# Heron's Formula

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Perimeter and Semi Perimeter of Triangles

Perimeter and Semi-Perimeter

The concept of perimeter is applicable to many real-life situations. We need to calculate the perimeter: to find the length of barbed wire required for fencing the boundary of a garden; to find the length of the walkway around a swimming pool; and even to find the length of fabric required for stitching around the edge of a blanket or quilt.
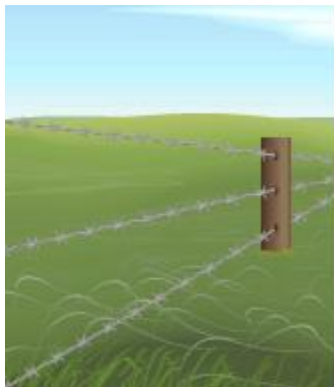
Here, we will introduce a new term called **semi-perimeter** which, as the name itself implies, means half the perimeter.

In this lesson, we will learn how to find the perimeter and semi-perimeter of a triangle.
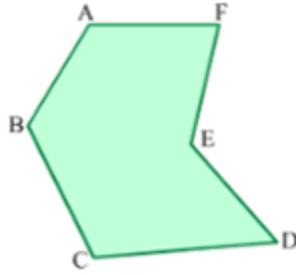
Perimeter of a Triangle

Perimeter is the length of the boundary of a closed figure.

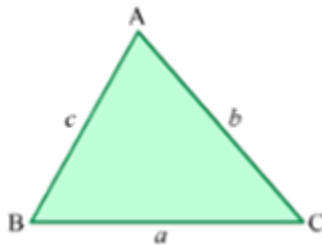**The perimeter of a polygon is the sum of the lengths of all its sides.**

For example, in case of a polygon ABCDEF:

Perimeter of ABCDEF = AB + BC + CD + DE + EF + AF

In case of a triangle ABC, with sides of lengths *a*, *b* and *c* units:

Perimeter of ABC = AB + BC + AC = *a* + *b* + *c*

Did You Know?

Perimeter is the combination of the Greek words *peri*, which means 'around', and *meter*, which means 'measure'.

Solved Examples

Easy

Example 1: Two sides of a triangle measure 5 cm and 7 cm respectively. If the perimeter of the triangle is 20 cm, find the length of the third side.

Solution:

Let the measure of the three sides of the triangle be *a*, *b* and *c*.

We know that:

a = 5 cm

b = 7 cm

Perimeter = 20 cm

We also know that:

Perimeter = *a* + *b* + *c*

⇒ 20 cm = 5 cm + 7 cm + *c*

⇒ *c* = 20 cm − (5 + 7) cm

= 20 cm − 12 cm

= 8 cm

Thus, the length of the third side is 8 cm.

Example 2: **Ashish has a clock with a triangular frame. Its dimensions are 20 cm, 24 cm and 24 cm. One of the edges of the frame is cracked. Ashish puts tape along the boundary twice. What is the length of the tape put around the frame?**

Solution:

Perimeter of the frame = 20 cm + 24 cm + 24 cm

= 68 cm

∴ Length of tape put around the frame = 2 × 68 cm

= 136 cm

= 1 m 36 cm

Medium

Example 1: **The two triangles shown in the figure have the same perimeter. What is the length of QR?**

Equilateral
triangle

Isosceles
triangle
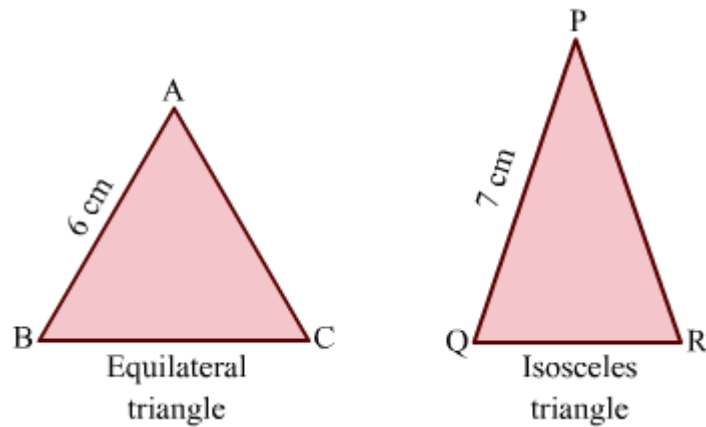
Solution:

We know that the sides of an equilateral triangle are equal in length.

