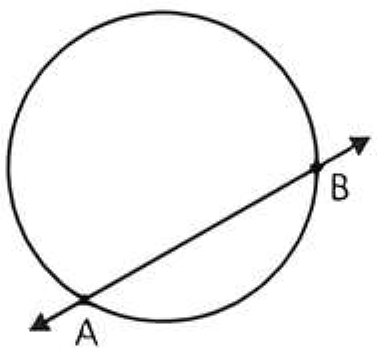
10 Circles

Fastrack Revision

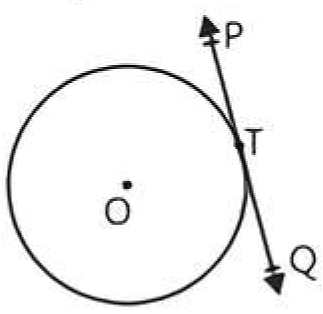
- **Circle:** A circle is a collection of all points in a plane which are at a constant distance from a fixed point. The fixed point is called **centre** and the constant distance is **radius**. The line joining two points on the circumference of the circle is **chord**.
- **Secant:** A line which intersects a circle in two distinct points is called a **secant** of the circle.



In figure, AB is the secant of the circle.

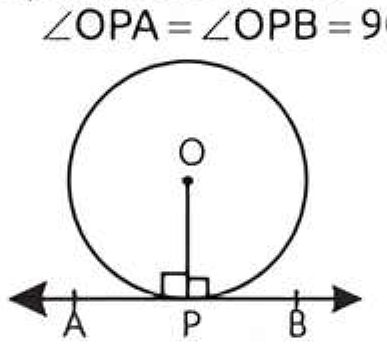
- **Tangent:** A line meets a circle only in one point is called a **tangent** to the circle at that point. The point at which the tangent line meets the circle is called the **point of contact**.

In figure, PQ is the tangent and T is the point of contact.



- The tangent at any point of a circle is perpendicular to the radius through the point of contact.

In figure, $\angle OPA = \angle OPB = 90°$



- **Number of Tangents to a Circle**
  1. No tangent can be drawn from a point inside the circle.
  2. Not more than one tangent can be drawn to a circle at a point on the circumference of the circle.
  3. Two tangents can be drawn to a circle from a point outside the circle. See figure (a).

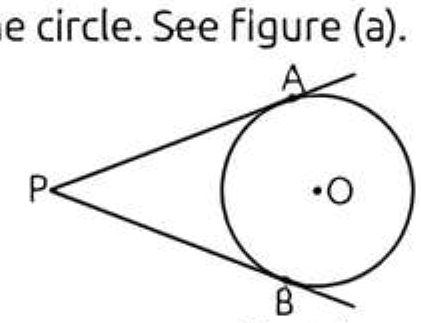


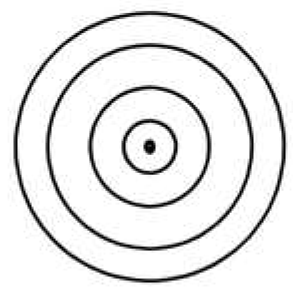
Fig. (a)

- **Length of a Tangent:** The length of the segment of the tangent from the external point P and the point of contact with the circle is called the length of a tangent from the point P to the circle. The length of two tangents drawn from the same external point to the circle are equal.

In fig. (a), PA = PB

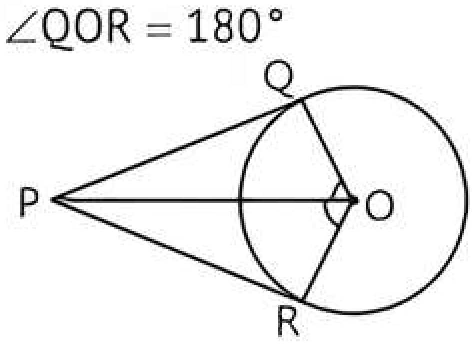
- **Concentric Circles:** Two or more circles having the same centre but different radii are called concentric circles.

In figure, circles are concentric.

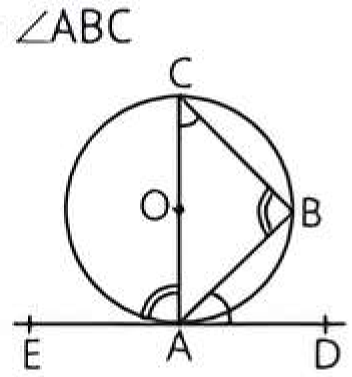


Knowledge BOOSTER

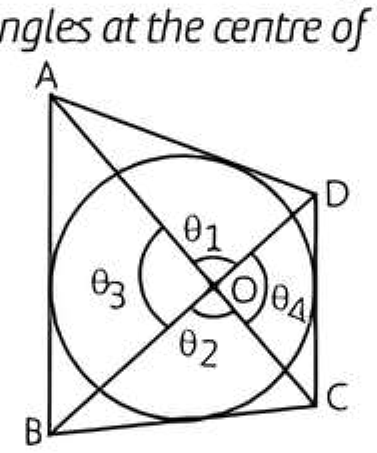
1. *A circle can have maximum two parallel tangents.*
2. *The distance between two parallel tangents to a circle is equal to the diameter of a circle.*
3. *The incentre of a triangle is the point where all the angle bisectors meet in the triangle.*
4. *If two tangents are drawn to a circle from an external point, then*
   (i) $\angle POQ = \angle POR$
   (ii) $\angle QPO = \angle RPO$
   (iii) $\angle QPR + \angle QOR = 180°$



5. *If a chord is drawn through a point of contact of a tangent to the circle then the angles formed by this chord from the tangent are equal to the angles of corresponding alternate segments.*
   i.e., $\angle BAD = \angle ACB$
   and $\angle EAC = \angle ABC$



6. *The opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.*
   In figure, $\theta_1 + \theta_2 = 180°$
   and $\theta_3 + \theta_4 = 180°$



Practice Exercise

Multiple Choice Questions

Q 1. The distance between two parallel tangents of a circle of diameter 7 cm is: *[CBSE 2023]*

a. 7 cm b. 14 cm

c. $\frac{7}{2}$ cm d. 28 cm

Q 2. Two parallel tangents are drawn to a circle at a distance of 10 cm, then the radius of circle is:

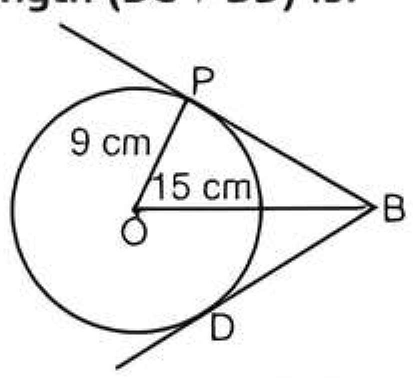
a. 3 cm b. 4 cm c. 5 cm d. 7 cm

Q 3. The length of tangent drawn to a circle of radius 9 cm from a point 41 cm from the centre is: *[CBSE 2023]*

a. 40 cm b. 9 cm

c. 41 cm d. 50 cm

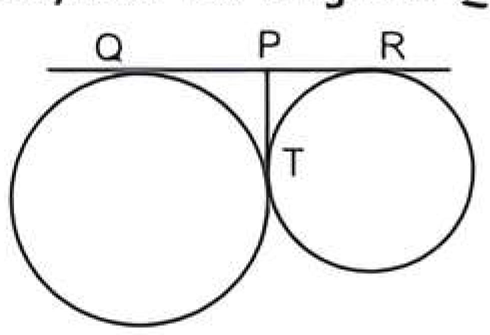
Q 4. In the given figure, BC and BD are tangents to the circle with centre O and radius 9 cm. If OB = 15 cm, then the length (BC + BD) is: *[CBSE 2023]*



a. 18 cm b. 12 cm

c. 24 cm d. 36 cm

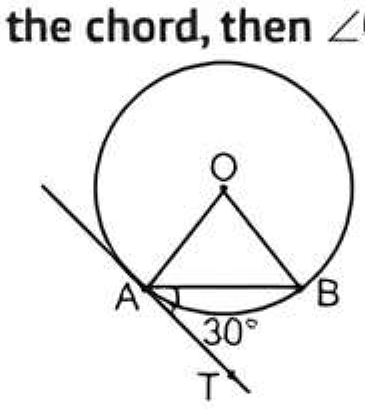
Q 5. In the given figure, QR is a common tangent to given circle. Tangent at T meets QR at P. If PQ = 5.5 cm, then the length of QR is:



a. 8 cm b. 10 cm

c. 11 cm d. 7 cm

Q 6. In the given figure, if O is the centre of a circle. AB is a chord and the tangent AT at A makes an angle of 30° with the chord, then ∠OAB is:



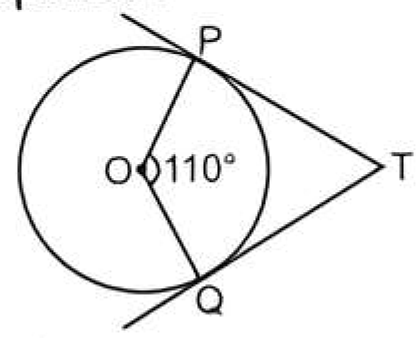
a. 40° b. 30° c. 60° d. 50°

Q 7. From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. The radius of the circle is: *[NCERT EXERCISE]*

a. 24.51 cm b. 12 cm

c. 15 cm d. 7 cm

Q 8. In figure, if TP and TQ are the two tangents to a circle with centre O so that ∠ POQ = 110°, then ∠ PTQ is equal to : *[CBSE SQP 2023-24]*

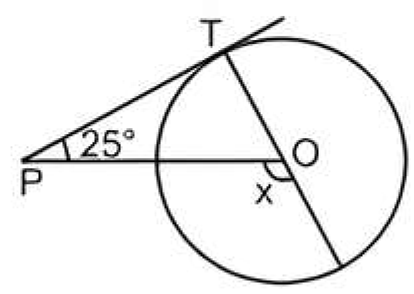


a. 60° b. 70° c. 80° d. 90°

Q 9. Two tangents PA and PB from a point P to a circle with centre O are inclined to each other at angle of 80°, then ∠POA is equal to: *[NCERT EXERCISE]*

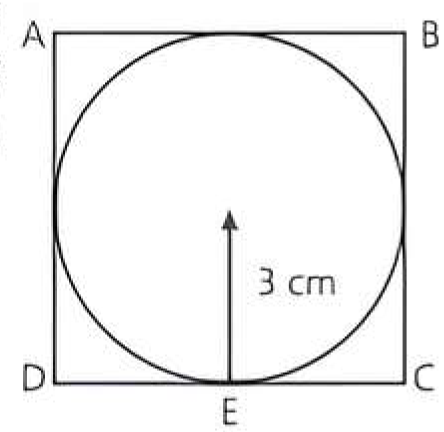
a. 80° b. 70° c. 60° d. 50°

Q 10. In the given figure, PT is a tangent at T to the circle with centre O. If ∠TPO = 25°, then x is equal to: *[CBSE 2023]*



a. 25° b. 65° c. 90° d. 115°

Q 11. A circle is inscribed in a square. The radius of inscribed circle is 3 cm, then the length of tangent is:



a. 3 cm

b. 9 cm

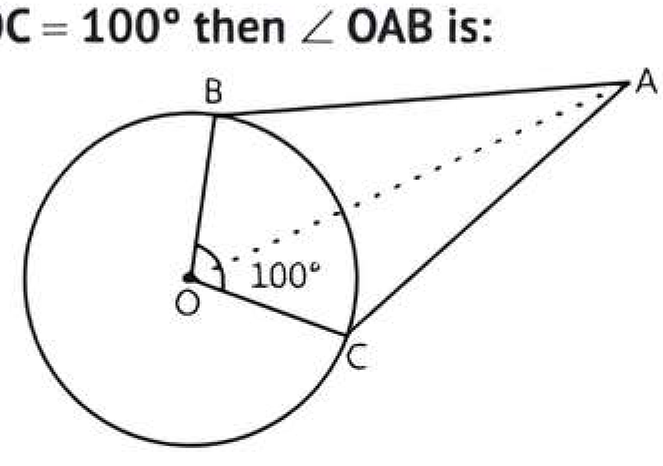
c. 6 cm

d. Can't be determined

Q 12. If radii of two concentric circles are 6 cm and 4 cm, the length of chord touches the smaller circle is:

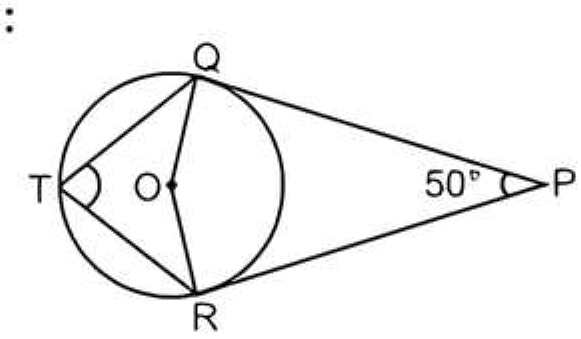
a. $\sqrt{5}$ cm b. $2\sqrt{5}$ cm c. $3\sqrt{5}$ cm d. $4\sqrt{5}$ cm

Q 13. In the given figure, if AB and AC are two tangents to a circle with centre O, so that ∠BOC = 100° then ∠ OAB is:



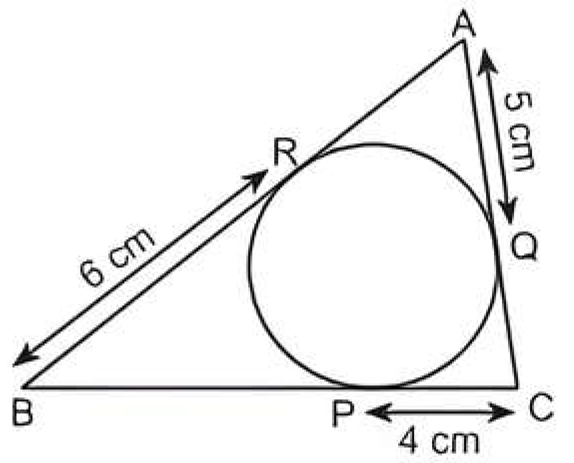
a. 70° b. 40° c. 60° d. 50°

Q 14. From a point P, two tangents PQ and PR are drawn to a circle with centre at O. T is a point on the major arc QR of the circle. If ∠ QPR = 50°, then ∠ QTR equals: *[CBSE 2023]*



a. 50° b. 130° c. 65° d. 90°

Q 15. In the given figure, the perimeter of $\triangle ABC$ is: *[CBSE 2023]*

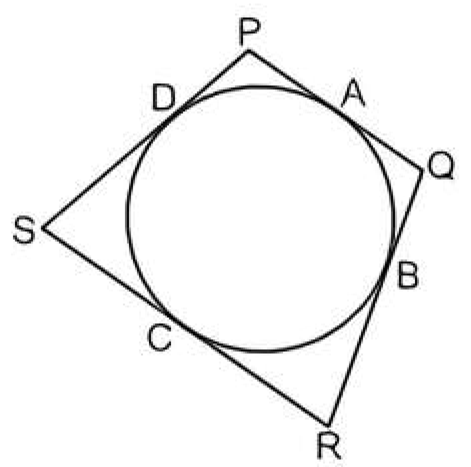


a. 30 cm b. 15 cm c. 45 cm d. 60 cm

Q 16. A quadrilateral PQRS is drawn to circumscribe a circle. If $PQ = 12$ cm, $QR = 15$ cm and $RS = 14$ cm, find the length of SP. *[CBSE SQP 2023-24]*

