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CHAPTER-10 STRAIGHT LINES.
EXERCISE 10.3

General Equation of a LINE:

Any equation of the form $Ax+By+C=0$, where A and B are not zero simultaneously is called General Linear Equation or General Equation of a Line.

Reduction of General Equation to Standard forms:

The General =n of line can be reduced to the standard forms:

(i) The given =n is $Ax+By+C=0$

$$\therefore \quad By = -Ax - C$$

Dividing throughout by $B(\neq 0)$ we get

$$y = -\frac{A}{B}x - \frac{C}{B}$$

Which is of the Slope-Intercept form $y = mx + c$

where $m = -\frac{A}{B}$ and $c = \frac{-C}{B}$

If $B=0$, then $Ax+C=0 \Rightarrow x = \frac{-C}{A}$ which is vertical. Whose slope is undefined and x-intercept is $-\frac{C}{A}$.

NOTE: $\because \ m = -\frac{A}{B}$

$\therefore$ Slope of line $Ax+By+C=0$ is

$$m = -\frac{\text{Coeff of } x}{\text{Coeff. of } y}.$$

(ii) The given =n is $Ax+By+C=0$

$$\therefore \quad Ax + By = -C$$

Dividing both sides by $-C(\neq 0)$ we get

$$\frac{Ax}{-C} + \frac{By}{-C} = 1$$

or $\frac{x}{-c/A} + \frac{y}{-c/B} = 1.$

Which is of Intercept form $\frac{x}{a} + \frac{y}{b} = 1$

Where $a = -\frac{c}{A}$; $b = -\frac{c}{B}$.

If $c = 0$ then $Ax + By = 0$, which is a line passing. Through origin and therefore, has zero intercepts on axes.

(iii) The given eqn is $Ax + By + c = 0$

Dividing through out by. $\sqrt{A^2+B^2}$, we get

$$\frac{A}{\sqrt{A^2+B^2}}x + \frac{B}{\sqrt{A^2+B^2}}y + \frac{c}{\sqrt{A^2+B^2}} = 0$$

$$\therefore \quad \frac{A}{\sqrt{A^2+B^2}}x + \frac{B}{\sqrt{A^2+B^2}} = \frac{-c}{\sqrt{A^2+B^2}} \quad -(1)$$

Two Cases arise :

Case I. If c is +ve then RHS of (1) is −ve

$\therefore$ Changing the sign throughout [To make RHS +ve].

$$\frac{-A}{\sqrt{A^2+B^2}}x - \frac{B}{\sqrt{A^2+B}} = \frac{c}{\sqrt{A^2+B^2}}$$

Which is of form. $x\cos\omega + y\sin\omega = p$.

Where $\cos\omega = \frac{-A}{\sqrt{A^2+B^2}}$; $\sin\omega = \frac{-B}{\sqrt{A^2+B^2}}$; $p = \frac{c}{\sqrt{A^2+B^2}}$.

Case II. If c is −ve, then RHS of (1) is +ve

$\therefore$ (1) is of required form.

Note that proper ~~sign~~ choise of signs is made So that the $p = \frac{c}{\sqrt{A^2+B^2}}$ should always be +ve as p is distance of line from origin Which can never be −ve.

Q.No 1: Reduce the following equations into slope-intercept form and find their slopes and Y-intercept.

(i) $x+7y=0$ (ii) $6x+3y-5=0$ (iii) $y=0$

Sol: (i) The given equation of line is

$$x+7y=0$$

$$\therefore \quad 7y=-x \quad \Rightarrow \quad y=-\frac{1}{7}x+0$$

Which is of form $y=mx+c$

Where $m=-\frac{1}{7}$; $c=0$

(ii) The given equation of line is

$$6x+3y-5=0$$

$$\therefore \quad 3y=+5-6x \quad \text{or} \quad y=\frac{-6}{3}x+\frac{5}{3}$$

$$\text{or} \quad y=-2x+\frac{5}{3}$$

Which is of form $y=mx+c$ where $m=-2$; $c=\frac{5}{3}$

(iii) The given eqn of line is $y=0$

$$\text{or} \quad y=0.x+0$$

Which is of form $y=mx+c$; $m=0$, $c=0$

Q.No 2: Reduce the following equations into intercept form and find their intercepts on axes.

