Term-II

CHAPTER 13

PROBABILITY

Syllabus

- ***Conditional probability, multiplication theorem on probability, independent events, total probability, Bayes' theorem, Random variable and its probability distribution.***

STAND ALONE MCQs (1 Mark each)

Q. 1. If A and B are two events such that $P(A) \neq 0$ and $P(B|A) = 1$, then

(A) $A \subset B$ **(B)** $B \subset A$

(C) $B = \varphi$ **(D)** $A = \varphi$

Ans. Option (A) is correct.

Explanation:

$$P(A) \neq 0$$

and $$P(B|A) = 1$$

$$P(B|A) = \frac{P(B \cap A)}{P(A)}$$

$$1 = \frac{P(B \cap A)}{P(A)}$$

$$P(A) = P(B \cap A)$$

$\therefore$ $$A \subset B$$

Q. 2. If $P(A|B) > P(A)$, then which of the following is correct :

(A) $P(B|A) < P(B)$

(B) $P(A \cap B) < P(A).P(B)$

(C) $P(B|A) > P(B)$

(D) $P(B|A) = P(B)$

Ans. Option (C) is correct.

Explanation:

$$P(A|B) > P(A)$$

$\Rightarrow$ $$\frac{P(A \cap B)}{P(B)} > P(A)$$

$\Rightarrow$ $$P(A \cap B) > P(A).P(B)$$

$\Rightarrow$ $$\frac{P(A \cap B)}{P(A)} > P(B)$$

$\Rightarrow$ $$P(B|A) > P(B)$$

Q. 3. If A and B are any two events such that $P(A) + P(B) - P(A \text{ and } B) = P(A)$, then

(A) $P(B|A) = 1$ **(B)** $P(A|B) = 1$

(C) $P(B|A) = 0$ **(D)** $P(B|A) = 0$

Ans. Option (B) is correct.

Explanation :

$$P(A) + P(B) - P(A \text{ and } B) = P(A)$$

$$\Rightarrow P(A) + P(B) - P(A \cap B) = P(A)$$

$\Rightarrow$ $$P(B) - P(A \cap B) = 0$$

$\Rightarrow$ $$P(A \cap B) = P(B)$$

$\therefore$ $$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B)}{P(B)} = 1$$

Q. 4. In a box containing 100 bulbs, 10 are defective. The probability that out of a sample of 5 bulbs, none is defective is

(A) 10^{-1} **(B)** $\left(\frac{1}{2}\right)^5$

(C) $\left(\frac{9}{10}\right)^5$ **(D)** $\frac{9}{10}$

Ans. Option (C) is correct.

Explanation : The repeated selections of defective bulbs from a box are Bernoulli trials. Let X denotes the number of defective bulbs out of a sample of 5 bulbs.

Probability of getting a defective bulb,

$$p = \frac{10}{100} = \frac{1}{10}$$

$$\therefore \quad q = 1 - p = 1 - \frac{1}{10} = \frac{9}{10}$$

Clearly, X has a binomial distribution with $n = 5$ and $p = \frac{1}{10}$.

$$\therefore \quad P(X = x) = {}^nC_x q^{n-x} p^x = {}^5C_x \left(\frac{9}{10}\right)^{5-x} \left(\frac{1}{10}\right)^x$$

P (none of the bulbs is defective) $= P(X=0)$

$$= {}^5C_0 \cdot \left(\frac{9}{10}\right)^5 = 1.\left(\frac{9}{10}\right)^5 = \left(\frac{9}{10}\right)^5$$

Q. 5. The mean of the numbers obtained on throwing a die having written 1 on three faces, 2 on two faces and 5 on one face is

(A) 1 **(B)** 2

(C) 5 **(D)** $\frac{8}{3}$

Ans. Option (B) is correct.

Explanation :

Let X be the random variable representing a number on the die.

The total number of observations is 6. Therefore,

$$P(X = 1) = \frac{3}{6} = \frac{1}{2}$$

$$P(X = 2) = \frac{2}{6} = \frac{1}{3}$$

$$P(X = 5) = \frac{1}{6}$$

Therefore, the probability distribution is as follows.

X	1	2	5
$P(X)$	1/2	1/3	1/6

$$\text{Mean} = E(X) = \sum p_i x_i = \frac{1}{2}\times 1 + \frac{1}{3}\times 2 + \frac{1}{6}\times 5 = \frac{1}{2} + \frac{2}{3} + \frac{5}{6} = \frac{3+4+5}{6} = \frac{12}{6} = 2$$

Q. 6. The probability of obtaining an even prime number on each die, when a pair of dice is rolled is

(A) 0 **(B)** $\frac{1}{3}$

(C) $\frac{1}{12}$ **(D)** $\frac{1}{36}$

Ans. Option (D) is correct.

Explanation : When two dices are rolled, the number of outcomes is 36. The only even prime number is 2.

Let E be the event of getting an even prime number on each die.

$\therefore \quad E = \{(2, 2)\}$

$\Rightarrow P(E) = \frac{1}{36}$

Q. 7. If $P(A) = 0.4$, $P(B) = 0.8$ and $P(B|A) = 0.6$, then $P(A \cup B)$ is equal to

(A) 0.24 **(B)** 0.3

(C) 0.48 **(D)** 0.96

Ans. Option (D) is correct.

Explanation :

Here,

$P(A) = 0.4$, $P(B) = 0.8$ and $P(A\,|\,B) = 0.6$

$$\because \quad P(B|A) = \frac{P(B \cap A)}{P(A)}$$

$$\Rightarrow \quad P(B \cap A) = P(B\,|\,A).P(A) = 0.6 \times 0.4 = 0.24$$

$$\because \quad P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.4 + 0.8 - 0.24 = 1.2 - 0.24 = 0.96$$

Q. 8. A box has 100 pens of which 10 are defective. What is the probability that out of a sample of 5 pens drawn one by one with replacement at most one is defective?

(A) $\left(\frac{9}{10}\right)^5$ **(B)** $\frac{1}{2}\left(\frac{9}{10}\right)^4$

(C) $\frac{1}{2}\left(\frac{9}{10}\right)^5$ **(D)** $\left(\frac{9}{10}\right)^5 + \frac{1}{2}\left(\frac{9}{10}\right)^4$

Ans. Option (D) is correct.

Explanation : Here,

$n = 5,\ p = \frac{10}{100} = \frac{1}{10}$ and $q = \frac{9}{10}$

$$r \leq 1$$

$$\Rightarrow \quad r = 0, 1$$

Also,

$$P(X = r) = {}^nC_r p^r q^{n-r}$$

$$P(X = r) = P(r = 0) + P(r = 1)$$

$$= {}^5C_0\left(\frac{1}{10}\right)^0\left(\frac{9}{10}\right)^5 + {}^5C_1\left(\frac{1}{10}\right)^1\left(\frac{9}{10}\right)^4$$

$$= \left(\frac{9}{10}\right)^5 + 5.\frac{1}{10}.\left(\frac{9}{10}\right)^4$$

$$= \left(\frac{9}{10}\right)^5 + \frac{1}{2}\left(\frac{9}{10}\right)^4$$

Q. 9. A and B are two students. Their chances of solving a problem correctly are $\frac{1}{3}$ and $\frac{1}{4}$, respectively. If the probability of their making a common error is, $\frac{1}{20}$ and they obtain the same answer, then the probability of their answer to be correct is

(A) $\frac{1}{12}$ **(B)** $\frac{1}{40}$

(C) $\frac{13}{120}$ **(D)** $\frac{10}{13}$

Ans. Option (D) is correct.

