Chapter-4

Practical Geometry

K2C9C7

In class VII you have learnt how to draw triangles. A triangle consists of 3 sides and 3 angles. To draw a particular triangle we need to have three sets of measurements of sides and angles (SSS or SAS or ASA).

In this chapter we are going to learn the method of drawing a four sided closed figure, namely, a *quadrilateral*. Now as three measurements are sufficient to draw a unique triangle can we draw a quadrilateral uniquely with four measurements ? Or, do we need more than four measurements ?

Let us clarify the concept with the help of the following discussion.

A quadrilateral is a closed figure which consists of four sides, four angles and two diagonals. Therefore the shape of a particular quadrilateral depends on these 10 measurements.

Suppose we have to draw a quadrilateral having sides measuring 3 cm, 5 cm, 6.5 cm and 7 cm respectively.

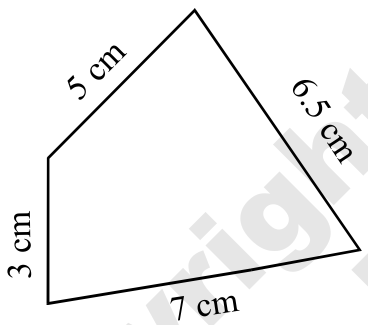
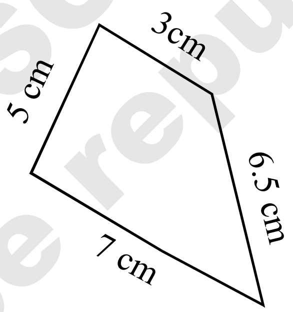


fig -i



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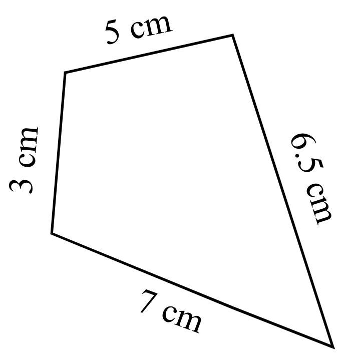


fig -iii

Now, the question is whether the three quadrilaterals drawn with the given measures as shown in the figure (i), (ii) and (iii) are same? Are not the angles and diagonal length of the three quadrilaterials different ? This means that it is not possible to draw a unique quadrilaterial by four sides only.

Again, as we can use the successive measures 5cm, 3cm, 5cm and 3cm to draw a rectangle (fig iv), in the same way we can also draw a parallelogram with the same measures (fig v). But, once the measure of a diagonal for fig (iv) or the measure of an angle for fig (v) is given, rectangle or the parallelogram becomes fixed.

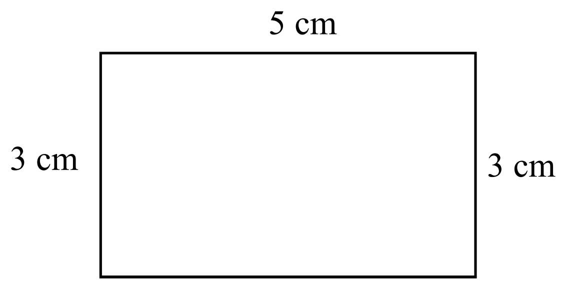


fig -iv

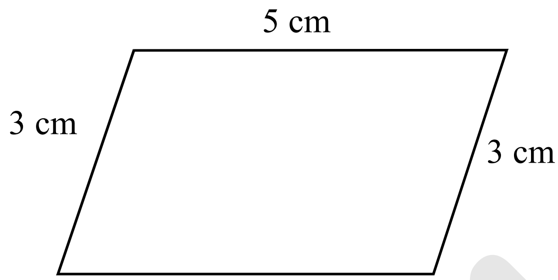


fig -v

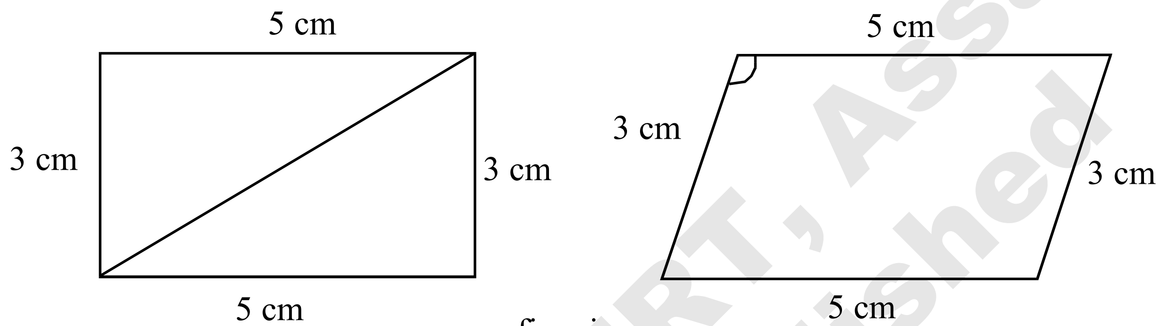


fig -vi

Therefore, we can say that to draw a quadrilateral uniquely, we need at least **five** measures of sides and angles.

