lecture 8
03/04/19

$$\left[\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho . u) + \frac{\partial}{\partial y}(\rho . v) + \frac{\partial}{\partial z}(\rho . w) = 0\right]$$

Assumption →

1. 3D - steady flow $\frac{\partial \rho}{\partial t} = 0$

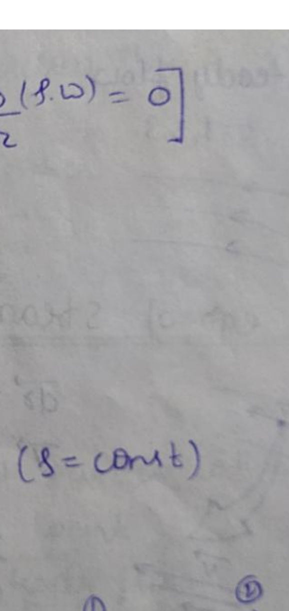
$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$$

2. 3D - steady incompressible flow (ρ = const)

$$\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = 0$$

3. 2D - steady incompressible flow

$$\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0$$



$$(\dot m_{in} - \dot m_{out})_x = \dot m_{storage}$$

$$\dot m_x - \left(\dot m_x + \frac{\partial \dot m_x}{\partial x} . dx\right) = \cancel{\frac{\partial \rho}{\partial t}}^{\,0} \, dV$$

(For steady flow)

$$-\frac{\partial \dot M}{\partial x} . dx = 0$$

$d\dot m = 0$. int it $\dot m$ = constant

$\rho A V$ = const

$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$ if $\rho_1 = \rho_2$

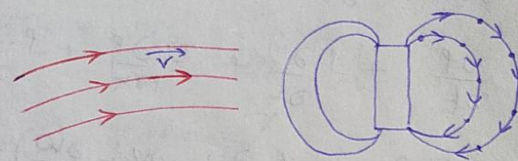
$$\boxed{A_1V_1 = A_2V_2}$$

Flow Representation :

→ Stream line

→ Path line

→ Streak line

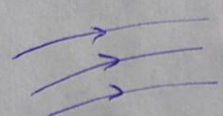


① S.L → Stream line is an imaginary curve drawn through a flowing field in such a way that a tangential to it at any point gives the dirⁿ of the $\underbrace{\text{instantaneous velocity}}_{\text{(instantaneous)}}$ of flow at that point

② These lines never cut each other in a flow

For steady flow

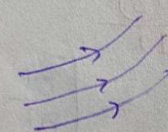
At . t = 1, 2, 3



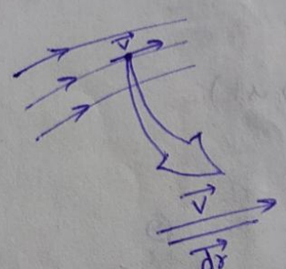
For unsteady flow

At (t) ~~+dt~~

At t+dt



the eqⁿ of stream line :



$\vec{dr} = dx\hat{i} + dy\hat{j} + dz\hat{k}$

$\vec{V} = U\hat{i} + V\hat{j} + W\hat{k}$

$\vec{dr} \times \vec{V} = 0$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ dx & dy & dz \\ U & V & W \end{vmatrix} = 0$$

$\hat{i}(Wdy - Vdz) - \hat{j}(Wdx - Udz) + \hat{k}(Vdx - Udy) = 0$

$Wdy - Vdz = 0$, $Wdx - Udz = 0$ and $Vdx - Udy = 0$

$$\boxed{\frac{dx}{U} = \frac{dy}{V} = \frac{dz}{W}}$$

[gnt it eqⁿ of stream line]

① For 2D incompressible flow the velocity field is $2x\hat{i} - 2y\hat{j}$ determine the eqⁿ of stream line passing from point (1,1)

$\vec{V} = 2x\hat{i} - 2y\hat{j}$

Streamline flow eqn

$$\frac{dx}{u} = \frac{dy}{v}$$

$$\frac{dx}{2x} = -\frac{dy}{-2y}$$

$$\int \frac{dx}{x} = -\int \frac{dy}{y},$$

$\log x = -\log(y) + \log(c)$

$\log x + \log(y) = \log c$

$\boxed{xy = c}$

at $(1,1)$ $c = 1$

then $\boxed{xy = 1}$

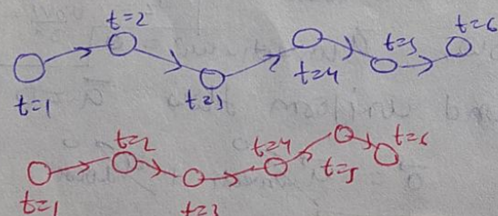
Path line → (lagrangian approach)