Explanation : Let E_1 = Event that both A and B solve the problem

$$\therefore \quad P(E_1) = \frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$$

Let E_2 = Event that both A and B got incorrect solution of the problem

$$\therefore \quad P(E_2) = \frac{2}{3} \times \frac{3}{4} = \frac{1}{2}$$

Let E = Event that they got same answer

Here,

$$P(E/E_1) = 1,$$

$$P(E/E_2) = \frac{1}{20}$$

$$P(E_1/E) = \frac{P(E_1 \cap E)}{P(E)}$$

$$= \frac{P(E_1).P(E/E_1)}{P(E_1).P(E/E_1) + P(E_2)P(E/E_1)}$$

$$= \frac{\frac{1}{12} \times 1}{\frac{1}{12} \times 1 + \frac{1}{2} \times \frac{1}{20}}$$

$$= \frac{\frac{1}{12}}{\frac{10+3}{120}}$$

$$= \frac{120}{12 \times 13}$$

$$= \frac{10}{13}$$

Q. 10. In a college, 30% students fail in physics, 25% fail in mathematics and 10% fail in both. One student is chosen at random. The probability that she fails in physics if she has failed in mathematics is

(A) $\frac{1}{10}$ **(B)** $\frac{2}{5}$

(C) $\frac{9}{20}$ **(D)** $\frac{1}{3}$

Ans. Option (B) is correct.

Explanation : Here,

$$P_{(Ph)} = \frac{30}{100} = \frac{3}{10},$$

$$P_{(M)} = \frac{25}{100} = \frac{1}{4}$$

and $P_{(M \cap Ph)} = \frac{10}{100} = \frac{1}{10}$

$$\therefore \quad P\left(\frac{Ph}{M}\right) = \frac{P(Ph \cap M)}{P(M)} = \frac{\frac{1}{10}}{\frac{1}{4}} = \frac{2}{5}$$

Q. 11. Two cards are drawn from a well shuffled deck of 52 playing cards with replacement. The probability, that both cards are queens, is

(A) $\frac{1}{13} \times \frac{1}{13}$ **(B)** $\frac{1}{13} + \frac{1}{13}$

(C) $\frac{1}{13} \times \frac{1}{17}$ **(D)** $\frac{1}{13} \times \frac{4}{51}$

Ans. Option (A) is correct.

Explanation :

Required probability $= \frac{4}{52} \times \frac{4}{52} = \frac{1}{13} \times \frac{1}{13}$

Q. 12. Two dice are thrown. If it is known that the sum of numbers on the dice was less than 6, the probability of getting a sum 3, is

(A) $\frac{1}{18}$ (B) $\frac{5}{18}$

(C) $\frac{1}{5}$ (D) $\frac{2}{5}$

Ans. Option (C) is correct.

Explanation : Let,

E_1 = Event that the sum of numbers on the dice was less than 6 and

E_2 = Event that the sum of numbers on the dice is 3.

$\therefore\ E_1 = \{(1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(2,3),(3,1),(3,2),(4,1)\}$

$\Rightarrow\quad n(E_1) = 10$

and $E_2 = \{(1,2),(2,1)\}$

$\Rightarrow\quad n(E_2) = 2$

$\therefore$ Required probability $= \frac{2}{10}$

$= \frac{1}{5}$

Q. 13. Eight coins are tossed together. The probability of getting exactly 3 heads is

(A) $\frac{1}{256}$ (B) $\frac{7}{32}$

(C) $\frac{5}{32}$ (D) $\frac{3}{32}$

Ans. Option (B) is correct.

Explanation :

We know that, probability distribution

$$P(X = r) = {}^nC_r(p)^r\, q^{n-r}$$

Here, $n = 8,\ r = 3,\ p = \frac{1}{2}$ and $q = \frac{1}{2}$

$$\therefore \text{ Required probability } = {}^8C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^{8-3}$$

$$= \frac{8!}{5!3!}\left(\frac{1}{2}\right)^8$$

$$= \frac{8\cdot7\cdot6}{3\cdot2}\cdot\frac{1}{2^8}$$

$$= \frac{7}{32}$$

Q. 14. A box contains 3 orange balls, 3 green balls and 2 blue balls. Three balls are drawn at random from the box without replacement. The probability of drawing 2 green balls and one blue ball is

(A) $\frac{3}{28}$ (B) $\frac{2}{21}$

(C) $\frac{1}{28}$ (D) $\frac{167}{168}$

Ans. Option (A) is correct.

Explanation :

Probability of drawing 2 green balls and one blue ball

$$= P(G)\cdot P(G)\cdot P(B) + P(B)\cdot P(G)\cdot P(G) + P(G)\cdot P(B)\cdot P(G)$$

$$= \frac{3}{8}\cdot\frac{2}{7}\cdot\frac{2}{6} + \frac{2}{8}\cdot\frac{3}{7}\cdot\frac{2}{6} + \frac{3}{8}\cdot\frac{2}{7}\cdot\frac{2}{6}$$

$$= \frac{3}{28}$$

Q. 15. A die is thrown and a card is selected at random from a deck of 52 playing cards. The probability of getting an even number on the die and a spade card is

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$

(C) $\frac{1}{8}$ (D) $\frac{3}{4}$

Ans. Option (C) is correct.

Explanation : Let,

E_1 = Event for getting an even number on die and

E_2 = Event that a spade card is selected

$\therefore\quad P(E_1) = \frac{3}{6} = \frac{1}{2}$

and $\quad P(E_2) = \frac{13}{52} = \frac{1}{4}$

Then, $\quad P(E_1 \cap E_2) = P(E_1)\cdot P(E_2) = \frac{1}{2}\cdot\frac{1}{4} = \frac{1}{8}$

Q. 16. Assume that in a family, each child is equally likely to be a boy or a girl. A family with three children is chosen at random. The probability that the eldest child is a girl given that the family has at least one girl is

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$

(C) $\frac{2}{3}$ (D) $\frac{4}{7}$

Ans. Option (D) is correct.

Explanation : We have,

S = {B, B, B), (G, G, G), (B, G, G), (G, B, G), (G, G, B), (G, B, B), (B, G, B), (B, B, G)}

E_1 = Event that a family has at least one girl, then

E_1 = {(G, B, B), (B, G, B), (B, B, G), (G, G, B), (B, G, G), (G, B, G), (G, G, G)}

E_2 = Event that the eldest child is a girl, then

E_2 = {(G, B, B), (G, G, B), (G, B, G), (G, G, G)}

$\therefore \quad E_1 \cap E_2 = \{(G,B,B),(G,G,B),(G,B,G),(G,G,G)\}$

$$\therefore \ P(E_2 \mid E_1) = \frac{P(E_1 \cap E_2)}{P(E_1)} = \frac{\frac{4}{8}}{\frac{7}{8}} = \frac{4}{7}$$

Q. 17. Two events *E* and *F* are independent. If *P(E)* = 0.3, $P(E \cup F) = 0.5$, then $P(E|F) - P(F|E)$ equals

(A) $\frac{2}{7}$ (B) $\frac{3}{35}$

(C) $\frac{1}{70}$ (D) $\frac{1}{7}$

Ans. Option (C) is correct.

