MATHEMATICS

(For 7th Class)

Punjab School Education Board

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Contents

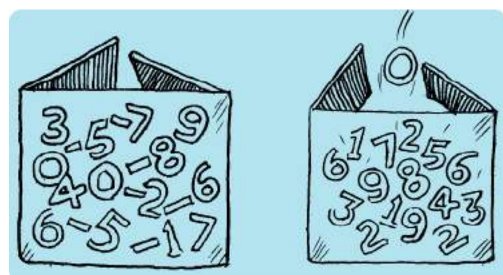
Chapter 1	Integers	1
Chapter 2	Fractions and Decimals	29
Chapter 3	Data Handling	57
Chapter 4	Simple Equations	77
Chapter 5	Lines and Angles	93
Chapter 6	The Triangle and its Properties	113
Chapter 7	Congruence of Triangles	133
Chapter 8	Comparing Quantities	153
Chapter 9	Rational Numbers	173
Chapter 10	Practical Geometry	193
Chapter 11	Perimeter and Area	205
Chapter 12	Algebraic Expressions	229
Chapter 13	Exponents and Powers	249
Chapter 14	Symmetry	265
Chapter 15	Visualising Solid Shapes	277
	Answers	293
	Brain-Teasers	311

Chapter 1

Integers

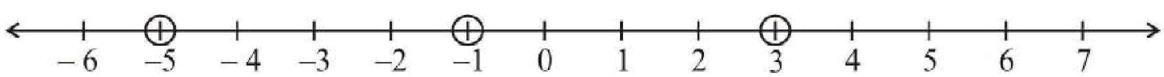
1.1 Introduction

We have learnt about whole numbers and integers in Class VI. We know that integers form a *bigger* collection of numbers which contains whole numbers and negative numbers. What other differences do you find between whole numbers and integers? In this chapter, we will study more about integers, their properties and operations. First of all, we will review and revise what we have done about integers in our previous class.



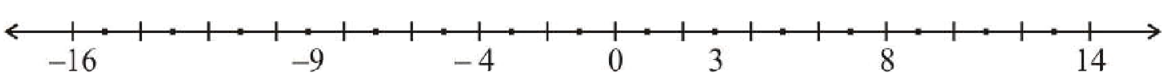
1.2 Recall

We know how to represent integers on a number line. Some integers are marked on the number line given below.



Can you write these marked integers in ascending order? The ascending order of these numbers is – 5, – 1, 3. Why did we choose – 5 as the smallest number?

Some points are marked with integers on the following number line. Write these integers in descending order.

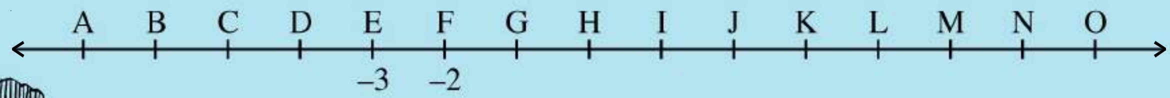


The descending order of these integers is 14, 8, 3, ...

The above number line has only a few integers filled. Write appropriate numbers at each dot.

Try These

1. A number line representing integers is given below



–3 and –2 are marked by E and F respectively. Which integers are marked by B, D, H, J, M and O?

2. Arrange 7, –5, 4, 0 and – 4 in ascending order and then mark them on a number line to check your answer.

We have done addition and subtraction of integers in our previous class. Read the following statements.

On a number line when we

(i) add a positive integer, we move to the right.

(ii) add a negative integer, we move to the left.

(iii) subtract a positive integer, we move to the left.

(iv) subtract a negative integer, we move to the right.

State whether the following statements are correct or incorrect. Correct those which are wrong:

(i) When two positive integers are added we get a positive integer.

(ii) When two negative integers are added we get a positive integer.

(iii) When a positive integer and a negative integer are added, we always get a negative integer.

(iv) Additive inverse of an integer 8 is (– 8) and additive inverse of (– 8) is 8.

(v) For subtraction, we add the additive inverse of the integer that is being subtracted, to the other integer.

(vi) (–10) + 3 = 10 – 3

(vii) 8 + (–7) – (– 4) = 8 + 7 – 4

Compare your answers with the answers given below:

(i) Correct. For example:

(a) 56 + 73 = 129 (b) 113 + 82 = 195 etc.

Construct five more examples in support of this statement.

(ii) Incorrect, since (– 6) + (– 7) = – 13, which is not a positive integer. The correct statement is: When two negative integers are added we get a negative integer.

For example,

(a) (– 56) + (– 73) = – 129 (b) (– 113) + (– 82) = – 195, etc.

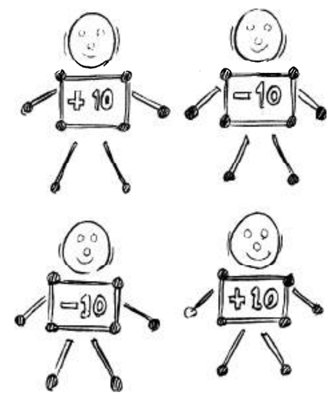
Construct five more examples on your own to verify this statement.

(iii) Incorrect, since – 9 + 16 = 7, which is not a negative integer. The correct statement is : When one positive and one negative integers are added, we take their difference and place the sign of the bigger integer. The bigger integer is decided by ignoring the signs of both the integers. For example:

(a) (– 56) + (73) = 17 (b) (– 113) + 82 = – 31

(c) 16 + (– 23) = – 7 (d) 125 + (– 101) = 24

Construct five more examples for verifying this statement.



(iv) Correct. Some other examples of additive inverse are as given below:

Integer	Additive inverse
10	–10
–10	10
76	–76
–76	76

Thus, the additive inverse of any integer a is $-a$ and additive inverse of $(-a)$ is a.

(v) Correct. Subtraction is opposite of addition and therefore, we add the additive inverse of the integer that is being subtracted, to the other integer. For example:

(a) 56 – 73 = 56 + additive inverse of 73 = 56 + (–73) = –17

(b) 56 – (–73) = 56 + additive inverse of (–73) = 56 + 73 = 129

(c) (–79) – 45 = (–79) + (– 45) = –124

(d) (–100) – (–172) = –100 + 172 = 72 etc.

Write atleast five such examples to verify this statement.

Thus, we find that for any two integers a and b,

$a - b = a$ + additive inverse of $b = a + (-b)$

and $a - (-b) = a$ + additive inverse of $(-b) = a + b$

(vi) Incorrect, since (–10) + 3 = –7 and 10 – 3 = 7

therefore, $(-10) + 3 \neq 10 - 3$

(vii) Incorrect, since, 8 + (–7) – (– 4) = 8 + (–7) + 4 = 1 + 4 = 5

and 8 + 7 – 4 = 15 – 4 = 11

However, 8 + (–7) – (– 4) = 8 – 7 + 4

TRY THESE

We have done various patterns with numbers in our previous class.

Can you find a pattern for each of the following? If yes, complete them:

(a) 7, 3, – 1, – 5, ______, ______, ______.

(b) – 2, – 4, – 6, – 8, ______, ______, ______.

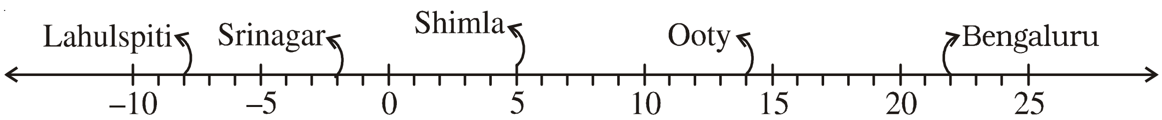
(c) 15, 10, 5, 0, ______, ______, ______.

(d) – 11, – 8, – 5, – 2, ______, ______, ______.

Make some more such patterns and ask your friends to complete them.

Exercise 1.1

1. Following number line shows the temperature in degree celsius (°C) at different places on a particular day.



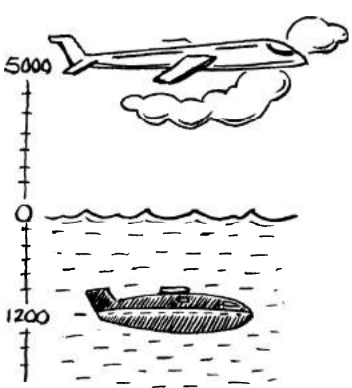
(a) Observe this number line and write the temperature of the places marked on it.

(b) What is the temperature difference between the hottest and the coldest places among the above?

(c) What is the temperature difference between Lahulspiti and Srinagar?

(d) Can we say temperature of Srinagar and Shimla taken together is less than the temperature at Shimla? Is it also less than the temperature at Srinagar?

2. In a quiz, positive marks are given for correct answers and negative marks are given for incorrect answers. If Jack's scores in five successive rounds were 25, – 5, – 10, 15 and 10, what was his total at the end?

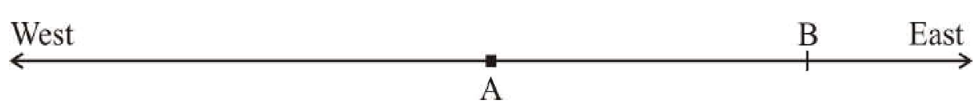


3. At Srinagar temperature was – 5°C on Monday and then it dropped by 2°C on Tuesday. What was the temperature of Srinagar on Tuesday? On Wednesday, it rose by 4°C. What was the temperature on this day?

4. A plane is flying at the height of 5000 m above the sea level. At a particular point, it is exactly above a submarine floating 1200 m below the sea level. What is the vertical distance between them?

5. Mohan deposits ₹ 2,000 in his bank account and withdraws ₹ 1,642 from it, the next day. If withdrawal of amount from the account is represented by a negative integer, then how will you represent the amount deposited? Find the balance in Mohan's account after the withdrawal.

6. Rita goes 20 km towards east from a point A to the point B. From B, she moves 30 km towards west along the same road. If the distance towards east is represented by a positive integer then, how will you represent the distance travelled towards west? By which integer will you represent her final position from A?



7. In a magic square each row, column and diagonal have the same sum. Check which of the following is a magic square.

(i)

5	–1	– 4
–5	–2	7
0	3	–3

(ii)

1	–10	0
– 4	–3	–2
– 6	4	–7

8. Verify $a - (-b) = a + b$ for the following values of a and b.

(i) $a = 21, b = 18$ (ii) $a = 118, b = 125$

(iii) $a = 75, b = 84$ (iv) $a = 28, b = 11$

9. Use the sign of >, < or = in the box to make the statements true.

(a) (– 8) + (– 4) ☐ (–8) – (– 4)

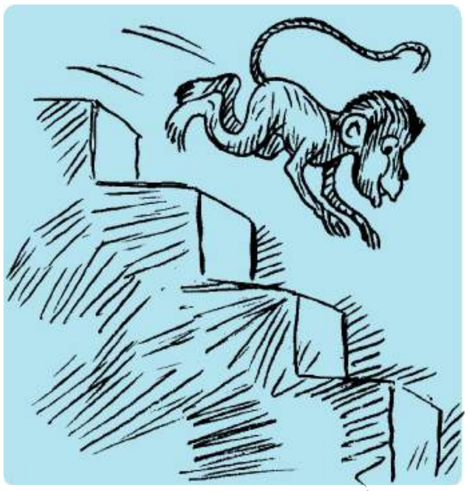
(b) (– 3) + 7 – (19) ☐ 15 – 8 + (– 9)

(c) 23 – 41 + 11 ☐ 23 – 41 – 11

(d) 39 + (– 24) – (15) ☐ 36 + (– 52) – (– 36)

(e) – 231 + 79 + 51 ☐ –399 + 159 + 81

10. A water tank has steps inside it. A monkey is sitting on the topmost step (i.e., the first step). The water level is at the ninth step.



(i) He jumps 3 steps down and then jumps back 2 steps up. In how many jumps will he reach the water level?

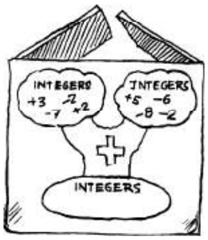
(ii) After drinking water, he wants to go back. For this, he jumps 4 steps up and then jumps back 2 steps down in every move. In how many jumps will he reach back the top step?

(iii) If the number of steps moved down is represented by negative integers and the number of steps moved up by positive integers, represent his moves in part (i) and (ii) by completing the following; (a) – 3 + 2 – ... = – 8 (b) 4 – 2 + ... = 8. In (a) the sum (– 8) represents going down by eight steps. So, what will the sum 8 in (b) represent?

1.3 Properties of Addition and Subtraction of Integers

1.3.1 Closure under Addition

We have learnt that sum of two whole numbers is again a whole number. For example, 17 + 24 = 41 which is again a whole number. We know that, this property is known as the closure property for addition of the whole numbers.



Let us see whether this property is true for integers or not.

Following are some pairs of integers. Observe the following table and complete it.

Statement	Observation
(i) 17 + 23 = 40	Result is an integer
(ii) (–10) + 3 = _____	_____________
(iii) (– 75) + 18 = _____	_____________
(iv) 19 + (– 25) = – 6	Result is an integer
(v) 27 + (– 27) = _____	_____________
(vi) (– 20) + 0 = _____	_____________
(vii) (– 35) + (– 10) = _____	_____________

What do you observe? Is the sum of two integers always an integer?

Did you find a pair of integers whose sum is not an integer?

Since addition of integers gives integers, we say **integers are closed under addition**.

In general, **for any two integers *a* and *b*, *a* + *b* is an integer.**

1.3.2 Closure under Subtraction

What happens when we subtract an integer from another integer? Can we say that their difference is also an integer?

Observe the following table and complete it:

Statement	Observation
(i) 7 – 9 = – 2	Result is an integer
(ii) 17 – (– 21) = _______	_____________
(iii) (– 8) – (–14) = 6	Result is an integer
(iv) (– 21) – (– 10) = _______	_____________
(v) 32 – (–17) = _______	_____________
(vi) (– 18) – (– 18) = _______	_____________
(vii) (– 29) – 0 = _______	_____________

What do you observe? Is there any pair of integers whose difference is not an integer? Can we say integers are closed under subtraction? Yes, we can see that *integers are closed under subtraction*.

Thus, *if a and b are two integers then a – b is also an intger*. Do the whole numbers satisfy this property?

1.3.3 Commutative Property

We know that 3 + 5 = 5 + 3 = 8, that is, the whole numbers can be added in any order. In other words, addition is commutative for whole numbers.

Can we say the same for integers also?

We have 5 + (– 6) = –1 and (– 6) + 5 = –1

So, 5 + (– 6) = (– 6) + 5

Are the following equal?

(i) (– 8) + (– 9) and (– 9) + (– 8)

(ii) (– 23) + 32 and 32 + (– 23)

(iii) (– 45) + 0 and 0 + (– 45)

Try this with five other pairs of integers. Do you find any pair of integers for which the sums are different when the order is changed? Certainly not. We say that *addition is commutative for integers*.

In general, for any two integers a and b, we can say

$$a + b = b + a$$

- We know that subtraction is not commutative for whole numbers. Is it commutative for integers?

  Consider the integers 5 and (–3).

  Is 5 – (–3) the same as (–3) –5? No, because 5 – (–3) = 5 + 3 = 8, and (–3) – 5 = – 3 – 5 = – 8.

  Take atleast five different pairs of integers and check this.

  We conclude that subtraction is not commutative for integers.

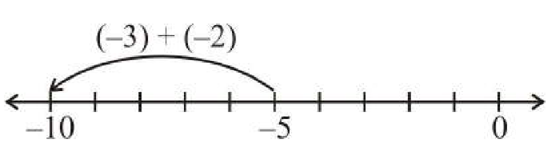
1.3.4 Associative Property

Observe the following examples:

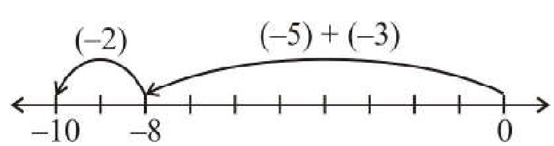
Consider the integers –3, –2 and –5.

Look at (–5) + [(–3) + (–2)] and [(–5) + (–3)] + (–2).

In the first sum (–3) and (–2) are grouped together and in the second (–5) and (–3) are grouped together. We will check whether we get different results.



(–5) + [(–3) + (–2)]



[(–5) + (–3)] + (–2)

In both the cases, we get –10.

i.e., $(-5) + [(-3) + (-2)] = [(-5) + (-2)] + (-3)$

Similarly consider –3 , 1 and –7.

$(-3) + [1 + (-7)] = -3 +$ __________ = __________

$[(-3) + 1] + (-7) = -2 +$ __________ = __________

Is $(-3) + [1 + (-7)]$ same as $[(-3) + 1] + (-7)$?

Take five more such examples. You will not find any example for which the sums are different. *Addition is associative for integers*.

In general for any integers *a*, *b* and *c*, we can say

$$a + (b + c) = (a + b) + c$$

1.3.5 Additive Identity

When we add zero to any whole number, we get the same whole number. Zero is an additive identity for whole numbers. Is it an additive identity again for integers also?

Observe the following and fill in the blanks:

(i) $(-8) + 0 = -8$ (ii) $0 + (-8) = -8$

(iii) $(-23) + 0 =$ _____ (iv) $0 + (-37) = -37$

(v) $0 + (-59) =$ _____ (vi) $0 +$ _____ $= -43$

(vii) $-61 +$ _____ $= -61$ (viii) _____ $+ 0 =$ _____

The above examples show that zero is an additive identity for integers.

You can verify it by adding zero to any other five integers.

In general, for any integer *a*

$$a + 0 = a = 0 + a$$

Try These

1. Write a pair of integers whose sum gives

(a) a negative integer (b) zero

(c) an integer smaller than both the integers. (d) an integer smaller than only one of the integers.

(e) an integer greater than both the integers.

2. Write a pair of integers whose difference gives

(a) a negative integer. (b) zero.

(c) an integer smaller than both the integers. (d) an integer greater than only one of the integers.

(e) an integer greater than both the integers.

EXAMPLE 1 Write down a pair of integers whose

(a) sum is –3 (b) difference is –5

(c) difference is 2 (d) sum is 0

SOLUTION

(a) (–1) + (–2) = –3 or (–5) + 2 = –3

(b) (–9) – (– 4) = –5 or (–2) – 3 = –5

(c) (–7) – (–9) = 2 or 1 – (–1) = 2

(d) (–10) + 10 = 0 or 5 + (–5) = 0

Can you write more pairs in these examples?

EXERCISE 1.2

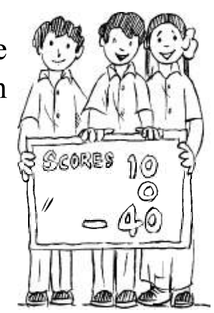
1. Write down a pair of integers whose:

   (a) sum is –7 (b) difference is –10 (c) sum is 0

2. (a) Write a pair of negative integers whose difference gives 8.

   (b) Write a negative integer and a positive integer whose sum is –5.

   (c) Write a negative integer and a positive integer whose difference is –3.



3. In a quiz, team A scored – 40, 10, 0 and team B scored 10, 0, – 40 in three successive rounds. Which team scored more? Can we say that we can add integers in any order?

4. Fill in the blanks to make the following statements true:

   (i) (–5) + (– 8) = (– 8) + (............)

   (ii) –53 + ... = –53

   (iii) 17 + ... = 0

   (iv) [13 + (– 12)] + (............) = 13 + [(–12) + (–7)]

   (v) (– 4) + [15 + (–3)] = [– 4 + 15] + ...

1.4 MULTIPLICATION OF INTEGERS

We can add and subtract integers. Let us now learn how to multiply integers.

1.4.1 Multiplication of a Positive and a Negative Integer

We know that multiplication of whole numbers is repeated addition. For example,

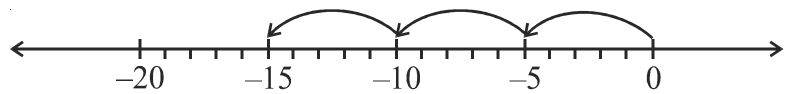
$$5 + 5 + 5 = 3 \times 5 = 15$$

Can you represent addition of integers in the same way?

TRY THESE

Find:
$4 \times (-8)$,
$8 \times (-2)$,
$3 \times (-7)$,
$10 \times (-1)$
using number line.

We have from the following number line, $(-5) + (-5) + (-5) = -15$

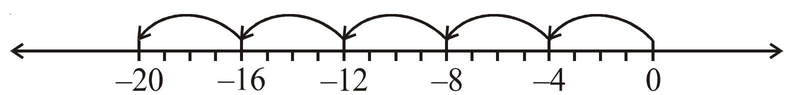


But we can also write

$$(-5) + (-5) + (-5) = 3 \times (-5)$$

Therefore, $3 \times (-5) = -15$

Similarly $(-4) + (-4) + (-4) + (-4) + (-4) = 5 \times (-4) = -20$



And $(-3) + (-3) + (-3) + (-3) =$ __________ $=$ __________

Also, $(-7) + (-7) + (-7) =$ __________ $=$ __________

Let us see how to find the product of a positive integer and a negative integer without using number line.

Let us find $3 \times (-5)$ in a different way. First find 3×5 and then put minus sign (–) before the product obtained. You get –15. That is we find $-(3 \times 5)$ to get –15.

Similarly, $5 \times (-4) = -(5 \times 4) = -20$.

Find in a similar way,

$4 \times (-8) =$ _____ $=$ _____, $3 \times (-7) =$ _____ $=$ _____

$6 \times (-5) =$ _____ $=$ _____, $2 \times (-9) =$ _____ $=$ _____

Using this method we thus have,

$10 \times (-43) =$ _____ $-(10 \times 43) = -430$

TRY THESE

Find:
(i) $6 \times (-19)$
(ii) $12 \times (-32)$
(iii) $7 \times (-22)$

Till now we multiplied integers as (positive integer) × (negative integer).

Let us now multiply them as (negative integer) × (positive integer).

We first find -3×5.

To find this, observe the following pattern:

We have,

$$3 \times 5 = 15$$
$$2 \times 5 = 10 = 15 - 5$$
$$1 \times 5 = 5 = 10 - 5$$
$$0 \times 5 = 0 = 5 - 5$$

So, $-1 \times 5 = 0 - 5 = -5$

$-2 \times 5 = -5 - 5 = -10$

$-3 \times 5 = -10 - 5 = -15$

We already have $3 \times (-5) = -15$

So we get $(-3) \times 5 = -15 = 3 \times (-5)$

Using such patterns, we also get $(-5) \times 4 = -20 = 5 \times (-4)$

Using patterns, find $(-4) \times 8$, $(-3) \times 7$, $(-6) \times 5$ and $(-2) \times 9$

Check whether, $(-4) \times 8 = 4 \times (-8)$, $(-3) \times 7 = 3 \times (-7)$, $(-6) \times 5 = 6 \times (-5)$

and $(-2) \times 9 = 2 \times (-9)$

Using this we get, $(-33) \times 5 = 33 \times (-5) = -165$

We thus find that while *multiplying a positive integer and a negative integer, we multiply them as whole numbers and put a minus sign* (–) *before the product. We thus get a negative integer.*

TRY THESE

1. Find: (a) $15 \times (-16)$ (b) $21 \times (-32)$
   (c) $(-42) \times 12$ (d) -55×15
2. Check if (a) $25 \times (-21) = (-25) \times 21$ (b) $(-23) \times 20 = 23 \times (-20)$

   Write five more such examples.

In general, for any two positive integers *a* and *b* we can say

$$a \times (-b) = (-a) \times b = -(a \times b)$$

1.4.2 Multiplication of two Negative Integers

Can you find the product $(-3) \times (-2)$?

Observe the following:

$-3 \times 4 = -12$

$-3 \times 3 = -9 = -12 - (-3)$

$-3 \times 2 = -6 = -9 - (-3)$

$-3 \times 1 = -3 = -6 - (-3)$

$-3 \times 0 = 0 = -3 - (-3)$

$-3 \times -1 = 0 - (-3) = 0 + 3 = 3$

$-3 \times -2 = 3 - (-3) = 3 + 3 = 6$

Do you see any pattern? Observe how the products change.

Based on this observation, complete the following:

$-3 \times -3 =$ _____ $-3 \times -4 =$ _____

Now observe these products and fill in the blanks:

$-4 \times 4 = -16$

$-4 \times 3 = -12 \quad = -16 + 4$

$-4 \times 2 =$ _____ $= -12 + 4$

$-4 \times 1 =$ _____

$-4 \times 0 =$ _____

$-4 \times (-1) =$ _____

$-4 \times (-2) =$ _____

$-4 \times (-3) =$ _____

TRY THESE

(i) Starting from $(-5) \times 4$, find $(-5) \times (-6)$

(ii) Starting from $(-6) \times 3$, find $(-6) \times (-7)$

From these patterns we observe that,

$(-3) \times (-1) = 3 = 3 \times 1$

$(-3) \times (-2) = 6 = 3 \times 2$

$(-3) \times (-3) = 9 = 3 \times 3$

and $(-4) \times (-1) = 4 = 4 \times 1$

So, $(-4) \times (-2) = 4 \times 2 =$ _____

$(-4) \times (-3) =$ _____ $=$ _____

So observing these products we can say that the *product of two negative integers is a positive integer. We multiply the two negative integers as whole numbers and put the positive sign before the product.*

Thus, we have $(-10) \times (-12) = 120$

Similarly $(-15) \times (-6) = 90$

In general, for any two positive integers *a* and *b*,

$$(-a) \times (-b) = a \times b$$

TRY THESE Find: $(-31) \times (-100)$, $(-25) \times (-72)$, $(-83) \times (-28)$

Game 1

(i) Take a board marked from –104 to 104 as shown in the figure.

(ii) Take a bag containing two blue and two red dice. Number of dots on the blue dice indicate positive integers and number of dots on the red dice indicate negative integers.

(iii) Every player will place his/her counter at zero.

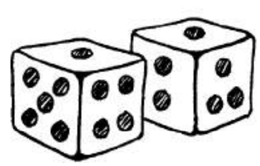
(iv) Each player will take out two dice at a time from the bag and throw them.

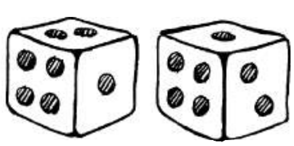
104	103	102	101	100	99	98	97	96	95	94
83	84	85	86	87	88	89	90	91	92	93
82	81	80	79	78	77	76	75	74	73	72
61	62	63	64	65	66	67	68	69	70	71
60	59	58	57	56	55	54	53	52	51	50
39	40	41	42	43	44	45	46	47	48	49
38	37	36	35	34	33	32	31	30	29	28
17	18	19	20	21	22	23	24	25	26	27
16	15	14	13	12	11	10	9	8	7	6
–5	– 4	–3	–2	–1	0	1	2	3	4	5
– 6	–7	– 8	–9	–10	–11	–12	–13	–14	–15	–16
–27	–26	–25	–24	–23	–22	–21	–20	–19	–18	–17
–28	–29	–30	–31	–32	–33	–34	–35	–36	–37	–38
– 49	–48	–47	–46	–45	–44	–43	–42	–41	–40	–39
– 50	–51	–52	–53	–54	–55	–56	–57	–58	–59	– 60
–71	–70	–69	–68	–67	–66	–65	–64	–63	–62	–61
–72	–73	–74	–75	–76	–77	–78	–79	–80	–81	–82
–93	–92	–91	–90	–89	–88	–87	–86	–85	–84	–83
–94	–95	–96	–97	–98	–99	–100	–101	–102	–103	–104

(v) After every throw, the player has to multiply the numbers marked on the dice.

(vi) If the product is a positive integer then the player will move his counter towards 104; if the product is a negative integer then the player will move his counter towards –104.

(vii) The player who reaches either -104 or 104 first is the winner.





1.4.3 Product of three or more Negative Integers

> Euler in his book Ankitung zur Algebra(1770), was one of the first mathematicians to attempt to prove $(-1) \times (-1) = 1$

We observed that the product of two negative integers is a positive integer. What will be the product of three negative integers? Four negative integers? Let us observe the following examples:

(a) $(-4) \times (-3) = 12$

(b) $(-4) \times (-3) \times (-2) = [(-4) \times (-3)] \times (-2) = 12 \times (-2) = -24$

(c) $(-4) \times (-3) \times (-2) \times (-1) = [(-4) \times (-3) \times (-2)] \times (-1) = (-24) \times (-1)$

(d) $(-5) \times [(-4) \times (-3) \times (-2) \times (-1)] = (-5) \times 24 = -120$

From the above products we observe that

(a) the product of two negative integers is a positive integer;

(b) the product of three negative integers is a negative integer.

(c) product of four negative integers is a positive integer.

What is the product of five negative integers in (d)?

So what will be the product of six negative integers?

We further see that in (a) and (c) above, the number of negative integers that are multiplied are even [two and four respectively] and the product obtained in (a) and (c) are positive integers. The number of negative integers that are multiplied in (b) and (d) is odd and the products obtained in (b) and (d) are negative integers.

> **A Special Case**
>
> Consider the following statements and the resultant products:
>
> $(-1) \times (-1) = +1$
>
> $(-1) \times (-1) \times (-1) = -1$
>
> $(-1) \times (-1) \times (-1) \times (-1) = +1$
>
> $(-1) \times (-1) \times (-1) \times (-1) \times (-1) = -1$
>
> This means that if the integer (-1) is multiplied even number of times, the product is $+1$ and if the integer (-1) is multiplied odd number of times, the product is -1. You can check this by making pairs of (-1) in the statement. This is useful in working out products of integers.

We find that *if the number of negative integers in a product is even, then the product is a positive integer; if the number of negative integers in a product is odd, then the product is a negative integer*.

Justify it by taking five more examples of each kind.

Think, Discuss and Write

(i) The product $(-9) \times (-5) \times (-6) \times (-3)$ is positive whereas the product $(-9) \times (-5) \times 6 \times (-3)$ is negative. Why?

(ii) What will be the sign of the product if we multiply together:

(a) 8 negative integers and 3 positive integers?

(b) 5 negative integers and 4 positive integers?

(c) (–1), twelve times?

(d) (–1), 2*m* times, *m* is a natural number?

1.5 Properties of Multiplication of Integers

1.5.1 Closure under Multiplication

1. Observe the following table and complete it:

Statement	Inference
(–20) × (–5) = 100	Product is an integer
(–15) × 17 = – 255	Product is an integer
(–30) × 12 = _____	
(–15) × (–23) = _____	
(–14) × (–13) = _____	
12 × (–30) = _____	

What do you observe? Can you find a pair of integers whose product is not an integer? No. This gives us an idea that the product of two integers is again an integer. So we can say that *integers are closed under multiplication*.

In general,

a × *b* is an integer, for all integers *a* and *b*.

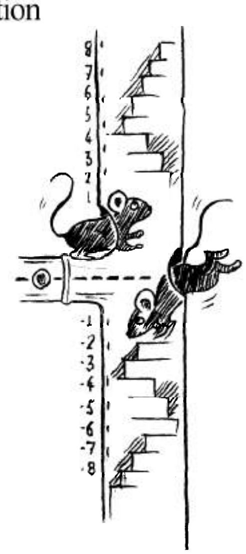
Find the product of five more pairs of integers and verify the above statement.

1.5.2 Commutativity of Multiplication

We know that multiplication is commutative for whole numbers. Can we say, multiplication is also commutative for integers?

Observe the following table and complete it:

Statement 1	Statement 2	Inference
3 × (– 4) = –12	(– 4) × 3 = –12	3 × (– 4) = (– 4) × 3
(–30) × 12 = _____	12 × (–30) = _____	
(–15) × (–10) = 150	(–10) × (–15) = 150	
(–35) × (–12) = _____	(–12) × (–35) =	
(–17) × 0 = _____		
__________ = _____	(–1) × (–15) =	



What are your observations? The above examples suggest *multiplication is commutative for integers*. Write five more such examples and verify.

In general, for any two integers *a* and *b*,

$$a \times b = b \times a$$

1.5.3 Multiplication by Zero

We know that any whole number when multiplied by zero gives zero. Observe the following products of negative integers and zero. These are obtained from the patterns done earlier.

(–3) × 0 = 0

0 × (– 4) = 0

– 5 × 0 = _____

0 × (– 6) = _____

This shows that the product of a negative integer and zero is zero.

In general, for any integer *a*,

$$a \times 0 = 0 \times a = 0$$

1.5.4 Multiplicative Identity

We know that 1 is the multiplicative identity for whole numbers.

Check that 1 is the multiplicative identity for integers as well. Observe the following products of integers with 1.

(–3) × 1 = –3	1 × 5 = 5
(– 4) × 1 = _____	1 × 8 = _____
1 × (–5) = _____	3 × 1 = _____
1 × (– 6) = _____	7 × 1 = _____

This shows that 1 is the multiplicative identity for integers also.

In general, for any integer *a* we have,

$$a \times 1 = 1 \times a = a$$

What happens when we multiply any integer with –1? Complete the following:

(–3) × (–1) = 3

3 × (–1) = –3

(– 6) × (–1) = _____

(–1) × 13 = _____

(–1) × (–25) = _____

18 × (–1) = _____

0 is the additive identity whereas 1 is the multiplicative identity for integers. We get additive inverse of an integer a when we multiply (–1) to a, i.e. $a \times (-1) = (-1) \times a = -a$

What do you observe?

Can we say –1 is a multiplicative identity of integers? No.

1.5.5 Associativity for Multiplication

Consider –3, –2 and 5.

Look at [(–3) × (–2)] × 5 and (–3) × [(–2) × 5].

In the first case (–3) and (–2) are grouped together and in the second (–2) and 5 are grouped together.

We see that [(–3) × (–2)] × 5 = 6 × 5 = 30

and (–3) × [(–2) × 5] = (–3) × (–10) = 30

So, we get the same answer in both the cases.

Thus, [(–3) × (–2)] × 5 = (–3) × [(–2) × 5]

Look at this and complete the products:

[(7) × (– 6)] × 4 = __________ × 4 = __________

7 × [(– 6) × 4] = 7 × __________ = __________

Is [7 × (– 6)] × 4 = 7 × [(– 6) × 4]?

Does the grouping of integers affect the product of integers? No.

In general, for any three integers *a*, *b* and *c*

$$(a \times b) \times c = a \times (b \times c)$$

Take any five values for *a*, *b* and *c* each and verify this property.

Thus, like whole numbers, *the product of three integers does not depend upon the grouping of integers and this is called the associative property for multiplication of integers.*

1.5.6 Distributive Property

We know

16 × (10 + 2) = (16 × 10) + (16 × 2) [Distributivity of multiplication over addition]

Let us check if this is true for integers also.

Observe the following:

(a) (–2) × (3 + 5) = –2 × 8 = –16

and [(–2) × 3] + [(–2) × 5] = (– 6) + (–10) = –16

So, (–2) × (3 + 5) = [(–2) × 3] + [(–2) × 5]

(b) (– 4) × [(–2) + 7] = (– 4) × 5 = –20

and [(– 4) × (–2)] + [(– 4) × 7] = 8 + (–28) = –20

So, (– 4) × [(–2) + 7] = [(– 4) × (–2)] + [(– 4) × 7]

(c) (– 8) × [(–2) + (–1)] = (– 8) × (–3) = 24

and [(– 8) × (–2)] + [(– 8) × (–1)] = 16 + 8 = 24

So, (– 8) × [(–2) + (–1)] = [(– 8) × (–2)] + [(– 8) × (–1)]

Can we say that the distributivity of multiplication over addition is true for integers also? Yes.

In general, for any integers a, b and c,

$$a \times (b + c) = a \times b + a \times c$$

Take atleast five different values for each of a, b and c and verify the above Distributive property.

TRY THESE

(i) Is $10 \times [(6 + (-2)] = 10 \times 6 + 10 \times (-2)$?

(ii) Is $(-15) \times [(-7) + (-1)] = (-15) \times (-7) + (-15) \times (-1)$?

Now consider the following:

Can we say $4 \times (3 - 8) = 4 \times 3 - 4 \times 8$?

Let us check:

$4 \times (3 - 8) = 4 \times (-5) = -20$

$4 \times 3 - 4 \times 8 = 12 - 32 = -20$

So, $4 \times (3 - 8) = 4 \times 3 - 4 \times 8$.

Look at the following:

$(-5) \times [(-4) - (-6)] = (-5) \times 2 = -10$

$[(-5) \times (-4)] - [(-5) \times (-6)] = 20 - 30 = -10$

So, $(-5) \times [(-4) - (-6)] = [(-5) \times (-4)] - [(-5) \times (-6)]$

Check this for $(-9) \times [10 - (-3)]$ and $[(-9) \times 10] - [(-9) \times (-3)]$

You will find that these are also equal.

In general, for any three integers a, b and c,

$$a \times (b - c) = a \times b - a \times c$$

Take atleast five different values for each of a, b and c and verify this property.

TRY THESE

(i) Is $10 \times (6 - (-2)] = 10 \times 6 - 10 \times (-2)$?

(ii) Is $(-15) \times [(-7) - (-1)] = (-15) \times (-7) - (-15) \times (-1)$?

1.5.7 Making Multiplication Easier

Consider the following:

(i) We can find $(-25) \times 37 \times 4$ as

$[(-25) \times 37] \times 4 = (-925) \times 4 = -3700$

Or, we can do it this way,

$(-25) \times 37 \times 4 = (-25) \times 4 \times 37 = [(-25) \times 4] \times 37 = (-100) \times 37 = -3700$

Which is the easier way?

Obviously the second way is easier because multiplication of (–25) and 4 gives –100 which is easier to multiply with 37. Note that the second way involves commutativity and associativity of integers.

So, we find that the commutativity, associativity and distributivity of integers help to make our calculations simpler. Let us further see how calculations can be made easier using these properties.

(ii) Find 16×12

16×12 can be written as $16 \times (10 + 2)$.

$16 \times 12 = 16 \times (10 + 2) = 16 \times 10 + 16 \times 2 = 160 + 32 = 192$

(iii) $(-23) \times 48 = (-23) \times [50 - 2] = (-23) \times 50 - (-23) \times 2 = (-1150) - (-46)$

$= -1104$

(iv) $(-35) \times (-98) = (-35) \times [(-100) + 2] = (-35) \times (-100) + (-35) \times 2$

$= 3500 + (-70) = 3430$

(v) $52 \times (-8) + (-52) \times 2$

$(-52) \times 2$ can also be written as $52 \times (-2)$.

Therefore, $52 \times (-8) + (-52) \times 2 = 52 \times (-8) + 52 \times (-2)$

$= 52 \times [(-8) + (-2)] = 52 \times [(-10)] = -520$

TRY THESE

Find $(-49) \times 18$; $(-25) \times (-31)$; $70 \times (-19) + (-1) \times 70$ using distributive property.

EXAMPLE 2 Find each of the following products:

(i) $(-18) \times (-10) \times 9$ (ii) $(-20) \times (-2) \times (-5) \times 7$

(iii) $(-1) \times (-5) \times (-4) \times (-6)$

SOLUTION

(i) $(-18) \times (-10) \times 9 = [(-18) \times (-10)] \times 9 = 180 \times 9 = 1620$

(ii) $(-20) \times (-2) \times (-5) \times 7 = -20 \times (-2 \times -5) \times 7 = [-20 \times 10] \times 7 = -1400$

(iii) $(-1) \times (-5) \times (-4) \times (-6) = [(-1) \times (-5)] \times [(-4) \times (-6)] = 5 \times 24 = 120$

EXAMPLE 3 Verify $(-30) \times [13 + (-3)] = [(-30) \times 13] + [(-30) \times (-3)]$

SOLUTION $(-30) \times [13 + (-3)] = (-30) \times 10 = -300$

$[(-30) \times 13] + [(-30) \times (-3)] = -390 + 90 = -300$

So, $(-30) \times [13 + (-3)] = [(-30) \times 13] + [(-30) \times (-3)]$

EXAMPLE 4 In a class test containing 15 questions, 4 marks are given for every correct answer and (–2) marks are given for every incorrect answer. (i) Gurpreet attempts all questions but only 9 of her answers are correct. What is her total score? (ii) One of her friends gets only 5 answers correct. What will be her score?

SOLUTION

(i) Marks given for one correct answer = 4

So, marks given for 9 correct answers = $4 \times 9 = 36$

Marks given for one incorrect answer = – 2

So, marks given for 6 = (15 – 9) incorrect answers = $(-2) \times 6 = -12$

Therefore, Gurpreet's total score = 36 + (–12) = 24

(ii) Marks given for one correct answer = 4

So, marks given for 5 correct answers = $4 \times 5 = 20$

Marks given for one incorrect answer = (–2)

So, marks given for 10 (=15 – 5) incorrect answers = $(-2) \times 10 = -20$

Therefore, her friend's total score = 20 + (–20) = 0

EXAMPLE 5 Suppose we represent the distance above the ground by a positive integer and that below the ground by a negative integer, then answer the following:

(i) An elevator descends into a mine shaft at the rate of 5 metre per minute. What will be its position after one hour?

(ii) If it begins to descend from 15 m above the ground, what will be its position after 45 minutes?

SOLUTION

(i) Since the elevator is going down, so the distance covered by it will be represented by a negative integer.

Change in position of the elevator in one minute = – 5 m

Position of the elevator after 60 minutes = $(-5) \times 60 = -300$ m, i.e., 300 m below down from the starting position of elevator.

(ii) Change in position of the elevator in 45 minutes = $(-5) \times 45 = -225$ m, i.e., 225 m below ground level.

So, the final position of the elevator = –225 + 15 = –210 m, i.e., 210 m below ground level.

EXERCISE 1.3

1. Find each of the following products:

(a) $3 \times (-1)$ (b) $(-1) \times 225$

(c) $(-21) \times (-30)$ (d) $(-316) \times (-1)$

(e) $(-15) \times 0 \times (-18)$ (f) $(-12) \times (-11) \times (10)$

(g) $9 \times (-3) \times (-6)$ (h) $(-18) \times (-5) \times (-4)$

(i) $(-1) \times (-2) \times (-3) \times 4$ (j) $(-3) \times (-6) \times (-2) \times (-1)$

2. Verify the following:

(a) $18 \times [7 + (-3)] = [18 \times 7] + [18 \times (-3)]$

(b) $(-21) \times [(-4) + (-6)] = [(-21) \times (-4)] + [(-21) \times (-6)]$

3. (i) For any integer a, what is $(-1) \times a$ equal to?

(ii) Determine the integer whose product with (-1) is

(a) -22 (b) 37 (c) 0

4. Starting from $(-1) \times 5$, write various products showing some pattern to show $(-1) \times (-1) = 1$.

5. Find the product, using suitable properties:

(a) $26 \times (-48) + (-48) \times (-36)$ (b) $8 \times 53 \times (-125)$

(c) $15 \times (-25) \times (-4) \times (-10)$ (d) $(-41) \times 102$

(e) $625 \times (-35) + (-625) \times 65$ (f) $7 \times (50 - 2)$

(g) $(-17) \times (-29)$ (h) $(-57) \times (-19) + 57$

6. A certain freezing process requires that room temperature be lowered from 40°C at the rate of 5°C every hour. What will be the room temperature 10 hours after the process begins?

7. In a class test containing 10 questions, 5 marks are awarded for every correct answer and (–2) marks are awarded for every incorrect answer and 0 for questions not attempted.

(i) Mohan gets four correct and six incorrect answers. What is his score?

(ii) Reshma gets five correct answers and five incorrect answers, what is her score?

(iii) Heena gets two correct and five incorrect answers out of seven questions she attempts. What is her score?

8. A cement company earns a profit of ₹ 8 per bag of white cement sold and a loss of ₹ 5 per bag of grey cement sold.

(a) The company sells 3,000 bags of white cement and 5,000 bags of grey cement in a month. What is its profit or loss?

(b) What is the number of white cement bags it must sell to have neither profit nor loss, if the number of grey bags sold is 6,400 bags.

9. Replace the blank with an integer to make it a true statement.

(a) (–3) × _____ = 27 (b) 5 × _____ = –35

(c) _____ × (– 8) = –56 (d) _____ × (–12) = 132

1.6 Division of Integers

We know that division is the inverse operation of multiplication. Let us see an example for whole numbers.

Since 3 × 5 = 15

So 15 ÷ 5 = 3 and 15 ÷ 3 = 5

Similarly, 4 × 3 = 12 gives 12 ÷ 4 = 3 and 12 ÷ 3 = 4

We can say for each multiplication statement of whole numbers there are two division statements.

Can you write multiplication statement and its corresponding divison statements for integers?

- Observe the following and complete it.

Multiplication Statement	Corresponding Division Statements	
2 × (– 6) = (–12)	(–12) ÷ (– 6) = 2 ,	(–12) ÷ 2 = (– 6)
(– 4) × 5 = (–20)	(–20) ÷ 5 = (– 4) ,	(–20) ÷ (– 4) = 5
(– 8) × (–9) = 72	72 ÷ _____ = _____ ,	72 ÷ _____ = _____
(–3) × (–7) = _____	_____ ÷ (–3) = _____ ,	_____________
(– 8) × 4 = _____	_____________ ,	_____________
5 × (– 9) = _____	_____________ ,	_____________
(–10) × (–5) =	_____________ ,	_____________

From the above we observe that :

(–12) ÷ 2 = (– 6)

(–20) ÷ 5 = (– 4)

(–32) ÷ 4 = (– 8)

(– 45) ÷ 5 = (– 9)

Try These

Find:

(a) (–100) ÷ 5 (b) (–81) ÷ 9

(c) (–75) ÷ 5 (d) (–32) ÷ 2

We observe that *when we divide a negative integer by a positive integer, we divide them as whole numbers and then put a minus sign (–) before the quotient. We, thus, get a negative integer*.

- We also observe that:

  72 ÷ (–8) = –9 and 50 ÷ (–10) = –5

  72 ÷ (–9) = – 8 50 ÷ (–5) = –10

So we can say that *when we divide a positive integer by a negative integer, we first divide them as whole numbers and then put a minus sign (–) before the quotient. That is, we get a negative integer.*

In general, for any two positive integers a and b

$$a \div (-b) = (-a) \div b \quad \text{where } b \neq 0$$

Can we say that

(– 48) ÷ 8 = 48 ÷ (– 8)?

Let us check. We know that

(– 48) ÷ 8 = – 6

and 48 ÷ (– 8) = – 6

So (– 48) ÷ 8 = 48 ÷ (– 8)

Check this for

(i) 90 ÷ (– 45) and (–90) ÷ 45

(ii) (–136) ÷ 4 and 136 ÷ (– 4)

TRY THESE

Find: (a) 125 ÷ (–25) (b) 80 ÷ (–5) (c) 64 ÷ (–16)

- Lastly, we observe that

  (–12) ÷ (– 6) = 2; (–20) ÷ (– 4) = 5; (–32) ÷ (– 8) = 4; (– 45) ÷ (–9) = 5

So, we can say that when we divide a negative integer by a negative integer, we first divide them as whole numbers and then put a positive sign (+). That is, we get a positive integer.

In general, for any two positive integers a and b

$$(-a) \div (-b) = a \div b \quad \text{where } b \neq 0$$

TRY THESE

Find: (a) (–36) ÷ (– 4) (b) (–201) ÷ (–3) (c) (–325) ÷ (–13)

1.7 PROPERTIES OF DIVISION OF INTEGERS

Observe the following table and complete it:

Statement	Inference	Statement	Inference
(– 8) ÷ (– 4) = 2	Result is an integer	$(-8) \div 3 = \frac{-8}{3}$	________________
$(-4) \div (-8) = \frac{-4}{-8}$	Result is not an integer	$3 \div (-8) = \frac{3}{-8}$	________________

What do you observe? We observe that integers are not closed under division.

Justify it by taking five more examples of your own.

- We know that division is not commutative for whole numbers. Let us check it for integers also.

You can see from the table that $(-8) \div (-4) \neq (-4) \div (-8)$.

Is $(-9) \div 3$ the same as $3 \div (-9)$?

Is $(-30) \div (-6)$ the same as $(-6) \div (-30)$?

Can we say that division is commutative for integers? No.

You can verify it by taking five more pairs of integers.

- Like whole numbers, any integer divided by zero is meaningless and zero divided by an integer other than zero is equal to zero i.e., *for any integer a,* $a \div 0$ *is not defined but* $0 \div a = 0$ *for* $a \neq 0$.

- When we divide a whole number by 1 it gives the same whole number. Let us check whether it is true for negative integers also.