① It represent the actual path travelled by a fluid particles over a period of time

② These lines make cut one another

Particle-①

Particle-②



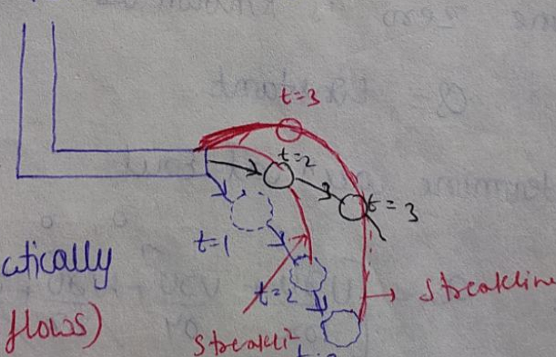
③ Streakline → It is locus of fluid particle at a distance, of which have cross through the same point

For unsteady flow:

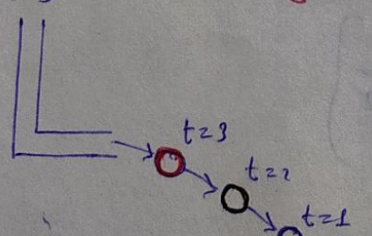
O⇒ Particle - I

O→ Particle - 2

O→ Particle - ③



Note In physical, all the line r different bt mathematically they may be same (steady flows)



Stream line = dirn of motion (velocity)

Pathline = motion of a particular fluid Particle

Streak line   Identification of location of different fluid Particle

Acceleration

$\vec a$ vector quantity

$\vec a = a_x\hat i + a_y\hat j + a_z\hat k$ $|\vec a| = \sqrt{a_x^2 + a_y^2 + a_z^2}$ units

$\vec a = \frac{d\vec V}{dt}$

$\partial\vec V = \frac{\partial\vec V}{\partial x}.dx + \frac{\partial\vec V}{\partial y}.dy + \frac{\partial\vec V}{\partial z}.dz + \frac{\partial\vec V}{\partial t}.dt$

$\vec V = f^n(x, y, z, t)$

$\frac{\partial\vec V}{\partial t} = \frac{\partial\vec V}{\partial x}.\frac{dx}{dt} + \frac{\partial\vec V}{\partial y}.\frac{dy}{dt} + \frac{\partial\vec V}{\partial z}.\frac{dz}{dt} + \frac{\partial\vec V}{\partial t}$

$\vec V = U\hat i + V\hat j + W\hat k$

$\left[\vec a = U\frac{\partial\vec V}{\partial x} + V\frac{\partial\vec V}{\partial y} + W\frac{\partial\vec V}{\partial z} + \frac{\partial\vec V}{\partial t}\right]$

$U\frac{\partial\vec V}{\partial x} + V\frac{\partial\vec V}{\partial y} + W\frac{\partial\vec V}{\partial z}$ = convective acceleration

$\frac{\partial\vec V}{\partial t}$ = temporal / local acc^n

$a_x = U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial y} + W\frac{\partial U}{\partial z} + \frac{\partial U}{\partial t} = \emptyset$

$a_y = U\frac{\partial V}{\partial x} + V\frac{\partial V}{\partial y} + W\frac{\partial V}{\partial z} + \frac{\partial V}{\partial t}$

$a_z = U\frac{\partial W}{\partial x} + V\frac{\partial W}{\partial y} + W\frac{\partial W}{\partial z} + \frac{\partial W}{\partial t}$

Theory **

1. For steady flow, $\vec a_{local} = 0$
2. For uniform flow, $\vec a_{convective\ acc^n} = 0$
3. For steady and uniform flow $\vec a = 0$ $[\vec a_{local} = 0$ and $\vec a_{conve} = 0]$

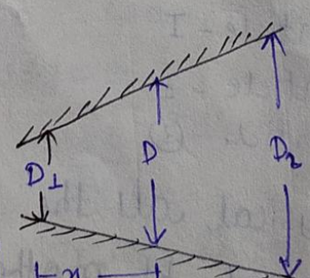
$\vec a = \cancel{\vec a_{convecti}}^{\,0} + \cancel{a_{local}}^{\,0}$

$\boxed{\vec a = 0}$

⇒ In a fluid flow system where the velocity of net flow become zero is known as stagnation flow.