Explanation : We have,

$$P(E) = 0.3$$

and $$P(E \cup F) = 0.5$$

Also, E and F are independent.
Now,

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

$$\Rightarrow \quad 0.5 = 0.3 + P(F) - 0.3P(F)$$

$$\Rightarrow \quad P(F) = \frac{0.5 - 0.3}{0.7} = \frac{2}{7}$$

$$\therefore \ P(E/F) - P(F/E) = P(E) - P(F)$$

(as E and F are independent)

$$= \frac{3}{10} - \frac{2}{7} = \frac{1}{70}$$

Q. 18. If *A* and *B* are two independent events with $P(A) = \frac{3}{5}$ and $P(B) = \frac{4}{5}$, then $P(A \cap B)$ equals

(A) $\frac{4}{15}$ (B) $\frac{8}{45}$

(C) $\frac{1}{3}$ (D) $\frac{2}{9}$

Ans. Option (D) is correct.

Explanation : Since *A* and *B* are independent events, *A′* and *B′* are also independent. Therefore,

$$P(A' \cap B') = P(A') \cdot (B')$$

$$= (1 - P(A)(1 - P(B))$$

$$= \left(1 - \frac{3}{5}\right)\left(1 - \frac{4}{9}\right)$$

$$= \frac{2}{5} \cdot \frac{5}{9}$$

$$= \frac{2}{9}$$

Q. 19. If *A* and *B* are such events that *P(A)* > 0 and *P(B)* ≠ 1, then *P* (*A*|*B*) equals

(A) $1 - P(A|B)$ (B) $1 - P(A|B)$

(C) $\frac{1 - P(A \cup B)}{P(B')}$ (D) $P(A)|P(B)$

Ans. Option (C) is correct.

Explanation : We have,

$$P(A) > 0 \text{ and } P(B) \neq 1$$

$$P(A'/B') = \frac{P(A' \cap B')}{P(B')} = \frac{1 - P(A \cup B)}{P(B')}$$

Q. 20. Let $P(A) = \frac{7}{13}$, $P(B) = \frac{9}{13}$ and $P(A \cap B) = \frac{4}{13}$. Then $P(A|B)$ is equal to

(A) $\frac{6}{13}$ (B) $\frac{4}{13}$

(C) $\frac{4}{9}$ (D) $\frac{5}{9}$

Ans. Option (D) is correct.

Explanation : Here,

$$P(A) = \frac{7}{13},$$

$$P(B) = \frac{9}{13}$$

and $$P(A \cap B) = \frac{4}{13}$$

$$P(A'|B) = \frac{P(A' \cap B)}{P(B)} = \frac{P(B) - P(A \cap B)}{P(B)}$$

$$= \frac{\frac{9}{13} - \frac{4}{13}}{\frac{9}{13}} = \frac{5}{9}$$

Q. 21. *A* and *B* are events such that *P(A)* = 0.4, *P(B)* = 0.3 and $P(A \cup B) = 0.5$. Then $P(B \cap A)$ equals

(A) $\frac{2}{3}$ (B) $\frac{1}{2}$

(C) $\frac{3}{10}$ (D) $\frac{1}{5}$

Ans. Option (D) is correct.

Explanation : We have,

$$P(A) = 0.4,$$

$$P(B) = 0.3$$

and $$P(A \cup B) = 0.5$$

Now, $$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow \quad P(A \cap B) = 0.4 + 0.3 - 0.5 = 0.2$$

$$
\begin{aligned}
P(A) &= 0.4, \\
P(B) &= 0.3 \\
\text{and} \quad P(A \cup B) &= 0.5 \\
\text{Now,} \quad P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\
\Rightarrow \quad P(A \cap B) &= 0.4 + 0.3 - 0.5 \\
&= 0.2 \\
\therefore \quad P(B' \cap A) &= P(A) - P(A \cap B) \\
&= 0.4 - 0.2 \\
&= 0.2 \\
&= \frac{1}{5}
\end{aligned}
$$

Q. 22. If $P(A) = \frac{2}{5}$ $P(B) = \frac{3}{10}$ and $P(A \cap B) = \frac{1}{5}$, then $P(A'|B').P(B'|A')$ is equal to

(A) $\frac{5}{6}$ **(B)** $\frac{5}{7}$

(C) $\frac{25}{42}$ **(D)** 1

Ans. Option (C) is correct.

Explanation : We have,

$$
\begin{aligned}
P(A) &= \frac{2}{5}, \\
P(B) &= \frac{3}{10} \\
\text{and} \quad P(A \cap B) &= \frac{1}{5} \\
P(A'|B') \cdot P(B'|A') &= \frac{P(A' \cap B')}{P(B')} \cdot \frac{P(A' \cap B')}{P(A')} \\
&= \frac{(P((A \cup B)'))^2}{P(A')P(B')} \\
&= \frac{(1 - P(A \cup B))^2}{(1 - P(A))(1 - P(B))} \\
&= \frac{(1 - P(A) + P(B) - P(A \cap B))^2}{(1 - P(A))(1 - P(B))} \\
&= \frac{\left[1 - \left(\frac{2}{5} + \frac{3}{10} - \frac{1}{5}\right)\right]^2}{\left(1 - \frac{1}{2}\right)\left(1 - \frac{3}{10}\right)} \\
&= \frac{\left(1 - \frac{1}{2}\right)^2}{\frac{3}{5} \cdot \frac{7}{10}} \\
&= \frac{25}{42}
\end{aligned}
$$

Q. 23. If $P(A) = \frac{4}{5}$ and $P(A \cap B) = \frac{7}{10}$, then $(P(B|A)$ is equal to

(A) $\frac{1}{10}$ **(B)** $\frac{1}{8}$

(C) $\frac{7}{8}$ **(D)** $\frac{17}{20}$

Ans. Option (C) is correct.

Explanation :

$$
\begin{aligned}
\because \quad P(A) &= \frac{4}{5}, \\
P(A \cap B) &= \frac{7}{10} \\
\therefore \quad P(B|A) &= \frac{P(A \cap B)}{P(A)} \\
&= \frac{\frac{7}{10}}{\frac{4}{5}} \\
&= \frac{7}{8}
\end{aligned}
$$

ASSERTION AND REASON BASED MCQs (1 Mark each)

Directions : In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false but R is True

Q. 1. Assertion (A): Let A and B be two events such that $P(A) = \frac{1}{5}$, while $P(A \text{ or } B) = \frac{1}{2}$. Let $P(B) = P$, then for $P = \frac{3}{8}$, A and B independent.

Reason (R) : For independent events,

$$
\begin{aligned}
P(A \cap B) &= P(A)\, P(B) \\
P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\
&= P(A) + P(B) - P(A)\, P(B) \\
&= \frac{1}{5} + P - \left(\frac{1}{5}\right)P \\
\Rightarrow \quad \frac{1}{2} &= \frac{1}{5} + \frac{4}{5}P \\
\Rightarrow \quad P &= \frac{3}{8}.
\end{aligned}
$$

Ans. Option (A) is correct.