  Observe the following :

$(-8) \div 1 = (-8)$	$(-11) \div 1 = -11$	$(-13) \div 1 = -13$
$(-25) \div 1 =$ ____	$(-37) \div 1 =$ ____	$(-48) \div 1 =$ ____

  This shows that negative integer divided by 1 gives the same negative integer. So, *any integer divided by 1 gives the same integer.*

  In general, for any integer a,

$$a \div 1 = a$$

- What happens when we divide any integer by (-1)? Complete the following table

$(-8) \div (-1) = 8$	$11 \div (-1) = -11$	$13 \div (-1) =$ ____
$(-25) \div (-1) =$ ____	$(-37) \div (-1) =$ ____	$-48 \div (-1) =$ ____

  What do you observe?

  We can say that if any integer is divided by (-1) it does not give the same integer.

TRY THESE

Is (i) $1 \div a = 1$?

(ii) $a \div (-1) = -a$? for any integer a.

Take different values of a and check.

- Can we say $[(-16) \div 4] \div (-2)$ is the same as $(-16) \div [4 \div (-2)]$?

  We know that $[(-16) \div 4] \div (-2) = (-4) \div (-2) = 2$

  and $(-16) \div [4 \div (-2)] = (-16) \div (-2) = 8$

  So $[(-16) \div 4] \div (-2) \neq (-16) \div [4 \div (-2)]$

  Can you say that division is associative for integers? No.

  Verify it by taking five more examples of your own.

EXAMPLE 6 In a test (+5) marks are given for every correct answer and (–2) marks are given for every incorrect answer. (i) Radhika answered all the questions and scored 30 marks though she got 10 correct answers. (ii) Jay also answered all the questions and scored (–12) marks though he got 4 correct answers. How many incorrect answers had they attempted?

Solution

(i) Marks given for one correct answer = 5

So, marks given for 10 correct answers = 5 × 10 = 50

Radhika's score = 30

Marks obtained for incorrect answers = 30 – 50 = – 20

Marks given for one incorrect answer = (–2)

Therefore, number of incorrect answers = (–20) ÷ (–2) = 10

(ii) Marks given for 4 correct answers = 5 × 4 = 20

Jay's score = –12

Marks obtained for incorrect answers = –12 – 20 = – 32

Marks given for one incorrect answer = (–2)

Therefore number of incorrect answers = (–32) ÷ (–2) = 16

Example 7 A shopkeeper earns a profit of ₹ 1 by selling one pen and incurs a loss of 40 paise per pencil while selling pencils of her old stock.

(i) In a particular month she incurs a loss of ₹ 5. In this period, she sold 45 pens. How many pencils did she sell in this period?

(ii) In the next month she earns neither profit nor loss. If she sold 70 pens, how many pencils did she sell?

Solution

(i) Profit earned by selling one pen = ₹ 1

Profit earned by selling 45 pens = ₹ 45, which we denote by + ₹ 45

Total loss given = ₹ 5, which we denote by – ₹ 5

Profit earned + Loss incurred = Total loss

Therefore, Loss incurred = Total Loss – Profit earned

= ₹ (– 5 – 45) = ₹ (–50) = –5000 paise

Loss incurred by selling one pencil = 40 paise which we write as –40 paise

So, number of pencils sold = (–5000) ÷ (– 40) = 125 pencils.

(ii) In the next month there is neither profit nor loss.

So, Profit earned + Loss incurred = 0

i.e., Profit earned = – Loss incurred.

Now, profit earned by selling 70 pens = ₹ 70

Hence, loss incurred by selling pencils = ₹ 70 which we indicate by – ₹ 70 or – 7,000 paise.

Total number of pencils sold = (–7000) ÷ (– 40) = 175 pencils.

Exercise 1.4

1. Evaluate each of the following:

    (a) (–30) ÷ 10 (b) 50 ÷ (–5) (c) (–36) ÷ (–9)

    (d) (– 49) ÷ (49) (e) 13 ÷ [(–2) + 1] (f) 0 ÷ (–12)

    (g) (–31) ÷ [(–30) + (–1)]

    (h) [(–36) ÷ 12] ÷ 3 (i) [(– 6) + 5)] ÷ [(–2) + 1]

2. Verify that $a \div (b + c) \neq (a \div b) + (a \div c)$ for each of the following values of a, b and c.

    (a) $a = 12, b = -4, c = 2$ (b) $a = (-10), b = 1, c = 1$

3. Fill in the blanks:

    (a) 369 ÷ ______ = 369 (b) (–75) ÷ ______ = –1

    (c) (–206) ÷ ______ = 1 (d) – 87 ÷ ______ = 87

    (e) ______ ÷ 1 = – 87 (f) ______ ÷ 48 = –1

    (g) 20 ÷ ______ = –2 (h) ______ ÷ (4) = –3

4. Write five pairs of integers (a, b) such that $a \div b = -3$. One such pair is (6, –2) because 6 ÷ (–2) = (–3).

5. The temperature at 12 noon was 10°C above zero. If it decreases at the rate of 2°C per hour until midnight, at what time would the temperature be 8°C below zero? What would be the temperature at mid-night?

6. In a class test (+ 3) marks are given for every correct answer and (–2) marks are given for every incorrect answer and no marks for not attempting any question. (i) Radhika scored 20 marks. If she has got 12 correct answers, how many questions has she attempted incorrectly? (ii) Mohini scores –5 marks in this test, though she has got 7 correct answers. How many questions has she attempted incorrectly?

7. An elevator descends into a mine shaft at the rate of 6 m/min. If the descent starts from 10 m above the ground level, how long will it take to reach – 350 m.

What have We Discussed?

1. Integers are a bigger collection of numbers which is formed by whole numbers and their negatives. These were introduced in Class VI.

2. You have studied in the earlier class, about the representation of integers on the number line and their addition and subtraction.

3. We now study the properties satisfied by addition and subtraction.

   (a) Integers are closed for addition and subtraction both. That is, $a + b$ and $a - b$ are again integers, where a and b are any integers.

   (b) Addition is commutative for integers, i.e., $a + b = b + a$ for all integers a and b.

   (c) Addition is associative for integers, i.e., $(a + b) + c = a + (b + c)$ for all integers a, b and c.

   (d) Integer 0 is the identity under addition. That is, $a + 0 = 0 + a = a$ for every integer a.

4. We studied, how integers could be multiplied, and found that product of a positive and a negative integer is a negative integer, whereas the product of two negative integers is a positive integer. For example, $-2 \times 7 = -14$ and $-3 \times -8 = 24$.

5. Product of even number of negative integers is positive, whereas the product of odd number of negative integers is negative.

6. Integers show some properties under multiplication.

   (a) Integers are closed under multiplication. That is, $a \times b$ is an integer for any two integers a and b.

   (b) Multiplication is commutative for integers. That is, $a \times b = b \times a$ for any integers a and b.

   (c) The integer 1 is the identity under multiplication, i.e., $1 \times a = a \times 1 = a$ for any integer a.

   (d) Multiplication is associative for integers, i.e., $(a \times b) \times c = a \times (b \times c)$ for any three integers a, b and c.

7. Under addition and multiplication, integers show a property called distributive property. That is, $a \times (b + c) = a \times b + a \times c$ for any three integers a, b and c.

8. The properties of commutativity, associativity under addition and multiplication, and the distributive property help us to make our calculations easier.
9. We also learnt how to divide integers. We found that,
   (a) When a positive integer is divided by a negative integer, the quotient obtained is a negative integer and vice-versa.
   (b) Division of a negative integer by another negative integer gives a positive integer as quotient.
10. For any integer a, we have
   (a) $a \div 0$ is not defined
   (b) $a \div 1 = a$

Fractions and Decimals

Chapter 2

2.1 Introduction

You have learnt fractions and decimals in earlier classes. The study of fractions included proper, improper and mixed fractions as well as their addition and subtraction. We also studied comparison of fractions, equivalent fractions, representation of fractions on the number line and ordering of fractions.

Our study of decimals included, their comparison, their representation on the number line and their addition and subtraction.

We shall now learn multiplication and division of fractions as well as of decimals.

2.2 How Well have You Learnt about Fractions?

A **proper fraction** is a fraction that represents a part of a whole. Is $\frac{7}{4}$ a proper fraction? Which is bigger, the numerator or the denominator?

An **improper fraction** is a combination of whole and a proper fraction. Is $\frac{7}{4}$ an improper fraction? Which is bigger here, the numerator or the denominator?

The improper fraction $\frac{7}{4}$ can be written as $1\frac{3}{4}$. This is a **mixed fraction**.

Can you write five examples each of proper, improper and mixed fractions?

Example 1 Write five equivalent fractions of $\frac{3}{5}$.

Solution One of the equivalent fractions of $\frac{3}{5}$ is

$\frac{3}{5} = \frac{3 \times 2}{5 \times 2} = \frac{6}{10}$. Find the other four.

EXAMPLE 2 Ramesh solved $\frac{2}{7}$ part of an exercise while Seema solved $\frac{4}{5}$ of it. Who solved lesser part?

SOLUTION In order to find who solved lesser part of the exercise, let us compare $\frac{2}{7}$ and $\frac{4}{5}$.

Converting them to like fractions we have, $\frac{2}{7}=\frac{10}{35}$, $\frac{4}{5}=\frac{28}{35}$.

Since $10 < 28$, so $\frac{10}{35} < \frac{28}{35}$.

Thus, $\frac{2}{7} < \frac{4}{5}$.

Ramesh solved lesser part than Seema.

EXAMPLE 3 Sameera purchased $3\frac{1}{2}$ kg apples and $4\frac{3}{4}$ kg oranges. What is the total weight of fruits purchased by her?

SOLUTION The total weight of the fruits $=\left(3\frac{1}{2}+4\frac{3}{4}\right)$ kg

$$=\left(\frac{7}{2}+\frac{19}{4}\right)\text{kg}=\left(\frac{14}{4}+\frac{19}{4}\right)\text{kg}$$

$$=\frac{33}{4}\text{kg}=8\frac{1}{4}\text{kg}$$

EXAMPLE 4 Suman studies for $5\frac{2}{3}$ hours daily. She devotes $2\frac{4}{5}$ hours of her time for Science and Mathematics. How much time does she devote for other subjects?

SOLUTION Total time of Suman's study $=5\frac{2}{3}\text{ h}=\frac{17}{3}\text{ h}$

Time devoted by her for Science and Mathematics $=2\frac{4}{5}=\frac{14}{5}\text{ h}$

Thus, time devoted by her for other subjects = $\left(\frac{17}{3}-\frac{14}{5}\right)$ h

$$= \left(\frac{17\times 5}{15}-\frac{14\times 3}{15}\right)\text{h} = \left(\frac{85-42}{15}\right)\text{h}$$

$$= \frac{43}{15}\text{ h} = 2\frac{13}{15}\text{ h}$$

EXERCISE 2.1

1. Solve:

(i) $2-\frac{3}{5}$ (ii) $4+\frac{7}{8}$ (iii) $\frac{3}{5}+\frac{2}{7}$ (iv) $\frac{9}{11}-\frac{4}{15}$

(v) $\frac{7}{10}+\frac{2}{5}+\frac{3}{2}$ (vi) $2\frac{2}{3}+3\frac{1}{2}$ (vii) $8\frac{1}{2}-3\frac{5}{8}$

2. Arrange the following in descending order:

(i) $\frac{2}{9},\frac{2}{3},\frac{8}{21}$ (ii) $\frac{1}{5},\frac{3}{7},\frac{7}{10}$.

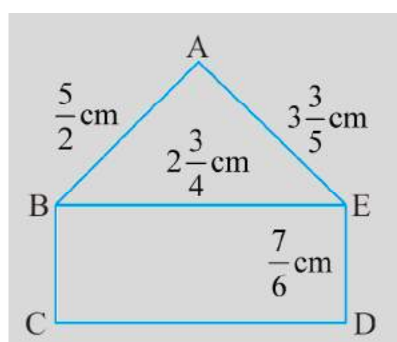
3. In a "magic square", the sum of the numbers in each row, in each column and along the diagonals is the same. Is this a magic square?

$\frac{4}{11}$	$\frac{9}{11}$	$\frac{2}{11}$
$\frac{3}{11}$	$\frac{5}{11}$	$\frac{7}{11}$
$\frac{8}{11}$	$\frac{1}{11}$	$\frac{6}{11}$

(Along the first row $\frac{4}{11}+\frac{9}{11}+\frac{2}{11}=\frac{15}{11}$).

4. A rectangular sheet of paper is $12\frac{1}{2}$ cm long and $10\frac{2}{3}$ cm wide. Find its perimeter.

5. Find the perimeters of (i) ΔABE (ii) the rectangle BCDE in this figure. Whose perimeter is greater?



6. Salil wants to put a picture in a frame. The picture is $7\frac{3}{5}$ cm wide.

To fit in the frame the picture cannot be more than $7\frac{3}{10}$ cm wide. How much should the picture be trimmed?

7. Ritu ate $\frac{3}{5}$ part of an apple and the remaining apple was eaten by her brother Somu. How much part of the apple did Somu eat? Who had the larger share? By how much?

8. Michael finished colouring a picture in $\frac{7}{12}$ hour. Vaibhav finished colouring the same picture in $\frac{3}{4}$ hour. Who worked longer? By what fraction was it longer?

2.3 Multiplication of Fractions

You know how to find the area of a rectangle. It is equal to length × breadth. If the length and breadth of a rectangle are 7 cm and 4 cm respectively, then what will be its area? Its area would be $7 \times 4 = 28$ cm^2.

What will be the area of the rectangle if its length and breadth are $7\frac{1}{2}$ cm and $3\frac{1}{2}$ cm respectively? You will say it will be $7\frac{1}{2} \times 3\frac{1}{2} = \frac{15}{2} \times \frac{7}{2}$ cm^2. The numbers $\frac{15}{2}$ and $\frac{7}{2}$ are fractions. To calculate the area of the given rectangle, we need to know how to multiply fractions. We shall learn that now.

2.3.1 Multiplication of a Fraction by a Whole Number

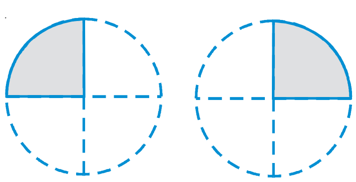


Fig 2.1

Observe the pictures at the left (Fig 2.1). Each shaded part is $\frac{1}{4}$ part of a circle. How much will the two shaded parts represent together? They will represent $\frac{1}{4} + \frac{1}{4} = 2 \times \frac{1}{4}$.

Combining the two shaded parts, we get Fig 2.2 . What part of a circle does the shaded part in Fig 2.2 represent? It represents $\frac{2}{4}$ part of a circle .

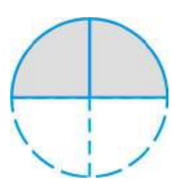


Fig 2.2

The shaded portions in Fig 2.1 taken together are the same as the shaded portion in Fig 2.2, i.e., we get Fig 2.3.

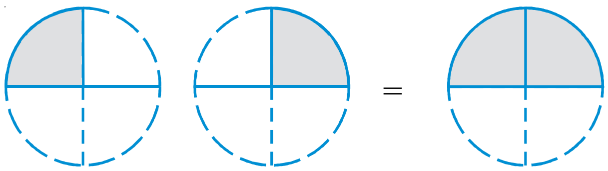


Fig 2.3

or $2 \times \frac{1}{4} = \frac{2}{4}$.

Can you now tell what this picture will represent? (Fig 2.4)

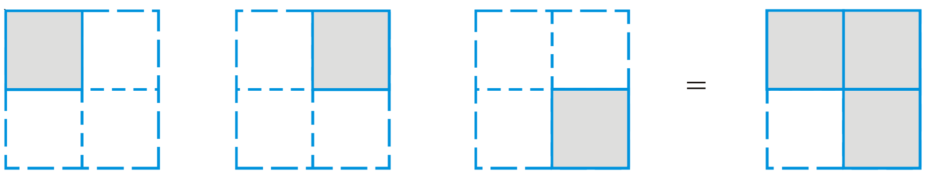


Fig 2.4

And this? (Fig 2.5)

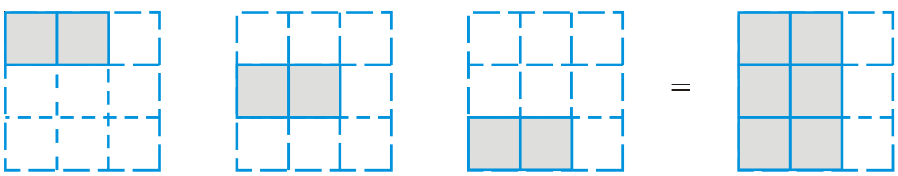


Fig 2.5

Let us now find $3 \times \frac{1}{2}$.

We have $3 \times \frac{1}{2} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{3}{2}$

We also have $\frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{1+1+1}{2} = \frac{3 \times 1}{2} = \frac{3}{2}$

So $3 \times \frac{1}{2} = \frac{3 \times 1}{2} = \frac{3}{2}$

Similarly $\frac{2}{3} \times 5 = \frac{2 \times 5}{3} = ?$

Can you tell $3 \times \frac{2}{7} = ?$ $4 \times \frac{3}{5} = ?$

The fractions that we considered till now, i.e., $\frac{1}{2}, \frac{2}{3}, \frac{2}{7}$ and $\frac{3}{5}$ were proper fractions.

For improper fractions also we have,

$$2 \times \frac{5}{3} = \frac{2 \times 5}{3} = \frac{10}{3}$$

Try, $3 \times \frac{8}{7} = ?$ $4 \times \frac{7}{5} = ?$

Thus, *to multiply a whole number with a proper or an improper fraction, we multiply the whole number with the numerator of the fraction, keeping the denominator same.*

TRY THESE

1. Find: (a) $\frac{2}{7} \times 3$ (b) $\frac{9}{7} \times 6$ (c) $3 \times \frac{1}{8}$ (d) $\frac{13}{11} \times 6$

If the product is an improper fraction express it as a mixed fraction.

2. Represent pictorially : $2 \times \frac{2}{5} = \frac{4}{5}$

TRY THESE

Find: (i) $5 \times 2\frac{3}{7}$

(ii) $1\frac{4}{9} \times 6$

To multiply a mixed fraction to a whole number, first convert the mixed fraction to an improper fraction and then multiply.

Therefore, $3 \times 2\frac{5}{7} = 3 \times \frac{19}{7} = \frac{57}{7} = 8\frac{1}{7}$.

Similarly, $2 \times 4\frac{2}{5} = 2 \times \frac{22}{5} = ?$

Fraction as an operator 'of'

Observe these figures (Fig 2.6)

The two squares are exactly similar.

Each shaded portion represents $\frac{1}{2}$ of 1.

So, both the shaded portions together will represent $\frac{1}{2}$ of 2.

Combine the 2 shaded $\frac{1}{2}$ parts. It represents 1.

So, we say $\frac{1}{2}$ of 2 is 1. We can also get it as $\frac{1}{2} \times 2 = 1$.

Thus, $\frac{1}{2}$ of $2 = \frac{1}{2} \times 2 = 1$

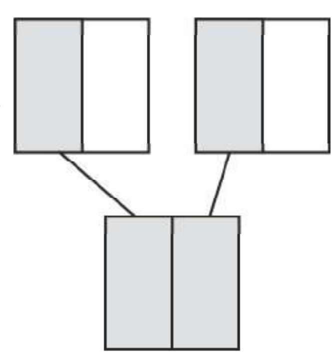


Fig 2.6

Also, look at these similar squares (Fig 2.7).

Each shaded portion represents $\frac{1}{2}$ of 1.

So, the three shaded portions represent $\frac{1}{2}$ of 3.

Combine the 3 shaded parts.

It represents $1\frac{1}{2}$ i.e., $\frac{3}{2}$.

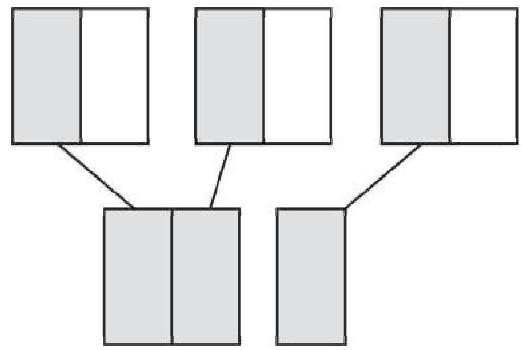


Fig 2.7

So, $\frac{1}{2}$ of 3 is $\frac{3}{2}$. Also, $\frac{1}{2} \times 3 = \frac{3}{2}$.

Thus, $\frac{1}{2}$ of $3 = \frac{1}{2} \times 3 = \frac{3}{2}$.

So we see that **'of' represents multiplication.**

Farida has 20 marbles. Reshma has $\frac{1}{5}$th of the number of marbles what Farida has. How many marbles Reshma has? As, 'of ' indicates multiplication, so, Reshma has $\frac{1}{5} \times 20 = 4$ marbles.

Similarly, we have $\frac{1}{2}$ of 16 is $\frac{1}{2} \times 16 = \frac{16}{2} = 8$.

TRY THESE

Can you tell, what is (i) $\frac{1}{2}$ of 10?, (ii) $\frac{1}{4}$ of 16?, (iii) $\frac{2}{5}$ of 25?

EXAMPLE 5 In a class of 40 students $\frac{1}{5}$ of the total number of studetns like to study English, $\frac{2}{5}$ of the total number like to study Mathematics and the remaining students like to study Science.

(i) How many students like to study English?

(ii) How many students like to study Mathematics?

(iii) What fraction of the total number of students like to study Science?

SOLUTION Total number of students in the class = 40.

(i) Of these $\frac{1}{5}$ of the total number of students like to study English.

Thus, the number of students who like to study English $= \frac{1}{5}$ of $40 = \frac{1}{5} \times 40 = 8$.

(ii) Try yourself.

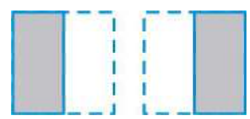
(iii) The number of students who like English and Mathematics $= 8 + 16 = 24$. Thus, the number of students who like Science $= 40 - 24 = 16$.

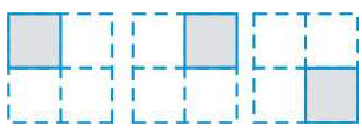
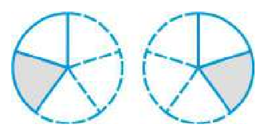
Thus, the required fraction is $\frac{16}{40}$.

EXERCISE 2.2

1. Which of the drawings (a) to (d) show :

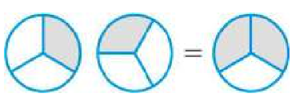
(i) $2 \times \frac{1}{5}$ (ii) $2 \times \frac{1}{2}$ (iii) $3 \times \frac{2}{3}$ (iv) $3 \times \frac{1}{4}$

(a)  (b) 

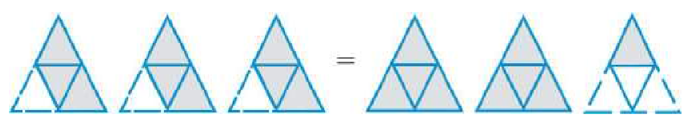
(c)  (d) 

2. Some pictures (a) to (c) are given below. Tell which of them show:

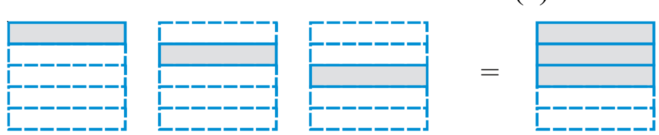
(i) $3 \times \frac{1}{5} = \frac{3}{5}$ (ii) $2 \times \frac{1}{3} = \frac{2}{3}$ (iii) $3 \times \frac{3}{4} = 2\frac{1}{4}$



(a)



(b)



(c)

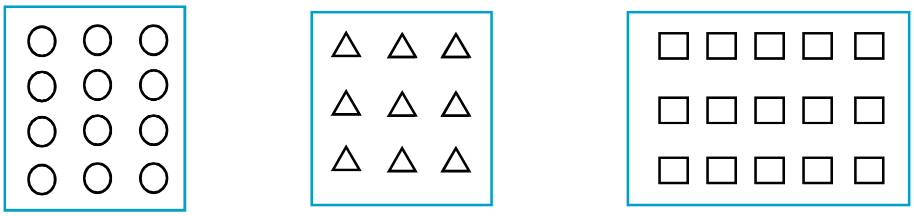
3. Multiply and reduce to lowest form and convert into a mixed fraction:

(i) $7 \times \frac{3}{5}$ (ii) $4 \times \frac{1}{3}$ (iii) $2 \times \frac{6}{7}$ (iv) $5 \times \frac{2}{9}$ (v) $\frac{2}{3} \times 4$

(vi) $\frac{5}{2} \times 6$ (vii) $11 \times \frac{4}{7}$ (viii) $20 \times \frac{4}{5}$ (ix) $13 \times \frac{1}{3}$ (x) $15 \times \frac{3}{5}$

4. Shade: (i) $\frac{1}{2}$ of the circles in box (a) (ii) $\frac{2}{3}$ of the triangles in box (b)

(iii) $\frac{3}{5}$ of the squares in box (c).



(a) (b) (c)

5. Find:

(a) $\frac{1}{2}$ of (i) 24 (ii) 46 (b) $\frac{2}{3}$ of (i) 18 (ii) 27

(c) $\frac{3}{4}$ of (i) 16 (ii) 36 (d) $\frac{4}{5}$ of (i) 20 (ii) 35

6. Multiply and express as a mixed fraction :

(a) $3 \times 5\frac{1}{5}$ (b) $5 \times 6\frac{3}{4}$ (c) $7 \times 2\frac{1}{4}$

(d) $4 \times 6\frac{1}{3}$ (e) $3\frac{1}{4} \times 6$ (f) $3\frac{2}{5} \times 8$

7. Find: (a) $\frac{1}{2}$ of (i) $2\frac{3}{4}$ (ii) $4\frac{2}{9}$ (b) $\frac{5}{8}$ of (i) $3\frac{5}{6}$ (ii) $9\frac{2}{3}$

8. Vidya and Pratap went for a picnic. Their mother gave them a water bottle that contained 5 litres of water. Vidya consumed $\frac{2}{5}$ of the water. Pratap consumed the remaining water.

(i) How much water did Vidya drink?

(ii) What fraction of the total quantity of water did Pratap drink?

2.3.2 Multiplication of a Fraction by a Fraction

Farida had a 9 cm long strip of ribbon. She cut this strip into four equal parts. How did she do it? She folded the strip twice. What fraction of the total length will each part represent?

Each part will be $\frac{9}{4}$ of the strip. She took one part and divided it in two equal parts by folding the part once. What will one of the pieces represent? It will represent $\frac{1}{2}$ of $\frac{9}{4}$ or $\frac{1}{2} \times \frac{9}{4}$.

Let us now see how to find the product of two fractions like $\frac{1}{2} \times \frac{9}{4}$.

To do this we first learn to find the products like $\frac{1}{2} \times \frac{1}{3}$.

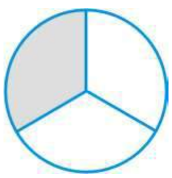


Fig 2.8

(a) How do we find $\frac{1}{3}$ of a whole? We divide the whole in three equal parts. Each of the three parts represents $\frac{1}{3}$ of the whole. Take one part of these three parts, and shade it as shown in Fig 2.8.

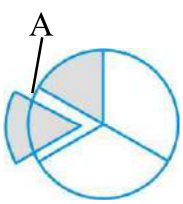


Fig 2.9

(b) How will you find $\frac{1}{2}$ of this shaded part? Divide this one-third ($\frac{1}{3}$) shaded part into two equal parts. Each of these two parts represents $\frac{1}{2}$ of $\frac{1}{3}$ i.e., $\frac{1}{2} \times \frac{1}{3}$ (Fig 2.9).

Take out 1 part of these two and name it 'A'. 'A' represents $\frac{1}{2} \times \frac{1}{3}$.

(c) What fraction is 'A' of the whole? For this, divide each of the remaining $\frac{1}{3}$ parts also in two equal parts. How many such equal parts do you have now?

There are six such equal parts. 'A' is one of these parts.

So, 'A' is $\frac{1}{6}$ of the whole. Thus, $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$.

How did we decide that 'A' was $\frac{1}{6}$ of the whole? The whole was divided in $6 = 2 \times 3$ parts and $1 = 1 \times 1$ part was taken out of it.

Thus, $$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6} = \frac{1 \times 1}{2 \times 3}$$

or $$\frac{1}{2} \times \frac{1}{3} = \frac{1 \times 1}{2 \times 3}$$

The value of $\frac{1}{3}\times\frac{1}{2}$ can be found in a similar way. Divide the whole into two equal parts and then divide one of these parts in three equal parts. Take one of these parts. This will represent $\frac{1}{3}\times\frac{1}{2}$ i.e., $\frac{1}{6}$.

Therefore $\frac{1}{3}\times\frac{1}{2}=\frac{1}{6}=\frac{1\times 1}{3\times 2}$ as discussed earlier.

Hence $\frac{1}{2}\times\frac{1}{3}=\frac{1}{3}\times\frac{1}{2}=\frac{1}{6}$

Find $\frac{1}{3}\times\frac{1}{4}$ and $\frac{1}{4}\times\frac{1}{3}$; $\frac{1}{2}\times\frac{1}{5}$ and $\frac{1}{5}\times\frac{1}{2}$ and check whether you get

$$\frac{1}{3}\times\frac{1}{4}=\frac{1}{4}\times\frac{1}{3};\ \frac{1}{2}\times\frac{1}{5}=\frac{1}{5}\times\frac{1}{2}$$

TRY THESE

Fill in these boxes:

(i) $\frac{1}{2}\times\frac{1}{7}=\frac{1\times 1}{2\times 7}=\square$

(ii) $\frac{1}{5}\times\frac{1}{7}=\square=\square$

(iii) $\frac{1}{7}\times\frac{1}{2}=\square=\square$

(iv) $\frac{1}{7}\times\frac{1}{5}=\square=\square$

EXAMPLE 6 Sushant reads $\frac{1}{3}$ part of a book in 1 hour. How much part of the book will he read in $2\frac{1}{5}$ hours?

SOLUTION The part of the book read by Sushant in 1 hour $=\frac{1}{3}$.

So, the part of the book read by him in $2\frac{1}{5}$ hours $=2\frac{1}{5}\times\frac{1}{3}$

$$=\frac{11}{5}\times\frac{1}{3}=\frac{11\times 1}{5\times 3}=\frac{11}{15}$$

Let us now find $\frac{1}{2}\times\frac{5}{3}$. We know that $\frac{5}{3}=\frac{1}{3}\times 5$.

So, $\frac{1}{2}\times\frac{5}{3}=\frac{1}{2}\times\frac{1}{3}\times 5=\frac{1}{6}\times 5=\frac{5}{6}$

Also, $\frac{5}{6} = \frac{1 \times 5}{2 \times 3}$. Thus, $\frac{1}{2} \times \frac{5}{3} = \frac{1 \times 5}{2 \times 3} = \frac{5}{6}$.

This is also shown by the figures drawn below. Each of these five equal shapes (Fig 2.10) are parts of five similar circles. Take one such shape. To obtain this shape we first divide a circle in three equal parts. Further divide each of these three parts in two equal parts. One part out of it is the shape we considered. What will it represent?

It will represent $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$. The total of such parts would be $5 \times \frac{1}{6} = \frac{5}{6}$.

Fig 2.10

TRY THESE

Find: $\frac{1}{3} \times \frac{4}{5}$; $\frac{2}{3} \times \frac{1}{5}$

Similarly $\frac{3}{5} \times \frac{1}{7} = \frac{3 \times 1}{5 \times 7} = \frac{3}{35}$.

We can thus find $\frac{2}{3} \times \frac{7}{5}$ as $\frac{2}{3} \times \frac{7}{5} = \frac{2 \times 7}{3 \times 5} = \frac{14}{15}$.

So, we find that we multiply two fractions as $\frac{\text{Product of Numerators}}{\text{Product of Denominators}}$.

TRY THESE

Find: $\frac{8}{3} \times \frac{4}{7}$; $\frac{3}{4} \times \frac{2}{3}$.

Value of the Products

You have seen that the product of two whole numbers is bigger than each of the two whole numbers. For example, $3 \times 4 = 12$ and $12 > 4$, $12 > 3$. What happens to the value of the product when we multiply two fractions?

Let us first consider the product of two proper fractions.

We have,

$\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$	$\frac{8}{15} < \frac{2}{3}, \frac{8}{15} < \frac{4}{5}$	Product is less than each of the fractions
$\frac{1}{5} \times \frac{2}{7}$ = ---------	--------,--------	--------------------------------------
$\frac{3}{5} \times \frac{\square}{8} = \frac{21}{40}$	--------,--------	--------------------------------------
$\frac{2}{\square} \times \frac{4}{9} = \frac{8}{45}$	--------,--------	--------------------------------------

You will find that *when two proper fractions are multiplied, the product is less than each of the fractions*. Or, *we say the value of the product of two proper fractions is smaller than each of the two fractions.*

Check this by constructing five more examples.

Let us now multiply two improper fractions.

$\frac{7}{3} \times \frac{5}{2} = \frac{35}{6}$	$\frac{35}{6} > \frac{7}{3}, \frac{35}{6} > \frac{5}{2}$	Product is greater than each of the fractions
$\frac{6}{5} \times \frac{\square}{3} = \frac{24}{15}$	--------,--------	--
$\frac{9}{2} \times \frac{7}{\square} = \frac{63}{8}$	--------,--------	--
$\frac{3}{\square} \times \frac{8}{7} = \frac{24}{14}$	--------,--------	--

We find that *the product of two improper fractions is greater than each of the two fractions.*

Or, *the value of the product of two improper fractions is more than each of the two fractions*.

Construct five more examples for yourself and verify the above statement.

Let us now multiply a proper and an improper fraction, say $\frac{2}{3}$ and $\frac{7}{5}$.

We have $\frac{2}{3} \times \frac{7}{5} = \frac{14}{15}$. Here, $\frac{14}{15} < \frac{7}{5}$ and $\frac{14}{15} > \frac{2}{3}$

The product obtained is less than the improper fraction and greater than the proper fraction involved in the multiplication.

Check it for $\frac{6}{5} \times \frac{2}{7}$, $\frac{8}{3} \times \frac{4}{5}$.

Exercise 2.3

1. Find:

(i) $\frac{1}{4}$ of (a) $\frac{1}{4}$ (b) $\frac{3}{5}$ (c) $\frac{4}{3}$

(ii) $\frac{1}{7}$ of (a) $\frac{2}{9}$ (b) $\frac{6}{5}$ (c) $\frac{3}{10}$

2. Multiply and reduce to lowest form (if possible) :

(i) $\frac{2}{3} \times 2\frac{2}{3}$ (ii) $\frac{2}{7} \times \frac{7}{9}$ (iii) $\frac{3}{8} \times \frac{6}{4}$ (iv) $\frac{9}{5} \times \frac{3}{5}$

(v) $\frac{1}{3} \times \frac{15}{8}$ (vi) $\frac{11}{2} \times \frac{3}{10}$ (vii) $\frac{4}{5} \times \frac{12}{7}$

3. Multiply the following fractions:

(i) $\frac{2}{5} \times 5\frac{1}{4}$ (ii) $6\frac{2}{5} \times \frac{7}{9}$ (iii) $\frac{3}{2} \times 5\frac{1}{3}$ (iv) $\frac{5}{6} \times 2\frac{3}{7}$

(v) $3\frac{2}{5} \times \frac{4}{7}$ (vi) $2\frac{3}{5} \times 3$ (vii) $3\frac{4}{7} \times \frac{3}{5}$

4. Which is greater:

(i) $\frac{2}{7}$ of $\frac{3}{4}$ or $\frac{3}{5}$ of $\frac{5}{8}$ (ii) $\frac{1}{2}$ of $\frac{6}{7}$ or $\frac{2}{3}$ of $\frac{3}{7}$

5. Saili plants 4 saplings, in a row, in her garden. The distance between two adjacent saplings is $\frac{3}{4}$ m. Find the distance between the first and the last sapling.

6. Lipika reads a book for $1\frac{3}{4}$ hours everyday. She reads the entire book in 6 days. How many hours in all were required by her to read the book?

7. A car runs 16 km using 1 litre of petrol. How much distance will it cover using $2\frac{3}{4}$ litres of petrol.

8. (a) (i) Provide the number in the box $\square$, such that $\frac{2}{3} \times \square = \frac{10}{30}$.

(ii) The simplest form of the number obtained in $\square$ is _____.

(b) (i) Provide the number in the box $\square$, such that $\frac{3}{5} \times \square = \frac{24}{75}$.

(ii) The simplest form of the number obtained in $\square$ is _____.

2.4 Division of Fractions

John has a paper strip of length 6 cm. He cuts this strip in smaller strips of length 2 cm each. You know that he would get $6 \div 2 = 3$ strips.

John cuts another strip of length 6 cm into smaller strips of length $\frac{3}{2}$ cm each. How many strips will he get now? He will get $6 \div \frac{3}{2}$ strips.

A paper strip of length $\frac{15}{2}$ cm can be cut into smaller strips of length $\frac{3}{2}$ cm each to give $\frac{15}{2} \div \frac{3}{2}$ pieces.

So, we are required to divide a whole number by a fraction or a fraction by another fraction. Let us see how to do that.

2.4.1 Division of Whole Number by a Fraction

Let us find $1 \div \frac{1}{2}$.

We divide a whole into a number of equal parts such that each part is half of the whole. The number of such half ($\frac{1}{2}$) parts would be $1 \div \frac{1}{2}$. Observe the figure (Fig 2.11). How many half parts do you see?

There are two half parts.

So, $1 \div \frac{1}{2} = 2$. Also, $1 \times \frac{2}{1} = 1 \times 2 = 2$. Thus, $1 \div \frac{1}{2} = 1 \times \frac{2}{1}$

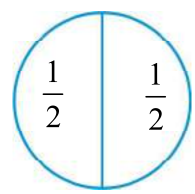


Fig 2.11

Similarly, $3 \div \frac{1}{4}$ = number of $\frac{1}{4}$ parts obtained when each of the 3 whole, are divided into $\frac{1}{4}$ equal parts = 12 (From Fig 2.12)

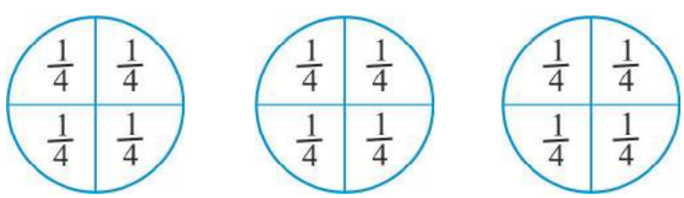


Fig 2.12

Observe also that, $3 \times \frac{4}{1} = 3 \times 4 = 12$. Thus, $3 \div \frac{1}{4} = 3 \times \frac{4}{1} = 12$.

Find in a similar way, $3 \div \frac{1}{2}$ and $3 \times \frac{2}{1}$.

Reciprocal of a fraction

The number $\frac{2}{1}$ can be obtained by interchanging the numerator and denominator of $\frac{1}{2}$ or by inverting $\frac{1}{2}$. Similarly, $\frac{3}{1}$ is obtained by inverting $\frac{1}{3}$.

Let us first see about the inverting of such numbers.

Observe these products and fill in the blanks :

$7 \times \frac{1}{7} = 1$	$\frac{5}{4} \times \frac{4}{5} =$ ---------
$\frac{1}{9} \times 9 =$ ------	$\frac{2}{7} \times$ ------- $= 1$
$\frac{2}{3} \times \frac{3}{2} = \frac{2 \times 3}{3 \times 2} = \frac{6}{6} = 1$	------ $\times \frac{5}{9} = 1$

Multiply five more such pairs.

The non-zero numbers whose product with each other is 1, are called the reciprocals of each other. So reciprocal of $\frac{5}{9}$ is $\frac{9}{5}$ and the reciprocal of $\frac{9}{5}$ is $\frac{5}{9}$. What is the receiprocal of $\frac{1}{9}$? $\frac{2}{7}$?

You will see that the reciprocal of $\frac{2}{3}$ is obtained by inverting it. You get $\frac{3}{2}$.

Think, Discuss and Write

(i) Will the reciprocal of a proper fraction be again a proper fraction?

(ii) Will the reciprocal of an improper fraction be again an improper fraction?

Therefore, we can say that

$$1 \div \frac{1}{2} = 1 \times \frac{2}{1} = 1 \times \text{ reciprocal of } \frac{1}{2}.$$

$$3 \div \frac{1}{4} = 3 \times \frac{4}{1} = 3 \times \text{ reciprocal of } \frac{1}{4}.$$

$$3 \div \frac{1}{2} = \text{------} = \text{----------------------}.$$

$$\text{So, } 2 \div \frac{3}{4} = 2 \times \text{ reciprocal of } \frac{3}{4} = 2 \times \frac{4}{3}.$$

$$5 \div \frac{2}{9} = 5 \times \text{------------------} = 5 \times \text{-------------}$$

Thus, *to divide a whole number by any fraction, multiply that whole number by the reciprocal of that fraction.*

TRY THESE

Find : (i) $7 \div \frac{2}{5}$ (ii) $6 \div \frac{4}{7}$ (iii) $2 \div \frac{8}{9}$

- *While dividing a whole number by a mixed fraction, first convert the mixed fraction into improper fraction and then solve it.*

Thus, $4 \div 2\frac{2}{5} = 4 \div \frac{12}{5} = ?$ Also, $5 \div 3\frac{1}{3} = 3 \div \frac{10}{3} = ?$

TRY THESE

Find: (i) $6 \div 5\frac{1}{3}$

(ii) $7 \div 2\frac{4}{7}$

2.4.2 Division of a Fraction by a Whole Number

- What will be $\frac{3}{4} \div 3$?

Based on our earlier observations we have: $\frac{3}{4} \div 3 = \frac{3}{4} \div \frac{3}{1} = \frac{3}{4} \times \frac{1}{3} = \frac{3}{12} = \frac{1}{4}$

So, $\frac{2}{3} \div 7 = \frac{2}{3} \times \frac{1}{7} = ?$ What is $\frac{5}{7} \div 6$, $\frac{2}{7} \div 8$?

- While dividing mixed fractions by whole numbers, convert the mixed fractions into improper fractions. That is,

$2\frac{2}{3} \div 5 = \frac{8}{3} \div 5 =$ ------ ; $4\frac{2}{5} \div 3 =$ ------ $=$ ------; $2\frac{3}{5} \div 2 =$ ------ $=$ ------

2.4.3 Division of a Fraction by Another Fraction

We can now find $\frac{1}{3} \div \frac{6}{5}$.

$\frac{1}{3} \div \frac{6}{5} = \frac{1}{3} \times$ reciprocal of $\frac{6}{5} = \frac{1}{3} \times \frac{6}{5} = \frac{2}{5}$.

Similarly, $\frac{8}{5} \div \frac{2}{3} = \frac{8}{5} \times$ reciprocal of $\frac{2}{3} = ?$ and, $\frac{1}{2} \div \frac{3}{4} = ?$

TRY THESE

Find: (i) $\frac{3}{5} \div \frac{1}{2}$ (ii) $\frac{1}{2} \div \frac{3}{5}$ (iii) $2\frac{1}{2} \div \frac{3}{5}$ (iv) $5\frac{1}{6} \div \frac{9}{2}$

EXERCISE 2.4

1. Find:

   (i) $12 \div \frac{3}{4}$ (ii) $14 \div \frac{5}{6}$ (iii) $8 \div \frac{7}{3}$ (iv) $4 \div \frac{8}{3}$

   (v) $3 \div 2\frac{1}{3}$ (vi) $5 \div 3\frac{4}{7}$

2. Find the reciprocal of each of the following fractions. Classify the reciprocals as proper fractions, improper fractions and whole numbers.

   (i) $\frac{3}{7}$ (ii) $\frac{5}{8}$ (iii) $\frac{9}{7}$ (iv) $\frac{6}{5}$

   (v) $\frac{12}{7}$ (vi) $\frac{1}{8}$ (vii) $\frac{1}{11}$

3. Find:

   (i) $\frac{7}{3} \div 2$ (ii) $\frac{4}{9} \div 5$ (iii) $\frac{6}{13} \div 7$ (iv) $4\frac{1}{3} \div 3$

   (v) $3\frac{1}{2} \div 4$ (vi) $4\frac{3}{7} \div 7$

4. Find:

   (i) $\frac{2}{5} \div \frac{1}{2}$ (ii) $\frac{4}{9} \div \frac{2}{3}$ (iii) $\frac{3}{7} \div \frac{8}{7}$ (iv) $2\frac{1}{3} \div \frac{3}{5}$ (v) $3\frac{1}{2} \div \frac{8}{3}$

   (vi) $\frac{2}{5} \div 1\frac{1}{2}$ (vii) $3\frac{1}{5} \div 1\frac{2}{3}$ (viii) $2\frac{1}{5} \div 1\frac{1}{5}$

2.5 How Well have you Learnt about Decimal Numbers

You have learnt about decimal numbers in the earlier classes. Let us briefly recall them here. Look at the following table and fill up the blank spaces.

Hundreds (100)	Tens (10)	Ones (1)	Tenths $\left(\frac{1}{10}\right)$	Hundredths $\left(\frac{1}{100}\right)$	Thousandths $\left(\frac{1}{1000}\right)$	Number
2	5	3	1	4	7	253.147
6	2	9	3	2	1	....
0	4	3	1	9	2	....
....	1	4	2	5	1	514.251
2	....	6	5	1	2	236.512
....	2	....	5	....	3	724.503
6	....	4	....	2	....	614.326
0	1	0	5	3	0	....

In the table, you wrote the decimal number, given its place-value expansion. You can do the reverse, too. That is, given the number you can write its expanded form. For example, $253.417 = 2 \times 100 + 5 \times 10 + 3 \times 1 + 4 \times \left(\frac{1}{10}\right) + 1 \times \left(\frac{1}{100}\right) + 7 \times \left(\frac{1}{1000}\right)$.

John has ₹ 15.50 and Salma has ₹ 15.75. Who has more money? To find this we need to compare the decimal numbers 15.50 and 15.75. To do this, we first compare the digits on the left of the decimal point, starting from the leftmost digit. Here both the digits 1 and 5, to the left of the decimal point, are same. So we compare the digits on the right of the decimal point starting from the tenths place. We find that $5 < 7$, so we say $15.50 < 15.75$. Thus, Salma has more money than John.

If the digits at the tenths place are also same then compare the digits at the hundredths place and so on.

Now compare quickly, 35.63 and 35.67; 20.1 and 20.01; 19.36 and 29.36.

While converting lower units of money, length and weight, to their higher units, we are required to use decimals. For example, 3 paise = ₹ $\frac{3}{100}$ = ₹ 0.03, 5g = $\frac{5}{1000}$ kg = 0.005 kg, 7 cm = 0.07 m.

Write 75 paise = ₹ ______, 250 g = _____ kg, 85 cm = _____m.

We also know how to add and subtract decimals. Thus, 21.36 + 37.35 is

$$\begin{array}{r} 21.36 \\ +\ 37.35 \\ \hline 58.71 \end{array}$$

What is the value of 0.19 + 2.3 ?

The difference 29.35 – 4.56 is

$$\begin{array}{r} 29.35 \\ -\ 04.56 \\ \hline 24.79 \end{array}$$

Tell the value of 39.87 – 21.98.

EXERCISE 2.5

1. Which is greater?
   (i) 0.5 or 0.05 (ii) 0.7 or 0.5 (iii) 7 or 0.7
   (iv) 1.37 or 1.49 (v) 2.03 or 2.30 (vi) 0.8 or 0.88.
2. Express as rupees using decimals :
   (i) 7 paise (ii) 7 rupees 7 paise (iii) 77 rupees 77 paise
   (iv) 50 paise (v) 235 paise.
3. (i) Express 5 cm in metre and kilometre (ii) Express 35 mm in cm, m and km

4. Express in kg:

(i) 200 g (ii) 3470 g (iii) 4 kg 8 g

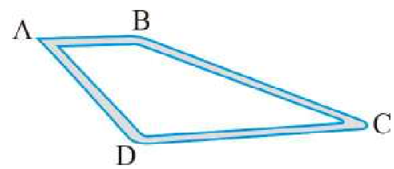
5. Write the following decimal numbers in the expanded form:

(i) 20.03 (ii) 2.03 (iii) 200.03 (iv) 2.034

6. Write the place value of 2 in the following decimal numbers:

(i) 2.56 (ii) 21.37 (iii) 10.25 (iv) 9.42 (v) 63.352.