(i) $3x+2y-12=0$ (ii) $4x-3y=6$ (iii) $3y+2=0$

Soln (i) The given eqn of line is

$$3x+2y-12=0$$

$$\text{or} \quad 3x+2y=12$$

$$\text{or} \quad \frac{3x}{12}+\frac{2y}{12}=1$$

$$\text{or} \quad \frac{x}{4}+\frac{y}{6}=1$$ Which is of form $\frac{x}{a}+\frac{y}{b}=1$

Where x-intercept $a=4$

y-intercept $b=6$.

(ii) The given =n of line is $4x-3y=6$

or $\frac{4x}{6}-\frac{3y}{6}=1$

or $\frac{2x}{3}-\frac{y}{2}=1$

or $\frac{x}{3/2}+\frac{y}{-2}=1$ which is of intercept form

$\frac{x}{a}+\frac{y}{b}=1$ where $a=\frac{3}{2}$; $b=-2$

(iii) The given eqn of line is $3y+2=0$

or $3y=-2$ or $y=\frac{-2}{3}$

This is a st. line parallel to x-axis at a distance of $\frac{2}{3}$ below x-axis

∴ Intercept on y-axis $=-\frac{2}{3}$ and there is no intercept on x-axis.

QNo3: Reduce the following =ns into Normal form. Find their perpendicular distance from the origin and angle between perpendicular and the +ve x-axis.

(i) $x-\sqrt{3}y+8=0$ (ii) $y-2=0$ (iii) $x-y=4$.

Sol: (i) The given eqn. is

$x-\sqrt{3}y+8=0$

or $x-\sqrt{3}y=-8$

or $-x+\sqrt{3}y=8$. [Always make RHS +ve]

Dividing both sides by $\sqrt{(-1)^2+(\sqrt{3})^2}=\sqrt{1+3}=2$ we get.

$-\frac{1}{2}x+\frac{\sqrt{3}}{2}y=4$.

which is of normal form $x\cos\omega+y\sin\omega=p$ where

$\cos\omega=-\frac{1}{2}$; $\sin\omega=\frac{\sqrt{3}}{2}$; $p=4$.

∴ $\omega=120°$ and $p=4$.

(ii) The given eqn of line is $y-2=0$

or $y=2$

or $0x+1\cdot y=2$

Dividing both sides by $\sqrt{(0)^2+(1)^2}=1$.

$0x+1y=2$.

Which is of Normal form $x\cos\omega+y\sin\omega=p$.

Where $\cos\omega=0$, $\sin\omega=1$, $p=2$

ie $\omega=90^\circ$ and $p=2$.

(iii) The given eqn of line is $x-y=4$.

Dividing both sides by $\sqrt{(1)^2+(-1)^2}$ we get

$$\frac{1}{\sqrt{2}}x-\frac{1}{\sqrt{2}}y=\frac{4}{\sqrt{2}}$$

Which is of Normal form $x\cos\omega+y\sin\omega=p$.

Where $\cos\omega=\frac{1}{\sqrt{2}}$ $\sin\omega=-\frac{1}{\sqrt{2}}$; $p=\frac{4}{\sqrt{2}}=2\sqrt{2}$.

