4 LIFE MATHEMATICS

Learning Objectives

B1Q6R5

- To recall the concepts of percentage, profit, loss and simple interest.
- To solve problems involving percentage, applications of percentage in profit, loss, overhead expenses, discount and GST.
- To know what compound interest is and to be able to find the compound interest through patterns and formula and use them in simple problems.
- To find the difference between compound interest and simple interest for 2 years and 3 years.
- To recall direct and inverse proportions.
- To know about compound variation and do problems on it.
- To solve time and work problems.

4.1 Introduction

The following conversation happens in a Math class of VIII Std.

Teacher : Dear students, money collection is being made for the Flag Day and so far 32 out of 40 students in VII Std and 42 out of 50 students from our class have contributed. Can anyone of you say, whose contribution is better?

Sankar : Teacher, 32 out of 40 is $\frac{32}{40}$ and 42 out of 50 is $\frac{42}{50}$. The corresponding like fractions of them are $\frac{160}{200}$ and $\frac{168}{200}$ respectively. So, our class students' contribution is better.

Teacher : Very Good, Sankar. Students, is there any other way to compare?

Bumrah : Yes Teacher, percentages will help. Here, $\frac{32}{40} = \frac{32}{40} \times 100 = 80\ \%$ and $\frac{42}{50} = \frac{42}{50} \times 100 = 84\%$ Clearly, our class contribution is 4 % more than VII Std.

Teacher : Well done, Bumrah. You are exactly correct. Can anyone of you say where the use of percentages is seen more?

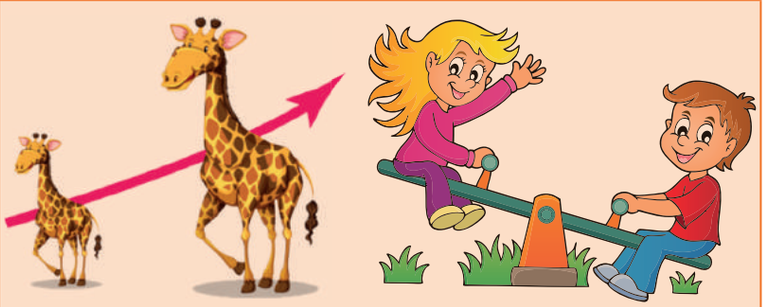
Bhuvi : Yes Teacher. My father told me that percentages are used in the calculation of profit, loss, discount, calculation of tax (eg: GST), interest on investment, growth in population and depreciation of machines and almost everywhere where comparison is made. He also had told me that it is an easy tool which is helpful to compare values.

Teacher : Well said, Bhuvi. These are the topics what we are going to see in this chapter using percentages.

The above conversation leads the way to know the application of percentages in various situations and in the commonly seen problems in our day-to-day life. We will also see direct and inverse proportions, compound variation and time and work problems topics later.

MATHEMATICS ALIVE – LIFE MATHEMATICS IN REALITY

Money grows faster by compound interest.



The growth of a giraffe over time is an example for direct proportion and see-saw is an example for inverse proportion

Try these

Find the indicated percentage value of the given numbers.

Number / %	60	240	660	852	1200
10 %					
20 %					
25 %					
$33\frac{1}{3}$ %					

4.2 Applications of Percentage in Word Problems

We know that Per Cent means per hundred or out of a hundred. It is denoted by the symbol %. x% denotes the fraction $\frac{x}{100}$. It is very useful in comparing quantities easily. Let us see its uses in the following word problems.

Example 4.1

If x % of 600 is 450, then find the value of x.

Solution:

Given, x% of $600 = 450$

$$\frac{x}{100} \times 600 = 450$$

$$x = \frac{450}{6}$$

$$x = 75$$

Example 4.2

When a number is decreased by 25 % , it becomes 120. Find the number.

Solution:

Let the number be x.

Given, $x - \frac{25}{100}x = 120$

$$\frac{100x - 25x}{100} = 120$$

$$\frac{75x}{100} = 120$$

$$\Rightarrow x = \frac{120 \times 100}{75}$$

$$\Rightarrow x = 160$$

If we start with a quantity A and then decrease that quantity by x%, we will get the decreased quantity as,

$$D = \left(1 - \frac{x}{100}\right) A$$

Aliter: Using the above formula,

$$D = \left(1 - \frac{25}{100}\right)A \Rightarrow 120 = \frac{75}{100} \times A$$

$$A = 120 \times \frac{100}{75} = 160$$

Example 4.3

Akila scored 80 % of marks in an examination. If her score was 576 marks, then find the maximum marks of the examination.

Solution:

Let the maximum marks be x.

Now, 80 % of $x = 576$

$$\frac{80}{100} \times x = 576$$

$$\Rightarrow x = 576 \times \frac{100}{80} = 720 \text{ marks}$$

Therefore, the maximum marks of the examination = 720.

Example 4.4

If the price of Orid dhall after 20% increase is ₹96 per kg, then find the original price of Orid dhall per kg.

Solution:

Let the original price of Orid dhall be ₹ x.

New price after 20% increase

$$= x + \frac{20}{100}x = \frac{120x}{100}$$

Given that, $\frac{120x}{100} = 96$

$$\therefore x = \frac{96 \times 100}{120}$$

$\therefore$ Original price of Orid dhall per kg, x = ₹80

If we start with a quantity A and then increase that quantity by x%, we will get the increased quantity as,

$$I = \left(1 + \frac{x}{100}\right) A$$

Aliter: Using the above formula,

$$I = \left(1 + \frac{20}{100}\right)A$$

$$\Rightarrow 96 = \frac{120}{100} \times A$$

$$A = 96 \times \frac{100}{120}$$

A = ₹80

Try these

1. What percentage of a day is 10 hours?
2. Divide ₹350 among P, Q and R such that P gets 50% of what Q gets and Q gets 50% of what R gets.

Think

With a lot of pride, the traffic police commissioner of a city reported that the accidents had decreased by 200% in one year. He came up with this number by stating that the increase in accidents from 200 to 600 is clearly a 200% rise and now that it had gone down from 600 last year to 200 this year should be a 200% fall. Is this decrease from 600 to 200, the same 200% as reported by him? Justify.

Example 4.5

The income of a person is increased by 10% and then decreased by 10%. Find the change in his income.

Solution:

Let his initial income be ₹ x.

Income after 10% increase is

$$₹\,x + \left(\frac{10}{100} \times x\right) = ₹\,\frac{110}{100}\,x \text{ (or) } ₹\,\frac{11}{10}\,x$$

Now, income after 10% decrease is,

$$₹\,\frac{11x}{10} - \frac{10}{100}\left(\frac{11x}{10}\right)$$

$$\frac{11x}{10} - \frac{11x}{100} = \frac{110x - 11x}{100} = ₹\,\frac{99x}{100}$$

$\therefore$ Net change in his income $= x - \frac{99x}{100} = \frac{x}{100}$

$\therefore$ Percentage change $= \frac{\frac{x}{100}}{x} \times 100\% = 1\%$

That is, income of the person is reduced by 1%.

(or)

Aliter:

Let his income be ₹100

Income after 10% increase is

$$100 + 100 \times \frac{10}{100} = ₹110.$$

Now, income after 10% decrease is,

$$110 - 110 \times \frac{10}{100} = 110 - 11 = ₹99$$

$\therefore$ Net change in his income = 100 – 99 = 1

$\therefore$ Percentage change $= \frac{1}{100} \times 100\% = 1\%$.

That is, income of the person is reduced by 1%.

Note

For any given number, when it is increased first by x % and then decreased by y %, then the value of the number is increased or decreased by $\left(x + y + \frac{xy}{100}\right)\%$. Use 'negative' sign for decrease and also assume 'decrease' if the sign is negative. Use this note to check the answer for the Example 4.5.

Exercise 4.1

1. **Fill in the blanks:**
   (i) If 30 % of x is 150, then x is _____.
   (ii) 2 minutes is _____ % to an hour.
   (iii) If x % of x = 25, then x = _____.
   (iv) In a school of 1400 students, there are 420 girls. The percentage of boys in the school is _____.
   (v) 0.5252 is _____ % .

2. **Rewrite each underlined part using percentage language.**
   (i) One half of the cake is distributed to the children.
   (ii) Aparna scored 7.5 points out of 10 in a competition.
   (iii) The statue was made of pure silver.
   (iv) 48 out of 50 students participated in sports.
   (v) Only 2 persons out of 3 will be selected in the interview.