7. Dinesh went from place A to place B and from there to place C. A is 7.5 km from B and B is 12.7 km from C. Ayub went from place A to place D and from there to place C. D is 9.3 km from A and C is 11.8 km from D. Who travelled more and by how much?



8. Shyama bought 5 kg 300 g apples and 3 kg 250 g mangoes. Sarala bought 4 kg 800 g oranges and 4 kg 150 g bananas. Who bought more fruits?

9. How much less is 28 km than 42.6 km?

2.6 Multiplication of Decimal Numbers

Reshma purchased 1.5kg vegetable at the rate of ₹ 8.50 per kg. How much money should she pay? Certainly it would be ₹ (8.50 × 1.50). Both 8.5 and 1.5 are decimal numbers. So, we have come across a situation where we need to know how to multiply two decimals. Let us now learn the multiplication of two decimal numbers.

First we find 0.1 × 0.1.

Now, $0.1 = \frac{1}{10}$. So, $0.1 \times 0.1 = \frac{1}{10} \times \frac{1}{10} = \frac{1 \times 1}{10 \times 10} = \frac{1}{100} = 0.01$.

Let us see it's pictorial representation (Fig 2.13)

The fraction $\frac{1}{10}$ represents 1 part out of 10 equal parts.

The shaded part in the picture represents $\frac{1}{10}$.

We know that,

$\frac{1}{10} \times \frac{1}{10}$ means $\frac{1}{10}$ of $\frac{1}{10}$. So, divide this $\frac{1}{10}^{\text{th}}$ part into 10 equal parts and take one part out of it.

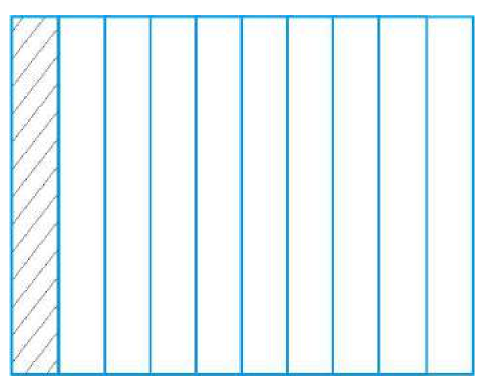


Fig 2.13

Thus, we have, (Fig 2.14).

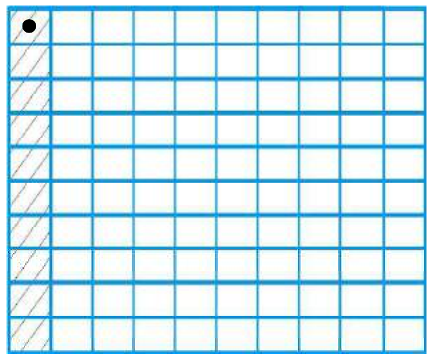


Fig 2.14

The dotted square is one part out of 10 of the $\frac{1}{10}$ th part. That is, it represents $\frac{1}{10} \times \frac{1}{10}$ or 0.1×0.1.

Can the dotted square be represented in some other way?

How many small squares do you find in Fig 2.14?

There are 100 small squares. So the dotted square represents one out of 100 or 0.01.

Hence, $0.1 \times 0.1 = 0.01$.

Note that 0.1 occurs two times in the product. In 0.1 there is one digit to the right of the decimal point. In 0.01 there are two digits (i.e., 1 + 1) to the right of the decimal point.

Let us now find 0.2×0.3.

We have, $0.2 \times 0.3 = \frac{2}{10} \times \frac{3}{10}$

As we did for $\frac{1}{10} \times \frac{1}{10}$, let us divide the square into 10 equal parts and take three parts out of it, to get $\frac{3}{10}$. Again divide each of these three equal parts into 10 equal parts and take two from each. We get $\frac{2}{10} \times \frac{3}{10}$.

The dotted squares represent $\frac{2}{10} \times \frac{3}{10}$ or 0.2×0.3. (Fig 2.15)

Since there are 6 dotted squares out of 100, so they also reprsent 0.06.

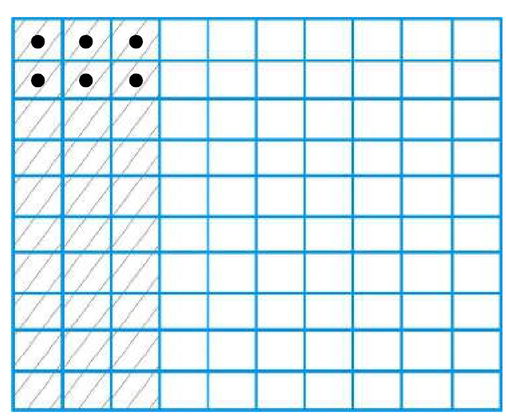


Fig 2.15

Thus, 0.2 × 0.3 = 0.06.

Observe that 2 × 3 = 6 and the number of digits to the right of the decimal point in 0.06 is 2 (= 1 + 1).

Check whether this applies to 0.1 × 0.1 also.

Find 0.2 × 0.4 by applying these observations.

While finding 0.1 × 0.1 and 0.2 × 0.3, you might have noticed that first we multiplied them as whole numbers ignoring the decimal point. In 0.1 × 0.1, we found 01 × 01 or 1 × 1. Similarly in 0.2 × 0.3 we found 02 × 03 or 2 × 3.

Then, we counted the number of digits starting from the rightmost digit and moved towards left. We then put the decimal point there. The number of digits to be counted is obtained by adding the number of digits to the right of the decimal point in the decimal numbers that are being multiplied.

Let us now find 1.2 × 2.5.

Multiply 12 and 25. We get 300. Both, in 1.2 and 2.5, there is 1 digit to the right of the decimal point. So, count 1 + 1 = 2 digits from the rightmost digit (i.e., 0) in 300 and move towards left. We get 3.00 or 3.

Find in a similar way 1.5 × 1.6, 2.4 × 4.2.

While multiplying 2.5 and 1.25, you will first multiply 25 and 125. For placing the decimal in the product obtained, you will count 1 + 2 = 3 (Why?) digits starting from the rightmost digit. Thus, 2.5 × 1.25 = 3.225

Find 2.7 × 1.35.

TRY THESE

1. Find: (i) 2.7 × 4 (ii) 1.8 × 1.2 (iii) 2.3 × 4.35
2. Arrange the products obtained in (1) in descending order.

EXAMPLE 7 The side of an equilateral triangle is 3.5 cm. Find its perimeter.

SOLUTION All the sides of an equilateral triangle are equal.

So, length of each side = 3.5 cm

Thus, perimeter = 3 × 3.5 cm = 10.5 cm

EXAMPLE 8 The length of a rectangle is 7.1 cm and its breadth is 2.5 cm. What is the area of the rectangle?

SOLUTION Length of the rectangle = 7.1 cm

Breadth of the rectangle = 2.5 cm

Therefore, area of the rectangle = 7.1 × 2.5 cm^2 = 17.75 cm^2

2.6.1 Multiplication of Decimal Numbers by 10, 100 and 1000

Reshma observed that 2.3 = $\frac{23}{10}$ whereas 2.35 = $\frac{235}{100}$. Thus, she found that depending on the position of the decimal point the decimal number can be converted to a fraction with denominator 10 or 100. She wondered what would happen if a decimal number is multiplied by 10 or 100 or 1000.

Let us see if we can find a pattern of multiplying numbers by 10 or 100 or 1000.

Have a look at the table given below and fill in the blanks:

$1.76 \times 10 = \frac{176}{100} \times 10 = 17.6$	$2.35 \times 10 =$ ___	$12.356 \times 10 =$ ___
$1.76 \times 100 = \frac{176}{100} \times 100 = 176$ or 176.0	$2.35 \times 100 =$ ___	$12.356 \times 100 =$ ___
$1.76 \times 1000 = \frac{176}{100} \times 1000 = 1760$ or 1760.0	$2.35 \times 1000 =$ ___	$12.356 \times 1000 =$ ___
$0.5 \times 10 = \frac{5}{10} \times 10 = 5$; $0.5 \times 100 =$ ___ ; $0.5 \times 1000 =$ ___		

Observe the shift of the decimal point of the products in the table. Here the numbers are multiplied by 10,100 and 1000. In 1.76 × 10 = 17.6, the digits are same i.e., 1, 7 and 6. Do you observe this in other products also? Observe 1.76 and 17.6. To which side has the decimal point shifted, right or left? The decimal point has shifted to the right by one place. Note that 10 has one zero over 1.

In 1.76×100 = 176.0, observe 1.76 and 176.0. To which side and by how many digits has the decimal point shifted? The decimal point has shifted to the right by two places.

Note that 100 has two zeros over one.

Do you observe similar shifting of decimal point in other products also?

So we say, *when a decimal number is multiplied by 10, 100 or 1000, the digits in the product are same as in the decimal number but the decimal point in the product is shifted to the right by as, many of places as there are zeros over one.*

Based on these observations we can now say

0.07 × 10 = 0.7, 0.07 × 100 = 7 and 0.07 × 1000 = 70.

Can you now tell 2.97 × 10 = ? 2.97 × 100 = ? 2.97 × 1000 = ?

Can you now help Reshma to find the total amount i.e., ₹ 8.50 × 150, that she has to pay?

TRY THESE

Find: (i) 0.3 × 10
(ii) 1.2 × 100
(iii) 56.3 × 1000

EXERCISE 2.6

1. Find:
   (i) 0.2×6 (ii) 8×4.6 (iii) 2.71×5 (iv) 20.1×4
   (v) 0.05×7 (vi) 211.02×4 (vii) 2×0.86
2. Find the area of rectangle whose length is 5.7cm and breadth is 3 cm.
3. Find:
   (i) 1.3×10 (ii) 36.8×10 (iii) 153.7×10 (iv) 168.07×10
   (v) 31.1×100 (vi) 156.1×100 (vii) 3.62×100 (viii) 43.07×100
   (ix) 0.5×10 (x) 0.08×10 (xi) 0.9×100 (xii) 0.03×1000
4. A two-wheeler covers a distance of 55.3 km in one litre of petrol. How much distance will it cover in 10 litres of petrol?
5. Find:
   (i) 2.5×0.3 (ii) 0.1×51.7 (iii) 0.2×316.8 (iv) 1.3×3.1
   (v) 0.5×0.05 (vi) 11.2×0.15 (vii) 1.07×0.02
   (viii) 10.05×1.05 (ix) 101.01×0.01 (x) 100.01×1.1

2.7 DIVISION OF DECIMAL NUMBERS

Savita was preparing a design to decorate her classroom. She needed a few coloured strips of paper of length 1.9 cm each. She had a strip of coloured paper of length 9.5 cm. How many pieces of the required length will she get out of this strip? She thought it would be $\frac{9.5}{1.9}$ cm. Is she correct?

Both 9.5 and 1.9 are decimal numbers. So we need to know the division of decimal numbers too!

2.7.1 Division by 10, 100 and 1000

Let us find the division of a decimal number by 10, 100 and 1000.
Consider $31.5 \div 10$.

$$31.5 \div 10 = \frac{315}{10} \times \frac{1}{10} = \frac{315}{100} = 3.15$$

Similarly, $$31.5 \div 100 = \frac{315}{10} \times \frac{1}{100} = \frac{315}{1000} = 0.315$$

Let us see if we can find a pattern for dividing numbers by 10, 100 or 1000. This may help us in dividing numbers by 10, 100 or 1000 in a shorter way.

31.5 ÷ 10 = 3.15	231.5 ÷ 10 =___	1.5 ÷ 10 =___	29.36 ÷ 10 =___
31.5 ÷ 100 = 0.315	231.5 ÷ 10 =___	1.5 ÷ 100 =___	29.36 ÷ 100 =___
31.5 ÷1000 = 0.0315	231.5 ÷ 1000 =___	1.5 ÷ 1000 =___	29.36 ÷1000 =___

Take $31.5 \div 10 = 3.15$. In 31.5 and 3.15, the digits are same i.e., 3, 1, and 5 but the decimal point has shifted in the quotient. To which side and by how many digits? The decimal point has shifted to the left by one place. Note that 10 has one zero over 1.

Try These

Find: (i) $235.4 \div 10$
(ii) $235.4 \div 100$
(iii) $235.4 \div 1000$

Consider now $31.5 \div 100 = 0.315$. In 31.5 and 0.315 the digits are same, but what about the decimal point in the quotient? It has shifted to the left by two places. Note that 100 has two zeros over1.

So we can say that, *while dividing a number by 10, 100 or 1000, the digits of the number and the quotient are same but the decimal point in the quotient shifts to the left by as many places as there are zeros over 1.* Using this observation let us now quickly find: $2.38 \div 10 = 0.238$, $2.38 \div 100 = 0.0238$, $2.38 \div 1000 = 0.00238$

2.7.2 Division of a Decimal Number by a Whole Number

Let us find $\frac{6.4}{2}$. Remember we also write it as $6.4 \div 2$.

Try These

(i) $35.7 \div 3 = ?$;
(ii) $25.5 \div 3 = ?$

So, $6.4 \div 2 = \frac{64}{10} \div 2 = \frac{64}{10} \times \frac{1}{2}$ as learnt in fractions.

$$= \frac{64 \times 1}{10 \times 2} = \frac{1 \times 64}{10 \times 2} = \frac{1}{10} \times \frac{64}{2} = \frac{1}{10} \times 32 = \frac{32}{10} = 3.2$$

Or, let us first divide 64 by 2. We get 32. There is one digit to the right of the decimal point in 6.4. Place the decimal in 32 such that there would be one digit to its right. We get 3.2 again.

To find $19.5 \div 5$, first find $195 \div 5$. We get 39. There is one digit to the right of the decimal point in 19.5. Place the decimal point in 39 such that there would be one digit to its right. You will get 3.9.

Try These

(i) $43.15 \div 5 = ?$;
(ii) $82.44 \div 6 = ?$

Now, $12.96 \div 4 = \frac{1296}{100} \div 4 = \frac{1296}{100} \times \frac{1}{4} = \frac{1}{100} \times \frac{1296}{4} = \frac{1}{100} \times 324 = 3.24$

Or, divide 1296 by 4. You get 324. There are two digits to the right of the decimal in 12.96. Making similar placement of the decimal in 324, you will get 3.24.

Note that here and in the next section, we have considered only those divisions in which, ignoring the decimal, the number would be completely divisible by another number to give remainder zero. Like, in $19.5 \div 5$, the number 195 when divided by 5, leaves remainder zero.

Try These

Find: (i) $15.5 \div 5$
(ii) $126.35 \div 7$

However, there are situations in which the number may not be completely divisible by another number, i.e., we may not get remainder zero. For example, $195 \div 7$. We deal with such situations in later classes.

Thus, $40.86 \div 6 = 6.81$

EXAMPLE 9 Find the average of 4.2, 3.8 and 7.6.

SOLUTION The average of 4.2, 3.8 and 7.6 is $\frac{4.2+3.8+7.6}{3}=\frac{15.6}{3}=5.2$.

2.7.3 Division of a Decimal Number by another Decimal Number

Let us find $\frac{25.5}{0.5}$ i.e., $25.5 \div 0.5$.

We have $25.5 \div 0.5 = \frac{255}{10} \div \frac{5}{10} = \frac{255}{10} \times \frac{10}{5} = 51$. Thus, $25.5 \div 0.5 = 51$

What do you observe? For $\frac{25.5}{0.5}$, we find that there is one digit to the right of the decimal in 0.5. This could be converted to whole number by dividing by 10. Accordingly 25.5 was also converted to a fraction by dividing by 10.

Or, we say the decimal point was shifted by one place to the right in 0.5 to make it 5. So, there was a shift of one decimal point to the right in 25.5 also to make it 255.

Thus, $22.5 \div 1.5 = \frac{22.5}{1.5} = \frac{225}{15} = 15$

Find $\frac{20.3}{0.7}$ and $\frac{15.2}{0.8}$ in a similar way.

TRY THESE

Find: (i) $\frac{7.75}{0.25}$ (ii) $\frac{42.8}{0.02}$ (iii) $\frac{5.6}{1.4}$

Let us now find $20.55 \div 1.5$.

We can write it is as $205.5 \div 15$, as discussed above. We get 13.7. Find $\frac{3.96}{0.4}$, $\frac{2.31}{0.3}$.

Consider now, $\frac{33.725}{0.25}$. We can write it as $\frac{3372.5}{25}$ (How?) and we get the quotient as 134.9. How will you find $\frac{27}{0.03}$? We know that 27 can be written as 27.0.

So, $\frac{27}{0.03} = \frac{27.00}{0.03} = \frac{2700}{3} = ?$

EXAMPLE 10 Each side of a regular polygon is 2.5 cm in length. The perimeter of the polygon is 12.5cm. How many sides does the polygon have?

SOLUTION The perimeter of a regular polygon is the sum of the lengths of all its equal sides = 12.5 cm.

Length of each side = 2.5 cm. Thus, the number of sides = $\frac{12.5}{2.5} = \frac{125}{25} = 5$

The polygon has 5 sides.

EXAMPLE 11 A car covers a distance of 89.1 km in 2.2 hours. What is the average distance covered by it in 1 hour?

SOLUTION Distance covered by the car = 89.1 km.

Time required to cover this distance = 2.2 hours.

So distance covered by it in 1 hour $= \frac{89.1}{2.2} = \frac{891}{22} = 40.5$ km.

EXERCISE 2.7

1. Find:
(i) $0.4 \div 2$ (ii) $0.35 \div 5$ (iii) $2.48 \div 4$ (iv) $65.4 \div 6$
(v) $651.2 \div 4$ (vi) $14.49 \div 7$ (vii) $3.96 \div 4$ (viii) $0.80 \div 5$

2. Find:
(i) $4.8 \div 10$ (ii) $52.5 \div 10$ (iii) $0.7 \div 10$ (iv) $33.1 \div 10$
(v) $272.23 \div 10$ (vi) $0.56 \div 10$ (vii) $3.97 \div 10$

3. Find:
(i) $2.7 \div 100$ (ii) $0.3 \div 100$ (iii) $0.78 \div 100$
(iv) $432.6 \div 100$ (v) $23.6 \div 100$ (vi) $98.53 \div 100$

4. Find:
(i) $7.9 \div 1000$ (ii) $26.3 \div 1000$ (iii) $38.53 \div 1000$
(iv) $128.9 \div 1000$ (v) $0.5 \div 1000$

5. Find:
(i) $7 \div 3.5$ (ii) $36 \div 0.2$ (iii) $3.25 \div 0.5$ (iv) $30.94 \div 0.7$
(v) $0.5 \div 0.25$ (vi) $7.75 \div 0.25$ (vii) $76.5 \div 0.15$ (viii) $37.8 \div 1.4$
(ix) $2.73 \div 1.3$

6. A vehicle covers a distance of 43.2 km in 2.4 litres of petrol. How much distance will it cover in one litre of petrol?

WHAT HAVE WE DISCUSSED?

1. We have learnt about fractions and decimals alongwith the operations of addition and subtraction on them, in the earlier class.

2. We now study the operations of multiplication and division on fractions as well as on decimals.

3. We have learnt how to multiply fractions. Two fractions are multiplied by multiplying their numerators and denominators seperately and writing the product as $\frac{\text{product of numerators}}{\text{product of denominators}}$. For example, $\frac{2}{3} \times \frac{5}{7} = \frac{2 \times 5}{3 \times 7} = \frac{10}{21}$.

4. A fraction acts as an operator 'of'. For example, $\frac{1}{2}$ of 2 is $\frac{1}{2} \times 2 = 1$.

5. (a) The product of two proper fractions is less than each of the fractions that are multiplied.

(b) The product of a proper and an improper fraction is less than the improper fraction and greater than the proper fraction.

(c) The product of two imporper fractions is greater than the two fractions.

6. A reciprocal of a fraction is obtained by inverting it upside down.

7. We have seen how to divide two fractions.

(a) While dividing a whole number by a fraction, we multiply the whole number with the reciprocal of that fraction.

For example, $2 \div \frac{3}{5} = 2 \times \frac{5}{3} = \frac{10}{3}$

(b) While dividing a fraction by a whole number we multiply the fraction by the reciprocal of the whole number.

For example, $\frac{2}{3} \div 7 = \frac{2}{3} \times \frac{1}{7} = \frac{2}{21}$

(c) While dividing one fraction by another fraction, we multuiply the first fraction by the reciprocal of the other. So, $\frac{2}{3} \div \frac{5}{7} = \frac{2}{3} \times \frac{7}{5} = \frac{14}{15}$.

8. We also learnt how to multiply two decimal numbers. While multiplying two decimal numbers, first multiply them as whole numbers. Count the number of digits to the right of the decimal point in both the decimal numbers. Add the number of digits counted. Put the decimal point in the product by counting the digits from its rightmost place. The count should be the sum obtained earlier.

For example, $0.5 \times 0.7 = 0.35$

9. To multiply a decimal number by 10, 100 or 1000, we move the decimal point in the number to the right by as many places as there are zeros over 1.

Thus $0.53 \times 10 = 5.3$, $0.53 \times 100 = 53$, $0.53 \times 1000 = 530$

10. We have seen how to divide decimal numbers.

(a) To divide a decimal number by a whole number, we first divide them as whole numbers. Then place the decimal point in the quotient as in the decimal number.

For example, $8.4 \div 4 = 2.1$

Note that here we consider only those divisions in which the remainder is zero.

(b) To divide a decimal number by 10, 100 or 1000, shift the digits in the decimal number to the left by as many places as there are zeros over 1, to get the quotient.

So, $23.9 \div 10 = 2.39$,$23.9 \div 100 = 0.239$, $23.9 \div 1000 = 0.0239$

(c) While dividing two decimal numbers, first shift the decimal point to the right by equal number of places in both, to convert the divisor to a whole number. Then divide. Thus, $2.4 \div 0.2 = 24 \div 2 = 12$.

Chapter 3

Data Handling

3.1 Introduction

In your previous classes, you have dealt with various types of data. You have learnt to collect data, tabulate and put it in the form of bar graphs. The collection, recording and presentation of data help us organise our experiences and draw inferences from them. In this Chapter, we will take one more step towards learning how to do this. You will come across some more kinds of data and graphs. You have seen several kinds of data through newspapers, magazines, television and other sources. You also know that all data give us some sort of information. Let us look at some common forms of data that you come across:

Table 3.1

Temperatures of Cities as on 20.6.2006		
City	**Max.**	**Min.**
Ahmedabad	38°C	29°C
Amritsar	37°C	26°C
Bangalore	28°C	21°C
Chennai	36°C	27°C
Delhi	38°C	28°C
Jaipur	39°C	29°C
Jammu	41°C	26°C
Mumbai	32°C	27°C

Table 3.2

Football World Cup 2006	
Ukraine beat Saudi Arabia by	4 - 0
Spain beat Tunisia by	3 - 1
Switzerland beat Togo by	2 - 0

Table 3.3

Data Showing Weekly Absentees in a Class	
Monday	
Tuesday	
Wednesday	–
Thursday	
Friday	
Saturday	
	 represents one child

What do these collections of data tell you?

For example you can say that the highest maximum temperature was in Jammu on 20.06.2006 (Table 3.1) or we can say that, on Wednesday, no child was absent. (Table 3.3)

Can we organise and present these data in a different way, so that their analysis and interpretation becomes better? We shall address such questions in this Chapter.

3.2 Collecting Data

The data about the temperatures of cities (Table 3.1) can tell us many things, but it cannot tell us the city which had the highest maximum temperature during the year. To find that, we need to collect data regarding the highest maximum temperature reached in each of these cities during the year. In that case, the temperature chart of one particular date of the year, as given in Table 3.1 will not be sufficient.

This shows that a given collection of data may not give us a specific information related to that data. For this we need to collect data keeping in mind that specific information. In the above case the specific information needed by us, was about the highest maximum temperature of the cities during the year, which we could not get from Table 3.1

Thus, **before collecting data, we need to know what we would use it for.**

Given below are a few situations.

You want to study the

– Performance of your class in Mathematics.

– Performance of India in football or in cricket.

– Female literacy rate in a given area, or

– Number of children below the age of five in the families around you.

What kind of data would you need in the above situations? Unless and until you collect appropriate data, you cannot know the desired information. What is the appropriate data for each?

Discuss with your friends and identify the data you would need for each. Some of this data is easy to collect and some difficult.

3.3 Organisation of Data

When we collect data, we have to record and organise it. Why do we need to do that? Consider the following example.

Ms Neelam, class teacher wanted to find how children had performed in English. She writes down the marks obtained by the students in the following way:

23, 35, 48, 30, 25, 46, 13, 27, 32, 38

In this form, the data was not easy to understand. She also did not know whether her impression of the students matched their performance.

Neelam's colleague helped her organise the data in the following way (Table 3.4).

Table 3.4

Roll No.	Names	Marks Out of 50	Roll No.	Names	Marks Out of 50
1	Ajay	23	6	Govind	46
2	Armaan	35	7	Jay	13
3	Ashish	48	8	Kavita	27
4	Dipti	30	9	Manisha	32
5	Faizaan	25	10	Neeraj	38

In this form, Neelam was able to know which student has got how many marks. But she wanted more. Deepika suggested another way to organise this data (Table 3.5).

Table 3.5

Roll No.	Names	Marks Out of 50	Roll No.	Names	Marks Out of 50
3	Ashish	48	4	Dipti	30
6	Govind	46	8	Kavita	27
10	Neeraj	38	5	Faizaan	25
2	Armaan	35	1	Ajay	23
9	Manisha	32	7	Jay	13

Now Neelam was able to see who had done the best and who needed help.

Many kinds of data we come across are put in tabular form. Our school rolls, progress report, index in the notebooks, temperature record and many others are all in tabular form. Can you think of a few more data that you come across in tabular form?

When we put data in a proper table it becomes easy to understand and interpret.

Try These

Weigh (in kg) atleast 20 children (girls and boys) of your class. Organise the data, and answer the following questions using this data.

(i) Who is the heaviest of all? (ii) What is the most common weight?

(iii) What is the difference between your weight and that of your best friend?

3.4 Representative Values

You might be aware of the term *average* and would have come across statements involving the term 'average' in your day-to-day life:

- Isha spends on an average of about 5 hours daily for her studies.

- The average temperature at this time of the year is about 40 degree celsius.
- The average age of pupils in my class is 12 years.
- The average attendance of students in a school during its final examination was 98 per cent.

Many more of such statements could be there. Think about the statements given above. Do you think that the child in the first statement studies exactly for 5 hours daily? Or, is the temperature of the given place during that particular time always 40 degrees? Or, is the age of each pupil in that class 12 years? Obviously not.

Then what do these statements tell you?

By average we understand that Isha, usually, studies for 5 hours. On some days, she may study for less number of hours and on the other days she may study longer.

Similarly, the average temperature of 40 degree celsius, means that, very often, the temperature at this time of the year is around 40 degree celsius. Sometimes, it may be less than 40 degree celsius and at other times, it may be more than 40°C.

Thus, we realise that average is a number that represents or shows the central tendency of a group of observations or data. Since average lies between the highest and the lowest value of the given data so, we say average is a measure of the central tendency of the group of data. Different forms of data need different forms of representative or central value to describe it. One of these representative values is the "**Arithmetic mean**". You will learn about the other representative values in the later part of the chapter.

3.5 Arithmetic Mean

The most common representative value of a group of data is the **arithmetic mean** or the **mean**. To understand this in a better way, let us look at the following example:

Two vessels contain 20 litres and 60 litres of milk respectively. What is the amount that each vessel would have, if both share the milk equally? When we ask this question we are seeking the arithmetic mean.

In the above case, the average or the arithmetic mean would be

$$\frac{\text{Total quantity of milk}}{\text{Number of vessels}} = \frac{20+60}{2} \text{ litres} = 40 \text{ litres.}$$

Thus, each vessels would have 40 litres of milk.

The average or Arithmetic Mean (A.M.) or simply mean is defined as follows:

$$\text{mean} = \frac{\text{Sum of all observations}}{\text{number of observations}}$$

Consider these examples.

Example 1 Ashish studies for 4 hours, 5 hours and 3 hours respectively on three consecutive days. How many hours does he study daily on an average?

SOLUTION The average study time of Ashish would be

$$\frac{\text{Total number of study hours}}{\text{Number of days for which he studied}} = \frac{4+5+3}{3} \text{ hours} = 4 \text{ hours per day}$$

Thus, we can say that Ashish studies for 4 hours daily on an average.

EXAMPLE 2 A batsman scored the following number of runs in six innings:

36, 35, 50, 46, 60, 55

Calculate the mean runs scored by him in an inning.

SOLUTION Total runs = 36 + 35 + 50 + 46 + 60 + 55 = 282.

To find the mean, we find the sum of all the observations and divide it by the number of observations.

Therefore, in this case, mean $= \frac{282}{6} = 47$. Thus, the mean runs scored in an inning are 47.

Where does the arithmetic mean lie

TRY THESE

How would you find the average of your study hours for the whole week?

THINK, DISCUSS AND WRITE

Consider the data in the above examples and think on the following:

- Is the mean bigger than each of the observations?
- Is it smaller than each observation?

Discuss with your friends. Frame one more example of this type and answer the same questions.

You will find that the mean lies inbetween the greatest and the smallest observations.

In particular, the mean of two numbers will always lie between the two numbers.

For example the mean of 5 and 11 is $\frac{5+11}{2} = 8$, which lies between 5 and 11.

Can you use this idea to show that between any two fractional numbers, you can find as many fractional numbers as you like. For example between $\frac{1}{2}$ and $\frac{1}{4}$ you have their average $\frac{\frac{1}{2}+\frac{1}{4}}{2} = \frac{3}{8}$ and then between $\frac{1}{2}$ and $\frac{3}{8}$, you have their average $\frac{7}{16}$ and so on.

TRY THESE

1. Find the mean of your sleeping hours during one week.
2. Find atleast 5 numbers between $\frac{1}{2}$ and $\frac{1}{3}$.

3.5.1 Range

The difference between the highest and the lowest observation gives us an idea of the spread of the observations. This can be found by subtracting the lowest observation from the highest observation. We call the result the **range** of the observation. Look at the following example:

EXAMPLE 3 The ages in years of 10 teachers of a school are:

32, 41, 28, 54, 35, 26, 23, 33, 38, 40

(i) What is the age of the oldest teacher and that of the youngest teacher?

(ii) What is the range of the ages of the teachers?

(iii) What is the mean age of these teachers?

SOLUTION

(i) Arranging the ages in ascending order, we get:

23, 26, 28, 32, 33, 35, 38, 40, 41, 54

We find that the age of the oldest teacher is 54 years and the age of the youngest teacher is 23 years.

(ii) Range of the ages of the teachers = (54 – 23) years = 31 years

(iii) Mean age of the teachers

$$= \frac{23+26+28+32+33+35+38+40+41+54}{10} \text{ years}$$

$$= \frac{350}{10} \text{ years } = 35 \text{ years}$$

EXERCISE 3.1

1. Find the range of heights of any ten students of your class.
2. Organise the following marks in a class assessment, in a tabular form.

   4, 6, 7, 5, 3, 5, 4, 5, 2, 6, 2, 5, 1, 9, 6, 5, 8, 4, 6, 7

   (i) Which number is the highest? (ii) Which number is the lowest?

   (iii) What is the range of the data? (iv) Find the arithmetic mean.
3. Find the mean of the first five whole numbers.
4. A cricketer scores the following runs in eight innings:

   58, 76, 40, 35, 46, 45, 0, 100.

   Find the mean score.

5. Following table shows the points of each player scored in four games:

Player	Game 1	Game 2	Game 3	Game 4
A	14	16	10	10
B	0	8	6	4
C	8	11	Did not Play	13

Now answer the following questions:

(i) Find the mean to determine A's average number of points scored per game.

(ii) To find the mean number of points per game for C, would you divide the total points by 3 or by 4? Why?

(iii) B played in all the four games. How would you find the mean?

(iv) Who is the best performer?

6. The marks (out of 100) obtained by a group of students in a science test are 85, 76, 90, 85, 39, 48, 56, 95, 81 and 75. Find the:

(i) Highest and the lowest marks obtained by the students.

(ii) Range of the marks obtained.

(iii) Mean marks obtained by the group.

7. The enrolment in a school during six consecutive years was as follows:

1555, 1670, 1750, 2013, 2540, 2820

Find the mean enrolment of the school for this period.

8. The rainfall (in mm) in a city on 7 days of a certain week was recorded as follows:

Day	**Mon**	**Tue**	**Wed**	**Thurs**	**Fri**	**Sat**	**Sun**
Rainfall (in mm)	0.0	12.2	2.1	0.0	20.5	5.5	1.0

(i) Find the range of the rainfall in the above data.

(ii) Find the mean rainfall for the week.

(iii) On how many days was the rainfall less than the mean rainfall.

9. The heights of 10 girls were measured in cm and the results are as follows:

135, 150, 139, 128, 151, 132, 146, 149, 143, 141.

(i) What is the height of the tallest girl? (ii) What is the height of the shortest girl?

(iii) What is the range of the data? (iv) What is the mean height of the girls?

(v) How many girls have heights more than the mean height.

3.6 Mode

As we have said Mean is not the only measure of central tendency or the only form of representative value. For different requirements from a data, other measures of central tendencies are used.

Look at the following example

To find out the weekly demand for different sizes of shirt, a shopkeeper kept records of sales of sizes 90 cm, 95 cm, 100 cm, 105 cm, 110 cm. Following is the record for a week:

Size (in inches)	90 cm	95 cm	100 cm	105 cm	110 cm	Total
Number of Shirts Sold	8	22	32	37	6	105

If he found the mean number of shirts sold, do you think that he would be able to decide which shirt sizes to keep in stock?

$$\text{Mean of total shirts sold} = \frac{\text{Total number of shirts sold}}{\text{Number of different sizes of shirts}} = \frac{105}{5} = 21$$

Should he obtain 21 shirts of each size? If he does so, will he be able to cater to the needs of the customers?

The shopkeeper, on looking at the record, decides to procure shirts of sizes 95 cm, 100 cm, 105 cm. He decided to postpone the procurement of the shirts of other sizes because of their small number of buyers.

Look at another example

The owner of a readymade dress shop says, "The most popular size of dress I sell is the size 90 cm.

Observe that here also, the owner is concerned about the number of shirts of different sizes sold. She is however looking at the shirt size that is sold the most. This is another representative value for the data. The highest occuring event is the sale of size 90 cm.This representative value is called the **mode** of the data.

The mode of a set of observations is the observation that occurs most often.

Example 4 Find the mode of the given set of numbers: 1, 1, 2, 4, 3, 2, 1, 2, 2, 4

Solution Arranging the numbers with same values together, we get

1, 1, 1, 2, 2, 2, 2, 3, 4, 4

Mode of this data is 2 because it occurs more frequently than other observations.

3.6.1 Mode of Large Data

Putting the same observations together and counting them is not easy if the number of observations is large. In such cases we tabulate the data. Tabulation can begin by putting tally marks and finding the frequency, as you did in your previous class.

Look at the following example:

EXAMPLE 5 Following are the margins of victory in the football matches of a league.

1, 3, 2, 5, 1, 4, 6, 2, 5, 2, 2, 2, 4, 1, 2, 3, 1, 1, 2, 3, 2,
6, 4, 3, 2, 1, 1, 4, 2, 1, 5, 3, 3, 2, 3, 2, 4, 2, 1, 2

Find the mode of this data.

SOLUTION Let us put the data in a tabular form:

Margins of Victory	Tally Bars	Number of Matches
1	~~IIII~~ IIII	9
2	~~IIII~~ ~~IIII~~ IIII	14
3	~~IIII~~ II	7
4	~~IIII~~	5
5	III	3
6	II	2
	Total	**40**

Looking at the table, we can quickly say that 2 is the 'mode' since 2 has occured the highest number of times. Thus, most of the matches have been won with a victory margin of 2 goals.

TRY THESE

Find the mode of

(i) 2, 6, 5, 3, 0, 3, 4, 3, 2, 4, 5, 2, 4

(ii) 2, 14, 16, 12, 14, 14, 16, 14, 10, 14, 18, 14

THINK, DISCUSS AND WRITE

Can a set of numbers have more than one mode?

EXAMPLE 6 Find the mode of the numbers: 2, 2, 2, 3, 3, 4, 5, 5, 5, 6, 6, 8

SOLUTION Here, 2 and 5 both occur three times. Therefore, they both are modes of the data.

DO THIS

1. Record the age in years of all your classmates. Tabulate the data and find the mode.
2. Record the heights in centimetres of your classmates and find the mode.

TRY THESE

1. Find the mode of the following data:
12, 14, 12, 16, 15, 13, 14, 18, 19, 12, 14, 15, 16, 15, 16, 16, 15,
17, 13, 16, 16, 15, 15, 13, 15, 17, 15, 14, 15, 13, 15, 14

2. Heights (in cm) of 25 children are given below:

168, 165, 163, 160, 163, 161, 162, 164, 163, 162, 164, 163, 160, 163, 160, 165, 163, 162, 163, 164, 163, 160, 165, 163, 162

What is the mode of their heights? What do we understand by mode here?

Whereas mean gives us the average of all observations of the data, the mode gives that observation which occurs most frequently in the data.

Let us consider the following examples:

(a) You have to decide upon the number of chapattis needed for 25 people called for a feast.

(b) A shopkeeper selling shirts has decided to replenish her stock.

(c) We need to find the height of the door needed in our house.

(d) When going on a picnic, if only one fruit can be bought for everyone, which is the fruit that we would get.

In which of these situations can we use the mode as a good estimate?

Consider the first statement. Suppose the number of chapattis needed by each person is 2, 3, 2, 3, 2, 1, 2, 3, 2, 2, 4, 2, 2, 3, 2, 4, 4, 2, 3, 2, 4, 2, 4, 3, 5

The mode of the data is 2 chapattis. If we use mode as the representative value for this data, then we need 50 chapattis only, 2 for each of the 25 persons. However the total number would clearly be inadequate. Would **mean** be an appropriate representative value?

For the third statement the height of the door is related to the height of the persons using that door. Suppose there are 5 children and 4 adults using the door and the height of each of 5 children is around 135 cm. The mode for the heights is 135 cm. Should we get a door that is 144 cm high? Would all the adults be able to go through that door? It is clear that mode is not the appropriate representative value for this data. Would **mean** be an appropriate representative value here?

Why not? Which representative value of height should be used to decide the doorheight?

Similarly analyse the rest of the statements and find the representative value useful for that issue.

Try These

Discuss with your friends and give

(a) Two situations where mean would be an appropriate representative value to use, and

(b) Two situations where mode would be an appropriate representative value to use.

3.7 Median

We have seen that in some situations, arithmetic mean is an appropriate measure of central tendency whereas in some other situations, mode is the appropriate measure of central tendency.

Let us now look at another example. Consider a group of 17 students with the following heights (in cm): 106, 110, 123, 125, 117, 120, 112, 115, 110, 120, 115, 102, 115, 115, 109, 115, 101.

The games teacher wants to divide the class into two groups so that each group has equal number of students, one group has students with height lesser than a particular height and the other group has students with heights greater than the particular height. How would she do that?

Let us see the various options she has:

(i) She can find the mean. The mean is

$$\frac{106+110+123+125+117+120+112+115+110+120+115+102+115+115+109+115+101}{17}$$

$$=\frac{1930}{17}=113.5$$

So, if the teacher divides the students into two groups on the basis of this mean height, such that one group has students of height less than the mean height and the other group has students with height more than the mean height, then the groups would be of unequal size. They would have 7 and 10 members respectively.

(ii) The second option for her is to find mode. The observation with highest frequency is 115 cm, which would be taken as mode.

There are 7 children below the mode and 10 children at the mode and above the mode. Therefore, we cannot divide the group into equal parts.

Let us therefore think of an alternative representative value or measure of central tendency. For doing this we again look at the given heights (in cm) of students and arrange them in ascending order. We have the following observations:

101, 102, 106, 109, 110, 110, 112, 115, 115, 115, 115, 115, 117, 120, 120, 123, 125

The middle value in this data is 115 because this value divides the students into two equal groups of 8 students each. This value is called as **Median**. Median refers to the value which lies in the middle of the data (when arranged in an increasing or decreasing order) with half of the observations above it and the other half below it. The games teacher decides to keep the middle student as a refree in the game.

Here, we consider only those cases where number of observations is odd.

Thus, in a given data, arranged in ascending or descending order, the **median** gives us the middle observation.

Try These

Your friend found the median and the mode of a given data. Describe and correct your friends error if any:

35, 32, 35, 42, 38, 32, 34

Median = 42, Mode = 32

Note that in general, we may not get the same value for median and mode.

Thus we realise that mean, mode and median are the numbers that are the representative values of a group of observations or data. They lie between the minimum and maximum values of the data. They are also called the measures of the central tendency.

EXAMPLE 7 Find the median of the data: 24, 36, 46, 17, 18, 25, 35

SOLUTION We arrange the data in ascending order, we get 17, 18, 24, 25, 35, 36, 46
Median is the middle observation. Therefore 25 is the median.

EXERCISE 3.2

1. The scores in mathematics test (out of 25) of 15 students is as follows:

   19, 25, 23, 20, 9, 20, 15, 10, 5, 16, 25, 20, 24, 12, 20

   Find the mode and median of this data. Are they same?
2. The runs scored in a cricket match by 11 players is as follows:

   6, 15, 120, 50, 100, 80, 10, 15, 8, 10, 15

   Find the mean, mode and median of this data. Are the three same?
3. The weights (in kg.) of 15 students of a class are:

   38, 42, 35, 37, 45, 50, 32, 43, 43, 40, 36, 38, 43, 38, 47

   (i) Find the mode and median of this data.

   (ii) Is there more than one mode?
4. Find the mode and median of the data: 13, 16, 12, 14, 19, 12, 14, 13, 14
5. Tell whether the statement is true or false:

   (i) The mode is always one of the numbers in a data.

   (ii) The mean is one of the numbers in a data.

   (iii) The median is always one of the numbers in a data.

   (iv) The data 6, 4, 3, 8, 9, 12, 13, 9 has mean 9.

3.8 USE OF BAR GRAPHS WITH A DIFFERENT PURPOSE

We have seen last year how information collected could be first arranged in a frequency distribution table and then this information could be put as a visual representation in the form of pictographs or bar graphs. You can look at the bar graphs and make deductions about the data. You can also get information based on these bar graphs. For example, you can say that the mode is the longest bar if the bar represents the frequency.

3.8.1 Choosing a Scale

We know that a bar graph is a representation of numbers using bars of uniform width and the lengths of the bars depend upon the frequency and the scale you have chosen. For example, in a bar graph where numbers in units are to be shown, the graph represents one unit length for one observation and if it has to show numbers in tens or hundreds, one unit length can represent 10 or 100 observations. Consider the following examples:

EXAMPLE 8 Two hundred students of 6th and 7th classes were asked to name their favourite colour so as to decide upon what should be the colour of their school building. The results are shown in the following table. Represent the given data on a bar graph.

Favourite Colour	Red	Green	Blue	Yellow	Orange
Number of Students	43	19	55	49	34

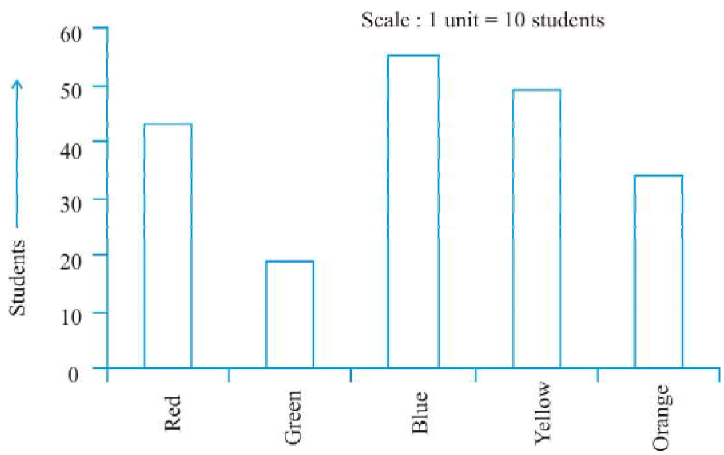
Answer the following questions with the help of the bar graph:

(i) Which is the most preferred colour and which is the least preferred?

(ii) How many colours are there in all? What are they?

SOLUTION Choose a suitable scale as follows:

Start the scale at 0. The greatest value in the data is 55, so end the scale at a value greater than 55, such as 60. Use equal divisions along the axes, such as increments of 10. You know that all the bars would lie between 0 and 60. We choose the scale such that the length between 0 and 60 is neither too long nor too small. Here we take 1 unit for 10 students.



We then draw and label the graph as shown.

From the bar graph we conclude that

(i) Blue is the most preferred colour (Because the bar representing Blue is the tallest).

(ii) Green is the least preferred colour. (Because the bar representing Green is the shortest).

(iii) There are five colours. They are Red, Green, Blue, Yellow and Orange. (These are observed on the horizontal line)

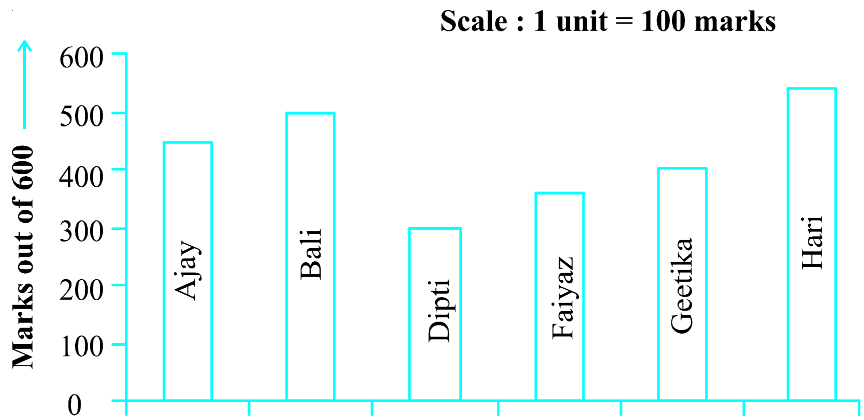
EXAMPLE 9 Following data gives total marks (out of 600) obtained by six children of a particular class. Represent the data on a bar graph.

Students	Ajay	Bali	Dipti	Faiyaz	Geetika	Hari
Marks Obtained	450	500	300	360	400	540

SOLUTION

(i) To choose an appropriate scale we make equal divisions taking increments of 100. Thus 1 unit will represent 100 marks. (What would be the difficulty if we choose one unit to represent 10 marks?)

(ii) Now represent the data on the bar graph.



Drawing double bar graph

Consider the following two collections of data giving the average daily hours of sunshine in two cities Aberdeen and Margate for all the twelve months of the year. These cities are near the south pole and hence have only a few hours of sunshine each day.

In Margate												
	Jan.	**Feb.**	**Mar.**	**April**	**May**	**June**	**July**	**Aug.**	**Sept.**	**Oct.**	**Nov.**	**Dec.**
Average hours of Sunshine	2	$3\frac{1}{4}$	4	4	$7\frac{3}{4}$	8	$7\frac{1}{2}$	7	$6\frac{1}{4}$	6	4	2
In Aberdeen												
Average hours of Sunshine	$1\frac{1}{2}$	3	$3\frac{1}{2}$	6	$5\frac{1}{2}$	$6\frac{1}{2}$	$5\frac{1}{2}$	5	$4\frac{1}{2}$	4	3	$1\frac{3}{4}$

By drawing individual bar graphs you could answer questions like

(i) In which month does each city has maximum sunlight? or

(ii) In which months does each city has minimum sunlight?

However, to answer questions like "In a particular month, which city has more sunshine hours", we need to compare the average hours of sunshine of both the cities. To do this we will learn to draw what is called a double bar graph giving the information of both cities side-by-side.

This bar graph (Fig 3.1) shows the average sunshine of both the cities.

For each month we have two bars, the heights of which give the average hours of sunshine in each city. From this we can infer that except for the month of April, there is always more sunshine in Margate than in Aberdeen. You could put together a similiar bar graph for your area or for your city.