∴ AB = BC = AC = 6 cm

Perimeter of ΔABC = AB + BC + AC

= 6 cm + 6 cm + 6 cm

= 18 cm

We know that two sides of an isosceles triangle are equal.

In ΔPQR, PQ = PR = 7 cm.

Perimeter of ΔPQR = PQ + QR + PR

= 7 cm + QR + 7 cm

= 14 cm + QR

It is given that:

Perimeter of ΔABC = Perimeter of ΔPQR

18 cm = 14 cm + QR

18 cm − 14 cm = QR

∴ QR = 4 cm

Example 2: **Ravi paid Rs 1200 for fencing his triangular park of dimensions 7 m, 8 m and 9 m. Find the cost of fencing per metre.**

Solution:

Perimeter of the triangular park = 7 m + 8 m + 9 m

= 24 m

Cost of fencing 24 m = Rs 1200

∴ Cost of fencing 1 m = $Rs\dfrac{1200}{24}$

= Rs 50

Thus, the cost of fencing is Rs 50 per metre.

Activity

Follow these steps to verify the perimeter of a triangle.

(1) Take a wire of length, say, l cm.

(2) Bend the wire to make a triangle.

(3) Now, measure the length of each side of this triangle.

**(4)** Add the lengths to get the perimeter of the triangle.

You will observe that the perimeter of the triangle is equal to the length of the wire.

Show a wire of length l cm which is bending to form a triangle.

Semi-Perimeter of a Triangle

The word 'semi' means 'half'. So, the semi-perimeter of a triangle is half the perimeter of the triangle.

The perimeter of a triangle with sides a, b and c is $(a + b + c)$.
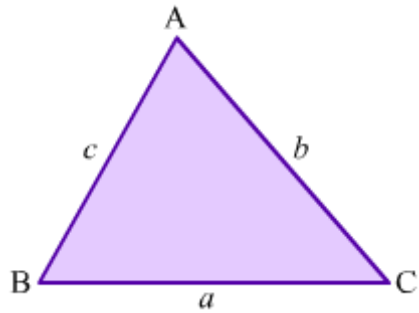
Therefore, the semi-perimeter (s) of this triangle is $\dfrac{a+b+c}{2}$.

The semi-perimeter of a triangle is used for calculating the area of the triangle when the length of the altitude is not known.

Solved Examples

Easy

Example 1: The sides of a triangular field are 4 m, 7 m and 9 m. Calculate its semi-perimeter.

Solution:

Let the sides of the field be a, b and c.

We know that a = 4 m, b = 7 m and c = 9 m.

∴ Semi-perimeter, $s = \dfrac{a+b+c}{2}$

$$= \left(\dfrac{4+7+9}{2}\right) m$$

$$= 10\,m$$

Thus, the semi-perimeter of the field is 10 m.

Example 2: The semi-perimeter of a triangle is 25.5 cm. Two sides of this triangle are 8 cm and 11 cm. What is the length of the third side?

Solution:

Let the sides of the triangle be a, b and c.

It is given that a = 8 cm and b = 11 cm.

Also, semi-perimeter, s = 25.5 cm

We know that:

$$s = \frac{a+b+c}{2}$$

$$\Rightarrow 25.5\,cm = \frac{(8+11)\,cm + c}{2}$$
$$\Rightarrow 51\,cm = 19\,cm + c$$
$$\therefore c = (51-19)\,cm$$
$$= 32\,cm$$

Thus, the length of the third side of the triangle is 32 cm.

Medium

Example 1: The sides of a triangle are in the ratio 5 : 3 : 4 and its semi-perimeter is 48 cm. Find the length of each side.

Solution:

Let the sides of the triangle be a, b and c.

It is given that the sides are in the ratio 5 : 3 : 4.

So, a = 5x, b = 3x and c = 4x.

Also, the semi-perimeter (s) of the triangle is 48 cm.