3. 48 is 32% of which number?
4. What is 25% of 30% of 400?
5. If a car is sold for ₹200000 from its original price of ₹300000, then find the percentage of decrease in the value of the car.

6. If the difference between 75% of a number and 60% of the same number is 82.5, then find 20% of that number.
7. A number when increased by 18% gives 236. Find the number.
8. A number when decreased by 20% gives 80. Find the number.
9. A number is increased by 25% and then decreased by 20%. Find the percentage change in that number.
10. The ratio of boys and girls in a class is 5:3. If 16% of boys and 8% of girls failed in an examination, then find the percentage of passed students.

Objective Type Questions

11. 12% of 250 litre is the same as _____of 150 litre.
    (A) 10% (B) 15% (C) 20% (D) 30%
12. If three candidates A, B and C in a school election got 153,245 and 102 votes respectively, then the percentage of votes got by the winner is_____.
    (A) 48% (B) 49% (C) 50% (D) 45%
13. 15% of 25% of 10000 =_____.
    (A) 375 B) 400 C) 425 (D) 475
14. When 60 is subtracted from 60% of a number to give 60, the number is
    (A) 60 (B) 100 (C) 150 (D) 200
15. If 48% of 48 = 64% of x, then x =
    (A) 64 (B) 56 (C) 42 (D) 36

4.3 Profit, Loss, Discount, Overhead Expenses and GST

4.3.1 Profit and Loss

Try these

1. If the selling price of an article is less than the cost price of the article, then there is a _____.
2. An article costing ₹5000 is sold for ₹4850. Is there a profit or loss? What percentage is it?
3. If the ratio of cost price and the selling price of an article is 5:7, then the profit/ gain is_____%.

Cost Price (C.P)

The amount for which an article is bought is called its ***Cost Price*** (C.P).

Selling Price (S.P)

The amount for which an article is sold is called its ***Selling Price*** (S.P).

Profit or Gain

When the *S.P* is more than the *C.P,* then there is a ***profit or gain***.

Therefore, Profit/Gain = S.P – C.P

Loss

When the *S.P* is less than the *C.P,* then there is a ***loss***.

Therefore, Loss = C.P – S.P

It is to be noted that the profit or loss is always calculated on the cost price.

Formulae:

(i) Profit or Gain % $= \left(\frac{Profit}{C.P} \times 100\right)\%$

(ii) Loss % $= \left(\frac{Loss}{C.P} \times 100\right)\%$

(iii) Selling Price, S.P $= \frac{(100 + Profit\%)}{100} \times$ C.P (or) Cost Price, C.P $= \frac{100}{(100 + Profit\%)} \times$ S.P

(iv) Selling Price, S.P $= \frac{(100 - Loss\%)}{100} \times$ C.P (or) Cost Price, C.P $= \frac{100}{(100 - Loss\%)} \times$ S.P

4.3.2 Discount

During the month of Aadi and festival seasons, shopkeepers offer a certain percentage of rebate on the marked price of the articles in order to increase the sales and also to clear the old stock. This rebate is known as ***discount***.

Marked price

In big shops and departmental stores, we see that every product is tagged with a card and a price marked on it. This price marked on the card is called the ***marked price***.

Based on this marked price, the shopkeeper offers a certain percentage of discount. The price payable by the customer after deduction of discount is called the ***selling price***.

That is, Selling Price = Marked Price – Discount

4.3.3 Overhead Expenses

Traders, retailers and shopkeepers are involved in the buying and selling of goods. Sometimes, when articles like machinery, furniture, electronic items etc., are bought, a few expenses may happen on their repairs, transportation and labour charges etc., These expenses are included in the cost price and are called as ***overhead expenses.***

Total Cost Price = Cost Price + Overhead Expenses

$$\text{Discount \%} = \frac{\text{Discount}}{\text{marked price}} \times 100\%$$

4.3.4 Goods and Services Tax (GST)

The goods and services tax (GST) is the only common tax in India levied on almost all the goods and the services meant for domestic consumption. The GST is remitted by the traders and the consumers alike and is also one of the main sources of income to both the Central and State Governments.There are 3 types of GST namely Central GST (CGST), State GST (SGST) and Integrated GST (IGST). For union territories, there is UTGST.

The GST is shared by the Central and the State Governments equally. There are also many products like Egg, Honey, Milk, Salt etc., which are exempted from GST. Products like Petrol, Diesel etc., do not come under GST and they are taxed separately. The GST council has fitted over 1300 goods and 500 services under four tax slabs namely 5%, 12%, 18% and 28%.

Example 4.6

Ranjith bought a washing machine for ₹16150 and paid ₹1350 for its transportation. Then, he sold it for ₹19250. Find his gain or loss percentage.

Solution:

Total C.P of the washing machine

= C.P + Overhead Expenses

= 16150 + 1350 = ₹17500

S.P = ₹19250

Here, S.P > C.P. Hence, there is a gain.

$$\text{Gain \%} = \left(\frac{Gain}{C.P} \times 100\right)\% = \left(\frac{19250 - 17500}{17500} \times 100\right)\%$$

$$= \left(\frac{1750}{17500} \times 100\right)\% = 10\%$$

Example 4.7

If the selling price of an LED TV is equal to $\frac{5}{4}$ of its cost price, then find the gain / profit percentage.

Solution:

Let the C.P of the LED TV be ₹x.

$$\therefore \text{ S.P} = \frac{5}{4}x$$

$$\text{Profit} = \text{S.P} - \text{C.P} = \frac{5}{4}x - x = \frac{x}{4}$$

$$\text{Profit \%} = \left(\frac{Profit}{C.P} \times 100\right)\%$$

$$\therefore \text{ Profit \%} = \left(\frac{x/4}{x} \times 100\right)\% = \left(\frac{1}{4} \times 100\right)\% = 25\%$$

Example 4.8

The cost price of 16 boxes of strawberries is equal to the selling price of 20 boxes of strawberries. Find the gain or loss percentage.

Solution:

Let the C.P of one strawberry box be ₹ x .

C.P of 20 strawberry boxes = $20x$ and

S.P of 20 strawberry boxes = C.P of 16 strawberry boxes = $16x$ (given)

Here, S.P < C.P, hence there is a loss.

$$\text{Loss} = \text{C.P} - \text{S.P} = 20x - 16x = 4x$$

$$\text{Loss \%} = \left(\frac{Loss}{C.P} \times 100\right)\%$$

$$= \left(\frac{4x}{20x} \times 100\right)\%$$

$$\therefore \text{ Loss \%} = 20 \%$$

If the cost price of x articles = selling price of y articles, then profit % $= \left(\frac{x-y}{y} \times 100\right)\%$ .If the answer is negative, then it is treated as loss. Check and verify this for Example 4.8.

Example 4.9

By selling a bicycle for ₹4275, a shopkeeper loses 5%. For how much should he sell it to have a profit of 5%?

Solution:

S.P of the bicycle = ₹4275

Loss = 5%

$$\therefore \text{ C.P} = \frac{100}{(100 - Loss\%)} \times \text{ S.P}$$

$$= \frac{100}{95} \times 4275 = ₹4500$$

Now, C.P = ₹4500 and the desired profit = 5%

$$\therefore \text{ Desired S.P } = \frac{(100 + Profit\%)}{100} \times \text{C.P}$$

$$= \frac{100+5}{100} \times 4500 = 105 \times 45 = ₹4725$$

$\therefore$ To have a profit of 5%, he should sell the bicycle for ₹4725.

Example 4.10

The price of a rain coat was slashed from ₹1060 to ₹901 by a shopkeeper in the rainy season to boost the sales. Find the rate of discount given by him.

Solution:

Discount = Marked Price –Selling Price

= 1060 – 901

= ₹159

$$\therefore \text{ Discount \%} = \left(\frac{159}{1060} \times 100\right)\%$$

= 15%

Think

A shopkeeper marks the price of a marker board 15% above the cost price and then allows a discount of 15% on the marked price. Does he gain or lose in the transaction?

Example 4.11

Find the single discount in percentage which is equivalent to two successive discounts of 25% and 20% given on an article.

Solution:

Let the marked price of an article be ₹100.

