = 6 cm, then the value of AC will be:



A. 4.0 cm

B. 4.5 cm

C. 5 cm

D. 5.5 cm

Answer

Given,

AB = 6 cm, BD = 8 cm, DC = 6 cm

In ΔABC, AD is the internal angle bisector of angle A.

By internal angle bisector theorem, the internal angle bisector divides the opposite side in the ratio of other two sides.

$$\frac{AC}{AB} = \frac{DC}{BD}$$

Putting given values,

$$\Rightarrow AC = AB \times \frac{DC}{BD} = 6 \times \frac{6}{8}$$

$$\Rightarrow AC = 4.5 \text{ cm}$$

3. Question

If figure, if DE || BC, then the value of x will be:

A. √5

B. √6

C. √3

D. √7

Answer

Given,

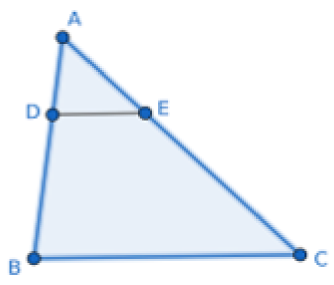
AD = x + 4

DB = x + 3

AE = 2x – 1

EC = x + 1

In ΔADE & ΔABC,



DE||BC

By BPT,

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Putting the values,

$$\frac{x + 4}{x + 3} = \frac{2x - 1}{x + 1}$$

$\Rightarrow (x + 4)(x + 1) = (2x - 1)(x + 3)$

$\Rightarrow x^2 + 5x + 4 = 2x^2 + 5x - 3$

$\Rightarrow x^2 + 4 = 2x^2 - 3$

$\Rightarrow x^2 = 7$

$\Rightarrow x = \sqrt{7}$

4. Question

In figure, if AB = 3.4 cm, BD = 4 cm, BC = 10 cm, then the value of AC will be:

A. 5.1 cm

B. 3.4 cm

C. 6 cm

D. 5.3 cm

Answer

Given,

AB = 3.4 cm, BD = 4 cm, BC = 10 cm

DC = BC – BD = 10 – 4 = 6 cm

In ΔACB, AD is the internal angle bisector of angle A.

By internal angle bisector theorem,

$$\frac{AC}{AB} = \frac{DC}{BD}$$

Putting given values,

$$\Rightarrow AC = AB \times \frac{DC}{BD} = 3.4 \times \frac{6}{4}$$

$$\Rightarrow AC = 5.1 \text{ cm}$$

5. Question

The areas of two similar triangles are respectively 25 cm^2 and 36 cm^2. If the median of the smaller triangle is 10 cm, then the corresponding median of the larger triangle will be:

A. 12 cm

B. 15 cm

C. 10 cm

D. 18 cm

Answer

The ratio of the areas of two similar triangles is equal to the ratio of squares of their corresponding medians.

$$\Rightarrow \frac{\text{area of } \Delta 1}{\text{area of } \Delta 2} = \left(\frac{\text{median of } \Delta 1}{\text{median of } \Delta 2}\right)^2 = \frac{25}{36}$$

$$\Rightarrow \frac{\text{median of } \Delta 1}{\text{median of } \Delta 2} = \sqrt{\frac{25}{36}} = \frac{5}{6}$$

$$\Rightarrow \text{median of } \Delta 2 = \frac{6}{5} \times \text{median of } \Delta 1$$

$$\Rightarrow \text{median of } \Delta 2 = \frac{6}{5} \times 10 = 12 \text{ cm}$$

6. Question

In a trapezium ABCD, AB || CD and its diagonals meet at point O. If AB = 6 cm and DC = 3 cm then the ratio of the areas of ΔAOB and ΔCOD will be:

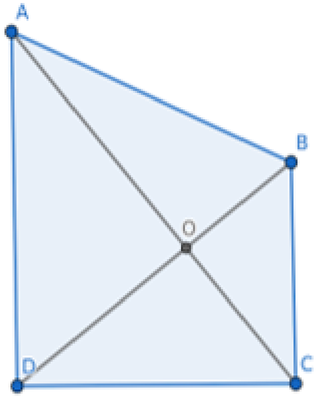
A. 4 : 1

B. 1 : 2

C. 2 : 1

D. 1 : 4

Answer



In ΔAOB and ΔCOD,

∠O = ∠O |vertically opposite angle

As AB||CD

∠BAO = ∠OCD |alternate ∠s

∠OBA = ∠ODC |alternate ∠s

Thus, ΔAOB~ΔCOD

For two similar triangles, the ratio of their area is equal to the square of the ratio of their corresponding sides.