We know that:

$$s = \frac{a+b+c}{2}$$

$$\Rightarrow 48\,\text{cm} = \frac{5x + 3x + 4x}{2}$$
$$\Rightarrow 48\,\text{cm} \times 2 = 12x$$
$$\Rightarrow x = \frac{96}{12}\,\text{cm} = 8\,\text{cm}$$

Now,

$$5x = 5 \times 8\,\text{cm} = 40\,\text{cm}$$
$$3x = 3 \times 8\,\text{cm} = 24\,\text{cm}$$
$$4x = 4 \times 8\,\text{cm} = 32\,\text{cm}$$

Thus, the sides of the triangle are 40 cm, 24 cm and 32 cm.

Example 2: The pairs of the sides of a triangle add up to 6 cm, 5 cm and 7 cm. What is its semi-perimeter?

Solution:

Let the sides of the triangle be *a*, *b* and *c*.

According to the given condition:

$$a + b = 6\,\text{cm} \quad \text{...(1)}$$
$$b + c = 5\,\text{cm} \quad \text{...(2)}$$
$$c + a = 7\,\text{cm} \quad \text{...(3)}$$

On adding equations (1), (2) and (3), we get:

$$(a + b) + (b + c) + (c + a) = (6 + 5 + 7)\,\text{cm}$$
$$\Rightarrow 2(a + b + c) = 18\,\text{cm}$$
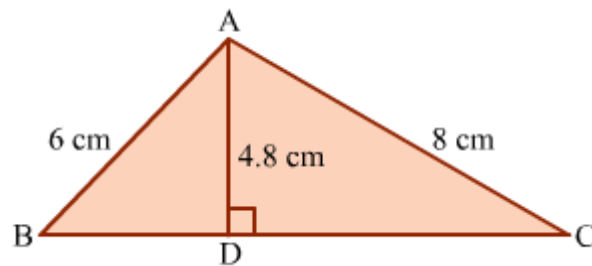$$\Rightarrow a + b + c = 9\,\text{cm}$$

$$\text{Semi-perimeter, } s = \frac{a + b + c}{2}$$
$$= \frac{9}{2}\,\text{cm}$$
$$= 4.5\,\text{cm}$$

Thus, the semi-perimeter of the triangle is 4.5 cm.

Hard

Example 1: In a ΔABC, AC is 8 cm, AB is 6 cm and the length of the perpendicular drawn from A to D is 4.8 cm. Find the semi-perimeter of ΔABC if its area is 24 cm².
Solution:

Area of ΔABC = 24 cm²

$$\Rightarrow \frac{1}{2} \times BC \times AD = 24\,cm^2$$

$$\Rightarrow \frac{1}{2} \times BC \times 4.8\,cm = 24\,cm^2$$

$$\therefore BC = 10\,cm$$

Semi-perimeter of **ΔABC,** $\quad s = \dfrac{AB + BC + AC}{2}$

$$\Rightarrow s = \left(\frac{6 + 8 + 10}{2} \right) cm$$

$$= 12\,cm$$

Areas of Triangles Using Heron's Formula

Area of a Triangle

Kishan has a triangular field with sides 30 m, 30 m and 20 m. Can we find the area of his field using this information?

Yes, we can. The area of any triangle, when all its sides are known, can be calculated using Heron's formula.

We can use Heron's formula:

• To find the area of a triangle when the lengths of all its sides are given.

• To find the area of a quadrilateral by dividing it into two triangles.

• To calculate the area of a cyclic quadrilateral when the lengths of all its sides are given.

In this lesson, we will learn how to find the area of a triangle using Heron's formula.

