# Class X Session 2023-24
# Subject - Mathematics (Basic)
# Sample Question Paper - 9

**Time Allowed: 3 hours** **Maximum Marks: 80**

**General Instructions:**

1. This Question Paper has 5 Sections A, B, C, D and E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with sub- parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take $\pi = \frac{22}{7}$ wherever required if not stated.

**Section A**

1. The product of a rational number and an irrational number is **[1]**

a) both rational and irrational number b) none of these

c) an irrational number only d) a rational number only

2. If two numbers do not have common factor (other than 1), then they are called **[1]**

a) prime numbers b) co-prime numbers

c) composite numbers d) twin primes

3. If the equation $x^2 - ax + 1 = 0$ has two distinct roots, then **[1]**

a) a > 2 b) a > 2 or a < -2

c) a = 2 d) a < 2

4. 5 years hence, the age of a man shall be 3 times the age of his son while 5 years earlier the age of the man was 7 times the age of his son. The present age of the man is **[1]**

a) 50 years b) 45 years

c) 47 years d) 40 years

5. One of the roots of the quadratic equation $a^2x^2 - 2abx + 2b^2 = 0$ is **[1]**

a) $\frac{-2b}{a}$ b) $\frac{-2a}{b}$

c) $\frac{2b}{a}$ d) $\frac{2a}{b}$

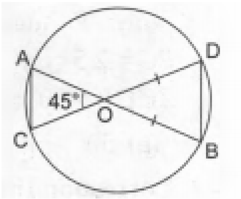
6. The perpendicular bisector of the line segment joining the points A (1, 5) and B (4, 6) cuts the y-axis at **[1]**

a) (0, -13)
b) (0, 12)
c) (0, 13)
d) (13, 0)

7. If $\triangle$ABC and $\triangle$DEF are similar such that 2 AB = DE and BC = 8cm, then EF = **[1]**

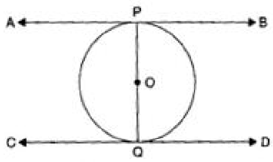
a) 16 cm
b) 8 cm
c) 12 cm
d) 4 cm

8. In the given figure, O is the point of intersection of two chords AB and CD such that OB = OD and $\angle$AOC = 45°. Then, $\triangle$OAC and $\triangle$ODB are **[1]**




a) equilateral and similar
b) equilateral but not similar
c) isosceles but not similar
d) isosceles and similar

9. The distance between two parallel tangents of a circle of radius 3 cm is **[1]**




a) 6 cm
b) 3 cm
c) 4.5 cm
d) 12 cm

10. $\frac{\tan\theta}{\sec\theta-1}+\frac{\tan\theta}{\sec\theta+1}$ is equal to **[1]**

a) 2 cosec$\theta$
b) 2 tan$\theta$ sec$\theta$
c) 2 sec$\theta$
d) 2 tan$\theta$

11. A ramp for disabled people in a hospital must slope at not more than $30^{o}$. If the height of the ramp has to be 1 m, then the length of the ramp be **[1]**

a) 3 m
b) 1 m
c) 2 m
d) $\sqrt{3}$ m

12. If tan θ =$\sqrt{3}$, then sec θ = **[1]**

a) $\sqrt{\frac{3}{2}}$
b) 2
c) $\frac{2}{\sqrt{3}}$
d) $\frac{1}{\sqrt{3}}$

13. The perimeter (in cm) of a square circumscribing a circle of radius a cm, is **[1]**

a) 16a
b) 8a
c) 2a
d) 4a

14. If the difference between the circumference and the radius of a circle is 37 cm, then circumference of the circle is (in cm **[1]**

a) 14
b) 44

c) 154 d) 7

15. If two different dice are rolled together, the probability of getting an even number **[1]**

a) $\frac{1}{2}$ b) $\frac{1}{4}$

c) $\frac{1}{36}$ d) $\frac{1}{6}$

16. The median and mode of a frequency distribution are 26 and 29 respectively. Then, the mean is **[1]**

a) 28.4 b) 22.5

c) 25.8 d) 24.5

17. An icecream cone has hemispherical top. If the height of the cone is 9 cm and base radius is 2.5 cm, then the volume of icecream is **[1]**

a) 91.67 $cm^3$ b) 96.67 $cm^3$

c) 90.67 $cm^3$ d) 91.76 $cm^3$

18. If the mean of observations $x_1, x_2, ..., x_n$ is $\overline{x}$, then the mean of $x_1 + a, x_2 + a, ..., x_n + a$ is: **[1]**

a) $\overline{x}$ - a b) $\frac{\overline{x}}{a}$

c) a$\overline{x}$ d) $\overline{x}$ + a

19. **Assertion (A):** The points (k, 2 - 2k), (-k +1, 2k) and (-4 - k, 6 - 2k) are collinear if k = $\frac{1}{2}$ **[1]**

**Reason (R):** Three points A, B and C are collinear in the same straight line if AB + BC = AC

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** H.C.F. of 12 and 77 is 1. **[1]**

**Reason (R):** L.C.M. of two coprime numbers is equal to their product.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

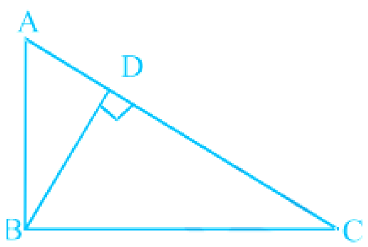
c) A is true but R is false.

d) A is false but R is true.

## Section B

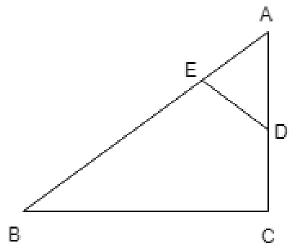
21. The taxi charges in a city consist of a fixed charge together with the charge for the distance covered. For a distance of 10 km, the charge paid is ₹ 105 and for a journey of 15 km the charge paid is ₹ 155. What are the fixed charges and the charge per km? How much does a person have to pay for travelling a distance of 25 km? Find them by substitution method. **[2]**

22. In the given figure, ABC is a triangle, right angled at B and BD ⊥ AC. If AD = 4 cm and CD = 5 cm, find BD and AB. **[2]**

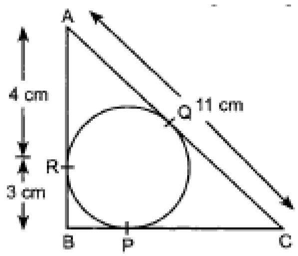



OR

In $\triangle ABC$, if $\angle ADE = \angle B$ , then prove that $\triangle ADE \sim \triangle ABC$. Also, if AD = 7.6 cm, AE = 7.2 cm, BE = 4.2 cm and BC = 8.4 cm, find DE.