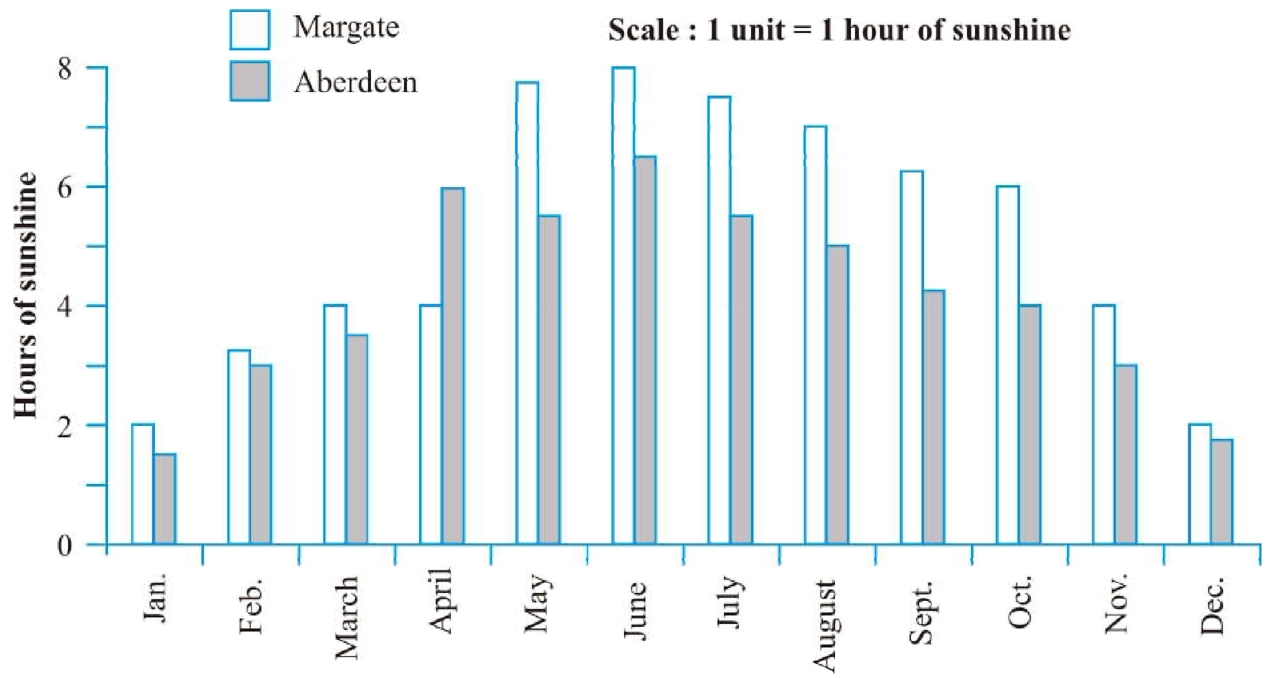
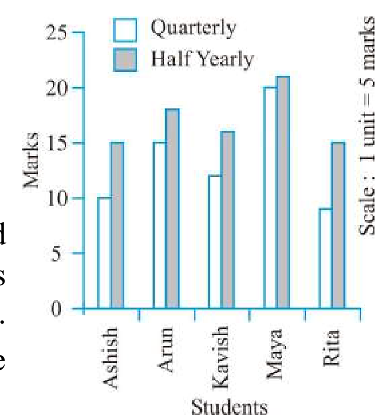


Fig 3.1

Let us look at another example more related to us.

EXAMPLE 10 A mathematics teacher wants to see, whether the new technique of teaching she applied after quarterly test was effective or not. She takes the scores of the 5 weakest children in the quarterly test (out of 25) and in the half yearly test (out of 25):

Students	**Ashish**	**Arun**	**Kavish**	**Maya**	**Rita**
Quarterly	10	15	12	20	9
Half yearly	15	18	16	21	15



SOLUTION She draws the adjoining double bar graph and finds a marked improvement in most of the students, the teacher decides that she should continue to use the new technique of teaching.

Can you think of a few more situations where you could use double bar graphs?

TRY THESE

1. The bar graph (Fig 3.2) shows the result of a survey to test water resistant watches made by different companies.

   Each of these companies claimed that their watches were water resistant. After a test the above results were revealed.

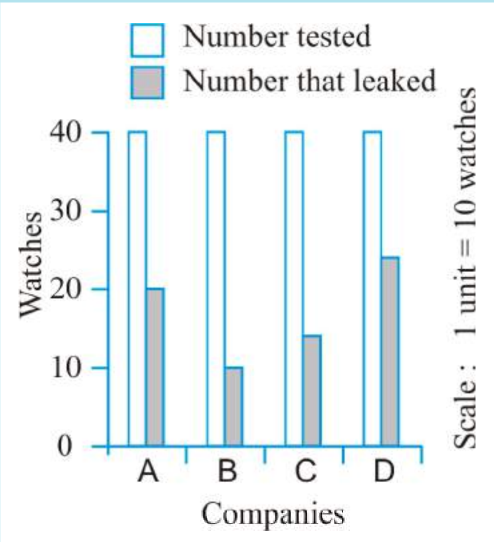


Fig 3.2

(a) Can you work out a fraction of the number of watches that leaked to the number tested for each company?

(b) Could you tell on this basis which company has better watches?

2. Sale of English and Hindi books in the years 1995, 1996, 1997 and 1998 are given below:

Years	**1995**	**1996**	**1997**	**1998**
English	350	400	450	620
Hindi	500	525	600	650

Draw a double bar graph and answer the following questions:

(a) In which year was the difference in the sale of the two language books least?.

(b) Can you say that the demand for English books rose faster? Justify.

Exercise 3.3

1. Use the bar graph (Fig 3.3) to answer the following questions.

(a) Which is the most popular pet? (b) How many students have dog as a pet?

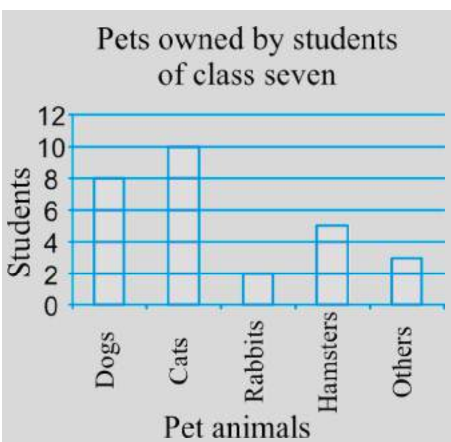


Fig 3.3

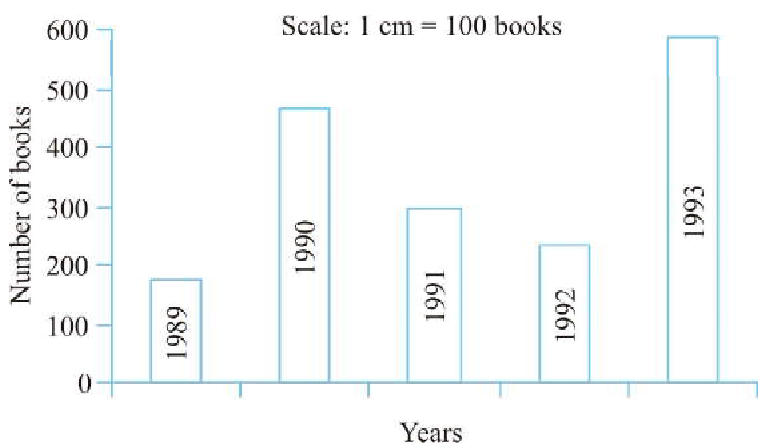


Fig 3.4

2. Read the bar graph (Fig 3.4) which shows the number of books sold by a bookstore during five consecutive years and answer the following questions:

(i) About how many books were sold in 1989? 1990? 1992?

(ii) In which year were about 475 books sold? About 225 books sold?

(iii) In which years were fewer than 250 books sold?

(iv) Can you explain how you would estimate the number of books sold in 1989?

3. Number of children in six different classes are given below. Represent the data on a bar graph.

Class	**Fifth**	**Sixth**	**Seventh**	**Eighth**	**Ninth**	**Tenth**
Number of Children	135	120	95	100	90	80

(a) How would you choose a scale?

(b) Answer the following questions:

(i) Which class has the maximum number of children? And the minimum?

(ii) Find the ratio of students of class sixth to the students of class eight.

4. The performance of a student in 1st Term and 2nd Term is given. Draw a double bar graph choosing appropriate scale and answer the following:

Subject	**English**	**Hindi**	**Maths**	**Science**	**S. Science**
1st Term (M.M. 100)	67	72	88	81	73
2nd Term (M.M. 100)	70	65	95	85	75

(i) In which subject, has the child improved his performance the most?

(ii) In which subject is the improvement the least?

(iii) Has the performance gone down in any subject?

5. Consider this data collected from a survey of a colony.

Favourite Sport	**Cricket**	**Basket Ball**	**Swimming**	**Hockey**	**Athletics**
Watching	1240	470	510	430	250
Participating	620	320	320	250	105

(i) Draw a double bar graph choosing an appropriate scale. What do you infer from the bar graph?

(ii) Which sport is most popular?

(iii) Which is more preferred, watching or participating in sports?

6. Take the data giving the minimum and the maximum temperature of various cities given in the beginning of this Chapter (Table 3.1). Plot a double bar graph using the data and answer the following:

(i) Which city has the largest difference in the minimum and maximum temperature on the given date?

(ii) Which is the hottest city and which is the coldest city?

(iii) Name two cities where maximum temperature of one was less than the minimum temperature of the other.

(iv) Name the city which has the least difference between its minimum and the maximum temperature.

TRY THESE

Think of some situations, atleast 3 examples of each, that are certain to happen, some that are impossible and some that may or may not happen i.e., situations that have some chance of happening.

3.9 CHANCE AND PROBABILITY

These words often come up in our daily life. We often say, "there is no chance of it raining today" and also say things like "it is quite probable that India will win the World Cup." Let us try and understand these terms a bit more. Consider the statements;

(i) The Sun coming up from the West (ii) An ant growing to 3 m height.

(iii) If you take a cube of larger volume its side will also be larger.

(iv) If you take a circle with larger area then it's radius will also be larger.

(v) India winning the next test series.

If we look at the statements given above you would say that the Sun coming up from the West is impossible, an ant growing to 3 m is also not possible. On the other hand if the circle is of a larger area it is certain that it will have a larger radius. You can say the same about the larger volume of the cube and the larger side. On the other hand India can win the next test series or lose it. Both are possible.

3.9.1 Chance

If you toss a coin, can you always correctly predict what you will get? Try tossing a coin and predicting the outcome each time. Write your observations in the following table:

Toss Number	Prediction	Outcome

Do this 10 times. Look at the observed outcomes. Can you see a pattern in them? What do you get after each head? Is it that you get head all the time? Repeat the observation for another 10 tosses and write the observations in the table.

You will find that the observations show no clear pattern. In the table below we give you observations generated in 25 tosses by Sushila and Salma. Here H represents Head and T represents Tail.

Numbers	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Outcome	H	T	T	H	T	T	T	H	T	T	H	H	H	H	H
Numbers	16	17	18	19	20	21	22	23	24	25					
Outcome	T	T	H	T	T	T	T	T	T	T					

What does this data tell you? Can you find a predictable pattern for head and tail? Clearly there is no fixed pattern of occurrence of head and tail. When you throw the coin each time the outcome of every throw can be either head or tail. It is a matter of chance that in one particular throw you get either of these.

In the above data, count the number of heads and the number of tails. Throw the coin some more times and keep recording what you obtain. Find out the total number of times you get a head and the total number of times you get a tail.

You also might have played with a die. The die has six faces. When you throw a die, can you predict the number that will be obtained? While playing ludo or snake and ladders you may have often wished that in a throw you get a particular outcome.

Does the die always fall according to your wishes? Take a die and throw it 150 times and fill the data in the following table:

Number on Die	Tally Marks	Number of Times it Occured
1		
2		

Make a tally mark each time you get the outcome, against the appropriate number. For example in the first throw you get 5. Put a tally in front of 5. The next throw gives you 1. Make a tally for 1. Keep on putting tally marks for the appropriate number. Repeat this exercise for 150 throws and find out the number of each outcome for 150 throws.

Make bar graph using the above data showing the number of times 1, 2, 3, 4, 5, 6 have occured in the data.

Try These

(Do in a group)

1. Toss a coin 100 times and record the data. Find the number of times heads and tails occur in it.
2. Aftaab threw a die 250 times and got the following table. Draw a bar graph for this data.

Number on the Die	Tally Marks
1	卌 卌 卌 卌 卌 卌 III
2	卌 卌 卌 卌 卌 卌 卌 卌
3	卌 卌 卌 卌 卌 卌 卌 卌 卌 II
4	卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 II
5	卌 卌 卌 卌 卌 卌 卌 III
6	卌 卌 卌 卌 卌 卌 卌 卌

3. Throw a die 100 times and record the data. Find the number of times 1, 2, 3, 4, 5, 6 occur.

What is probability?

We know that when a coin is thrown, it has two possible outcomes, Head or Tail and for a die we have 6 possible outcomes. We also know from experience that for a coin, Head or Tail is equally likely to be obtained. We say that the probability of getting Head or Tail is equal and is $\frac{1}{2}$ for each.

For a die, possibility of getting either of 1, 2, 3, 4, 5 or 6 is equal. That is for a die there are 6 equally likely possible outcomes. We say each of 1, 2, 3, 4, 5, 6 has one-sixth ($\frac{1}{6}$) probability. We will learn about this in the later classes. But from what we

TRY THESE

Construct or think of five situations where outcomes do not have equal chances.

have done, it may perhaps be obvious that events that have many possibilities can have probability between 0 and 1. Those which have no chance of happening have probability 0 and those that are bound to happen have probability 1.

Given any situation we need to understand the different possible outcomes and study the possible chances for each outcome. It may be possible that the outcomes may not have equal chance of occuring unlike the cases of the coin and die. For example, if a container has 15 red balls and 9 white balls and if a ball is pulled out without seeing, the chances of getting a red ball are much more. Can you see why? How many times are the chances of getting a red ball than getting a white ball, probabilities for both being between 0 and 1.

EXERCISE 3.4

1. Tell whether the following is certain to happen, impossible, can happen but not certain.
   (i) You are older today than yesterday. (ii) A tossed coin will land heads up.
   (iii) A die when tossed shall land up with 8 on top.
   (iv) The next traffic light seen will be green. (v) Tomorrow will be a cloudy day.
2. There are 6 marbles in a box with numbers from 1 to 6 marked on each of them.
   (i) What is the probability of drawing a marble with number 2?
   (ii) What is the probability of drawing a marble with number 5?
3. A coin is flipped to decide which team starts the game. What is the probability that your team will start?

WHAT HAVE WE DISCUSSED?

1. The collection, recording and presentation of data help us organise our experiences and draw inferences from them.
2. Before collecting data we need to know what we would use it for.
3. The data that is collected needs to be organised in a proper table, so that it becomes easy to understand and interpret.
4. Average is a number that represents or shows the central tendency of a group of observations or data.
5. Arithmetic mean is one of the representative values of data.
6. Mode is another form of central tendency or representative value. The mode of a set of observations is the observation that occurs most often.
7. Median is also a form of representative value. It refers to the value which lies in the middle of the data with half of the observations above it and the other half below it.
8. A bar graph is a representation of numbers using bars of uniform widths.
9. Double bar graphs help to compare two collections of data at a glance.
10. There are situations in our life, that are certain to happen, some that are impossible and some that may or may not happen. The situation that may or may not happen has a chance of happening.

Chapter 4

Simple Equations

4.1 A Mind-reading Game!

The teacher has said that she would be starting a new chapter in mathematics and it is going to be simple equations. Appu, Sarita and Ameena have revised what they learnt in algebra chapter in Class VI. Have you? Appu, Sarita and Ameena are excited because they have constructed a game which they call mind reader and they want to present it to the whole class.

The teacher appreciates their enthusiasm and invites them to present their game. Ameena begins; she asks Sara to think of a number, multiply it by 4 and add 5 to the product. Then, she asks Sara to tell the result. She says it is 65. Ameena instantly declares that the number Sara had thought of is 15. Sara nods. The whole class including Sara is surprised.

It is Appu's turn now. He asks Balu to think of a number, multiply it by 10 and subtract 20 from the product. He then asks Balu what his result is? Balu says it is 50. Appu immediately tells the number thought by Balu. It is 7, Balu confirms it.

Everybody wants to know how the 'mind reader' presented by Appu, Sarita and Ameena works. Can you see how it works? After studying this chapter and chapter 12, you will very well know how the game works.

4.2 Setting up of an Equation

Let us take Ameena's example. Ameena asks Sara to think of a number. Ameena does not know the number. For her, it could be anything 1, 2, 3, . . ., 11, . . . , 100, ... Let us denote this unknown number by a letter, say x. You may use y or t or some other letter in place of x. It does not matter which letter we use to denote the unknown number Sara has thought of. When Sara multiplies the number by 4, she gets $4x$. She then adds 5 to the product, which gives $4x + 5$. The value of $(4x + 5)$ depends on the value of x. Thus if $x = 1$, $4x + 5 = 4 \times 1 + 5 = 9$. This means that if Sara had 1 in her mind, her result would have been 9. Similarly, if she thought of 5, then for $x = 5$, $4x + 5 = 4 \times 5 + 5 = 25$; Thus if Sara had chosen 5, the result would have been 25.

To find the number thought by Sara let us work backward from her answer 65. We have to find x such that

$$4x + 5 = 65 \qquad (4.1)$$

Solution to the equation will give us the number which Sara held in her mind.

Let us similarly look at Appu's example. Let us call the number Balu chose as y. Appu asks Balu to multiply the number by 10 and subtract 20 from the product. That is, from y, Balu first gets $10y$ and from there $(10y – 20)$. The result is known to be 50.

Therefore, $$10y – 20 = 50 \qquad (4.2)$$

The solution of this equation will give us the number Balu had thought of.

4.3 Review of what We Know

Note, (4.1) and (4.2) are equations. Let us recall what we learnt about equations in Class VI. *An equation is a condition on a variable.* In equation (4.1), the variable is x; in equation (4.2), the variable is y.

The word *variable* means something that can vary, i.e. change. A **variable** *takes on different numerical values*; *its value is not fixed. Variables are denoted usually by letters of the alphabets, such as x, y, z, l, m, n, p, etc*. *From variables*, *we form expressions*. The expressions are formed by performing operations like addition, subtraction, multiplication and division on the variables. From x, we formed the expression $(4x + 5)$. For this, first we multiplied x by 4 and then added 5 to the product. Similarly, from y, we formed the expression $(10y – 20)$. For this, we multiplied y by 10 and then subtracted 20 from the product. All these are examples of expressions.

The value of an expression thus formed depends upon the chosen value of the variable. As we have already seen, when $x = 1$, $4x + 5 = 9$; when $x = 5$, $4x + 5 = 25$. Similarly,

when $x = 15$, $4x + 5 = 4×15 + 5 = 65$;

when $x = 0$, $4x + 5 = 4 × 0 + 5 = 5$; and so on.

Equation (4.1) is a condition on the variable x. It states that the value of the expression $(4x + 5)$ is 65. The condition is satisfied when $x = 15$. It is the solution to the equation $4x + 5 = 65$. When $x = 5$, $4x + 5 = 25$ and not 65. Thus $x = 5$ is not a solution to the equation. Similarly, $x = 0$ is not a solution to the equation. No value of x other than 15 satisfies the condition $4x + 5 = 65$.

Try These

The value of the expression $(10y – 20)$ depends on the value of y. Verify this by giving five different values to y and finding for each y the value of $(10y – 20)$. From the different values of $(10y – 20)$ you obtain, do you see a solution to $10y – 20 = 50$? If there is no solution, try giving more values to y and find whether the condition $10y – 20 = 50$ is met.

4.4 What Equation is?

In an equation there is always an **equality** sign. The equality sign shows that the value of the expression to the left of the sign (the left hand side or LHS) is equal to the value of the expression to the right of the sign (the right hand side or RHS). In equation (4.1), the LHS is $(4x + 5)$ and the RHS is 65. In equation (4.2), the LHS is $(10y - 20)$ and the RHS is 50.

> If there is some sign other than the equality sign between the LHS and the RHS, it is not an equation. Thus, $4x + 5 > 65$ is not an equation.
>
> It says that, the value of $(4x + 5)$ is greater than 65.
>
> Similarly, $4x + 5 < 65$ is not an equation. It says that the value of $(4x + 5)$ is smaller than 65.

In equations, we often find that the RHS is just a number. In Equation (4.1), it is 65 and in equation (4.2), it is 50. But this need not be always so. The RHS of an equation may be an expression containing the variable. For example, the equation

$$4x + 5 = 6x - 25$$

has the expression $(4x + 5)$ on the left and $(6x - 25)$ on the right of the equality sign.

In short, an equation is a condition on a variable. The condition is that two expressions should have equal value. Note that at least one of the two expressions must contain the variable.

We also note a simple and useful property of equations. The equation $4x + 5 = 65$ is the same as $65 = 4x + 5$. Similarly, the equation $6x - 25 = 4x + 5$ is the same as $4x + 5 = 6x - 25$. *An equation remains the same, when the expressions on the left and on the right are interchanged.* This property is often useful in solving equations.

Example 1 Write the following statements in the form of equations:

(i) The sum of three times x and 11 is 32.

(ii) If you subtract 5 from 6 times a number, you get 7.

(iii) One fourth of m is 3 more than 7.

(iv) One third of a number plus 5 is 8.

Solution

(i) Three times x is $3x$.

Sum of $3x$ and 11 is $3x + 11$. The sum is 32.

The equation is $3x + 11 = 32$.

(ii) Let us say the number is z; z multiplied by 6 is $6z$.

Subtracting 5 from $6z$, one gets $6z - 5$. The result is 7.

The equation is $6z - 5 = 7$

(iii) One fourth of m is $\frac{m}{4}$.

It is greater than 7 by 3. This means the difference ($\frac{m}{4} - 7$) is 3.

The equation is $\frac{m}{4} - 7 = 3$.

(iv) Take the number to be n. One third of n is $\frac{n}{3}$.

This one-third plus 5 is $\frac{n}{3} + 5$. It is 8.

The equation is $\frac{n}{3} + 5 = 8$.

EXAMPLE 2 Convert the following equations in statement form:

(i) $x - 5 = 9$ (ii) $5p = 20$ (iii) $3n + 7 = 1$ (iv) $\frac{m}{5} - 2 = 6$

SOLUTION (i) Taking away 5 from x gives 9.

(ii) Five times a number p is 20.

(iii) Add 7 to three times n to get 1.

(iv) You get 6, when you subtract 2 from one-fifth of a number m.

What is important to note is that for a given equation, **not just one, but many** statement forms can be given. For example, for Equation (i) above, you can say:

Subtract 5 from x, you get 9.

or The number x is 5 more than 9.

or The number x is greater by 5 than 9.

or The difference between x and 5 is 9, and so on.

TRY THESE

Write atleast one other form for each equation (ii), (iii) and (iv).

EXAMPLE 3 Consider the following situation:

Raju's father's age is 5 years more than three times Raju's age. Raju's father is 44 years old. Set up an equation to find Raju's age.

SOLUTION We do not know Raju's age. Let us take it to be y years. Three times Raju's age is $3y$ years. Raju's father's age is 5 years more than $3y$; that is, Raju's father is $(3y + 5)$ years old. It is also given that Raju's father is 44 years old.

Therefore, $3y + 5 = 44$ (4.3)

This is an equation in y. It will give Raju's age when solved.

EXAMPLE 4 A shopkeeper sells mangoes in two types of boxes, one small and one large. A large box contains as many as 8 small boxes plus 4 loose mangoes. Set up an equation which gives the number of mangoes in each small box. The number of mangoes in a large box is given to be 100.

SOLUTION Let a small box contain m mangoes. A large box contains 4 more than 8 times m, that is, 8m + 4 mangoes. But this is given to be 100. Thus

$$8m + 4 = 100 \quad (4.4)$$

You can get the number of mangoes in a small box by solving this equation.

EXERCISE 4.1

1. Complete the last column of the table.

S. No.	Equation	Value	Say, whether the Equation is Satisfied. (Yes/ No)
(i)	$x + 3 = 0$	$x = 3$	
(ii)	$x + 3 = 0$	$x = 0$	
(iii)	$x + 3 = 0$	$x = -3$	
(iv)	$x - 7 = 1$	$x = 7$	
(v)	$x - 7 = 1$	$x = 8$	
(vi)	$5x = 25$	$x = 0$	
(vii)	$5x = 25$	$x = 5$	
(viii)	$5x = 25$	$x = -5$	
(ix)	$\frac{m}{3} = 2$	$m = -6$	
(x)	$\frac{m}{3} = 2$	$m = 0$	
(xi)	$\frac{m}{3} = 2$	$m = 6$	

2. Check whether the value given in the brackets is a solution to the given equation or not:

(a) $n + 5 = 19$ $(n = 1)$ (b) $7n + 5 = 19$ $(n = -2)$ (c) $7n + 5 = 19$ $(n = 2)$

(d) $4p - 3 = 13$ $(p = 1)$ (e) $4p - 3 = 13$ $(p = -4)$ (f) $4p - 3 = 13$ $(p = 0)$

3. Solve the following equations by trial and error method:

(i) $5p + 2 = 17$ (ii) $3m - 14 = 4$

4. Write equations for the following statements:

(i) The sum of numbers x and 4 is 9. (ii) 2 subtracted from y is 8.

(iii) Ten times a is 70. (iv) The number b divided by 5 gives 6.

(v) Three-fourth of t is 15. (vi) Seven times m plus 7 gets you 77.

(vii) One-fourth of a number x minus 4 gives 4.

(viii) If you take away 6 from 6 times y, you get 60.

(ix) If you add 3 to one-third of z, you get 30.

5. Write the following equations in statement forms:

(i) $p + 4 = 15$ (ii) $m - 7 = 3$ (iii) $2m = 7$ (iv) $\frac{m}{5} = 3$

(v) $\frac{3m}{5} = 6$ (vi) $3p + 4 = 25$ (vii) $4p - 2 = 18$ (viii) $\frac{p}{2} + 2 = 8$

6. Set up an equation in the following cases:

(i) Irfan says that he has 7 marbles more than five times the marbles Parmit has. Irfan has 37 marbles. (Take m to be the number of Parmit's marbles.)

(ii) Laxmi's father is 49 years old. He is 4 years older than three times Laxmi's age. (Take Laxmi's age to be y years.)

(iii) The teacher tells the class that the highest marks obtained by a student in her class is twice the lowest marks plus 7. The highest score is 87. (Take the lowest score to be l.)

(iv) In an isosceles triangle, the vertex angle is twice either base angle. (Let the base angle be b in degrees. Remember that the sum of angles of a triangle is 180 degrees).

4.4.1 Solving an Equation

Consider an equality $\qquad 8 - 3 = 4 + 1 \qquad$ (4.5)

The equality (4.5) holds, since both its sides are equal (each is equal to 5).

- Let us now add 2 to both sides; as a result

  LHS = $8 - 3 + 2 = 5 + 2 = 7$ $\qquad$ RHS = $4 + 1 + 2 = 5 + 2 = 7$.

  Again the equality holds (i.e., its LHS and RHS are equal).

  Thus *if we add the same number to both sides of an equality, it still holds.*

- Let us now subtract 2 from both the sides; as a result,

  LHS = $8 - 3 - 2 = 5 - 2 = 3$ $\qquad$ RHS = $4 + 1 - 2 = 5 - 2 = 3$.

  Again, the equality holds.

  Thus *if we subtract the same number from both sides of an equality, it still holds.*

- Similarly, *if we multiply or divide both sides of the equality by the same non-zero number, it still holds.*

  For example, let us multiply both the sides of the equality by 3, we get

  LHS = $3 \times (8 - 3) = 3 \times 5 = 15$, RHS = $3 \times (4 + 1) = 3 \times 5 = 15$.

  The equality holds.

  Let us now divide both sides of the equality by 2.

  $$\text{LHS} = (8 - 3) \div 2 = 5 \div 2 = \frac{5}{2}$$

  $$\text{RHS} = (4+1) \div 2 = 5 \div 2 = \frac{5}{2} = \text{LHS}$$

  Again, the equality holds.

If we take any other equality, we shall find the same conclusions.

Suppose, we do not observe these rules. Specificially, suppose we add different numbers, to the two sides of an equality. We shall find in this case that the equality does not hold (i.e., its both sides are not equal). For example, let us take again equality (4.5),

$$8 - 3 = 4 + 1$$

add 2 to the LHS and 3 to the RHS. The new LHS is $8 - 3 + 2 = 5 + 2 = 7$ and the new RHS is $4 + 1 + 3 = 5 + 3 = 8$. The equality does not hold, because the new LHS and RHS are not equal.

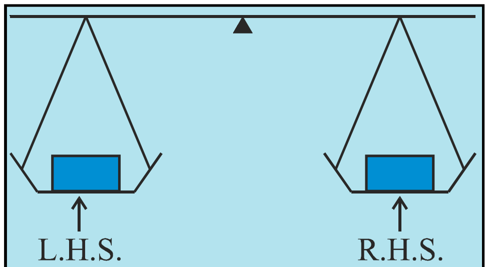
Thus if we fail to do the same mathematical operation on both sides of an equality, the equality does not hold.

The equality that involves variables is an equation.

These conclusions are also valid for equations, as in each equation variable represents a number only.

Often an equation is said to be like a weighing balance. Doing a mathematical operation on an equation is like adding weights to or removing weights from the pans of a weighing balance.

An equation is like a weighing balance with equal weights on both its pans, in which case the arm of the balance is exactly horizontal. If we add the same weights to both the pans, the arm remains horizontal. Similarly, if we remove the same weights from both the pans, the arm remains horizontal. On the other hand if we add different weights to the pans or remove different weights from them, the balance is tilted; that is, the arm of the balance does not remain horizontal.



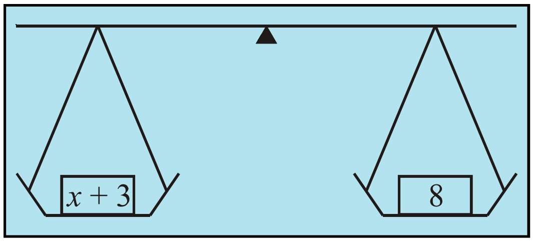
A balanced equation is like a weighing balance with equal weights in the two pans.

We use this principle for solving an equation. Here, ofcourse, the balance is imaginary and numbers can be used as weights that can be physically balanced against each other. This is the real purpose in presenting the principle. Let us take some examples.

- Consider the equation: $x + 3 = 8$ (4.6)

  We shall subtract 3 from both sides of this equation.

  The new LHS is $x + 3 - 3 = x$ and the new RHS is $8 - 3 = 5$



Why should we subtract 3, and not some other number? Try adding 3. Will it help? Why not?

It is because subtracting 3 reduces the LHS to x.

Since this does not disturb the balance, we have

$$\text{New LHS} = \text{New RHS} \quad \text{or} \quad x = 5$$

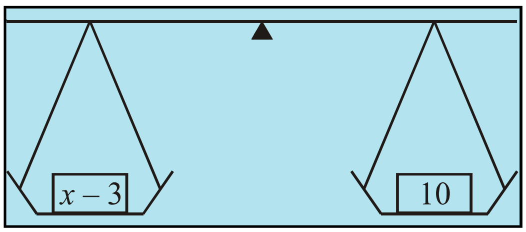
which is exactly what we want, the solution of the equation (4.6).

To confirm whether we are right, we shall put $x = 5$ in the original equation. We get LHS $= x + 3 = 5 + 3 = 8$, which is equal to the RHS as required.

By doing the right mathematical operation (i.e., subtracting 3) on both the sides of the equation, we arrived at the solution of the equation.

- Let us look at another equation $\qquad x - 3 = 10 \qquad$ (4.7)

What should we do here? We should add 3 to both the sides, By doing so, we shall retain the balance and also the LHS will reduce to just x.



New LHS $= x - 3 + 3 = x$, New RHS $= 10 + 3 = 13$

Therefore, $x = 13$, which is the required solution.

By putting $x = 13$ in the original equation (4.7) we confirm that the solution is correct:

LHS of original equation $= x - 3 = 13 - 3 = 10$

This is equal to the RHS as required.

- Similarly, let us look at the equations

$$5y = 35 \qquad (4.8)$$

$$\frac{m}{2} = 5 \qquad (4.9)$$

In the first case, we shall divide both the sides by 5. This will give us just y on LHS

$$\text{New LHS} = \frac{5y}{5} = \frac{5 \times y}{5} = y\,, \qquad \text{New RHS} = \frac{35}{5} = \frac{5 \times 7}{5} = 7$$

Therefore, $\qquad y = 7$

This is the required solution. We can substitute $y = 7$ in Eq. (4.8) and check that it is satisfied.

In the second case, we shall multiply both sides by 2. This will give us just m on the LHS

The new LHS $= \frac{m}{2} \times 2 = m$. The new RHS $= 5 \times 2 = 10$.

Hence, $m = 10$ (It is the required solution. You can check whether the solution is correct).

One can see that in the above examples, the operation we need to perform depends on the equation. Our attempt should be to get the variable in the equation separated. Sometimes, for doing so we may have to carry out more than one mathematical operation. Let us solve some more equations with this in mind.

EXAMPLE 5 Solve: (a) $3n + 7 = 25 \qquad$ (4.10)

(b) $2p - 1 = 23 \qquad$ (4.11)

SOLUTION

(a) We go stepwise to separate the variable n on the LHS of the equation. The LHS is $3n + 7$. We shall first subtract 7 from it so that we get $3n$. From this, in the next step we shall divide by 3 to get n. Remember we must do the same operation on both sides of the equation. Therefore, subtracting 7 from both sides,

$$3n + 7 - 7 = 25 - 7 \qquad \text{(Step 1)}$$

or $\qquad 3n = 18$

Now divide both sides by 3,

$$\frac{3n}{3} = \frac{18}{3} \qquad \text{(Step 2)}$$

or $n = 6$, which is the solution.

(b) What should we do here? First we shall add 1 to both the sides:

$$2p - 1 + 1 = 23 + 1 \qquad \text{(Step 1)}$$

or $2p = 24$

Now divide both sides by 2, we get $\frac{2p}{2} = \frac{24}{2}$ (Step 2)

or $p = 12$, which is the solution.

One good practice you should develop is to check the solution you have obtained. Although we have not done this for (a) above, let us do it for this example.

Let us put the solution $p = 12$ back into the equation.

$$\text{LHS} = 2p - 1 = 2 \times 12 - 1 = 24 - 1$$
$$= 23 = \text{RHS}$$

The solution is thus checked for its correctness.

Why do you not check the solution of (a) also?

We are now in a position to go back to the mind-reading game presented by Appu, Sarita, and Ameena and understand how they got their answers. For this purpose, let us look at the equations (4.1) and (4.2) which correspond respectively to Ameena's and Appu's examples.

- First consider the equation $4x + 5 = 65$. (4.1)

  Subtracting 5 from both sides, $4x + 5 - 5 = 65 - 5$.

  i.e. $4x = 60$

  Divide both sides by 4; this will separate x. We get $\frac{4x}{4} = \frac{60}{4}$

  or $x = 15$, which is the solution. (Check, if it is correct.)

- Now consider, $10y - 20 = 50$ (4.2)

  Adding 20 to both sides, we get $10y - 20 + 20 = 50 + 20$ or $10y = 70$

  Dividing both sides by 10, we get $\frac{10y}{10} = \frac{70}{10}$

  or $y = 7$, which is the solution. (Check if it is correct.)

You will realise that exactly these were the answers given by Appu, Sarita and Ameena. They had learnt to set up equations and solve them. That is why they could construct their mind reader game and impress the whole class. We shall come back to this in Section 4.7.

EXERCISE 4.2

1. Give first the step you will use to separate the variable and then solve the equation:
   (a) $x - 1 = 0$ (b) $x + 1 = 0$ (c) $x - 1 = 5$ (d) $x + 6 = 2$
   (e) $y - 4 = -7$ (f) $y - 4 = 4$ (g) $y + 4 = 4$ (h) $y + 4 = -4$
2. Give first the step you will use to separate the variable and then solve the equation:
   (a) $3l = 42$ (b) $\frac{b}{2} = 6$ (c) $\frac{p}{7} = 4$ (d) $4x = 25$
   (e) $8y = 36$ (f) $\frac{z}{3} = \frac{5}{4}$ (g) $\frac{a}{5} = \frac{7}{15}$ (h) $20t = -10$
3. Give the steps you will use to separate the variable and then solve the equation:
   (a) $3n - 2 = 46$ (b) $5m + 7 = 17$ (c) $\frac{20p}{3} = 40$ (d) $\frac{3p}{10} = 6$
4. Solve the following equations:
   (a) $10p = 100$ (b) $10p + 10 = 100$ (c) $\frac{p}{4} = 5$ (d) $\frac{-p}{3} = 5$
   (e) $\frac{3p}{4} = 6$ (f) $3s = -9$ (g) $3s + 12 = 0$ (h) $3s = 0$
   (i) $2q = 6$ (j) $2q - 6 = 0$ (k) $2q + 6 = 0$ (l) $2q + 6 = 12$

4.5 More Equations

Let us practise solving some more equations. While solving these equations, we shall learn about transposing a number, i.e., moving it from one side to the other. We can transpose a number instead of adding or subtracting it from both sides of the equation.

Example 6 Solve: $12p - 5 = 25$ (4.12)

Solution

- Adding 5 on both sides of the equation,
  $12p - 5 + 5 = 25 + 5$ or $12p = 30$
- Dividing both sides by 12,
  $\frac{12p}{12} = \frac{30}{12}$ or $p = \frac{5}{2}$

> *Note, adding 5 to both sides is the same as changing side of (– 5).*
> $12p - 5 = 25$
> $12p = 25 + 5$
> *Changing side is called* ***transposing****. While transposing a number, we change its sign.*

Check Putting $p = \frac{5}{2}$ in the LHS of equation 4.12,

$$\text{LHS} = 12 \times \frac{5}{2} - 5 = 6 \times 5 - 5$$
$$= 30 - 5 = 25 = \text{RHS}$$

As we have seen, while solving equations one commonly used operation is adding or subtracting the same number on both sides of the equation. *Transposing a number (i.e., changing the side of the number) is the same as adding or subtracting the number from both sides.* In doing so, the sign of the number has to be changed. What applies to numbers also applies to expressions. Let us take two more examples of transposing.

Adding or Subtracting on both sides	**Transposing**
(i) $3p - 10 = 5$	(i) $3p - 10 = 5$
Add 10 to both sides	Transpose (–10) from LHS to RHS
	(On transposing – 10 becomes + 10).
$3p - 10 + 10 = 5 + 10$	$3p = 5 + 10$ or $3p = 15$
or $3p = 15$	
(ii) $5x + 12 = 27$	(ii) $5x + 12 = 27$
Subtract 12 from both sides	Transposing + 12
	(On transposing + 12 becomes – 12)
$5x + 12 - 12 = 27 - 12$	$5x = 27 - 12$
or $5x = 15$	or $5x = 15$

We shall now solve two more equations. As you can see they involve brackets, which have to be solved before proceeding.

EXAMPLE 7 Solve

(a) $4(m + 3) = 18$ (b) $-2(x + 3) = 8$

SOLUTION

(a) $4(m + 3) = 18$

Let us divide both the sides by 4. This will remove the brackets in the LHS We get,

$$m + 3 = \frac{18}{4} \quad \text{or} \quad m + 3 = \frac{9}{2}$$

or $m = \frac{9}{2} - 3$ (transposing 3 to RHS)

or $m = \frac{3}{2}$ (required solution) $\left(\text{as } \frac{9}{2} - 3 = \frac{9}{2} - \frac{6}{2} = \frac{3}{2}\right)$

Check LHS $= 4\left[\frac{3}{2} + 3\right] = 4 \times \frac{3}{2} + 4 \times 3 = 2 \times 3 + 4 \times 3$ [put $m = \frac{3}{2}$]

$= 6 + 12 = 18 =$ RHS

(b) $-2(x + 3) = 8$

We divide both sides by (– 2), so as to remove the brackets in the LHS, we get,

$$x + 3 = -\frac{8}{2} \quad \text{or} \quad x + 3 = -4$$

i.e., $x = -4 - 3$ (transposing 3 to RHS) or $x = -7$ (required solution)

Check LHS = $-2(-7+3) = -2(-4)$

$= 8 = \text{RHS}$ as required.

4.6 From Solution to Equation

Atul always thinks differently. He looks at successive steps that one takes to solve an equation. He wonders why not follow the reverse path:

Equation	⟶	Solution	(normal path)
Solution	⟶	Equation	(reverse path)

He follows the path given below:

Start with	↓	$x = 5$	↑	
Multiply both sides by 4,		$4x = 20$		Divide both sides by 4.
Subtract 3 from both sides,	↓	$4x - 3 = 17$	↑	Add 3 to both sides.

This has resulted in an equation. If we follow the reverse path with each step, as shown on the right, we get the solution of the equation.

Try These

Start with the same step $x = 5$ and make two different equations. Ask two of your classmates to solve the equations. Check whether they get the solution $x = 5$.

Hetal feels interested. She starts with the same first step and builds up another equation.

	$x = 5$
Multiply both sides by 3	$3x = 15$
Add 4 to both sides	$3x + 4 = 19$

Start with $y = 4$ and make two different equations. Ask three of your friends to do the same. Are their equations different from yours?

> *Is it not nice that not only can you solve an equation, but you can make equations? Further, did you notice that given an equation, you get one solution; but given a solution, you can make many equations?*

Now, Sara wants the class to know what she is thinking. She says, "I shall take Hetal's equation and put it into a statement form and that makes a puzzle. For example, think of a number; multiply it by 3 and add 4 to the product. Tell me the sum you get.

If the sum is 19, the equation Hetal got will give us the solution to the puzzle. In fact, we know it is 5, because Hetal started with it."

She turns to Appu, Ameena and Sarita to check whether they made their puzzle this way. All three say, "Yes!"

We now know how to create number puzzles and many other similar problems.

Try These

Try to make two number puzzles, one with the solution 11 and another with 100

EXERCISE 4.3

1. Solve the following equations:

(a) $2y+\frac{5}{2}=\frac{37}{2}$ (b) $5t + 28 = 10$ (c) $\frac{a}{5}+3=2$ (d) $\frac{q}{4}+7=5$

(e) $\frac{5}{2}x=-10$ (f) $\frac{5}{2}x=\frac{25}{4}$ (g) $7m+\frac{19}{2}=13$ (h) $6z + 10 = -2$

(i) $\frac{3l}{2}=\frac{2}{3}$ (j) $\frac{2b}{3}-5=3$

2. Solve the following equations:

(a) $2(x + 4) = 12$ (b) $3(n - 5) = 21$ (c) $3(n - 5) = - 21$

(d) $- 4(2 + x) = 8$ (e) $4(2 - x) = 8$

3. Solve the following equations:

(a) $4 = 5(p - 2)$ (b) $- 4 = 5(p - 2)$ (c) $16 = 4 + 3(t + 2)$

(d) $4 + 5(p - 1) = 34$ (e) $0 = 16 + 4(m - 6)$

4. (a) Construct 3 equations starting with $x = 2$

(b) Construct 3 equations starting with $x = - 2$

4.7 APPLICATIONS OF SIMPLE EQUATIONS TO PRACTICAL SITUATIONS

We have already seen examples in which we have taken statements in everyday language and converted them into simple equations. We also have learnt how to solve simple equations. Thus we are ready to solve puzzles/problems from practical situations. The method is first to form equations corresponding to such situations and then to solve those equations to give the solution to the puzzles/problems. We begin with what we have already seen [Example 1 (i) and (iii), Section 4.2].

EXAMPLE 8 The sum of three times a number and 11 is 32. Find the number.

SOLUTION

- If the unknown number is taken to be x, then three times the number is $3x$ and the sum of $3x$ and 11 is 32. That is, $3x + 11 = 32$

This equation was obtained earlier in Section 4.2, Example 1.

- To solve this equation, we transpose 11 to RHS, so that

  $3x = 32 - 11$ or $3x = 21$

  Now, divide both sides by 3

  So $x = \frac{21}{3} = 7$

The required number is 7. (We may check it by taking 3 times 7 and adding 11 to it. It gives 32 as required.)

EXAMPLE 9 Find a number, such that one-fourth of the number is 3 more than 7.

SOLUTION

- Let us take the unknown number to be y; one-fourth of y is $\frac{y}{4}$.

  This number $\left(\frac{y}{4}\right)$ is more than 7 by 3.

  Hence we get the equation for y as $\frac{y}{4} - 7 = 3$

TRY THESE

(i) When you multiply a number by 6 and subtract 5 from the product, you get 7. Can you tell what the number is?

(ii) What is that number one third of which added to 5 gives 8?

- To solve this equation, first transpose 7 to RHS We get,

  $\frac{y}{4} = 3 + 7 = 10.$

  We then multiply both sides of the equation by 4, to get

  $\frac{y}{4} \times 4 = 10 \times 4$ or $y = 40$ (the required number)

  Let us check the equation formed. Putting the value of y in the equation,

  LHS $= \frac{40}{4} - 7 = 10 - 7 = 3 =$ RHS, as required.

EXAMPLE 10 Raju's father's age is 5 years more than three times Raju's age. Find Raju's age, if his father is 44 years old.

SOLUTION

- As given in Example 3 earlier, the equation that gives Raju's age is

  $$3y + 5 = 44$$

- To solve it, we first transpose 5, to get $3y = 44 - 5 = 39$

  Dividing both sides by 3, we get $y = 13$

  That is, Raju's age is 13 years. (You may check the answer.)

TRY THESE

There are two types of boxes containing mangoes. Each box of the larger type contains 4 more mangoes than the number of mangoes contained in 8 boxes of the smaller type. Each larger box contains 100 mangoes. Find the number of mangoes contained in the smaller box?

EXERCISE 4.4

1. Set up equations and solve them to find the unknown numbers in the following cases:
   (a) Add 4 to eight times a number; you get 60.
   (b) One-fifth of a number minus 4 gives 3.
   (c) If I take three-fourths of a number and add 3 to it, I get 21.
   (d) When I subtracted 11 from twice a number, the result was 15.
   (e) Munna subtracts thrice the number of notebooks he has from 50, he finds the result to be 8.
   (f) Ibenhal thinks of a number. If she adds 19 to it and divides the sum by 5, she will get 8.
   (g) Anwar thinks of a number. If he takes away 7 from $\frac{5}{2}$ of the number, the result is 23.
2. Solve the following:
   (a) The teacher tells the class that the highest marks obtained by a student in her class is twice the lowest marks plus 7. The highest score is 87. What is the lowest score?
   (b) In an isosceles triangle, the base angles are equal. The vertex angle is 40°. What are the base angles of the triangle? (Remember, the sum of three angles of a triangle is 180°).
   (c) Sachin scored twice as many runs as Rahul. Together, their runs fell two short of a double century. How many runs did each one score?
3. Solve the following:
   (i) Irfan says that he has 7 marbles more than five times the marbles Parmit has. Irfan has 37 marbles. How many marbles does Parmit have?
   (ii) Laxmi's father is 49 years old. He is 4 years older than three times Laxmi's age. What is Laxmi's age?
   (iii) People of Sundargram planted trees in the village garden. Some of the trees were fruit trees. The number of non-fruit trees were two more than three times the number of fruit trees. What was the number of fruit trees planted if the number of non-fruit trees planted was 77?
4. Solve the following riddle:

I am a number,
Tell my identity!
Take me seven times over
And add a fifty!
To reach a triple century
You still need forty!

What have We Discussed?