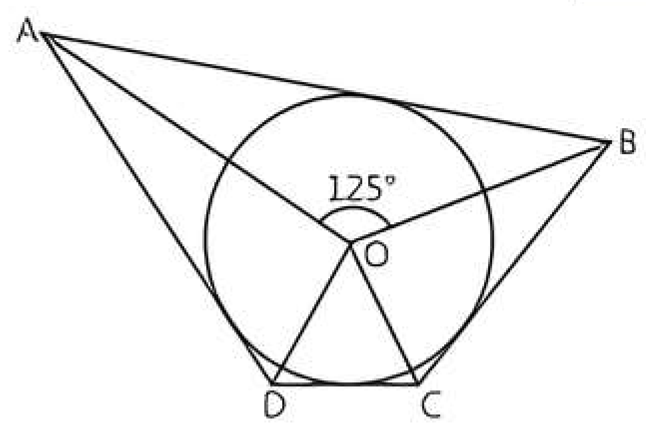
a. 15 cm b. 14 cm c. 12 cm d. 11 cm

Q 17. In the given figure, the quadrilateral PQRS circumscribes a circle. Here, PA + CS is equal to: *[CBSE 2023]*



a. QR b. PR c. PS d. PQ

Q 18. In the given figure, if $\angle AOB = 125°$, then $\angle COD$ is equal to: *[NCERT EXEMPLAR]*



a. 62.5° b. 45° c. 35° d. 55°

Assertion & Reason Type Questions

Directions (Q. Nos. 19-23): *In the following questions, a statement of assertion (A) is followed by a statement of a reason (R). Choose the correct option:*

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)

c. Assertion (A) is true but Reason (R) is false

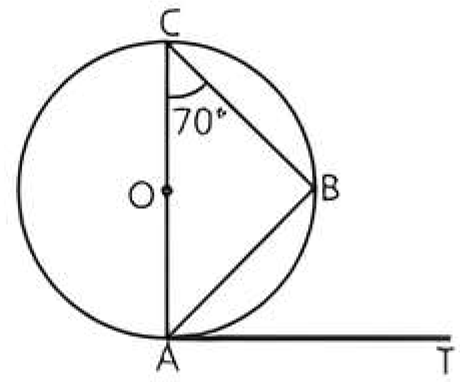
d. Assertion (A) is false but Reason (R) is true

Q 19. Assertion (A): A tangent to a circle is perpendicular to the radius through the point of contact.

Reason (R): The lengths of tangents drawn from the external point to a circle are equal. *[CBSE 2023]*

Q 20. Assertion (A): In the given figure, O is the centre of a circle and AT is a tangent at point A, then $\angle BAT = 70°$.

Reason (R): A straight line can intersect a circle at one point only.



Q 21. Assertion (A): Suppose the distance between two parallel tangents of a circle is 16 cm, then radius of a circle is 10 cm.

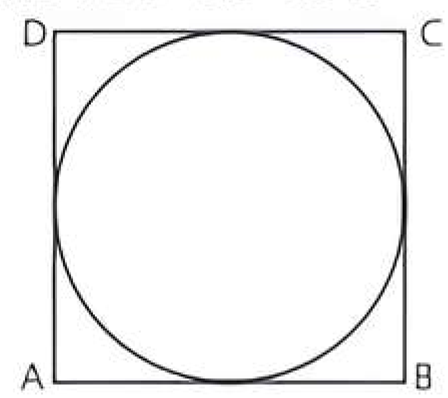
Reason (R): The distance between two parallel tangents of a circle is equal to the diameter of a circle.

Q 22. Assertion (A): If PA and PB are tangents drawn from an external point P to a circle with centre O, then the quadrilateral AOBP is cyclic.

Reason (R): The angle between two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the points of contact at the centre.

Q 23. Assertion (A): In the given figure, a quadrilateral ABCD is drawn to circumscribe a given circle, as shown. Then

$$AB + BC = AD + DC.$$



Reason (R): In two concentric circles, the chord of the larger circle, which touches the smaller circle, is bisected at the point of contact.

Fill in the Blanks Type Questions

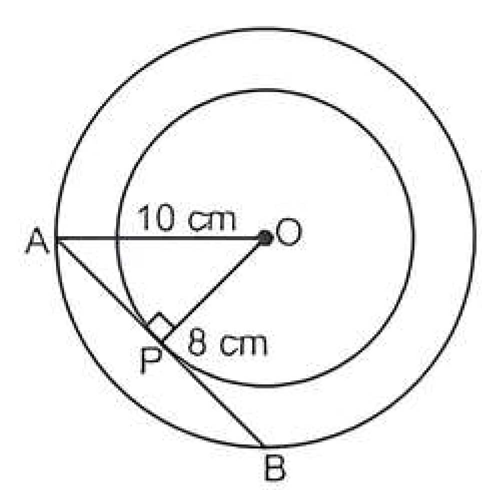
Q 24. A line which intersects a circle in two distinct points is called a .... of the circle. *[NCERT EXERCISE]*

Q 25. A circle can have maximum .... parallel tangents. *[NCERT EXERCISE]*

Q 26. The common point of a tangent and the circle is called point of .... *[NCERT EXERCISE]*

Q 27. A tangent at a point P on a circle of radius 5 cm meets a line through the centre O at a point Q, so that $OQ = 13$ cm, then length of PQ is ....

Q 28. In the given figure, the length PB = ... cm. *[CBSE 2020]*



True/False Type Questions

Q 29. If a point lies on a circle, then the number of tangents drawn from that point to the circle is 2.

Q 30. In two concentric circles, all chords of the outer circle, which touch the inner circle are of equal length.

Q 31. The tangent of a circle makes an angle of 90° with radius at point of contact.

Q 32. If tangents PA and PB from a point P to a circle with centre O are inclined to each other at an angle of 80°, then ∠APO is equal to 70°.

Q 33. If a chord AB subtends an angle of 60° at the centre of a circle, then angle between the tangents at A and B is also 60°.

Solutions

1. (a)

TiP

In two parallel tangents to a circle, the distance between two tangents is equal to the diameter of a circle.

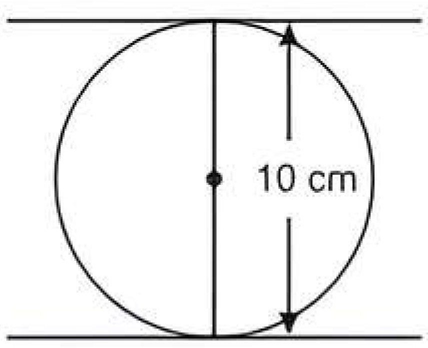
Given, Diameter of Circle = 7 cm

∴ Distance between two parallel tangents = diameter of circle = 7 cm.

2. (c)

TiP

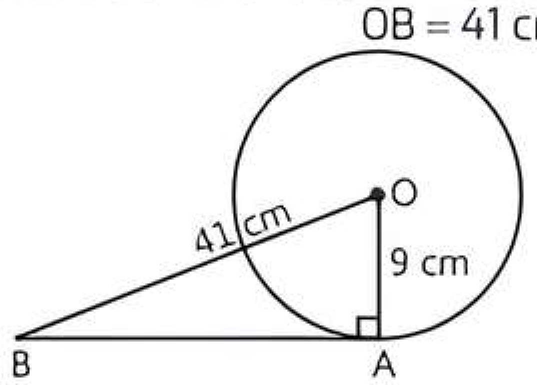
In two parallel tangents to a circle, the distance between two tangents is equal to the diameter of a circle.



Here, diameter of a circle, d = Distance between two parallel tangents = 10 cm

∴ Radius of circle, $r = \frac{d}{2} = \frac{10}{2} = 5$ cm

3. (a) Given radius of a circle, OA = 9 cm and OB = 41 cm



TR!CK

In right-angled triangle,

$$(Hypotenuse)^2 = (Base)^2 + (Perpendicular)^2$$

Since, tangent at any point of a circle is perpendicular to the radius through the point of contact.

i.e., $\angle OAB = 90°$

In right-angled ΔOAB, use Pythagoras theorem

$$(OB)^2 = (OA)^2 + (AB)^2$$

∴ $(41)^2 = (9)^2 + (AB)^2$

⇒ $AB = \sqrt{1681 - 81} = \sqrt{1600} = 40$ cm

4. (c)

TiP

Radius is perpendicular to the point of contact of tangents.

∴ $OC \perp BC$

⇒ $\angle OCB = 90°$

Now in right-angled ΔOCB,

$OB^2 = OC^2 + BC^2$ (by Pythagoras theorem)

⇒ $BC^2 = OB^2 - OC^2$

$= (15)^2 - (9)^2 = 225 - 81$

$= 144$

∴ $BC = 12$ cm

TiP

Tangents are drawn from an external point to a circle are equal in lengths.

∴ BC = BD = 12 cm

So, BC + BD = 12 + 12 = 24 cm

5. (c) Given PQ = 5.5 cm

The lengths of the tangents drawn from an external point to a circle are equal.

∴ PT = QP = 5.5 cm

Again P is an external point to a smaller circle. Therefore,

PR = PT = 5.5 cm

Now, length of tangent QR = PQ + PR

= 5.5 + 5.5 = 11 cm

6. (c)

TiP

Radius is perpendicular to the point of contact of tangent.

In the given figure,

$\angle OAT = 90°$

⇒ $\angle OAB + \angle BAT = 90°$

⇒ $\angle OAB + 30° = 90°$

⇒ $\angle OAB = 60°$

7. (d) In right-angled triangle OPQ,

$OP^2 + QP^2 = OQ^2$

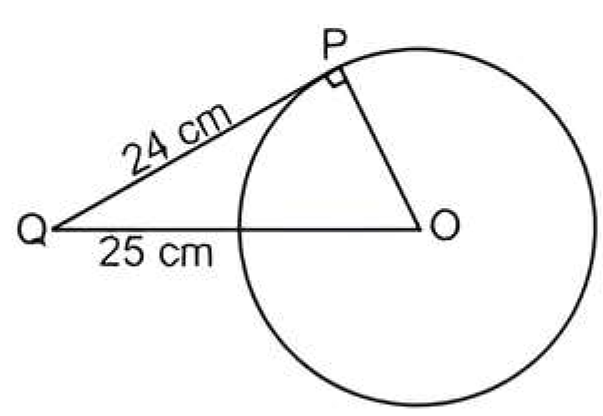
(by Pythagoras theorem)

⇒ $(OP)^2 + (24)^2 = (25)^2$

⇒ $(OP)^2 = 625 - 576$

⇒ $(OP)^2 = 49$

⇒ $OP = 7$ cm



8. (b)

TiP

Tangent is perpendicular to the radius through the point of contact.

Here, $\angle OPT = \angle OQT = 90^\circ$

Given $\angle POQ = 110^\circ$

In quadrilateral ABOC,

$\angle POQ + \angle OQT + \angle QTP + \angle TPO = 360^\circ$

$\Rightarrow \quad 110^\circ + 90^\circ + \angle QTP + 90^\circ = 360^\circ$

$\Rightarrow \quad \angle PTQ = 360^\circ - 290^\circ = 70^\circ$

9. (d) Since OP line bisect $\angle$ P. Therefore

$\angle APO = \angle BPO = 40^\circ$

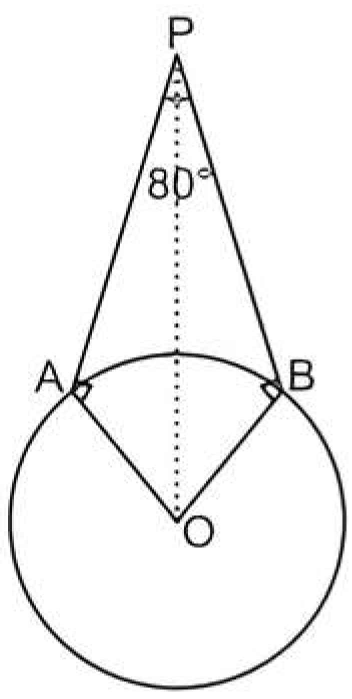
In right-angled ΔPAO,

$\angle PAO + \angle APO + \angle POA = 180^\circ$

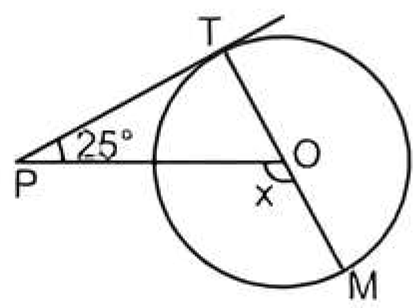
(angle sum property)

$\Rightarrow \quad 90^\circ + 40^\circ + \angle POA = 180^\circ$

$\Rightarrow \quad \angle POA = 180^\circ - 130^\circ = 50^\circ$



10. (d) Given, PT is a tangent at T to the circle with centre O.



$\angle TPO = 25^\circ$

Since, $\quad OT \perp PT$

$\therefore \quad \angle OTP = 90^\circ$

TiP

In a triangle, the exterior angle is equal to the sum of the two interior opposite angles.

In ΔPOT,

ext. $\angle POM = \angle OPT + \angle PTO$

$\Rightarrow \quad x = 25^\circ + 90^\circ = 115^\circ$

11. (c) Given radius of a circle, $r = 3$ cm. Therefore, diameter of a circle, $d = 2r = 2 \times 3 = 6$ cm.

Since, diameter of circle is equal to the side of a square.

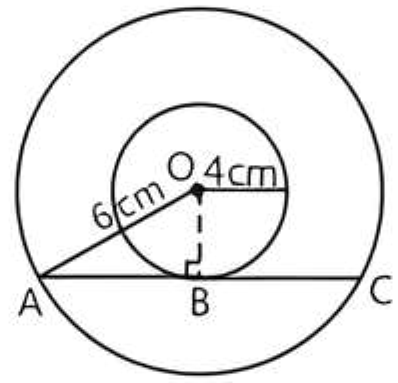
$\therefore$ Side of a square = Diameter of a circle

= 6 cm.

$\therefore$ The length of a tangent line = side of a square

= 6 cm.

12. (d) Given radius of bigger circle, OA = 6 cm and radius of smaller circle, OB = 4 cm.



TiP

In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

In right-angled ΔOBA,

$OA^2 = OB^2 + AB^2$ (by Pythagoras theorem)

$(6)^2 = (4)^2 + (AB)^2 \quad (\because OB = 4 \text{ cm})$

$\Rightarrow \quad (AB)^2 = \sqrt{36 - 16}$

$\Rightarrow \quad AB = \sqrt{20} = 2\sqrt{5}$ cm

$\therefore \quad AC = 2\,AB$

$= 2 \times 2\sqrt{5} = 4\sqrt{5}$ cm

13. (b)

TiP

Angle between radii and pair of tangent is supplementary.

Here, $\angle BOC + \angle BAC = 180^\circ \Rightarrow \angle BAC = 180^\circ - 100^\circ$

$\Rightarrow \quad \angle BAC = 80^\circ$

Also, line OA is bisector of $\angle$A.

$\therefore \quad \angle OAB = \frac{80^\circ}{2} = 40^\circ$

14. (c) PQ and PR are tangents from external point P to a circle.