$$\frac{\text{area of }\Delta\text{ABC}}{\text{area of }\Delta\text{COD}} = \left(\frac{\text{AB}}{\text{DC}}\right)^2 = \left(\frac{6}{3}\right)^2 = 4$$

$$\frac{\text{area of }\Delta\text{ABC}}{\text{area of }\Delta\text{COD}} = 4:1$$

7. Question

If in ΔABC and ΔDEF ∠A = 50°, ∠B = 70°, ∠C = 60°, ∠D = 60°, ∠E = 70° and ∠F = 50° then the correct of the following is:

A. ΔABC ~ ΔDEF

B. ΔABC ~ ΔEDF

C. ΔABC ~ ΔDFE

D. ΔABC ~ ΔFED

Answer

For similar triangles, the corresponding angles should be equal. Thus, the order of equal corresponding angles are angles A,B,C & angles F,E,D. So, ΔABC ~ ΔFED

8. Question

If ΔABC ~ ΔDEF and AB = 10 cm, DE = 8 cm then the ratio of area of ΔABC and area of ΔDEF will be:

A. 25 : 16

B. 16 : 25

C. 4 : 5

D. 5 : 4

Answer

To find: Ratio of Area of Similar trianglesGiven: Their corresponding sides AB = 10 cm and DE = 8 cm For similar triangles ABC & DEF, the ratio of the areas of two similar triangles is equal to the ratio of squares of their corresponding sides.For both triangles there corresponding sides will be AB and DE

$$\frac{\text{area of }\Delta ABC}{\text{area of }\Delta DEF} = \left(\frac{AB}{DE}\right)^2 = \left(\frac{10}{8}\right)^2$$

$\Rightarrow \frac{\text{area of }\Delta ABC}{\text{area of }\Delta DEF} = \frac{25}{16} = 25:16$Hence, Ratio of area of triangles ABC anf DEF is 25:16

9. Question

D and E are points on sides AB and AC of triangle ABC such that DE || BC and AD = 8 cm, AB = 12 cm and AE = 12 cm. Then the measure of CE will be:

A. 6 cm

B. 18 cm

C. 9 cm

D. 15 cm

Answer

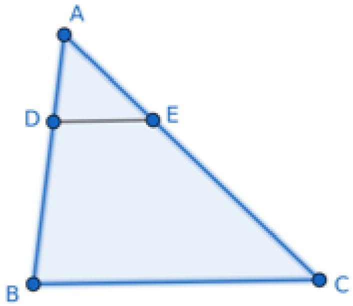
Given,

AD = 8 cm

AB = 12 cm

DB = AB – AD = 12 – 8 = 4 cm

AE = 12 cm



In ΔADE & ΔABC,

DE||BC

$$\Rightarrow \frac{AD}{DB} = \frac{AE}{EC}$$

Putting the values of AD,DB,AE,

$$\Rightarrow \frac{8}{4} = \frac{12}{EC}$$

$$\Rightarrow EC = \frac{4}{8} \times 12 = 6 \text{ cm}$$

10. Question

The shadow of 12 cm long vertical rod on the ground is 8 cm long. If at the same time the length of the shadow of a tower is 40 m, then the height of the tower will be:

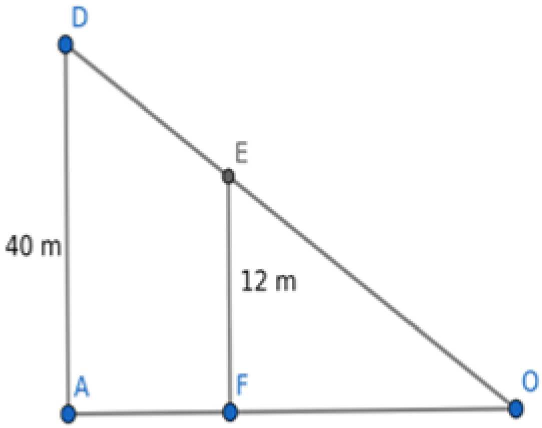
A. 60 m

B. 60 cm

C. 40 cm

D. 80 cm

Answer



The tower and the vertical rod are perpendicular to the ground and thus make a triangle like the above figure.

height of tower(AD) = 40 m

height of rod(EF) = 12 m

In ΔOEF & ΔOAD,

EF||AD

$$\frac{OE}{OA} = \frac{EF}{AD}$$

$$\Rightarrow AD = \frac{EF}{OE} \times OA = \frac{12}{8} \times 40 \text{ m}$$

⇒ AD = 60 m

11. Question

In ΔABC, D is a point on BC such that $\frac{AB}{AC} = \frac{BD}{DC}$ and ∠B = 70°, ∠C = 50° then find ∠BAD.