First discount of 25% $= 100 \times \frac{25}{100} = ₹25$

Price after first discount $= 100 - 25 = ₹75$

Second discount of 20% $= 75 \times \frac{20}{100} = ₹15$

Price after second discount = 75 – 15 = ₹60

$\therefore$ Net selling price = ₹60

$\therefore$ Single discount in percentage equivalent to two given successive discounts = (100–60)% = 40%.

Note

- If there are 2 successive discounts of a % and b % respectively, then
  $\text{S.P} = \left(1-\frac{a}{100}\right)\left(1-\frac{b}{100}\right)\times \text{M.P.}$
- Single discount in % equivalent to 3 successive discounts of a %, b % and c % respectively
  $= \left\{1-\left(1-\frac{a}{100}\right)\left(1-\frac{b}{100}\right)\left(1-\frac{c}{100}\right)\right\}\times 100\%$.

(Use this formula for the Example 4.11 and check the answer).

Example 4.12

A water heater is sold by a trader for ₹10502 including GST at 18%. Find the marked price of the water heater and GST.

Solution:

Let the marked price be ₹ x.

$$\text{Now, } x+\frac{18x}{100} = 10502$$

$$\frac{118x}{100} = 10502$$

$\therefore$ Marked price, x = ₹8900

GST at 18% = ₹10502 – ₹8900 = ₹1602

(or)

$$= 8900 \times \frac{18}{100} = ₹1602$$

Example 4.13

A family went to a hotel and spent ₹350 for food and paid extra 5% as GST. Calculate the CGST and SGST.

Solution:

Cost of food = ₹350

Extra 5% paid as GST is equally shared by the Central and the State Governments at 2.5% each.

$$\therefore \text{CGST} = \text{SGST} = 350 \times \frac{2.5}{100} = ₹8.75$$

Exercise 4.2

1. Fill in the blanks:

(i) Loss or gain percentage is always calculated on the_____.

(ii) A mobile phone is sold for ₹8400 at a gain of 20%. The cost price of the mobile phone is_____.

(iii) An article is sold for ₹555 at a loss of $7\frac{1}{2}$ %. The cost price of the article is _____.

(iv) A mixer grinder marked at ₹4500 is sold for ₹4140 after discount. The rate of discount is _____.

(v) The total bill amount of a shirt costing ₹575 and a T-shirt costing ₹325 with GST of 5% is_____.

2. If selling an article for ₹820 causes 10% loss on the selling price, then find its cost price.

3. If the profit earned on selling an article for ₹810 is the same as loss on selling it for ₹530, then find the cost price of the article.

4. If the selling price of 10 rulers is the same as the cost price of 15 rulers, then find the profit percentage.

5. Some articles are bought at 2 for ₹15 and sold at 3 for ₹25. Find the gain percentage.

6. By selling a speaker for ₹768, a man loses 20%. In order to gain 20%, how much should he sell the speaker?

7. Find the unknowns x, y and z.

S.No	Name of the item	Marked Price	Selling Price	Discount
(i)	Book	₹225	x	8 %
(ii)	LED TV	y	₹11970	5 %
(iii)	Digital clock	₹750	₹615	z

8. Find the total bill amount for the data given below:

S.No	Name of the item	Marked Price	Discount	GST
(i)	School bag	₹500	5 %	12 %
(ii)	Hair dryer	₹2000	10 %	28 %

9. A branded Air-Conditioner (AC) has a marked price of ₹38000. There are 2 options given for the customer.

(i) Selling Price is the same ₹38000 but with attractive gifts worth ₹3000

(or)

(ii) Discount of 8% on the marked price but no free gifts.

Which offer is better?

10. If a mattress is marked for ₹7500 and is available at two successive discounts of 10% and 20%, find the amount to be paid by the customer.

Objective Type Questions

11. A fruit vendor sells fruits for ₹200 gaining ₹40. His gain percentage is

(A) 20% (B) 22% (C) 25% (D) $16\frac{2}{3}\%$

12. By selling a flower pot for ₹528, a woman gains 20%. At what price should she sell it to gain 25%?

(A) ₹500 (B) ₹550 (C) ₹553 (D) ₹573

13. A man buys an article for ₹150 and makes overhead expenses which are 12% of the cost price. At what price must he sell it to gain 5%?

(A) ₹180 (B) ₹168 (C) ₹176.40 (D) ₹88.20

14. What is the marked price of a hat which is bought for ₹210 at 16% discount?

(A) ₹243 (B) ₹176 (C) ₹230 (D) ₹250

15. The single discount in % which is equivalent to two successive discounts of 20% and 25% is

(A) 40% (B) 45% (C) 5% (D) 22.5%

4.4 Compound Interest

The most powerful force in this universe is_____. How would you complete this quote? The world renowned physicist Albert Einstein completed this quote with the word **Compound Interest.**

When money is borrowed or deposited on simple interest ($I = \frac{PNR}{100}$), then the interest is calculated evenly on the principal throughout the loan or deposit period.

Try these

1. The formula to find the simple interest for a given principal is _____.
2. Find the simple interest on ₹900 for 73 days at 8% p.a.
3. In how many years will ₹2000 become ₹3600 at 10% p.a simple interest?

In post offices, banks, insurance companies and other financial institutions, there is another type of interest calculation on offer. Here, the interest accrued during the first time period (say 6 months) is added to the original principal and the amount so obtained is taken as the principal for the second time period (that is, the next 6 months) and this keeps going on, up to the fixed time agreement between the banker and the depositer.

After a certain period, the difference between the amount and the money deposited is called the compound interest which is abbreviated as C.I. Clearly, the compound interest will be more than the simple interest just because the principal keeps on changing for every time period.

We call the time period after which the interest is added to the principal, as the conversion period. For example, if the interest is compounded say quarterly, there will be four conversion periods in a year, every 3 months. In such cases, the interest rate will be one fourth of the annual rate and the number of times that interest will be compounded is four times the number of years.

In case of simple interest, the principal remains the same for the whole duration where as in case of compound interest, the principal keeps on changing as per the conversion period. The simple interest and the compound interest remains the same for the first conversion period.

Illustration 1

To find the compound interest on ₹20000 for 4 years at 10 % p.a compounded annually and compare it with the simple interest obtained for the same.

Calculating Compound Interest

Principal for the I year = ₹20000

Interest for the I year $\left(\frac{20000\times10\times1}{100}\right)$ = ₹ 2000

Amount at the end of I Year (P+I) = ₹22000

That is, Principal for the II year = ₹22000

Interest for the II year $\left(\frac{22000\times10\times1}{100}\right)$ = ₹ 2200

Amount at the end of II year = ₹24200

That is, Principal for the III year = ₹24200

Interest for the III year $\left(\frac{24200\times10\times1}{100}\right)$ = ₹ 2420

Amount at the end of III year = ₹26620

That is, Principal for the IV year = ₹26620

Interest for the IV year $\left(\frac{26620\times10\times1}{100}\right)$ = ₹ 2662

∴ Amount at the end of IV year = ₹29282

∴ Compound Interest for 4 years = A – P = 29282 – 20000 = ₹9282

Calculating Simple Interest

Recall that,

Simple Interest, $I = \frac{PNR}{100}$

Here, P = ₹20000

N = 4 years

R = 10%

$\therefore I = \frac{20000\times4\times10}{100}$

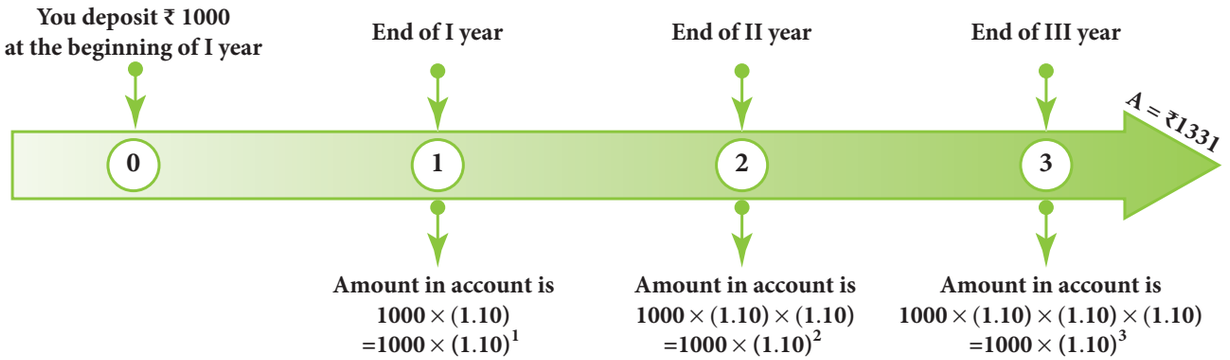
That is, I = ₹ 8000

What we observe from the calculation of C.I is a repeated multiplication by the factor 1.1 as 20000 (×1.1), 22000 (×1.1), 24200 (×1.1), 26620 (×1.1) for 4 years. We also note that the compound interest (₹9282) grows faster and is clearly more than the simple interest (₹8000) obtained. When the time period is longer, the above method is time

consuming. So, to save time and to find the amount and the compound interest easily we have a formula as explained in the following illustration.