1. An equation is a condition on a variable such that two expressions in the variable should have equal value.
2. The value of the variable for which the equation is satisfied is called the solution of the equation.
3. An equation remains the same if the LHS and the RHS are interchanged.
4. In case of the balanced equation, if we

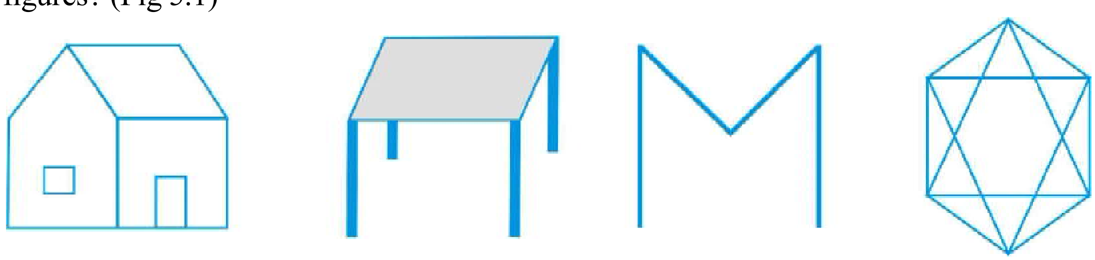
   (i) add the same number to both the sides, or (ii) subtract the same number from both the sides, or (iii) multiply both sides by the same number, or (iv) divide both sides by the same number, the balance remains undisturbed, i.e., the value of the LHS remains equal to the value of the RHS
5. The above property gives a systematic method of solving an equation. We carry out a series of identical mathematical operations on the two sides of the equation in such a way that on one of the sides we get just the variable. The last step is the solution of the equation.
6. Transposing means moving to the other side. Transposition of a number has the same effect as adding same number to (or subtracting the same number from) both sides of the equation. When you transpose a number from one side of the equation to the other side, you change its sign. For example, transposing +3 from the LHS to the RHS in equation $x + 3 = 8$ gives $x = 8 – 3$ (= 5). We can carry out the transposition of an expression in the same way as the transposition of a number.
7. We have learnt how to construct simple algebraic expressions corresponding to practical situations.
8. We also learnt how, using the technique of doing the same mathematical operation (for example adding the same number) on both sides, we could build an equation starting from its solution. Further, we also learnt that we could relate a given equation to some appropriate practical situation and build a practical word problem/puzzle from the equation.

Chapter 5

Lines and Angles

5.1 Introduction

You already know how to identify different lines, line segments and angles in a given shape. Can you identify the different line segments and angles formed in the following figures? (Fig 5.1)



(i) (ii) (iii) (iv)

Fig 5.1

Can you also identify whether the angles made are acute or obtuse or right?

Recall that a **line segment** has two end **points**. If we extend the two end points in either direction endlessly, we get a **line**. Thus, we can say that a line has no end points. On the other hand, recall that a ray has one end point (namely its starting point). For example, look at the figures given below:

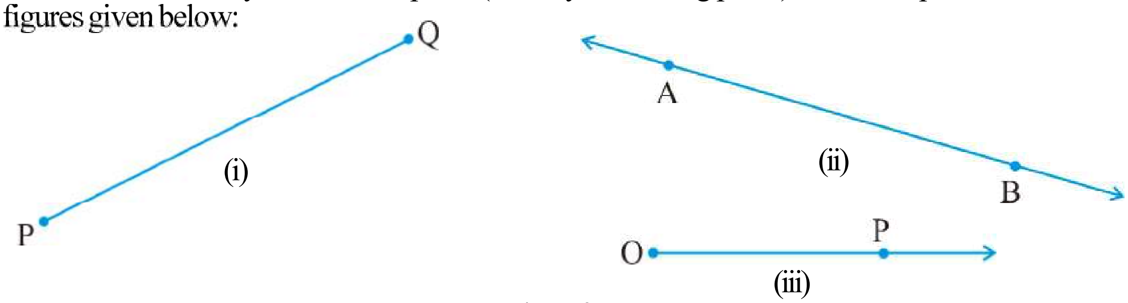


Fig 5.2

Here, Fig 5.2 (i) shows a **line segment,** Fig 5.2 (ii) shows a **line** and Fig 5.2 (iii) is that of a **ray**. A line segment PQ is generally denoted by the symbol $\overline{PQ}$, a line AB is denoted by the symbol $\overleftrightarrow{AB}$ and the ray OP is denoted by $\overrightarrow{OP}$. Give some examples of line segments and rays from your daily life and discuss them with your friends.

Again recall that an **angle** is formed when lines or line segments meet. In Fig 5.1, observe the corners. These corners are formed when two lines or line segments intersect at a point. For example, look at the figures given below:

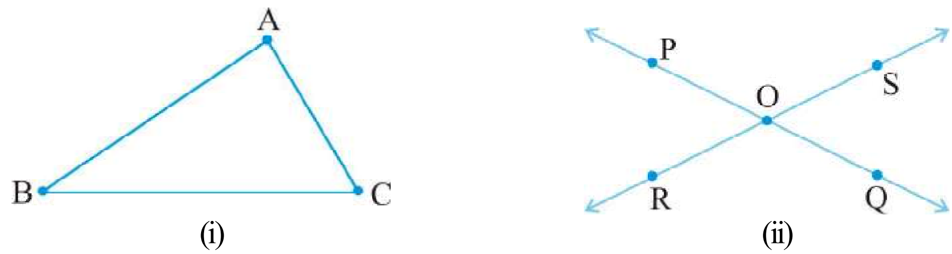


Fig 5.3

In Fig 5.3 (i) line segments AB and BC intersect at B to form angle ABC, and again line segments BC and AC intersect at C to form angle ACB and so on. Whereas, in Fig 5.3 (ii) lines PQ and RS intersect at O to form four angles POS, SOQ, QOR and ROP. An angle ABC is represented by the symbol ∠ABC. Thus, in Fig 5.3 (i), the three angles formed are ∠ABC, ∠BCA and ∠BAC, and in Fig 5.3 (ii), the four angles formed are ∠POS, ∠SOQ, ∠QOR and ∠POR. You have already studied how to classify the angles as acute, obtuse or right angle.

TRY THESE

List ten figures around you and identify the acute, obtuse and right angles found in them.

Note: While referring to the measure of an angle ABC, we shall write m∠ABC as simply ∠ABC. The context will make it clear, whether we are referring to the angle or its measure.

5.2 Related Angles

5.2.1 Complementary Angles

When the sum of the measures of two angles is 90°, the angles are called **complementary angles.**

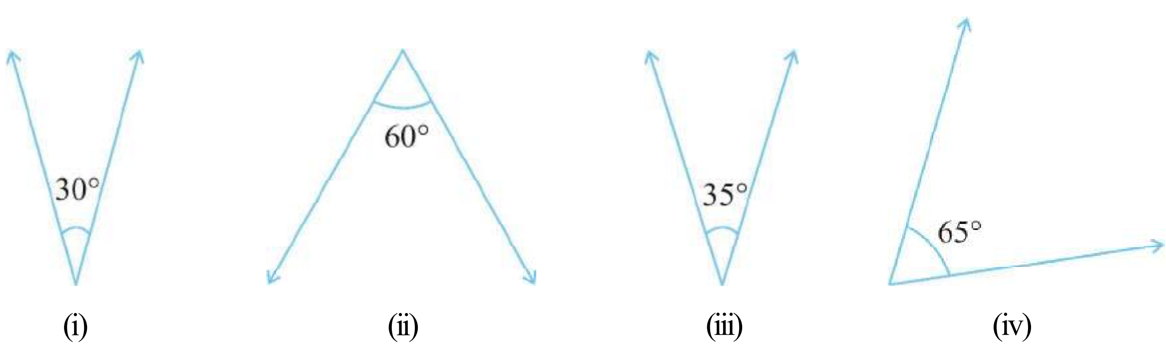


Fig 5.4

Whenever two angles are complementary, each angle is said to be the **complement** of the other angle. In the above diagram (Fig 5.4), the '30° angle' is the complement of the '60° angle' and vice versa.

Think, Discuss and Write

1. Can two acute angles be complement to each other?
2. Can two obtuse angles be complement to each other?
3. Can two right angles be complement to each other?

Try These

1. Which pairs of following angles are complementary? (Fig 5.5)

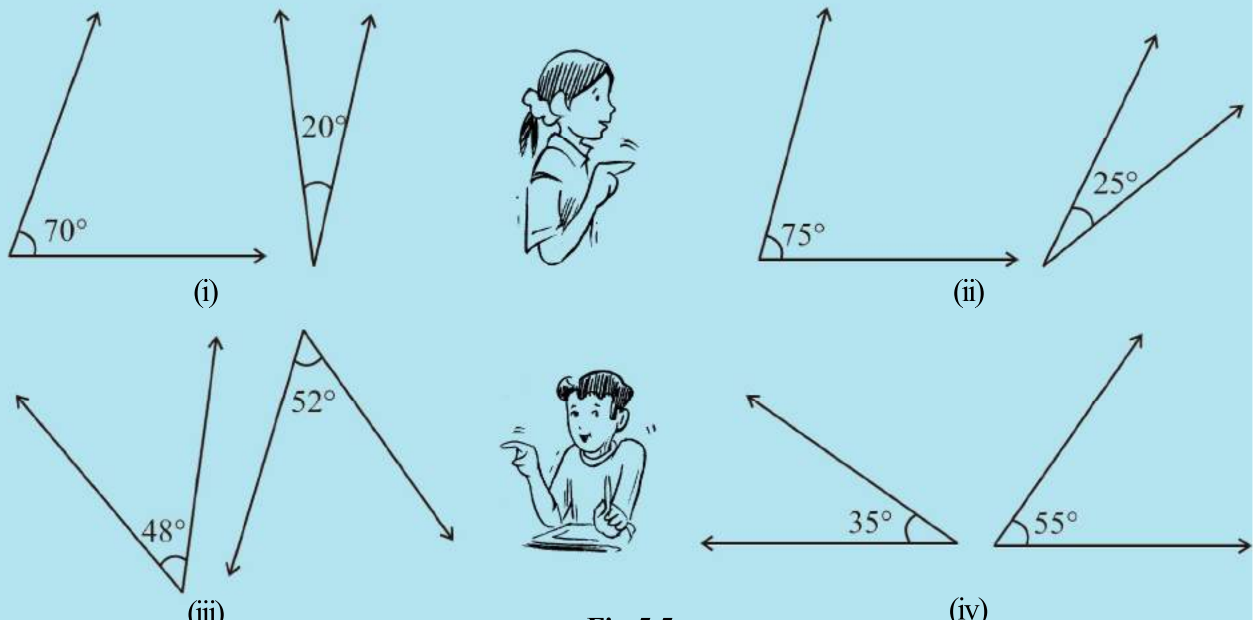
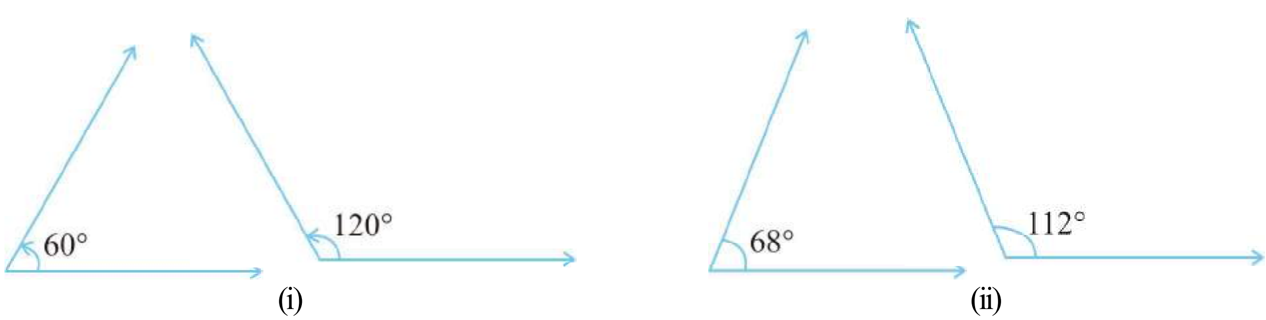


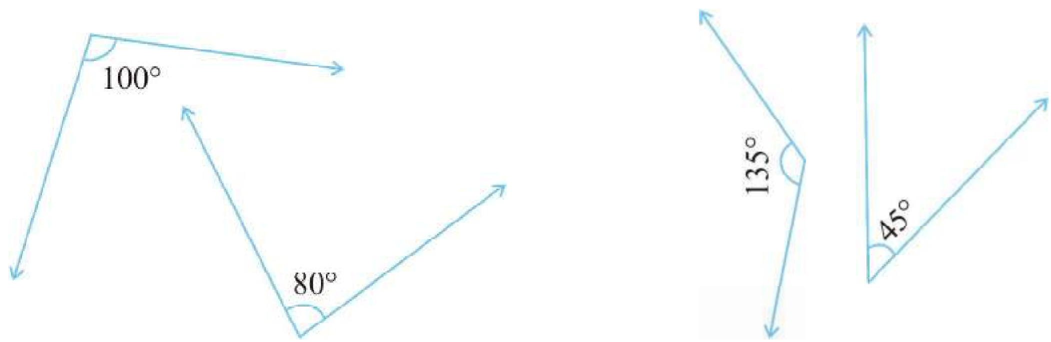
Fig 5.5

2. What is the measure of the complement of each of the following angles?
   (i) 45° (ii) 65° (iii) 41° (iv) 54°
3. The difference in the measures of two complementary angles is 12°. Find the measures of the angles.

5.2.2 Supplementary Angles

Let us now look at the following pairs of angles (Fig 5.6):





(iii) Fig 5.6 (iv)

Do you notice that the sum of the measures of the angles in each of the above pairs (Fig 5.6) comes out to be 180º? Such pairs of angles are called **supplementary angles**. When two angles are supplementary, each angle is said to be the **supplement** of the other.

Think, Discuss and Write

1. Can two obtuse angles be supplementary?
2. Can two acute angles be supplementary?
3. Can two right angles be supplementary?

Try These

1. Find the pairs of supplementary angles in Fig 5.7:

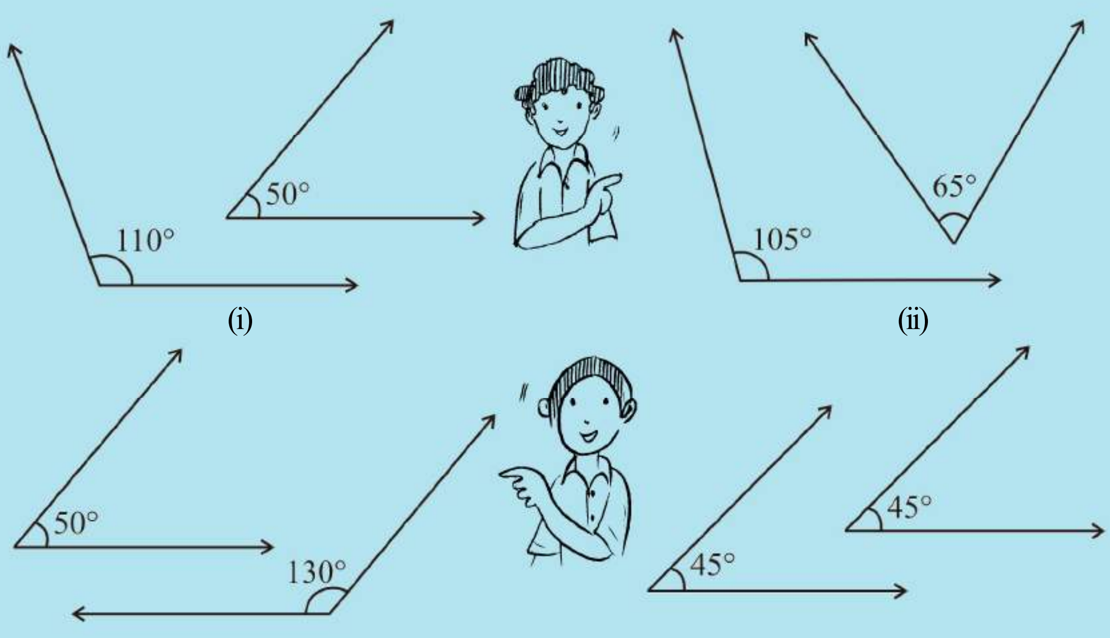


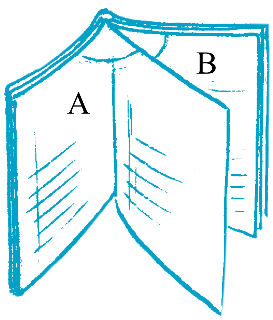
Fig 5.7

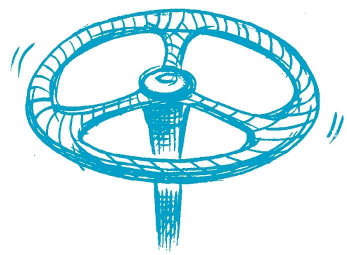
2. What will be the measure of the supplement of each one of the following angles?
(i) 100º (ii) 90º (iii) 55º (iv) 125º

3. Among two supplementary angles the measure of the larger angle is 44º more than the measure of the smaller. Find their measures.

5.2.3. Adjacent Angles

Look at the following figures:





When you open a book it looks like the above figure. In A and B, we find a pair of angles, placed next to each other.

Look at this steering wheel of a car. At the centre of the wheel you find three angles being formed, lying next to one another.

Fig 5.8

At both the vertices A and B, we find, a pair of angles are placed next to each other.

These angles are such that:

(i) they have a common vertex;

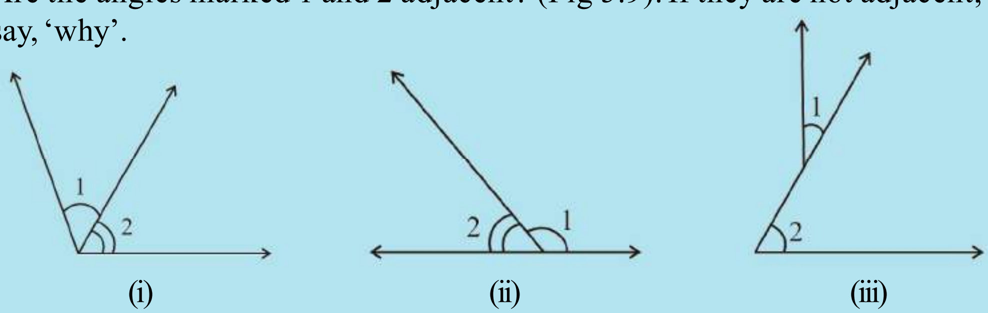
(ii) they have a common arm; and

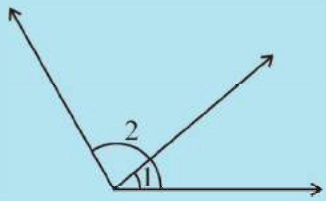
(iii) the non-common arms are on either side of the common arm.

Such pairs of angles are called **adjacent angles.** Adjacent angles have a common vertex and a common arm but no common interior points.

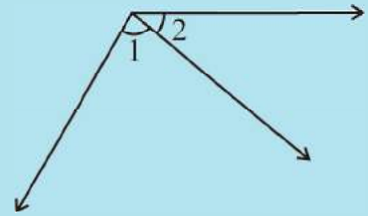
TRY THESE

1. Are the angles marked 1 and 2 adjacent? (Fig 5.9). If they are not adjacent, say, 'why'.





(iv)



(v)

Fig 5.9

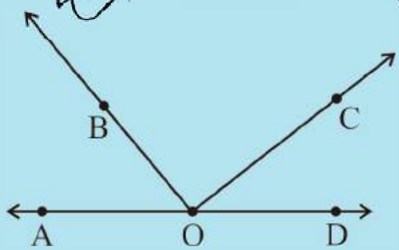


Fig 5.10

2. In the given Fig 5.10, are the following adjacent angles?

(a) ∠AOB and ∠BOC

(b) ∠BOD and ∠BOC

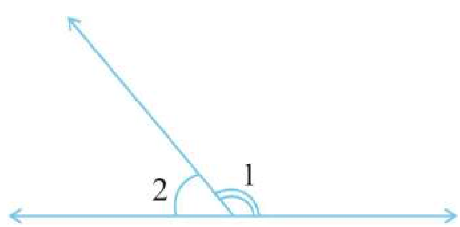
Justify your answer.

Think, Discuss and Write

1. Can two adjacent angles be supplementary?
2. Can two adjacent angles be complementary?
3. Can two obtuse angles be adjacent angles?
4. Can an acute angle be adjacent to an obtuse angle?

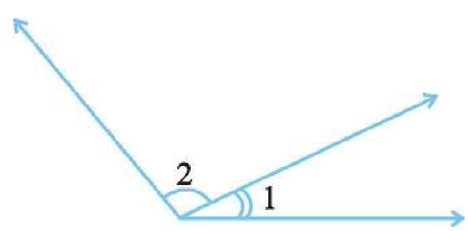
5.2.4 Linear Pair

A linear pair is a pair of adjacent angles whose non-common sides are opposite rays.



Are ∠1, ∠2 a linear pair? Yes

(i)



Are ∠1, ∠2 a linear pair? No! (Why?)

(ii)

Fig 5.11

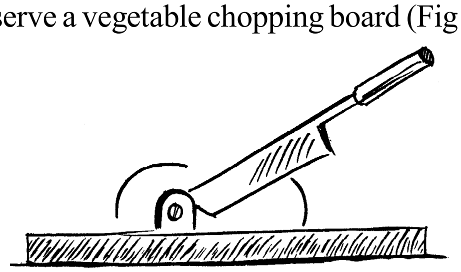
In Fig 5.11 (i) above, observe that the opposite rays (which are the non-common sides of ∠1 and ∠2) form a line. Thus, ∠1 + ∠2 amounts to 180°.

The angles in a linear pair are supplementary.

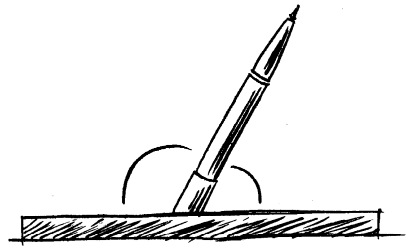
Have you noticed models of a linear pair in your environment?

Note carefully that a pair of supplementary angles form a linear pair when placed adjacent to each other. Do you find examples of linear pair in your daily life?

Observe a vegetable chopping board (Fig 5.12).



A vegetable chopping board
The chopping blade makes a linear pair of angles with the board.



A pen stand
The pen makes a linear pair of angles with the stand.

Fig 5.12

THINK, DISCUSS AND WRITE

1. Can two acute angles form a linear pair?
2. Can two obtuse angles form a linear pair?
3. Can two right angles form a linear pair?

TRY THESE

Check which of the following pairs of angles form a linear pair (Fig 5.13):

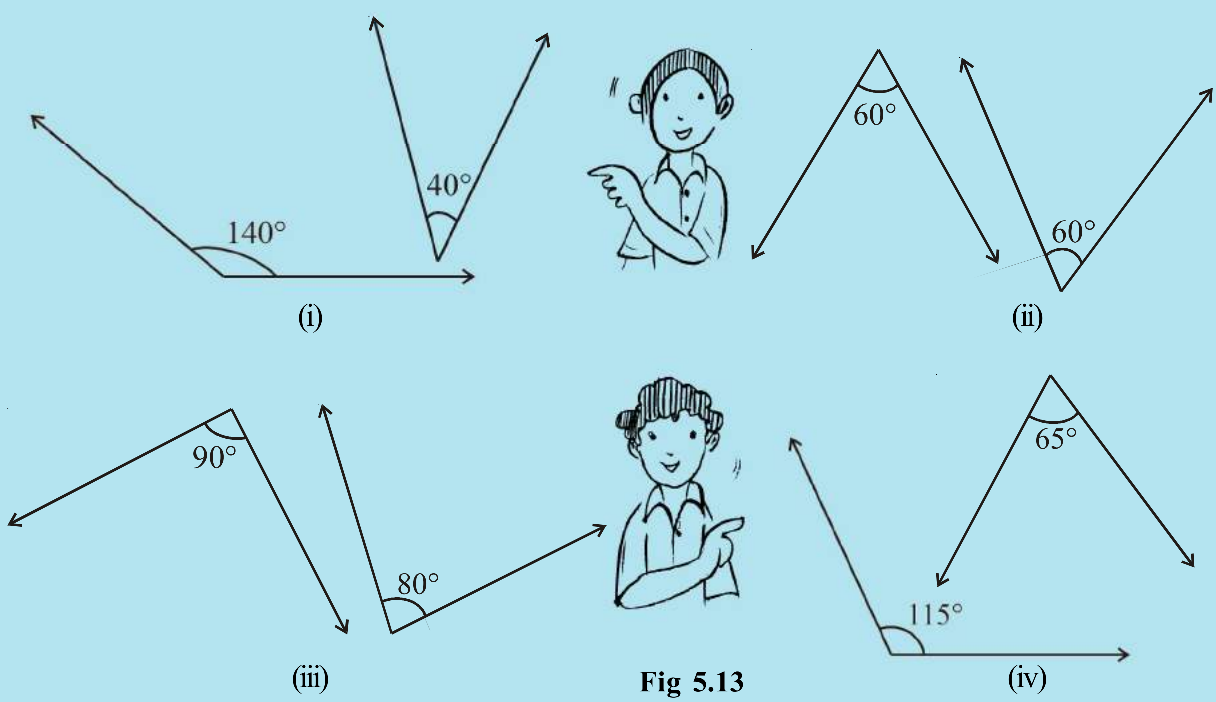


Fig 5.13

5.2.5 Vertically Opposite Angles

Next take two pencils and tie them with the help of a rubber band at the middle as shown (Fig 5.14).

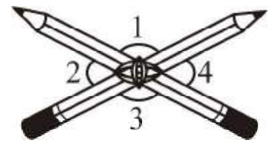


Fig 5.14

Look at the four angles formed ∠1, ∠2, ∠3 and ∠4.

∠1 is vertically opposite to ∠3.

and ∠2 is vertically opposite to ∠4.

We call ∠1 and ∠3, a pair of vertically opposite angles.

Can you name the other pair of vertically opposite angles?

Does ∠1 appear to be equal to ∠3? Does ∠2 appear to be equal to ∠4?

Before checking this, let us see some real life examples for vertically opposite angles (Fig 5.15).

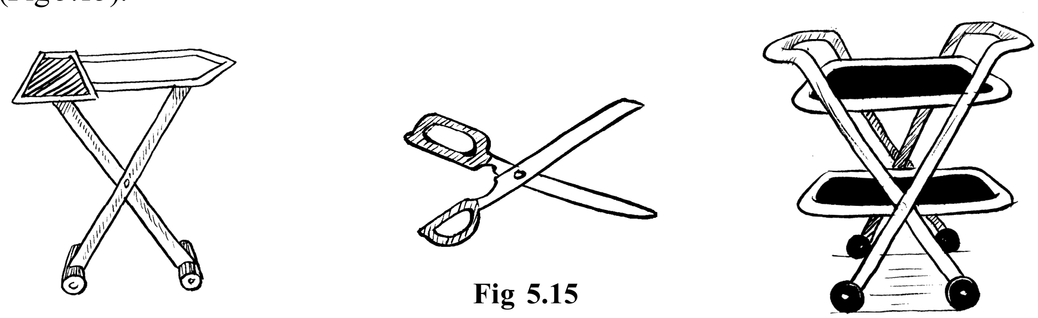


Fig 5.15

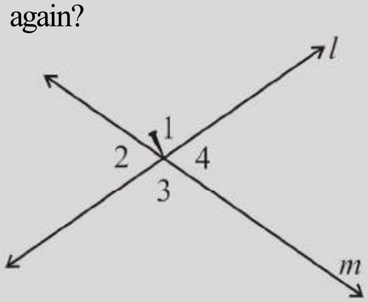
Do This

Draw two lines l and m, intersecting at a point. You can now mark ∠1, ∠2, ∠3 and ∠4 as in the Fig (5.16).

Take a tracecopy of the figure on a transparent sheet.

Place the copy on the original such that ∠1 matches with its copy, ∠2 matches with its copy, ... etc.

Fix a pin at the point of intersection. Rotate the copy by 180°. Do the lines coincide again?



can be rotated to get

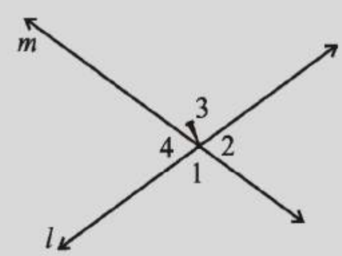


Fig 5.16

You find that ∠1 and ∠3 have interchanged their positions and so have ∠2 and ∠4. This has been done without disturbing the position of the lines.

Thus, ∠1 = ∠3 and ∠2 = ∠4.

We conclude that **when two lines intersect, the vertically opposite angles so formed are equal.**

Let us try to prove this using Geometrical Idea.

Let us consider two lines *l* and *m*. (Fig 5.17)

We can arrive at this result through logical reasoning as follows:

Let *l* and *m* be two lines, which intersect at O, making angles ∠1, ∠2, ∠3 and ∠4.

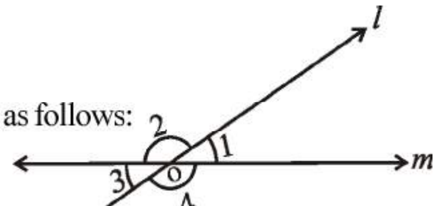


Fig 5.17

We want to prove that ∠1 = ∠3 and ∠2 = ∠4

Now, ∠1 = 180° – ∠2 (Because ∠1, ∠2 form a linear pair, so, ∠1 + ∠2 = 180°) (i)

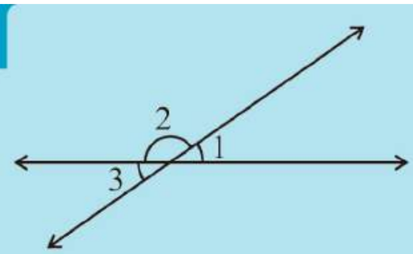
Similarly, ∠3 = 180° – ∠2 (Since ∠2, ∠3 form a linear pair, so, ∠2 + ∠3 = 180°) (ii)

Therfore, ∠1 = ∠3 [By (i) and (ii)]

Similarly, we can prove that ∠2 = ∠4, (Try it!)

TRY THESE

1. In the given figure, if ∠1 = 30°, find ∠2 and ∠3.
2. Give an example for vertically opposite angles in your surroundings.



EXAMPLE 1 In Fig (5.18) identify:

(i) Five pairs of adjacent angles. (ii) Three linear pairs.

(iii) Two pairs of vertically opposite angles.

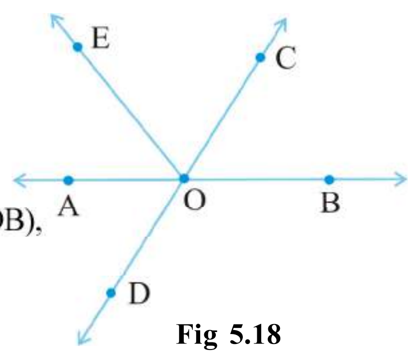


Fig 5.18

SOLUTION

(i) Five pairs of adjacent angles are (∠AOE, ∠EOC), (∠EOC, ∠COB), (∠AOC, ∠COB), (∠COB, ∠BOD), (∠EOB, ∠BOD)

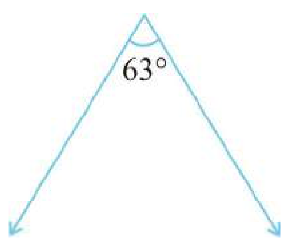
(ii) Linear pairs are (∠AOE, ∠EOB), (∠AOC, ∠COB), (∠COB, ∠BOD)

(iii) Vertically opposite angles are: (∠COB, ∠AOD), and (∠AOC, ∠BOD)

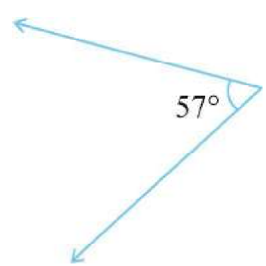
EXERCISE 5.1

1. Find the complement of each of the following angles:

(i)

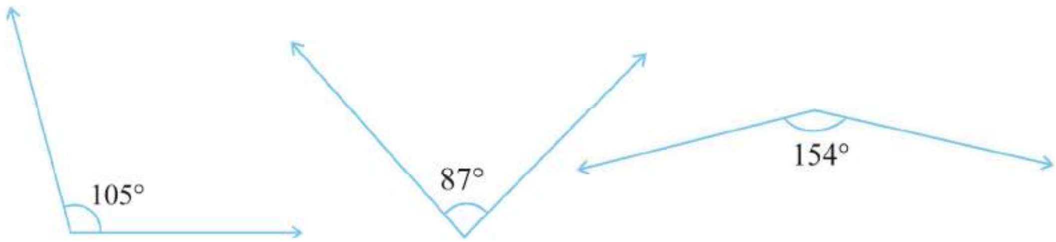


(ii)



(iii)

2. Find the supplement of each of the following angles:



(i) (ii) (iii)

3. Identify which of the following pairs of angles are complementary and which are supplementary.

(i) 65º, 115º (ii) 63º, 27º (iii) 112º, 68º

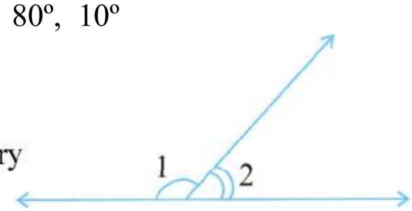
(iv) 130º, 50º (v) 45º, 45º (vi) 80º, 10º

4. Find the angle which is equal to its complement.

5. Find the angle which is equal to its supplement.

6. In the given figure, ∠1 and ∠2 are supplementary angles.

If ∠1 is decreased, what changes should take place in ∠2 so that both the angles still remain supplementary.



7. Can two angles be supplementary if both of them are:

(i) acute? (ii) obtuse? (iii) right?

8. An angle is greater than 45º. Is its complementary angle greater than 45º or equal to 45º or less than 45º?

9. In the adjoining figure:

(i) Is ∠1 adjacent to ∠2?

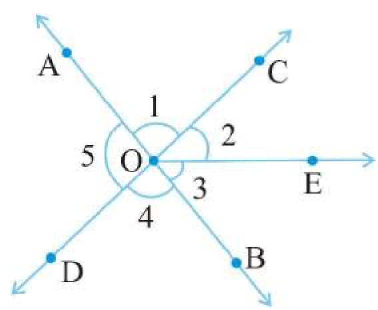
(ii) Is ∠AOC adjacent to ∠AOE?

(iii) Do ∠COE and ∠EOD form a linear pair?

(iv) Are ∠BOD and ∠DOA supplementary?

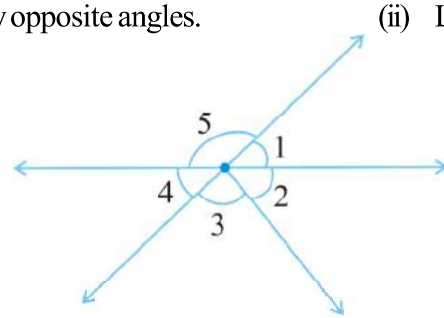
(v) Is ∠1 vertically opposite to ∠4?

(vi) What is the vertically opposite angle of ∠5?

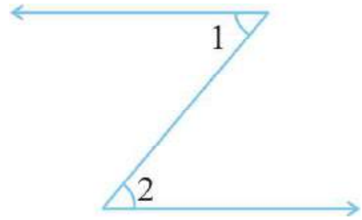


10. Indicate which pairs of angles are:

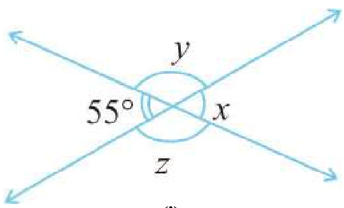
(i) Vertically opposite angles. (ii) Linear pairs.



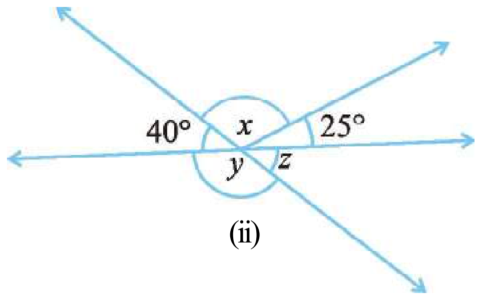
11. In the following figure, is $\angle 1$ adjacent to $\angle 2$? Give reasons.



12. Find the values of the angles x, y, and z in each of the following:



(i)



(ii)

13. Fill in the blanks:

(i) If two angles are complementary, then the sum of their measures is _______.

(ii) If two angles are supplementary, then the sum of their measures is ______.

(iii) Two angles forming a linear pair are _______________.

(iv) If two adjacent angles are supplementary, they form a ___________.

(v) If two lines intersect at a point, then the vertically opposite angles are always _____________.

(vi) If two lines intersect at a point, and if one pair of vertically opposite angles are acute angles, then the other pair of vertically opposite angles are ___________.

14. In the adjoining figure, name the following pairs of angles.

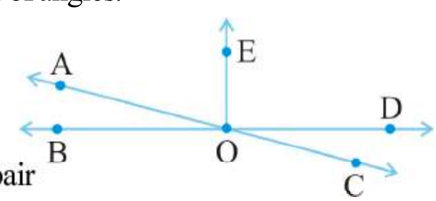
(i) Obtuse vertically opposite angles

(ii) Adjacent complementary angles

(iii) Equal supplementary angles

(iv) Unequal supplementary angles

(v) Adjacent angles that do not form a linear pair



5.3 Pairs of Lines

5.3.1 Intersecting Lines

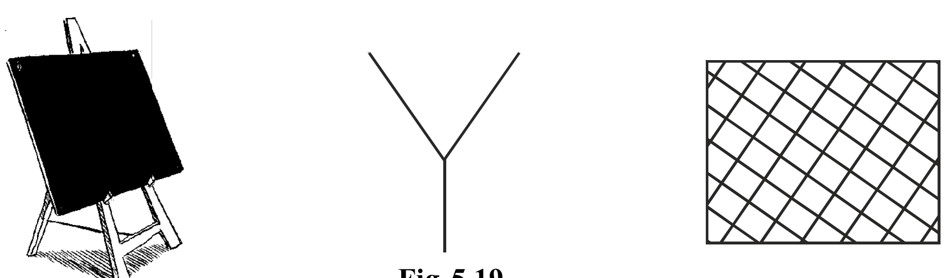


Fig 5.19

The blackboard on its stand, the letter Y made up of line segments and the grill-door of a window (Fig 5.19), what do all these have in common? They are examples of **intersecting lines**.

Two lines *l* and m intersect if they have a point in common. This common point O is their **point of intersection**.

Think, discuss and write

In Fig 5.20, AC and BE intersect at P.

AC and BC intersect at C, AC and EC intersect at C.

Try to find another ten pairs of intersecting line segments.

Should any two lines or line segments necessarily intersect? Can you find two pairs of non-intersecting line segments in the figure?

Can two lines intersect in more than one point? Think about it.

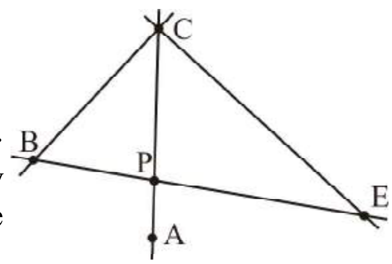


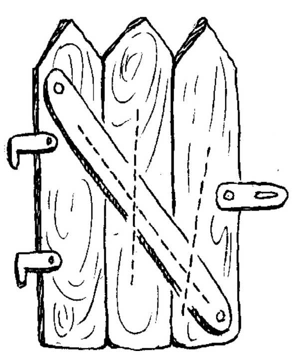
Fig 5.20

Try These

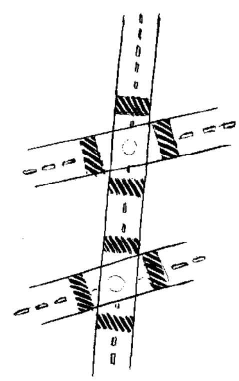
1. Find examples from your surroundings where lines intersect at right angles.
2. Find the measures of the angles made by the intersecting lines at the vertices of an equilateral triangle.
3. Draw any rectangle and find the measures of angles at the four vertices made by the intersecting lines.
4. If two lines intersect, do they always intersect at right angles?

5.3.2 Transversal

You might have seen a road crossing two or more roads or a railway line crossing several other lines (Fig 5.21). These give an idea of a transversal.



(i)



(ii)

Fig 5.21

A line that intersects two or more lines at **distinct** points is called a **transversal**.

In the Fig 5.22, *p* is a transversal to the lines *l* and *m*.

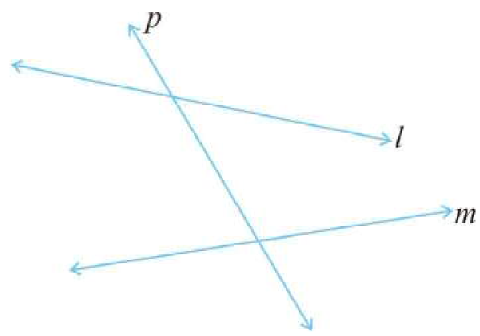


Fig 5.22

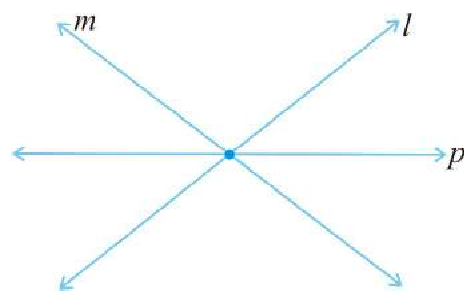


Fig 5.23

In Fig 5.23 the line *p* is not a transversal, although it cuts two lines *l* and *m*. Can you say, 'why'?

TRY THESE

1. Suppose two lines are given. How many transversals can you draw for these lines?
2. If a line is a transversal to three lines, how many points of intersections are there?
3. Try to identify a few transversals in your surroundings.

5.3.3. Angles made by a Transversal

In Fig 5.24, you see lines *l* and *m* cut by transversal *p*. The eight angles marked 1 to 8 have their special names:

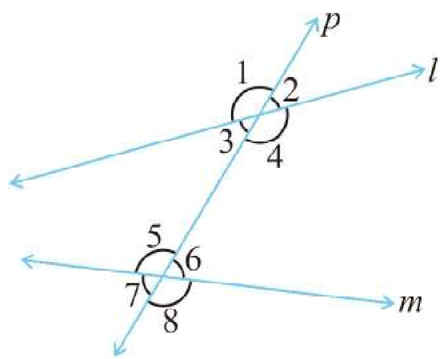


Fig 5.24

Interior angles	∠3, ∠4, ∠5, ∠6
Exterior angles	∠1, ∠2, ∠7, ∠8
Pairs of Corresponding angles	∠1 and ∠5, ∠2 and ∠6, ∠3 and ∠7, ∠4 and ∠8
Pairs of Alternate interior angles	∠3 and ∠6, ∠4 and ∠5
Pairs of Alternate exterior angles	∠1 and ∠8, ∠2 and ∠7
Pairs of interior angles on the same side of the transversal	∠3 and ∠5, ∠4 and ∠6

Note: Corresponding angles (like ∠1 and ∠5 in Fig 5.25) include

(i) different vertices (ii) are on the same side of the transversal and

(iii) are in ‘corresponding’ positions (above or below, left or right) relative to the two lines.

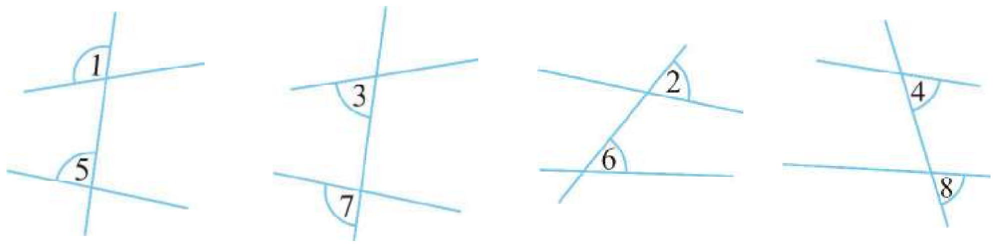


Fig 5.25

Alternate interior angles (like ∠3 and ∠6 in Fig 5.26)

(i) have different vertices

(ii) are on opposite sides of the transversal and

(iii) lie ‘between’ the two lines.

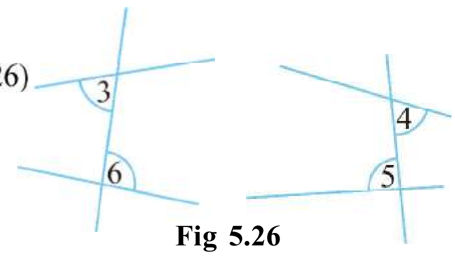
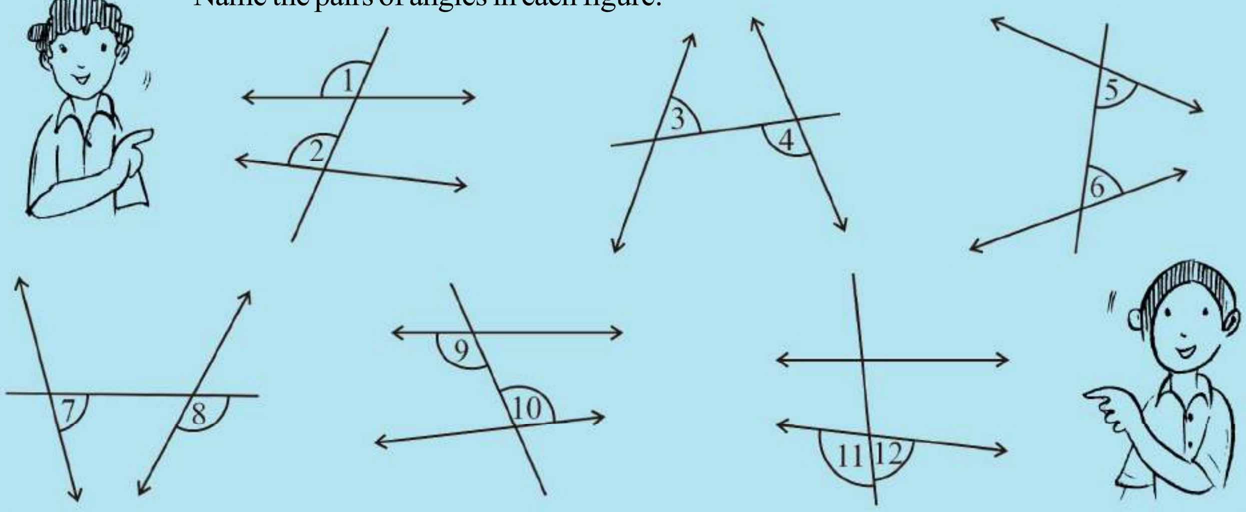


Fig 5.26

Try These

Name the pairs of angles in each figure:



5.3.4 Transversal of Parallel Lines

Do you remember what parallel lines are? They are lines on a plane that do not meet anywhere. Can you identify parallel lines in the following figures? (Fig 5.27)

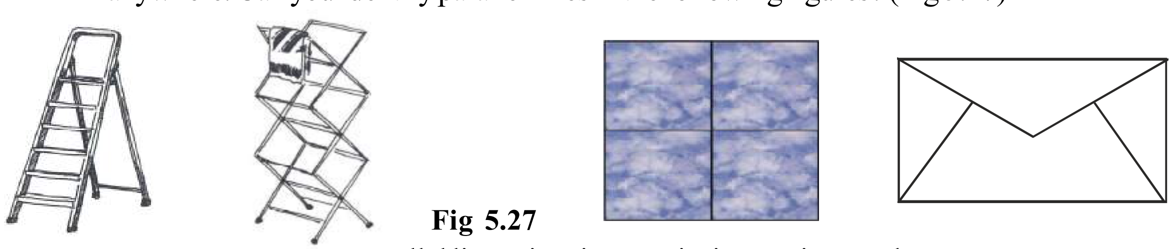


Fig 5.27

Transversals of parallel lines give rise to quite interesting results.

Do This

Take a ruled sheet of paper. Draw (in thick colour) two parallel lines *l* and *m*. Draw a transversal *t* to the lines *l* and *m*. Label ∠1 and ∠2 as shown [Fig 5.28(i)]. Place a tracing paper over the figure drawn. Trace the lines *l*, *m* and *t*. Slide the tracing paper along *t*, until *l* coincides with *m*. You find that ∠1 on the traced figure coincides with ∠2 of the original figure. In fact, you can see all the following results by similar tracing and sliding activity.