Now $\cos\omega=\frac{1}{\sqrt{2}}$ and $\sin\omega=-\frac{1}{\sqrt{2}}$

ie $\cos\omega=\cos\frac{\pi}{4}$

$\Rightarrow \omega=2n\pi\pm\frac{\pi}{4}$

$\Rightarrow \omega=\frac{\pi}{4},\frac{7\pi}{4}$

ie $\sin\omega=-\sin\frac{\pi}{4}=\sin\left(-\frac{\pi}{4}\right)$

ie $\omega=n\pi+(-1)^n\left(-\frac{\pi}{4}\right)$

$=\frac{5\pi}{4},\frac{7\pi}{4}$

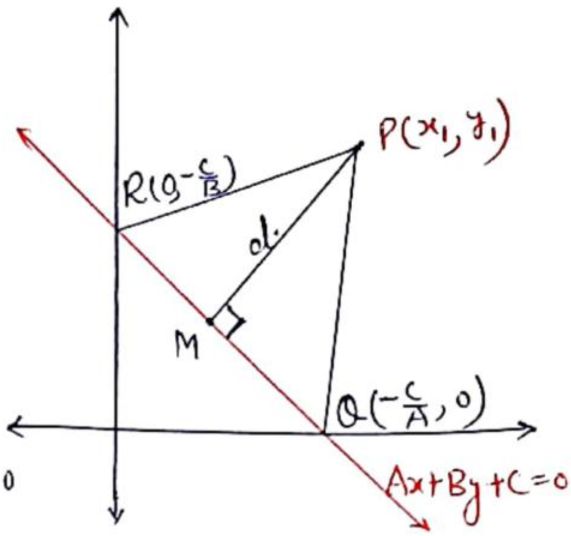
$\therefore$ $\cos\omega=\frac{1}{\sqrt{2}}$ and $\sin\omega=-\frac{1}{\sqrt{2}}$ are both satisfied

Simultaneously when $\theta=\frac{7\pi}{4}$ or $\frac{7\times180}{4}=315^\circ$.

$\therefore$ Normal form of eqn is

$$x\cos315^\circ+y\sin315^\circ=2\sqrt{2}.$$

PERPENDICULAR DISTANCE FORMULA : Distance of any Point $P(x_1, y_1)$ from given line $Ax+By+C=0$



Let $L: Ax+By+C=0$ be the given line meeting the axes in Q and R. Let $P(x_1, y_1)$ be the point and $PM \perp QR$. Join PQ and PR.

The eqn of QR is $Ax+By+C=0$

It meets x-axis in Q where $y=0$ ie at point $Q(-C/A, 0)$

It meets y-axis in R where $x=0$ ie at point $R(0, -\frac{C}{B})$

$$\therefore |RQ| = \sqrt{\left(0+\frac{C}{A}\right)^2+\left(-\frac{C}{B}-0\right)^2} = \sqrt{\frac{C^2}{A^2}+\frac{C^2}{B^2}} = \sqrt{C^2\left(\frac{1}{A^2}+\frac{1}{B^2}\right)}$$

$$= \sqrt{C^2\left(\frac{B^2+A^2}{A^2B^2}\right)} = \sqrt{\frac{C^2}{A^2B^2}(A^2+B^2)}$$

$$= \frac{|C|}{|A||B|}\sqrt{A^2+B^2}$$ (Using distance formula)

Also Area of $\Delta PQR = \frac{1}{2}\left|x_1\left(0+\frac{C}{B}\right)+\left(-\frac{C}{A}\right)\left(-\frac{C}{B}-y_1\right)+0(y_1-0)\right|$

$$= \frac{1}{2}\left|\frac{x_1C}{B}+y_1\frac{C}{A}+\frac{C^2}{AB}\right|$$

$$= \frac{1}{2}\left|C\left[\frac{x_1}{B}+\frac{y_1}{A}+\frac{C}{AB}\right]\right| = \frac{1}{2}\left|C\frac{(Ax_1+By_1+C)}{AB}\right|$$

$$= \frac{1}{2}\frac{|C|}{|A||B|}|Ax_1+By_1+C|$$

Now area of $\Delta PQR = \frac{1}{2} \times PM \times RQ$

$$\Rightarrow PM = \frac{2 \times \text{Area of } \Delta PQR}{RQ}$$

ie $PM = \dfrac{2 \times \frac{1}{2} \frac{|C|}{|A||B|} |Ax_1 + By_1 + C|}{\frac{|C|}{|A||B|} \sqrt{A^2+B^2}}$

ie $d = \dfrac{|Ax_1 + By_1 + C|}{\sqrt{A^2+B^2}}$

Thus The perpendicular distance of line $Ax+By+C=0$ from a point $P(x_1, y_1)$ is given by.

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2+B^2}}.$$

WORKING RULE TO FIND PERPENDICULAR DISTANCE:

(i) Make the equ of Line of form $Ax+By+C=0$
ie RHS $= 0$

(ii) In LHS Put Coordinates of point.