23. In figure, $\triangle$ABC is circumscribing a circle. Find the length of BC. **[2]**




24. Prove that: $\frac{\cos A}{1-\tan A} + \frac{\sin^2 A}{sinA-cosA}$ = sin A + cos A. **[2]**

25. A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding: **[2]**
    i. minor segment
    ii. major sector.

OR

A horse is tethered to one corner of a field which is in the shape of an equilateral triangle of side 12 m. If the length of the rope is 7 m, find the area of the field which the horse cannot graze. Take $\sqrt{3}$ = 1.732. Write the answer correct to 2 places of decimal.

**Section C**

26. A wine seller had three types of wine. 403 liters of 1st kind, 434 liters of 2nd kind and 465 liters of 3rd kind. Find the least possible number of casks of equal size in which different types of wine can be filled without mixing. **[3]**

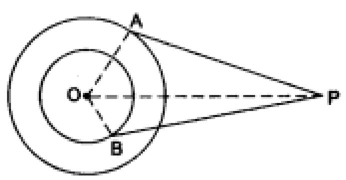
27. If $\alpha$, $\beta$ are the zeros of the polynomial $2x^2 - 4x + 5$. find the value of (i) $\alpha^2 + \beta^2$ (ii) $(\alpha - \beta)^2$. **[3]**

28. A and B have certain number of oranges A says to B " If you give me 10 of yours oranges I will have twice the number of oranges left with you". B replies, " If you give me 10 of your oranges I will have the same number of oranges as left with you". Find the number of oranges that A and B have separately. **[3]**

OR

Five years ago, Amit was thrice as old as Baljeet. Ten years hence, Amit shall be twice as old as Baljeet. What are their present ages?

29. In the given figure, O is the centre of two concentric circles of radii 4 cm and 6 cm respectively. PA and PB are tangents to the outer and inner circle respectively. If PA = 10 cm, find the length of PB up to one place of decimal. **[3]**




30. Prove that: sec A (1 - sin A) (sec A + tan A) = 1 **[3]**

OR

Prove that: $1+\frac{\cot^2\alpha}{1+cosec\alpha}= \ cosec\,\alpha$

31. Two dice are thrown simultaneously. What is the probability that **[3]**

i. 5 will not come on either of them?
ii. 5 will come up on atleast one?
iii. 5 will come up on both dice?

**Section D**

32. A train travelling at a uniform speed for 360 km,would have taken 48 minutes less to travel the same distance if its speed were 5 km/hour more. Find the original speed of the train. **[5]**

OR

If the roots of the quadratic equation $\left(c^2-ab\right)x^2-2\left(a^2-bc\right)x+b^2-ac=0$ in x are equal then show that either a = 0 or $a^3+b^3+c^3=3abc$

33. Let ABC be a triangle and D and E be two points on side AB such that AD = BE. If DP $\|$ BC and EQ $\|$ AC, then prove that PQ $\|$ AB. **[5]**

34. A solid toy is in the form of a hemisphere surmounted by a right circular cone. Height of the cone is 2 cm and the diameter of the base is 4 cm. If a right circular cylinder circumscribes the solid. Find how much more space it will cover. **[5]**

OR

A spherical glass vessel has a cylindrical neck 8 cm long and 1 cm in radius. The radius of the spherical part is 9 cm. Find the amount of water (in litres) it can hold, when filled completely.

35. The following table gives the distribution of the life time of 400 neon lamps: **[5]**

| **Lite time (in hours)** | **Number of lamps** |
|---|---|
| 1500-2000 | 14 |
| 2000-2500 | 56 |
| 2500-3000 | 60 |
| 3000-3500 | 86 |
| 3500-4000 | 74 |
| 4000-4500 | 62 |
| 4500-5000 | 48 |

Find the median life time of a lamp.

**Section E**

36. **Read the text carefully and answer the questions:** **[4]**

Elpis Technology is a TV manufacturer company. It produces smart TV sets not only for the Indian market but also exports them to many foreign countries. Their TV sets have been in demand every time but due to the Covid-19 pandemic, they are not getting sufficient spare parts, especially chips to accelerate the production. They have to work in a limited capacity due to the lack of raw materials.

(i) They produced 600 sets in the third year and 700 sets in the seventh year. Assuming that the production increases uniformly by a fixed number every year, find an increase in the production of TV every year.

(ii) They produced 600 sets in the third year and 700 sets in the seventh year. Assuming that the production increases uniformly by a fixed number every year, find in which year production of TV is 1000.

**OR**

They produced 600 sets in the third year and 700 sets in the seventh year. Assuming that the production increases uniformly by a fixed number every year, find the production in the 10th year.

(iii) They produced 600 sets in the third year and 700 sets in the seventh year. Assuming that the production increases uniformly by a fixed number every year, find the total production in first 7 years.

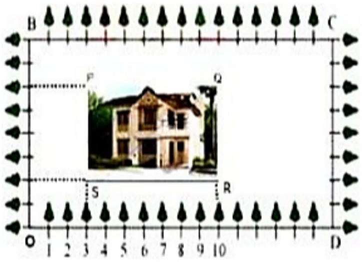
37. **Read the text carefully and answer the questions:** **[4]**

Using Cartesian Coordinates we mark a point on a graph by how far along and how far up it is.

The left-right (horizontal) direction is commonly called X-axis.

The up-down (vertical) direction is commonly called Y-axis.

In Green Park, New Delhi Suresh is having a rectangular plot ABCD as shown in the following figure. Sapling of Gulmohar is planted on the boundary at a distance of 1 m from each other. In the plot, Suresh builds his house in the rectangular area PQRS. In the remaining part of plot, Suresh wants to plant grass.




(i) Find the coordinates of the midpoints of the diagonal QS.

(ii) Find the length and breadth of rectangle PQRS?

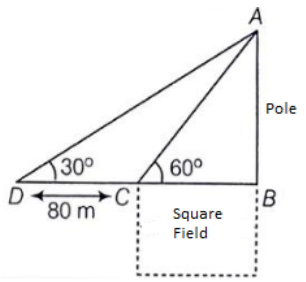
**OR**

Find Area of rectangle PQRS.

(iii) Find the diagonal of rectangle.