Illustration 2

To calculate the amount and compound interest for ₹1000 for 3 years at 10% p.a compound annually.



Flow chart showing 10% interest being compounded annually

This leads to the pattern for Amount as $P\left(1+\frac{r}{100}\right)$ for the I year, $P\left(1+\frac{r}{100}\right)\left(1+\frac{r}{100}\right)$ for the II year, $P\left(1+\frac{r}{100}\right)\left(1+\frac{r}{100}\right)\left(1+\frac{r}{100}\right)$ for the III year and so on. In general, $A = P\left(1+\frac{r}{100}\right)^n$ for the n^{th} year. Here, the amount after 3 years, $A = 1000\left(1+\frac{10}{100}\right)^3 = ₹1331$ and so, C.I = A – P = ₹331.

In 1626, Peter Minuit convinced the Wappinger Indians to sell him the Manhattan Island for 24 dollars. If the native Americans had put the 24 dollars into a bank account at 5% interest rate compounded monthly, by the year 2020 there would be well over 5.5 billion (550 crores) dollars in the account! This is the might of compound interest!

The following formulae will be helpful in calculating the compound interest easily for the following time periods.

(i) When the interest is compounded annually, we have

$$A = P\left(1+\frac{r}{100}\right)^n$$

where A is the amount, P is the principal, r is the rate of interest per annum and n is the number of years and we shall get the compound interest as C.I = Amount – Principal.

(ii) When the interest is compounded half yearly, we have

$$A = P\left(1+\frac{r}{200}\right)^{2n}$$

(iii) When the interest is compounded quarterly, we have

$$A = P\left(1+\frac{r}{400}\right)^{4n}$$

(iv) When the interest is compounded annually but rate of interest differs year by year, we have

$$A = P\left(1+\frac{a}{100}\right)\left(1+\frac{b}{100}\right)\left(1+\frac{c}{100}\right)\ldots$$

where a, b and c are interest rates for I, II and III years respectively.

(v) When interest is compounded annually but time period is in fraction say $a\frac{b}{c}$ years, we have

$$A = P\left(1+\frac{r}{100}\right)^{a}\left(1+\frac{\frac{b}{c}\times r}{100}\right)$$

(Use of calculators are permitted for lengthy calculations and also to verify answers).

Example 4.14

Find the C.I for the data given below:

(i) Principal = ₹4000, r = 5% p.a, n = 2 years, interest compounded annually.

(ii) Principal = ₹5000, r = 4% p.a, n = $1\frac{1}{2}$ years, interest compounded half-yearly.

(iii) Principal = ₹30000, r = 7% for I year, r = 8% for II year, compounded annually.

(iv) Principal = ₹10000, r = 8% p.a, n = $2\frac{3}{4}$ years, interest compounded yearly.

Solution:

(i) Amount, $A = P\left(1+\frac{r}{100}\right)^{n}$

$$= 4000\left(1+\frac{5}{100}\right)^{2}$$

$$= 4000\times\frac{21}{20}\times\frac{21}{20}$$

$$A = ₹4410$$

$$\therefore C.I = A - P = 4410 - 4000 = ₹410$$

(ii) Amount, $A = P\left(1+\frac{r}{200}\right)^{2n} = 5000\left(1+\frac{4}{200}\right)^{2\times\frac{3}{2}} = 5000\times\frac{51}{50}\times\frac{51}{50}\times\frac{51}{50}$

$$= 51 \times 10.2 \times 10.2$$

$$= ₹5306.04$$

$$\therefore C.I = A - P = 5306.04 - 5000$$

$$= ₹306.04$$

(iii) Amount, $A = P\left(1+\frac{a}{100}\right)\left(1+\frac{b}{100}\right)$

$= 30000\left(1+\frac{7}{100}\right)\left(1+\frac{8}{100}\right)$

$= 30000\times\frac{107}{100}\times\frac{108}{100}$

$A = ₹34668$

$\therefore$ C.I = A − P = 34668 − 30000 = ₹4668

(iv) Amount, $A = P\left(1+\frac{r}{100}\right)^{a}\left[1+\frac{\frac{b}{c}\times r}{100}\right] = 10000\left(1+\frac{8}{100}\right)^{2}\left[1+\frac{\frac{3}{4}\times 8}{100}\right]$

$= 10000\left(\frac{27}{25}\right)^{2}\left(\frac{53}{50}\right)$

$= 10000\times\frac{27}{25}\times\frac{27}{25}\times\frac{53}{50}$

$A = ₹12363.84$

$\therefore$ C.I = A − P $= 12363.84 - 10000 = ₹2363.84$

4.4.1 Applications of Compound Interest Formula

The compound interest formula is used in the following situations.

(i) To find the increase $\left[P\left(1+\frac{r}{100}\right)^{n}\right]$ or decrease $\left[P\left(1-\frac{r}{100}\right)^{n}\right]$ in population.

(ii) To find the growth of cells when the rate of growth is given.

(iii) To find the depreciation in the values of machines, vehicles, utility appliances etc.,

Example 4.15

The value of a motor cycle 2 years ago was ₹70000. It depreciates at the rate of 4% p.a. Find its present value.

Solution:

Depreciated value $= P\left(1-\frac{r}{100}\right)^{n}$

$= 70000\left(1-\frac{4}{100}\right)^{2}$

$= 70000\times\frac{96}{100}\times\frac{96}{100}$

$= ₹64512$

Example 4.16

The bacteria in a culture grows by 5% in the first hour, decreases by 8% in the second hour and again increases by 10% in the third hour. Find the count of the bacteria at the end of 3 hours, if its initial count was 10000.

Solution:

Bacteria at the end of 3 hours

$$A = P\left(1+\frac{a}{100}\right)\left(1+\frac{b}{100}\right)\left(1+\frac{c}{100}\right)$$

$$= 10000\left(1+\frac{5}{100}\right)\left(1-\frac{8}{100}\right)\left(1+\frac{10}{100}\right) \text{ ('–' because 'decrease')}$$

$$= 10000 \times \frac{105}{100} \times \frac{92}{100} \times \frac{110}{100}$$

$$A = ₹10626$$

Example 4.17

The population of a town is increasing at the rate of 6% p.a. It was 238765 in the year 2018. Find the population in the year 2016 and 2020.

Solution:

Let the population in 2016 be P.

Then, $A = P\left(1+\frac{r}{100}\right)^n$

$$\Rightarrow 238765 = P\left(1+\frac{6}{100}\right)^2 = P\left(\frac{53}{50}\right)^2$$

$$\Rightarrow P = 238765 \times \frac{50}{53} \times \frac{50}{53}$$

$$\therefore P = 212500$$

Let the population in 2020 be A

Then, $A = P\left(1+\frac{r}{100}\right)^n$

$$\therefore A = 238765\left(1+\frac{6}{100}\right)^2$$

$$= 238765 \times \frac{53}{50} \times \frac{53}{50}$$

$$= 95.506 \times 53 \times 53$$

$$A = 268276$$

$\therefore$ The population in the year 2016 was 212500 and that in the year 2020 will be 268276.

4.4.2 Difference between C.I and S.I

- There is no difference in C.I and S.I for the first year or the first conversion period.
- For 2 years, the difference in C.I and S.I is

$$\text{C.I} - \text{S.I} = \text{P}\left(\frac{r}{100}\right)^2$$

- For 3 years, the difference in C.I and S.I is

$$\text{C.I} - \text{S.I} = \text{P}\left(\frac{r}{100}\right)^2\left(3+\frac{r}{100}\right)$$

Example 4.18

Find the difference in C.I and S.I for

(i) P = ₹5000, r = 4% p.a, n = 2 years.