Explanation: Assertion (A) and Reason (R) both are correct and Reason (R) is the correct explanation of Assertion (A).

Q. 2. Assertion (A) : If A and B are two mutually exclusive events with $P(\bar{A}) = \frac{5}{6}$ and $P(B) = \frac{1}{3}$. Then $P(A/\bar{B})$ is equal to $\frac{1}{4}$.

Reason (R) : If A and B are two events such that $P(A) = 0.2$, $P(B) = 0.6$ and $P(A|B) = 0.2$ then the value of $P(A|\bar{B})$ is 0.2.

Ans. Option (B) is correct.

> ***Explanation:*** Assertion (A) is correct.
>
> $$P(A|\bar{B}) = \frac{P(A \cap \bar{B})}{P(\bar{B})}$$
>
> $$P(A|\bar{B}) = \frac{P(A)}{P(\bar{B})}$$
>
> [since, given A and B are two mutually exclusive events]
>
> $$P\left(\frac{A}{\bar{B}}\right) = \frac{\left(1-\frac{5}{6}\right)}{\left(1-\frac{1}{3}\right)}$$
>
> $$= \frac{\frac{1}{6}}{\frac{2}{3}}$$
>
> $$= \frac{1}{4}$$
>
> Reason (R) is also correct.
>
> For independent events,
>
> $$P(A|\bar{B}) = P(A)$$
>
> $$= 0.2.$$

Q. 3. Assertion (A) : Let A and B be two events such that the occurrence of A implies occurrence of B, but not vice-versa, then the correct relation between $P(A)$ and $P(B)$ is $P(B) \geq P(A)$.

Reason (R) : Here, according to the given statement

$$A \subseteq B$$

$$P(B) = P(A \cup (A \cap B))$$

$$(\because A \cap B = A)$$

$$= P(A) + P(A \cap B)$$

Therefore, $P(B) \geq P(A)$

Ans. Option (A) is correct.

> ***Explanation:*** Assertion (A) and Reason (R) both are correct and Reason (R) is the correct explanation of Assertion (A).

Q. 4. Assertion (A) : If $A \subset B$ and $B \subset A$ then, $P(A) = P(B)$.

Reason (R) : If $A \subset B$ then $P(\bar{A}) \leq P(\bar{B})$.

Ans. Option (C) is correct.

> ***Explanation :*** Assertion (A) is correct.
>
> $A \subset B$ and $B \subset A \Rightarrow A = B$
>
> Hence, $P(A) = P(B)$.
>
> But (R) is wrong.
>
> $$A \subset B \Rightarrow \bar{B} \subset \bar{A}$$
>
> Therefore, $P(\bar{A}) \geq P(\bar{B})$

Q. 5. Assertion (A) : The probability of an impossible event is 1.

Reason (R) : If A is a perfect subset of B and $P(A) < P(B)$, then $P(B - A)$ is equal to $P(B) - P(A)$.

Ans. Option (D) is correct.

> ***Explanation :*** Assertion (A) is wrong.
>
> If the probability of an event is 0, then it is called as an impossible event.
>
> But Reason (R) is correct.
>
> From Basic Theorem of Probability,
>
> $P(B - A) = P(B) - P(A)$, this is true only if the condition given in the question is true.

Q. 6. Assertion (A) : If $A = A_1 \cup A_2 ... \cup A_n$, where $A_1 ... A_n$ are mutually exclusive events then

$$\sum_{i=1}^{n} P(A_i) = P(A)$$

Reason (R) :

Given, $A = A_1 \cup A_2 ... \cup A_n$

Since $A_1 ... A_n$ are mutually exclusive

$$P(A) = P(A_1) + P(A_2) + ... + P(A_n)$$

Therefore $P(A) = \sum_{i=1}^{n} P(A_i)$

Ans. Option (B) is correct.

> ***Explanation:*** Assertion (A) and Reason (R) both are correct and Reason (R) is the correct explanation of Assertion (A).

CASE-BASED MCQs

Attempt any four sub-parts from each question. Each sub-part carries 1 mark.

I. Read the following text and answer the following questions on the basis of the same:

A coach is training 3 players. He observes that the player A can hit a target 4 times in 5 shots, player B can hit 3 times in 4 shots and the player C can hit 2 times in 3 shots. **[CBSE QB 2021]**

Q. 1. Let the target is hit by A, B: the target is hit by B and, C: the target is hit by A and C. Then, the probability that A, B and, C all will hit, is

(A) $\frac{4}{5}$ **(B)** $\frac{3}{5}$

(C) $\frac{2}{5}$ **(D)** $\frac{1}{5}$

Ans. Option (C) is correct.

Explanation:

$P(A) = \frac{4}{5},\ P(B) = \frac{3}{4},\ P(C) = \frac{2}{3}$

Probability that A, B and C all will hit the target

$$= P(A \cap B \cap C)$$
$$= P(A)P(B)P(C)$$
$$= \frac{4}{5} \times \frac{3}{4} \times \frac{2}{3}$$
$$= \frac{2}{5}$$

Q. 2. What is the probability that B, C will hit and A will lose?

(A) $\frac{1}{10}$ **(B)** $\frac{3}{10}$

(C) $\frac{7}{10}$ **(D)** $\frac{4}{10}$

Ans. Option (A) is correct.

Explanation:

$$P(\bar{A}) = 1 - \frac{4}{5}$$
$$= \frac{1}{5}$$

Probability that B, C will hit and A will lose

$$= P(\bar{A} \cap B \cap C)$$
$$= P(\bar{A}).P(B).P(C)$$
$$= \frac{1}{5} \times \frac{3}{4} \times \frac{2}{3}$$
$$= \frac{1}{10}$$

Q. 3. What is the probability that 'any two of A, B and C will hit?

(A) $\frac{1}{30}$ **(B)** $\frac{11}{30}$

(C) $\frac{17}{30}$ **(D)** $\frac{13}{30}$

Ans. Option (D) is correct.

Explanation:

$$P(\bar{B}) = 1 - \frac{3}{4}$$
$$= \frac{1}{4},$$
$$P(\bar{C}) = 1 - \frac{2}{3}$$
$$= \frac{1}{3}$$

Probability that any two of A, B and C will hit

$$= P(\bar{A})P(B)P(C) + P(A)P(\bar{B})P(C) + P(A)P(B)P(\bar{C})$$
$$= \frac{1}{5} \times \frac{3}{4} \times \frac{2}{3} + \frac{4}{5} \times \frac{1}{4} \times \frac{2}{3} + \frac{4}{5} \times \frac{3}{4} \times \frac{1}{3}$$
$$= \frac{1}{10} + \frac{2}{15} + \frac{1}{5}$$
$$= \frac{3+4+6}{30}$$
$$= \frac{13}{30}$$

Q. 4. What is the probability that 'none of them will hit the target'?

(A) $\frac{1}{30}$ **(B)** $\frac{1}{60}$

(C) $\frac{1}{15}$ **(D)** $\frac{2}{15}$

Ans. Option (B) is correct.