Given, $\angle QPR = 50^\circ$

TiPs

- *Angle between radii and pair of tangent is supplementary.*
- *The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.*

Here, $\quad \angle QPR + \angle QOR = 180^\circ$

$\Rightarrow \quad 50^\circ + \angle QOR = 180^\circ$

$\Rightarrow \quad \angle QOR = 180^\circ - 50^\circ = 130^\circ$

Now, $\quad \angle QTR = \frac{1}{2}\angle QOR = \frac{1}{2} \times 130^\circ$

$\therefore \quad \angle QTR = 65^\circ$

15. (a)

TiP

Tangents are drawn from an external point to a circle are equal in lengths.

$\therefore \quad AR = AQ = 5$ cm

$BP = BR = 6$ cm

and $\quad CQ = CP = 4$ cm.

So, perimeter of $\Delta ABC = AB + BC + CA$

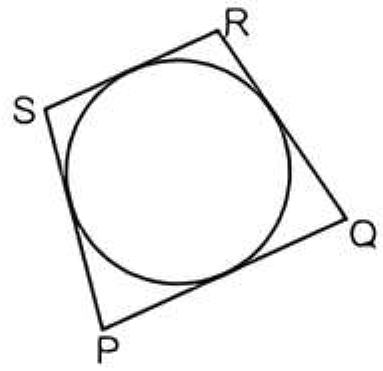
$= (AR + BR) + (BP + CP) + (CQ + AQ)$

$= (5 + 6) + (6 + 4) + (4 + 5)$

$= 11 + 10 + 9 = 30$ cm.

16. (d) Given, PQ = 12 cm, QR = 15 cm and RS = 14 cm

Also, a quadrilateral PQRS is drawn to circumscribe a circle.



We know that, when a quadrilateral PQRS is drawn to circumscribe a given circle then,

$$PQ + RS = SP + QR$$

$$\therefore \quad SP = PQ + RS - QR$$

$$= 12 + 14 - 15 = 26 - 15 = 11$$

So, length of SP is 11 cm.

17. (c) Given, the quadrilateral PQRS circumscribes a circle.

TiP

The length of two tangents drawn from an external point are equal.

We know that, if a quadrilateral PQRS is drawn to circumscribe a given circle then,

$$PQ + RS = SP + QR$$

$$\Rightarrow (PA + AQ) + (SC + CR) = (PD + SD) + (RB + BQ)$$

$$\Rightarrow \quad PA + SC + (AQ + CR) = (PD + SD) + (CR + AQ)$$

$$(\because AQ = BQ \text{ and } CR = RB)$$

$$\Rightarrow \quad PA + SC = PS \quad (\because PS = PD + SD)$$

18. (d)

TR!CK

The opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Given $\angle AOB = 125°$

Here, $\angle AOB + \angle COD = 180°$

$\therefore \quad 125° + \angle COD = 180°$

$\Rightarrow \quad \angle COD = 55°$.

19. (b) **Assertion (A):** It is true that a tangent to a circle is perpendicular to the radius through the point of contact.

Reason (R): It is also true that the lengths of tangents drawn from the external point to a circle are equal.

Thus, both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

20. (c) **Assertion (A):**

TR!CK

If a chord is drawn through a point of contact of a tangent to the circle then the angles formed by this chord from the tangents are equal to the angles of corresponding alternate segments.

Here, $\angle BAT = \angle ACB$

(by alternate segment theorem)

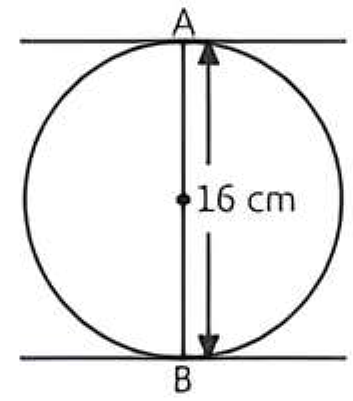
$\therefore \quad \angle BAT = 70° \quad (\because \angle ACB = 70°, \text{ given})$

So, Assertion (A) is true.

Reason (R): Any straight line can intersect a circle at two points.

So, Reason (R) is false.

21. (d) **Assertion (A):** The distance between two parallel tangents is equal to the diameter of a circle.



$\therefore \quad d = AB = 16$ cm

Now, radius of a circle, $r = \dfrac{d}{2} = \dfrac{16}{2}$

$= 8$ cm

So, Assertion (A) is false.

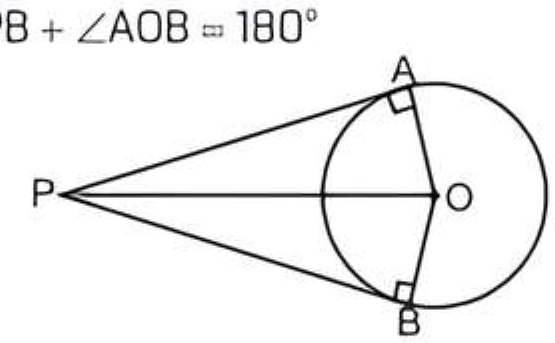
Reason (R): It is also true that the distance between two parallel tangents is equal to the diameter of a circle.

So, Reason (R) is true.

Hence, Assertion (A) is false but Reason (R) is true.

22. (a) **Assertion (A):** We know that, the angle between two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the points of contact at the centre.

i.e., $\angle APB + \angle AOB = 180°$ ...(1)



Also, the tangent at any point of a circle is perpendicular to the radius through the point of contact.

i.e., $PA \perp OA \Rightarrow \angle OAP = 90°$

and $PB \perp OB \Rightarrow \angle OBP = 90°$

$\therefore \angle OAP + \angle OBP = 90° + 90° = 180°$ ...(2)

If the sum of a pair of opposite angles of a quadrilateral is 180° then quadrilateral is cyclic.

From eqs. (1) and (2), we get

Quadrilateral AOBP is cyclic.

So, Assertion (A) is true.

Reason (R): It is a true statement also.

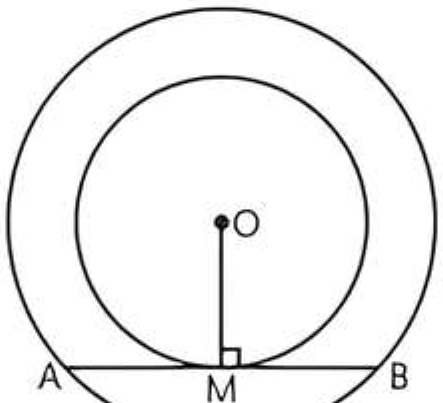
Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

23. (d) **Assertion (A):** If a quadrilateral ABCD is drawn to circumscribe a circle, then

$$AB + CD = AD + BC$$

So, Assertion (A) is false.

Reason (R): We have two concentric circles with O is the centre of concentric circles and AB is the tangent.



Since, $OM \perp AB$

$\therefore \quad AM = MB$

($\because$ perpendicular drawn from centre O to the chord AB bisect the chord AB)

So, Reason (R) is true.

Hence, Assertion (A) is false but Reason (R) is true.

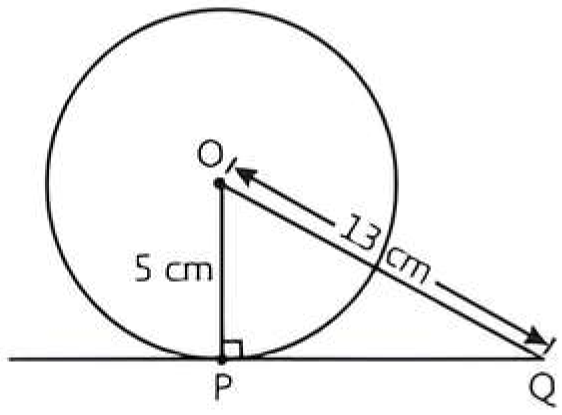
24. secant

25. two

26. contact

27. Given, PQ is a tangent to the circle at point P and radius of circle

$OP = 5$ cm



Also, it is given

$OQ = 13$ cm.

TiP

A line drawn from centre O to the point of contact at point P is perpendicular.

In right-angled Δ OPQ, use Pythagoras theorem,

$$PQ = \sqrt{(OQ)^2 - (OP)^2} = \sqrt{(13)^2 - (5)^2} = \sqrt{169 - 25}$$

$$= \sqrt{144} = 12 \text{ cm}$$

Hence, length of PQ is 12 cm.

28. In right-angled ΔAPO, use Pythagoras theorem

$$AP = \sqrt{(OA)^2 - (OP)^2} = \sqrt{(10)^2 - (8)^2}$$

$$= \sqrt{100 - 64} = \sqrt{36} = 6 \text{ cm}$$

TR!CK

A line drawn from centre of circle to the chord, it bisects the chord.

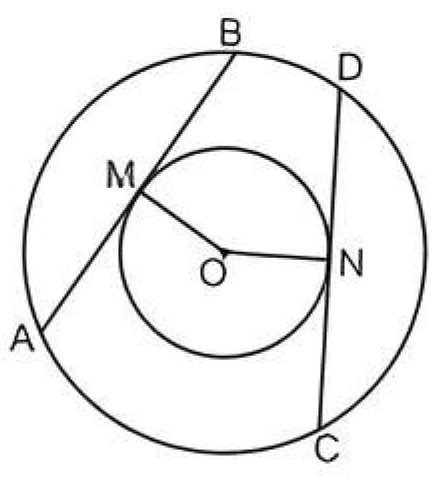
Since, $OP \perp AB$

$\therefore$ $PB = AP$

$\Rightarrow$ $PB = 6$ cm.

29. False, because not more one tangent can be drawn to a circle at a point on the circumference of the circle.

30. Suppose, AB and CD are two chords of larger circle touches the inner circle at M and N.



Here, $OM = ON$ (radii of circle)

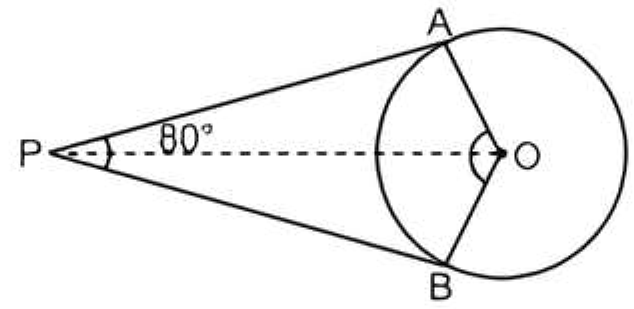
Since, AB and CD are two chords of a bigger circle and are equidistant from the centre,

So, $AB = CD$

Similarly, we can say that all chords of outer circle touch the inner circle are of equal length.

31. True

32. Given, tangents PA and PB are inclined an angle $80°$ *i.e.*, $\angle APB = 80°$.



As point joining the external point of pair of tangent P to the centre O, it bisects the angle.

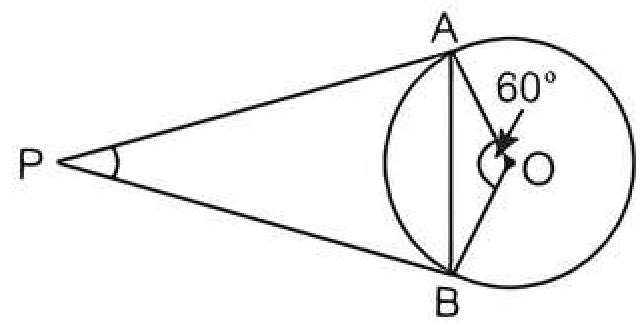
$$\therefore \quad \angle APO = \frac{1}{2}\angle APB = \frac{1}{2} \times 80° = 40°$$

Hence, given statement is false.

33.

TR!CK

Angle subtended by the pair of tangents to the centre of circle is supplementary.



Here, $\angle APB + \angle AOB = 180°$

$\therefore$ $\angle APB + 60° = 180°$

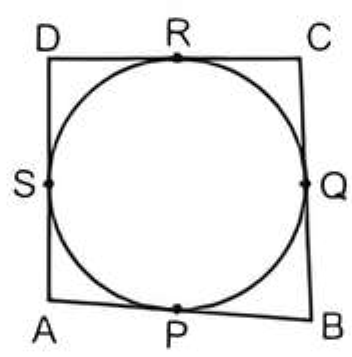
$\Rightarrow$ $\angle APB = 120°$

Hence, given statement is false.

Case Study Based Questions

Case Study 1

In a park, four poles are standing at positions A, B, C and D around the fountain such that the cloth joining the poles AB, BC, CD and DA touches the fountain at P, Q, R and S respectively as shown in the figure.



Based on the above information, solve the following questions:

Q 1. If O is the centre of the circular fountain, then $\angle$OSA =

a. $60°$ b. $90°$ c. $45°$ d. None of these

Q 2. Which of the following is correct?

a. AS = AP b. BP = BQ

c. CQ = CR d. All of these

Q 3. If DR = 7 cm and AD = 11 cm, then AP =

a. 4 cm b. 18 cm

c. 7 cm d. 11 cm

Q 4. If O is the centre of the fountain, with $\angle$QCR = 60°, then $\angle$QOR =

a. $60°$ b. $120°$

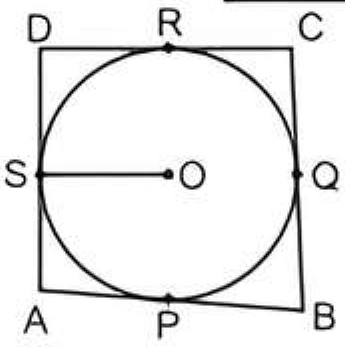
c. $90°$ d. $30°$

Q 5. Which of the following is correct?

a. $AB + BC = CD + DA$ b. $AB + AD = BC + CD$
c. $AB + CD = AD + BC$ d. All of these

Solutions

1. (b)



Here, OS the is radius of circle.

Since, radius at the point of contact is perpendicular to tangent.

So, $\angle OSA = 90°$

So, option (b) is correct.

2. (d) Since, length of tangents drawn from an external point to a circle are equal.

$\therefore$ $AS = AP$, $BP = BQ$,

$CQ = CR$ and $DR = DS$ ...(1)

So, option (d) is correct.

3. (a) $AP = AS = AD - DS = AD - DR$ [using *eq.* (1)]

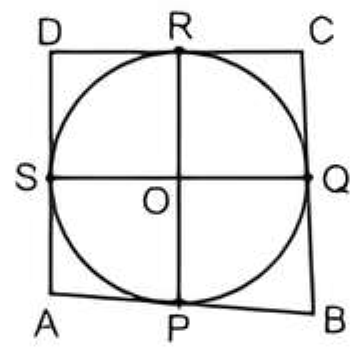
$= 11 - 7 = 4$ cm

So, option (a) is correct.

4. (b) In quadrilateral OQCR, $\angle QCR = 60°$ (Given)

And $\angle OQC = \angle ORC = 90°$

(Since, radius at the point of contact is perpendicular to tangent.)



$\angle QOR = 360° - 90° - 90° - 60°$

$= 120°$

So, option (b) is correct.