Heron's Formula

Heron's formula can be used to find the area of any triangle in terms of the lengths of its sides. Let *a, b* and *c* denote the lengths of the sides of a ΔABC.
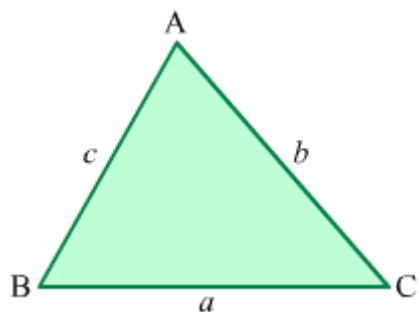
Perimeter of ΔABC = *a* + *b* + *c*

⇒ Semi-perimeter (*s*) of ΔABC = $\dfrac{a+b+c}{2}$

∴ Area of ΔABC = $\sqrt{s(s-a)(s-b)(s-c)}$

Concept Builder

Area of a triangle: If any side of a triangle is taken as the base and a perpendicular is drawn to it from the opposite vertex, then the area of the triangle is given as follows:

$\text{Area} = \dfrac{1}{2} \times \text{Base} \times \text{Height}$

Know Your Scientist

Heron
Born:10 AD **Died**:75 AD Heron (or Hero) of Alexandria, Greece was a mathematician and engineer. The proof of the formula named after him can be found in his book *Metrica*, written in 60 AD. Heron has written so much on mathematics and physics that he can be described as an 'encyclopaedic writer' in these fields.

Solved Examples

Easy

Example 1: **Find the area of a triangle with sides 12 cm, 16 cm and 20 cm.**

Solution:

Let the sides of the triangle be a, b and c.

In this case, a = 12 cm, b = 16 cm and c = 20 cm.

Semi-perimeter (s) of the triangle $= \dfrac{a+b+c}{2}$

$$= \left(\dfrac{12+16+20}{2}\right) \text{cm}$$
$$= 24 \text{ cm}$$

Using Heron's formula,

Area of the triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{24(24-12)(24-16)(24-20)} \text{ cm}^2$$

$= \sqrt{9216}$ cm²

$= 96$ cm²

Example 2: **Find the area of the triangle shown in the figure using the formula** $\dfrac{1}{2} \times \text{Base} \times \text{Height}$ **and Heron's formula. Compare the results obtained.**
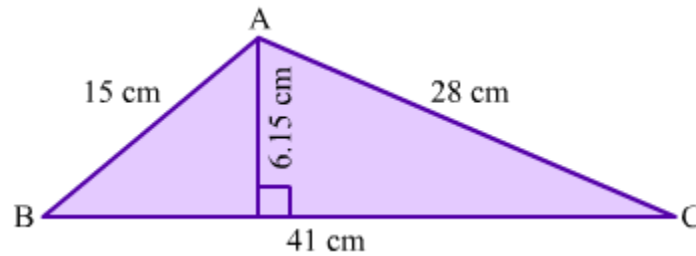
Solution:

Let the sides of ΔABC be *a*, *b* and *c*.

In this case, *a* = 15 cm, *b* = 41 cm and *c* = 28 cm.

Semi-perimeter (*s*) of ΔABC $= \dfrac{a+b+c}{2}$

$= \left(\dfrac{15+41+28}{2} \right)$ cm

$= 42$ cm

Using Heron's formula,

Area of ΔABC $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{42(42-15)(42-41)(42-28)}$ cm²

$= \sqrt{42 \times 27 \times 1 \times 14}$ cm²

$= \sqrt{15876}$ cm²

$= 126$ cm²

Now, we will calculate the area of ΔABC by using the formula: $\frac{1}{2} \times \text{Base} \times \text{Height}$

In this case,

Base = 41 cm

Height = 6.15 cm

$\therefore \text{Area of } \Delta ABC = \frac{1}{2} \times 41 \times 6.15 \text{ cm}^2$

= 126.075 cm² ≈ 126 cm²

Thus, the area of the triangle is found to be the same on using both the methods.

Medium

Example 1: Two sides of a triangular field are 8 m and 11 m, and the semi-perimeter is 16 m. Find the area of the field.

Solution:

Let the sides of the field be *a*, *b* and *c*.

In this case, *a* = 8 m and *b* = 11 m.

It is given that the semi-perimeter (*s*) of the field is 16 m.