(i) ∠1 = ∠2 (ii) ∠3 = ∠4 (iii) ∠5 = ∠6 (iv) ∠7 = ∠8

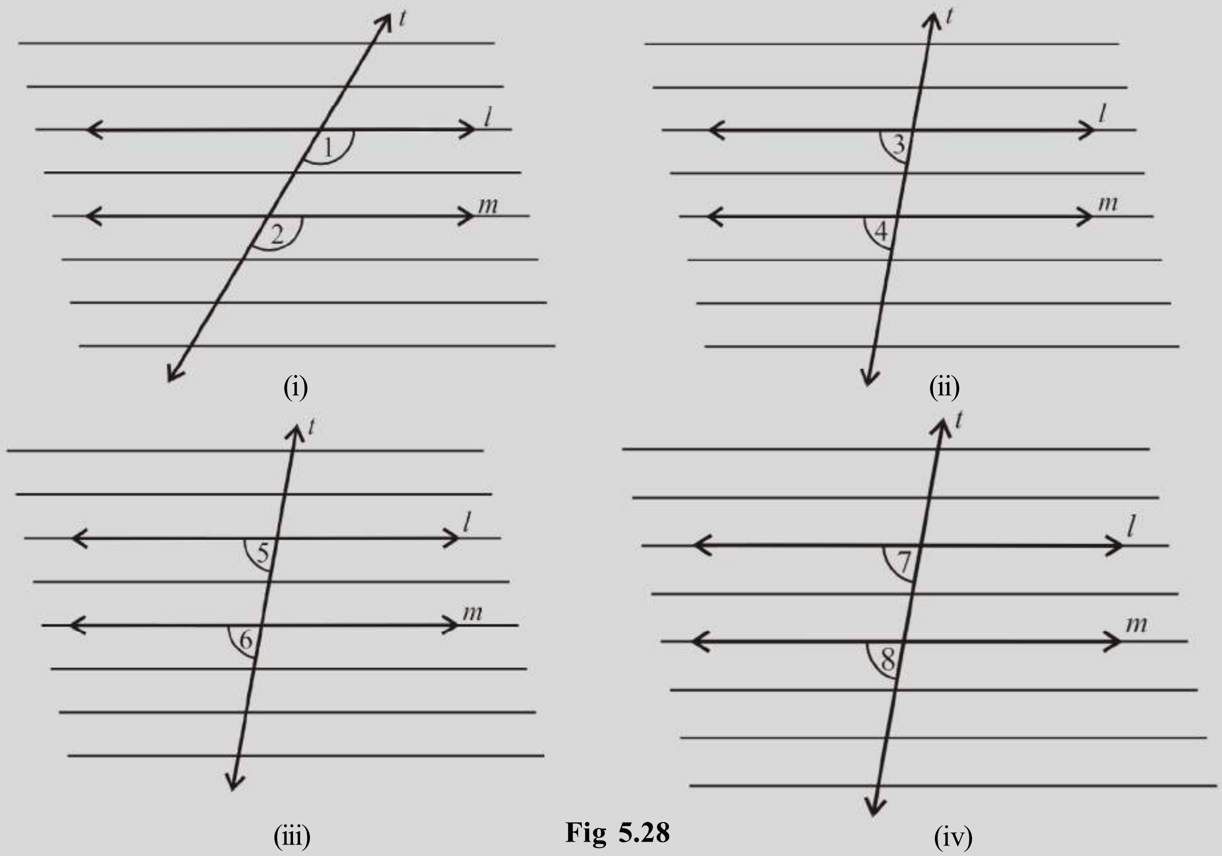


Fig 5.28

This activity illustrates the following fact:

> If two parallel lines are cut by a transversal, each pair of corresponding angles are equal in measure.

We use this result to get another interesting result. Look at Fig 5.29.

When *t* cuts the parallel lines, *l*, *m,* we get, ∠3 = ∠7 (vertically opposite angles).

But ∠7 = ∠8 (corresponding angles). Therefore, ∠3 = ∠8

You can similarly show that $\angle 1 = \angle 6$. Thus, we have the following result :

> If two parallel lines are cut by a transversal, each pair of alternate interior angles are equal.

This second result leads to another interesting property. Again, from Fig 5.29.

$\angle 3 + \angle 1 = 180°$ ($\angle 3$ and $\angle 1$ form a linear pair)

But $\angle 1 = \angle 6$ (A pair of alternate interior angles)

Therefore, we can say that $\angle 3 + \angle 6 = 180°$.

Similarly, $\angle 1 + \angle 8 = 180°$. Thus, we obtain the following result:

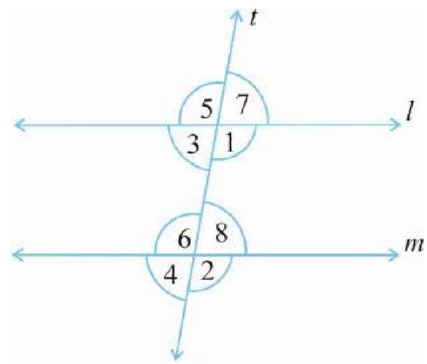
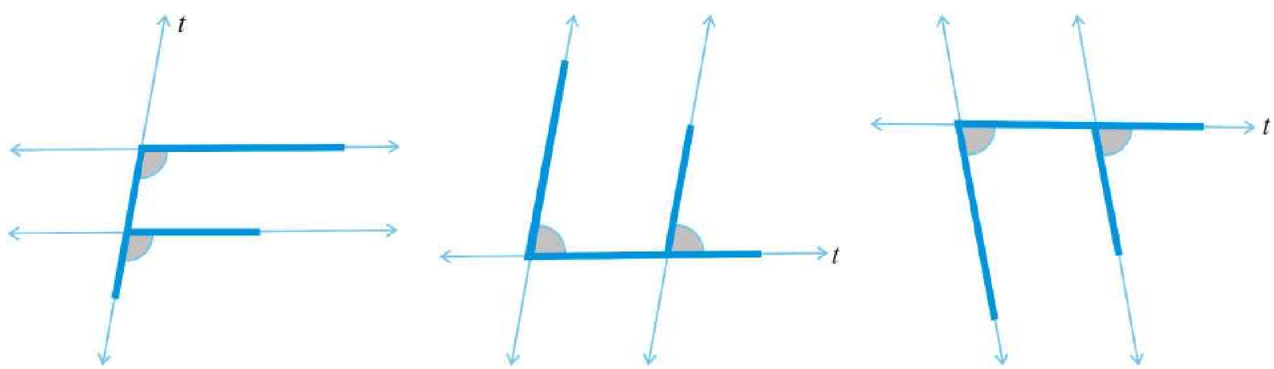


Fig 5.29

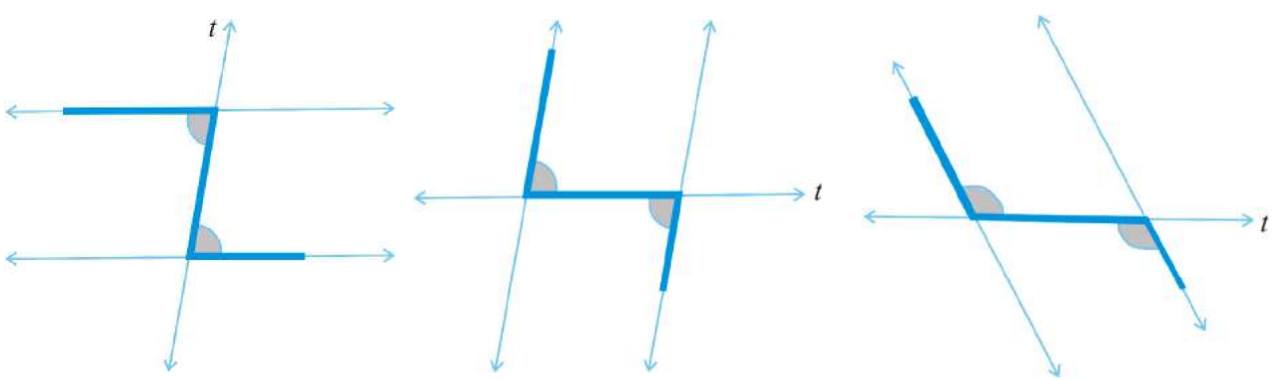
> If two parallel lines are cut by a transversal, then each pair of interior angles on the same side of the transversal are supplementary.

You can very easily remember these results if you can look for relevant 'shapes'.

The F-shape stands for corresponding angles:



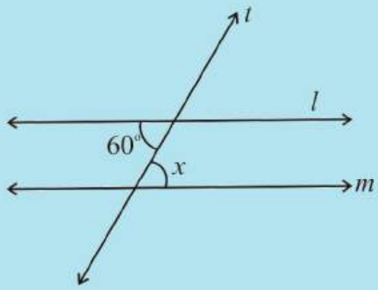
The Z - shape stands for alternate angles.



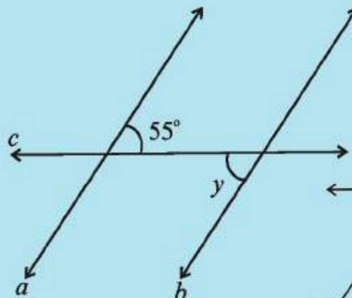
Do This

Draw a pair of parallel lines and a transversal. Verify the above three statements by actually measuring the angles.

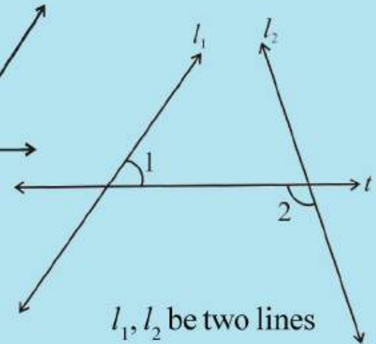
Try These



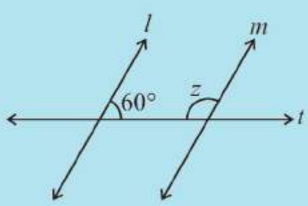
Lines $l \parallel m$;
t is a transversal
$\angle x = ?$



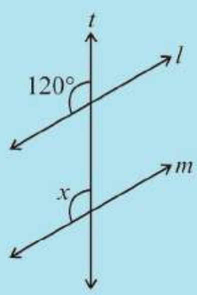
Lines $a \parallel b$;
c is a transversal
$\angle y = ?$



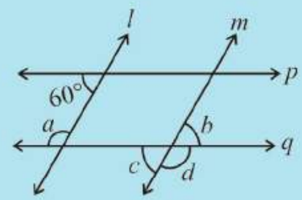
l_1, l_2 be two lines
t is a transversal
Is $\angle 1 = \angle 2$?



Lines $l \parallel m$;
t is a transversal
$\angle z = ?$



Lines $l \parallel m$;
t is a transversal
$\angle x = ?$



Lines $l \parallel m$, $p \parallel q$;
Find a, b, c, d

5.4 Checking for Parallel Lines

If two lines are parallel, then you know that a transversal gives rise to pairs of equal corresponding angles, equal alternate interior angles and interior angles on the same side of the transversal being supplementary.

When two lines are given, is there any method to check if they are parallel or not? You need this skill in many life-oriented situations.

A draftsman uses a carpenter's square and a straight edge (ruler) to draw these segments (Fig 5.30). He claims they are parallel. How?

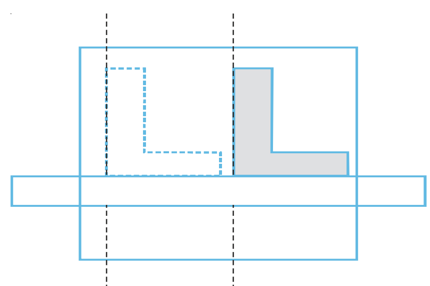


Fig 5.30

Are you able to see that he has kept the corresponding angles to be equal? (What is the transversal here?)

Thus, *when a transversal cuts two lines, such that pairs of corresponding angles are equal, then the lines have to be parallel*.

Look at the letter Z(Fig 5.31). The horizontal segments here are parallel, because the alternate angles are equal.

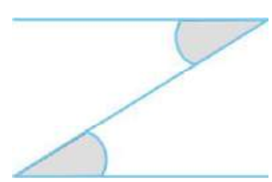


Fig 5.31

When a transversal cuts two lines, such that pairs of alternate interior angles are equal, the lines have to be parallel.

Draw a line l (Fig 5.32).

Draw a line m, perpendicular to l. Again draw a line p, such that p is perpendicular to m.

Thus, p is perpendicular to a perpendicular to l.

You find $p \parallel l$. How? This is because you draw p such that $\angle 1 + \angle 2 = 180^{\circ}$.

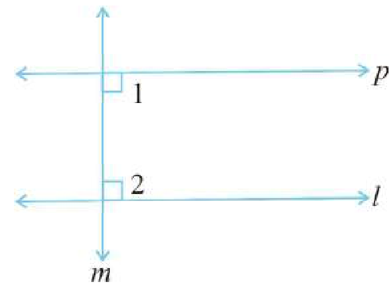
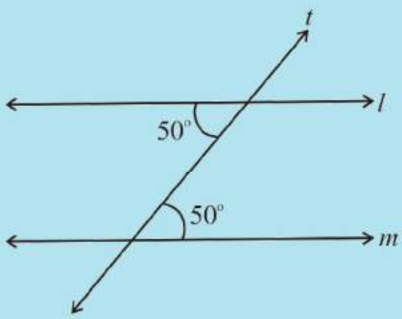


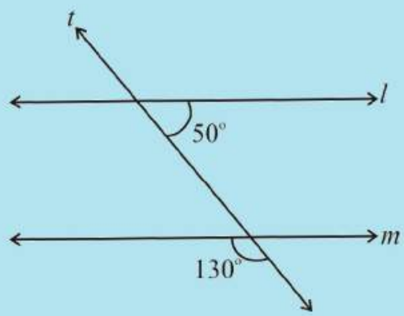
Fig 5.32

Thus, *when a transversal cuts two lines, such that pairs of interior angles on the same side of the transversal are supplementary, the lines have to be parallel*.

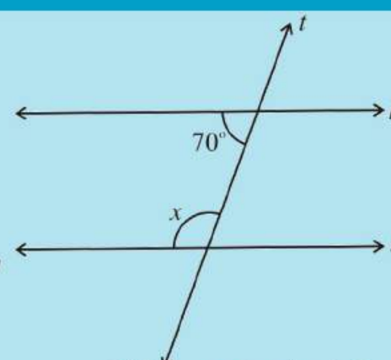
Try These



Is $l \parallel m$? Why?



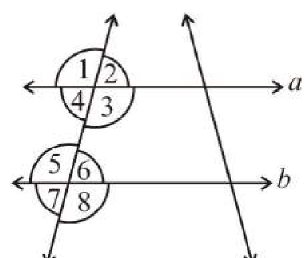
Is $l \parallel m$? Why?



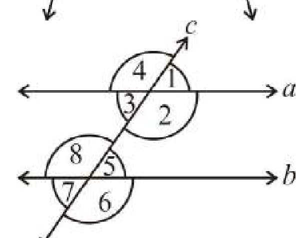
If $l \parallel m$, what is $\angle x$?

Exercise 5.2

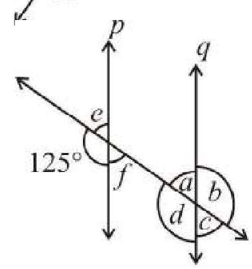
1. State the property that is used in each of the following statements?
   (i) If $a \parallel b$, then $\angle 1 = \angle 5$.
   (ii) If $\angle 4 = \angle 6$, then $a \parallel b$.
   (iii) If $\angle 4 + \angle 5 = 180°$, then $a \parallel b$.



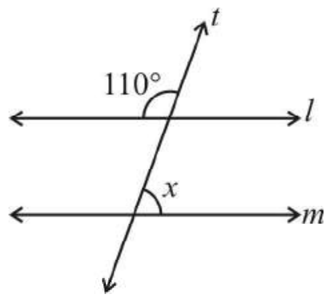
2. In the adjoining figure, identify
   (i) the pairs of corresponding angles.
   (ii) the pairs of alternate interior angles.
   (iii) the pairs of interior angles on the same side of the transversal.
   (iv) the vertically opposite angles.



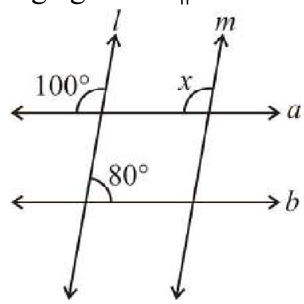
3. In the adjoining figure, $p \parallel q$. Find the unknown angles.



4. Find the value of x in each of the following figures if $l \parallel m$.

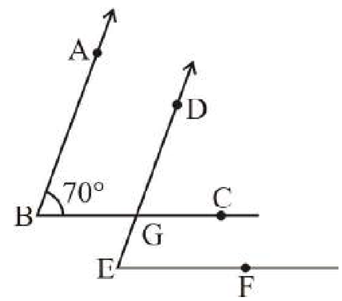


(i)

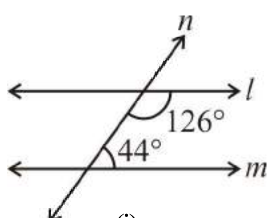


(ii)

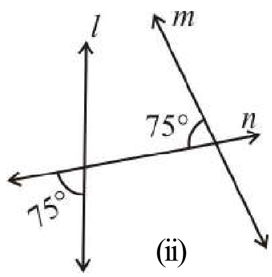
5. In the given figure, the arms of two angles are parallel.
If ∠ABC = 70º, then find
(i) ∠DGC
(ii) ∠DEF



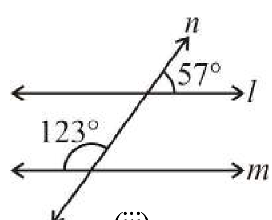
6. In the given figures below, decide whether l is parallel to m.



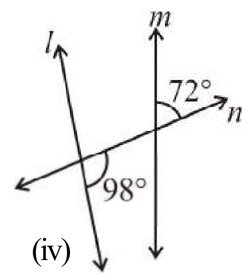
(i)



(ii)



(iii)



(iv)

What have We Discussed?

1. We recall that (i) A line-segment has two end points.
(ii) A ray has only one end point (its vertex); and
(iii) A line has no end points on either side.

2. An angle is formed when two lines (or rays or line-segments) meet.

Pairs of Angles	**Condition**
Two complementary angles	Measures add up to 90°
Two supplementary angles	Measures add up to 180°
Two adjacent angles	Have a common vertex and a common arm but no common interior.
Linear pair	Adjacent and supplementary

3. When two lines l and m meet, we say they *intersect*; the meeting point is called the point of intersection.
When lines drawn on a sheet of paper do not meet, however far produced, we call them to be *parallel* lines.

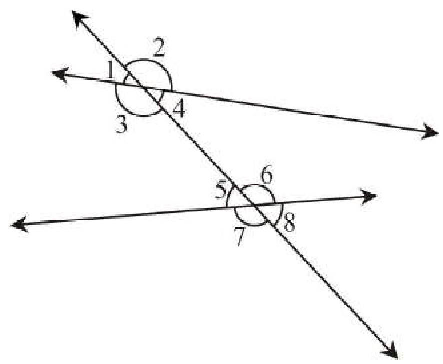
4. (i) When two lines intersect (looking like the letter X) we have two pairs of opposite angles. They are called *vertically opposite angles*. They are equal in measure.

(ii) A transversal is a line that intersects two or more lines at distinct points.

(iii) A transversal gives rise to several types of angles.

(iv) In the figure, we have

Types of Angles	Angles Shown
Interior	$\angle 3, \angle 4, \angle 5, \angle 6$
Exterior	$\angle 1, \angle 2, \angle 7, \angle 8$
Corresponding	$\angle 1$ and $\angle 5$, $\angle 2$ and $\angle 6$, $\angle 3$ and $\angle 7$, $\angle 4$ and $\angle 8$
Alternate interior	$\angle 3$ and $\angle 6$, $\angle 4$ and $\angle 5$
Alternate exterior	$\angle 1$ and $\angle 8$, $\angle 2$ and $\angle 7$
Interior, on the same side of transversal	$\angle 3$ and $\angle 5$, $\angle 4$ and $\angle 6$



(v) When a transversal cuts two *parallel* lines, we have the following interesting relationships:

Each pair of corresponding angles are equal.

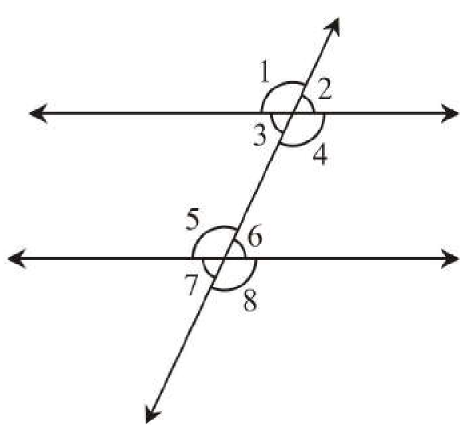
$\angle 1 = \angle 5, \angle 3 = \angle 7, \angle 2 = \angle 6, \angle 4 = \angle 8$

Each pair of alternate interior angles are equal.

$\angle 3 = \angle 6, \angle 4 = \angle 5$

Each pair of interior angles on the same side of transversal are supplementary.

$\angle 3 + \angle 5 = 180°, \angle 4 + \angle 6 = 180°$



Chapter 6

The Triangle and its Properties

6.1 Introduction

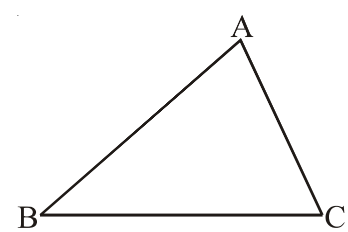


Fig 6.1

A triangle, you have seen, is a simple closed curve made of three line segments. It has three vertices, three sides and three angles.

Here is ΔABC (Fig 6.1). It has

Sides: $\overline{AB}$, $\overline{BC}$, $\overline{CA}$

Angles: $\angle BAC$, $\angle ABC$, $\angle BCA$

Vertices: A, B, C

The side opposite to the vertex A is BC. Can you name the angle opposite to the side AB?

You know how to classify triangles based on the (i) sides (ii) angles.

(i) Based on Sides: Scalene, Isosceles and Equilateral triangles.

(ii) Based on Angles: Acute-angled, Obtuse-angled and Right-angled triangles.

Make paper-cut models of the above triangular shapes. Compare your models with those of your friends and discuss about them.

Try These

1. Write the six elements (i.e., the 3 sides and the 3 angles) of ΔABC.
2. Write the:
   - (i) Side opposite to the vertex Q of ΔPQR
   - (ii) Angle opposite to the side LM of ΔLMN
   - (iii) Vertex opposite to the side RT of ΔRST
3. Look at Fig 6.2 and classify each of the triangles according to its
   - (a) Sides
   - (b) Angles

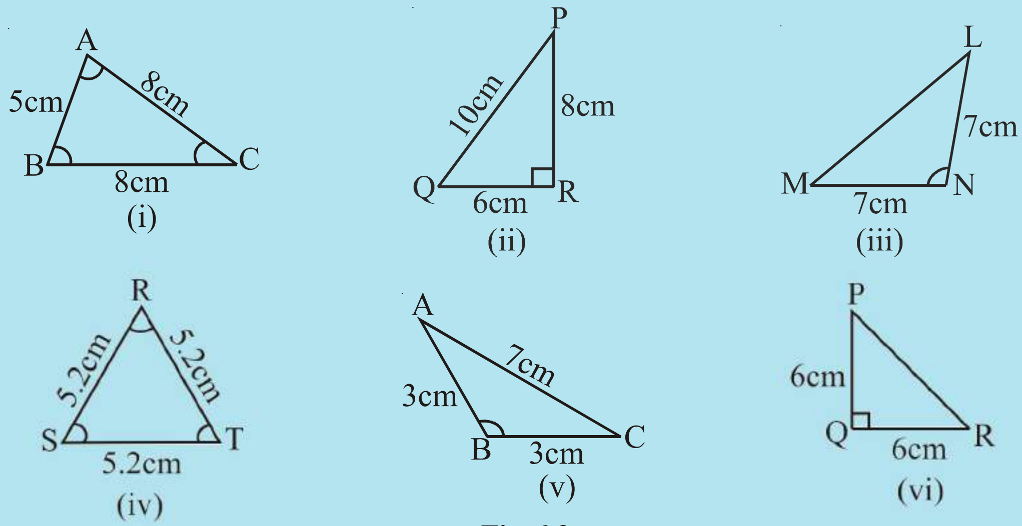


Fig 6.2

Now, let us try to explore something more about triangles.

6.2 Medians of a Triangle

Given a line segment, you know how to find its perpendicular bisector by paper folding. Cut out a triangle ABC from a piece of paper (Fig 6.3). Consider any one of its sides, say, $\overline{BC}$. By paper-folding, locate the perpendicular bisector of $\overline{BC}$. The folded crease meets $\overline{BC}$ at D, its mid-point. Join $\overline{AD}$.

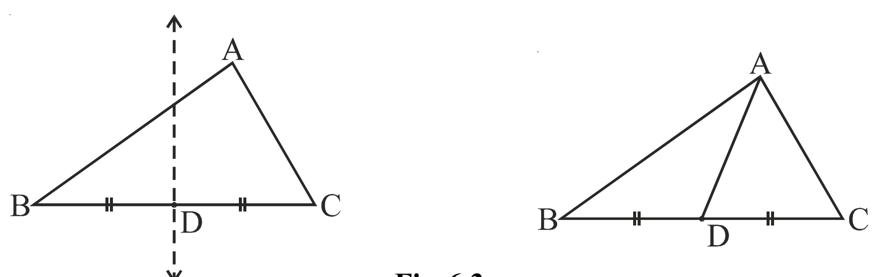


Fig 6.3

The line segment AD, joining the mid-point of $\overline{BC}$ to its opposite vertex A is called a **median** of the triangle.

Consider the sides $\overline{AB}$ and $\overline{CA}$ and find two more medians of the triangle.

A median connects a vertex of a triangle to the mid-point of the opposite side.

Think, Discuss and Write

1. How many medians can a triangle have?
2. Does a median lie wholly in the interior of the triangle? (If you think that this is not true, draw a figure to show such a case).

6.3 Altitudes of a Triangle

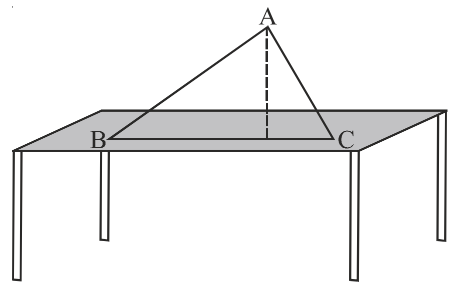


Fig 6.4

Make a triangular shaped cardboard ABC. Place it upright on a table. How 'tall' is the triangle? The **height** is the distance from vertex A (in the Fig 6.4) to the base $\overline{BC}$.

From A to $\overline{BC}$, you can think of many line segments (see the next Fig 6.5). Which among them will represent its height?

The **height** is given by the line segment that starts from A, comes straight down to $\overline{BC}$, and is perpendicular to $\overline{BC}$. This line segment $\overline{AL}$ is an **altitude** of the triangle.

An altitude has one end point at a vertex of the triangle and the other on the line containing the opposite side. Through each vertex, an altitude can be drawn.

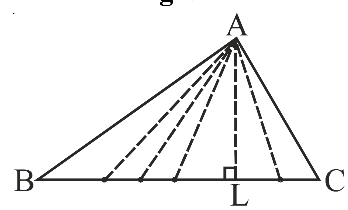
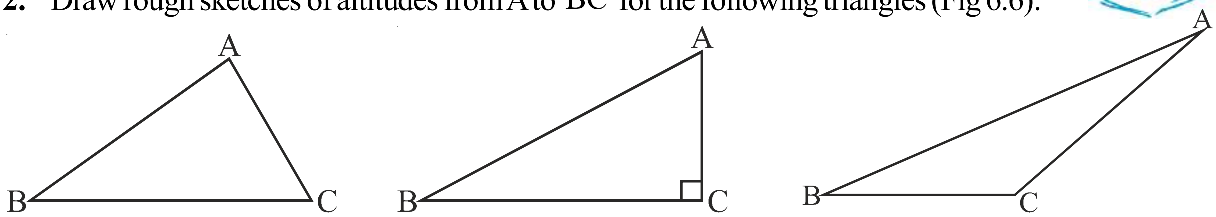


Fig 6.5

Think, Discuss and Write

1. How many altitudes can a triangle have?
2. Draw rough sketches of altitudes from A to $\overline{BC}$ for the following triangles (Fig 6.6):



Acute-angled (i) | Right-angled (ii) | Obtuse-angled (iii)

Fig 6.6

3. Will an altitude always lie in the interior of a triangle? If you think that this need not be true, draw a rough sketch to show such a case.
4. Can you think of a triangle in which two altitudes of the triangle are two of its sides?
5. Can the altitude and median be same for a triangle?

(Hint: For Q.No. 4 and 5, investigate by drawing the altitudes for every type of triangle).

Do This

Take several cut-outs of

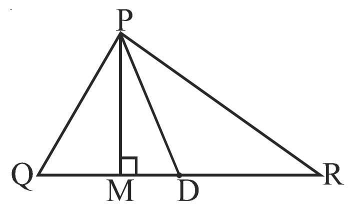
(i) an equilateral triangle (ii) an isosceles triangle and

(iii) a scalene triangle.

Find their altitudes and medians. Do you find anything special about them? Discuss it with your friends.

Exercise 6.1

1. In Δ PQR, D is the mid-point of $\overline{QR}$.



$\overline{PM}$ is ________________.

PD is ________________.

Is QM = MR?

2. Draw rough sketches for the following:
   (a) In ΔABC, BE is a median.
   (b) In ΔPQR, PQ and PR are altitudes of the triangle.
   (c) In ΔXYZ, YL is an altitude in the exterior of the triangle.
3. Verify by drawing a diagram if the median and altitude of an isosceles triangle can be same.

6.4 Exterior Angle of a Triangle and its Property

Do This

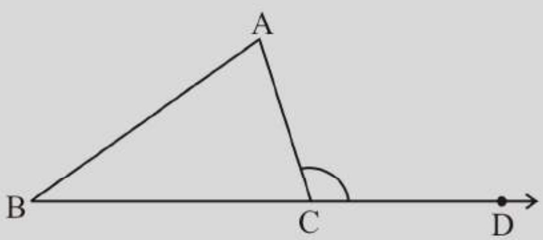


Fig 6.7

1. Draw a triangle ABC and produce one of its sides, say BC as shown in Fig 6.7. Observe the angle ACD formed at the point C. This angle lies in the exterior of ΔABC. We call it an **exterior angle** of the ΔABC formed at vertex C.

   Clearly ∠BCA is an adjacent angle to ∠ACD. The remaining two angles of the triangle namely ∠A and ∠B are called the two **interior opposite angles** or the two remote interior angles of ∠ACD. Now cut out (or make trace copies of) ∠A and ∠B and place them adjacent to each other as shown in Fig 6.8.

   Do these two pieces together entirely cover ∠ACD?

   Can you say that

   $m\angle ACD = m\angle A + m\angle B$?

2. As done earlier, draw a triangle ABC and form an exterior angle ACD. Now take a protractor and measure ∠ACD, ∠A and ∠B.

   Find the sum ∠A + ∠B and compare it with the measure of ∠ACD. Do you observe that ∠ACD is equal (or nearly equal, if there is an error in measurement) to ∠A + ∠B?

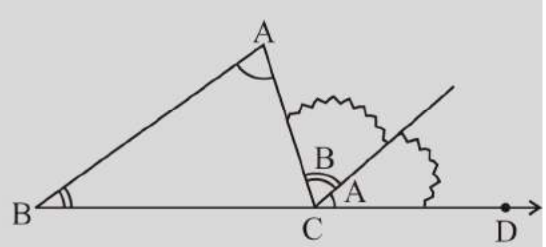


Fig 6.8

You may repeat the two activities as mentioned by drawing some more triangles along with their exterior angles. Every time, you will find that the exterior angle of a triangle is equal to the sum of its two interior opposite angles.

A logical step-by-step argument can further confirm this fact.

An exterior angle of a triangle is equal to the sum of its interior opposite angles.

Given: Consider ΔABC.

$\angle ACD$ is an exterior angle.

To Show: $m\angle ACD = m\angle A + m\angle B$

Through C draw $\overline{CE}$, parallel to $\overline{BA}$.

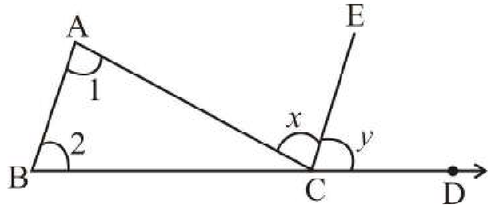


Fig 6.9

Justification

Steps	Reasons
(a) $\angle 1 = \angle x$	$\overline{BA} \parallel \overline{CE}$ and $\overline{AC}$ is a transversal. Therefore, alternate angles should be equal.
(b) $\angle 2 = \angle y$	$\overline{BA} \parallel \overline{CE}$ and $\overline{BD}$ is a transversal. Therefore, corresponding angles should be equal.
(c) $\angle 1 + \angle 2 = \angle x + \angle y$	
(d) Now, $\angle x + \angle y = m\ \angle ACD$	From Fig 6.9
Hence, $\angle 1 + \angle 2 = \angle ACD$	

The above relation between an exterior angle and its two interior opposite angles is referred to as the **Exterior Angle Property of a triangle.**

THINK, DISCUSS AND WRITE

1. Exterior angles can be formed for a triangle in many ways. Three of them are shown here (Fig 6.10)

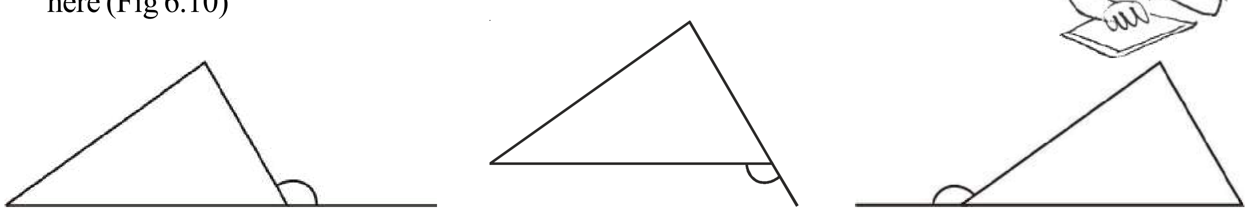


Fig 6.10

There are three more ways of getting exterior angles. Try to produce those rough sketches.

2. Are the exterior angles formed at each vertex of a triangle equal?
3. What can you say about the sum of an exterior angle of a triangle and its adjacent interior angle?

EXAMPLE 1 Find angle x in Fig 6.11.

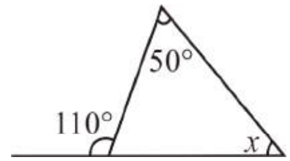


Fig 6.11

SOLUTION Sum of interior opposite angles = Exterior angle

or $50° + x = 110°$

or $x = 60°$

THINK, DISCUSS AND WRITE

1. What can you say about each of the interior opposite angles, when the exterior angle is
   (i) a right angle? (ii) an obtuse angle? (iii) an acute angle?
2. Can the exterior angle of a triangle be a straight angle?

TRY THESE

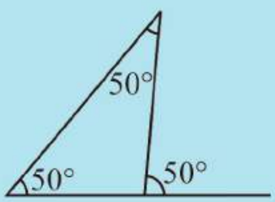
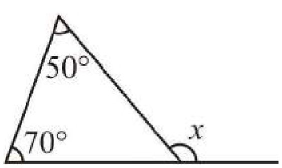


Fig 6.12

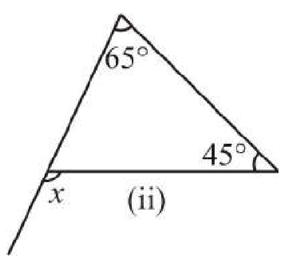
1. An exterior angle of a triangle is of measure 70º and one of its interior opposite angles is of measure 25º. Find the measure of the other interior opposite angle.
2. The two interior opposite angles of an exterior angle of a triangle are 60º and 80º. Find the measure of the exterior angle.
3. Is something wrong in this diagram (Fig 6.12)? Comment.

EXERCISE 6.2

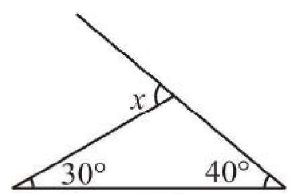
1. Find the value of the unknown exterior angle x in the following diagrams:



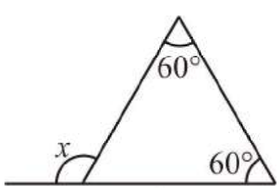
(i)



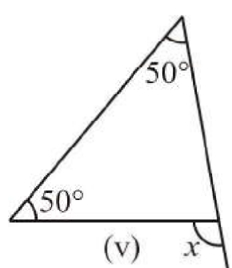
(ii)



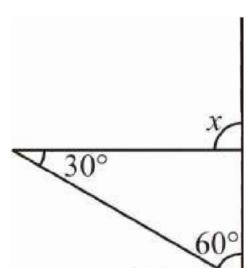
(iii)



(iv)

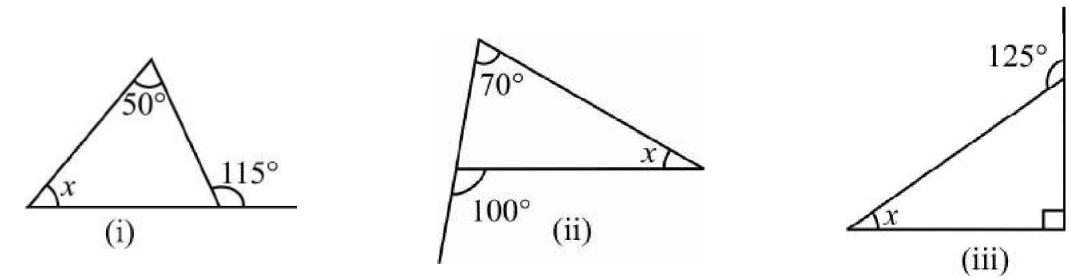


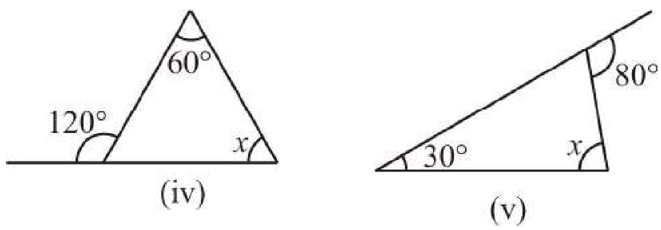
(v)

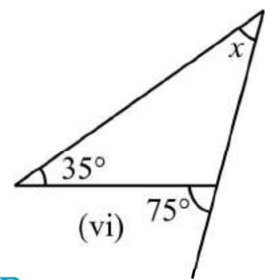


(vi)

2. Find the value of the unknown interior angle *x* in the following figures:



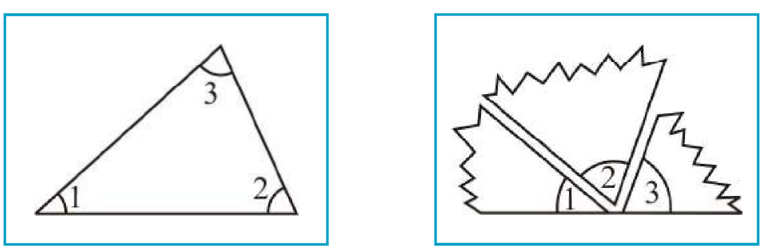




6.5 Angle Sum Property of a Triangle

There is a remarkable property connecting the three angles of a triangle. You are going to see this through the following four activities.

1. Draw a triangle. Cut on the three angles. Rearrange them as shown in Fig 6.13 (i), (ii). The three angles now constitute one angle. This angle is a straight angle and so has measure 180°.



(i) (ii)

Fig 6.13

Thus, the sum of the measures of the three angles of a triangle is 180°.

2. The same fact you can observe in a different way also. Take three copies of any triangle, say ΔABC (Fig 6.14).

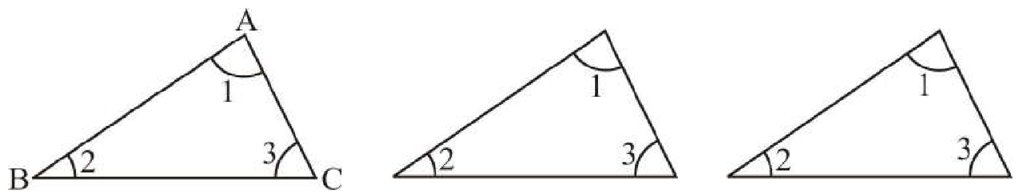


Fig 6.14

Arrange them as in Fig 6.15.

What do you observe about $\angle 1 + \angle 2 + \angle 3$?

(Do you also see the 'exterior angle property'?)

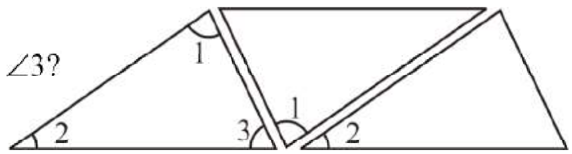


Fig 6.15

3. Take a piece of paper and cut out a triangle, say, ΔABC (Fig 6.16).

   Make the altitude AM by folding ΔABC such that it passes through A.

   Fold now the three corners such that all the three vertices A, B and C touch at M.

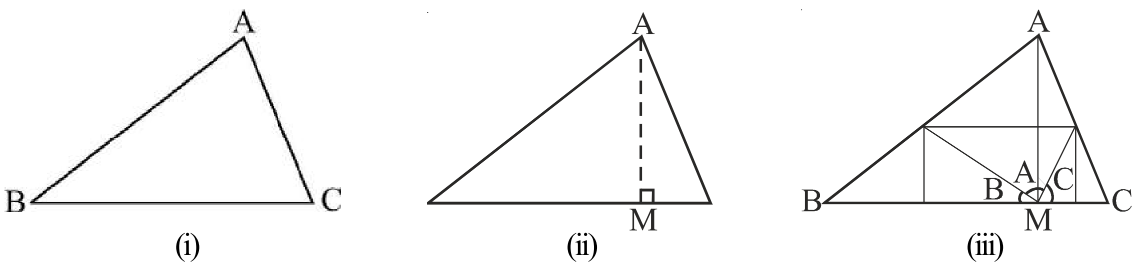


Fig 6.16

You find that all the three angles form together a straight angle. This again shows that the sum of the measures of the three angles of a triangle is 180°.

4. Draw any three triangles, say ΔABC, ΔPQR and ΔXYZ in your notebook.

   Use your protractor and measure each of the angles of these triangles.

   Tabulate your results

Name of Δ	Measures of Angles			Sum of the Measures of the three Angles
ΔABC	$m\angle A =$	$m\angle B =$	$m\angle C =$	$m\angle A + m\angle B + m\angle C =$
ΔPQR	$m\angle P =$	$m\angle Q =$	$m\angle R =$	$m\angle P + m\angle Q + m\angle R =$
ΔXYZ	$m\angle X =$	$m\angle Y =$	$m\angle Z =$	$m\angle X + m\angle Y + m\angle Z =$

Allowing marginal errors in measurement, you will find that the last column always gives 180° (or nearly 180°).

When perfect precision is possible, this will also show that the sum of the measures of the three angles of a triangle is 180°.

You are now ready to give a formal justification of your assertion through logical argument.

Statement **The total measure of the three angles of a triangle is 180°.**

To justify this let us use the exterior angle property of a triangle.

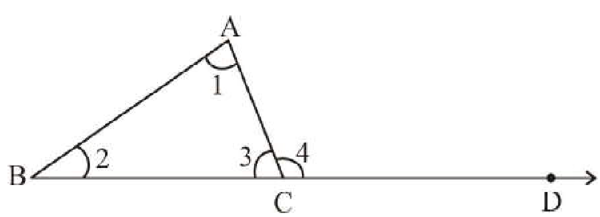


Fig 6.17

Given $\angle 1, \angle 2, \angle 3$ are angles of ΔABC (Fig 6.17).

$\angle 4$ is the exterior angle when BC is extended to D.

Justification $\angle 1 + \angle 2 = \angle 4$ (by exterior angle property)

$\angle 1 + \angle 2 + \angle 3 = \angle 4 + \angle 3$ (adding $\angle 3$ to both the sides)

But $\angle 4$ and $\angle 3$ form a linear pair so it is 180°. Therefore, $\angle 1 + \angle 2 + \angle 3 = 180°$.

Let us see how we can use this property in a number of ways.

EXAMPLE 2 In the given figure (Fig 6.18) find $m\angle P$.

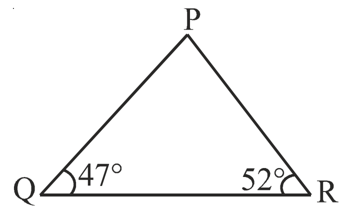


Fig 6.18

SOLUTION By angle sum property of a triangle,

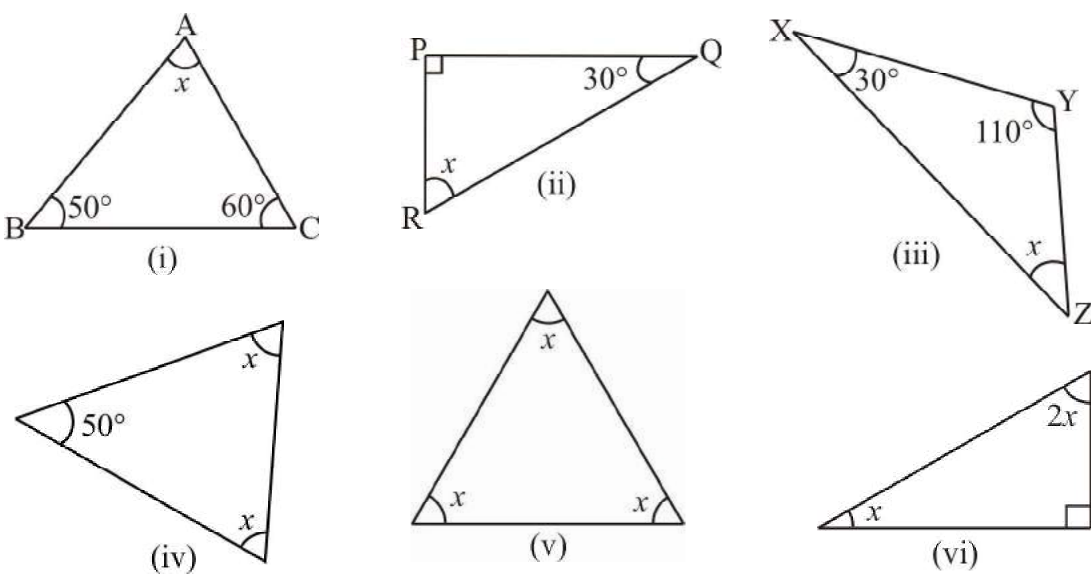
$$m\angle P + 47° + 52° = 180°$$

Therefore $$m\angle P = 180° – 47° – 52°$$

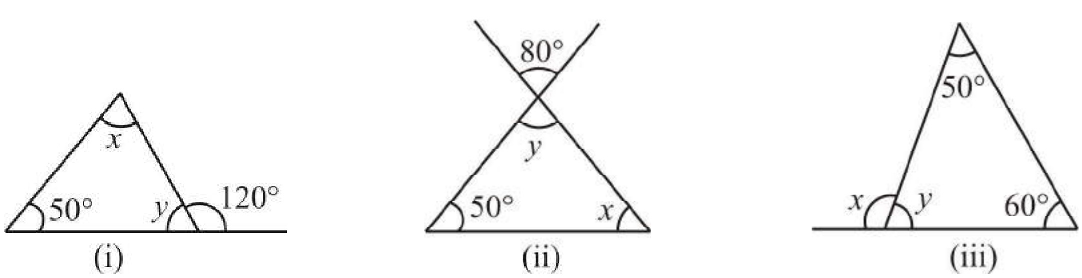
$$= 180° – 99° = 81°$$

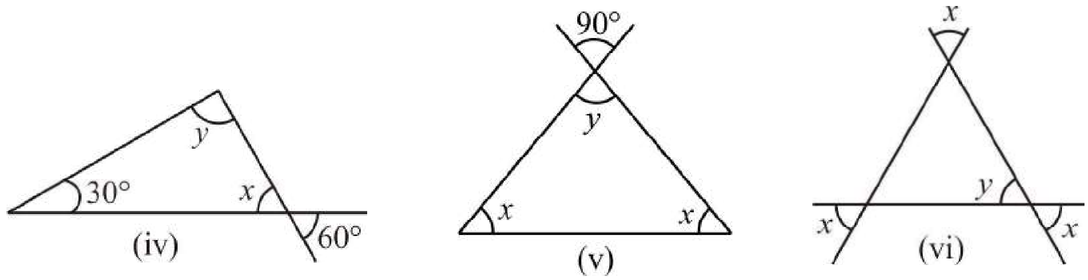
EXERCISE 6.3

1. Find the value of the unknown x in the following diagrams:



2. Find the values of the unknowns x and y in the following diagrams:





Try These

1. Two angles of a triangle are 30º and 80º. Find the third angle.
2. One of the angles of a triangle is 80º and the other two angles are equal. Find the measure of each of the equal angles.
3. The three angles of a triangle are in the ratio 1:2:1. Find all the angles of the triangle. Classify the triangle in two different ways.