5. (c) From eq. (1), we have $AS = AP$, $DS = DR$,

$BQ = BP$ and $CQ = CR$

Adding all above equations, we get

$AS + DS + BQ + CQ = AP + DR + BP + CR$

$\Rightarrow AD + BC = AB + CD$

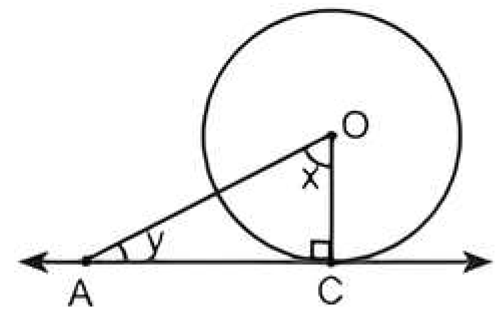
So, option (c) is correct.

Case Study 2

For class 10 students, a teacher planned a game for the revision of chapter circles with some questions written on the board, which are to be answered by the students. For each correct answer, a student will get a reward. Some of the questions are given below.

Based on the given information, solve the following questions:

Q 1. In the given figure, $x + y =$

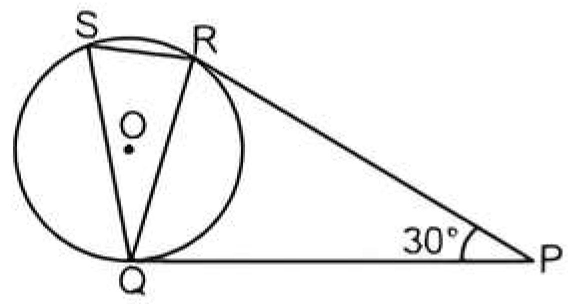


a. 60° b. 90° c. 120° d. 145°

Q 2. If PA and PB are two tangents drawn to a circle with centre O from P such that $\angle PBA = 50°$, then $\angle OAB =$

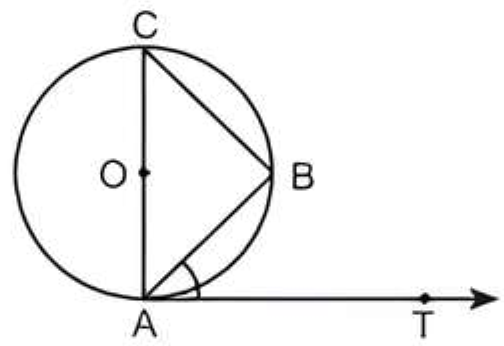
a. 50° b. 25° c. 40° d. 130°

Q 3. In the given figure, PQ and PR are two tangents to the circle, then $\angle ROQ =$



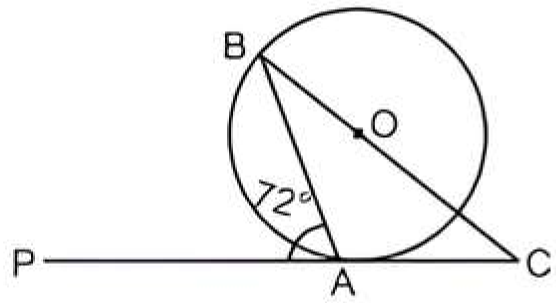
a. 30° b. 60° c. 105° d. 150°

Q 4. In the given figure, AB is a chord of the circle and AOC is its diameter such that $\angle ACB = 55°$, then $\angle BAT =$



a. 35° b. 55° c. 125° d. 110°

Q 5. In the given figure, if PC is the tangent at A of the circle with $\angle PAB = 72°$ and $\angle AOB = 132°$, then $\angle ABC =$



a. 18° b. 30°
c. 60° d. Can't be determined

Solutions

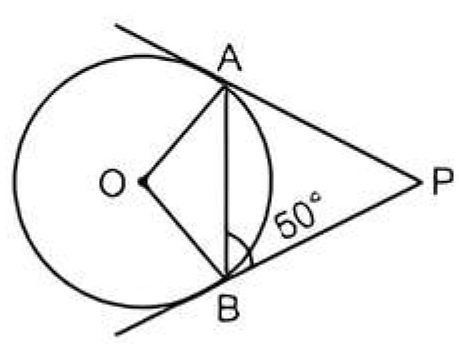
1. (b) In ΔOAC, $\angle OCA = 90°$

Since, radius at the point of contact is perpendicular to tangent.

$\therefore \angle OAC + \angle AOC = 90° \Rightarrow x + y = 90°$

So, option (b) is correct.

2. (c)



Since, $OB \perp PB$ (since, radius at the point of contact is perpendicular to tangent) and $\angle PBA = 50°$ (Given)

$\angle OBA = 90° - 50° = 40°$

Also, $OA = OB$ (radii of circle)

$\therefore \angle OAB = \angle OBA = 40°$

(angle opposite to equal sides are equal)

So, option (c) is correct.

3. (d) In quadrilateral OQPR,

$\angle ROQ + \angle RPQ = 180°$

($\because$ Angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the point of contact at the centre)

$\therefore \angle ROQ = 180° - 30° = 150°$

So, option (d) is correct.

4. (b) Here, $\angle ABC = 90°$ (angle in a semicircle)

Now, In ΔABC,

$\angle BAC + \angle ACB + \angle ABC = 180°$

(by angle sum property of triangle)

$\Rightarrow \angle BAC + 55° + 90° = 180°$

$\Rightarrow \angle BAC = 180° - 145° = 35°$

Also, $\angle OAT = 90°$ ($\because$ radius at the point of contact is perpendicular to tangent)

$\Rightarrow \angle BAT + \angle OAB = 90°$

$\Rightarrow \angle BAT = 90° - 35°$ ($\because \angle CAB = \angle OAB$)

$= 55°$

So, option (b) is correct.

5. (b) Here, $\angle PAB = 72°$

$\because$ $\angle OAP = 90°$ ($\because OA \perp AP$)

$\therefore$ $\angle OAB + \angle PAB = 90°$

$\Rightarrow \angle OAB = 90° - 72° = 18°$

Also, $\angle AOB = 132°$ (given)

Now in ΔOAB,

$\angle ABO + \angle BAO + \angle AOB = 180°$

(by angle sum property of triangle)

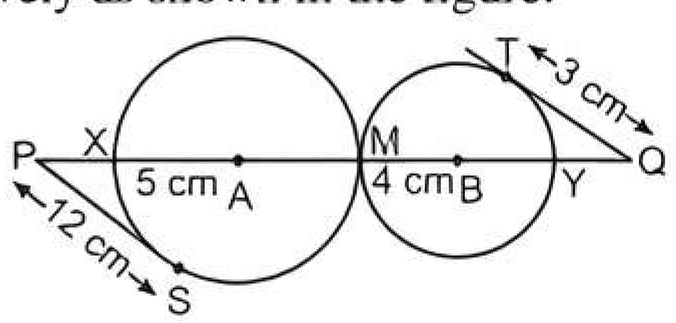
$\angle ABO = 180° - 132° - 18° = 30°$

$\therefore \angle ABC + \angle ABO = 30°$

So, option (b) is correct.

Case Study 3

In a math class–IX, the teacher draws two circles that touch each other externally at point M with centres A and B and radii 5 cm and 4 cm respectively as shown in the figure.



Based on the above information, solve the following questions:

Q 1. Find the value of PX.

Q 2. Find the value of QY.

Q 3. Show that $PS^2 = PM \cdot PX$.

Or

Show that $TQ^2 = YQ.MQ$

Solutions

1. Here, $AS = 5$ cm and $BT = 4$ cm ($\because$ radii of circles)

Since, radius at point of contact is perpendicular to tangent.

$\therefore$ $AS \perp PS$

$\Rightarrow$ $\angle ASP = 90°$

In right-angled ΔASP,

$PA^2 = PS^2 + AS^2$ (by Pythagoras theorem)

$\Rightarrow$ $PA^2 = (12)^2 + (5)^2$

$\Rightarrow$ $PA = \sqrt{144 + 25} = \sqrt{169} = 13$ cm

$\because$ $PX = PA - XA$

$\therefore$ $PX = 13 - 5 = 8$ cm ($\because$ radius, $XA = 5$ cm)

2. $\because$ $BT \perp TQ$

$\Rightarrow$ $\angle BTQ = 90°$

In right-angled ΔBTQ,

$BQ^2 = TQ^2 + BT^2$ (by Pythagoras theorem)

$\Rightarrow$ $BQ^2 = (3)^2 + (4)^2$

$\Rightarrow$ $BQ = \sqrt{9 + 16} = \sqrt{25} = 5$ cm

$\because$ $QY = BQ - BY$

$\therefore$ $QY = 5 - 4 = 1$ cm ($\because$ radius, $BY = 4$ cm)

3. In right-angled ΔASP,

$PS^2 = PA^2 - AS^2$

$= PA^2 - AM^2$ [$\because AS = AM$ (radii)]

$= (PA + AM)(PA - AM)$

$= (PA + AM)(PA - AX)$

$= PM \cdot PX$ [$\because AM = AX$ (radii)]

Hence proved.

Or

In right-angled ΔMTQ,

$TQ^2 = BQ^2 - TB^2$

$\therefore$ $= (BQ - TB)(BQ + TB)$ [$\because TB = MB$ (radii)]

$= (BQ - MB)(BQ + MB)$

$= (BQ - BY)\, MQ$ [$\because MB = BY$ (radii)]

$= YQ.MQ$ ($\because BQ + MB = MQ$, $BQ - BY = YQ$)

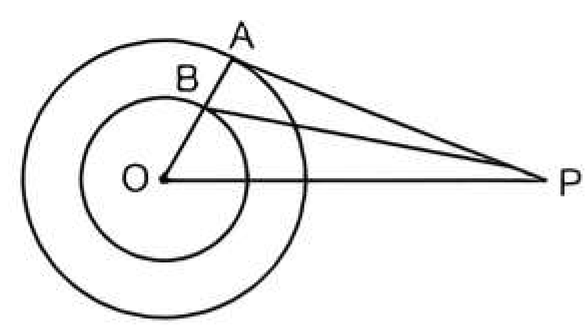
Hence proved.

Case Study 4

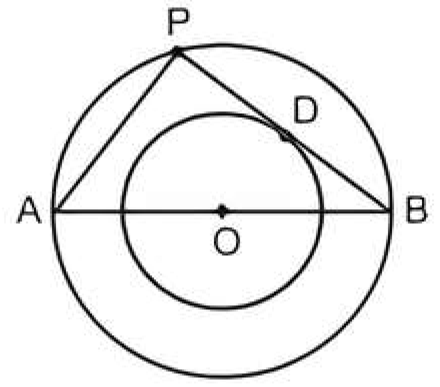
If a tangent is drawn to a circle from an external point, then the radius at the point of contact is perpendicular to the tangent.

Based on the above information, solve the following questions:

Q 1. In the given figure, O is the centre of two concentric circles. From an external point P tangents PA and PB are drawn to these circles such that PA = 6 cm and PB = 8 cm. If OP = 10 cm, then find the value of AB.



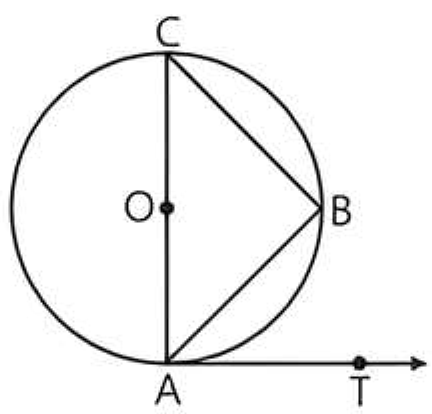
Q 2. The diameter of two concentric circles are 10 cm and 6 cm. AB is a diameter of the bigger circle and BD is the tangent to the smaller circle touching it at D and intersecting the larger circle at P on producing. Find the length of BP.



Q 3. Two concentric circles are such that the difference between their radii is 4 cm and the length of the chord of the larger circle which touches the smaller circle is 24 cm. Then find the radius of the smaller circle.

Or

If AB is a chord of a circle with centre O, AOC is a diameter and AT is the tangent at A as shown in figure. Prove that $\angle BAT = \angle ACB$.



Solutions

1. Since, radius is perpendicular to the tangent.

   $\therefore OB \perp BP$ and $OA \perp AP$

   Now in right-angled ΔOBP and ΔOAP,

   Here, $OP^2 - PB^2 = OB^2$ and $OP^2 - PA^2 = OA^2$

   (by Pythagoras theorem)

   $$OB = \sqrt{100-64} = \sqrt{36} = 6 \text{ cm}$$

   $(\because OP = 10 \text{ cm and } PB = 8 \text{ cm})$

   and $OA = \sqrt{100-36} = \sqrt{64} = 8 \text{ cm}$ $\quad (\because PA = 6 \text{ cm})$

   $\therefore \quad AB = OA - OB = 8 - 6 = 2 \text{ cm}$

2. Since, radius is perpendicular to the tangent

   $\therefore OD \perp BP$

   Given, $AB = 10$ cm

   $\Rightarrow OB = 10/2 = 5 \text{ cm}$ and $OD = \frac{6}{2} = 3 \text{ cm}$

   Now in right-angled ΔODB,

   $OB^2 = OD^2 + BD^2 \Rightarrow \quad BD = \sqrt{OB^2 - OD^2}$

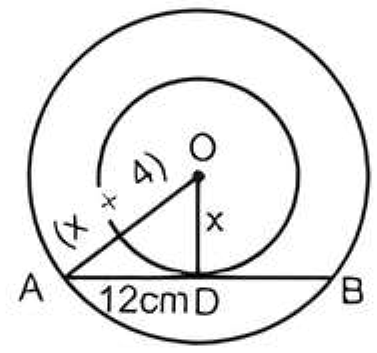
   (by Pythagoras theorem)

   $$\Rightarrow BD = \sqrt{25-9} = \sqrt{16} = 4 \text{ cm}$$

   Since, chord BP is bisected by radius OD.

   $\therefore BP = 2\,BD = 2 \times 4 = 8$ cm.

3. Let x be the radius of smaller circle, then $(x + 4)$ be the radius of larger circle.



   Since, radius is perpendicular to the tangent.

   $\therefore OD \perp AB$

   Now in right-angled ΔODA,

   $\therefore \quad OA^2 = OD^2 + AD^2$ (by Pythagoras theorem)

   $\Rightarrow (x+4)^2 = x^2 + 12^2$

   $\Rightarrow 8x + 16 = 144$

   $\Rightarrow \quad x = 16$ cm

   Or

   Since, AC is a diameter, so the angle in a semi-circle will be 90°.

   $\therefore \quad \angle ABC = 90^\circ$

   In ΔABC,

   $\angle CAB + \angle ABC + \angle ACB = 180^\circ$

   (sum of interior angles of a triangle)

   $\Rightarrow \quad \angle CAB + \angle ACB = 180^\circ - 90^\circ = 90^\circ$ ...(1)

   Since, the diameter of the circle is the perpendicular to the tangent.

   i.e., $\quad CA \perp AT$

   $\therefore \quad \angle CAT = 90^\circ$

   $\Rightarrow \quad \angle CAB + \angle BAT = 90^\circ$ ...(2)

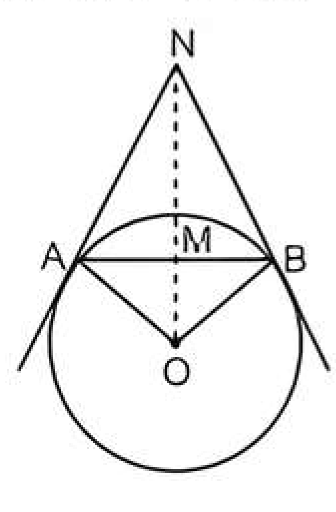
   From (1) and (2), we get

   $\angle CAB + \angle ACB = \angle CAB + \angle BAT$

   $\Rightarrow \quad \angle ACB = \angle BAT$ **Hence proved.**

Case Study 5

Circles play an important part in our life. When a circular object is hung on the wall with a chord at nail N, the chords NA and NB work like tangents. Observe the figure, given that $\angle ANO = 30^\circ$ and $OA = 5$ cm. **[CBSE 2023]**



Based on the above information, solve the following questions:

Q 1. Find the distance AN.