(iii) Divide the result by $\sqrt{(\text{Coeff of } x)^2 + (\text{Coeff of } y)^2}$

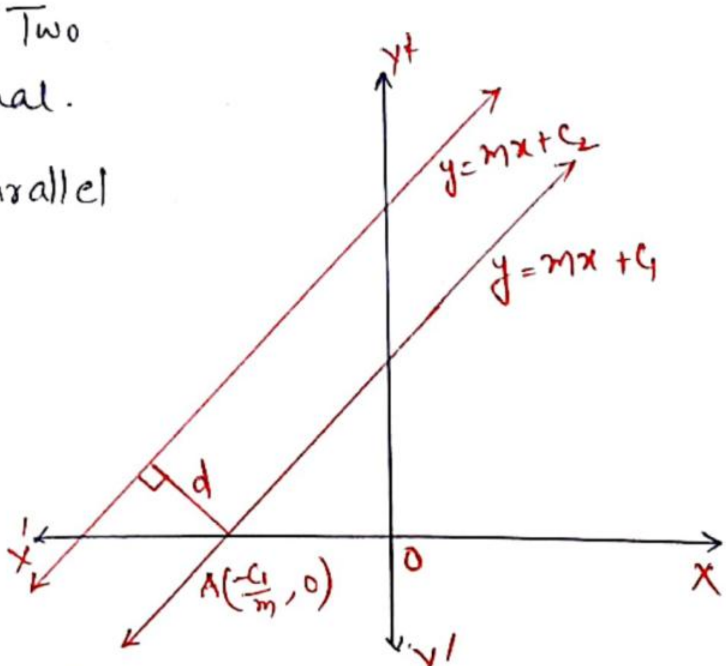
DISTANCE BETWEEN TWO PARALLEL LINES:

We know that slopes of Two parallel lines are equal.

Let the =ns of two parallel lines be

$y = mx + c_1$ — (i)

$y = mx + c_2$ — (ii)



Line (i) will Intersect x-axis at the point $A\left(-\frac{c_1}{m}, 0\right)$ as shown in figure.

Distance between two lines is equal to the length of the tr. from A to the line (2)

$\therefore$ Distance of point $A\left(-\frac{c_1}{m}, 0\right)$ on line (1) from line $y = mx + c_2$ i.e $mx - y + c_2 = 0$ will be

$$d = \frac{\left|m\left(-\frac{c_1}{m}\right) - 0 + c_2\right|}{\sqrt{(m)^2 + (-1)^2}} \quad \text{or} \quad d = \frac{|c_1 - c_2|}{\sqrt{1+m^2}}$$

Note: If the lines are given in General form.

i.e $Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0$

then $$d = \frac{|C_1 - C_2|}{\sqrt{A^2+B^2}}.$$

Q No 4: Find the distance of the point $(-1,1)$ from $12(x+6) = 5(y-2)$

Sol: The given line is $12(x+6) = 5(y-2)$

or $12x + 72 = 5y - 10$

or $12x - 5y + 82 = 0$

$\therefore$ By formula. $$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2+B^2}}$$

The distance of point $(-1,1)$ from $12x - 5y + 82 = 0$

$$d = \frac{|12(-1) - 5(+1) + 82|}{\sqrt{(12)^2 + (-5)^2}} = \frac{|-12-5+82|}{\sqrt{144+25}}$$

$$= \frac{65}{\sqrt{169}} = \frac{65}{13} = 5 \text{ units}.$$

Q No. 5: Find the points on x-axis whose distances from the line $\frac{x}{3}+\frac{y}{4}=1$ are 4 unit.