Think, Discuss and Write

1. Can you have a triangle with two right angles?
2. Can you have a triangle with two obtuse angles?
3. Can you have a triangle with two acute angles?
4. Can you have a triangle with all the three angles greater than 60º?
5. Can you have a triangle with all the three angles equal to 60º?
6. Can you have a triangle with all the three angles less than 60º?

6.6 Two Special Triangles : Equilateral and Isosceles

A triangle in which all the three sides are of equal lengths is called an equilateral triangle.

Take two copies of an equilateral triangle ABC (Fig 6.19). Keep one of them fixed. Place the second triangle on it. It fits exactly into the first. Turn it round in any way and still they fit with one another exactly. Are you able to see that when the three sides of a triangle have equal lengths then the three angles are also of the same size?

We conclude that in an equilateral triangle:

(i) all sides have same length.

(ii) each angle has measure 60°.

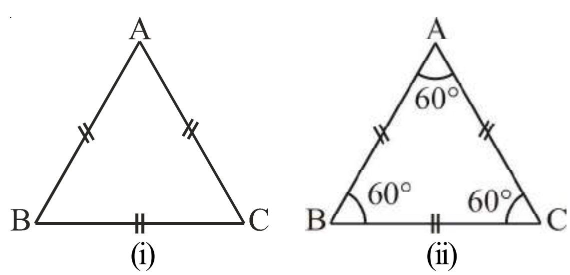


Fig 6.19

A triangle in which two sides are of equal lengths is called an isosceles triangle.

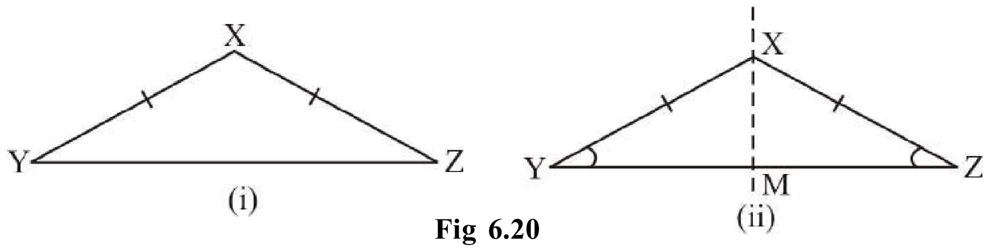


Fig 6.20

From a piece of paper cut out an isosceles triangle XYZ, with XY=XZ (Fig 6.20). Fold it such that Z lies on Y. The line XM through X is now the axis of symmetry (which you will read in Chapter 14). You find that $\angle$Y and $\angle$Z fit on each other exactly. XY and XZ are called equal sides; YZ is called the base; $\angle$Y and $\angle$Z are called base angles and these are also equal.

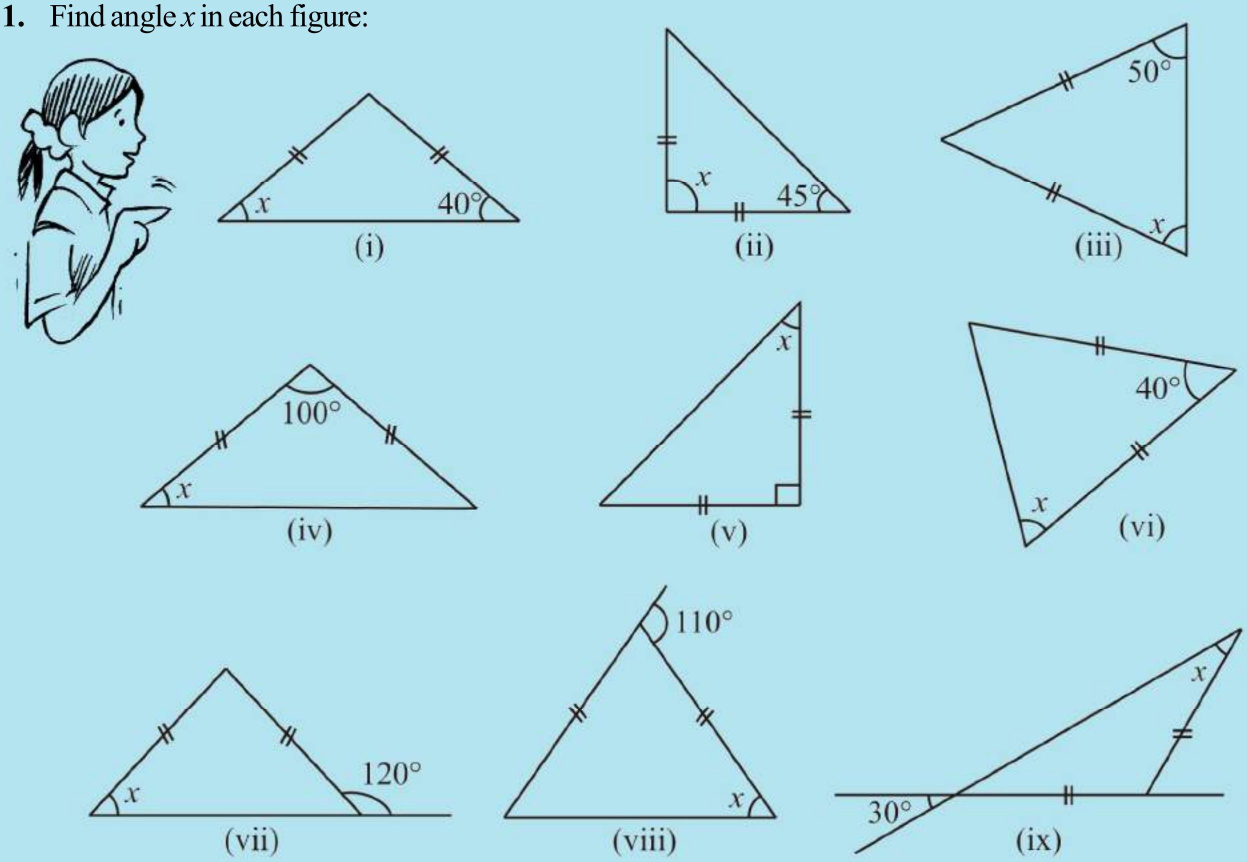
Thus, in an isosceles triangle:

(i) two sides have same length.

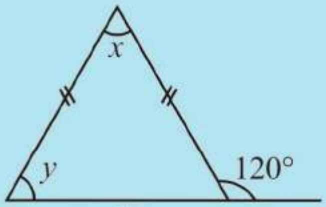
(ii) base angles opposite to the equal sides are equal.

Try These

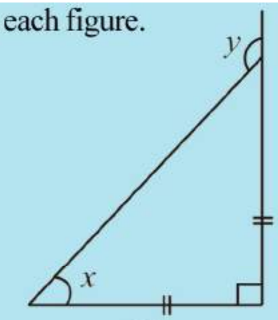
1. Find angle x in each figure:



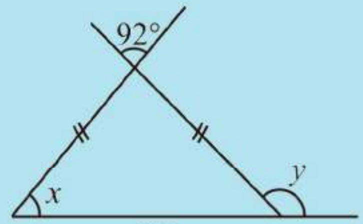
2. Find angles x and y in each figure.



(i)



(ii)



(iii)

6.7 Sum of the Lengths of Two Sides of a Triangle

1.Mark three non-collinear spots A, B and C in your playground. Using lime powder mark the paths AB, BC and AC.

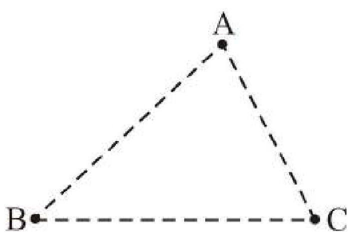


Fig 6.21

Ask your friend to start from A and reach C, walking along one or more of these paths. She can, for example, walk first along $\overline{AB}$ and then along $\overline{BC}$ to reach C; or she can walk straight along $\overline{AC}$. She will naturally prefer the direct path AC. If she takes the other path ($\overline{AB}$ and then $\overline{BC}$), she will have to walk more. In other words,

$$AB + BC > AC \quad \text{(i)}$$

Similarly, if one were to start from B and go to A, he or she will not take the route $\overline{BC}$ and $\overline{CA}$ but will prefer $\overline{BA}$ This is because

$$BC + CA > AB \quad \text{(ii)}$$

By a similar argument, you find that

$$CA + AB > BC \quad \text{(iii)}$$

These observations suggest that **the sum of the lengths of any two sides of a triangle is greater than the third side**.

2. Collect fifteen small sticks (or strips) of different lengths, say, 6 cm, 7 cm, 8 cm, 9 cm, ..., 20 cm.

Take any three of these sticks and try to form a triangle. Repeat this by choosing different combinations of three sticks.

Suppose you first choose two sticks of length 6 cm and 12 cm. Your third stick has to be of length more than 12 – 6 = 6 cm and less than 12 + 6 = 18 cm. Try this and find out why it is so.

To form a triangle you will need any three sticks such that the sum of the lengths of any two of them will always be greater than the length of the third stick.

This also suggests that the sum of the lengths of any two sides of a triangle is greater than the third side.

3. Draw any three triangles, say ΔABC, ΔPQR and ΔXYZ in your notebook (Fig 6.22).

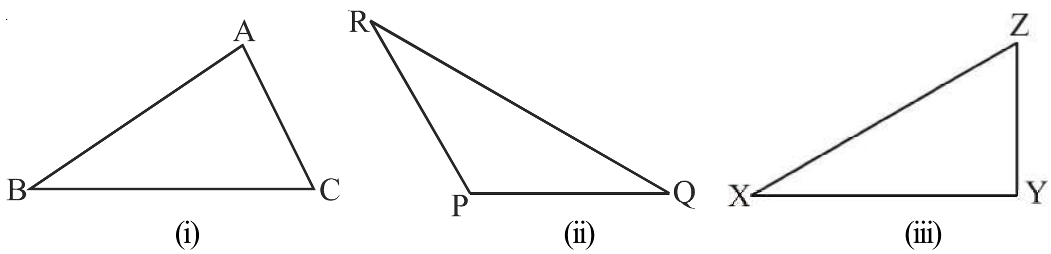


Fig 6.22

Use your ruler to find the lengths of their sides and then tabulate your results as follows:

Name of Δ	Lengths of Sides	Is this True?	
Δ ABC	AB ___	AB – BC < CA	(Yes/No)
		___ + ___ > ___	
	BC ___	BC – CA < AB	(Yes/No)
		___ + ___ > ___	
	CA ___	CA – AB < BC	(Yes/No)
		___ + ___ > ___	
Δ PQR	PQ ___	PQ – QR < RP	(Yes/No)
		___ + ___ > ___	
	QR ___	QR – RP < PQ	(Yes/No)
		___ + ___ > ___	
	RP ___	RP – PQ < QR	(Yes/No)
		___ + ___ > ___	
Δ XYZ	XY ___	XY – YZ < ZX	(Yes/No)
		___ + ___ > ___	
	YZ ___	YZ – ZX < XY	(Yes/No)
		___ + ___ > ___	
	ZX ___	ZX – XY < YZ	(Yes/No)
		___ + ___ > ___	

This also strengthens our earlier guess. Therefore, we conclude that **sum of the lengths of any two sides of a triangle is greater than the length of the third side.**

We also find that the difference between the length of any two sides of a triangle is smaller than the length of the third side.

EXAMPLE 3 Is there a triangle whose sides have lengths 10.2 cm, 5.8 cm and 4.5 cm?

SOLUTION Suppose such a triangle is possible. Then the sum of the lengths of any two sides would be greater than the length of the third side. Let us check this.

Is 4.5 + 5.8 >10.2?	Yes
Is 5.8 + 10.2 > 4.5?	Yes
Is 10.2 + 4.5 > 5.8?	Yes

Therefore, the triangle is possible.

EXAMPLE 4 The lengths of two sides of a triangle are 6 cm and 8 cm. Between which two numbers can length of the third side fall?

SOLUTION We know that the sum of two sides of a triangle is always greater than the third.

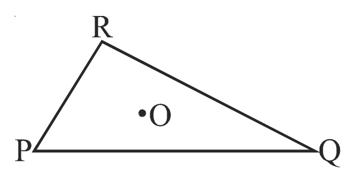
Therefore, third side has to be less than the sum of the two sides. The third side is thus, less than 8 + 6 = 14 cm.

The side cannot be less than the difference of the two sides. Thus, the third side has to be more than 8 – 6 = 2 cm.

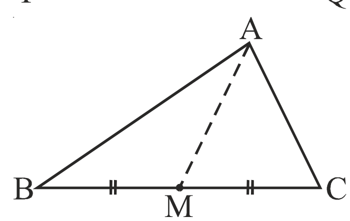
The length of the third side could be any length greater than 2 and less than 14 cm.

EXERCISE 6.4

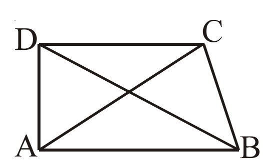
1. Is it possible to have a triangle with the following sides?
   (i) 2 cm, 3 cm, 5 cm (ii) 3 cm, 6 cm, 7 cm
   (iii) 6 cm, 3 cm, 2 cm



2. Take any point O in the interior of a triangle PQR. Is
   (i) OP + OQ > PQ?
   (ii) OQ + OR > QR?
   (iii) OR + OP > RP?



3. AM is a median of a triangle ABC.
   Is AB + BC + CA > 2 AM?
   (Consider the sides of triangles ΔABM and ΔAMC.)



4. ABCD is a quadrilateral.
   Is AB + BC + CD + DA > AC + BD?
5. ABCD is quadrilateral. Is
   AB + BC + CD + DA < 2 (AC + BD)?

6. The lengths of two sides of a triangle are 12 cm and 15 cm. Between what two measures should the length of the third side fall?

THINK, DISCUSS AND WRITE

1. Is the sum of any two angles of a triangle always greater than the third angle?

6.8 RIGHT-ANGLED TRIANGLES AND PYTHAGORAS PROPERTY

Pythagoras, a Greek philosopher of sixth century B.C. is said to have found a very important and useful property of right-angled triangles given in this section. The property is, hence, named after him. In fact, this property was known to people of many other countries too. The Indian mathematician Baudhayan has also given an equivalent form of this property. We now try to explain the Pythagoras property.

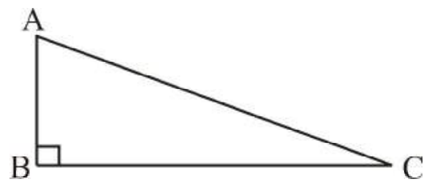


Fig 6.23

In a right-angled triangle, the sides have some special names. The side opposite to the right angle is called the **hypotenuse**; the other two sides are known as the **legs** of the right-angled triangle.

In ΔABC (Fig 6.23), the right-angle is at B. So, AC is the hypotenuse. $\overline{AB}$ and $\overline{BC}$ are the legs of ΔABC.

Make eight identical copies of right angled triangle of any size you prefer. For example, you make a right-angled triangle whose hypotenuse is *a* units long and the legs are of lengths *b* units and *c* units (Fig 6.24).

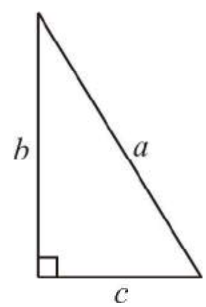
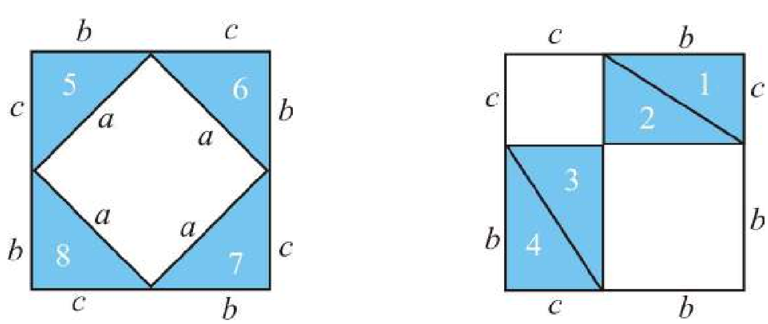


Fig 6.24

Draw two identical squares on a sheet with sides of lengths $b + c$.

You are to place four triangles in one square and the remaining four triangles in the other square, as shown in the following diagram (Fig 6.25).



Square A

Square B

Fig 6.25

The squares are identical; the eight triangles inserted are also identical.

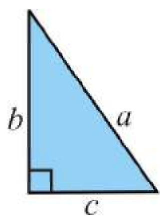
Hence the uncovered area of square A = Uncovered area of square B.

i.e., Area of inner square of square A = The total area of two uncovered squares in square B.

$$a^2 = b^2 + c^2$$

This is Pythagoras property. It may be stated as follows:

> In a right-angled triangle,
> the square on the hypotenuse = sum of the squares on the legs.



Pythagoras property is a very useful tool in mathematics. It is formally proved as a theorem in later classes. You should be clear about its meaning.

It says that for any right-angled triangle, the area of the square on the hypotenuse is equal to the sum of the areas of the squares on the legs.

Draw a right triangle, preferably on a square sheet, construct squares on its sides, compute the area of these squares and verify the theorem practically (Fig 6.26).

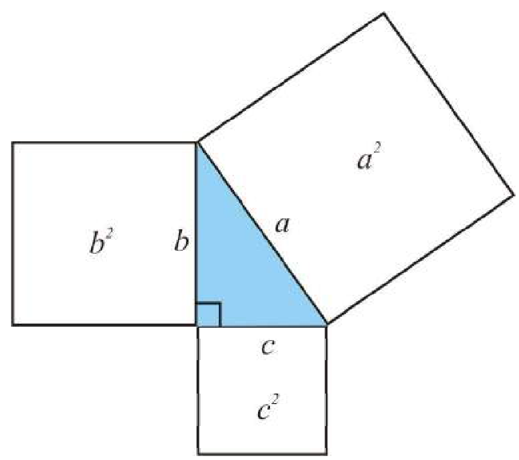


Fig 6.26

If you have a right-angled triangle, the Pythagoras property holds. If the Pythagoras property holds for some triangle, will the triangle be right-angled? (Such problems are known as converse problems). We will try to answer this. Now, we will show that, if there is a triangle such that sum of the squares on two of its sides is equal to the square of the third side, it must be a right-angled triangle.

Do This

1. Have cut-outs of squares with sides 4 cm, 5 cm, 6 cm long. Arrange to get a triangular shape by placing the corners of the squares suitably as shown in the figure (Fig 6.27). Trace out the triangle formed. Measure each angle of the triangle. You find that there is no right angle at all.

   In fact, in this case each angle will be acute! Note that $4^2 + 5^2 \neq 6^2$, $5^2 + 6^2 \neq 4^2$ and $6^2 + 4^2 \neq 5^2$.

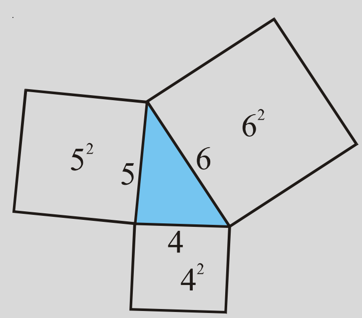


Fig 6.27

2. Repeat the above activity with squares whose sides have lengths 4 cm, 5 cm and 7 cm. You get an obtuse-angled triangle! Note that

$$4^2 + 5^2 \neq 7^2 \text{ etc.}$$

This shows that Pythagoras property holds if and only if the triangle is right-angled. Hence we get this fact:

> If the Pythagoras property holds, the triangle must be right-angled.

EXAMPLE 5 Determine whether the triangle whose lengths of sides are 3 cm, 4 cm, 5 cm is a right-angled triangle.

SOLUTION $3^2 = 3 \times 3 = 9$; $4^2 = 4 \times 4 = 16$; $5^2 = 5 \times 5 = 25$

We find $3^2 + 4^2 = 5^2$.

Therefore, the triangle is right-angled.

Note: In any right-angled triangle, the hypotenuse happens to be the longest side. In this example, the side with length 5 cm is the hypotenuse.

EXAMPLE 6 Δ ABC is right-angled at C. If AC = 5 cm and BC = 12 cm find the length of AB.

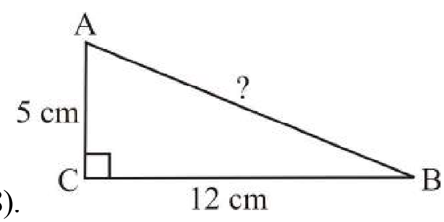


Fig 6.28

SOLUTION A rough figure will help us (Fig 6.28).

By Pythagoras property,

$$AB^2 = AC^2 + BC^2$$
$$= 5^2 + 12^2 = 25 + 144 = 169 = 13^2$$

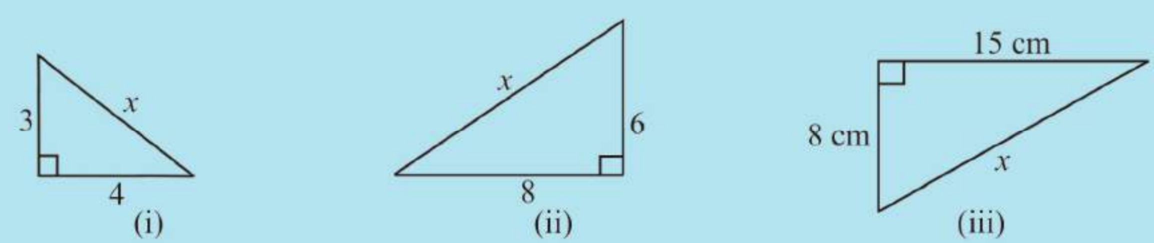
or $AB^2 = 13^2$. So, AB = 13

or the length of AB is 13 cm.

Note: To identify perfect squares, you may use prime factorisation technique.

TRY THESE

Find the unknown length x in the following figures (Fig 6.29):



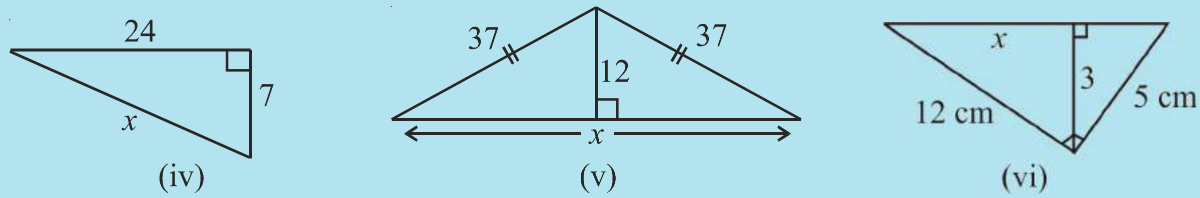
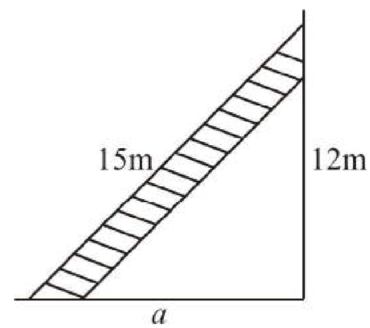


Fig 6.29

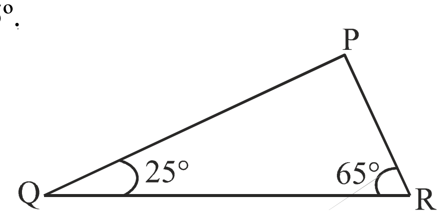
Exercise 6.5

1. PQR is a triangle, right-angled at P. If PQ = 10 cm and PR = 24 cm, find QR.
2. ABC is a triangle, right-angled at C. If AB = 25 cm and AC = 7 cm, find BC.
3. A 15 m long ladder reached a window 12 m high from the ground on placing it against a wall at a distance *a*. Find the distance of the foot of the ladder from the wall.



4. Which of the following can be the sides of a right triangle?
   (i) 2.5 cm, 6.5 cm, 6 cm.
   (ii) 2 cm, 2 cm, 5 cm.
   (iii) 1.5 cm, 2cm, 2.5 cm.

   In the case of right-angled triangles, identify the right angles.
5. A tree is broken at a height of 5 m from the ground and its top touches the ground at a distance of 12 m from the base of the tree. Find the original height of the tree.
6. Angles Q and R of a ΔPQR are 25º and 65º. Write which of the following is true:
   (i) $PQ^2 + QR^2 = RP^2$
   (ii) $PQ^2 + RP^2 = QR^2$
   (iii) $RP^2 + QR^2 = PQ^2$



7. Find the perimeter of the rectangle whose length is 40 cm and a diagonal is 41 cm.
8. The diagonals of a rhombus measure 16 cm and 30 cm. Find its perimeter.

Think, Discuss and Write

1. Which is the longest side in the triangle PQR, right-angled at P?
2. Which is the longest side in the triangle ABC, right-angled at B?
3. Which is the longest side of a right triangle?
4. 'The diagonal of a rectangle produce by itself the same area as produced by its length and breadth'– This is Baudhayan Theorem. Compare it with the Pythagoras property.

Do This

Enrichment activity

There are many proofs for Pythagoras theorem, using 'dissection' and 'rearrangement' procedure. Try to collect a few of them and draw charts explaining them.

What have We Discussed?

1. The **six elements** of a triangle are its **three angles** and the **three sides**.
2. The line segment joining a vertex of a triangle to the mid point of its opposite side is called a **median** of the triangle. A triangle has 3 medians.
3. The perpendicular line segment from a vertex of a triangle to its opposite side is called an **altitude** of the triangle. A triangle has 3 altitudes.
4. An **exterior angle** of a triangle is formed, when a side of a triangle is produced. At each vertex, you have two ways of forming an exterior angle.
5. A property of exterior angles:

   The measure of any exterior angle of a triangle is equal to the sum of the measures of its interior opposite angles.
6. The angle sum property of a triangle:

   The total measure of the three angles of a triangle is 180°.
7. A triangle is said to be **equilateral,** if each one of its sides has the same length.

   In an equilateral triangle, each angle has measure 60°.
8. A triangle is said to be **isosceles,** if atleast any two of its sides are of same length.

   The non-equal side of an isosceles triangle is called its **base**; the base angles of an isosceles triangle have equal measure.
9. Property of the lengths of sides of a triangle:

   The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

   The difference between the lengths of any two sides is smaller than the length of the third side.

This property is useful to know if it is possible to draw a triangle when the lengths of the three sides are known.

10. In a right angled triangle, the side opposite to the right angle is called the **hypotenuse** and the other two sides are called its **legs**.

11. **Pythagoras property**:

    In a right-angled triangle,

    the square on the hypotenuse = the sum of the squares on its legs.

    If a triangle is not right-angled, this property does not hold good. This property is useful to decide whether a given triangle is right-angled or not.

Congruence of Triangles

Chapter 7

7.1 Introduction

You are now ready to learn a very important geometrical idea, **Congruence**. In particular, you will study a lot about congruence of triangles.

To understand what congruence is, we turn to some activities.

Do This

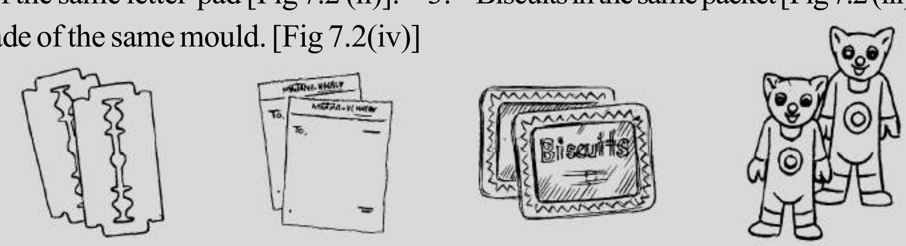
Take two stamps (Fig 7.1) of same denomination. Place one stamp over the other. What do you observe?

Fig 7.1

One stamp covers the other completely and exactly. This means that the two stamps are of the same shape and same size. Such objects are said to be congruent. The two stamps used by you are congruent to one another. Congruent objects are exact copies of one another.

Can you, now, say if the following objects are congruent or not?

1. Shaving blades of the same company [Fig 7.2 (i)].
2. Sheets of the same letter-pad [Fig 7.2 (ii)]. 3. Biscuits in the same packet [Fig 7.2 (iii)].
4. Toys made of the same mould. [Fig 7.2(iv)]



(i) (ii) (iii) (iv)

Fig 7.2

The relation of two objects being congruent is called **congruence.** For the present, we will deal with plane figures only, although congruence is a general idea applicable to three-dimensional shapes also. We will try to learn a precise meaning of the congruence of plane figures already known.

7.2 Congruence of Plane Figures

Look at the two figures given here (Fig 7.3). Are they congruent?

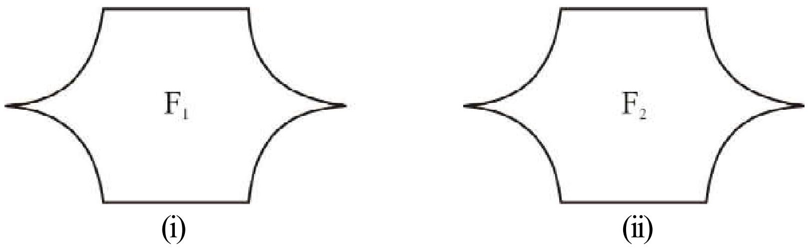


Fig 7.3

You can use the method of superposition. Take a trace-copy of one of them and place it over the other. If the figures cover each other completely, they are congruent. Alternatively, you may cut out one of them and place it over the other. Beware! You are not allowed to bend, twist or stretch the figure that is cut out (or traced out).

In Fig 7.3, if figure F_1 is congruent to figure F_2, we write $F_1 \cong F_2$.

7.3 Congruence Among Line Segments

When are two line segments congruent? Observe the two pairs of line segments given here (Fig 7.4).

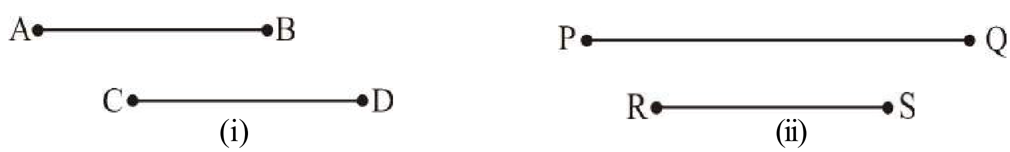


Fig 7.4

Use the 'trace-copy' superposition method for the pair of line segments in [Fig 7.4(i)]. Copy $\overline{CD}$ and place it on $\overline{AB}$. You find that $\overline{CD}$ covers $\overline{AB}$, with C on A and D on B. Hence, the line segments are congruent. We write $\overline{AB} \cong \overline{CD}$.

Repeat this activity for the pair of line segments in [Fig 7.4(ii)]. What do you find? They are not congruent. How do you know it? It is because the line segments do not coincide when placed one over other.

You should have by now noticed that the pair of line segments in [Fig 7.4(i)] matched with each other because they had same length; and this was not the case in [Fig 7.4(ii)].

If two line segments have the same (i.e., equal) length, they are congruent. Also, if two line segments are congruent, they have the same length.

In view of the above fact, when two line segments are congruent, we sometimes just say that the line segments are equal; and we also write AB = CD. (What we actually mean is $\overline{AB} \cong \overline{CD}$).

7.4 Congruence of Angles

Look at the four angles given here (Fig 7.5).

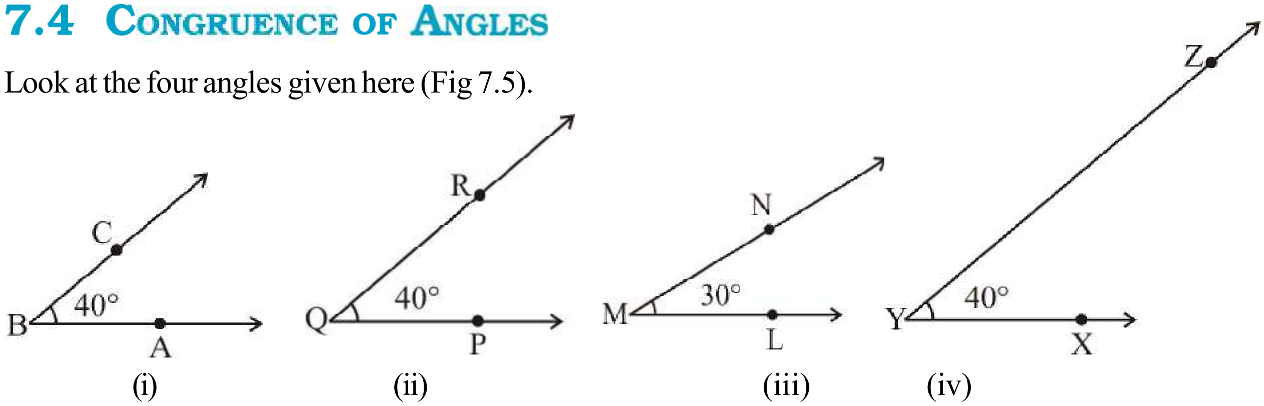


Fig 7.5

Make a trace-copy of ∠PQR. Try to superpose it on ∠ABC. For this, first place Q on B and $\overrightarrow{QP}$ along $\overrightarrow{BA}$. Where does $\overrightarrow{QR}$ fall? It falls on $\overrightarrow{BC}$.

Thus, ∠PQR matches exactly with ∠ABC.

That is, ∠ABC and ∠PQR are congruent.

(Note that the measurement of these two congruent angles are same).

We write $\angle ABC \cong \angle PQR$ (i)

or $m\angle ABC = m\ \angle PQR$ (In this case, measure is 40°).

Now, you take a trace-copy of ∠LMN. Try to superpose it on ∠ABC. Place M on B and $\overrightarrow{ML}$ along $\overrightarrow{BA}$. Does $\overrightarrow{MN}$ fall on $\overrightarrow{BC}$? No, in this case it does not happen. You find that ∠ABC and ∠LMN do not cover each other exactly. So, they are not congruent.

(Note that, in this case, the measures of ∠ABC and ∠LMN are not equal).

What about angles ∠XYZ and ∠ABC? The rays $\overrightarrow{YX}$ and $\overrightarrow{YZ}$, respectively appear [in Fig 7.5 (iv)] to be longer than $\overrightarrow{BA}$ and $\overrightarrow{BC}$. You may, hence, think that ∠ABC is 'smaller' than ∠XYZ. But remember that the rays in the figure only indicate the direction and not any length. On superposition, you will find that these two angles are also congruent.

We write $\angle ABC \cong \angle XYZ$ (ii)

or $m\angle ABC = m\angle XYZ$

In view of (i) and (ii), we may even write

$$\angle ABC \cong \angle PQR \cong \angle XYZ$$

If two angles have the same measure, they are congruent. Also, if two angles are congruent, their measures are same.

As in the case of line segments, congruency of angles entirely depends on the equality of their measures. So, to say that two angles are congruent, we sometimes just say that the angles are equal; and we write

$$\angle ABC = \angle PQR \text{ (to mean } \angle ABC \cong \angle PQR).$$

7.5 Congruence of Triangles

We saw that two line segments are congruent where one of them, is just a copy of the other. Similarly, two angles are congruent if one of them is a copy of the other. We extend this idea to triangles.

Two triangles are congruent if they are copies of each other and when superposed, they cover each other exactly.

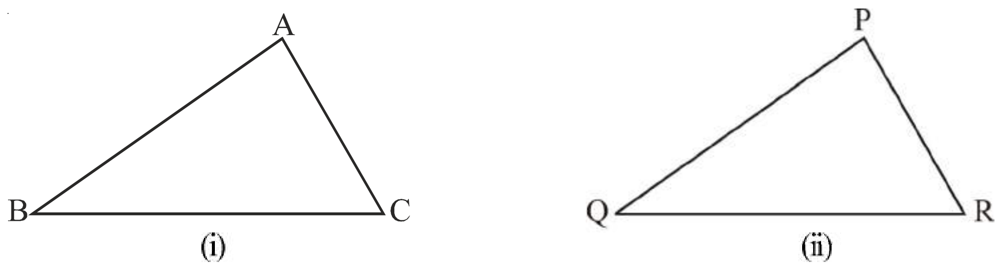


Fig 7.6

ΔABC and ΔPQR have the same size and shape. They are congruent. So, we would express this as

$$\Delta ABC \cong \Delta PQR$$

This means that, when you place ΔPQR on ΔABC, P falls on A, Q falls on B and R falls on C, also $\overline{PQ}$ falls along $\overline{AB}$, $\overline{QR}$ falls along $\overline{BC}$ and $\overline{PR}$ falls along $\overline{AC}$. If, under a given correspondence, two triangles are congruent, then their corresponding parts (i.e., angles and sides) that match one another are equal. Thus, in these two congruent triangles, we have:

Corresponding vertices : A and P, B and Q, C and R.

Corresponding sides : $\overline{AB}$ and $\overline{PQ}$, $\overline{BC}$ and $\overline{QR}$, $\overline{AC}$ and $\overline{PR}$.

Corresponding angles : $\angle A$ and $\angle P$, $\angle B$ and $\angle Q$, $\angle C$ and $\angle R$.

If you place ΔPQR on ΔABC such that P falls on B, then, should the other vertices also correspond suitably? *It need not happen*! Take trace, copies of the triangles and try to find out.

This shows that while talking about congruence of triangles, not only the measures of angles and lengths of sides matter, but also the matching of vertices. In the above case, the correspondence is

$$A \leftrightarrow P, \quad B \leftrightarrow Q, \quad C \leftrightarrow R$$

We may write this as $ABC \leftrightarrow PQR$

EXAMPLE 1 ΔABC and ΔPQR are congruent under the correspondence:

$$ABC \leftrightarrow RQP$$

Write the parts of ΔABC that correspond to

(i) $\overline{PQ}$ (ii) $\angle Q$ (iii) $\overline{RP}$

SOLUTION For better understanding of the correspondence, let us use a diagram (Fig 7.7).

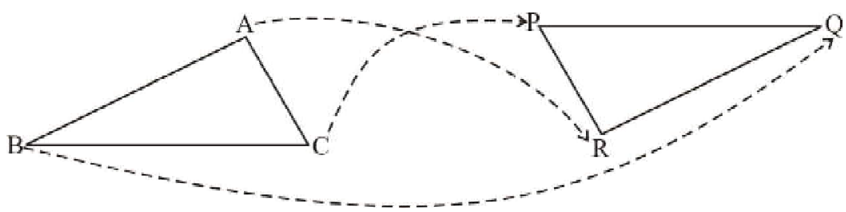


Fig 7.7

The correspondence is $ABC \leftrightarrow RQP$. This means

$A \leftrightarrow R$; $B \leftrightarrow Q$; and $C \leftrightarrow P$.

So, (i) $\overline{PQ} \leftrightarrow \overline{CB}$ (ii) $\angle Q \leftrightarrow \angle B$ and (iii) $\overline{RP} \leftrightarrow \overline{AC}$

THINK, DISCUSS AND WRITE

When two triangles, say ABC and PQR are given, there are, in all, six possible matchings or correspondences. Two of them are

(i) $ABC \leftrightarrow PQR$ and (ii) $ABC \leftrightarrow QRP$.

Find the other four correspondences by using two cutouts of triangles. Will all these correspondences lead to congruence? Think about it.

EXERCISE 7.1

1. Complete the following statements:

(a) Two line segments are congruent if ___________.

(b) Among two congruent angles, one has a measure of 70°; the measure of the other angle is ___________.

(c) When we write $\angle A = \angle B$, we actually mean ___________.

2. Give any two real-life examples for congruent shapes.

3. If $\Delta ABC \cong \Delta FED$ under the correspondence $ABC \leftrightarrow FED$, write all the corresponding congruent parts of the triangles.

4. If $\Delta DEF \cong \Delta BCA$, write the part(s) of ΔBCA that correspond to

(i) $\angle E$ (ii) $\overline{EF}$ (iii) $\angle F$ (iv) $\overline{DF}$

7.6 Criteria for Congruence of Triangles

We make use of triangular structures and patterns frequently in day-to-day life. So, it is rewarding to find out when two triangular shapes will be congruent. If you have two triangles drawn in your notebook and want to verify if they are congruent, you cannot everytime cut out one of them and use method of superposition. Instead, if we can judge congruency in terms of approrpriate measures, it would be quite useful. Let us try to do this.

A Game

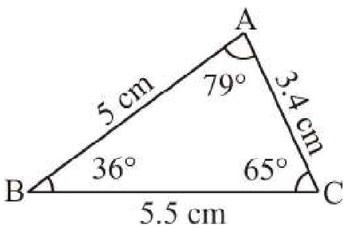


Fig 7.8
Triangle drawn by Appu

Appu and Tippu play a game. Appu has drawn a triangle ABC (Fig 7.8) and has noted the length of each of its sides and measure of each of its angles. Tippu has not seen it. Appu challenges Tippu if he can draw a copy of his ΔABC based on bits of information that Appu would give. Tippu attempts to draw a triangle congruent to ΔABC, using the information provided by Appu. The game starts. Carefully observe their conversation and their games.

SSS Game

Appu : One side of ΔABC is 5.5 cm.

Tippu : With this information, I can draw any number of triangles (Fig 7.9) but they need not be copies of ΔABC. The triangle I draw may be obtuse-angled or right-angled or acute-angled. For example, here are a few.

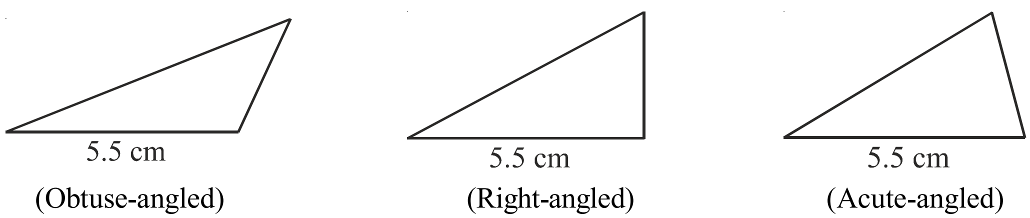


Fig 7.9

I have used some arbitrary lengths for other sides. This gives me many triangles with length of base 5.5 cm.

So, giving only one side-length will not help me to produce a copy of ΔABC.

Appu : Okay. I will give you the length of one more side. Take two sides of ΔABC to be of lengths 5.5 cm and 3.4 cm.

Tippu : Even this will not be sufficient for the purpose. I can draw several triangles (Fig 7.10) with the given information which may not be copies of ΔABC. Here are a few to support my argument:

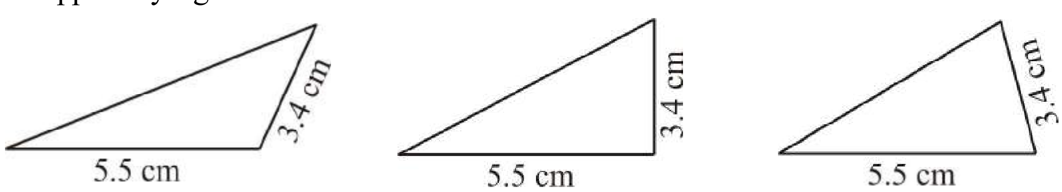


Fig 7.10

One cannot draw an exact copy of your triangle, if only the lengths of two sides are given.

Appu : Alright. Let me give the lengths of all the three sides. In ΔABC, I have AB = 5cm, BC = 5.5 cm and AC = 3.4 cm.

Tippu : I think it should be possible. Let me try now.

First I draw a rough figure so that I can remember the lengths easily.

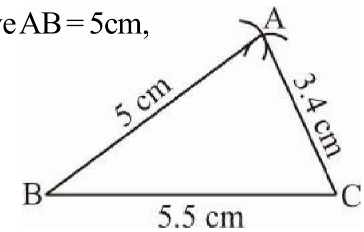


Fig 7.11

I draw $\overline{BC}$ with length 5.5 cm.

With B as centre, I draw an arc of radius 5 cm. The point A has to be somewhere on this arc. With C as centre, I draw an arc of radius 3.4 cm. The point A has to be on this arc also.

So, A lies on both the arcs drawn. This means A is the point of intersection of the arcs.

I know now the positions of points A, B and C. Aha! I can join them and get ΔABC (Fig 7.11).

Appu : Excellent. So, to draw a copy of a given ΔABC (i.e., to draw a triangle congruent to ΔABC), we need the lengths of three sides. Shall we call this condition as side-side-side criterion?

Tippu : Why not we call it SSS criterion, to be short?

SSS Congruence criterion:

If under a given correspondence, the three sides of one triangle are equal to the three corresponding sides of another triangle, then the triangles are congruent.

EXAMPLE 2 In triangles ABC and PQR, AB = 3.5 cm, BC = 7.1 cm, AC = 5 cm, PQ = 7.1 cm, QR = 5 cm and PR = 3.5 cm. Examine whether the two triangles are congruent or not. If yes, write the congruence relation in symbolic form.

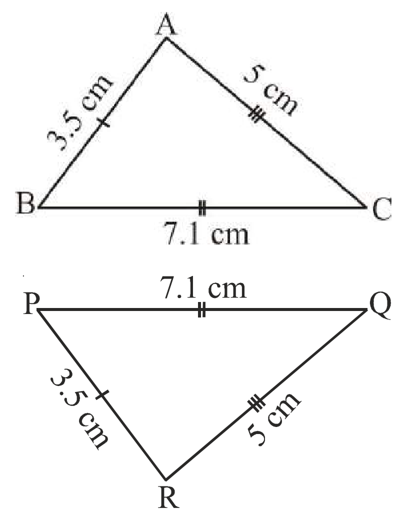


Fig 7.12

SOLUTION Here, AB = PR (= 3.5 cm),

BC = PQ (= 7.1 cm)

and AC = QR (= 5 cm)

This shows that the three sides of one triangle are equal to the three sides of the other triangle. So, by SSS congruence rule, the two triangles are congruent. From the above three equality relations, it can be easily seen that A ↔ R, B ↔ P and C ↔ Q.

So, we have ΔABC ≅ ΔRPQ

Important note: The order of the letters in the names of congruent triangles displays the corresponding relationships. Thus, when you write ΔABC ≅ ΔRPQ, you would know that A lies on R, B on P, C on Q, $\overline{AB}$ along $\overline{RP}$, $\overline{BC}$ along $\overline{PQ}$ and $\overline{AC}$ along $\overline{RQ}$.

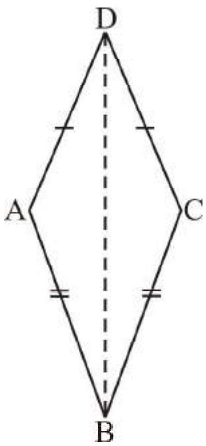


Fig 7.13

EXAMPLE 3 In Fig 7.13, AD = CD and AB = CB.

(i) State the three pairs of equal parts in ΔABD and ΔCBD.

(ii) Is ΔABD ≅ ΔCBD? Why or why not?

(iii) Does BD bisect ∠ABC? Give reasons.

SOLUTION

(i) In ΔABD and ΔCBD, the three pairs of equal parts are as given below:

AB = CB (Given)

AD = CD (Given)

and BD = BD (Common in both)

(ii) From (i) above, ΔABD ≅ ΔCBD (By SSS congruence rule)

(iii) ∠ABD = ∠CBD (Corresponding parts of congruent triangles)

So, BD bisects ∠ABC.