Pb-(18) Q = constant

Determine acc^n at Exit



Sol^n = $a_x = \left[U\frac{\partial U}{\partial x} + \cancel{V\frac{\partial U}{\partial y}}^{\,0} + \cancel{W\frac{\partial U}{\partial z}}^{\,0} + \cancel{\frac{\partial U}{\partial t}}^{\,0}\right]$

$a_x = U.\frac{\partial U}{\partial x}$ $\left\{ U = \frac{Q}{\frac{\pi}{4}D^2} \right\}$

$$a_x = \frac{Q}{\frac{\pi}{4}D^2}\cdot\frac{\partial}{\partial x}\cdot\frac{Q}{\frac{\pi}{4}D^2} = \frac{Q^2}{\frac{\pi^2}{16}\times D^2}\,\frac{\partial}{\partial x}\left(\frac{1}{D^2}\right)$$

(by geometory) $a_x = \frac{D_2-D_1}{L} = \frac{D-D_1}{x}$ $\quad D = D_1 + B.x$

B (assume)

$$a_x = \frac{16}{\pi^2}\frac{Q^2}{(D_1+B.x)^2}\,\frac{\partial}{\partial x}\frac{1}{(D_1+Bx)^2}$$

$$a_x = \frac{16}{\pi^2}\frac{Q^2}{(D_1+B.x)^2}\times\frac{(-2)}{(D_1+Bx)^2}\cdot\times B$$

$$a_x = \frac{-32\,Q^2\,(D_2-D_1)}{\pi^2\,(D_1+B.x)^5.\,L}$$

At exit $x = L$

$$a_x = \frac{-32\,Q^2\,(D_2-D_1)}{\pi^2(D_1+L.B)^5.\,L} = \frac{-32\,Q^2\,(D_2-D_1)}{\pi^2\times\left[D_1+L\left(\frac{D_2-D_1}{L}\right)\right]^5}$$

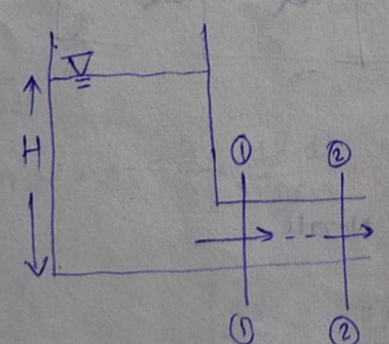
$$a_x = \frac{-32\,Q^2\,(D_2-D_1)}{\pi^2.\cancel{b}\times(\cancel{D_1}+D_2-\cancel{D_1})^5.\,L} = \frac{-32\,Q^2\,(D_2-D_1)}{\pi^2.\,L\,(D_2)^5.\cancel{L}}$$

$$a_x = \frac{-32\,Q^2\,(r_2-r_1)2}{\pi^2.\cancel{2}\,(r_2)^5.\,L\times 8} = \frac{32^2\,Q^2\,(r_1-r_2)2}{\pi^2\times L.\,(r_2)^5\times\cancel{16}}$$

$$a_x = \frac{2\,Q^2\,(r_1-r_2)^2}{\pi^2.\,L\,(r_2)^5.\cancel{2}} = \boxed{\frac{2\,Q^2\,(r_1-R_2)}{\pi^2.\,L.\,R_2^5}}$$

$$\frac{\left(\frac{m^3}{s}\right)^2(m)}{m.\,m^5} \Rightarrow \frac{m}{s^2}$$

Case-①



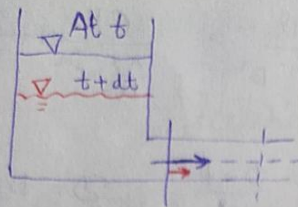
$A_1V_1 \quad A_2V_2 \qquad (V_1 = V_2)$

$(A_1 = A_2)$

$$\vec{a} = \cancel{\vec{a}_{convective}}^{\,0} + \cancel{\vec{a}_{local}}^{\,0}$$