Q 2. Find the measure of $\angle AOB$.

Q 3. Find the total length of chords NA, NB and the chord AB.

Or

Name the type of quadrilateral OANB. Justify your answer.

Solutions

1.

TiP

Tangent is perpendicular to the radius through the point of contact of circle.

Here $OA \perp AN$

$\therefore$ $\angle OAN = 90^\circ$

Given, $\angle ANO = 30^\circ$ and $OA = 5$ cm.

In right-angled ΔOAN,

$$\tan \angle ANO = \frac{OA}{AN} \Rightarrow \tan 30^\circ = \frac{5}{AN}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{5}{AN}$$

$$\Rightarrow AN = 5\sqrt{3} \text{ cm}$$

2.

TiP

If two tangents are drawn from an external point to a circle, then the line joining that external point to the centre of circle makes equal angles from two tangents.

$\therefore$ $\angle ANO = \angle BNO = 30^\circ$

$\Rightarrow$ $\angle ANB = 2 \times \angle ANO = 2 \times 30^\circ = 60^\circ$

$\because$ $OA \perp AN$ and $OB \perp BN$

$\therefore$ $\angle OAN = \angle OBN = 90^\circ$

Now in quadrilateral OANB,

$\angle AOB + \angle OAN + \angle OBN + \angle ANB = 360^\circ$

$\Rightarrow$ $\angle AOB + 90^\circ + 90^\circ + 60^\circ = 360^\circ$

$\therefore$ $\angle AOB = 360^\circ - 240^\circ = 120^\circ$

3.

TiP

Angles opposite to equal sides of a triangle is also equal.

In ΔAOB,

$OA = OB$ (radii of circle)

$\Rightarrow$ $\angle OAB = \angle OBA = \theta$ (Say)

$\therefore$ $\angle OAB + \angle OBA + \angle AOB = 180^\circ$

(by angle sum property)

$\Rightarrow$ $\theta + \theta + 120^\circ = 180^\circ$ $(\because \angle AOB = 120^\circ)$

$\Rightarrow$ $2\theta = 60^\circ \Rightarrow \theta = 30^\circ$

$\therefore$ $\angle OAB = \angle OBA = 30^\circ$

$\because$ $\angle OAN = \angle OAB + \angle BAN$

$\therefore$ $90^\circ = 30^\circ + \angle BAN$

$\Rightarrow$ $\angle BAN = 90^\circ - 30^\circ = 60^\circ$

Similarly, $\angle ABN = 60^\circ$

$\because$ $\angle ANB = \angle BAN = \angle ABN = 60^\circ$

$\therefore$ Δ ANB is an equilateral triangle.

$\therefore$ Total length of chords = NA + NB + AB

$(\because AN = BN = AB = 5\sqrt{3}$ cm$)$

$= 5\sqrt{3} + 5\sqrt{3} + 5\sqrt{3}$

$= 15\sqrt{3}$ cm

Or

TiPs

- *Kite has two pairs of adjacent equal sides.*
- *Kite has one pair of equal opposite angle.*
- *In a kite, diagonals are perpendicular to each other with the longer diagonal bisecting the shorter one.*

From above parts,

$\angle OAN = \angle OBN = 90^\circ$

But $\angle AOB \neq \angle ANB$

Also, $AN = BN = 5\sqrt{3}$ cm

($\because$ the length of two tangents drawn from an external point of a circle are equal)

and $OA = OB = 5$ cm (Radii)

In quadrilateral OANB,

longer diagonal ON bisect shorter diagonal AB perpendicularly.

($\because$ the perpendicular from the centre of a circle to a chord bisect the chord)

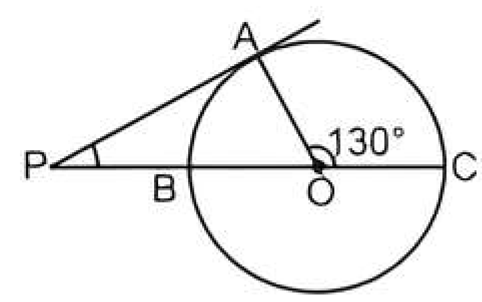
Hence, the special name of quadrilateral OANB is kite.

Very Short Answer Type Questions

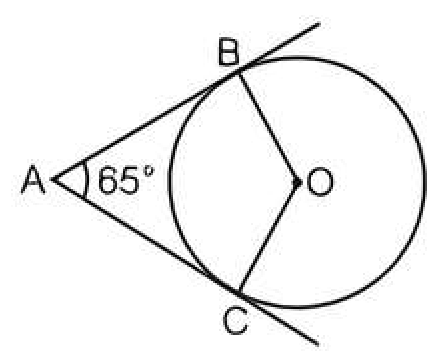
Q 1. The length of a tangent from a point A at distance 5 cm from the centre of the circle is 4 cm. Find the radius of the circle. *[CBSE SQP 2023-24]*

Q 2. If the angle between two tangents drawn from an external point P to a circle of radius *a* and centre O, is 60°, then find the length of OP. *[CBSE 2017]*

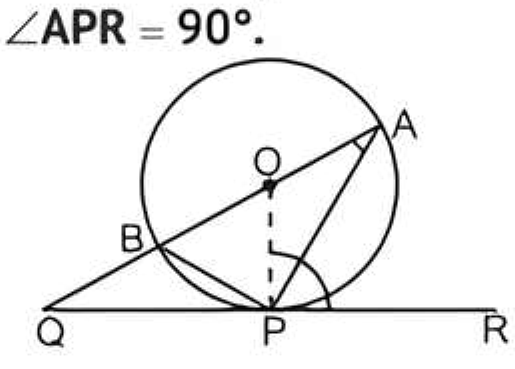
Q 3. In the given figure, PA is a tangent to the circle drawn from the external point P and PBC is the secant to the circle with BC as diameter. If $\angle$AOC = 130°, then find the measure of $\angle$APB, where O is the centre of the circle. *[CBSE 2023]*



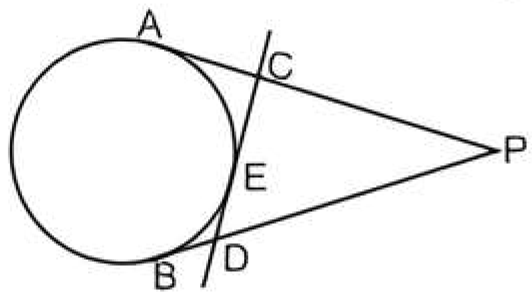
Q 4. In the given figure, O is the centre of the circle. AB and AC are tangents drawn to the circle from point A. If $\angle$BAC = 65°, then find the measure of $\angle$BOC. *[CBSE 2023]*



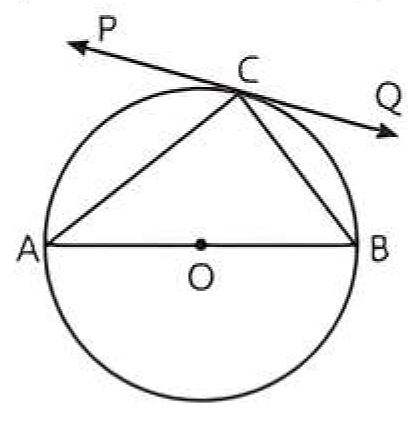
Q 5. In the given figure, O is the centre of the circle and QPR is a tangent to it at P. Prove that $\angle$QAP + $\angle$APR = 90°. *[CBSE 2023]*



Q 6. From an external point P, two tangents, PA and PB are drawn to a circle with centre O. At a point E on the circle, a tangent is drawn to intersect PA and PB at C and D, respectively. If PA = 10 cm, find the perimeter of $\triangle PCD$. *[CBSE SQP 2023-24]*



Q 7. In the given figure, PQ is a tangent at a point C to a circle with centre O. If AB is a diameter and $\angle PCA = 30°$, then find $\angle BCQ$.



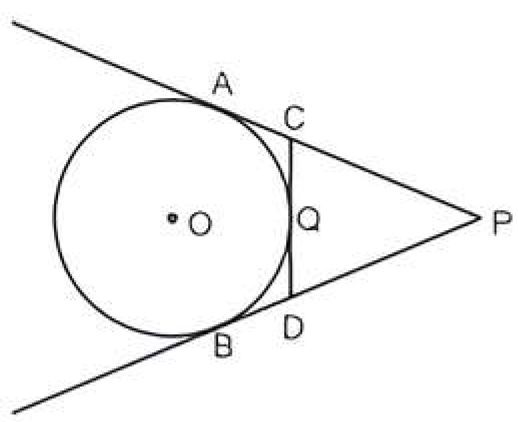
Q 8. From an external point P, tangents PA and PB are drawn to a circle with centre O. $\angle PAB = 50°$, then find $\angle AOB$. *[CBSE 2016]*

Short Answer Type-I Questions

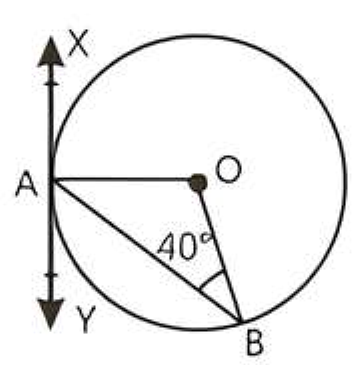
Q 1. Prove that tangents drawn at the ends of a diameter of a circle are parallel to each other.

[NCERT EXERCISE; CBSE 2019, 17]

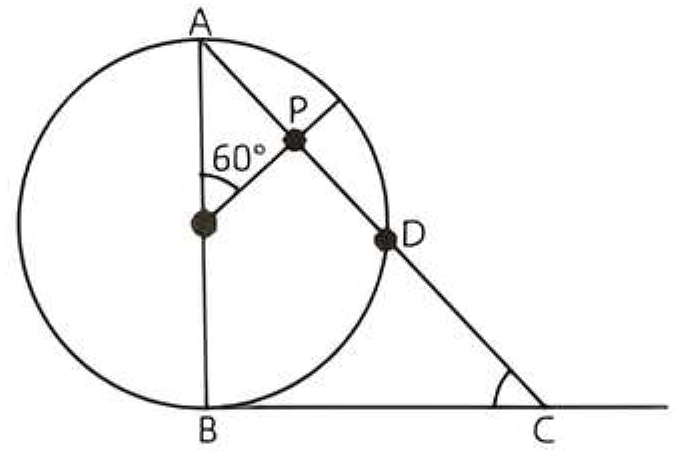
Q 2. In the given figure, PA and PB are tangents to the circle from an external point P. CD is another tangent touching the circle at Q. If PA = 12 cm, QC = QD = 3 cm, then find PC + PD. *[CBSE 2017]*



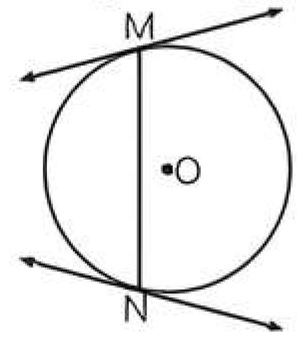
Q 3. In the given figure, XAY is a tangent to the circle centered at O. If $\angle ABO = 40°$, then find $\angle BAY$ and $\angle AOB$. *[CBSE 2022 Term-II]*



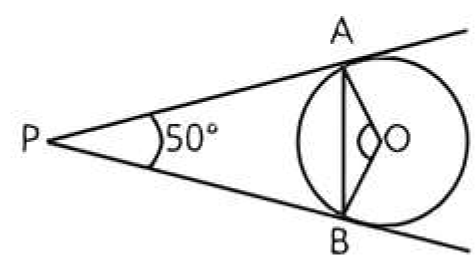
Q 4. In the given figure, AB is diameter of a circle centered at O. BC is tangent to the circle at B. If OP bisects the chord AD and $\angle AOP = 60°$, then find $\angle C$. *[CBSE 2022 Term-II]*



Q 5. Prove that tangents drawn at the ends of a chord of a circle make equal angles with the chord.



Q 6. In the given figure, PA and PB are tangents to the circle with centre O such that $\angle APB = 50°$. Write the measure of $\angle OAB$.



[CBSE 2015]

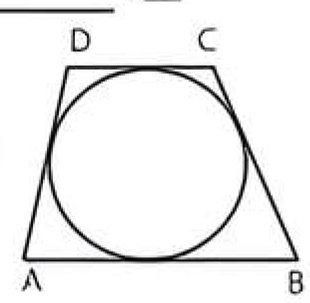
Q 7. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle. *[CBSE 2022 Term-II, CBSE 2023]*

Q 8. From an external point, two tangents are drawn to a circle. Prove that the line joining the external point to the centre of the circle bisects the angle between the two tangents. *[CBSE 2023]*

Short Answer Type-II Questions

Q 1. A quadrilateral ABCD is drawn to circumscribe a circle, as shown in the figure. Prove that

$$AB + CD = AD + BC$$



[NCERT EXERCISE; CBSE 2016,17, 23]

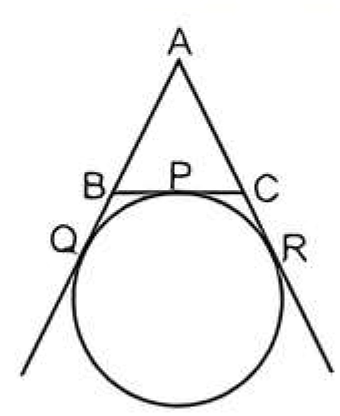
Or

A circle touches all the four sides of quadrilateral ABCD. Prove that AB + CD = AD + DA.

[CBSE SQP 2023-24]

Q 2. A circle touches the side BC of a $\triangle ABC$ at a point P and touches AB and AC when produced at Q and R respectively. Show that $AQ = \frac{1}{2}$ (Perimeter of $\triangle ABC) = \frac{1}{2}$ (BC + CA + AB).