TRY THESE

1. In Fig 7.14, lengths of the sides of the triangles are indicated. By applying the SSS congruence rule, state which pairs of triangles are congruent. In case of congruent triangles, write the result in symbolic form:

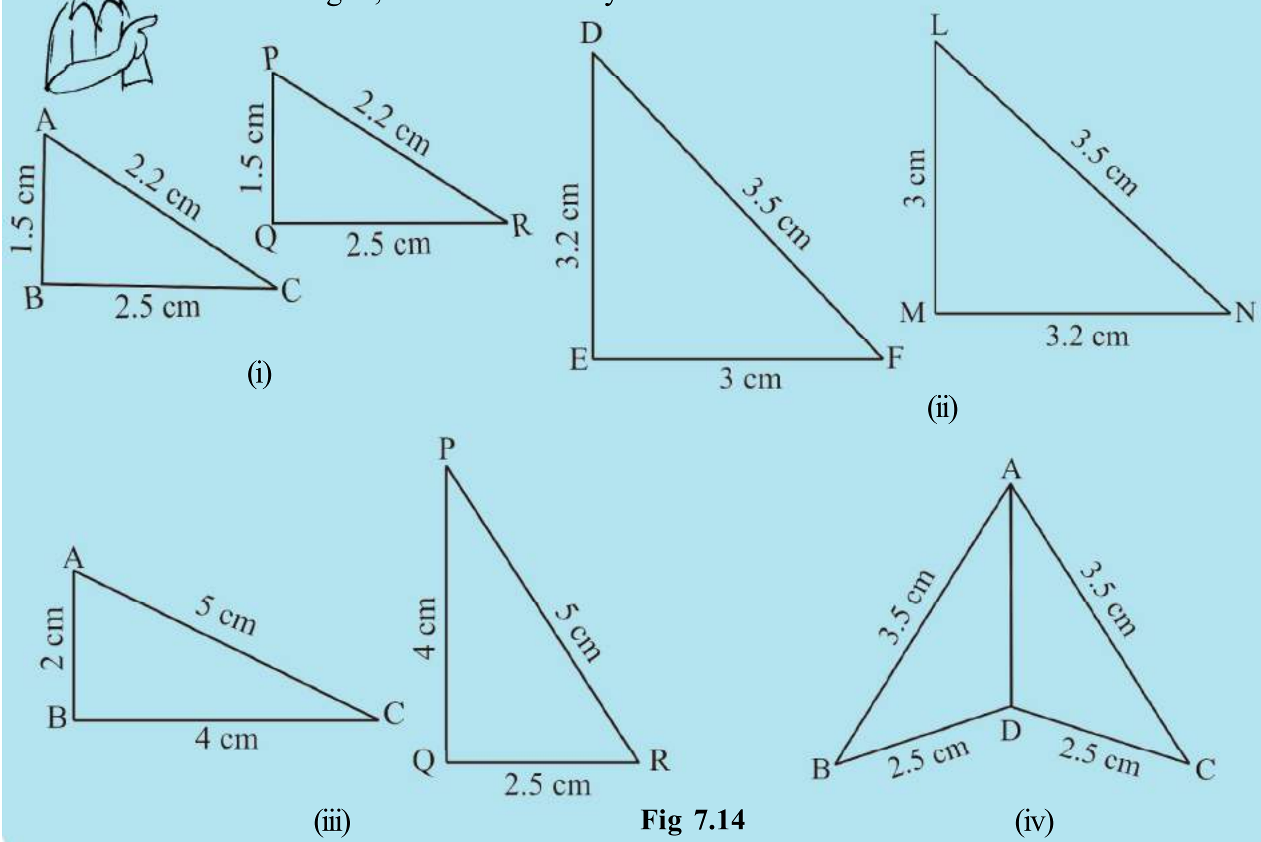


Fig 7.14

2. In Fig 7.15, AB = AC and D is the mid-point of $\overline{BC}$.
   (i) State the three pairs of equal parts in ΔADB and ΔADC.
   (ii) Is $\Delta ADB \cong \Delta ADC$? Give reasons.
   (iii) Is $\angle B = \angle C$? Why?

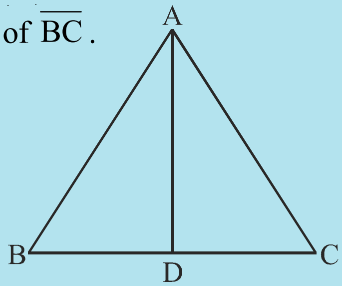


Fig 7.15

3. In Fig 7.16, AC = BD and AD = BC. Which of the following statements is meaningfully written?
   (i) $\Delta ABC \cong \Delta ABD$ (ii) $\Delta ABC \cong \Delta BAD$.

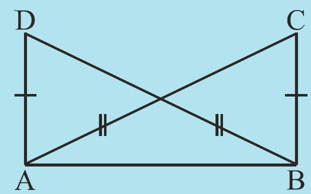


Fig 7.16

Think, Discuss and Write

ABC is an isosceles triangle with AB = AC (Fig 7.17).

Take a trace-copy of ΔABC and also name it as ΔABC.

(i) State the three pairs of equal parts in ΔABC and ΔACB.

(ii) Is $\Delta ABC \cong \Delta ACB$? Why or why not?

(iii) Is $\angle B = \angle C$? Why or why not?

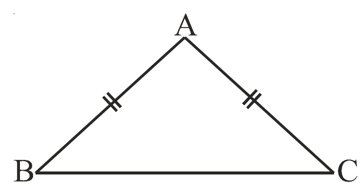


Fig 7.17

Appu and Tippu now turn to playing the game with a slight modification.

SAS Game

Appu : Let me now change the rules of the triangle-copying game.

Tippu : Right, go ahead.

Appu : You have already found that giving the length of only one side is useless.

Tippu : Of course, yes.

Appu : In that case, let me tell that in ΔABC, one side is 5.5 cm and one angle is 65°.

Tippu : This again is not sufficient for the job. I can find many triangles satisfying your information, but are not copies of ΔABC. For example, I have given here some of them (Fig 7.18):

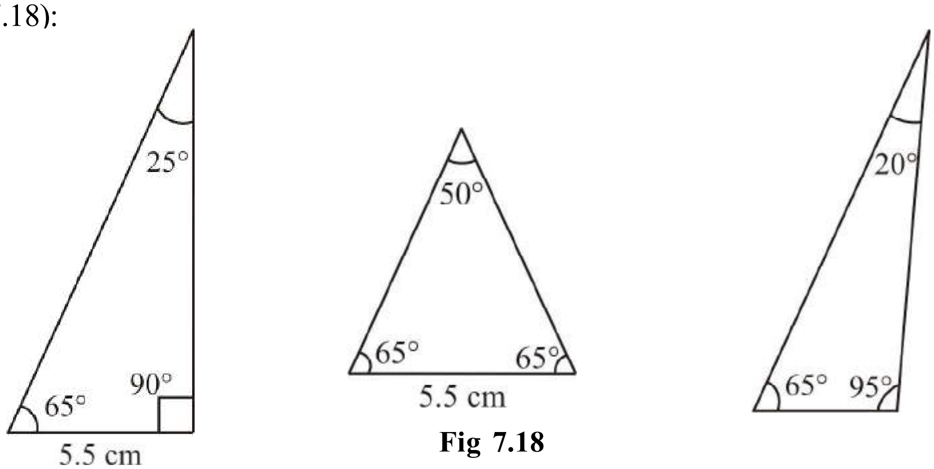


Fig 7.18

Appu : So, what shall we do?
Tippu : More information is needed.
Appu : Then, let me modify my earlier statement. In ΔABC, the length of two sides are 5.5 cm and 3.4 cm, and the angle between these two sides is 65°.

Tippu : This information should help me. Let me try. I draw first $\overline{BC}$ of length 5.5. cm [Fig 7.19 (i)]. Now I make 65° at C [Fig 7.19 (ii)].

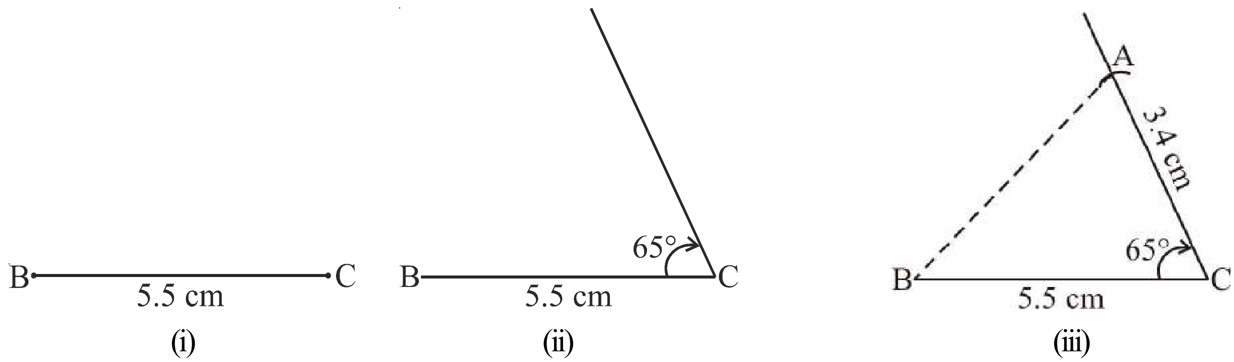


Fig 7.19

Yes, I got it, A must be 3.4 cm away from C along this angular line through C.
I draw an arc of 3.4 cm with C as centre. It cuts the 65° line at A.
Now, I join AB and get ΔABC [Fig 7.19(iii)].

Appu : You have used side-angle-side, where the angle is 'included' between the sides!
Tippu : Yes. How shall we name this criterion?
Appu : It is SAS criterion. Do you follow it?
Tippu : Yes, of course.

SAS Congruence criterion:
If under a correspondence, two sides and the angle included between them of a triangle are equal to two corresponding sides and the angle included between them of another triangle, then the triangles are congruent.

EXAMPLE 4 Given below are measurements of some parts of two triangles. Examine whether the two triangles are congruent or not, by using SAS congruence rule. If the triangles are congruent, write them in symbolic form.

	ΔABC	ΔDEF
(a)	AB = 7 cm, BC = 5 cm, $\angle B = 50°$	DE = 5 cm, EF = 7 cm, $\angle E = 50°$
(b)	AB = 4.5 cm, AC = 4 cm, $\angle A = 60°$	DE = 4 cm, FD = 4.5 cm, $\angle D = 55°$
(c)	BC = 6 cm, AC = 4 cm, $\angle B = 35°$	DF = 4 cm, EF = 6 cm, $\angle E = 35°$

(It will be always helpful to draw a rough figure, mark the measurements and then probe the question).

SOLUTION

(a) Here, AB = EF (= 7 cm), BC = DE (= 5 cm) and included ∠B = included ∠E (= 50°). Also, A ↔ F B ↔ E and C ↔ D. Therefore, ΔABC ≅ ΔFED (By SAS congruence rule) (Fig 7.20)

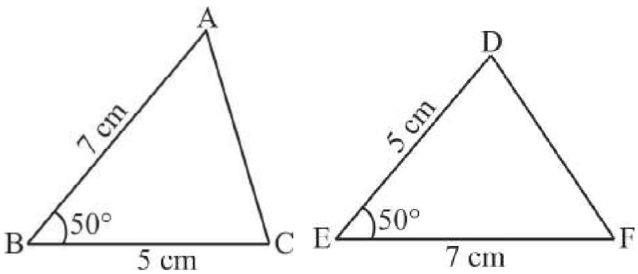


Fig 7.20

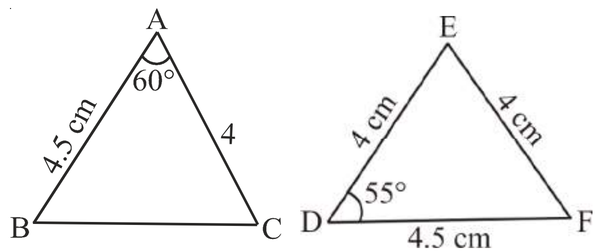


Fig 7.21

(b) Here, AB = FD and AC = DE (Fig 7.21). But included ∠A ≠ included ∠D. So, we cannot say that the triangles are congruent.

(c) Here, BC = EF, AC = DF and ∠B = ∠E. But ∠B is not the included angle between the sides AC and BC. Similarly, ∠E is not the included angle between the sides EF and DF. So, SAS congruence rule cannot be applied and we cannot conclude that the two triangles are congruent.

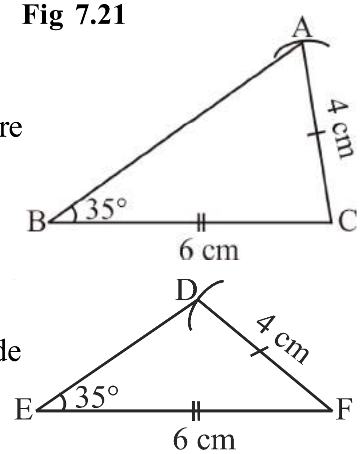


Fig 7.22

EXAMPLE 5 In Fig 7.23, AB = AC and AD is the bisector of ∠BAC.

(i) State three pairs of equal parts in triangles ADB and ADC.

(ii) Is ΔADB ≅ ΔADC? Give reasons.

(iii) Is ∠B = ∠C? Give reasons.

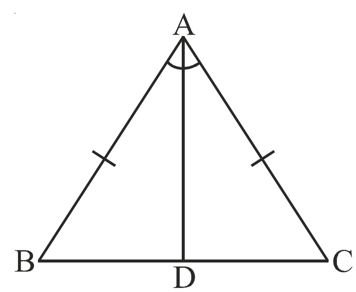


Fig 7.23

SOLUTION

(i) The three pairs of equal parts are as follows:
AB = AC (Given)
∠BAD = ∠CAD (AD bisects ∠BAC) and AD = AD (common)

(ii) Yes, ΔADB ≅ ΔADC (By SAS congruence rule)

(iii) ∠B = ∠C (Corresponding parts of congruent triangles)

TRY THESE

1. Which angle is included between the sides $\overline{DE}$ and $\overline{EF}$ of ΔDEF?
2. By applying SAS congruence rule, you want to establish that ΔPQR ≅ ΔFED. It is given that PQ = FE and RP = DF. What additional information is needed to establish the congruence?

3. In Fig 7.24, measures of some parts of the triangles are indicated. By applying SAS congruence rule, state the pairs of congruent triangles, if any, in each case. In case of congruent triangles, write them in symbolic form.

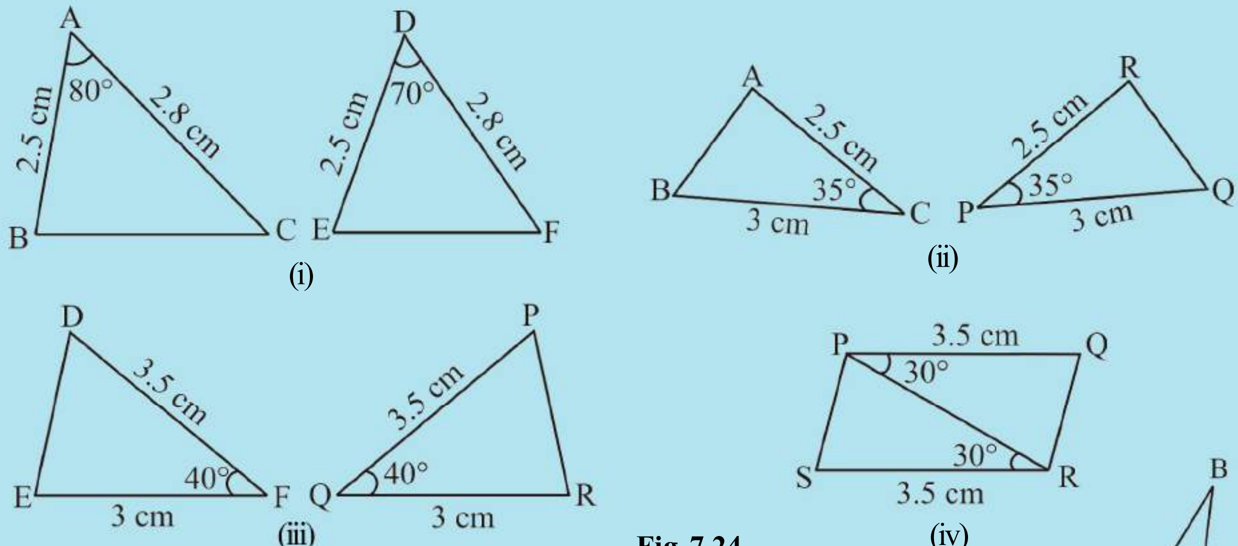


Fig 7.24

4. In Fig 7.25, $\overline{AB}$ and $\overline{CD}$ bisect each other at O.
   (i) State the three pairs of equal parts in two triangles AOC and BOD.
   (ii) Which of the following statements are true?
      (a) $\Delta AOC \cong \Delta DOB$
      (b) $\Delta AOC \cong \Delta BOD$

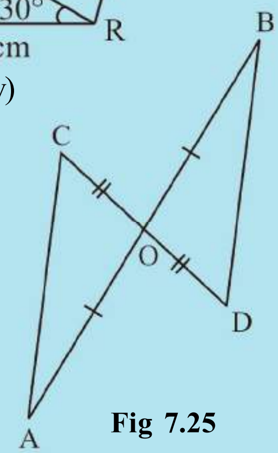


Fig 7.25

ASA Game

Can you draw Appu's triangle, if you know
   (i) only one of its angles?
   (ii) only two of its angles?
   (iii) two angles and any one side?
   (iv) two angles and the side included between them?

Attempts to solve the above questions lead us to the following criterion:

ASA Congruence criterion:
If under a correspondence, two angles and the included side of a triangle are equal to two corresponding angles and the included side of another triangle, then the triangles are congruent.

EXAMPLE 6 By applying ASA congruence rule, it is to be established that $\Delta ABC \cong \Delta QRP$ and it is given that BC = RP. What additional information is needed to establish the congruence?

SOLUTION For ASA congruence rule, we need the two angles between which the two sides BC and RP are included. So, the additional information is as follows:

$$\angle B = \angle R$$

and $$\angle C = \angle P$$

EXAMPLE 7 In Fig 7.26, can you use ASA congruence rule and conclude that $\Delta AOC \cong \Delta BOD$?

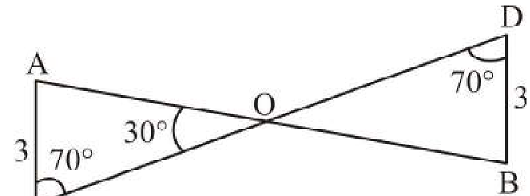


Fig 7.26

SOLUTION In the two triangles AOC and BOD, $\angle C = \angle D$ (each 70°)

Also, $\angle AOC = \angle BOD = 30°$ (vertically opposite angles)

So, $\angle A$ of $\Delta AOC = 180° – (70° + 30°) = 80°$

(using angle sum property of a triangle)

Similarly, $\angle B$ of $\Delta BOD = 180° – (70° + 30°) = 80°$

Thus, we have $\angle A = \angle B$, AC = BD and $\angle C = \angle D$

Now, side AC is between $\angle A$ and $\angle C$ and side BD is between $\angle B$ and $\angle D$.

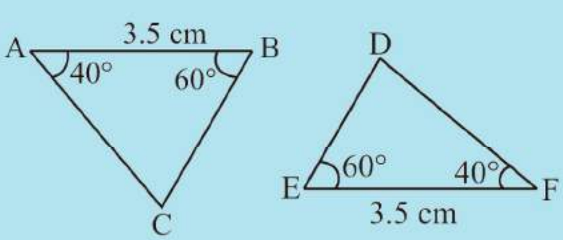
So, by ASA congruence rule, $\Delta AOC \cong \Delta BOD$.

Remark

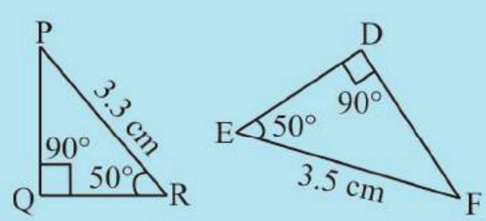
Given two angles of a triangle, you can always find the third angle of the triangle. So, whenever, two angles and one side of one triangle are equal to the corresponding two angles and one side of another triangle, you may convert it into 'two angles and the included side' form of congruence and then apply the ASA congruence rule.

TRY THESE

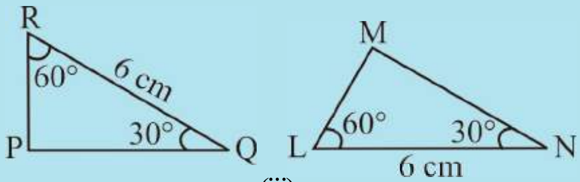
1. What is the side included between the angles M and N of ΔMNP?
2. You want to establish $\Delta DEF \cong \Delta MNP$, using the ASA congruence rule. You are given that $\angle D = \angle M$ and $\angle F = \angle P$. What information is needed to establish the congruence? (Draw a rough figure and then try!)
3. In Fig 7.27, measures of some parts are indicated. By applying ASA congruence rule, state which pairs of triangles are congruent. In case of congruence, write the result in symoblic form.



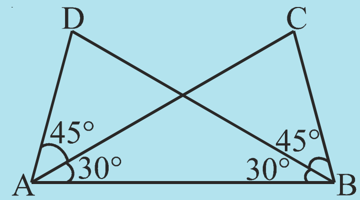
(i)



(ii)



(iii)



(iv)

Fig 7.27

4. Given below are measurements of some parts of two triangles. Examine whether the two triangles are congruent or not, by ASA congruence rule. In case of congruence, write it in symbolic form.

	Δ**DEF**	Δ**PQR**
(i)	$\angle D = 60^\circ$, $\angle F = 80^\circ$, DF = 5 cm	$\angle Q = 60^\circ$, $\angle R = 80^\circ$, QR = 5 cm
(ii)	$\angle D = 60^\circ$, $\angle F = 80^\circ$, DF = 6 cm	$\angle Q = 60^\circ$, $\angle R = 80^\circ$, QP = 6 cm
(iii)	$\angle E = 80^\circ$, $\angle F = 30^\circ$, EF = 5 cm	$\angle P = 80^\circ$, PQ = 5 cm, $\angle R = 30^\circ$

5. In Fig 7.28, ray AZ bisects $\angle DAB$ as well as $\angle DCB$.

(i) State the three pairs of equal parts in triangles BAC and DAC.

(ii) Is $\Delta BAC \cong \Delta DAC$? Give reasons.

(iii) Is AB = AD? Justify your answer.

(iv) Is CD = CB? Give reasons.

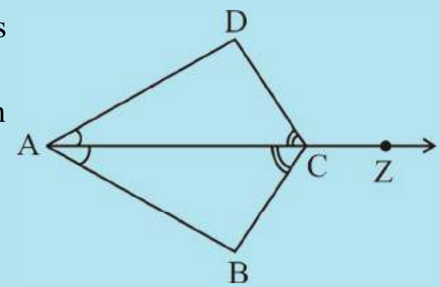


Fig 7.28

7.7 Congruence Among Right-Angled Triangles

Congruence in the case of two right triangles deserves special attention. In such triangles, obviously, the right angles are equal. So, the congruence criterion becomes easy.

Can you draw ΔABC (shown in Fig 7.29) with $\angle B = 90^\circ$, if

(i) only BC is known?

(ii) only $\angle C$ is known?

(iii) $\angle A$ and $\angle C$ are known?

(iv) AB and BC are known?

(v) AC and one of AB or BC are known?

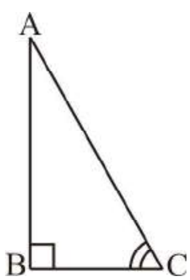


Fig 7.29

Try these with rough sketches. You will find that (iv) and (v) help you to draw the triangle. But case (iv) is simply the SAS condition. Case (v) is something new. This leads to the following criterion:

RHS Congruence criterion:

If under a correspondence, the hypotenuse and one side of a right-angled triangle are respectively equal to the hypotenuse and one side of another right-angled triangle, then the triangles are congruent.

Why do we call this 'RHS' congruence? Think about it.

EXAMPLE 8 Given below are measurements of some parts of two triangles. Examine whether the two triangles are congruent or not, using RHS congruence rule. In case of congruent triangles, write the result in symbolic form:

	ΔABC	ΔPQR
(i)	∠B = 90°, AC = 8 cm, AB = 3 cm	∠P = 90°, PR = 3 cm, QR = 8 cm
(ii)	∠A = 90°, AC = 5 cm, BC = 9 cm	∠Q = 90°, PR = 8 cm, PQ = 5 cm

SOLUTION

(i) Here, ∠B = ∠P = 90º,
hypotenuse, AC = hypotenuse, RQ (= 8 cm) and
side AB = side RP (= 3 cm)
So, ΔABC ≅ ΔRPQ (By RHS Congruence rule). [Fig 7.30(i)]

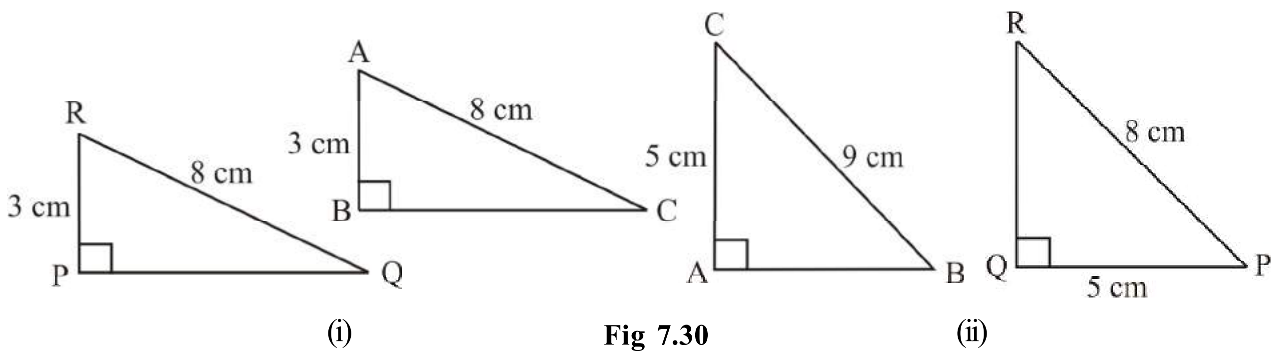


Fig 7.30

(ii) Here, ∠A = ∠Q (= 90°) and
side AC = side PQ (= 5 cm).
But hypotenuse BC ≠ hypotenuse PR [Fig 7.30(ii)]
So, the triangles are not congruent.

EXAMPLE 9 In Fig 7.31, DA ⊥ AB, CB ⊥ AB and AC = BD.
State the three pairs of equal parts in ΔABC and ΔDAB.
Which of the following statements is meaningful?

(i) ΔABC ≅ ΔBAD (ii) ΔABC ≅ ΔABD

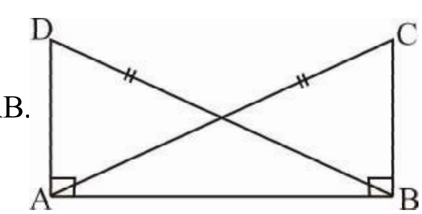


Fig 7.31

SOLUTION The three pairs of equal parts are:

∠ABC = ∠BAD (= 90°)
AC = BD (Given)
AB = BA (Common side)

From the above, ΔABC ≅ ΔBAD (By RHS congruence rule).
So, statement (i) is true

Statement (ii) is not meaningful, in the sense that the correspondence among the vertices is not satisfied.

TRY THESE

1. In Fig 7.32, measures of some parts of triangles are given.By applying RHS congruence rule, state which pairs of triangles are congruent. In case of congruent triangles, write the result in symbolic form.

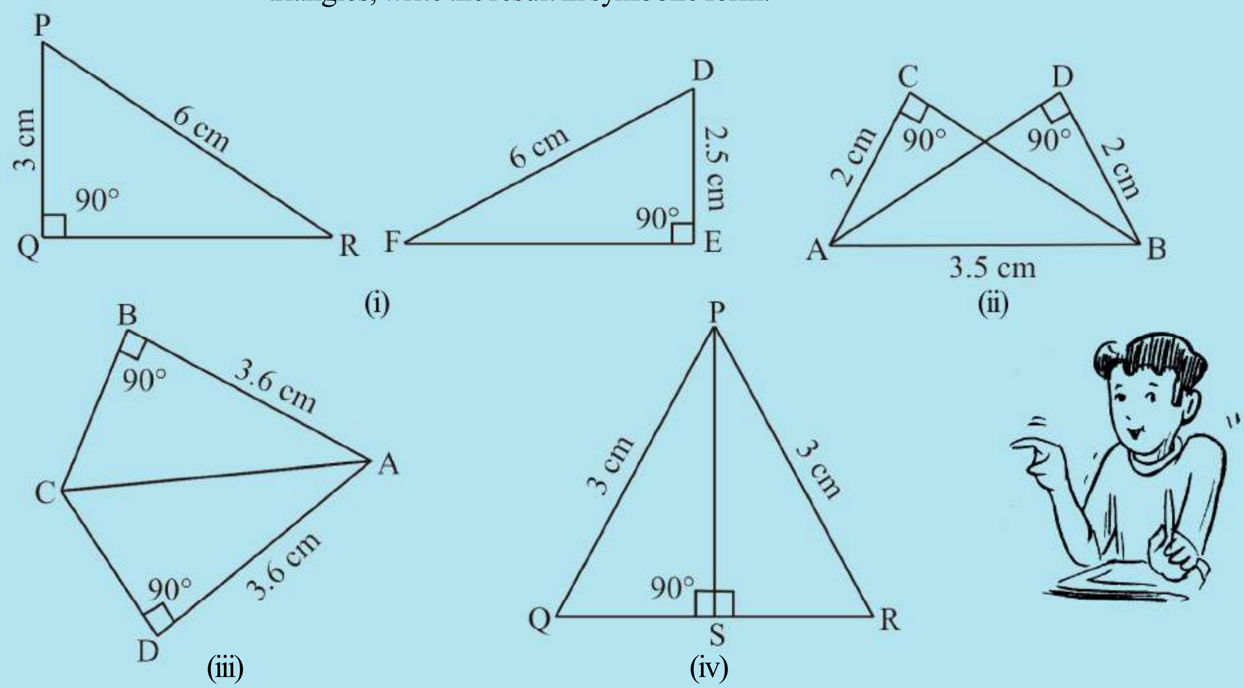


Fig 7.32

2. It is to be established by RHS congruence rule that $\Delta ABC \cong \Delta RPQ$. What additional information is needed, if it is given that

   $\angle B = \angle P = 90°$ and $AB = RP$?

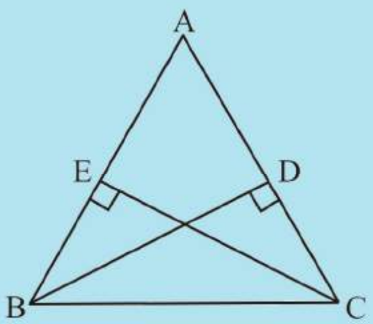


Fig 7.33

3. In Fig 7.33, BD and CE are altitudes of ΔABC such that $BD = CE$.
   (i) State the three pairs of equal parts in ΔCBD and ΔBCE.
   (ii) Is $\Delta CBD \cong \Delta BCE$? Why or why not?
   (iii) Is $\angle DCB = \angle EBC$? Why or why not?

4. ABC is an isosceles triangle with $AB = AC$ and AD is one of its altitudes (Fig 7.34).
   (i) State the three pairs of equal parts in ΔADB and ΔADC.
   (ii) Is $\Delta ADB \cong \Delta ADC$? Why or why not?
   (iii) Is $\angle B = \angle C$? Why or why not?
   (iv) Is $BD = CD$? Why or why not?

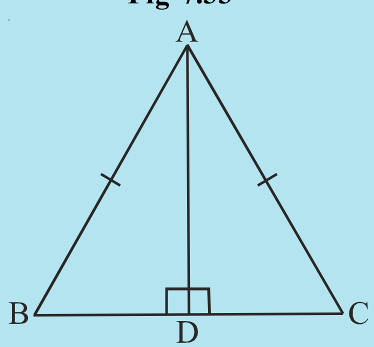


Fig 7.34

We now turn to examples and problems based on the criteria seen so far.

EXERCISE 7.2

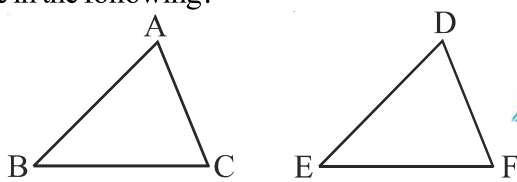
1. Which congruence criterion do you use in the following?

   (a) **Given:** AC = DF

   AB = DE

   BC = EF

   So, ΔABC ≅ ΔDEF

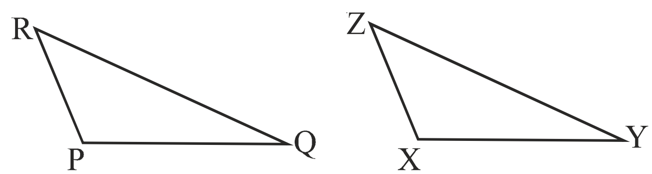


   (b) **Given:** ZX = RP

   RQ = ZY

   ∠PRQ = ∠XZY

   So, ΔPQR ≅ ΔXYZ

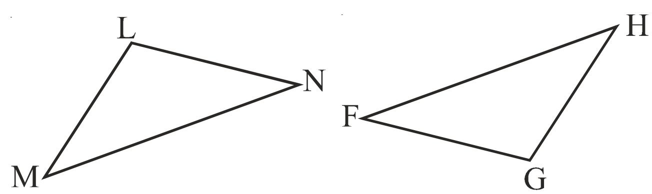


   (c) **Given:** ∠MLN = ∠FGH

   ∠NML = ∠GFH

   ML = FG

   So, ΔLMN ≅ ΔGFH

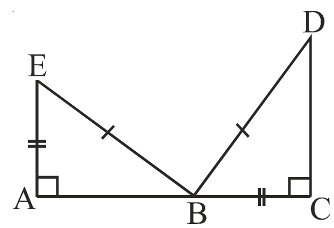


   (d) **Given:** EB = DB

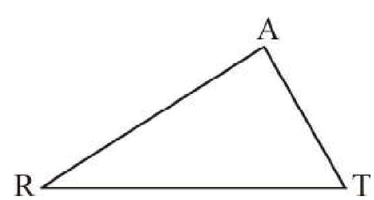
   AE = BC

   ∠A = ∠C = 90°

   So, ΔABE ≅ ΔCDB



2. You want to show that ΔART ≅ ΔPEN,

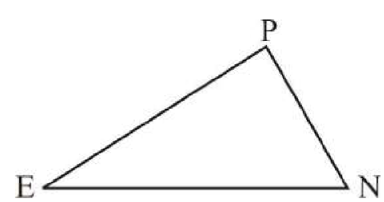


   (a) If you have to use SSS criterion, then you need to show

   (i) AR = (ii) RT = (iii) AT =

   (b) If it is given that ∠T = ∠N and you are to use SAS criterion, you need to have

   (i) RT = and (ii) PN =

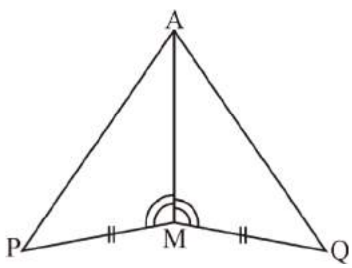


   (c) If it is given that AT = PN and you are to use ASA criterion, you need to have

   (i) ? (ii) ?

3. You have to show that $\Delta AMP \cong \Delta AMQ$.

In the following proof, supply the missing reasons.



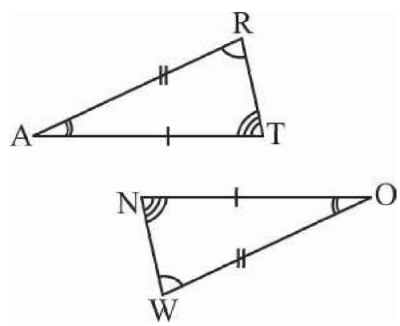
Steps	Reasons
(i) PM = QM	(i) ...
(ii) $\angle PMA = \angle QMA$	(ii) ...
(iii) AM = AM	(iii) ...
(iv) $\Delta AMP \cong \Delta AMQ$	(iv) ...

4. In ΔABC, $\angle A = 30°$, $\angle B = 40°$ and $\angle C = 110°$

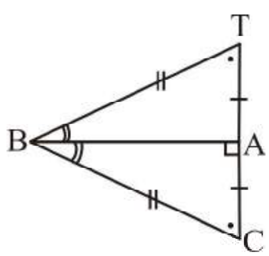
In ΔPQR, $\angle P = 30°$, $\angle Q = 40°$ and $\angle R = 110°$

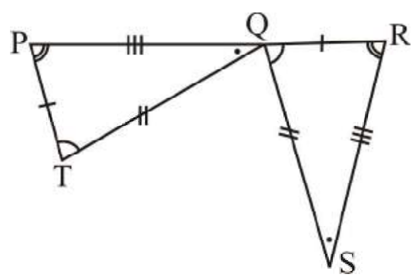
A student says that $\Delta ABC \cong \Delta PQR$ by AAA congruence criterion. Is he justified? Why or why not?

5. In the figure, the two triangles are congruent. The corresponding parts are marked. We can write $\Delta RAT \cong$?



6. Complete the congruence statement:





$\Delta BCA \cong$? $\Delta QRS \cong$?

7. In a squared sheet, draw two triangles of equal areas such that

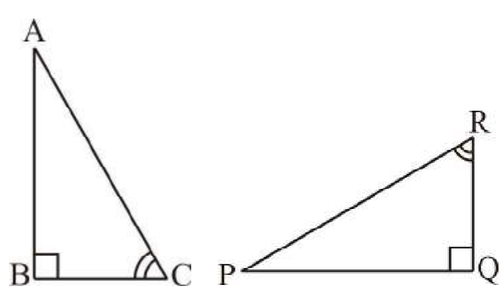
(i) the triangles are congruent.

(ii) the triangles are not congruent.

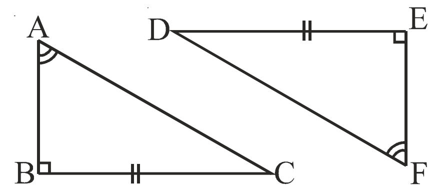
What can you say about their perimeters?

8. Draw a rough sketch of two triangles such that they have five pairs of congruent parts but still the triangles are not congruent.

9. If ΔABC and ΔPQR are to be congruent, name one additional pair of corresponding parts. What criterion did you use?



10. Explain, why
$\Delta ABC \cong \Delta FED$.



Enrichment activity

We saw that superposition is a useful method to test congruence of plane figures. We discussed conditions for congruence of line segments, angles and triangles. You can now try to extend this idea to other plane figures as well.

1. Consider cut-outs of different sizes of squares. Use the method of superposition to find out the condition for congruence of squares. How does the idea of 'corresponding parts' under congruence apply? Are there corresponding sides? Are there corresponding diagonals?
2. What happens if you take circles? What is the condition for congruence of two circles? Again, you can use the method of superposition. Investigate.
3. Try to extend this idea to other plane figures like regular hexagons, etc.
4. Take two congruent copies of a triangle. By paper folding, investigate if they have equal altitudes. Do they have equal medians? What can you say about their perimeters and areas?

What have We Discussed?

1. Congruent objects are exact copies of one another.

2. The method of superposition examines the congruence of plane figures.

3. Two plane figures, say, F_1 and F_2 are congruent if the trace-copy of F_1 fits exactly on that of F_2. We write this as $F_1 \cong F_2$.

4. Two line segments, say, $\overline{AB}$ and $\overline{CD}$, are congruent if they have equal lengths. We write this as $\overline{AB} \cong \overline{CD}$. However, it is common to write it as $\overline{AB} = \overline{CD}$.

5. Two angles, say, $\angle ABC$ and $\angle PQR$, are congruent if their measures are equal. We write this as $\angle ABC \cong \angle PQR$ or as $m\angle ABC = m\angle PQR$. However, in practice, it is common to write it as $\angle ABC = \angle PQR$.

6. SSS Congruence of two triangles:

Under a given correspondence, two triangles are congruent if the three sides of the one are equal to the three corresponding sides of the other.

7. SAS Congruence of two triangles:

Under a given correspondence, two triangles are congruent if two sides and the angle included between them in one of the triangles are equal to the corresponding sides and the angle included between them of the other triangle.

8. ASA Congruence of two triangles:

Under a given correspondence, two triangles are congruent if two angles and the side included between them in one of the triangles are equal to the corresponding angles and the side included between them of the other triangle.

9. RHS Congruence of two right-angled triangles:

Under a given correspondence, two right-angled triangles are congruent if the hypotenuse and a leg of one of the triangles are equal to the hypotenuse and the corresponding leg of the other triangle.

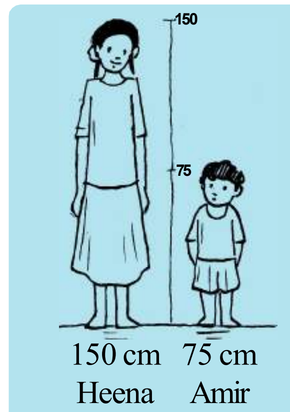
10. There is no such thing as AAA Congruence of two triangles:

Two triangles with equal corresponding angles need not be congruent. In such a correspondence, one of them can be an enlarged copy of the other. (They would be congruent only if they are exact copies of one another).

Chapter 8

Comparing Quantities

8.1 Introduction



In our daily life, there are many occasions when we compare two quantities. Suppose we are comparing heights of Heena and Amir. We find that

1. Heena is two times taller than Amir.

Or

2. Amir's height is $\frac{1}{2}$ of Heena's height.

Consider another example, where 20 marbles are divided between Rita and Amit such that Rita has 12 marbles and Amit has 8 marbles. We say,

1. Rita has $\frac{3}{2}$ times the marbles that Amit has.

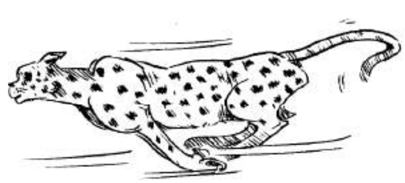
Or

2. Amit has $\frac{2}{3}$ part of what Rita has.

Yet another example is where we compare speeds of a Cheetah and a Man.
The speed of a Cheetah is 6 times the speed of a Man.

Or

The speed of a Man is $\frac{1}{6}$ of the speed of the Cheetah.



Speed of Cheetah
120 km per hour

Speed of Man
20 km per hour

Do you remember comparisons like this? In Class VI, we have learnt to make comparisons by saying how many times one quantity is of the other. Here, we see that it can also be inverted and written as what part one quantity is of the other.

In the given cases, we write the ratio of the heights as :

Heena's height : Amir's height is 150 : 75 or 2 : 1.

Can you now write the ratios for the other comparisons?

These are relative comparisons and could be same for two different situations.

If Heena's height was 150 cm and Amir's was 100 cm, then the ratio of their heights would be,

Heena's height : Amir's height = $150 : 100 = \frac{150}{100} = \frac{3}{2}$ or 3 : 2.

This is same as the ratio for Rita's to Amit's share of marbles.

Thus, we see that the ratio for two different comparisons may be the same. Remember that *to compare two quantities, the units must be the same.*

Example 1 Find the ratio of 3 km to 300 m.

Solution First convert both the distances to the same unit.

So, 3 km = 3 × 1000 m = 3000 m.

Thus, the required ratio, 3 km : 300 m is 3000 : 300 = 10 : 1.

8.2 Equivalent Ratios

Different ratios can also be compared with each other to know whether they are equivalent or not. To do this, we need to write the ratios in the form of fractions and then compare them by converting them to like fractions. If these like fractions are equal, we say the given ratios are equivalent.

Example 2 Are the ratios 1:2 and 2:3 equivalent?

Solution To check this, we need to know whether $\frac{1}{2} = \frac{2}{3}$.

We have, $\frac{1}{2} = \frac{1 \times 3}{2 \times 3} = \frac{3}{6}$; $\frac{2}{3} = \frac{2 \times 2}{3 \times 2} = \frac{4}{6}$

We find that $\frac{3}{6} < \frac{4}{6}$, which means that $\frac{1}{2} < \frac{2}{3}$.

Therefore, the ratio 1:2 is not equivalent to the ratio 2:3.

Use of such comparisons can be seen by the following example.

Example 3 Following is the performance of a cricket team in the matches it played:

Year	Wins	Losses
Last year	8	2
This year	4	2

In which year was the record better?

How can you say so?

SOLUTION Last year, Wins: Losses = 8 : 2 = 4 : 1

This year, Wins: Losses = 4 : 2 = 2 : 1

Obviously, 4 : 1 > 2 : 1 (In fractional form, $\frac{4}{1} > \frac{2}{1}$)

Hence, we can say that the team performed better last year.

In Class VI, we have also seen the importance of equivalent ratios. The ratios which are equivalent are said to be in proportion. Let us recall the use of proportions.

Keeping things in proportion and getting solutions

Aruna made a sketch of the building she lives in and drew sketch of her mother standing beside the building.

Mona said, "There seems to be something wrong with the drawing"

Can you say what is wrong? How can you say this?

In this case, the ratio of heights in the drawing should be the same as the ratio of actual heights. That is

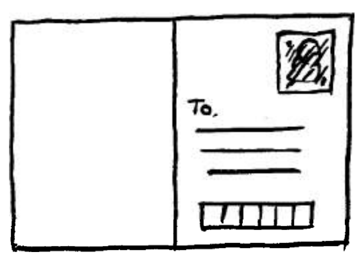
$$\frac{\text{Actual height of building}}{\text{Actual height of mother}} = \frac{\text{Height of building in drawing}}{\text{Height of mother in the drawing}}.$$

Only then would these be in proportion. Often when proportions are maintained, the drawing seems pleasing to the eye.

Another example where proportions are used is in the making of national flags.

Do you know that the flags are always made in a fixed ratio of length to its breadth? These may be different for different countries but are mostly around 1.5 : 1 or 1.7 : 1.

We can take an approximate value of this ratio as 3 : 2. Even the Indian post card is around the same ratio.



Now, can you say whether a card with length 4.5 cm and breadth 3.0 cm is near to this ratio. That is we need to ask, is 4.5 : 3.0 equivalent to 3 : 2?

We note that $4.5 : 3.0 = \frac{4.5}{3.0} = \frac{45}{30} = \frac{3}{2}$

Hence, we see that 4.5 : 3.0 is equivalent to 3 : 2.

We see a wide use of such proportions in real life. Can you think of some more situations?

We have also learnt a method in the earlier classes known as *Unitary Method* in which we first find the value of one unit and then the value of the required number of units.

Let us see how both the above methods help us to achieve the same thing.

EXAMPLE 4 A map is given with a scale of 2 cm = 1000 km. What is the actual distance between the two places in kms, if the distance in the map is 2.5 cm?

SOLUTION

Arun does it like this	**Meera does it like this**
Let distance = x km then, $1000 : x = 2 : 2.5$ $\frac{1000}{x} = \frac{2}{2.5}$ $\frac{1000 \times x \times 2.5}{x} = \frac{2}{2.5} \times x \times 2.5$ $1000 \times 2.5 = x \times 2$ $x = 1250$	2 cm means 1000 km. So, 1 cm means $\frac{1000}{2}$ km Hence, 2.5 cm means $\frac{1000}{2} \times 2.5$ km = 1250 km

Arun has solved it by equating ratios to make proportions and then by solving the equation. Meera has first found the distance that corresponds to 1 cm and then used that to find what 2.5 cm would correspond to. She used the unitary method.

Let us solve some more examples using the unitary method.

EXAMPLE 5 6 bowls cost ₹ 90. What would be the cost of 10 such bowls?

SOLUTION Cost of 6 bowls is ₹ 90.

Therefore, cost of 1 bowl = ₹ $\frac{90}{6}$

Hence, cost of 10 bowls = ₹ $\frac{90}{6} \times 10$ = ₹ 150

EXAMPLE 6 The car that I own can go 150 km with 25 litres of petrol. How far can it go with 30 litres of petrol?

SOLUTION With 25 litres of petrol, the car goes 150 km.

With 1 litre the car will go $\frac{150}{25}$ km.

Hence, with 30 litres of petrol it would go $\frac{150}{25} \times 30$ km = 180 km

In this method, we first found the value for one unit or the unit rate. This is done by the comparison of two different properties. For example, when you compare total cost to number of items, we get cost per item or if you take distance travelled to time taken, we get distance per unit time.

Thus, you can see that we often use per to mean for each.

For example, km per hour, children per teacher etc., denote unit rates.