We know that $s = \frac{a+b+c}{2}$

$\Rightarrow 16\,\text{m} = \frac{(8+11)\,\text{m}+c}{2}$
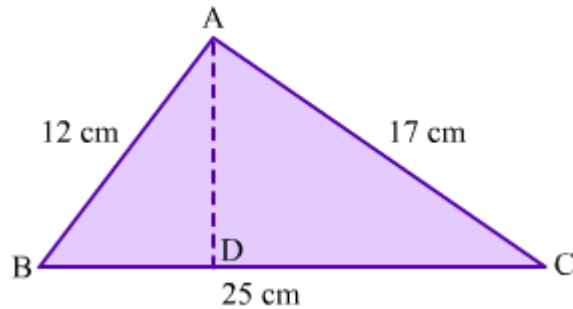
$\Rightarrow 32\,\text{m} = 19\,\text{m}+c$

$\therefore c = (32-19)\,\text{m} = 13\,\text{m}$

Using Heron's formula,

Area of the field = $\sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{16(16-8)(16-11)(16-13)} \, \text{m}^2$$
$$= \sqrt{16 \times 8 \times 5 \times 3} \, \text{m}^2$$
$$= \sqrt{1920} \, \text{m}^2$$
$$= 8\sqrt{30} \, \text{m}^2$$

Example 2: What is the height of the triangle shown in the figure?

Solution:

Let the sides of ΔABC be a, b and c.

In this case, a = 12 cm, b = 25 cm and c = 17 cm

Semi-perimeter (s) of ΔABC $= \dfrac{a+b+c}{2}$

$$= \left(\dfrac{12+25+17}{2} \right) \text{cm}$$
$$= 27 \text{cm}$$

Using Heron's formula,

Area of ΔABC $= \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{27(27-12)(27-25)(27-17)} \, \text{cm}^2$$
$$= \sqrt{27 \times 15 \times 2 \times 10} \, \text{cm}^2$$
$$= \sqrt{8100} \, \text{cm}^2$$
$$= 90 \text{cm}^2$$

We know that:

$$\text{Area of } \triangle ABC = \frac{1}{2} \times \text{Base} \times \text{Height}$$

$$\Rightarrow 90 \, cm^2 = \frac{1}{2} \times 25 \, cm \times AD$$

$$\Rightarrow AD = \frac{90 \times 2}{25} \, cm = 7.2 \, cm$$

Thus, the height of ΔABC is 7.2 cm.

Hard

Example 1: A floor is made up of 20 triangular tiles, each with sides 40 cm, 24 cm and 32 cm. Find the cost of polishing the floor at the rate of 25 paise per cm².

Solution:

Let the sides of each tile be *a*, *b* and *c*.

In this case, *a* = 40 cm, *b* = 24 cm and *c* = 32 cm.

Semi-perimeter (*s*) of each tile $= \dfrac{a+b+c}{2}$

$$= \left(\frac{40+24+32}{2} \right) cm$$

$$= 48 \, cm$$

Using Heron's formula,

Area of each tile $= \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{48(48-40)(48-24)(48-32)} \, cm^2$$
$$= \sqrt{48 \times 8 \times 24 \times 16} \, cm^2$$
$$= \sqrt{147456} \, cm^2$$
$$= 384 cm^2$$

∴ Area of 20 tiles = (384 × 20) cm² = 7680 cm²

Thus, the area of the floor is 7680 cm².

Now, cost of polishing 1 cm^2 = 25 paise = Rs 0.25

∴ Total cost of polishing the floor = Rs (7680 × 0.25) = Rs 1920

Example 2: The difference between the semi-perimeter of ΔABC and each of its sides are 8 cm, 7 cm and 5 cm. What is the area of ΔABC?

Solution:

Let the sides of ΔABC be *a*, *b* and *c*.

Semi perimeter (*s*) of ΔABC $= \dfrac{a+b+c}{2}$

It is given that:

$$s-a=8\,\text{cm} \quad \text{...(1)}$$
$$s-b=7\,\text{cm} \quad \text{...(2)}$$
$$s-c=5\,\text{cm} \quad \text{...(3)}$$

On adding equations (1), (2) and (3), we get:

$$3s-(a+b+c)=20\,\text{cm}$$
$$\Rightarrow 3s-2s=20\,\text{cm} \qquad \left(\because a+b+c=2s\right)$$
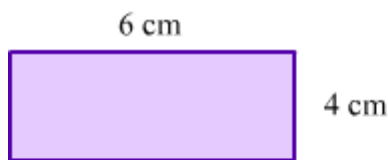$$\Rightarrow s=20\,\text{cm}$$