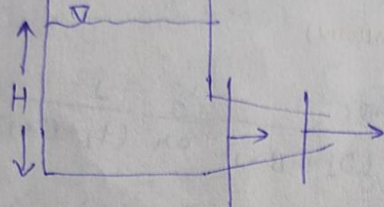
$$\boxed{\vec{a} = 0}$$

Case-②



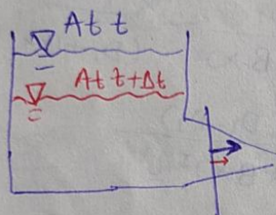
$\vec{a} = \cancel{\vec{a}_{conve}}^{0} + \vec{a}_{loc}$

Case-③



$\vec{a} = \vec{a}_{convec} + \cancel{\vec{a}_{loca}}^{0}$

Case-④

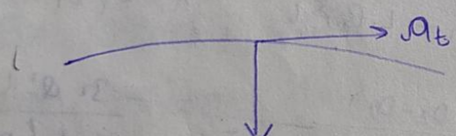


$\vec{a} = \vec{a}_{convec} + \vec{a}_{loc}$

Motion on curved path:

a_t (tangential $A c c^n$) ⇒ due to change in magnitude

a_N (Normal acc^n) ⇒ due of change in dir^n

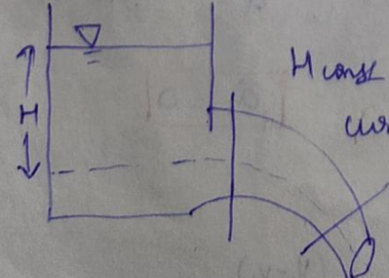


$a_N = \frac{V^2}{r}$

r = Radius of curvature

S.No.			H const	H varying
1	convective	tangential	~~H const~~ ✓✓	~~H varying~~ ✓✓
		Normal	✓✓	✓✓
2	Local	tangential	$\propto$	✓✓
		Normal	$\propto$	$\propto$

Case ⑤

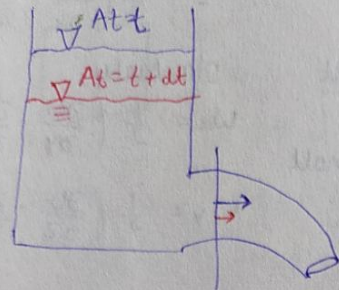


H const

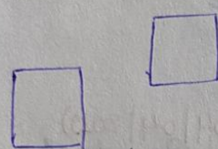
curve Nozzle

Case - 6

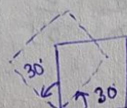
[If dimension remain constant then tange (x)...]



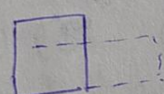
Rotational Parameter in flow →



Translation



(Pure Rotation) $\left(\frac{30+30}{2}\right)$



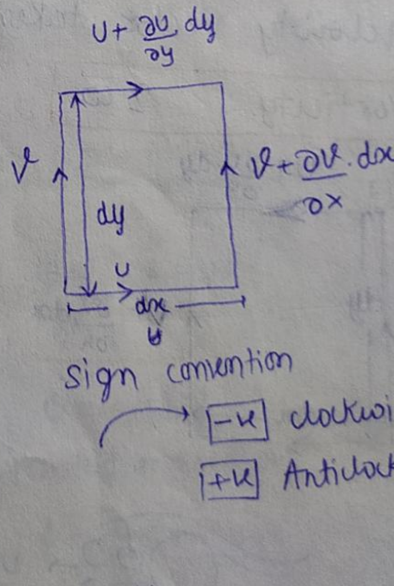
linear deformation

Shear deformation

angular velocity

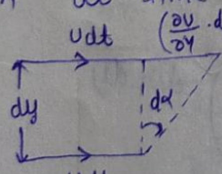
Angular Velocity ($\vec{\omega}$)

In x-y plane $\quad U + \frac{\partial U}{\partial y} . dy$



contribution of U - velocity in rotation :-

In dt time interval



sign convention

$\boxed{-ve}$ clockwise

$\boxed{+ve}$ Anticlockwise

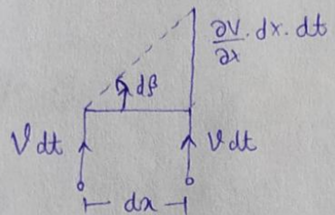
$\tan d\alpha = \dfrac{-\frac{\partial U}{\partial y} . \cancel{dy} . dt}{\cancel{dy}}$