4.1 Construction of a Quadrilateral

From our preceeding discussion we have understood that we need at least five measurements to draw a unique quadrilateral. A unique quadrilateral can be constructed with the help of the following measures–

- When four sides and one diagonal are given.
- When two diagonals and three sides are given.
- When four sides and one angle are given.
- When two adjacent sides and three angles are given.
- When three sides and two included angles are given.
- When other special properties are known.

Now we will try to construct a quadrilateral by measures as given above.

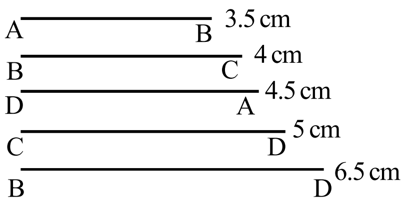
4.1.1 Construction of a quadrilateral when the length of four sides and one diagonal are given

Example : Construct a quadrilateral ABCD where

AB = 3.5 cm, BC = 4 cm, CD = 5 cm
DA = 4.5 cm and BD = 6.5 cm

Solution : [As it will help us in visualising the quadrilateral, first we draw a rough sketch and mark the measurements]. (fig vii).

Step 1 : First we will draw five line segments using a drawing scale.



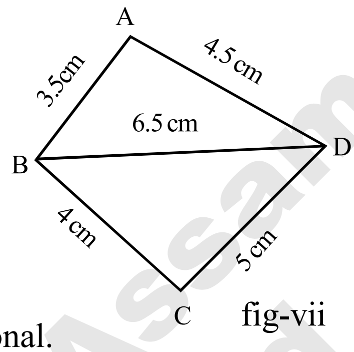


fig-vii

Step 2 : Now we are going to fix the position of the diagonal. To do this, we draw a line segment of length slightly greater than 6.5 cm with the help of the scale. From the rough diagram, it follows that the diagonal BD= 6.5 cm. Next taking B as the centre we draw an arc of radius 6.5 cm. We assume the arc to cut BD at D

So, BD=6.5 cm (fig. viii).

fig-viii

[The five line segments we have drawn in step 1 are for reference in the next steps. In stead, we could have drawn the line segment BD of length 6.5 cm with the help of scale]

Step 3 : Now we locate the point A. Taking B as the centre we draw an arc of radius 3.5 (fig ix) and taking D as centre we draw another arc of radius 4.5 cm . The two arcs intersect at A (fig x). BA and DA arc joined (fig-x)

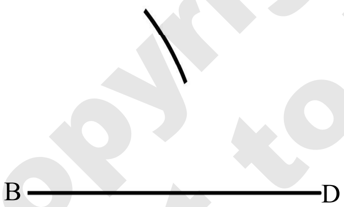


fig-ix

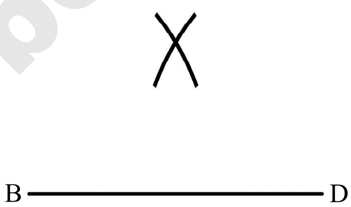


fig-x

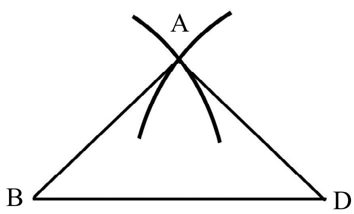


fig-xi

Step 4 : Next we locate the point C. The point C would be on the side opposite to A, with reference to BD. So taking B as centre we draw an arc of radius 4 cm and taking D as centre we draw another arc of radius 5 cm on the opposite side of A (fig-xiii). The point of intersection of the two arcs is C. Next we join BC and CD to complete ABCD (fig xiv) . Thus, ABCD is the required quadrilateral.