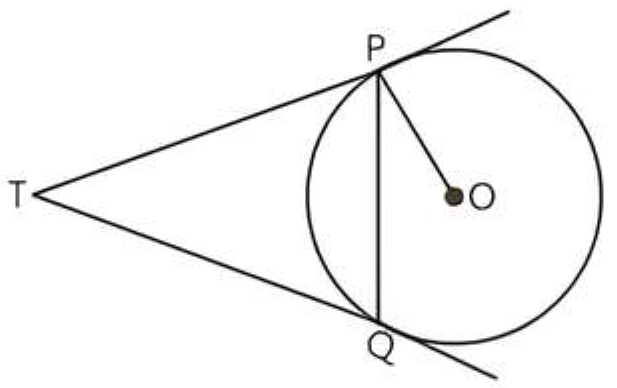
[NCERT EXEMPLAR; CBSE 2023, 20]



Q 3. Prove that the tangents drawn from an external point to a circle are equal in length. *[CBSE 2023]*

Q 4. Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre. *[CBSE 2023]*

Q 5. Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that $\angle PTQ = 2\angle OPQ$. *[NCERT EXERCISE; CBSE SQP 2022 Term-II, CBSE SQP 2023-24; CBSE 2023, 17]*



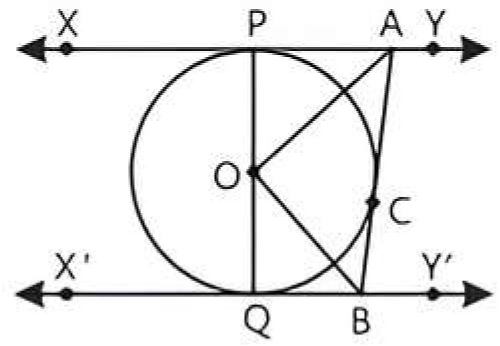
Q 6. In the given figure, PA and PB are tangents to a circle from an external point P such that PA = 4 cm and $\angle BAC = 135°$. Find the length of chord AB. *[CBSE 2017]*

Q 7. In the given figure, XY and X′Y′ are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X′Y′ at B.

Prove that $\angle AOB = 90°$. *[NCERT EXERCISE; CBSE 2017]*

Or

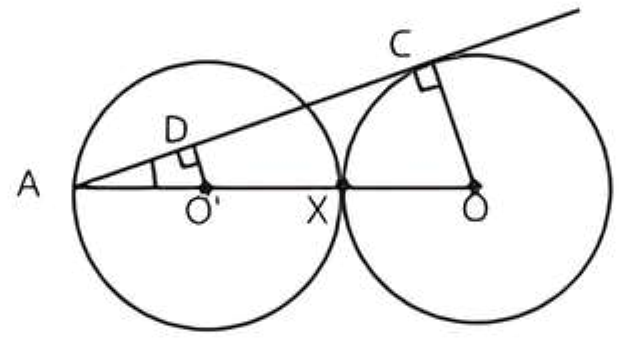
What is the measure of $\angle AOB$? *[CBSE SQP 2022-23]*

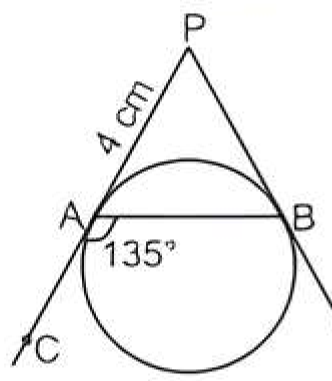


Long Answer Type Questions

Q 1. Prove that the parallelogram circumscribing a circle is a rhombus. *[NCERT EXERCISE; CBSE 2022 Term-II, CBSE SQP 2022-23, CBSE 2023]*

Q 2. In the adjoining figure, two equal circles with centres O and O′, touch each other at X, produce OO′ to meet the circle with centre O′ at A. AC is tangent to the circle with centre O, at the point C. O′D is perpendicular to AC. Find the ratio of $\frac{DO'}{CO}$. *[CBSE 2016]*





Q 3. Prove that the opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle. *[NCERT EXERCISE; CBSE 2017]*

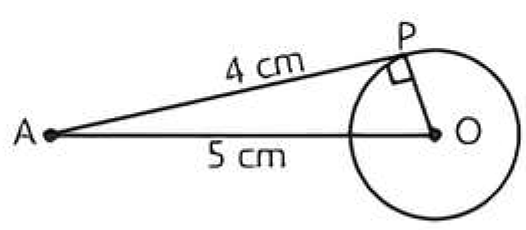
Solutions

Very Short Answer Type Questions

1.

TiP

A tangent to a circle is perpendicular to the radius through the point of contact.



∵ $OP \perp AP$

∴ $\angle OPA = 90°$

Now, in right-angled $\triangle OPA$,

$$(OA)^2 = (AP)^2 + (OP)^2$$

(by pythagoras theorem)

⇒ $(5)^2 = (4)^2 + (OP)^2$

⇒ $OP^2 = (5)^2 - (4)2$

$= 25 - 16 = 9$

∴ $OP = 3$ cm

So, required radius of circle is 3 cm.

2. Two tangents *PA* and *PB* are drawn from an external point *P* to the circle.

Given, $\angle APB = 60°$

TR!CK

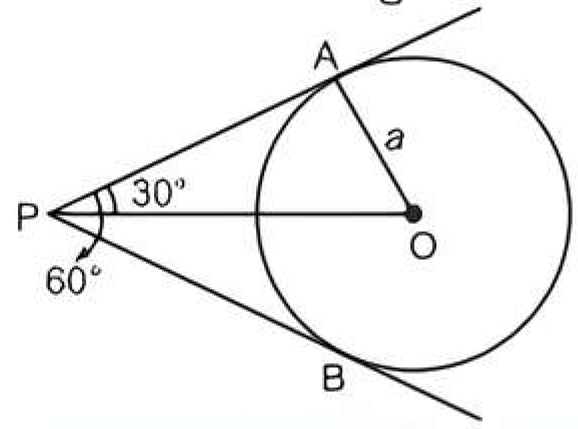
If two tangents are drawn from an external point to a circle, then the line joining that external point to the centre of circle bisect the angle between the tangents.

Join OA and OP.

∵ PA is a tangent, therefore

$OA \perp PA$

We know that OP is the angle bisector of $\angle APB$.



∴ $\angle OPA = \frac{1}{2}\angle APB = \frac{1}{2} \times 60° = 30°$

In right-angled ΔOAP,

$$\sin 30° = \frac{OA}{OP} \Rightarrow \frac{1}{2} = \frac{a}{OP}$$

$OP = 2a$

3. Given, PA is a tangent to the circle drawn from the external point P and PBC is the secant to the circle with BC as diameter.

Also, $\angle AOC = 130°$.

TiP

Tangent at any point of a circle is perpendicular to the radius through the point of contact.

In ΔPOA, $\angle PAO = 90°$

TR!CK

Exterior angle of a triangle = Sum of interior opposite angles.

Now, $\angle APO + \angle PAO = \angle AOC$

$\therefore$ $\angle APO = 130° - 90° = 40°$

So, $\angle APB = \angle APO = 40°$

4. Since, tangent is perpendicular to the radius through the point of contact.

$\therefore$ $\angle OBA = \angle OCA = 90°$

Now, $\angle OBA + \angle BAC + \angle OCA + \angle BOC = 360°$

(angle sum property of quadrilateral)

$\Rightarrow$ $90° + 65° + 90° + \angle BOC = 360°$

$\Rightarrow$ $\angle BOC = 360° - 245° = 115°$

COMMON ERR(!)R

Some students are not known with the circle properties, i.e., they could not well identify $\angle OBA = \angle OCA = 90°$ (angle between radius and tangents)

5.

TiP

If a chord is drawn through a point of contact of a tangent to the circle then the angle formed by this chord from the tangent are equal to the angles of corresponding alternate segments.

Here, $\angle BPQ = \angle QAP$ ...(1)

(by alternate segment theorem)

Since, AOB is a diameter of the circle.

$\therefore$ $\angle APB = 90°$ ...(2)

(angle of semicircle)

From figure, QPR is a tangent *i.e.*, a straight line.

$\therefore$ $\angle BPQ + \angle APB + \angle APR = 180°$

$\Rightarrow$ $\angle BPQ + 90° + \angle APR = 180°$ [from eq. (1)]

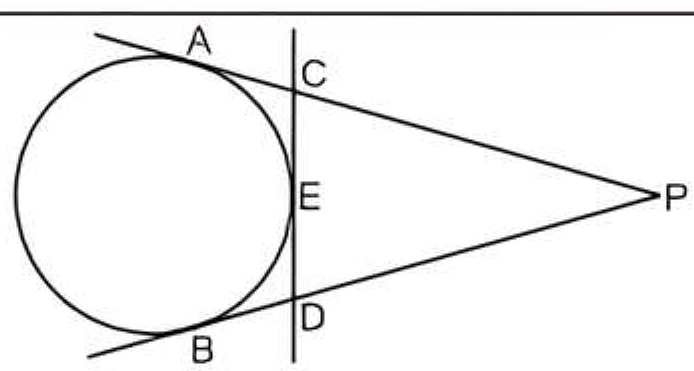
$\Rightarrow$ $\angle QAP + \angle APR = 180° - 90° = 90°$

[from eq. (2)] **Hence proved.**

6. Since BC touches the circle at R.

TiP

Perimeter of any triangle is equal to sum of all its three sides.



$\therefore$ Perimeter of $\Delta PCD = PC + CD + PD$

$= PC + (CE + ED) + PD$

TR!CK

Tangents are drawn from an external point to a circle are equal in lengths.

$= (PC + CE) + (ED + PD)$

$= (PC + CA) + (DB + PD)$

($\because$ CE = CA and ED = DB)

$= PA + PB$ ($\because$ PA = PC + CA and PB = PD + DB)

$= PA + PA$ ($\because$ AP = BP)

$= 2PA = 2 \times 10$ ($\because$ PA = 10 cm, given)

$= 20$ cm

7. Given, $\angle PCA = 30°$

TR!CK

Diameter of a circle subtends right angled to the circumference of a circle.

Here, AB is a diameter of a circle.

Therefore $\angle ACB = 90°$.

Since PQ is a straight line. Therefore sum of all angles of one side is equal to 180°.

i.e., $\angle PCA + \angle ACB + \angle BCQ = 180°$

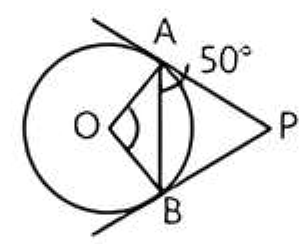
$\Rightarrow$ $30° + 90° + \angle BCQ = 180°$

$\Rightarrow$ $\angle BCQ = 60°$

8. Since, tangents drawn from external point are equal.

$\therefore$ PA = PB

$\therefore$ $\angle PBA = \angle PAB = 50°$ (angles opposite to equal sides are equal)



In ΔAPB,

$\angle APB + \angle PBA + \angle PAB = 180°$

(by angle sum property of a triangle)

$\Rightarrow$ $\angle APB = 180° - 50° - 50° = 80°$

In cyclic quadrilateral OAPB,

$\angle AOB + \angle APB = 180°$ ($\because$ sum of opposite angles of a cyclic quadrilateral is 180°)

$\Rightarrow$ $\angle AOB + 80° = 180°$

$\therefore$ $\angle AOB = 180° - 80° = 100°$

COMMON ERR(!)R

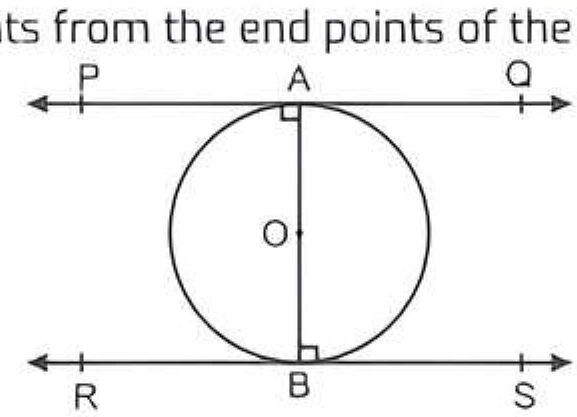
Some students could not apply the appropriate theorem of circle to find out the unknown angles.

Short Answer Type-I Questions

1. Given: AB is the diameter of a circle with centre O. Tangents are PAQ and RBS from the end points of the diameter on the circle.

To Prove: PQ || RS

Proof: $\because$ AB is the diameter and PAQ and RBS are tangents from the end points of the diameter.



TIP

Tangent is perpendicular to the radius through the point of contact of circle.

Here, $\angle PAB = 90°$ and $\angle ABS = 90°$

But $\angle PAB$ and $\angle ABS$ are the same alternate angle made by cutting the transverse line AB to lines PQ and RS.

So, PQ || RS **Hence proved.**

2.

TIP

Tangents drawn from an external point to a circle are equal in lengths.

Here, $PA = PB = 12$ cm ...(1)

$QC = AC = 3$ cm ...(2)

and $QD = BD = 3$ cm ...(3)

($\because$ PA = 12 cm, QC = QD = 3 cm)

Now, $PC + PD = (PA - AC) + (PB - BD)$ (from figure)

$= (12 - 3) + (12 - 3)$

(from eqs. (1), (2) and (3))

$= 9 + 9 = 18$ cm

3. In the given figure,

$OB = OA$ (radii of a circle)

$\Rightarrow \angle BAO = \angle ABO$ (angles opposite to equal sides of a triangle are equal)

$\Rightarrow \angle BAO = 40°$

Since tangent is perpendicular to the radius through the point of contact of circle.

i.e., $OA \perp XY$

$\therefore \angle OAY = 90°$

$\Rightarrow \angle OAB + \angle BAY = 90°$

$\Rightarrow 40° + \angle BAY = 90°$

$\therefore \angle BAY = 90° - 40° = 50°$

In ΔOAB, use angle sum property of a triangle.

$\angle AOB + \angle BAO + \angle ABO = 180°$

$\Rightarrow \angle AOB + 40° + 40° = 180°$

$\Rightarrow \angle AOB = 100°$

4. Given, OP bisect AD.

Since, the line drawn through the centre of a circle to bisect a chord is perpendicular to the chord.

$\therefore OP \perp AD$

i.e., $\angle OPA = 90°$

In ΔOPA,

$\angle AOP + \angle OPA + \angle OAP = 180°$

(by angle sum property of triangle)

$\Rightarrow 60° + 90° + \angle OAP = 180°$

$\Rightarrow \angle OAP = 30°$

$\Rightarrow \angle BAC = \angle OAP = 30°$

Again in ΔABC,

$\angle BAC + \angle ABC + \angle ACB = 180°$

(by angle sum property of triangle)

$\Rightarrow 30° + 90° + \angle ACB = 180°$

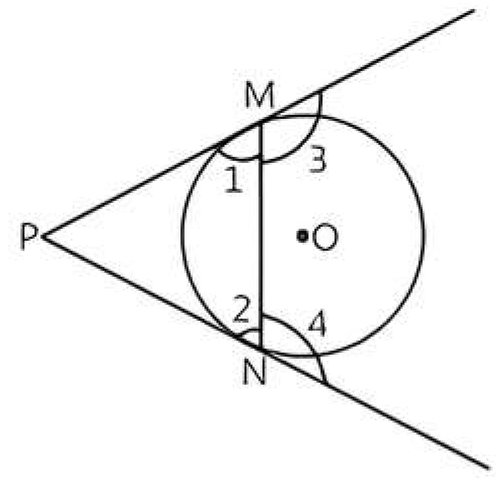
($\because$ radius is perpendicular to the tangent $\therefore \angle OBC = \angle ABC = 90°$)

$\Rightarrow \angle ACB = 60°$

5. Let tangents at points M and N extends it and intersect these tangents at point P.

TIP

Tangents drawn from an external point to the circle are equal in lengths.



$PM = PN \Rightarrow \angle 2 = \angle 1$

($\because$ angles opposite to equal sides of a triangle are equal)

$\Rightarrow 180° - \angle 4 = 180° - \angle 3$ (by linear pair)

$\Rightarrow \angle 3 = \angle 4$

Hence, tangents drawn at the ends of a chord of a circle make equal angles, with the chord.

Hence proved.