Answer

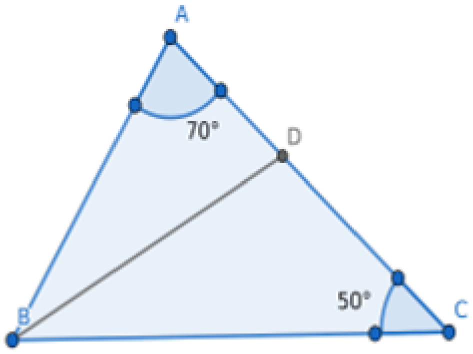
Given,

$$\frac{AB}{AC} = \frac{BD}{DC}$$

∠B = 70°

∠C = 50°

⇒ ∠A = 180° – (70 + 50)° = 60°



By Converse of Internal Angle Bisector Theorem, if the side opposite to the angle is in the ratio of the other two sides, then the side dividing side opposite to the angle is the internal angle bisector of the triangle.

Thus, $\angle BAD = \frac{\angle A}{2} = \frac{60°}{2} = 30°$

12. Question

If in ΔABC DE || BC and AD = 6 cm, DB = 9 cm and AE = 8 cm, then find AC.

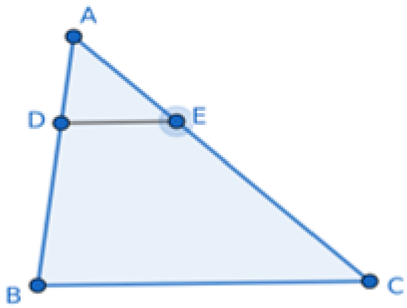
Answer

Given,

AD = 6 cm

DB = 9 cm

AE = 8 cm



In ΔADE & ΔABC,

DE||BC

$$\Rightarrow \frac{AD}{DB} = \frac{AE}{EC}$$

Putting the values of AD,DB,AE,

$$\Rightarrow \frac{6}{9} = \frac{8}{EC}$$

$$\Rightarrow EC = \frac{8}{6} \times 9 = 12 \text{ cm}$$

AC = AE + EC

⇒ AC = 8 + 12 = 20 cm

13. Question

If in ΔABC AD is the bisector of ∠A and AB = 8 cm, BD = 5 cm and DC = 4 cm then find AC.

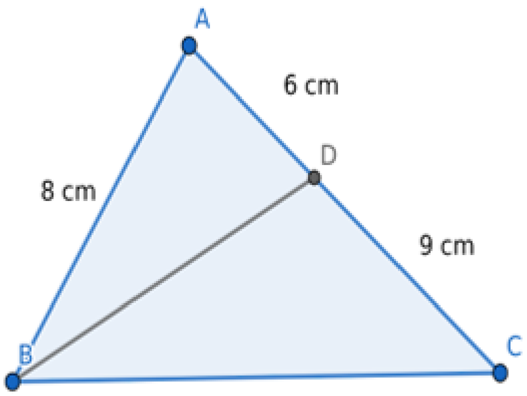
Answer

Given,

AB = 8 cm

BD = 5 cm

DC = 4 cm



By internal angle bisector theorem, the bisector of internal angle of a triangle divides the side opposite to the angle in the ratio of the other two sides.

In ΔABD & ΔADC,

∠A has internal bisector AD

$$\Rightarrow \frac{AB}{AC} = \frac{BD}{DC}$$

Putting the values of AB,BD and DC,

$$\Rightarrow \frac{8}{AC} = \frac{5}{4}$$

$$\Rightarrow AC = \frac{32}{5} = 6.4 \text{ cm}$$

14. Question

If the ratio of heights of two similar triangles be 4 : 9 then find the ratio of the areas of both the triangles.

Answer

The ratio of the areas of two similar triangles is equal to the ratio of squares of their corresponding heights.

$$\Rightarrow \frac{\text{area of } \Delta 1}{\text{area of } \Delta 2} = \left(\frac{\text{height of } \Delta 1}{\text{height of } \Delta 2}\right)^2$$

$$\frac{\text{area of } \Delta 1}{\text{area of } \Delta 2} = \left(\frac{4}{9}\right)^2 = 16:81$$