{ $d\alpha$ ⇒ very-2 small
$\tan d\alpha \approx d\alpha$

$$\boxed{\frac{d\alpha}{dt} = -\frac{\partial U}{\partial y}}$$

Contribution of v-velocity in Rotation →

In dt time interval -



$\tan d\beta = \frac{\partial v}{\partial x}.\frac{\cancel{dx}.dt}{\cancel{dx}}$

{ $d\beta$ = very-2 small

$$\boxed{\frac{d\beta}{dt} = \frac{\partial v}{\partial x}}$$

$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$

similarly,

$\omega_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)$

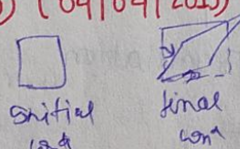
$\omega_y = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)$

$\vec{\omega} = \omega_x\hat{i} + \omega_y\hat{j} + \omega_z\hat{k}$

$\vec{\omega} = \frac{1}{2}[\vec{\nabla}\times\vec{V}] = \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ & & \end{vmatrix}$

Rate of shear strain :- (lecture 9) (04/04/2019)

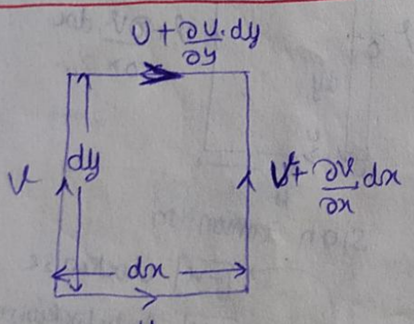
In x-y plane $= \frac{1}{2}\left[\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right]$



circulation: (Γ) [capital Gamma]

It is defined as the line integration of velocity vector taken along a close loop

Vorticity : $2\vec{\omega}$

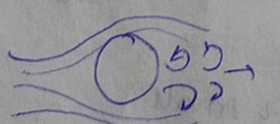


$\Gamma = \oint \vec{V}.\vec{dr}$

In x-y Plane

$\Gamma = u\,dx + \left(v + \frac{\partial v}{\partial x}.dx\right)dy - \left(u + \frac{\partial u}{\partial y}.dy\right)dx - (v\,dy)$

$$\boxed{\Gamma = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)dx.dy}$$



Pb-(22)

$u = 2x + 3y$

$v = 2y$

$r = 2$ units

sol^n

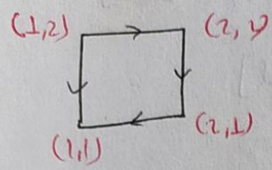
$\Gamma = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)dx.dy$

$= (0 - 3)(\pi r^2)$

$= -3 \times \pi (2)^2$

(Pb) $u = x^2$

$v = -2xy$



Sol^n $\Gamma = \int_{x=1}^{x=2} u.dx + \int_{y=1}^{y=2} (v.dy) + \int_{x=2}^{x=1} u.dx + \int_{y=2}^{y=1} v.dy$

$\boxed{y=1}$ $\boxed{x=2}$ $\boxed{y=2}$ $\boxed{x=1}$

$= \int_1^2 x^2 dx + \int_1^2 -2(2)y.dy + \int_2^1 x^2.dx + \int_2^1 -2(1).y\,dy$

$= \cancel{\int_1^2 x^2 dx} - 4\int_1^2 y.dy + \cancel{\int_2^1 x^2.dx} - 2\int_2^1 y.dy$

$= -4\left[\frac{y^2}{2}\right]_1^2 - 2\left[\frac{y^2}{2}\right]_2^1$

$= -\frac{4}{2}[4-1] - \frac{2}{2}[1-4]$

$= -2(3) - 1(-3)$

$= -6+3 = -3$

→ velocity Potential fun^n (ϕ)

→ Stream fun^n (ψ)

(V.P.F) (ϕ) In general V.P.F is defined as a fun^n of space and time in a flow such that the negative derivative of this fun^n W.R.T any dir^n gives the velocity of fluid flow in that dir^n.

$$\begin{bmatrix} U = -\frac{\partial \phi}{\partial x} \\ V = -\frac{\partial \phi}{\partial y} \\ W = -\frac{\partial \phi}{\partial z} \end{bmatrix}$$