6. Since, tangents drawn from an external point to a circle are equal in lengths.

$\therefore PA = PB$

TIP

Tangent is perpendicular to the radius through the point of contact of circle.

Here, $\angle OAP = \angle OBP = 90°$

$\because OB = OA$ (radii)

$\therefore \angle OAB = \angle OBA$ ...(1)

(angles opposite to equal sides of a triangle are equal)

By using angle sum property of quadrilateral, OAPB,

$\angle OAP + \angle AOB + \angle OBP + \angle APB = 360°$

$\Rightarrow 90° + \angle AOB + 90° + 50° = 360°$

$\Rightarrow \angle AOB = 360° - 230° = 130°$ ...(2)

In ΔOAB,

$\angle AOB + \angle OAB + \angle OBA = 180°$

(angle sum property of triangle)

$\Rightarrow 130° + \angle OAB + \angle OAB = 180°$ (from eq. (1))

$\Rightarrow 2\angle OAB = 180° - 130° = 50°$

$\therefore \angle OAB = 25°$

COMMON ERROR

Some students could not apply the appropriate theorem of circle to find out the unknown angles.

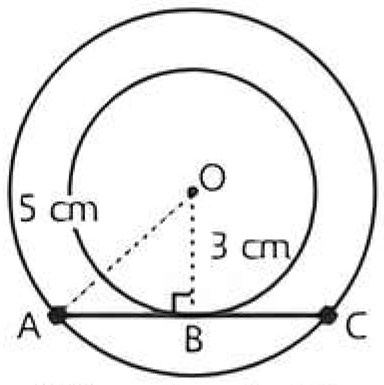
7. Given, radius of bigger circle OA = 5 cm and radius of smaller circle OB = 3 cm.

In right angled ΔOBA,

$OA^2 = OB^2 + AB^2$

$(5)^2 = (3)^2 + AB^2$

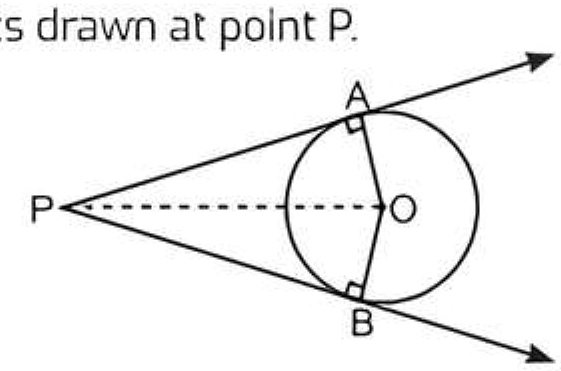
$\Rightarrow AB = \sqrt{25 - 9} = \sqrt{16} = 4$



$\therefore$ The length of the chord of larger circle,

$AC = 2\,AB = 2 \times 4 = 8$ cm.

8. **Given:** O is the centre of a circle and P is a point at a distance OP from the centre. PA and PB are two tangents drawn at point P.



To Prove: $\angle OPA = \angle OPB$

Construction: Join OA, OP and OB.

Proof: In ΔOAP and ΔOBP, we see that

$OA = OB$ (radii of a circle)

$OP = OP$ (common arm)

$PA = PB$ (tangents drawn from an external point to a circle are of equal length)

$\therefore$ $\Delta OAP \cong \Delta OPB$ (from SSS congruency)

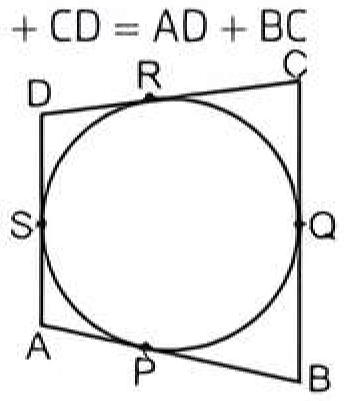
$\Rightarrow$ $\angle OPA = \angle OPB$ (by CPCT)

Which shows the line joining the external point to the centre of the circle bisects the angle between the two tangents. **Hence proved.**

Short Answer Type-II Questions

1. **Given :** A quadrilateral ABCD circumscribes a circle.

To Prove : $AB + CD = AD + BC$



Proof: $\because$ The lengths of tangents drawn from an external point to a circle are equal.

$\therefore$ $DR = DS$ ...(1)

$CR = CQ$ ...(2)

$BP = BQ$ ...(3)

$AP = AS$ ...(4)

On adding all these equations, we get

$$DR + CR + BP + AP = DS + CQ + BQ + AS$$

$$(DR + CR) + (BP + AP) = (DS + AS) + (CQ + BQ)$$

$\Rightarrow$ $CD + AB = AD + BC$

Hence proved.

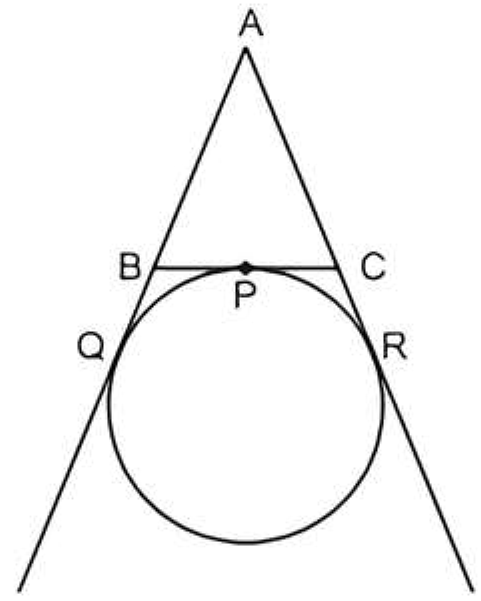
2. Given, a circle touches the side *BC* of a triangle *ABC* at *P* and extended sides AB and AC at *Q* and *R* respectively.

Since, the length of tangents drawn from an external point to a circle are equal.

$\therefore$ $AQ = AR$ ...(1)

$BQ = BP$ ...(2)

and $CR = CP$ ...(3)



> **TIP**
> *Perimeter of any triangle is equal to sum of all its three sides.*

So, perimeter of $\Delta ABC = AB + BC + AC$

$= AB + BP + PC + AC$ ($\because$ $BC = BP + PC$)

$= (AB + BQ) + (CR + AC)$ [from eqs. (2) and (3)]

$= AQ + AR$ ($\because$ $AQ = AB + BQ$, $AR = CR + AC$)

$= AQ + AQ$ [from eq. (1)]

$= 2AQ$

$\therefore$ $AQ = \dfrac{1}{2} \times$ Perimeter of $\Delta ABC = \dfrac{1}{2}(BC + CA + AB)$

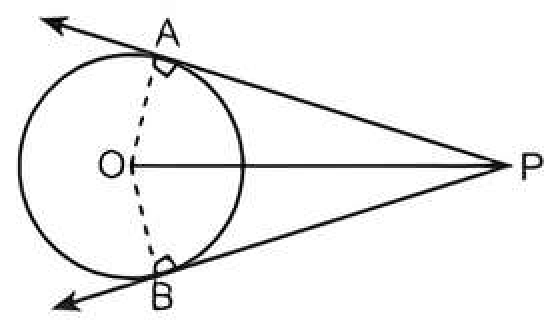
Hence proved.

3. **Given:** O is the centre of a circle and P is a point at OP distance from the centre. Two tangents PA and PB are drawn from point P to the circle.

To Prove: $PA = PB$

Construction: Draw the line segments OA and OB.

Proof: $\because$ OA and OB are the radii of the given circle and PA and PB are tangents.



$\therefore$ $OA \perp PA$ and $OB \perp PB$

$\therefore$ ΔOAP and ΔOBP are right angled triangle.

$\therefore$ In right ΔOAP and ΔOBP

$OA = OB$ ($\because$ radii of a circle)

$OP = OP$ (common arm of two triangles)

$\therefore$ From R.H.S. rule,

$\Delta OAP \cong \Delta OBP$

$\therefore$ $PA = PB$ (C.P.C.T.) **Proved.**

Alternate Method

In right ΔOAP,

$OP^2 = OA^2 + AP^2$

(from Pythagoras theorem)

$\Rightarrow$ $AP^2 = OP^2 - OA^2$

$\Rightarrow$ $AP^2 = OP^2 - OB^2$ [$\because$ OA = OB (radii)]

$\Rightarrow$ $AP^2 = (OB^2 + BP^2) - OB^2$

($\because$ In right ΔOBP, from Pythagoras theorem, $OP^2 = OB^2 + BP^2$)

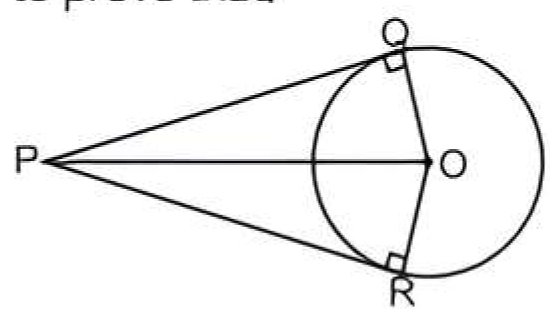
$\Rightarrow \qquad AP^2 = OB^2 + BP^2 - OB^2$

$\Rightarrow \qquad AP^2 = BP^2 \quad \Rightarrow \quad AP = BP$

Hence proved.

4. Let PQ and PR be two tangents drawn from an external point P to a circle with centre O.

We have to prove that,



$\angle QOR = 180^\circ - \angle QPR$

or $\quad \angle QOR + \angle QPR = 180^\circ$

In right ΔOQP and ΔORP

$PQ = PR$

(tangents drawn from an external point are equal)

$OQ = OR$ (radius of circle)

$OP = OP$ (common)

Therefore, by SSS criterion of congruence,

$\Delta OQP \cong ORP$

$\Rightarrow \qquad \angle QPO = \angle RPO$

and $\qquad \angle POQ = \angle POR$ (by CPCT)

$\Rightarrow \qquad \angle QPR = 2\,\angle OPQ$ ...(1)

and $\qquad \angle QOR = 2\,\angle POQ$

In ΔOPQ,

$\angle QPO + \angle QOP = 90^\circ \qquad (\therefore \angle OQP = 90^\circ)$

$\Rightarrow \qquad QOP = 90^\circ - \angle QPO$

$\Rightarrow \qquad 2\angle QOP = 180^\circ - 2\,\angle QPO$

(multiplying both sides by 2)

$\Rightarrow \qquad \angle QOR = 180^\circ - \angle QPR$ [from eq. (1)]

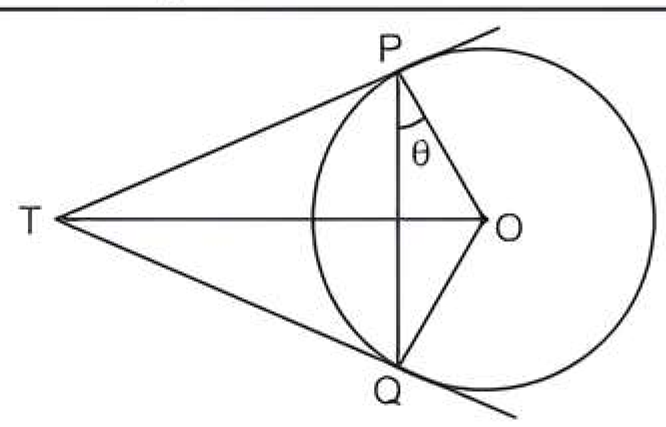
$\Rightarrow \qquad \angle QOR + \angle QPR = 180^\circ$ **Hence proved.**

5. Let $\angle OPQ = \theta$, then

$TP \perp OP \qquad (\because \text{tangent} \perp \text{radius})$

TiPs

- *Tangent is perpendicular to the radius through the point of contact of circle.*
- *Angles opposite to equal sides of a triangle is also equal.*
- *Length of two tangents drawn from an external point of a circle are equal.*



$\therefore \qquad \angle OPT = 90^\circ$

$\Rightarrow \qquad \angle OPQ + \angle TPQ = 90^\circ$

$\Rightarrow \qquad \angle TPQ = 90^\circ - \theta$

$\because \qquad TP = TQ$

$\therefore \qquad \angle TQP = \angle TPQ = 90^\circ - \theta$

Now, In ΔTPQ,

$\angle PTQ + \angle TPQ + \angle TQP = 180^\circ$

($\because$ sum of internal angles in a triangle is 180°)

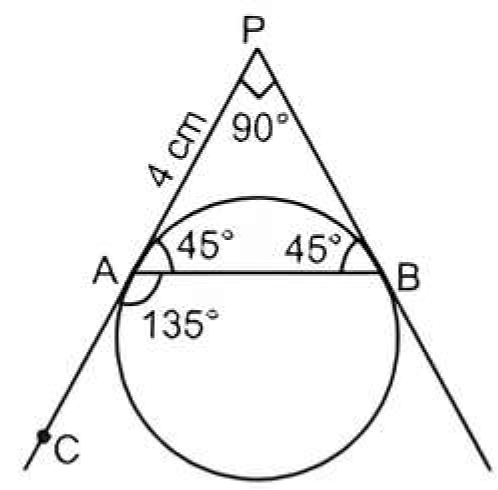
$\Rightarrow \qquad \angle PTQ + 90^\circ - \theta + 90^\circ - \theta = 180^\circ$

$\Rightarrow \qquad \angle PTQ = 2\theta = 2\,\angle OPQ$ **Hence proved.**

6. It is given that PA and PB are tangents drawn from an external point P to the circle.

$\therefore \qquad PA = PB = 4 \text{ cm}$

($\because$ lengths of tangents drawn from an external point to a circle are equal)



Also, $\qquad \angle BAC = 135^\circ$

Since, $\angle BAC + \angle PAB = 180^\circ$ (linear pair)

$\therefore \qquad \angle PAB = 180^\circ - 135^\circ = 45^\circ$

In ΔPAB, $\qquad PA = PB$

TiP

Angles opposite to equal sides of a triangle is also equal.

$\therefore \qquad \angle PBA = \angle PAB = 45^\circ$

Also $\qquad \angle PBA + \angle PAB + \angle APB = 180^\circ$

(angle sum property of triangle)

$\Rightarrow \qquad 45^\circ + 45^\circ + \angle APB = 180^\circ$

$\Rightarrow \qquad \angle APB = 180^\circ - 45^\circ - 45^\circ = 90^\circ$

So, ΔAPB is a right triangle right angled at P. Using Pythagoras theorem, we have

TR!CK

In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

$AB^2 = PA^2 + PB^2 = \sqrt{(4)^2 + (4)^2}$

$= \sqrt{16 + 16} = \sqrt{32}$

$\therefore \qquad AB = 4\sqrt{2} \text{ cm}$

Hence, the length of the chord AB is $4\sqrt{2}$ cm.

7.

TiP

Know about the circle and related angle theorem, cyclic theorem, tangent and secant theorem thoroughly.

Given: XY and X′Y′ are two parallel tangents to a circle with centre O. AB is another tangent at C meeting XY and X′Y′ at A and B respectively.

To Prove: $\angle AOB = 90^\circ$

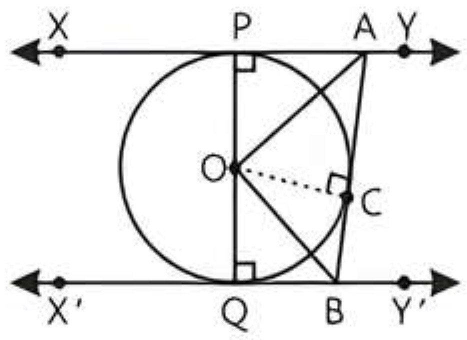
Construction: Join point O to C.