Sol: The given line is $\frac{x}{3}+\frac{y}{4}=1$ or $4x+3y=12$

or $4x+3y-12=0$

Now Let $A(x_1, 0)$ be any point on x-axis

ATQ.
$$\frac{|4x_1+3(0)-12|}{\sqrt{(4)^2+(3)^2}} = 4$$

$$\Rightarrow \frac{|4x_1-12|}{\sqrt{16+9}} = 4 \Rightarrow \frac{|4x_1-12|}{5} = 4$$

$$\Rightarrow |4x_1-12| = 20 \Rightarrow 4x_1-12 = \pm 20$$

$$\Rightarrow 4x_1-12 = 20 \quad \text{or} \quad 4x_1-12=-20$$

$$\Rightarrow 4x_1 = 32 \quad \text{or} \quad 4x_1 = -8$$

$$\Rightarrow x_1 = 8 \quad \text{or} \quad x_1 = -2$$

$\therefore$ Required pts on x-axis are $(8, 0)$ and $(-2, 0)$

Q No 6: Find the distance between the parallel lines.

(i) $15x+8y-34=0$ and $15x+8y+31=0$

(ii) $l(x+y)+p=0$ and $l(x+y)-r=0$

Sol: (i) The given lines are $15x+8y+34=0$ and $15x+8y+31=0$

$\therefore$ By Formula. $d = \frac{|c_1-c_2|}{\sqrt{A^2+B^2}}$

$$d = \frac{|-34-31|}{\sqrt{(15)^2+(8)^2}} = \frac{|-65|}{\sqrt{225+64}} = \frac{65}{\sqrt{289}} = \frac{65}{17}.$$

(ii) The given lines are $lx+ly+p=0$ and $lx+ly-r=0$

$\therefore$ By formula $d = \frac{|c_1-c_2|}{\sqrt{A^2+B^2}}$

$$d = \frac{|p-(-r)|}{\sqrt{l^2+l^2}} = \frac{|p+r|}{\sqrt{2l^2}} = \frac{1}{\sqrt{2}}\left|\frac{p+r}{l}\right|$$

QNo 7: Find the equation of line parallel to $3x-4y+2=0$ and passing through $(-2,3)$.

Soln: The given line is $3x-4y+2=0$

$$\text{slope of line} = -\frac{\text{Coeff of } x}{\text{Coeff of } y} = -\frac{3}{-4} = \frac{3}{4}.$$

Slope of line parallel to given line $= \frac{3}{4}$.

$\therefore$ =n of reqd. line using point-slope form.

$$y-3 = \frac{3}{4}(x+2) \quad \text{i.e} \quad 4(y-3) = 3(x+2)$$

$$\text{i.e.} \quad 4y-12 = 3x+6 \quad \text{or.} \quad 3x+6-4y+12=0$$

$$\text{or.} \quad 3x-4y+18=0.$$

QNo. 8: Find the equation of line $\perp$r to the line $x-7y+5=0$ and having x-intercept. 3.

Sol.: The given line is $x-7y+5=0$.

$$\text{Slope of line} = -\frac{1}{-7} = \frac{1}{7}.$$

Slope of line perpendicular to given line $= -7$.

$\therefore$ Slope of reqd line is -7 and is passing through the point $(3,0)$.

$\therefore$ =n of line will be

$$y-0 = -7(x-3) \quad \text{i.e} \quad y = -7x+21$$

$$\text{i.e} \quad 7x+y-21=0$$

QNo 9: Find the angles between the lines $\sqrt{3}x+y=1$ and $x+\sqrt{3}y=1$

Sol. Let m_1 be the slope of line $\sqrt{3}x+y=1 \Rightarrow m = \frac{\sqrt{3}}{1} = -\sqrt{3}$.

and m_2 be the slope of line $x+\sqrt{3}y=1 \Rightarrow m_2 = -\frac{1}{\sqrt{3}}$.

Let θ be the angle between the lines

$$\text{Then} \quad \tan\theta = \left|\frac{-\sqrt{3}+\frac{1}{\sqrt{3}}}{1+(-\sqrt{3})(-\frac{1}{\sqrt{3}})}\right| \qquad \left[\because \tan\theta = \left|\frac{m_1-m_2}{1+m_1m_2}\right|\right]$$

$$= \left|\frac{\frac{-3+1}{\sqrt{3}}}{1+1}\right| = \left|\frac{-2}{2\sqrt{3}}\right| = \pm\frac{1}{\sqrt{3}}.$$

$$\Rightarrow \theta = 30^\circ, 150^\circ.$$

Q No 10: The line through the points (h,3) and (4,1) intersects the line $7x-9y-19=0$ at right angle. find the value of h.