38. **Read the text carefully and answer the questions:** **[4]**

Basant Kumar is a farmer in a remote village of Rajasthan. He has a small square farm land. He wants to do fencing of the land so that stray animals may not enter his farmland. For this, he wants to get the perimeter of the land. There is a pole at one corner of this field. He wants to hang an effigy on the top of it to keep birds away. He standing in one corner of his square field and observes that the angle subtended by the pole in the corner just diagonally opposite to this corner is $60^o$. When he retires 80 m from the corner, along the same straight line, he finds the angle to be $30^o$.




(i) Find the height of the pole too so that he can arrange a ladder accordingly to put an effigy on the pole.

(ii) Find the length of his square field so that he can buy material to do the fencing work accordingly.

**OR**

Find the Distance from Farmer at position D and top of the pole?

(iii) Find the Distance from Farmer at position C and top of the pole?

# Solution

**Section A**

1. **(a)** both rational and irrational number

   **Explanation:** The product of a rational number and an irrational number can be either a rational number or an irrational number.

   e.g $\sqrt{5} \times \sqrt{2} = \sqrt{10}$ which is irrational

   but $\sqrt{8} \times \sqrt{2} = \sqrt{16} = 4$ which is a rational number

   Thus, the product of two irrational numbers can be either rational or irrational

   similarly, the product of rational and irrational numbers can be either rational or irrational

   $5 \times \sqrt{2} = 5\sqrt{2}$ which is irrational.

   but $0 \times \sqrt{3} = 0$ which is rational.

2. 

   **(b)** co-prime numbers

   **Explanation:** If two numbers do not have a common factor (other than 1), then they are called co-primer numbers. We know that two numbers are coprime if their common factor (greatest common divisor) is 1. e.g. co-prime of 12 are 11, 13.

3. 

   **(b)** a > 2 or a < -2

   **Explanation:** In the equation $x^2 - ax + 1 = 0$

   a = 1, b = -a, c = 1

   $D = b^2 - 4ac = (-a)^2 - 4 \times 1 \times 1 = a^2 - 4$

   Roots are distinct

   D > 0

   $\Rightarrow a^2 - 4 > 0$

   $\Rightarrow a^2 > 4$

   $\Rightarrow a^2 > (2)^2$

   $\Rightarrow a > 2$ or $a < -2$

4. 

   **(d)** 40 years

   **Explanation:** Let us assume the present age of men be x years

   Also, the present age of his son be y years

   According to question, after 5 years:

   $(x + 5) = 3 (y + 5)$

   $x + 5 = 3y + 15$

   $x - 3y = 10$ …(i)

   Also, five years ago:

   $(x - 5) = 7 (y - 5)$

   $x - 5 = 7y - 35$

   $x - 7y = - 30$ …(ii)

   Now, on subtracting (i) from (ii) we get:

   $- 4y = - 40$

   $y = 10$

   Putting the value of y in (i), we get

   $x - 3 \times 10 = 10$

   $x - 30 = 10$

   $x = 10 + 30$

   $x = 40$

   $\therefore$ The present age of men is 40 years

5.

**(c)** $\frac{2b}{a}$

**Explanation:** ⇒ $a^2x^2$ - 2abx + $2b^2$ = 0

⇒ ax(ax - 2b) - b(ax - 2b) = 0

⇒ (ax - b)(ax - 2b) = 0

⇒ ax - b = 0 and ax - 2b = 0

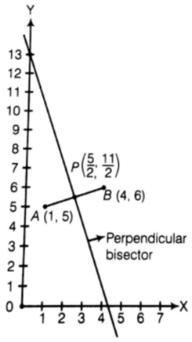
⇒ x = $\frac{b}{a}$ and x = $\frac{2b}{a}$

6.

**(c)** (0, 13)

**Explanation:**

First, we have to plot the points of the line segment on the paper and join them.




As we know that the perpendicular bisector of line segment AB, perpendicular at AB and passes through the mid-point of AB.

Let P be the mid-point of AB

Now find the mid-point,

Mid-point of AB = $\frac{1+4}{2}, \frac{5+6}{2}$

∵ Mid-point of line segment passes through the points ($x_1$, $y_1$) and ($x_2$, $y_2$)

= $\left[\frac{(x_1+x_2)}{2}, \frac{(y_1+y_2)}{2}\right]$

$\Rightarrow P = \frac{5}{2}, \frac{11}{2}$

Find the slope of the bisector:

Slop of the given line = $\frac{(y_1-y_2)}{(x_1-x_2)}$

Slope = $\frac{5-6}{1-4} = \frac{1}{3}$

Slope of given line multiplied by slope of bisector = - 1

Slope of bisector = $\frac{-1}{\frac{1}{3}} = \frac{-3}{1}$

= - 3

Now, we find the bisector's formula by using the point slope form;

Which is;

$-3 = \frac{\frac{11}{2}-y}{\frac{3}{2}-x} = \frac{5.5-y}{2.5-x}$

- 3(2.5 – x) = 5.5 - y

- 7.5 + 3x = 5.5 - y3x + y - 13 = 0

Transform the formula into slope - intercept form

3x + y - 13 = 0y = - 3x + 13

because, slope - intercept form is y = mx + c,

Where, m is the slope and c is the y - intercept

Thus, perpendicular bisector cuts the y - axis at (0, 13)

So, the required point is (0, 13).

7. **(a)** 16 cm

**Explanation:** $\triangle ABC \sim \Delta DEF$

2 AB = DE, BC = 8 cm

$\frac{AB}{DE} = \frac{1}{2}$

∵ $\triangle ABC \sim \triangle DEF$

$\therefore \frac{AB}{DE} = \frac{BC}{EF}$
$\Rightarrow \frac{1}{2} = \frac{8}{EF} \Rightarrow EF = 2 \times 8 = 16$
Hence EF = 16 cm

8.

**(d)** isosceles and similar

**Explanation:** In the given figure, O is the point of intersection of two chords AB and CD.

OB = OD and $\angle$AOC = 45°

$\angle$B = $\angle$D (Angles opposite to equal sides)

$\angle$A = $\angle$D, $\angle$C = $\angle$B (Angles in the same segment)

and $\angle$AOC = $\angle$BOD = 45° each

$\triangle$OAC $\sim$ $\triangle$ODB (AAA axiom)

OA = OC (Sides opposite to equal angles)

$\triangle$OAC and $\triangle$ODB are isosceles and similar.