Boundation with ϕ. continuity eq^n For 2D - steady, incompressible flow →

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial}{\partial x}\left(-\frac{\partial \phi}{\partial x}\right) + \frac{\partial}{\partial y}\left(-\frac{\partial \phi}{\partial y}\right) = 0$$

$$-\frac{\partial^2 \phi}{\partial x^2} - \frac{\partial^2 \phi}{\partial y^2} = 0$$

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$

→ $\boxed{\nabla^2 \phi = 0}$

Laplace eq^n

ϕ must satisfy Laplace eq^n

(Pb) $\phi = 2x + 3y$

$\boxed{\nabla^2 \phi = 0}$

ϕ = valid

irrotational Flow

(ii) $\phi = 4x^2 - 5y^2$

$\nabla^2 \phi = 2$

$\neq 0$

ϕ = invalid

Physical significance

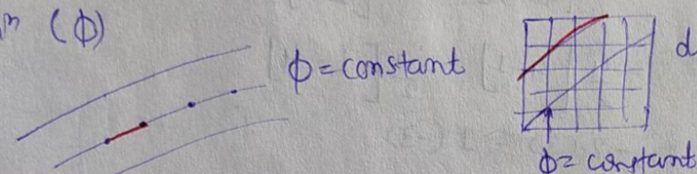
$$\omega_z = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right] = \frac{1}{2}\left[\frac{\partial}{\partial x}\left(-\frac{\partial \phi}{\partial y}\right) - \frac{\partial}{\partial y}\left(-\frac{\partial \phi}{\partial x}\right)\right]$$

$$= 0$$

[ϕ exist → irrotational]

Equ^n of equipotential fun^n line:

These line (curve) are formed by Joining the different point adding having the same value of velocity potential fun^n (ϕ)



Ex: $2x+3y$

for 2D steady incompressible flow

$$\phi = f^n(x, y)$$

$$d\phi = \frac{\partial \phi}{\partial x}.dx + \frac{\partial \phi}{\partial y}.dy = -U dx - V dy$$

$$0 = -U dx - V dy, \qquad U dx + v dy = 0$$

$$\boxed{\left.\frac{dy}{dx}\right|_{\phi = const} = \frac{-U}{V}}$$

Stream fun^n: (ψ) In general

it is a fun^n of space and time in a 2D (x-y plane) define in such a way i.e. continuity eq^n satisfy and flow is posibble.

continuity eq^n

For 2D - steady, incompressible flow

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0$$

If $U = -\dfrac{\partial \psi}{\partial y}$, $v = \dfrac{\partial \psi}{\partial x}$ then, $\dfrac{\partial}{\partial x}\left(-\dfrac{\partial \psi}{\partial y}\right) + \dfrac{\partial}{\partial y}\left(\dfrac{\partial \psi}{\partial x}\right) = 0$

(0=0)

continuity satisfied

check

$$\omega_z = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right] = \frac{1}{2}\left[\frac{\partial}{\partial x}\left(\frac{\partial \psi}{\partial x}\right) - \frac{\partial}{\partial y}\left(\frac{-\partial \psi}{\partial y}\right)\right] = \frac{1}{2}\left[\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2}\right]$$

$$= \frac{1}{2}\left|\nabla^2 \psi\right|$$

[If $\nabla^2 \psi = 0$, Irrotational flow]

[If $\nabla^2 \psi \neq 0$, Rotational flow]

Pb- 34 $\psi = px^2 + qy^2$

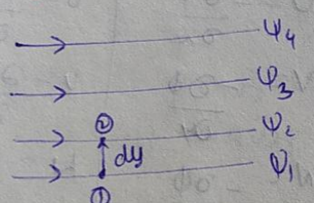
If ϕ exist means irrotational flow

$\nabla^2 \psi = 0$

$2p + 2q = 0$

$\boxed{p = -q}$

Physical significance :



Assume width = 1

For 2D - steady incompressible flow

$\psi = f^n(x, y)$

Math. $d\psi = \underbrace{\frac{\partial \psi}{\partial x}}_{v} dx + \underbrace{\frac{\partial \psi}{\partial y}}_{-u} dy$

$d\psi = -u(dy \cdot 1)$

$-d\psi = dQ$ per unit width

Int it,

$-\int_1^2 d\psi = Q$ per unit width

$\boxed{\psi_1 - \psi_2 = Q \text{ per unit width}}$

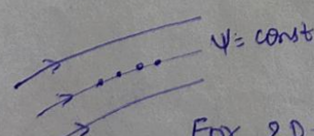
Pb 30 $\psi_1 = \frac{3}{2}(9-1)$

$\psi_2 = 0$

$|\psi_1 - \psi_2| = 12$ units

discharge (+ve)

Eqn of Equi stream funn line :- these line (curves) are formed by joining the different point having the same value of steam funn (ψ).