(ii) P = ₹8000, r = 5% p.a, n = 3 years.

Solution:

(i) For 2 years, $\text{C.I} - \text{S.I} = \text{P}\left(\frac{r}{100}\right)^2 = 5000 \times \frac{4}{100} \times \frac{4}{100} = ₹8$

(ii) For 3 years, $\text{C.I} - \text{S.I} = \text{P}\left(\frac{r}{100}\right)^2\left(3+\frac{r}{100}\right)$

$$= 8000 \times \frac{5}{100} \times \frac{5}{100} \times \left(3+\frac{5}{100}\right)$$

$$= 20 \times \frac{61}{20} = ₹61$$

Exercise 4.3

W4I6H4

1. **Fill in the blanks:**

(i) The compound interest on ₹5000 at 12% p.a for 2 years, compounded annually is _____.

(ii) The compound interest on ₹8000 at 10% p.a for 1 year, compounded half yearly is _____.

(iii) The annual rate of growth in population of a town is 10%. If its present population is 26620, then the population 3 years ago was_____.

(iv) If the compound interest is calculated quarterly, the amount is found using the formula _____.

(v) The difference between the C.I and S.I for 2 years for a principal of ₹5000 at the rate of interest 8% p.a is _____.

2. **Say True or False.**

   (i) Depreciation value is calculated by the formula, $P\left(1-\frac{r}{100}\right)^n$.

   (ii) If the present population of a city is P and it increases at the rate of $r\%$ p.a, then the population n years ago would be $P\left(1+\frac{r}{100}\right)^n$.

   (iii) The present value of a machine is ₹16800. It depreciates at 25% p.a. Its worth after 2 years is ₹9450.

   (iv) The time taken for ₹1000 to become ₹1331 at 20% p.a, compounded annually is 3 years.

   (v) The compound interest on ₹16000 for 9 months at 20% p.a, compounded quarterly is ₹2522.

3. Find the compound interest on ₹3200 at 2.5% p.a for 2 years, compounded annually.

4. Find the compound interest for $2\frac{1}{2}$ years on ₹4000 at 10% p.a, if the interest is compounded yearly.

5. A principal becomes ₹2028 in 2 years at 4% p.a compound interest. Find the principal.

6. In how many years will ₹3375 become ₹4096 at $13\frac{1}{3}$ % p.a if the interest is compounded half-yearly?

7. Find the C.I on ₹15000 for 3 years if the rates of interest are 15%, 20% and 25% for the I, II and III years respectively.

8. Find the difference between C.I and S.I on ₹5000 for 1 year at 2% p.a, if the interest is compounded half yearly.

9. Find the rate of interest if the difference between C.I and S.I on ₹8000 compounded annually for 2 years is ₹20.

10. Find the principal if the difference between C.I and S.I on it at 15% p.a for 3 years is ₹1134.

Objective Type Questions

11. The number of conversion periods in a year, if the interest on a principal is compounded every two months is_____.

    (A) 2 (B) 4 (C) 6 (D) 12

12. The time taken for ₹4400 to become ₹4851 at 10%, compounded half yearly is _____.

    (A) 6 months (B) 1 year (C) 1 years (D) 2 years

13. The cost of a machine is ₹18000 and it depreciates at $16\frac{2}{3}$% annually. Its value after 2 years will be_____.

    (A) ₹12000 (B) ₹12500 (C) ₹15000 (D) ₹16500

14. The sum which amounts to ₹2662 at 10% p.a in 3 years, compounded yearly is_____.

(A) ₹2000 (B) ₹1800 (C) ₹1500 (D) ₹2500

15. The difference between compound and simple interest on a certain sum of money for 2 years at 2% p.a is ₹1. The sum of money is _____ .

(A) ₹2000 (B) ₹1500 (C) ₹3000 (D) ₹2500

4.5 Compound Variation

Before we could learn about compound variation, let us recall the concepts on direct and inverse proportions.

If two quantities are such that an increase or decrease in one quantity makes a corresponding increase or decrease (same effect) in the other quantity, then they are said to be in direct proportion or said to vary directly. In other words, x and y are said to vary directly if $\boldsymbol{y=kx}$ always, where k is called the proportionality constant and $k > 0$ assuming that y depends on x and so $\boldsymbol{k = \frac{y}{x}}$.

For example, let us assume that one of you plan to give 2 pens to each of your friends in the birthday party. Then the number of pens to be bought will be in direct proportion with the number of friends who will attend the party. Isn't it? The following table will help you understand this clearly.

Number of friends (x)	1	2	5	12	15
Number of pens (y)	2	4	10	24	30

In this case, we find that the proportionality constant, $k = \frac{y}{x} = \frac{2}{1} = \frac{4}{2} = \frac{10}{5} = \frac{24}{12} = \frac{30}{15} = 2$.

Few more examples of Direct Proportion:

1. Distance –Time (under constant speed): If the distance increases, then the time taken to reach that distance will also increase and vice- versa.
2. Purchase – Spending: If the purchase on utilities for a family during the festival time increases, then the spending limit also increases and vice versa.
3. WorkTime – Earnings: If the number of hours worked is less, then the pay earned will also be less and vice-versa.

Similarly, if two quantities are such that an increase or decrease in one quantity makes a corresponding decrease or increase (opposite effect) in the other quantity, then they are said to be in inverse (indirect) proportion or said to vary inversely. In other words, x and y are said to vary inversely, if $\boldsymbol{xy = k}$ always, where k is called proportionality constant and $k > 0$.

For example, let us assume that a class of 30 students in a school walks on streets in a village for health awareness campaign in an orderly manner, then we can see an inverse proportion in the number of rows and columns they walk. The following table will help you understand this clearly.

Number of students in columns (x)	1	2	3	5	6
Number of students in rows (y)	30	15	10	6	5

We can map a few of these arrangements as  (5 rows/6 columns) and  (6 rows/5 columns) and see the opposite variations in rows and columns.

In this case, we find that the proportionality constant, k is 30.

Few more examples of Inverse Proportion:

1. Price – Consumption: If the price of consumable products increases, then naturally its consumption will decrease and vice-versa.
2. Workers – Time: If more workers are employed to complete a work, then the time taken to complete the work will be less and vice-versa.
3. Speed – Time (Fixed Distance): If we travel with less speed, then the time taken to cover a given distance will be more and vice-versa.

Now, use the concept of direct and inverse proportions and try to answer the following questions:

Try these

1. Classify the given examples as direct or inverse proportion:
   (i) Weight of pulses to their cost.
   (ii) Distance travelled by bus to the price of ticket.
   (iii) Speed of the athelete to cover a certain distance.
   (iv) Number of workers employed to complete a construction in a specified time.
   (v) Area of a circle to its radius.
2. A student can type 21 pages in 15 minutes. At the same rate, how long will it take the student to type 84 pages?
3. If 35 women can do a piece of work in 16 days, in how many days will 28 women do the same work?

Let us now see what a Compound Variation is? There will be problems which may involve a chain of two or more variations in them. This is called as **Compound Variation**. The different possibilities of two variations are:

Direct-Direct, Direct-Inverse, Inverse-Direct, Inverse-Inverse.

Note

There are situations where neither direct proportion nor indirect proportion can be applied. For example, if one can see a parrot at a distance through one eye, it does not mean that he can see two parrots at the same distance through both the eyes. Also, if it takes 5 minutes to fry a vadai, it does not mean that it will take 100 minutes to fry 20 vadais!

Let us now solve a few problems on compound variation. Here, we compare the known quantity with the unknown (x). There are a few methods in practice by which problems on compound variation are solved. They are given as follows:

4.5.1 Proportion Method

In this method, we shall compare the given data and find whether they are in direct or inverse proportion. By finding the proportion, we can use the fact that

the product of the extremes = the product of the means

and get the value of the unknown (x).

4.5.2 Multiplicative Factor Method

Illustration:

Men	Hours	Days
D $\begin{matrix} a \\ x \end{matrix}$ I	D $\begin{matrix} c \\ d \end{matrix}$	$\begin{matrix} e \\ f \end{matrix}$ I

Here, the unknown (x) is in men and so it is compared to the known, namely hours and days. If men and hours are in direct proportion (D), then take the multiplying factor as $\frac{d}{c}$ (take the reciprocal). Also, if men and days are in inverse proportion (I), then take the multiplying factor as $\frac{e}{f}$ (no change). Thus, we can find the unknown (x) in men as $x = a \times \frac{d}{c} \times \frac{e}{f}$.