Soln: Let m_1 be the slope of line $7x-9y-19=0$

$$\therefore m_1 = -\frac{7}{-9} = \frac{7}{9}$$

and let m_2 be the slope of line passing through (h,3) and (4,1) $\quad \therefore m_2 = \frac{1-3}{4-h} = \frac{-2}{4-h}$.

Since the two lines are $\perp r$

$$\therefore m_1 m_2 = -1$$

$$\Rightarrow \frac{7}{9}\left(\frac{-2}{4-h}\right) = -1$$

$$\Rightarrow \frac{14}{9(4-h)} = 1 \Rightarrow 14 = 9(4-h)$$

$$\Rightarrow 14 = 36-9h$$

$$\Rightarrow 9h = 36-14$$

$$\Rightarrow h = \frac{22}{9}$$

Q No 11: Prove that the line through the point (x_1, y_1) and parallel to $Ax+By+C=0$ is $A(x-x_1)+B(y-y_1)=0$

Soln: slope of line $Ax+By+C=0$ is

$$m = -\frac{A}{B}.$$

$\therefore$ Slope of line $\parallel$ to given line will also be

$$m = -\frac{A}{B}.$$

$\therefore$ By. Point slope formula

Eqn of line passing through (x_1, y_1) having slope $-\frac{A}{B}$ will be

$$y - y_1 = \frac{-A}{B}(x-x_1)$$

ie $B(y-y_1) = -(A)(x-x_1)$

$$\Rightarrow A(x-x_1) + B(y-y_1) = 0$$

Hence proved.

Q.No12: Two lines passing through the point (2,3) intersects each other at an angle of 60°. If slope of one line is 2. Find the slope of other line.

Soln Here $\theta = 60°$. Let $m_1 = 2$ and m_2 be slope of other line

Now $\tan\theta = \pm \frac{m_1 - m_2}{1 + m_1 m_2}$

$\Rightarrow \tan 60° = \pm \frac{2 - m_2}{1 + 2m_2}$

$\Rightarrow \sqrt{3} = \pm \frac{2 - m_2}{1 + 2m_2} \Rightarrow 2 - m_2 = \pm\sqrt{3}(1 + 2m_2)$

$\Rightarrow 2 - m_2 = \sqrt{3}(1 + 2m_2)$ or $2 - m_2 = -\sqrt{3}(1 + 2m_2)$

$\Rightarrow 2 - m_2 = \sqrt{3} + 2\sqrt{3}m_2$ or $2 - m_2 = -\sqrt{3} - 2\sqrt{3}m_2$

$\Rightarrow m_2(1 + 2\sqrt{3}) = 2 - \sqrt{3}$ or $m_2(1 - 2\sqrt{3}) = 2 + \sqrt{3}$

$\Rightarrow m_2 = \frac{2 - \sqrt{3}}{1 + 2\sqrt{3}}$ or $m_2 = \frac{2 + \sqrt{3}}{1 - 2\sqrt{3}}$

$\therefore$ Equation of line through (2, 3) and having slope $\frac{2 - \sqrt{3}}{1 + 2\sqrt{3}}$

is $y - 3 = \frac{2 - \sqrt{3}}{1 + 2\sqrt{3}}(x - 2)$

or $(1 + 2\sqrt{3})y - 3(1 + 2\sqrt{3}) = (2 - \sqrt{3})x - 2(2 - \sqrt{3})$

or $(\sqrt{3} - 2)x + (1 + 2\sqrt{3})y = -1 + 8\sqrt{3}$

and Eqn of line through (2,3) and having slope $\frac{2 + \sqrt{3}}{1 - 2\sqrt{3}}$ is