Proof: In ΔOPA and ΔOCA,

$OP = OC$ (radii of the same circle)

$AP = AC$

(tangents drawn from an external point A to the circle are equal)



$AO = AO$ (common side)

$\therefore$ $\Delta OPA \sim \Delta OCA$ (by SSS similarity)

$\Rightarrow$ $\angle POA = \angle COA$ (by CPCT) ...(1)

Similarly, $\Delta OQB \sim \Delta OCB$

$\Rightarrow$ $\angle QOB = \angle COB$ (by CPCT) ...(2)

Since, POQ is a diameter of the circle. So, sum of all adjacent angles lie on this line is 180°.

$\therefore$ $\angle POA + \angle COA + \angle COB + \angle QOB = 180°$

From eqs. (1) and (2), it can be observed that

$2\angle COA + 2\angle COB = 180°$

$\Rightarrow$ $\angle COA + \angle COB = 90°$

$\Rightarrow$ $\angle AOB = 90°$ **Hence proved.**

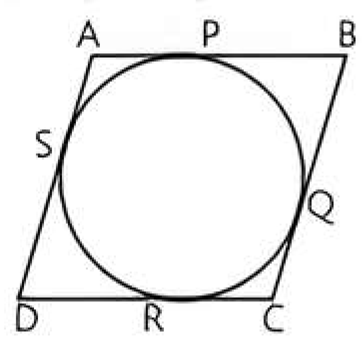
COMMON ERR(!)R

Some candidates could not apply the appropriate theorems of circle to find out the unknown angles.

Long Answer Type Questions

1. **Given:** ABCD is a parallelogram circumscribing a circle.

To Prove: ABCD is a rhombus.



Proof: In parallelogram ABCD,

TiP

In a parallelogram, opposite sides are equal in length.

$AB = CD$ ...(1)

and $BC = AD$ ...(2)

Since, the length of tangents drawn from an external point to a circle are equal

$\therefore$ $DR = DS,$ ...(3)

$CR = CQ,$ ...(4)

$BP = BQ$ ...(5)

and $AP = AS$ ...(6)

Adding eqs. (3), (4), (5) and (6), we get

$$DR + CR + BP + AP = DS + CQ + BQ + AS$$

$$(DR + CR) + (BP + AP) = (DS + AS) + (CQ + BQ)$$

$CD + AB = AD + BC$ ...(7)

On putting the values of eqs. (1) and (2) in eq. (7), we get

$$2AB = 2BC$$

$AB = BC$ ...(8)

From eqs. (1), (2) and (3), we get

$$AB = BC = CD = DA$$

Hence, ABCD is a rhombus. **Hence proved.**

2. Given, two equal circles with centres O and O', touch each other at point X. OO' is produced to meet the circle with centre O' at the point A.

$\because$ $O'D \perp AC$ and $OC \perp AC$

$\therefore$ $\angle ACO = 90°$ and $\angle ADO' = 90°$

TiP

Tangent is perpendicular to the radius through the point of contact of circle.

Now, In $\Delta AO'D$ and ΔAOC,

$\angle O'AD = \angle OAC$ (common angle)

$\angle ADO = \angle ACO$ (each 90°)

$\Delta AO'D \sim \Delta AOC$ (by AA similarity)

$$\frac{AO'}{AO} = \frac{DO'}{CO}$$

(corresponding sides are proportional)

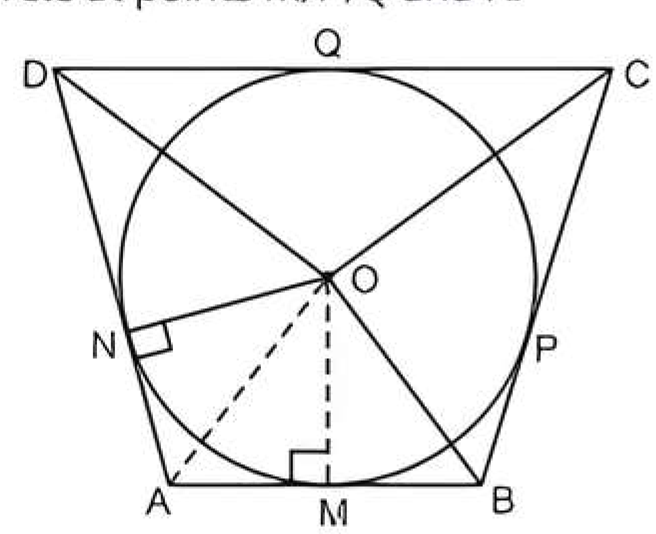
Let $AO' = O'X = XO = r$

Then $AO = AO' + O'X + XO = r + r + r = 3r$

$\therefore$ $\frac{AO'}{AO} = \frac{DO'}{CO} = \frac{r}{3r}$

Hence, $\frac{DO'}{CO} = \frac{1}{3}$

3. **Given:** A quadrilateral ABCD is circumscribed in a circle with centre O whose sides AB, BC, CD and DA touch the circle at points M, P, Q and N.



To Prove: $\angle AOB + \angle COD = 90°$

Construction: Join the points of contact M and N to O.

Proof: Let $\angle A = 2\alpha$, $\angle B = 2\beta$,

$\angle C = 2\gamma$, $\angle D = 2\delta$

In ΔOAM and ΔOAN,

$\angle OMA = \angle ONA$ (each right angle)

$OM = ON$ (radii of a circle)

$OA = OA$ (common side)

$\therefore$ Both triangles are congruent.

i.e., $\Delta OAM \cong \Delta OAN$ (from RHS congruency)

$\Rightarrow \angle OAM = \angle OAN = \frac{1}{2}(\angle A) = \frac{1}{2}(2\alpha) = \alpha$

(from CPCT)

$\Rightarrow \angle OAB = \angle OAD = \alpha$

Similarly, $\angle OBA = \angle OBC = \beta$

$\angle OCB = \angle OCD = \gamma$

and $\angle ODA = \angle ODC = \delta$

TiP

The sum of interior angles of a triangle is 180°.

In ΔAOB,

$\angle AOB = 180° - \angle OAB - \angle OBA$

(by angle sum property of a triangle)

$= 180° - \alpha - \beta = 180° - (\alpha + \beta)$ ...(1)

and $\angle COD = 180° - \angle OCD - \angle ODC$

$= 180° - \gamma - \delta = 180° - (\gamma + \delta)$ ...(2)

Adding eqs. (1) and (2),

$\angle AOB + \angle COD = \{180° - (\alpha + \beta)\} + \{180° - (\gamma + \delta)\}$

$= 360° - (\alpha + \beta + \gamma + \delta)$ ...(3)

But $\angle A + \angle B + \angle C + \angle D = 360°$

TiP

The sum of interior angles of a quadrilateral is 360°.

$\Rightarrow 2\alpha + 2\beta + 2\gamma + 2\delta = 360°$

$\Rightarrow \alpha + \beta + \gamma + \delta = 180°$

Therefore from eq. (3),

$\angle AOB + \angle COD = 360° - 180° = 180°$

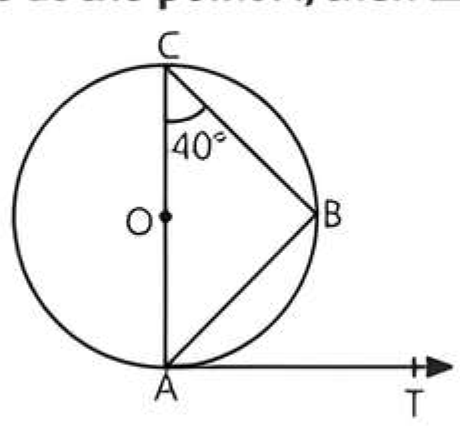
Chapter Test

Multiple Choice Questions

Q 1. If radii of two concentric circles are 8 cm and 10 cm, the length of the chord touches the smaller circle is:

a. 5 cm b. 7 cm c. 6 cm d. 12 cm

Q 2. In the figure, AB is a chord of the circle and AOC is its diameter such that $\angle ACB = 40°$. If AT is tangent to the circle at the point A, then $\angle BAT$ is equal to:



a. 60° b. 20° c. 40° d. 30°

Assertion and Reason Type Questions

Directions (Q.Nos. 3-4): *In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option:*

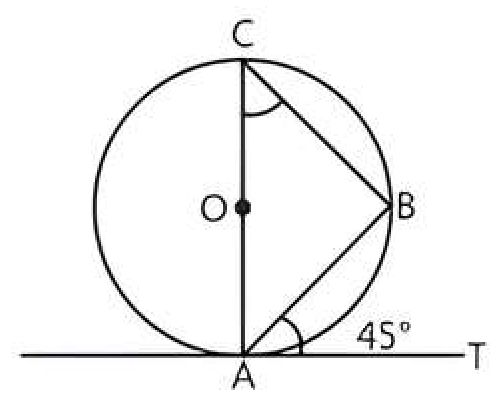
a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)

c. Assertion (A) is true but Reason (R) is false

d. Assertion (A) is false but Reason (R) is true

Q 3. Assertion (A): In the given figure, O is the centre of a circle and AT is a tangent at point A, then $\angle ACB$ is 45°.



Reason (R): Diameter of a circle is always perpendicular to the tangent line.

Q 4. Assertion (A): If the distance between two parallel tangents of a circle is 24 cm, then radius of a circle is 12 cm.

Reason (R): The distance between two parallel tangents of a circle is equal to twice the diameter of a circle.

Fill in the Blanks

Q 5. There are exactly two tangents to be drawn on a circle, if a point lying .... the circle.

Q 6. A line intersecting a circle at two points is said to be a ....

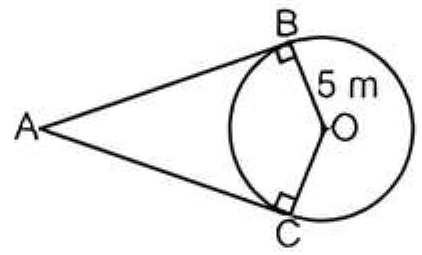
True/False

Q 7. If angle between two tangents drawn from a point P to a circle of radius *a* and centre O is 90°, then $OP = a\sqrt{2}$.

Q 8. The centre of the circle lies on the bisector of the angle between the two tangents.

Case Study Based Question

Q 9. There is a circular field of radius 5 m. A person was starting a walk along the tangents of the circular field. Two paths are connected by the tangents of circle AB and AC which is shown in the figure.



The distance of the point from where tangents are drawn *i.e.*, A to O is 13 m. A person running along path BA and AC *i.e.*, person starts from B and stops at C.

Based on the above information, solve the following questions:

(i) Find the length of AB.

(ii) Find the total distance travelled by the person.

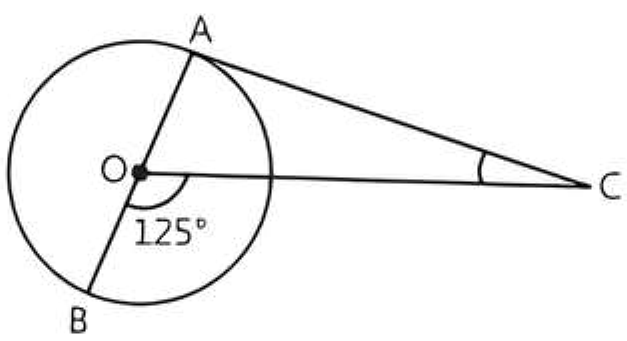
(iii) If $\angle OAB = 60°$, then find the value of $\angle BOA$ is:

Or

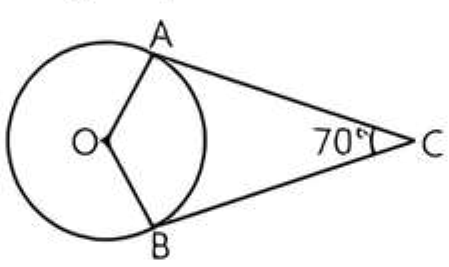
Find the measure of $\angle BOC$.

Very Short Answer Type Questions

Q 10. In the given figure, AOB is a diameter of a circle with centre O and AC is a tangent to the circle at A. If $\angle BOC = 125°$, then find $\angle ACO$.



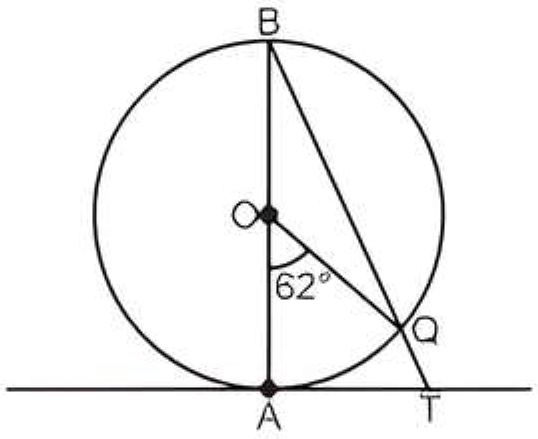
Q 11. In the given figure, find $\angle AOB$.



Short Answer Type-I Questions

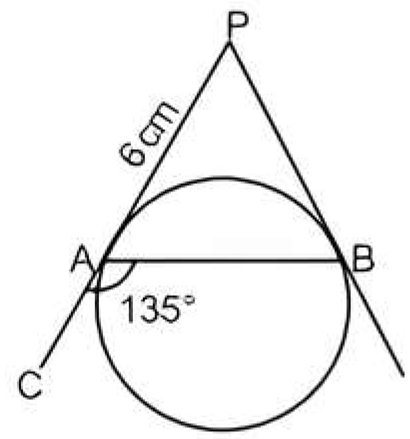
Q 12. If two tangents are inclined at an angle 120° are drawn to a circle of radius 6 cm, then find the length of each tangent.

Q 13. In the given figure, AB is the diameter of a circle with centre O and AT is a tangent. If AOQ = 62°, find $\angle ATQ$.

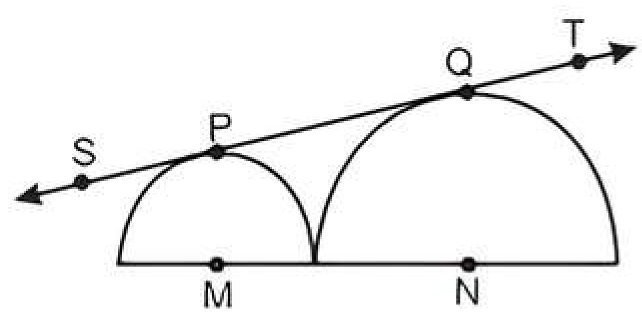


Short Answer Type-II Questions

Q 14. In the given figure, PA and PB are tangents to a circle from an external point P such that PA = 6 cm and $\angle BAC = 135°$. Find the length of the chord AB.



Q 15. In the figure below, M and N are the centres of two semi-circles having radii 9 cm and 16 cm respectively. ST is a common tangent. Find the length of PQ.



Long Answer Type Question

Q 16. In the given figure, PA and PB are tangents to the circle from an external point P. CD is another tangent touching the circle at Q. If PA = 12 cm, QC = QD = 3 cm, then find PC + CD.