CBSE Test Paper 05

CH-11 Conic Sections

1. The line y = c is a tangent to the parabola x^2 = y - 1 if c is equal to
   a. c = 1
   b. 0
   c. 2 a
   d. a
2. The locus of a point which moves so that its distance from a fixed point, called focus, bears a constant ratio, which is less than unity, to its distance from a fixed line, called the directrix is called
   a. a parabola
   b. an ellipse
   c. a circle
   d. a hyperbola
3. locus of the point of intersection of the lines x = sec θ + tan θ and y = sec θ – tan θ is
   a. an ellipse
   b. none of these
   c. a parabola
   d. a straight line
4. The straight line 3 x + 4 y = 20 and the circle $x^2 + y^2 = 16$
   a. none of these
   b. intersect in two distinct points
   c. neither touch nor intersect in two points
   d. touch each other
5. The centre of a circle passing through the points (0, 0), (1, 0) and touching the circle $x^2 + y^2 = 9$ is
   a. $\left(\frac{1}{2}, -\sqrt{2}\right)$
   b. $\left(\frac{1}{2}, \frac{1}{2}\right)$
   c. $\left(\frac{1}{2}, -\sqrt{2}\right)$
   d. $\left(\frac{1}{2}, \frac{3}{2}\right)$
6. Fill in the blanks:

Two hyperbolas such that transverse and conjugate axes of one hyperbola are respectively the conjugate and transverse axes of each other are called ________ hyperbolas of each other.

7. Fill in the blanks:

   The equation of right handed parabola is of the form ________.

8. Find the locus of the point of intersection of the lines $\sqrt{3}x - y - 4\sqrt{3}\lambda$ = 0 and $\sqrt{3}\lambda x + \lambda y - 4\sqrt{3}$ for different values of λ.

9. Find the equation of the parabola that satisfies the given conditions: Focus (0, - 3) directrix y = 3

10. Find the equation of a parabola with vertex at the origin, the axis along the x-axis and passing through (2,3).

11. Find the equation of a circle with centre (3, - 2) and touching the X-axis.

12. Find the equation of ellipse having b = 3, c= 4, centre at origin, foci on the x-axis.

13. Find the equation of the circle, whose end points of a diameter are A (1, 5) and B (- 1, 3).

14. Find the equation of the ellipse whose focus is (1, - 2), the directrix 3x - 2y + 5 = 0 and eccentricity equal to 1/2.

15. Find the equation of the ellipse, whose foci are (0, $\pm$ 4) and e = $\frac{4}{5}$

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Solution

1. (a) c = 1

   Explanation: putting the value y=c into parabola,we get

   x^2=c-1

   or x^2-(c-1)=0

   here discriminat=$\sqrt{4c-4}$

   line y=c is tangent when discriminant is equal to 0.

   putting disriminant =0 we get c=1.

   (0, c) will be a point on the parabola.

2. (b) an ellipse

   Explanation: For an ellipse e < 1
3. (d) a straight line

   Explanation: After solving the equations we will get x+y = 2sec which represents a linear equation.
4. (d) touch each other

   Explanation: distance from the origin to the line is equal to the radius.

   $\frac{0+0+20}{\sqrt{3^2+4^2}}$ = 4. The radius of the circle is 4.

   Hence the line and circle touches each other.

5. (c) $\left(\frac{1}{2},-\sqrt{2}\right)$

   Explanation: Since the circle passes through (0,0) the equation reduces to c= 0 -----(1)

Since it passes through (1,0),

1 + 2g + c = 0

This implies g = -1/2

Since the circle touches the circle $x^2 + y^2 = 9$, their radii should be equal

$2\sqrt{g^2 + f^2 + c} = 3$

Substituting the values and simplifying we get f = $\pm\sqrt{2}$

Hence the centre is (1/2, $-\sqrt{2}$)

6. conjugate

7. $y^2 = 4ax$, a>0

8. Let (h, k) be the point of intersection of the given lines. Then,
   $\sqrt{3}h - k - 4\sqrt{3}\lambda = 0$ and $\sqrt{3}\lambda h + \lambda k - 4\sqrt{3} = 0$
   $\Rightarrow \sqrt{3}h - k = 4\sqrt{3}\lambda$ and $\lambda(\sqrt{3}h + k) = 4\sqrt{3}$
   $\Rightarrow (\sqrt{3}h - k)\lambda(\sqrt{3}h + k) = (4\sqrt{3}\lambda)(4\sqrt{3})$ [multiplying both]
   $\Rightarrow 3h^2 - k^2 = 48$
   Hence, the locus of (h, k) is $3x^2 - y^2 = 48$

9. Since the focus (0, - 3) lies on the y-axis, therefore y-axis is the axis of parabola. Also the directrix is y = 3 i.e. y = a and focus (0, - 3) i.e. (0, -a). So the parabola is of the form $x^2 = -4ay$.
   The required equation of parabola is
   $x^2 = -4 \times 3y \Rightarrow x^2 = -12x$

10. Let the equation of parabola be
    $y^2 = 4ax$ ....(i) [$\because$ axis along x - axis]
    If passes through (2, 3)
    $\therefore (3)^2 = 4 \times a \times 2$