9. **(a)** 6 cm

**Explanation:** Since the distance between two parallel tangents of a circle is equal to the diameter of the circle.

Given: Radius (OP) = 3 cm

$\therefore$ Diameter = 2 $\times$ Radius = 2 $\times$ 3 = 6 cm

10. **(a)** 2 cosec$\theta$

**Explanation:** We have, $\frac{\tan\theta}{\sec\theta-1} + \frac{\tan\theta}{\sec\theta+1}$
$= \tan\theta\left(\frac{1}{\sec\theta-1} + \frac{1}{\sec\theta+1}\right)$
$= \frac{\tan\theta(\sec\theta+1+\sec\theta-1)}{(\sec\theta-1)(\sec\theta+1)}$
$= \frac{\tan\theta\times2\sec\theta}{\sec^2\theta-1} = \frac{2\tan\theta\sec\theta}{\tan^2\theta}$
$= \frac{2\sec\theta}{\tan\theta} = \frac{2\times\cos\theta}{\cos\theta\times\sin\theta} = \frac{2}{\sin\theta}$
$= 2cosec\theta$

11.

**(c)** 2 m

**Explanation:** Let the height of the ramp be AB = 1 m, the slope of the ramp AC and angle of elevation = $\theta$ = $30^o$

In triangle ABC,

$\sin^o 30$ = $\frac{AB}{AC}$
$\Rightarrow \frac{1}{2} = \frac{1}{AC}$
$\Rightarrow$ AC = 2 meters

Therefore, the length of the ramp is 2 m.

12.

**(b)** 2

**Explanation:** Since $\sec\theta = \sqrt{1+\tan^2\theta}$

$\therefore\sec\theta = \sqrt{1+\left(\sqrt{3}\right)^2}$

= $\sqrt{1+3} = \sqrt{4}$ = 2

13.

**(b)** 8a

**Explanation:** It is given that a square circumscribes a circle of radius a cm. Side of the square = Diameter of thecircle

⇒ Side of the square=2a

∴Perimeter of the square =4×2a=8a

14.

**(b)** 44

**Explanation:** Difference between circumference and radius of the circle = 37 cm

Let r be the radius of the circle.

∴2πr−r=37cmr(2π−1)=37cmr(2×227−1)=37cmr×377=37cmr=7cm

∴ Circumference of the circle =2πr=2×227×7cm=44cm

15.

**(b)** $\frac{1}{4}$

**Explanation:** Rolling two different dice, Number of total events = 6 × 6 = 36

Number of even number on both dice are {(2,2), (2,4), (2,6), (4,2), (4,4), (4,6), (6,2), (6,4), (6,6) }= 9

$\therefore Probability = \frac{9}{36} = \frac{1}{4}$

16.

**(d)** 24.5

**Explanation:** Median = 26

Mode = 29

Mode = 3Median - 2Mean

Hence, $Mean = \frac{\text{3Median - Mode}}{2}$

$= \frac{3(26)-29}{2}$

$= \frac{78-29}{2}$

$= \frac{49}{2}$

= 24.5

17. **(a)** 91.67 $cm^3$

**Explanation:** Height of ice-cream cone is 9 cm and radius of the hemispherical top is 2.5 cm.

Now, Volume of ice-cream cone = Volume of cone + volume of Hemispherical top

$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$

$= \frac{1}{3}\pi r^2(h + 2r)$

$= \frac{1}{3} \times \frac{22}{7} \times 2.5 \times 2.5(9 + 5)$

$= \frac{1}{3} \times \frac{22}{7} \times 2.5 \times 2.5 \times 14$

= 91.67 $cm^3$

18.

**(d)** $\overline{x}$ + a

**Explanation:** Mean of observations $x_1, x_2, ..., x_n$ is $\overline{x}$

i.e, $\bar{x} = \frac{x_1+x_2+x_3\cdots+x_n}{n}$

Now, $(x_1 + a) + (x_2 + a) + (x_3 + a) + ... +(x_n + a)$

$= x_1 + x_2 + x_3 + ... x_n + na$

$\therefore$ Mean of $(x_1 + a), (x_2 + a), (x_3 + a), ..., (x_n + a)$

$= \frac{(x_1+x_2+x_3...+x_n)+na}{n}$

$= \frac{(x_1+x_2+x_3...+x_n)}{n} + \frac{na}{n}$

$= \bar{x} + \frac{na}{n} = \bar{x}$ + a

19. **(a)** Both A and R are true and R is the correct explanation of A.

**Explanation:** Both A and R are true and R is the correct explanation of A.

20.

**(b)** Both A and R are true but R is not the correct explanation of A.

**Explanation:** Yes 12 and 17 are coprime numbers and H.C.F. of coprimes is always 1.

**Section B**

21. Let fixed charge be ₹x and the charge per km be ₹y.

For a distance of 10 km, the charge paid is ₹105 .

x + 10y = 105 ....(i)

For a journey of 15 km the charge paid is ₹155

x + 15y = 155 .... (ii)

From eqn. (i), x = 105 -10y ...(iii)

On substituting x from eqn. (iii) in eqn. (ii), 105 - 10y + 15y = 155

$\Rightarrow$ 5y = 155 - 105

$\Rightarrow$ 5y = 50

$\Rightarrow$ y = 10

Put y = 10 in (iii)

x = 105 - 10(10)

$\Rightarrow$ x = 105 - 100

$\therefore$ x = 5

Hence, fixed charges = ₹5

Rate per km = ₹10

Amount to be paid for travelling 25 km

= ₹5 + ₹10 $\times$ 25

= ₹5 + ₹250

= ₹255

22. Here $\triangle \mathrm{ADB} \sim \triangle \mathrm{BDC}$

$\therefore \quad \frac{\mathrm{AD}}{\mathrm{BD}} = \frac{\mathrm{BD}}{\mathrm{CD}}$

$\Rightarrow \quad AD \times CD = BD \times BD$

4 $\times$ 5 = $BD^2$

$\Rightarrow$ BD = $2\sqrt{5}$ cm

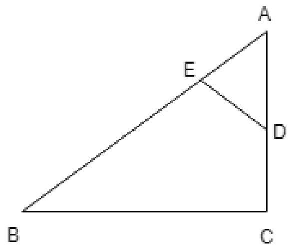
In right $\triangle$BDA

$AB^2 = BD^2 + AD^2$

$\Rightarrow AB^2 = (2\sqrt{5})^2 + (4)^2$

$\Rightarrow AB^2 = 36 \Rightarrow$ AB = 6 cm

OR




In $\triangle ADE$ and $\triangle ABC$,

$\angle ADE = \angle ABC$ [$\because$ given]