Using Heron's formula,

$$\begin{aligned}
\text{Area of } \Delta ABC &= \sqrt{s(s-a)(s-b)(s-c)}\\
&= \sqrt{20\times8\times7\times5}\,\text{cm}^2\\
&= \sqrt{5600}\,\text{cm}^2\\
&= 20\sqrt{14}\,\text{cm}^2
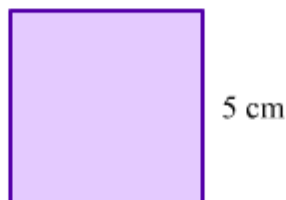\end{aligned}$$

Areas of Quadrilaterals Using Heron's Formula

Area of Quadrilaterals

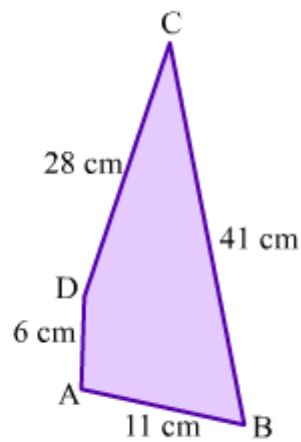Take a look at these figures.

(i)   **(ii)**   **(iii)**

All of them are quadrilaterals. Now, we know how to find the areas of some basic quadrilaterals such as a rectangle (figure i), a square (figure ii), a parallelogram and a rhombus. However, a quadrilateral such as figure iii does not exhibit any special property. To find the area of such a quadrilateral, we make use of the concept that the diagonal of a quadrilateral bisects it into two triangles.

In this lesson, we will learn how to find the area of a quadrilateral by adding the areas of the two triangles formed by its diagonal.

Whiz Kid

The area of a cyclic quadrilateral with sides *a*, *b*, *c* and *d* can be given as

$$\sqrt{(s-a)(s-b)(s-c)(s-d)} \text{ , where } s = \frac{a+b+c+d}{2}.$$

The above is known as **Brahmagupta's generalization**, named after the Indian mathematician Brahmagupta. Heron's formula has been generalized from this.

Know Your Scientist

Brahmagupta

Born:598 AD **Died**:668 AD Brahmagupta was an Indian mathematician and astronomer. His famous book *Brahma-sphuta-sidd'hanta* is mainly a work on astronomy; however, two of its chapters deal with mathematics. Included in this work is the method for finding the area of a cyclic quadrilateral when the lengths of its four sides are given.

$$\text{Area of a cyclic quadrilateral} = \sqrt{(s-a)(s-b)(s-c)(s-d)}$$

As the above only deals with cyclic quadrilaterals, it is called Brahmagupta's generalization, rather than Brahmagupta's formula.
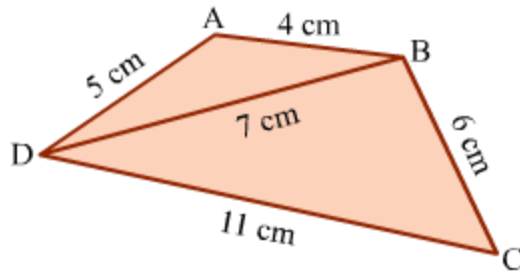
Solved Examples

Easy

Example 1: **Find the area of a quadrilateral ABCD, where AB = 4 cm, BC = 6 cm, CD = 11 cm,**
AD = 5 cm and BD = 7 cm.

Solution:

The quadrilateral ABCD with the given sides is shown in the figure.

The diagonal BD divides the quadrilateral into ΔABD and ΔBCD.

Let the sides of both the triangles be *a, b* and *c.*

In case of ΔABD, *a* = 4 cm, *b* = 7 cm and *c* = 5 cm.