$\Rightarrow$ 9 = 8a

$\Rightarrow$ a = $\frac{9}{8}$

Putting the value of a in equation (i), we get

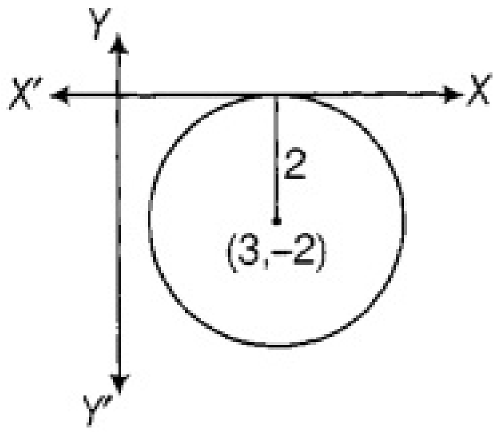
$y^2 = 4 \times \frac{9}{8} \times x$

$\Rightarrow y^2 = \frac{9}{2} \times x$

$\Rightarrow 2y^2 = 9x$

Hence, the required equation of parabola is $2y^2 = 9x$

11. Here, centre of a circle (3,- 2) with radius is 2 units.



Hence, equation of circle is $(x - 3)^2 + (y + 2)^2 = (2)^2$

[$\because$ equation of circle having centre (h, k) and radius r is

$(x - h)^2 + (y - k)^2 = r^2$]

$\Rightarrow x^2 - 6x + 9 + y^2 + 4y + 4 = 4$

$\Rightarrow x^2 + y^2 - 6x + 4y + 9 = 0$

12. The foci lie on x-axis

So the equation of ellipse in standard form is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

We know that $c^2 = a^2 - b^2$

$\therefore (4)^2 = a^2 - (3)^2 \Rightarrow a^2 = 16 + 9 = 25$

Thus equation of required ellipse is

$\frac{x^2}{25} + \frac{y^2}{9} = 1$

13. Given, endpoints of a diameter are A (1, 5) and B (- 1, 3).

$\therefore$ Mid-point of AB = $\left(\frac{1-1}{2}, \frac{5+3}{2}\right)$

= (0, 4) = (h, k) [say]

Now, radius = $\frac{1}{2}$ (distance between A and B)

= $\frac{1}{2}\left[\sqrt{(-1-1)^2+(3-5)^2}\right]$

[∵ distance = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$]

= $\frac{1}{2}\left[\sqrt{(-2)^2+(-2)^2}\right]=\frac{2\sqrt{2}}{2}$

= $\sqrt{2}$

Hence, equation of circle having centre (0, 4) and radius $\sqrt{2}$ is

$(x - 0)^2 + (y - 4)^2 = (\sqrt{2})^2$

$\Rightarrow x^2 + y^2 + 16 - 8y = 2$

$\Rightarrow x^2 + y^2 - 8y + 14 = 0$

14. Let P(x,y) be any point on the ellipse whose focus is S(1,−2) and eccentricity e = $\frac{1}{2}$. Let PM be perpendicular from P on the directrix. Then,

SP = ePM

$\Rightarrow$ SP = $\frac{1}{2}$ (PM)

$\Rightarrow SP^2 = \frac{1}{4}(PM)^2$

$\Rightarrow 4SP^2 = (PM)^2$

$\Rightarrow 4[(x - 1)^2 + (y + 2)^2] = \left[\frac{3x-2y+5}{\sqrt{(3)^2+(-2)^2}}\right]^2$

$\Rightarrow 4[x^2 + 1 - 2x + y^2 + 4 + 4y] = \frac{(3x-2y+5)^2}{(\sqrt{13})^2}$

$\Rightarrow 4[x^2 + y^2 - 2x + 4y + 5] = \frac{(3x-2y+5)^2}{13}$

$\Rightarrow 52[x^2 + y^2 - 2x + 4y + 5] = (3x - 2y + 5)^2$

$\Rightarrow 52x^2 + 52y^2 - 104x + 208y + 260 = (3x - 2y + 5)^2$

$\Rightarrow 52x^2 + 52y^2 - 104x + 208y + 260 = (3x)^2 + (-2y)^2 + (5)^2 + 2\times 3x \times (-2y) + 2 \times(-2y) \times 5$ $+ 2\times 5\times 3x$ ...[∵$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$]

$\Rightarrow 52x^2 + 52y^2 - 104x + 208y + 260 = 9x^2 + 4y^2 + 25 - 12xy - 20y + 30x$

$\Rightarrow 52x^2 - 9x^2 + 52y^2 - 4y^2 + 12xy - 104x - 30x + 208y + 20y + 260 - 25 = 0$

$\Rightarrow 43x^2 + 48y^2 + 12xy - 134x + 228y + 235 = 0$

This is the required equation of the ellipse.

15. Given, foci are (0, ± 4)

Let the equation of ellipse be $\frac{x^2}{b^2} + \frac{y^2}{a^2}$ = 1, a > b

Its foci are (0, ± ae)

$\therefore$ ae = 4

Also, e = $\frac{4}{5}$ [given]

$\therefore$ a $\left(\frac{4}{5}\right)$ = 4

$\Rightarrow$ a = 5

Now, $b^2 = a^2 (1 - e^2) = 25 (1 - \frac{16}{25}) = 25 \times \frac{9}{25}$

$\Rightarrow$ $b^2 = 9$

Hence, equation of ellipse is $\frac{x^2}{9} + \frac{y^2}{25}$ = 1.