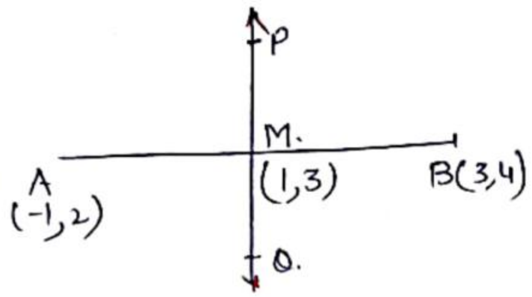
$y - 3 = \frac{2 + \sqrt{3}}{1 - 2\sqrt{3}}(x - 2)$

or. $(1 - 2\sqrt{3})y - 3(1 - 2\sqrt{3}) = (2 + \sqrt{3})x - 2(2 + \sqrt{3})$

or. $(2 + \sqrt{3})x + (2\sqrt{3} - 1)y = 8\sqrt{3} + 1$

Q No 13 Find the =n of right bisector of line segment joining the points (3, 4) and (−1, 2)

Sol. Let AB be the line segment joining the points A(−1, 2) and B(3, 4)



The slope of AB $= \frac{4-2}{3+1} = \frac{2}{4} = \frac{1}{2}.$

Now mid-point M of AB is $M\left(\frac{-1+3}{2}, \frac{2+4}{2}\right)$ ie $M(1, 3)$

Since Right bisector PQ of AB is the line ⊥r to AB and passing through M. ∴ slope of PQ = −2

∴ =n of PQ is $y-3 = -2(x-1)$

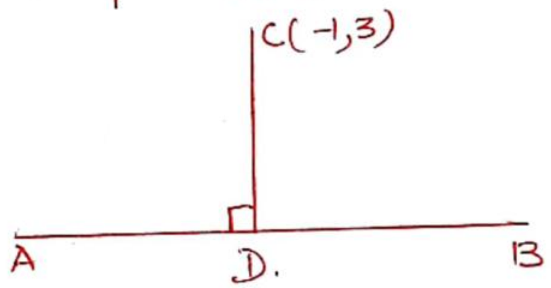
ie $y-3 = -2x+2$

ie $2x+y-5=0$

Q No. 14: Find the coordinates of foot of ⊥r from (−1, 3) to the line $3x-4y+6=0$

Soln. Let the given line is AB

ie $3x-4y+6=0$ ---(1)



Let D be the foot of ⊥r from C(−1, 3) on AB.

The slope of CD $= -\frac{1}{\text{slope of AB}} = -\frac{1}{\frac{-3}{-4}} = -\frac{4}{3}$

∴ Equation of CD is

$$y-3 = -\frac{4}{3}(x+1)$$

ie $4x+3y-5=0$ ---(2)

Solving (1) and (2) for point of intersection D

$$\frac{x}{20+48} = \frac{y}{-64+15} = \frac{1}{9+16}$$

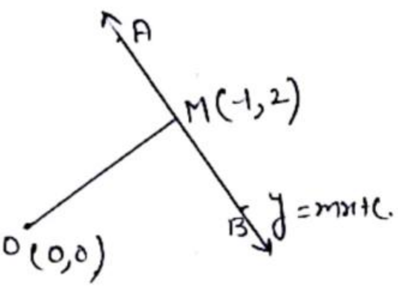
$$\therefore \frac{x}{68} = \frac{y}{-49} = \frac{1}{25}$$

$\Rightarrow x = \frac{65}{25}, \; y = -49/25$

∴ Foot of ⊥r D is $D\left(\frac{68}{25}, -\frac{49}{25}\right)$

Q No 15: The perpendicular from the origin to the line $y=mx+c$ meets it at point $(-1,2)$ find the value of m and c.

Sol.: Let $M(-1,2)$ be the foot of $\perp r$ from the origin $O(0,0)$ to line $y=mx+c$



The slope of $OM = \frac{2-0}{-1-0} = -2$.

The slope of $y=mx+c$

is $m = -\left(\frac{1}{-2}\right) = \frac{1}{2}$ (since $OM \perp AB$)

Since $M(-1,2)$ lies on $y=mx+c$

$\therefore \quad 2 = \frac{1}{2}(-1)+c \Rightarrow c = 2+\frac{1}{2} = \frac{5}{2}$

Q No 16: If p and q are lengths of $\perp r$ from origin to the lines $x\cos\theta - y\sin\theta = k\cos 2\theta$ and $x\sec\theta + y\,\mathrm{cosec}\,\theta = k$ resp. Prove that $p^2+4q^2=k^2$.