Explanation: Probability that none of them will hit the target

$$= P(\bar{A} \cap \bar{B} \cap \bar{C})$$
$$= P(\bar{A}).P(\bar{B}).P(\bar{C})$$
$$= \frac{1}{5} \times \frac{1}{4} \times \frac{1}{3}$$
$$= \frac{1}{60}$$

Q. 5. What is the probability that at least one of A, B or C will hit the target?

(A) $\frac{59}{60}$ **(B)** $\frac{2}{5}$

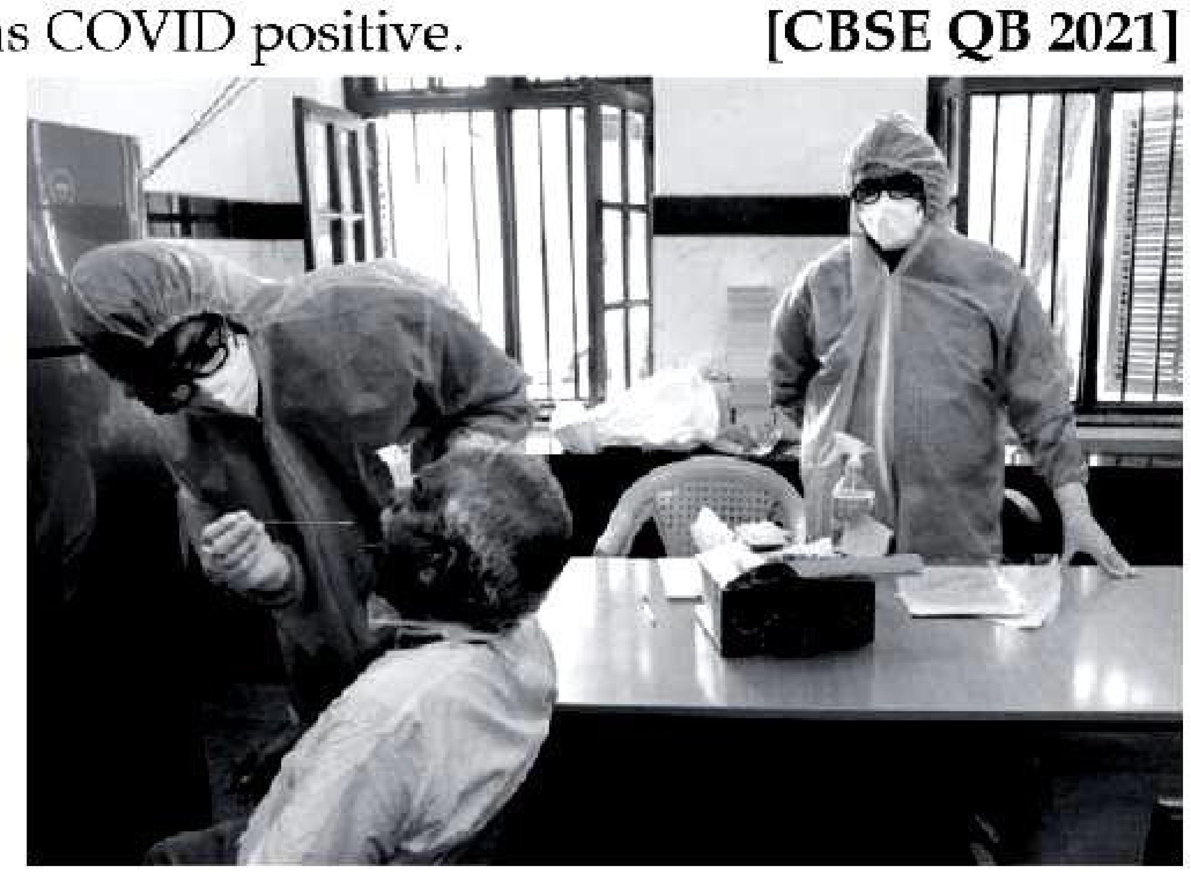
(C) $\frac{3}{5}$ **(D)** $\frac{1}{60}$

Ans. Option (A) is correct.

II. Read the following text and answer the following questions on the basis of the same:

The reliability of a COVID PCR test is specified as follows:

Of people having COVID, 90% of the test detects the disease but 10% goes undetecte. Of people free of COVID, 99% of the test is judged COVID negative but 1% are diagnosed as showing COVID positive. From a large population of which only 0.1% have COVID, one person is selected at random, given the COVID PCR test, and the pathologist reports him/her as COVID positive. **[CBSE QB 2021]**



Q. 1. What is the probability of the 'person to be tested as COVID positive' given that 'he is actually having COVID'?

(A) 0.001 **(B)** 0.1

(C) 0.8 **(D)** 0.9

Ans. Option (D) is correct.

> ***Explanation:***
> E = Person selected has Covid
> F = Does not have Covid
> G = Test judge Covid positive
> Probability of the person to be tested as Covid positive given that he is actually having Covid
> $= P(G/E) = 90\% = \frac{90}{100} = 0.9$

Q. 2. What is the probability of the 'person to be tested as COVID positive' given that 'he is actually not having COVID'?

(A) 0.01 **(B)** 0.99

(C) 0.1 **(D)** 0.001

Ans. Option (A) is correct.

> ***Explanation:*** Probability of person to be tested as Covid positive given that he is actually not having Covid
> $= P(G/E) = 1\% = \frac{1}{100} = 0.01$

Q. 3. What is the probability that the 'person is actually not having COVID'?

(A) 0.998 **(B)** 0.999

(C) 0.001 **(D)** 0.111

Ans. Option (B) is correct.

> ***Explanation:***
> $P(E) = 1 - P(E)$
> $= 1 - 0.001 \quad \left[\because P(E) = 0.1\% = \frac{0.1}{100} = 0.001\right]$
> $= 0.999$

Q. 4. What is the probability that the 'person is actually having COVID given that 'he is tested as COVID positive'?

(A) 0.83 **(B)** 0.0803

(C) 0.083 **(D)** 0.089

Ans. Option (C) is correct.

> ***Explanation:***
> $$P(E/G) = \frac{0.001 \times 0.9}{0.001 \times 0.9 + 0.999 \times 0.01}$$
> $$= \frac{9 \times 10^{-4}}{9 \times 10^{-4} + 99.9 \times 10^{-4}}$$
> $$= \frac{9 \times 10^{-4}}{10^{-4}(9 + 99.9)}$$
> $$= \frac{9}{108.9}$$
> $= 0.083$ (approx)