$\angle DAE = \angle BAC$ [$\because$ common angle]

So, $\triangle ADE \sim \triangle ABC$ [by AA similarity criterion]

Then , $\frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC}$ [since, corresponding sides of similar triangles are proportional]

$\Rightarrow \frac{AD}{AE+EB} = \frac{DE}{8.4}$

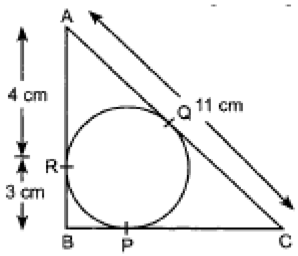
$\Rightarrow \frac{7.6}{7.2+4.2} = \frac{DE}{8.4}$ [$\because$AB = AE + BE]

$\Rightarrow \frac{7.6}{11.4} \times 8.4 = DE$

$\Rightarrow 0.66 \times 8.4 = DE$

$\Rightarrow DE = 5.6cm$

23. Given,




$AR = 4\ cm.$

Also, $AR = AQ \Rightarrow AQ = 4cm$

Now, $QC = AC - AQ$

$= 11cm - 4cm = 7cm$ ...(i)

Also, $BP = BR$

$\therefore$ BP = 3 cm and PC = QC

$\therefore$ PC = 7 cm [From (i)]

$BC = BP + PC$
$= 3cm + 7cm$
$= 10cm$

24. L.H.S = $\frac{\cos A}{1-\tan A} + \frac{\sin^2 A}{sinA - cosA}$

= $\frac{cos^2 A}{cosA - sinA} + \frac{sin^2 A}{cosA - sinA}$ [by putting tanA = $\frac{\sin A}{\cos A}$ ]

= $\frac{cos^2 A - sin^2 A}{cosA - sinA}$

= $\frac{(cosA - sinA)(cosA + sinA)}{cosA - sinA}$

= cos A + sin A

= R.H.S

25. i. r = 10 cm, $\theta = 90^\circ$

Area of minor sector = $\frac{\theta}{360} \times \pi r^2$

$= \frac{90}{360} \times 3.14 \times 10 \times 10 = 78.5\text{cm}^2$

Area of $\triangle OAB = \frac{OA \times OB}{2}$

$= \frac{10 \times 10}{2} = 50\text{cm}^2$

$\therefore$ Area of the minor segment

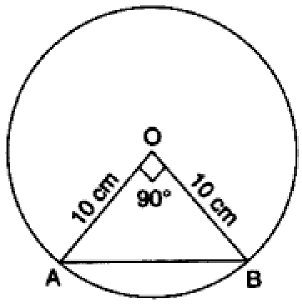
= Area of minor sector - Area of $\triangle OAB$

$= 78.5\text{cm}^2 - 50\text{cm}^2 = 28.5\text{cm}^2$

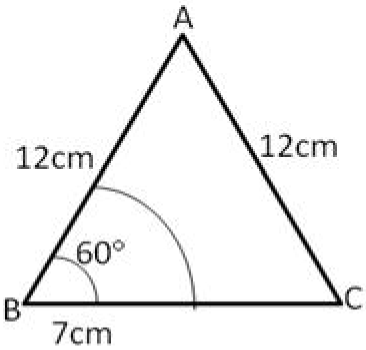
ii. Area of major sector = $\pi x^2 - 28.5$

$= 3.14 \times 10 \times 10 - 28.5$

$= 314 - 28.5 = 285.5\text{cm}^2$




OR




Area which cannot be grazed = (area of equilateral $\triangle ABC$ - (area of the sector with r = 7m,$\theta = 60^\circ$ )

$= \left[\frac{\sqrt{3}}{4} \times (12)^2 - \frac{22}{7} \times (7)^2 \times \frac{60}{360}\right] \text{m}^2$

$= \left[(\sqrt{3} \times 12 \times 3) - \frac{(22 \times 7)}{6}\right]$

= 62.35 - 25.66 $m^2$

= 36.68 $m^2$

**Section C**

26. For the least possible number of casks of equal size, the size of each cask must be of the greatest volume.

To get the greatest volume of each cask, we have to find the largest number which exactly divides 403, 434 and 465. That is nothing but the H.C.F of (403, 434, 465)

The H.C.F of (403, 434, 465) = 31 liters

Each cask must be of the volume 31 liters.

Req. No. of casks is

= $(\frac{403}{31}) + (\frac{434}{31}) + (\frac{465}{31})$

= 13 + 14 + 15

= 42

Hence, the least possible number of casks of equal size required is 42.

27. P (x) = $2x^2 - 4x + 5$

Here, a = 2 , b = – 4, c = 5

Let zeroes be $\alpha$, β

Sum of zeroes $\alpha + \beta = \frac{-b}{a} = \frac{-(-4)}{2} = 2$

Product of zeroes $\alpha \times \beta = \frac{c}{a} = \frac{5}{2}$

i. $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= (2)^2 - 2(\frac{5}{2})$

= 4 – 5

$\Rightarrow \alpha^2 + \beta^2 = -1$

ii. $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

$= (2)^2 - 4(\frac{5}{2})$

= 4 – 2(5)

= 4 – 10

= - 6

$(\alpha - \beta)^2 = -6$

28. Let the number of oranges with A and B separately be x and y respectively.

Then, according to the question,

x + 10 = 2(y - 10)

⇒ x + 10 = 2y - 20

⇒ x - 2y = - 20 - 10

⇒ x - 2y = - 30 ...(1)

and x - 10 = y + 10

⇒ x - y = 10 + 10

⇒ x - y = 20 ...(2)

Subtracting equation (1) from equations (2), we get

y = 50

substituting this value of y in equation (2), we get

x - 50 = 20

⇒ x = 50 + 20

⇒ x = 70

So the solution of the equations (1) and (2) is x = 70 and y = 50.

Hence, the number of oranges with A and B separately and 70 and 50 respectively.

**Verification.** Substituting x = 70, y = 50,

We find that both the equations (1) and (2) are satisfied as shown below:

x - 2y = 70 - 2(50) = 70 - 100 = - 30

x - y = 70 - 50 = 20

Hence, the solution is correct.

OR

Let the present ages of Baljeet and Amit be x years and y years respectively.