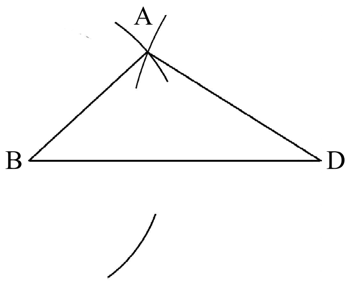


fig-xii

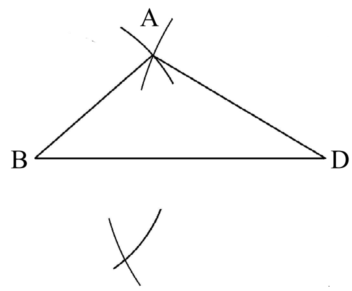


fig-xiii

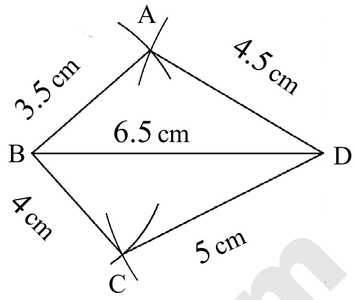


fig-xiv

4.1.2 Given two diagonals and three sides to construct the quadrilateral

Example (i) : Construct a quadrilateral PQRS where

QR = 7.5cm, PR = 6cm, PS = 6cm,

RS = 5cm and QS = 10cm

[As we did in the above example, here also we first draw a rough sketch of the quadrilateral to get an idea about the positions of the sides and the diagonal's]. From the rough sketch we see that, the lengths of the three sides QR, RS and PS and the two diagonals PR (6cm) and QS (10 cm) are given. (fig.xv).

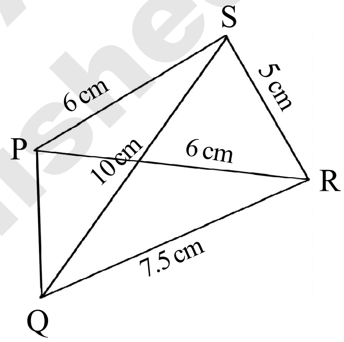
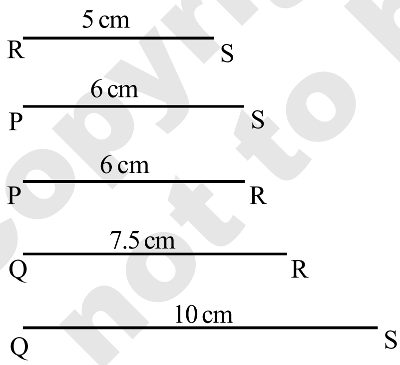


fig-xv

Solution :

Step 1



With the help of a marking scale we draw the line segments of lengths 5cm, 6cm, 6cm, 7.5 cm and 10 cm and to fix the position of the diagonal we take a line segment PX sligthly greater than 6 cm. Taking P as the centre we draw an arc of radius 6 cm (equal to PR) which cuts PX at R. (fig-xvi)

fig-xvi

Step 2 : Again taking P as the centre we draw an arc of radius 6cm to the upper side of PR and taking R as the centre we draw another arc of raidus 5 cm on the same side of PR, so that the two arcs intersect at the point S. (Since PS = 6 cm, RS = 5 cm, S common point) fig- xvii)

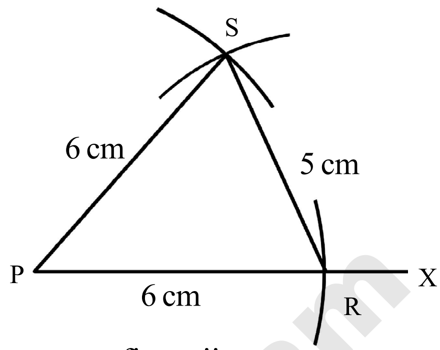


fig-xvii

Step 3 : Next we are going to locate the point Q which will be on the opposite side of S (since QS is diagonal) We take S as the centre and draw an arc of radius 10 cm (lower side of PR, opposite to S) and again taking R as the centre we draw another arc of radius 7.5 cm (opposite to S). The point of intersection of these two arcs is Q (since QS =10 cm, RQ =7.5cm, Q common point) -(fig-xviii)