4.5.3 Formula Method

Identify the data from the given statement as Persons (P), Days (D), Hours (H) and Work (W) and use the formula,

$$\frac{P_1 \times D_1 \times H_1}{W_1} = \frac{P_2 \times D_2 \times H_2}{W_2}$$

where the suffix 1 contains the complete data from the first statement of the given problem and the suffix 2 contains the unknown data to be found out in the second statement of the problem. That is, this formula says, P_1 men doing W_1 units of work in D_1 days working H_1 hours per day is the same as P_2 men doing W_2 units of work in D_2 days working H_2 hours per day. Identifying the work W_1 and W_2 correctly is more important in these problems. This method will be easy for finding the unknown (x) quickly.

Example 4.19 **(Direct – Direct Variation)**

If a company pays ₹6 lakh for 15 workers for 20 days, how much would it need to pay for 5 workers for 12 days?

Solution:

Proportion Method

Workers	Pay (Work)	Days
D $\begin{matrix} 15 \\ 5 \end{matrix}$	D $\begin{matrix} 6 \\ x \end{matrix}$ D	$\begin{matrix} 20 \\ 12 \end{matrix}$ D

Here, the unknown is the pay (x). It is to be compared with the workers and the days.

Step 1:

Here, less days means less pay. So, it is in direct proportion.

$\therefore$ The proportion is $20 : 12 :: 6 : x$ $\longrightarrow$ (1)

Step 2:

Also, less workers means less pay. So, it is in direct proportion again.

$\therefore$ The proportion is $15 : 5 :: 6 : x$ $\longrightarrow$ (2)

Step 3:

Combining (1) and (2),

$$\left.\begin{matrix} 20 : 12 \\ 15 : 5 \end{matrix}\right\} :: 6 : x$$

We know that, **the product of the extremes = the product of the means**

Extremes		Means		Extremes
20	:	12 : 6	:	x
15	:	5		

So, $20 \times 15 \times x = 12 \times 6 \times 5 \Rightarrow x = \dfrac{12 \times 6 \times 5}{20 \times 15}$ = ₹ 1.2 lakh.

Multiplicative Factor Method

Workers	Pay (Work)	Days
15	6	20
D	D	D
5	x	12

Here, the unknown is the pay (x). It is to be compared with the workers and the days.

Step 1:

Here, less days means less pay. So, it is in direct proportion.

$\therefore$ The multiplying factor is $\dfrac{12}{20}$ (take the reciprocal).

Step 2:

Also, less workers means less pay. So, it is in direct proportion again.

$\therefore$ The multiplying factor is $\dfrac{5}{15}$ (take the reciprocal).

Step 3:

$\therefore x = 6 \times \dfrac{12}{20} \times \dfrac{5}{15}$ = ₹ 1.2 lakh.

Formula Method

Here, $P_1 = 15, D_1 = 20$ and $W_1 = 6$. Also, $P_2 = 5, D_2 = 12$ and $W_2 = x$

Using the formula, $\dfrac{P_1 \times D_1}{W_1} = \dfrac{P_2 \times D_2}{W_2}$

$$\Rightarrow \frac{15\times 20}{6} = \frac{5\times 12}{x}$$

$$\Rightarrow x = \frac{5\times 12\times 6}{15\times 20} = ₹\ 1.2 \text{ lakh.}$$

Example 4.20 **(Direct – Inverse Variation)**

A mat of length 180 m is made by 15 women in 12 days. How long will it take for 32 women to make a mat of length 512 m?

Solution:

Proportion Method

Length (Work)	Women	Days
D 180 / 512	15 / I 32	12 / D I / x

Here, the unknown is the days (x). It is to be compared with the length and the women.

Step 1:

Here, more length means more days. So, it is in direct proportion.

$\therefore$ The proportion is $180 : 512 :: 12 : x$ ⟶ (1)

Step 2:

Also, more women means less days. So, it is in inverse proportion.

$\therefore$ The proportion is $32 : 15 :: 12 : x$ ⟶ (2)

Step 3:

Combining (1) and (2),

$$\left.\begin{matrix} 180 : 512 \\ 32 : 15 \end{matrix}\right\} :: 12 : x$$

We know that, **the product of the extremes = the product of the means**

Extremes	:	Means	:	Extremes
180 / 32	:	512 : 12 / 15	:	x

So, $180\times 32\times x = 512\times 12\times 15 \Rightarrow x = \frac{512\times 12\times 15}{180\times 32} = 16$ days.

Multiplicative Factor Method

Length (Work)	Women	Days
D 180 / 512	15 / I 32	12 / D I / x

Here, the unknown is the days (x). It is to be compared with the length and the women.

Step 1:

Here, more length means more days. So, it is in direct proportion.

$\therefore$ The multiplying factor is $\frac{512}{180}$ (take the reciprocal).

Step 2:

Also, more women means less days. So, it is in inverse proportion.

$\therefore$ The multiplying factor is $\frac{15}{32}$ (no change).

Step 3:

$\therefore x = 12 \times \frac{512}{180} \times \frac{15}{32} = 16$ days.

Formula Method:

Here, $P_1 = 15$, $D_1 = 12$ and $W_1 = 180$ and $P_2 = 32$, $D_2 = x$ and $W_2 = 512$

Using the formula, $\frac{P_1 \times D_1}{W_1} = \frac{P_2 \times D_2}{W_2}$

$$\Rightarrow \frac{15 \times 12}{180} = \frac{32 \times x}{512}$$

$$\Rightarrow 1 = \frac{32 \times x}{512} \Rightarrow x = \frac{512}{32} = 16 \text{ days.}$$

Remark: Students may answer in any of the three given methods dealt here.

Try these

1. If x and y vary directly, find k when $x = y = 5$.
2. If x and y vary inversely, find the constant of proportionality when $x = 64$ and $y = 0.75$

Activity

Draw a circle of a given radius. Then, draw its radii in such a way that the angles between any two consecutive pair of radii are equal. Start drawing 3 radii and end with drawing 12 radii in the circle. List and prepare a table for the number of radii to the angle between a pair of consecutive radii and check whether they are in inverse proportion. What is the proportionality constant?

Example 4.21 **(Inverse – Direct Variation)**

If 81 students can do a painting on a wall of length 448 m in 56 days, then how many students can do the painting on a similar type of wall of length 160 m in 27 days?

Solution:

Multiplicative Factor Method

Students	Days	Length of the wall (Work)
81 I D x	56 I 27	448 D 160

Here, the unknown is the students (x). It is to be compared with the days and the length of the wall.

Step 1:

Here, less days means more students. So, it is in inverse proportion.

$\therefore$ The multiplying factor is $\frac{56}{27}$.

Step 2:

Also, less length means less students. So, it is in direct proportion.

$\therefore$ The multiplying factor is $\frac{160}{448}$.

Step 3:

$$\therefore x = 81 \times \frac{56}{27} \times \frac{160}{448}$$

$x = 60$ students.

Formula Method

Here, $P_1 = 81$, $D_1 = 56$ and $W_1 = 448$ and $P_2 = x$, $D_2 = 27$ and $W_2 = 160$

Using the formula, $\frac{P_1 \times D_1}{W_1} = \frac{P_2 \times D_2}{W_2}$

$$\Rightarrow \frac{81 \times 56}{448} = \frac{x \times 27}{160}$$

$$\Rightarrow x = \frac{81 \times 56}{448} \times \frac{160}{27} \Rightarrow x = 60 \text{ students.}$$

Example 4.22 **(Inverse – Inverse Variation)**

If 48 men working 7 hours a day can do a work in 24 days, then in how many days will 28 men working 8 hours a day can complete the same work?

Solution:

Multiplicative Factor Method

Men	Hours	Days
48	7	24
I	I	I
28	8	x

Here, the unknown is the days (x). It is to be compared with the men and the hours.

Step 1:

Here, less men means more days. So, it is in inverse proportion.

$\therefore$ The multiplying factor is $\frac{48}{28}$.

Step 2:

Also, more hours means less days. So, it is in inverse proportion again.

$\therefore$ The multiplying factor is $\frac{7}{8}$.