Then,

Baljeet's age 5 years ago = (x - 5) years

and Amit's age 5 years ago = (y - 5) years

∴ (y - 5) = 3(x - 5) ⇒ 3x - y = 10 ....(i)

Baljeet's age 10 years hence = (x +10) years

Amit's age 10 years hence = (y +10) years

$\therefore$ (y + 10) = 2(x + 10) $\Rightarrow$ 2x - y = - 10 ....(ii)

On subtracting (ii) from (i), we get x = 20.

Putting x = 20 in (i), we get

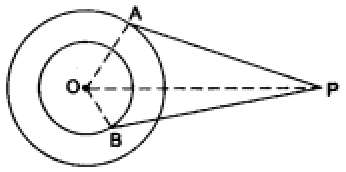
(3 $\times$ 20) - y = 10 $\Rightarrow$ y = 60 - 10 = 50.

$\therefore$ x = 20 and y = 50.

Hence, Baljeet's present age = 20 years

and Amit's present age = 50 years.

29.




Given, O is the center of two concentric circles of radii OA = 6 cm and OB = 4 cm. PA and PB are the two tangents to the outer and inner circles respectively and PA = 10 cm.

Now, tangent drawn from an external point is perpendicular to the radius at the point of contact.

$\therefore \angle OAP = \angle OBP = 90^\circ$

$\therefore$ From right - angled $\Delta OAP, OP^2 = OA^2 + PA^2$

$\Rightarrow OP = \sqrt{OA^2 + PA^2}$

$\Rightarrow OP = \sqrt{6^2 + 10^2}$

$\Rightarrow OP = \sqrt{136}$cm

$\therefore$ From right - angled $\Delta OBP, OP^2 = OB^2 + PB^2$

$\Rightarrow PB = \sqrt{OP^2 - OB^2}$

$\Rightarrow PB = \sqrt{136 - 16}$

$\Rightarrow PB = \sqrt{120}$cm

$\Rightarrow PB = 10.9$cm

$\therefore$ The length of PB is 10.9 cm.

30. L.H.S.

= sec A(1 - sin A)(sec A + tan A)

=$\frac{1}{cos A}$(1 - sin A)( $\frac{1}{cos A}$ + $\frac{sin A}{cos A}$)

=$\frac{(1-sin A)}{cos A}$($\frac{1+sin A}{cos A}$)

= $\frac{(1-\sin A)(1+\sin A)}{\cos A \times \cos A}$

= $\frac{(1^2-\sin^2 A)}{\cos^2 A}$ .[ Since, (a - b ) (a + b ) = $a^2$ - $b^2$ ]

= $\frac{(1-\sin^2 A)}{\cos^2 A}$

= $\frac{cos^2 A}{cos^2 A}$

= 1

=RHS

Hence, proved.

OR

L.H.S: $1 + \frac{\cot^2 \alpha}{1+ cosec\alpha}$

$= 1 + \frac{\cos^2 \alpha / \sin^2 \alpha}{1+1/\sin \alpha}$

[$\because \cot^2\alpha = \frac{\cos^2\alpha}{\sin^2\alpha}$ and $cosec\,\alpha = \frac{1}{\sin \alpha}$ ]

$\left[\because \cot^2 \alpha = \frac{\cos^2 \alpha}{\sin^2 \alpha} \text{ and } cosec\alpha = \frac{1}{\sin \alpha}\right]$

$= 1 + \frac{\cos^2 \alpha / \sin^2 \alpha}{\frac{\sin \alpha + 1}{\sin \alpha}}$

$= 1 + \frac{\cos^2 \alpha}{\sin \alpha (1+\sin \alpha)}$

$= \frac{\sin \alpha + \sin^2 \alpha + \cos^2 \alpha}{\sin \alpha + \sin^2 \alpha}$

$= \frac{1+\sin \alpha}{\sin \alpha (1+\sin \alpha)}$ $\left[\because \sin^2 \alpha + \cos^2 \alpha = 1\right]$

$= \frac{1}{\sin \alpha} = cosec\alpha$ $\left[\because \frac{1}{\sin \alpha} = cosec\alpha\right]$

31. When two dice are tossed together, The total no of outcomes = 36

Let A, B and C are the events with 5 neither on both dice, 5 on at least on one of dice and 5 on both dice respectively.

The outcomes having 5 are: 51, 52, 53, 54, 55, 56, 15, 25, 35, 45, 65

Total no of outcomes with 5 are 11:

i. Outcomes without 5 on either of dice = 36 - 11 = 25

So outcomes favouring A = 25

So P(A) = $\frac{25}{36}$

ii. Outcomes with 5 on at least on one of dice = 11

So outcomes favouring B = 11

So P(B) = $\frac{11}{36}$

iii. Outcomes with 5 on both of dice is (5,5) = 1

So outcomes favouring C = 1

So P(C) = $\frac{1}{36}$

## Section D

32. Given that a train travelling at a uniform speed for 360 km

Let the original speed of the train be x km/hr

Time taken $= \frac{\text{Distance}}{\text{Speed}} = \frac{360}{x}$

Time taken at increased speed $= \frac{360}{x+5}$ hours.

According to the question

$\frac{360}{x} - \frac{360}{x+5} = \frac{48}{60}$

$360\left[\frac{1}{x} - \frac{1}{x+5}\right] = \frac{4}{5}$

$or, \frac{360(x+5-x)}{x^2+5x} = \frac{4}{5}$

$or, \frac{1800}{x^2+5x} = \frac{4}{5}$

$\Rightarrow \quad x^2 + 5x - 2250 = 0$

$\Rightarrow \quad x^2 + (50-45)x - 2250 = 0$

$\Rightarrow \quad x^2 + 50x - 45x - 2250 = 0$

$\Rightarrow \quad (x+50)(x-45) = 0$

Either x = - 50 or x = 45

As speed cannot be negative

$\therefore$ Original speed of train = 45 km/hr.