Q. 5. What is the probability that the 'person selected will be diagnosed as COVID positive'?

(A) 0.1089 **(B)** 0.01089

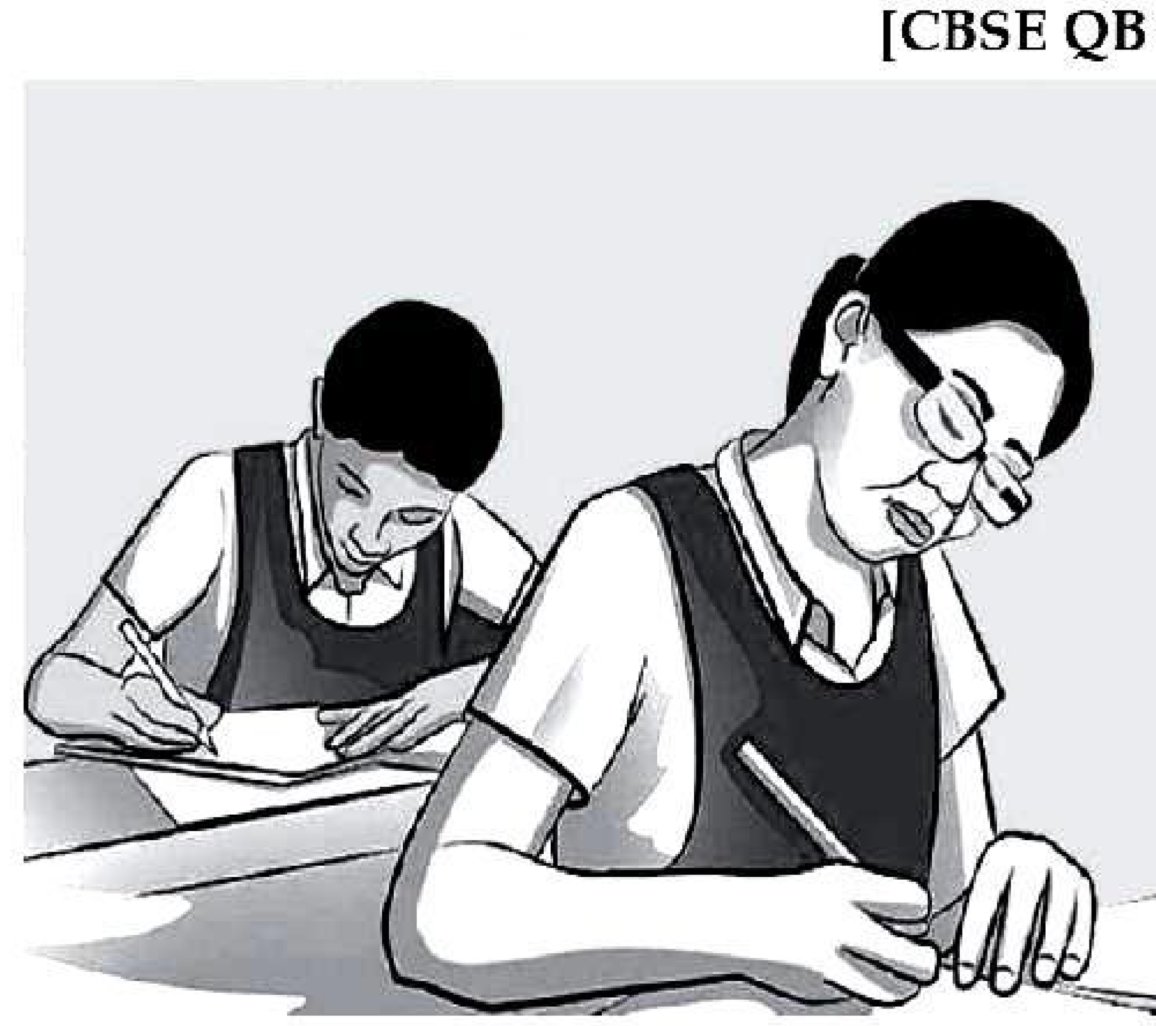
(C) 0.0189 **(D)** 0.189

Ans. Option (B) is correct.

III. Read the following text and answer the following questions on the basis of the same:

In answering a question on a multiple choice test for class XII, a student either knows the answer or guesses. Let $\frac{3}{5}$ be the probability that he knows the answer and $\frac{2}{5}$ be the probability that he guesses. Assume that a student who guesses at the answer will be correct with probability $\frac{1}{3}$. Let E_1, E_2, E be the events that the student knows the answer, guesses the answer and answers correctly respectively.

[CBSE QB 2021]



Q. 1. What is the value of $P(E_1)$?

(A) $\frac{2}{5}$ **(B)** $\frac{1}{3}$

(C) 1 **(D)** $\frac{3}{5}$

Ans. Option (D) is correct.

Q. 2. Value of $P(E|E_1)$ is

(A) $\frac{1}{3}$ **(B)** 1

(C) $\frac{2}{3}$ **(D)** $\frac{4}{5}$

Ans. Option (B) is correct.

> ***Explanation:***
> $P(E_1/E) = 1$

Q. 3. $\sum_{k=1}^{k=2} P(E|E_k)\, P(E_k)$ Equals

(A) $\frac{11}{15}$ **(B)** $\frac{4}{15}$

(C) $\frac{1}{5}$ **(D)** 1

Ans. Option (A) is correct.

Explanation:

$$\sum_{k=1}^{k=2} P(E|E_k)P(E_K) = P(E|E_1)P(E) + P(E|E_2) + P(E_2)$$

$$= 1 \times \frac{1}{3} + \frac{1}{3} \times \frac{2}{5}$$

$$= \frac{11}{15}$$

Q. 4. Value of $\sum_{k=1}^{k=2} P(E_k)$

(A) $\frac{1}{3}$ **(B)** $\frac{1}{5}$

(C) 1 **(D)** $\frac{3}{5}$

Ans. Option (C) is correct.

Explanation:

$$\sum_{k=1}^{k=2} P(E_k) = P(E_1) + P(E_2)$$

$$= \frac{3}{5} + \frac{2}{5}$$

$$= \frac{5}{5}$$

$$= 1$$

Q. 5. What is the probability that the student knows the answer given that he answered it correctly?

(A) $\frac{2}{11}$ **(B)** $\frac{5}{3}$

(C) $\frac{9}{11}$ **(D)** $\frac{13}{3}$

Ans. Option (C) is correct.

Explanation:

$$P(E|E_1) = \frac{P(E_1).P(E_1|E)}{P(E_1).P(E_1|E) + P(E_2)P(E|E_2)}$$

$$= \frac{\frac{3}{5} \times 1}{\frac{3}{5} \times 1 + \frac{2}{5} \times \frac{1}{3}}$$

$$= \frac{\frac{3}{5}}{\frac{11}{15}}$$

$$= \frac{9}{11}$$

IV. Read the following text and answer the following questions on the basis of the same:

In an office three employees Vinay, Sonia and Iqbal process incoming copies of a certain form. Vinay process 50% of the forms. Sonia processes 20% and Iqbal the remaining 30% of the forms. Vinay has an error rate of 0.06, Sonia has an error rate of 0.04 and Iqbal has an error rate of 0.03.

[CBSE QB 2021]

Q. 1. The conditional probability that an error is committed in processing given that Sonia processed the form is :

(A) 0.0210 **(B)** 0.04

(C) 0.47 **(D)** 0.06

Ans. Option (B) is correct.

Q. 2. The probability that Sonia processed the form and committed an error is :

(A) 0.005 **(B)** 0.006

(C) 0.008 **(D)** 0.68

Ans. Option (C) is correct.