Step 3:

$\therefore x = 24 \times \frac{48}{28} \times \frac{7}{8} = 36$ days.

Formula Method

Here, $P_1 = 48$, $D_1 = 24$, $H_1 = 7$ and $W_1 = 1$ (Why?)

$P_2 = 28$, $D_2 = x$, $H_2 = 8$ and $W_2 = 1$ (Why?)

Using the formula, $\frac{P_1 \times D_1 \times H_1}{W_1} = \frac{P_2 \times D_2 \times H_2}{W_2}$

$$\Rightarrow \frac{48 \times 24 \times 7}{1} = \frac{28 \times x \times 8}{1}$$

$$\Rightarrow x = \frac{48 \times 24 \times 7}{28 \times 8} = 36 \text{ days.}$$

Try these

Identify the different variations present in the following questions:

1. 24 men can make 48 articles in 12 days. Then, 6 men can make _____ articles in 6 days.
2. 15 workers can lay a road of length 4 km in 4 hours. Then, _____ workers can lay a road of length 8 km in 8 hours.
3. 25 women working 12 hours a day can complete a work in 36 days. Then, 20 women must work _____ hours a day to complete the same work in 30 days.
4. In a camp there are 420 kg of rice sufficient for 98 persons for 45 days. The number of days that 60 kg of rice will last for 42 persons is _____.

4.6 Time and Work

How would you find the answer for the following question?

If Kani can finish a given work in 2 hours and Viji in 3 hours, then in what time can they finish it working together? The answer for this question will be known here in this section.

Work to be done is usually considered as one unit. Work can be in any form like building a wall, making a road, filling or emptying a tank or even eating a certain amount of food.

Time is measured in hours, days etc., Certain assumptions are made that the work so done is uniform and each person shares the same work time in case of group work in completing the work.

Unitary Method

If two persons X and Y can complete some work individually in a and b days, then their one day's work will be $\frac{1}{a}$ and $\frac{1}{b}$ respectively.

Working together, their one day's work $= \frac{1}{a}+\frac{1}{b} = \frac{a+b}{ab}$ and so,

X and Y together can complete the work in $\frac{ab}{a+b}$ days.

Example 4.23

A and B together can do a piece of work in 16 days and A alone can do it in 48 days. How long will B take to complete the work?

Solution:

$(A+B)$'s 1 day's work $= \frac{1}{16}$

A's 1 day's work $= \frac{1}{48}$

$\therefore$ B's 1 day's work $= \frac{1}{16}-\frac{1}{48}$

$= \frac{3-1}{48} = \frac{2}{48} = \frac{1}{24}$

$\therefore$ B alone can complete the work in 24 days.

Note

The time taken to complete a work or task depends on various factors such as **number of persons**, their **capacity** to do the work, the **amount of work** and the **time spent per day** for the completion of work.

Example 4.24

P and Q can do a piece of work in 20 days and 30 days respectively. They started the work together and Q left after some days of work and P finished the remaining work in 5 days. After how many days from the start did Q leave?

S6A5L1

Solution:

P's 1 day's work $= \frac{1}{20}$ and Q's 1 day's work $= \frac{1}{30}$

P's work for 5 days $= \frac{1}{20}\times 5 = \frac{5}{20} = \frac{1}{4}$

Therefore, the remaining work $= 1-\frac{1}{4} = \frac{3}{4}$ (as the total work is always 1). This remaining work was done by both P and Q.

Work done by P and Q in a day $= \frac{1}{20}+\frac{1}{30} = \frac{5}{60} = \frac{1}{12}$

Therefore, the number of days they worked together $= \frac{3/4}{1/12} = \frac{3}{4}\times\frac{12}{1} = 9$ days.

So, Q left after 9 days from the day the work started.

Example 4.25

A works 3 times as fast as B and is able to complete a work in 24 days less than the days taken by B. Find the time in which they can complete the work together.

Solution:

If B does the work in 3 days, then A will do it in 1 day. That is, the difference is 2 days. Here, given that the difference between A and B in completing the work is 24 days. Therefore, A will take $\frac{24}{2} = 12$ days and B will take $3 \times 12 = 36$ days to complete the work separately.

Hence, the time taken by A and B together to complete the work = $\frac{ab}{a+b}$ days.

$$= \frac{12 \times 36}{12 + 36} = \frac{12 \times 36}{48} = 9 \text{ days.}$$

Note

If A is $\frac{a}{b}$ times as good a worker as B, then A will only $\frac{b}{a}$ of the time taken by B to complete the work. Try to solve the above Example 4.25 using this.

4.6.1 Sharing of the money for work

When a group of people do some work together, based on their individual work they get a share of money themselves. In general, **money earned** is shared by people, who worked together, in the ratio of the **total work** done by each of them.

DO YOU KNOW?

- If the ratio of the time taken by A and B in doing a work is $x : y$, then the ratio of work done by A and B is $\frac{1}{x} : \frac{1}{y} = y : x$. This is the ratio for their separate wages too.
- If three persons A, B and C can do a work in x, y and z days respectively, then the ratio in which their wages will be distributed to them is $\frac{1}{x} : \frac{1}{y} : \frac{1}{z}$.

Example 4.26

X, Y and Z can do a piece of job in 4, 6 and 10 days respectively. If X,Y and Z work together to complete, then find their separate shares if they will be paid ₹ 31000 for completing the job.

Solution:

Since they all work together for the same number of days $\left(\frac{60}{31}\right)$, the ratio in which they share the money is equal to the ratio of their work done per day.

That is, it is equal to $\frac{1}{4} : \frac{1}{6} : \frac{1}{10} = \frac{15}{60} : \frac{10}{60} : \frac{6}{60} = 15 : 10 : 6$

Here, the total parts = 15 + 10 + 6 = 31

Hence, A's share = $\frac{15}{31} \times 31000 = ₹15000$, B's share = $\frac{10}{31} \times 31000 = ₹10000$ and

C's share is ₹31000 − (₹15000 + ₹10000)= ₹6000.

Try these

1. Vikram can do one-third of a work in p days. He can do $\frac{3}{4}$ th of the same work in _____ days.
2. If m persons can complete a work in n days, then $4m$ persons can complete the work in _____days and $\frac{m}{4}$ persons can complete the same work in _____ days.

Exercise 4.4

S6A5L1

1. **Fill in the blanks:**
   (i) A can finish a job in 3 days whereas B finishes it in 6 days. The time taken to complete the job working together is _____days.
   (ii) If 5 persons can do 5 jobs in 5 days, then 50 persons can do 50 jobs in _____ days.
   (iii) A can do a work in 24 days. If A and B together can finish the work in 6 days, then B alone can finish the work in _____ days.
   (iv) A alone can do a piece of work in 35 days. If B is 40% more efficient than A, then B will finish the work in _____days.
   (v) A alone can do a work in 10 days and B alone in 15 days. They undertook the work for ₹200000. The amount that A will get is _____.
2. 210 men working 12 hours a day can finish a job in 18 days. How many men are required to finish the job in 20 days working 14 hours a day?
3. A cement factory makes 7000 cement bags in 12 days with the help of 36 machines. How many bags can be made in 18 days using 24 machines?
4. A soap factory produces 9600 soaps in 6 days working 15 hours a day. In how many days will it produce 14400 soaps working 3 more hours a day?
5. If 6 container lorries can transport 135 tonnes of goods in 5 days, how many more lorries are required to transport 180 tonnes of goods in 4 days?
6. A can do a piece of work in 12 hours, B and C can do it 3 hours whereas A and C can do it in 6 hours. How long will B alone take to do the same work?
7. A and B can do a piece of work in 12 days, while B and C can do it in 15 days whereas A and C can do it in 20 days. How long would each take to do the same work?
8. Carpenter A takes 15 minutes to fit the parts of a chair while Carpenter B takes 3 minutes more than A to do the same work. Working together, how long will it take for them to fit the parts for 22 chairs?

9. A can do a work in 45 days. He works at it for 15 days and then, B alone finishes the remaining work in 24 days. Find the time taken to complete 80% of the work, if they work together.

10. A is thrice as fast as B. If B can do a piece of work in 24 days, then find the number of days they will take to complete the work together.

Exercise 4.5

Miscellaneous Practice Problems

1. A fruit vendor bought some mangoes of which 10% were rotten. He sold $33\frac{1}{3}$% of the rest. Find the total number of mangoes bought by him initially, if he still has 240 mangoes with him.