For 2D - steady, incompressible flow,

$\psi = f^n(x, y)$

Math, $\cancel{d\psi}^{\,0} = \underbrace{\frac{\partial \psi}{\partial x}}_{v} dx + \underbrace{\frac{\partial \psi}{\partial y}}_{-u} . dy$

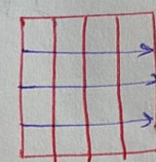
$0 = v.dx - u.dy$

$U\,dy = v\,dx$ $\qquad \boxed{\dfrac{dx}{U} = \dfrac{dy}{v}}$

$$\left.\frac{dy}{dx}\right|_{\psi \to const} = \frac{v}{U}$$

Note: $\left.\dfrac{dy}{dx}\right|_{\phi = const} \times \left.\dfrac{dy}{dx}\right|_{\psi \to const.} = -\dfrac{U}{V} \times \dfrac{V}{U} = -1$

[Equi-potential lies and equi stream funn lies are orthogonal to each other]



Flow net

Cauchy- Rieman eqn :

For irrotational flow

$$U = \boxed{-\frac{\partial \phi}{\partial x} = -\frac{\partial \psi}{\partial y}} \quad ***$$

$$v = \boxed{-\frac{\partial \phi}{\partial y} = +\frac{\partial \psi}{\partial x}}$$

Sign

$U = -\dfrac{\partial \phi}{\partial x}$	$U = \dfrac{\partial \phi}{\partial x}$
$V = -\dfrac{\partial \phi}{\partial y}$	$V = \dfrac{\partial \phi}{\partial y}$
$W = -\dfrac{\partial \phi}{\partial z}$	$W = \dfrac{\partial \phi}{\partial z}$
$U = -\dfrac{\partial \psi}{\partial y}$	$U = \dfrac{\partial \psi}{\partial y}$
$U = \dfrac{\partial \psi}{\partial x}$	$V = \dfrac{\partial \psi}{\partial x}$

continuity eqn

$$\left(\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y}\right) = 0$$

Pb (48) For 2D Incompressible flow field

$U = \dfrac{y^3}{3} + 2x - x^2 y$

$v = xy^2 - 2y - \dfrac{x^3}{3}$

Soln For 2D incompressible flow

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0$$

$$(2 - 2xy) + (2xy - 2) = 0$$

$$0 = 0$$

continuity satisfied

Flow is Possible

Obtain ψ

$$-\frac{\partial \psi}{\partial y} = U$$

$$\frac{\partial \psi}{\partial y} = -\frac{y^3}{3} - 2x + x^2 y$$

Int it $\psi = \frac{-y^4}{12} - 2xy + \frac{x^2y^2}{2} + f(x) + \text{constant}$

$\frac{\partial \psi}{\partial x} = v$

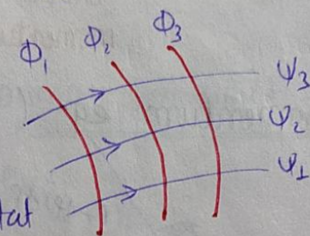
$\frac{\partial \psi}{\partial x} = xy^2 - 2y - \frac{x^3}{3}$

Int it :

$\psi = \frac{x^2y^2}{2} - 2xy - \frac{x^4}{12} + f(y) + \text{const.}$

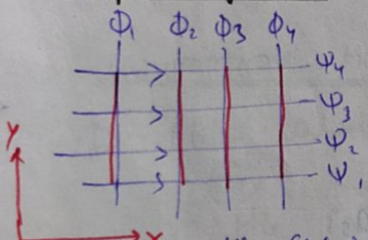
$$\left[\psi = \frac{x^2y^2}{2} - 2xy - \frac{y^4}{12} - \frac{x^4}{12} + \text{const}\right]$$

Flow Net : for irrotational Flow.