Explanation:

P (sonia processed the form and committed an error) = 20% × 0.4

$$= \frac{20}{100} \times 0.04$$

$$= \frac{1}{5} \times 0.04$$

$$= 0.008$$

Q. 3. The total probability of committing an error in processing the form is :

(A) 0 **(B)** 0.047

(C) 0.234 **(D)** 1

Ans. Option (B) is correct.

Q. 4. The manager of the company wants to do a quality check. During inspection he selects a form at random from the days output of processed forms. If the form selected at random has an error, the probability that the form is NOT processed by Vinay is :

(A) 1 **(B)** $\frac{30}{47}$

(C) $\frac{20}{47}$ **(D)** $\frac{17}{47}$

Ans. Option (D) is correct.

Q. 5. Let A be the event of committing an error in processing the form and let E_1, E_2 and E_3 be the events that Vinay, Sonia and Iqbal processed the form. The value of $\sum_{i=1}^{3} P(E_i|A) = 1$ is :

(A) 0 **(B)** 0.03

(C) 0.06 **(D)** 1

Ans. Option (D) is correct.

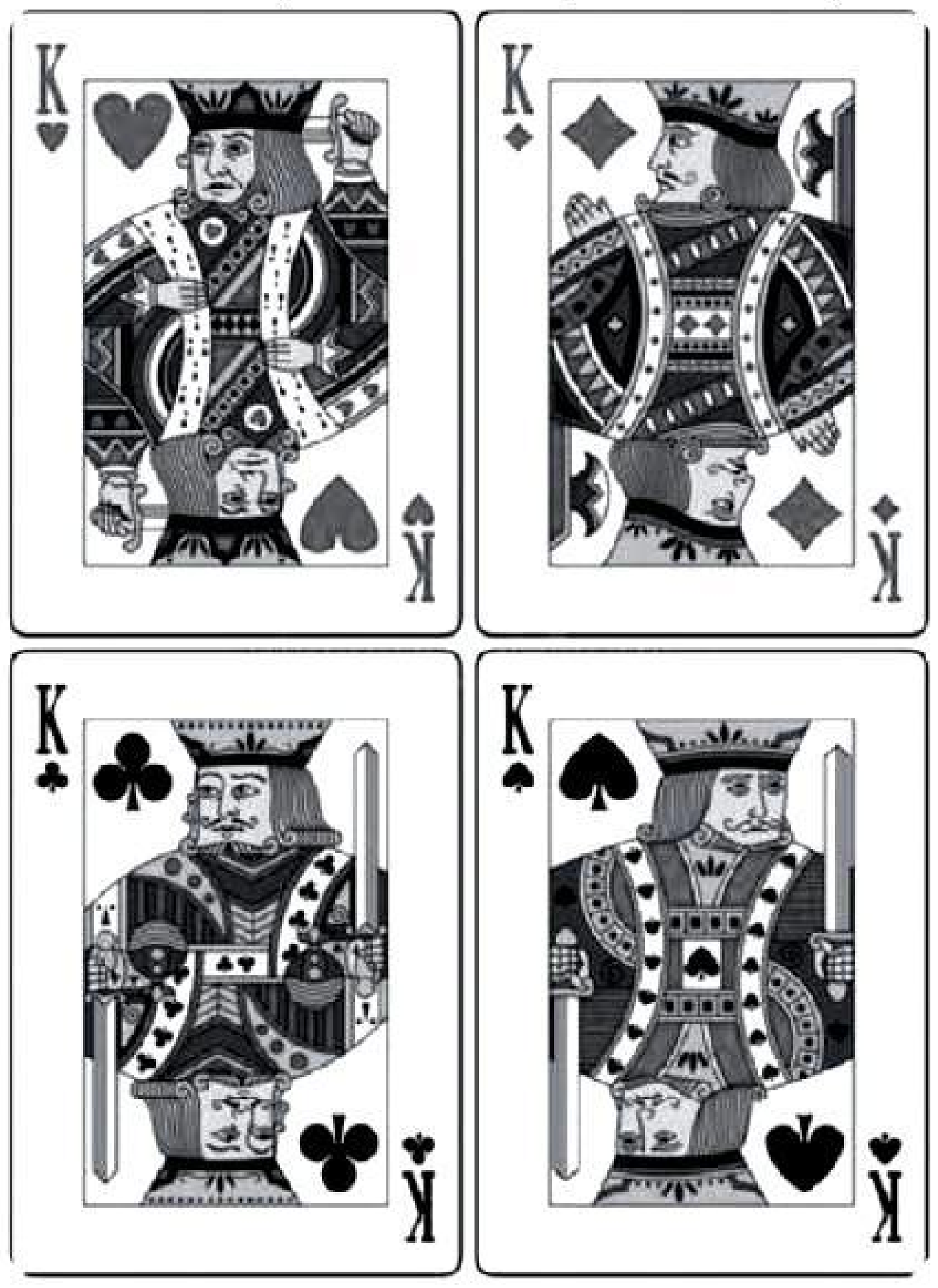
Explanation:

$$\sum_{i=1}^{3} P\left(\frac{E_i}{A}\right) = P\left(\frac{E_1}{A}\right) + P\left(\frac{E_2}{A}\right) + P\left(\frac{E_3}{A}\right) = 1$$

[∵ sum of all occurrence of an event is equal to 1]

V. Read the following text and answer the following questions on the basis of the same:

A group of people start playing cards. And as we know a well shuffled pack of cards contains a total of 52 cards. Then 2 cards are drawn simultaneously (or successively without replacement).



Q. 1. If x = no. of kings = 0, 1, 2. Then $P(x = 0)$ = ?

(A) $\frac{188}{221}$ **(B)** $\frac{198}{223}$

(C) $\frac{197}{290}$ **(D)** $\frac{187}{221}$

Ans. Option (A) is correct.

Explanation:

$$P(x = 0) = \frac{48}{52} \times \frac{47}{51} = \frac{188}{221}$$

Q. 2. If x = no. of kings = 0, 1, 2. Then $P(x = 1)$ = ?

(A) $\frac{32}{229}$ **(B)** $\frac{32}{227}$

(C) $\frac{32}{221}$ **(D)** $\frac{32}{219}$

Ans. Option (C) is correct.

Explanation:

$$P(x = 1) = \frac{48}{52} \times \frac{47}{51} = \frac{188}{221}$$

Q. 3. If x = no. of kings = 0, 1, 2. Then $P(x = 2)$ = ?

(a) $\frac{2}{219}$ **(B)** $\frac{1}{221}$

(C) $\frac{3}{209}$ **(D)** $\frac{1}{209}$

Ans. Option (B) is correct.

Explanation:

$$P(x = 2) = \frac{4}{52} \times \frac{3}{51} = \frac{1}{221}$$

Q. 4. Find the mean of the number of kings ?

(A) $\frac{2}{13}$ **(B)** $\frac{1}{13}$

(C) $\frac{1}{17}$ **(D)** $\frac{2}{17}$

Ans. Option (A) is correct.