2. A student gets 31% marks in an examination but fails by 12 marks. If the pass percentage is 35%, find the maximum marks of the examination.

3. Sultana bought the following things from a general store. Calculate the total bill amount paid by her.

   (i) Medicines costing ₹800 with GST at 5%

   (ii) Cosmetics costing ₹650 with GST at 12%

   (iii) Cereals costing ₹900 with GST at 0%

   (iv) Sunglass costing ₹1750 with GST at18%

   (v) Air Conditioner costing ₹28500 with GST at 28%

4. P's income is 25% more than that of Q. By what percentage is Q's income less than P's?

5. Vaidegi sold two sarees for ₹2200 each. On one she gains 10% and on the other she loses 12%. Find her total gain or loss percentage in the sale of the sarees.

6. If 32 men working 12 hours a day can do a work in 15 days, then how many men working 10 hours a day can do double that work in 24 days?

7. Amutha can weave a saree in 18 days. Anjali is twice as good a weaver as Amutha.If both of them weave together, then in how many days can they complete weaving the saree?

8. P and Q can do a piece of work in 12 days and 15 days respectively. P started the work alone and then after 3 days, Q joined him till the work was completed. How long did the work last?

Challenging Problems

9. If the numerator of a fraction is increased by 50% and the denominator is decreased by 20%, then it becomes $\frac{3}{5}$. Find the original fraction.

10. Gopi sold a laptop at 12% gain. If it had been sold for ₹1200 more, the gain would have been 20%. Find the cost price of the laptop.

11. A shopkeeper gives two successive discounts on an article whose marked price is ₹180 and selling price is ₹108. Find the first discount percentage if the second discount is 25%.

12. Find the rate of compound interest at which a principal becomes 1.69 times itself in 2 years.

13. A small–scale company undertakes an agreement to make 540 motor pumps in 150 days and employs 40 men for the work. After 75 days, the company could make only 180 motor pumps. How many more men should the company employ so that the work is completed on time as per the agreement?

14. P alone can do $\frac{1}{2}$ of a work in 6 days and Q alone can do $\frac{2}{3}$ of the same work in 4 days. In how many days will they finish $\frac{3}{4}$ of the work, working together?

15. X alone can do a piece of work in 6 days and Y alone in 8 days. X and Y undertook the work for ₹48000. With the help of Z, they completed the work in 3 days. How much is Z's share?

SUMMARY

- When the S.P is more than the C.P, then there is a gain or profit. *Profit/Gain* = *S.P – C.P.*
- When the S.P is less than the C.P, then there is a loss. *Loss* = *C.P – S.P.*
- The profit or loss is always calculated on the cost price.
- Selling price = Marked price – Discount.
- **Formulae**

(i) Profit or Gain % $= \left(\frac{Profit}{C.P} \times 100\right)\%$

(ii) Loss % $= \left(\frac{Loss}{C.P} \times 100\right)\%$

(iii) Selling Price, S.P $= \frac{(100 + Profit\%)}{100} \times$ C.P (or) Cost Price, C.P $= \frac{100}{(100 + Profit\%)} \times$ S.P

(iv) Selling Price, S.P $= \frac{(100 - Loss\%)}{100} \times$ C.P (or) Cost Price, C.P $= \frac{100}{(100 - Loss\%)} \times$ S.P

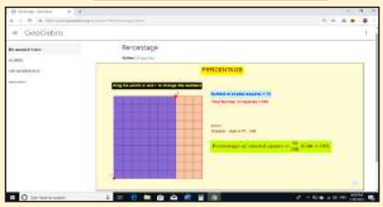
- When the interest is compounded annually, $A = P\left(1+\frac{r}{100}\right)^{n}$.
- When the interest is compounded half yearly, $A = P\left(1+\frac{r}{200}\right)^{2n}$.
- When the interest is compounded quarterly, $A = P\left(1+\frac{r}{400}\right)^{4n}$.
- When the interest is compounded annually but rate of interest differs year by year, $A = P\left(1+\frac{a}{100}\right)\left(1+\frac{b}{100}\right)\left(1+\frac{c}{100}\right)\ldots$ where a, b and c are interest rates for the I, II and III years respectively.
- When interest is compounded annually but time period is in fraction say $a\frac{b}{c}$ years, $A = P\left(1+\frac{r}{100}\right)^{a}\left(1+\frac{\frac{b}{c}\times r}{100}\right)$.
- C.I = A–P (Amount – Principal).
- The simple interest and the compound interest remains the same for the first year or the first conversion period.
- For 2 years, the difference in C.I and S.I is $C.I - S.I = P\left(\frac{r}{100}\right)^{2}$.
- For 3 years, the difference in C.I and S.I is $C.I - S.I = P\left(\frac{r}{100}\right)^{2}\left(3+\frac{r}{100}\right)$.
- x and y are said to vary directly if $\boldsymbol{y=kx}$ always, where k is called the proportionality constant and $k > 0$ assuming that y depends on x and so $\boldsymbol{k = \frac{y}{x}}$.
- x and y are said to vary inversely, if $\boldsymbol{xy = k}$ always, where k is called the proportionality constant and $k > 0$.
- There will be problems which involve a chain of two or more variations in them. This is called as compound variation.
- By finding the proportion, we can use the fact that the product of the extremes is equal to the product of the means to find the unknown (x) in the problem.
- By using the formula $\frac{P_1 \times D_1 \times H_1}{W_1} = \frac{P_2 \times D_2 \times H_2}{W_2}$, we can find the unknown (x).
- We can also find the unknown (x) by Multiplicative Factor Method.
- If two persons X and Y can complete some work individually in $\boldsymbol{a}$ and $\boldsymbol{b}$ days, their one day's work will be $\boldsymbol{\frac{1}{a}}$ and $\boldsymbol{\frac{1}{b}}$ respectively and X and Y together can complete the work in $\boldsymbol{\frac{ab}{a+b}}$ days.

ICT CORNER

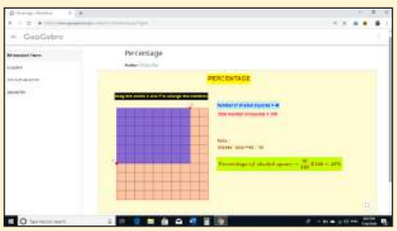
Step-1 Open the Browser type the URL Link given below (or) Scan the QR Code. GeoGebra work book named "LIFE MATHEMATICS" will open. Click on the worksheet named "Percentage".

Step-2 In the given worksheet you can drag the red points E and F to change the Blue rectangle. Find the ratio of Blue with the whole by counting the squares and check the ratio and percentage.

Expected Outcome



Step 1



Step 2

B358_8_MATHS_EM

Go through the remaining worksheets given for this chapter

Browse in the link

Life Mathematics:

https://www.geogebra.org/m/fqxbd7rz#chapter/409575 or Scan the QR Code.

*Pictures are indicatives only.

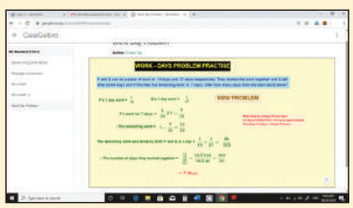
*If browser requires, allow Flash Player or Java Script to load the page.

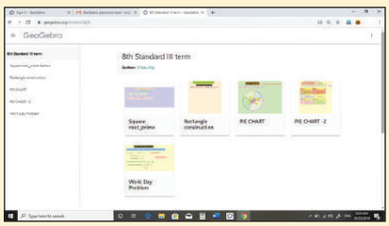
ICT CORNER

Step-1 Open the Browser type the URL Link given below (or) Scan the QR Code. GeoGebra work sheet named "8th Standard III term" will open. Select the work sheet named "Work Day Problem"

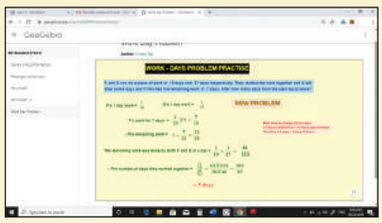
Step-2 Click on " NEW PROBLEM". Check the calculation and work out yourself.

Expected Outcome





Step 1



Step 2

B358_8_MATHS_EM

Browse in the link

Life Mathematics:

https://www.geogebra.org/m/xmm5kj9r or Scan the QR Code.