Soln.: The eqns of two lines are $x\sec\theta + y\,\mathrm{cosec}\,\theta - k = 0$ —(1)

and $x\cos\theta - y\sin\theta - k\cos 2\theta = 0$ —(2)

Now $p = \perp r$. distance of origin $O(0,0)$ from line (2)

$$= \frac{|-k\cos 2\theta|}{\sqrt{\cos^2\theta+\sin^2\theta}} = |k\cos 2\theta|.$$

and $q = \perp r$ distance of origin $O(0,0)$ from line (1)

$$= \frac{|-k|}{\sqrt{\sec^2\theta+\mathrm{cosec}^2\theta}} = \frac{|k|}{\sqrt{\frac{1}{\cos^2\theta}+\frac{1}{\sin^2\theta}}} = \frac{|k|}{\sqrt{\frac{\sin^2\theta+\cos^2\theta}{\sin^2\theta\cos^2\theta}}}$$

$$= \frac{|k|}{\frac{1}{\sin\theta\cos\theta}} = |k|\sin\theta\cos\theta = \frac{|k|}{2}\,2\sin\theta\cos\theta$$

$$= \frac{1}{2}|k|\sin 2\theta$$

Now $p^2+4q^2 = k^2\cos^2 2\theta + 4\times\frac{k^2}{4}\cdot\sin^2 2\theta$

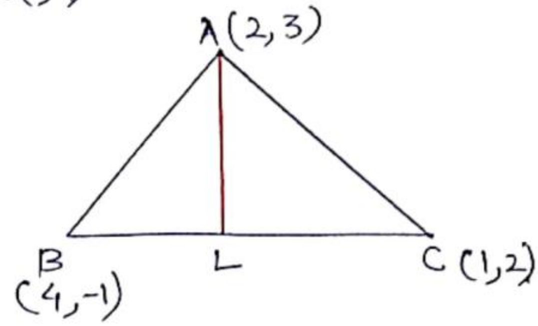
$= k^2(\cos^2 2\theta + \sin^2 2\theta) = k^2$

ie $p^2+4q^2=k^2$.

Q.No. 17: In the triangle ΔABC with vertices $A(2,3)$ $B(4,-1)$ and $C(1,2)$ find $=$n and length of altitude from vertex A.

Sol: Given vertices are $A(2,3)$; $B(4,-1)$; $C(1,2)$

From A draw $AL \perp BC$.



Now Eqn of BC is

$$y-(-1) = \frac{2-(-1)}{1-4}(x-4) \quad \left[\text{Two-point form}\right]$$

or $y+1 = \frac{3}{-3}(x-4)$

or $y+1 = -x+4$ or $x+y-3=0$

$\therefore$ Length of altitude from $A = AL = \frac{|2+3-3|}{\sqrt{(1)^2+(1)^2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$.

Also slope of $AL = -\frac{1}{\text{slope of BC}} = -\frac{1}{-1} = +1$.

$\therefore$ Eqn of AL is

$y-3 = 1(x-2)$ i.e. $y-3 = x-2$ i.e. $x-y+1=0$

Q.No. 18: If p is the length of $\perp$r from origin to the line whose intercepts on axes are a and b then show that $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$

Soln.: The equation of line with intercepts a and b is

$$\frac{x}{a} + \frac{y}{b} = 1 \quad \text{or} \quad \frac{x}{a} + \frac{y}{b} - 1 = 0 \quad \text{---} (1)$$

$\therefore$ Length of Origin $O(0,0)$ from (1) is

$$p = \frac{|0+0-1|}{\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2}} = \frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}}$$

$$\Rightarrow \sqrt{\frac{1}{a^2}+\frac{1}{b^2}} = \frac{1}{p}$$

$$\Rightarrow \frac{1}{a^2}+\frac{1}{b^2} = \frac{1}{p^2}$$

or $$\frac{1}{p^2} = \frac{1}{a^2}+\frac{1}{b^2}.$$