OR

We, $A = \left(c^2 - ab\right)$, B$= -2(a^2 - bc), C = b^2 - ac$

For real equal roots, $D = B^2 - 4AC = 0$

$\Rightarrow \quad \left[-2\left(a^2 - bc\right)\right]^2 - 4\left(c^2 - ab\right)\left(b^2 - ac\right) = 0$

$\Rightarrow 4\left(a^4 + b^2c^2 - 2a^2bc\right) - 4\left(b^2c^2 - c^3a - ab^3 - a^2bc\right) = 0$

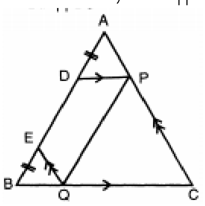
$\Rightarrow 4\left[a^4 + b^2c^2 - 2x^2bc - b^2c^2 + c^3a + ab^3 - a^2bc\right) = 0$

$\Rightarrow 4\left[a^4 + ac^3 + ab^3 - 3a^2bc\right] = 0$

$\Rightarrow a\left(a^3 + c^3 + b^3 - 3abc\right) = 0$

$\Rightarrow a = 0$ or $a^3 + b^3 + c^3 = 3abc$

33. In $\Delta ABC,\ DP||BC$




$\frac{AD}{DB} = \frac{AP}{PC}$, (BPT) ...(i)

Similarly, in $\Delta ABC$

$EQ||AC$

or, $\frac{BQ}{QC} = \frac{BE}{EA}$ ......(ii)

$EA = ED + DA = ED + BE = BD$

Then eqn. (ii) becomes, $\frac{BQ}{QC} = \frac{AD}{BD}$ ......(iii)

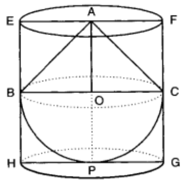
From (i) and (iii)

$\frac{AP}{PC} = \frac{BQ}{QC}$

Hence by converse of BPT

$PQ \| AB$ Hence proved

34. Let BPC be the hemisphere and ABC be the cone mounted on the base of the hemisphere. Let EFGH be the right circular cylinder circumscribing the given toy.




We have,

Given radius of cone, cylinder and hemisphere (r) = $\frac{4}{2}$ = 2 cm

Height of cone (l) = 2 cm

Height of cylinder (h) = 4 cm

Now, Volume of the right circular cylinder = $\pi r^2 h = \pi \times 2^2 \times 4 \text{cm}^3 = 16\pi \text{cm}^3$

Volume of the solid toy = $\left\{\frac{2}{3}\pi \times 2^3 + \frac{1}{3}\pi \times 2^2 \times 2\right\} \text{cm}^3 = 8\pi \text{cm}^3$

$\therefore$ Required space = Volume of the right circular cylinder - Volume of the toy

$= 16\pi \text{cm}^3 - 8\pi \text{cm}^3 = 8\pi \text{cm}^3$.

Hence, the right circular cylinder covers $8\pi \text{cm}^3$ more space than the solid toy.

So, remaining volume of cylinder when toy is inserted in it = $8\pi$ cm$^3$

OR

The volume of the spherical vessel is

calculated by the given formula

V = $\frac{4}{3}\pi \times r^3$

Now,

V = $\frac{4}{3} \times \frac{22}{7} \times 9 \times 9 \times 9$

V = 3,054.85 cm$^3$

The volume of the cylinder neck is calculated by the given formula.

V = $\pi \times R^2 \times h$

Now,

V = $\frac{22}{7} \times 1 \times 1 \times 8$

V = 25.14 cm$^3$

The total volume of the vessel is equal to the volume of the spherical shell and the volume of its cylindrical neck.

3054.85 + 25.14 = 3, 080 cm$^3$

The total volume of the vessel is 3,080 cm$^3$.

As we know,

1 L = 1000 cm$^3$

$\frac{3080}{1000}$ = 3.080 L

Thus, the amount of water (in litres) it can hold is 3.080 L.

35.

| **Life time** | **Number of lamps ($f_i$)** | **Cumulative frequency** |
|---|---|---|
| 1500-2000 | 14 | 14 |
| 2000-2500 | 56 | 14 + 56 = 70 |
| 2500-300 | 60 | 70 + 60 = 130 |
| 3000-3500 | 86 | 130 + 86 = 216 |
| 3500-4000 | 74 | 216 + 74 = 290 |

| | | |
|---|---|---|
| 4000-4500 | 62 | 290 + 62 = 352 |
| 4500-5000 | 48 | 352 + 48 = 400 |
| | 400 | |

N = 400

Now we may observe that cumulative frequency just greater than $\frac{n}{2}$ (ie., $\frac{400}{2}$ = 200) is 216

Median class = 3000 - 3500

Median = l + $\left(\frac{\frac{n}{2}-cf}{f}\right) \times$ h

Here,

l = Lower limit of median class

F = Cumulative frequency of class prior to median class.

f = Frequency of median class.

h = Class size.

Lower limit (l) of median class = 3000

Frequency (f) of median class 86

Cumulative frequency (cf) of class preceding median class = 130

Class size (h) = 500

Median = 3000 + $\left(\frac{200-130}{86}\right) \times$ 500

= 3000 + $\frac{70\times500}{86}$

= 3406.98

**Section E**

36. **Read the text carefully and answer the questions:**

Elpis Technology is a TV manufacturer company. It produces smart TV sets not only for the Indian market but also exports them to many foreign countries. Their TV sets have been in demand every time but due to the Covid-19 pandemic, they are not getting sufficient spare parts, especially chips to accelerate the production. They have to work in a limited capacity due to the lack of raw materials.

(i) Since the production increases uniformly by a fixed number every year. Therefore, the sequence formed by the production in different years is an A.P. Let a be the first term and d be the common difference of the A.P. formed i.e., 'a' denotes the production in the first year and d denotes the number of units by which the production increases every year.

We have, $a_3$ = 600 and

$a_3$ = 600

$\Rightarrow$ 600 = a + 2d

$\Rightarrow$ a = 600 - 2d ...(i)

$\Rightarrow$ $a_7$ = 700

$\Rightarrow$ $a_7$ = 700

$\Rightarrow$ 700 = a + 6d

$\Rightarrow$ a = 700 - 6d ...(ii)

From (i) and (ii)

600 - 2d = 700 - 6d

$\Rightarrow$ 4d = 100

$\Rightarrow$ d = 25

(ii) Since the production increases uniformly by a fixed number every year. Therefore, the sequence formed by the production in different years is an A.P. Let a be the first term and d be the common difference of the A.P. formed i.e., 'a' denotes the production in the first year and d denotes the number of units by which the production increases every year.