It is a graphical representat-ion of equipotential equi, fun^n of line (stream)

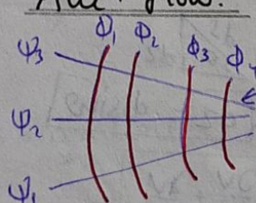
uniform flow →



$\psi = f^n(y)$

$\phi = f^n(x)$

Acc^n Flow:



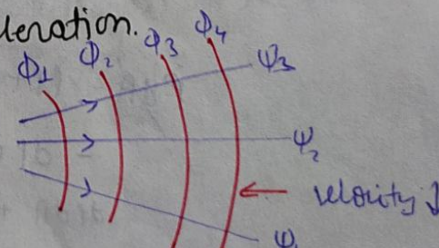
Extra

$|\psi_1 - \psi_3| = Q_{\text{per unit width}}$

$Q = \text{constant}$

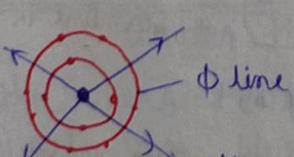
$Q = A.V$

Deceleration.



Source

(Radially outward)



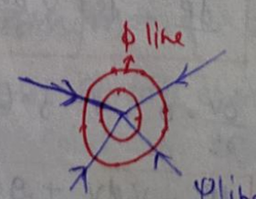
(ψ line never cut ϕ line)

$\psi = f^n(\theta)$

$\phi = f^n(\ln r)$ (radius)

Sink

(Radially inward)

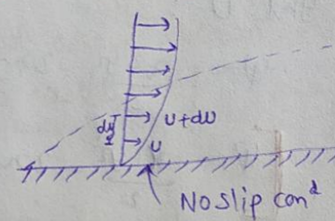


Fluid Dynamic (chapter)

Study of motion of fluid with considering the basic cause of motion ($F_{external}$)

Flow over a flat plate:

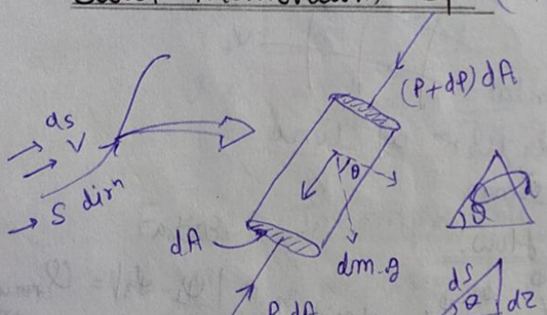
Inviscid Region of fluid flow [$F_{vis} \approx 0$]



$F_p + F_g = F_i$ (Euler momentum eqn)

$F_p + F_g + F_{vis} = F_i$ (Navier's stoke momentum eqn)

Euler momentum eqn (flow along stream line)



$$\boxed{dz = ds\sin\theta}$$

$$\boxed{a_s = V.\frac{\partial V}{\partial s} + \frac{\partial V}{\partial t}} \quad **$$

$$\cancel{P dA} - (\cancel{P} + dP)dA - dm.g\sin\theta = dm\,a_s$$

$$-dP.dA - dm.g\sin\theta = \cancel{dm\,a_s}\ 0$$

$$dP\,dA + \rho(dA.ds)\left[V.\frac{\partial V}{\partial s} + \frac{\partial V}{\partial t}\right] + \rho(dA\,\underbrace{ds\,g\sin\theta}_{dz}) = 0$$

Divide by ρ

$$\boxed{\frac{dP}{\rho} + ds\left[V\frac{\partial V}{\partial s} + \frac{\partial V}{\partial t}\right] + g\,dz = 0}$$

" Euler momentum eqn along a streamline in the dirn of Flow

→ steady Flow $\frac{\partial V}{\partial t} = 0$, $V = f^n(s,t)$ multiply by ρ

$$\boxed{\frac{dP}{\rho} + V.dV + g\,dz = 0}$$

Euler momentum eqn for steady flow along a stream line in the flow dirn

Int. it

$$\int \frac{dP}{\rho} + \frac{V^2}{2} + gz = const$$

Incompressible Flow → $\rho \to$ const

$$\frac{P}{\rho} + \frac{V^2}{2} + g.z = \text{constant}$$

$$\boxed{P + \frac{1}{2}\rho V^2 + \rho g.z = \text{const}}$$ Beraullis eqn