Semi-perimeter (s) of ΔABD $= \dfrac{a+b+c}{2}$

$$= \left(\dfrac{4+7+5}{2}\right) cm$$

$$= 8\,cm$$

Using Heron's formula,

Area of ΔABD $= \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{8(8-4)(8-7)(8-5)}\,cm^2$$
$$= \sqrt{8\times4\times1\times3}\,cm^2$$
$$= 4\sqrt{6}\,cm^2$$

In case of ΔBCD, *a* = 6 cm, *b* = 7 cm and *c* = 11 cm.

Semi-perimeter (s) of ΔBCD $= \dfrac{a+b+c}{2}$

$$= \left(\dfrac{6+7+11}{2}\right) cm$$

$$= 12\,cm$$

Using Heron's formula,

Area of ΔBCD $= \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{12(12-6)(12-7)(12-11)} \text{ cm}^2$$
$$= \sqrt{12 \times 6 \times 5 \times 1} \text{ cm}^2$$
$$= 6\sqrt{10} \text{ cm}^2$$

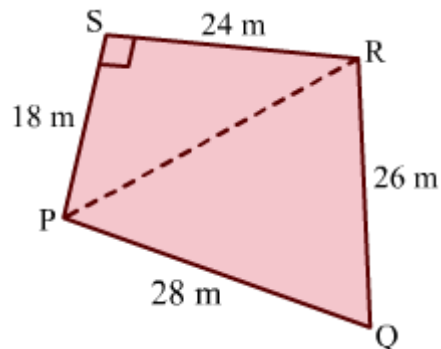∴ **Area of quadrilateral ABCD = Area of ΔABD + Area of ΔBCD**

$$= (4\sqrt{6} + 6\sqrt{10}) \text{ cm}^2$$

Medium

**Example 1: Find the area of a quadrilateral PQRS, where PS = 18 m, RS = 24 m, QR = 26 m,
PQ = 28 m and ∠S = 90°.**

Solution:

The quadrilateral PQRS with the given sides is shown in the figure.

The diagonal PR divides the quadrilateral into ΔPSR and ΔPQR.

In the right-angled ΔPSR, PS = 18 m, RS = 24 m and ∠S = 90°.

By applying Pythagoras theorem, we obtain

$$(PR)^2 = (PS)^2 + (RS)^2$$
$$= [(18)^2 + (24)^2] \text{ m}^2$$
$$= (324 + 576) \text{ m}^2$$
$$= 900 \text{ m}^2$$
$$\therefore PR = \sqrt{900} \text{ m}$$
$$= 30 \text{ m}$$

Now, area of ΔPSR = $\dfrac{1}{2} \times \text{Base} \times \text{Height}$

$= \dfrac{1}{2} \times PS \times RS$

$= \dfrac{1}{2} \times 18 \times 24 \, m^2$

$= 216 \, m^2$

Let the sides of ΔPQR be a, b and c, where a = 28 m, b = 26 m and c = 30 m.

Semi-perimeter (s) of ΔPQR $= \dfrac{a+b+c}{2}$

$= \left(\dfrac{28+26+30}{2} \right) m$

$= 42 \, m$

Using Heron's formula,

Area of ΔPQR = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{42(42-28)(42-26)(42-30)} \, m^2$

$= \sqrt{42 \times 14 \times 16 \times 12} \, m^2$

$= \sqrt{112896} \, m^2$

$= 336 \, m^2$

∴ Area of quadrilateral PQRS = Area of ΔPSR + Area of ΔPQR

= (216 + 336) m²

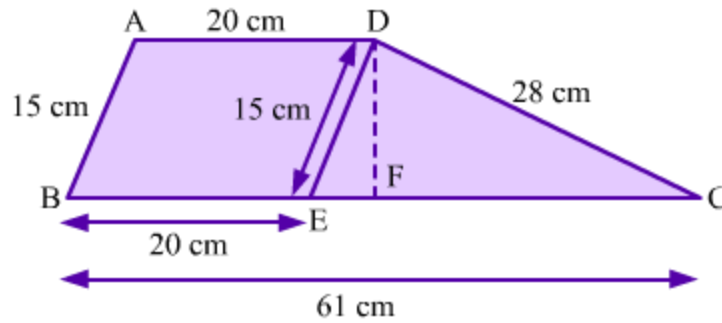= 552 m²

Hard

Example 1: What is the area of a trapezium whose parallel sides are 20 cm and 61 cm, and non-parallel sides are 15 cm and 28 cm?