We know that first term = a = 550 and common difference = d = 25

$a_n$ = 1000

$\Rightarrow$ 1000 = a + (n - 1)d
$\Rightarrow$ 1000 = 550 + 25n - 25
$\Rightarrow$ 1000 - 550 + 25 = 25n
$\Rightarrow$ 475 = 25n
$\Rightarrow$ n = $\frac{475}{25}$ = 19

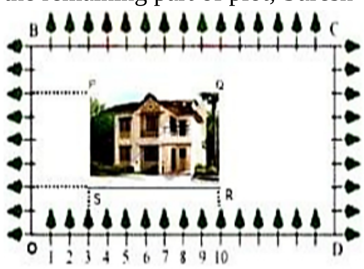
OR

Since the production increases uniformly by a fixed number every year. Therefore, the sequence formed by the production in different years is an A.P. Let a be the first term and d be the common difference of the A.P. formed i.e., 'a' denotes the production in the first year and d denotes the number of units by which the production increases every year. The production in the 10th term is given by $a_{10.}$ Therefore, production in the 10th year = $a_{10}$ = a + 9d = 550 + 9 $\times$ 25 = 775. So, production in 10th year is of 775 TV sets.

(iii) Since the production increases uniformly by a fixed number every year. Therefore, the sequence formed by the production in different years is an A.P. Let a be the first term and d be the common difference of the A.P. formed i.e., 'a' denotes the production in the first year and d denotes the number of units by which the production increases every year.
Total production in 7 years = Sum of 7 terms of the A.P. with first term a (= 550) and d (= 25).

$S_n = \frac{n}{2}[2a + (n-1)d]$
$\Rightarrow S_7 = \frac{7}{2}[2 \times 550 + (7-1)25]$
$\Rightarrow S_7 = \frac{7}{2}[2 \times 550 + (6) \times 25]$
$\Rightarrow S_7 = \frac{7}{2}[1100 + 150]$
$\Rightarrow$ $S_7$ = 4375

37. **Read the text carefully and answer the questions:**
Using Cartesian Coordinates we mark a point on a graph by how far along and how far up it is.
The left-right (horizontal) direction is commonly called X-axis.
The up-down (vertical) direction is commonly called Y-axis.
In Green Park, New Delhi Suresh is having a rectangular plot ABCD as shown in the following figure. Sapling of Gulmohar is planted on the boundary at a distance of 1 m from each other. In the plot, Suresh builds his house in the rectangular area PQRS. In the remaining part of plot, Suresh wants to plant grass.




(i) 


Middle point of QS = $\left(\frac{10+3}{2}, \frac{6+2}{2}\right)$
= (6.5, 4)

(ii) Length = RS = $\sqrt{(10-3)^2 + (2-2)^2}$
RS = $\sqrt{7^2 + 0}$
RS = 7 m
Breadth = RQ = $\sqrt{(10-10)^2 + (2-6)^2}$
$= \sqrt{0 + 16}$
= 4 m

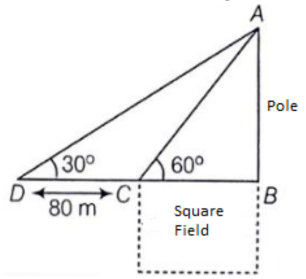
OR

Area of rectangle = l $\times$ b
= 7 $\times$ 4
= 28 $m^2$

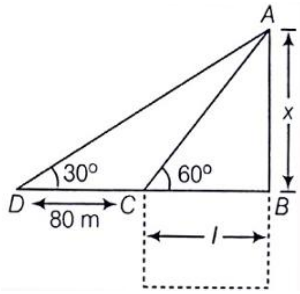
(iii) Diagonal = $\sqrt{l^2 + b^2}$
$= \sqrt{7^2 + 4^2}$
$= \sqrt{49 + 16}$
$= \sqrt{65}$

38. **Read the text carefully and answer the questions:**

Basant Kumar is a farmer in a remote village of Rajasthan. He has a small square farm land. He wants to do fencing of the land so that stray animals may not enter his farmland. For this, he wants to get the perimeter of the land. There is a pole at one corner of this field. He wants to hang an effigy on the top of it to keep birds away. He standing in one corner of his square field and observes that the angle subtended by the pole in the corner just diagonally opposite to this corner is $60^o$. When he retires 80 m from the corner, along the same straight line, he finds the angle to be $30^o$.




(i) The following figure can be drawn from the question:




Here AB is the pole of height x metres and BC is one side of the square field of length l metres.

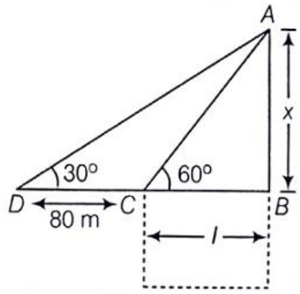
Now, l = 40 metres

We get,

$x = \sqrt{3}\,l = 40\sqrt{3} = 69.28$

Thus, height of the pole is 69.28 metres.

(ii) The following figure can be drawn from the question:




Here AB is the pole of height x metres and BC is one side of the square field of length l metres.

In $\triangle ABC$,

$\tan 60^o = \frac{x}{l}$

$\sqrt{3} = \frac{x}{l}$

$x = \sqrt{3}\,l$ ...(i)

Now, in $\triangle ABD$,

$\tan 30^o = \frac{x}{80+l}$

$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}l}{80+l}$ (From eq(i))

80 + l = 3l

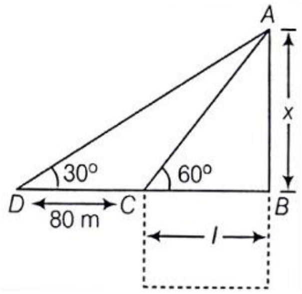
2l = 80

l = 40

Thus, length of the field is 40 metres.

OR

The following figure can be drawn from the question:




Here AB is the pole of height x metres and BC is one side of the square field of length l metres.

Distance from Farmer at position D and top of the pole is AD

In △ABC

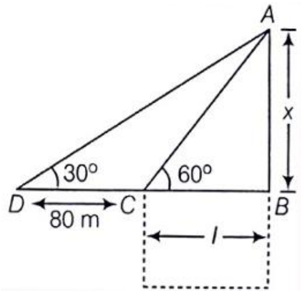
cos 30° = $\frac{DB}{AD}$

$\Rightarrow AD = \frac{DB}{\cos 30^0}$

$\Rightarrow AD = \frac{120}{\frac{\sqrt{2}}{2}} = \frac{240}{\sqrt{3}}$

⇒ AC = 138.56 m

(iii)The following figure can be drawn from the question:




Here AB is the pole of height x metres and BC is one side of the square field of length l metres.

Distance from Farmer at position C and top of the pole is AC.

In △ABC

cos 60° = $\frac{CB}{AC}$

$\Rightarrow AC = \frac{CB}{\cos 60^0}$

$\Rightarrow AC = \frac{40}{\frac{1}{2}}$

⇒ AC = 80 m