Explanation:

$$\text{Mean} = \Sigma x_i p_i$$
$$= \left(0 \times \frac{188}{221}\right) + \left(1 \times \frac{32}{221}\right) + \left(2 \times \frac{1}{221}\right)$$
$$= \frac{34}{221}$$
$$= \frac{2}{13}$$

Q. 5. Find the variance of the number of kings ?

(A) $\frac{400}{2873}$ **(B)** $\frac{400}{2877}$

(C) $\frac{400}{2879}$ **(D)** $\frac{400}{2871}$

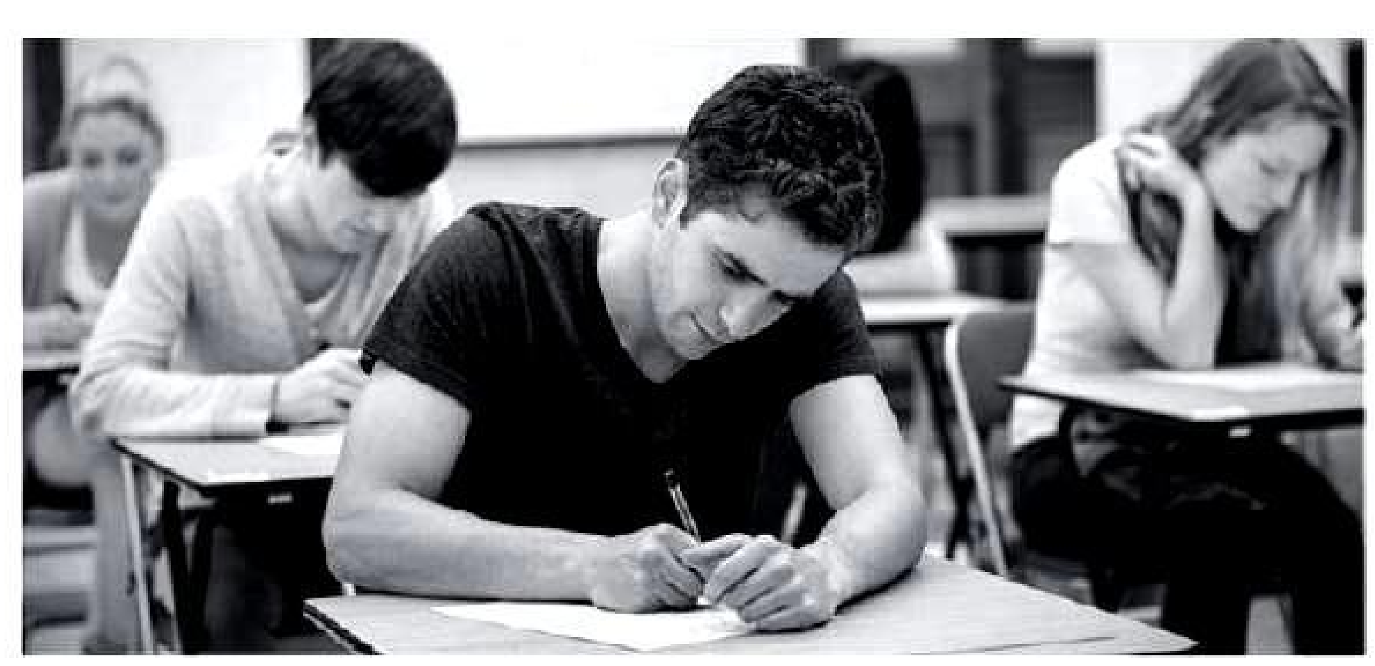
Ans. Option (A) is correct.

Explanation:

$$\text{Variance} = \Sigma x_i^2 p_i - (\Sigma x_i p_i)^2$$
$$\Sigma x_i^2 p_i = \left(0 \times \frac{188}{221}\right) + \left(1 \times \frac{32}{221}\right) + \left(4 \times \frac{1}{221}\right)$$
$$= \frac{36}{221}$$
$$\text{Variance} = \frac{36}{221} - \left(\frac{2}{13}\right)^2$$
$$= \frac{400}{2873}$$

VI. Read the following text and answer the following questions on the basis of the same:

Anand, Samanyu and Shah of SHORTCUTS classes were given a problem in Mathematics whose respective probabilities of solving it are $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$. They were asked to solve it independently.



Based on the above data, answer any four of the following questions.

Q. 1. The probability that Anand alone solves it is _______.

(A) $\frac{1}{4}$ **(B)** $\frac{3}{4}$

(C) $\frac{11}{24}$ **(D)** $\frac{17}{24}$

Ans. Option (A) is correct.

Explanation:

Let $A \rightarrow$ event that Anand solves

$B \rightarrow$ event that Samanyu solves

$C \rightarrow$ event that Shah solves

$$P(A) = \frac{1}{2},\ P(B) = \frac{1}{3},\ P(C) = \frac{1}{4}$$

$$\therefore P(A') = \frac{1}{2},\ P(B') = \frac{2}{3},\ P(C') = \frac{3}{4}$$

$$P(A \cap B' \cap C') = P(A)\,P(B')\,P(C') = \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} = \frac{1}{4}$$

Q. 2. The probability that the problem is not solved is _______.

(A) $\frac{1}{4}$ **(B)** $\frac{3}{4}$

(C) 0 **(D)** $\frac{11}{24}$

Ans. Option (A) is correct.

Explanation:

$$P(A' \cap B' \cap C') = P(A')\,P(B')\,P(C') = \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} = \frac{1}{4}$$

Q. 3. The probability that the problem is solved is _______.

(A) $\frac{1}{4}$ **(B)** $\frac{3}{4}$

(C) $\frac{17}{24}$ **(D)** $\frac{11}{24}$

Ans. Option (B) is correct.

Explanation:

$$P(A \cup B \cup C) = 1 - P(A')\,P(B')\,P(C') = 1 - \frac{1}{4} = \frac{3}{4}$$

Q. 4. The probability that exactly one of them solves it is _______.

(A) $\frac{1}{4}$ **(B)** $\frac{3}{4}$

(C) $\frac{17}{24}$ **(D)** $\frac{11}{24}$

Ans. Option (D) is correct.

Explanation:

$$P(A \cap B' \cap C') + P(A' \cap B \cap C') + P(A' \cap B' \cap C)$$

$$= P(A)\,P(B')\,P(C') + P(A')\,P(B)\,P(C') + P(A')\,P(B')\,P(C)$$

$$= \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} + \frac{1}{2} \times \frac{1}{3} \times \frac{3}{4} + \frac{1}{2} \times \frac{2}{3} \times \frac{1}{4}$$

$$= \frac{11}{24}$$

Q. 5. The probability that exactly two of them solves it is _______.

(A) $\frac{1}{4}$ **(B)** $\frac{3}{4}$

(C) $\frac{17}{24}$ **(D)** $\frac{11}{24}$

Ans. Option (A) is correct.

Explanation:

$$P(A \cap B \cap C') + P(A \cap B' \cap C) + P(A' \cap B \cap C)$$

$$= P(A)\,P(B)\,P(C') + P(A)\,P(B')\,P(C) + P(A')\,P(B)\,P(C)$$

$$= \frac{1}{2} \times \frac{1}{3} \times \frac{3}{4} + \frac{1}{2} \times \frac{2}{3} \times \frac{1}{4} + \frac{1}{2} \times \frac{1}{3} \times \frac{1}{4}$$

$$= \frac{6}{24}$$

$$= \frac{1}{4}$$