Solution:

The trapezium ABCD with the given sides is shown in the figure. Also shown is a line segment parallel to AB from D, cutting BC at E.

Clearly, ABED is a parallelogram as AB || DE and AD || BE.

∴AD = BE = 20 cm and AB = DE = 15 cm (∵ opposite sides of a parallelogram are equal)

Now, EC = BC − BE = (61 − 20) cm = 41 cm

Let the sides of ΔDEC be a, b and c, where a = 15 cm, b = 28 cm and c = 41 cm.

Semi-perimeter (s) of ΔDEC $= \dfrac{a+b+c}{2}$

$$= \left(\frac{15+28+41}{2}\right) cm$$

$$= 42\, cm$$

Using Heron's formula,

Area of ΔDEC $= \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{42(42-15)(42-28)(42-41)}\ cm^2$$
$$= \sqrt{42\times27\times14\times1}\ cm^2$$
$$= \sqrt{15876}\ cm^2$$
$$= 126\, cm^2$$

Since DF is the height of ΔDEC, we can give the area of the triangle as: $\dfrac{1}{2} \times EC \times DF$

$\Rightarrow 126$ cm² $= \dfrac{1}{2} \times 41\, cm \times DF$

$$\Rightarrow \therefore DF = \frac{126 \times 2}{41} cm = 6.15 cm$$

Thus, the distance between the parallel sides AD and BC is 6.15 cm.

Now, area of parallelogram ABED = Base × Height

= (20 × 6.15) cm²

= 123 cm²

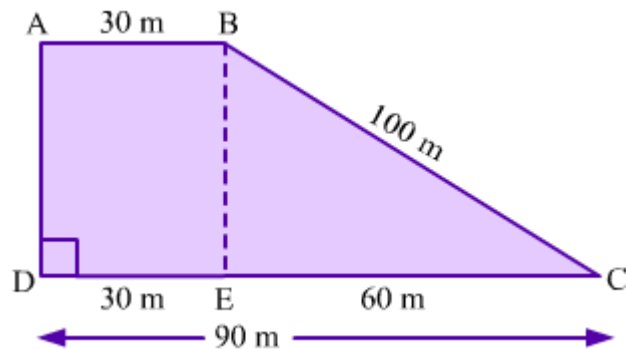∴ Area of trapezium ABCD = Area of parallelogram ABED + Area of ΔDEC

= (123 + 126) cm²

= 249 cm²

Example 2: A field is shaped like a trapezium, as shown in the figure. Find the cost of ploughing this field if it costs Rs 4 to plough a square metre.

Solution:

Area of trapezium ABCD = Area of rectangle ABED **+ Area of** ΔBEC

In right angled ΔBEC,

$$BE^2 + CE^2 = BC^2$$
$$\Rightarrow BE^2 = BC^2 - CE^2$$
$$= [(100)^2 - (60)^2] m^2$$
$$= 6400 m^2$$
$$\therefore BE = 80 m$$

Let the sides of ΔBEC be *a*, *b* and *c*, where *a* = 80 m, *b* = 60 m and *c* = 100 m.

Semi-perimeter (s) of ΔBEC $= \dfrac{a+b+c}{2}$

$$= \left(\dfrac{80+60+100}{2}\right) \text{m}$$

$$= 120 \text{ m}$$

Using Heron's formula,

Area of ΔBEC $= \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{120(120-80)(120-60)(120-100)}\ \text{m}^2$$
$$= \sqrt{120 \times 40 \times 60 \times 20}\ \text{m}^2$$
$$= \sqrt{5760000}\ \text{m}^2$$
$$= 2400\,\text{m}^2$$

Now, area of rectangle ABED $= 30\,\text{m} \times 80\,\text{m} = 2400\,\text{m}^2$

∴ **Area of trapezium ABCD =** Area of rectangle ABED **+ Area of** ΔBEC

= (2400 + 2400) m²

= 4800 m²

Cost of ploughing 1 m² = Rs 4

∴ Cost of ploughing 4800 m² = Rs (4 × 4800) = Rs 19200