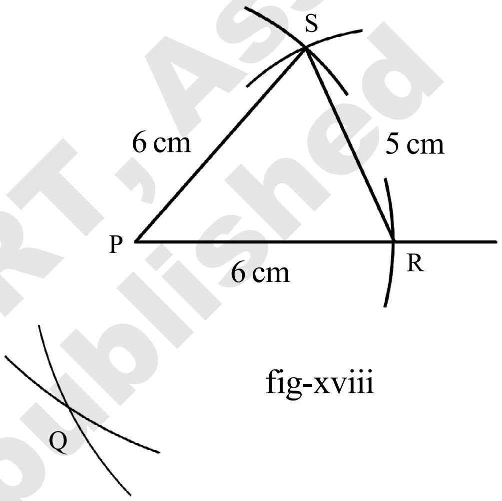


fig-xviii

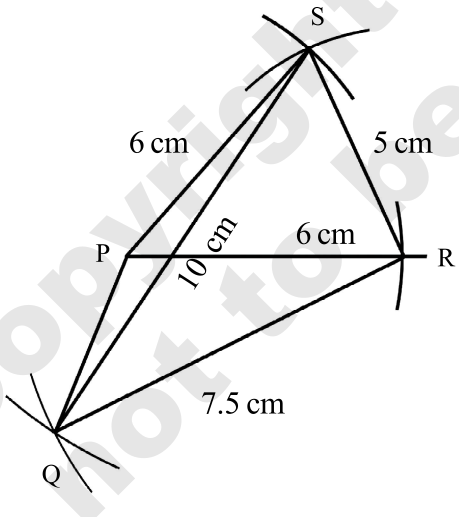
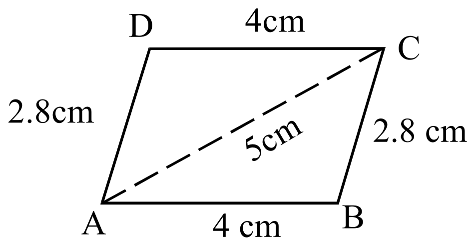


fig-xix

Step 4 : Now join PQ, QS and RQ and the quadrilateral PQRS (fig-xix) is completed.

Example (ii) : Construct a parallelogram ABCD where AB=4cm, BC=2.8 cm and AC=5 cm.

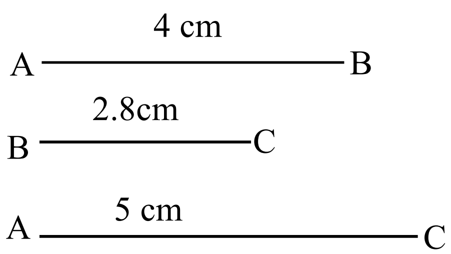


First draw a rough sketch of the parallelogram ABCD. The opposite sides of a parallelogram are parallel and equal. Therefore, we have, BC=AD=2.8 cm and AB = DC = 4 cm and AC=5 cm is the diagonal.

Given the four sides and a diagonal of quadrilateral try to construct the parallelogram.

Alternative Method of construction

Step 1 :



With the help of a marking scale draw the line segments measuring 4 cm, 2.8 cm and 5 cm. Next draw a line segment AX of length slightly greater than 4 cm, Taking A as the centre draw an arc of radius 4 cm which will cut AX at B.
So, AB = 4cm

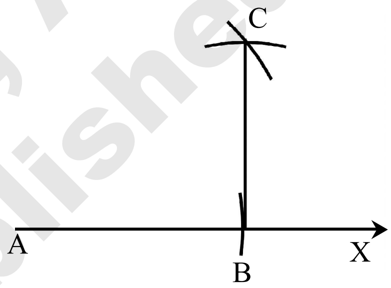


fig -xx

[We could have also started with AB=4cm. The three line segments we have drawn are for future reference only]

Now, taking B as the centre, draw an arc of radius 2.8 cm (fig-xx) and again taking A as centre draw another arc of radius 5 cm. The two arcs will intersect at C (since C is the common point of AC and BC). Now join AC and BC.

Step 2 : Now to locate the point D, take A as the centre and draw an arc of radius 2.8 cm. Next taking C as the centre draw another arc of radias 4 cm. The two arcs will intersect at the point D. Join AD and CD to complete the parallelogram (fig -xxi)

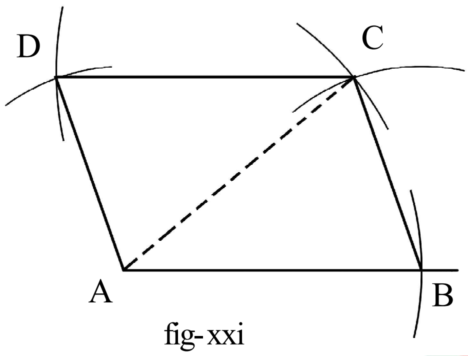


fig- xxi

Example (iii) : Construct a Rhombus ABCD where, diagonal AC= 6 cm and BD=8 cm.

Solution :

Step 1 : With the help of a marking scale draw a line segment AC measuring 6 cm.

Step 2 : Since the two diagonals of a Rhombus bisect each other at right angles, at their point of intersection, we will draw the perpendicular bisector of AC. This bisector cuts AC at O.

Step 3 : Now taking O as the centre draw two arcs of radius 4cm (half of BD) one above AC and the other below AC. The two arcs will cut the perpendicular bisector at B and D.

Step 4 : Join AB, AD, DC and CB to complete the Rhombus ABCD (fig-xxii)

[The sides AB, BC, CD, DA may be measured to check whether they are equal or not]

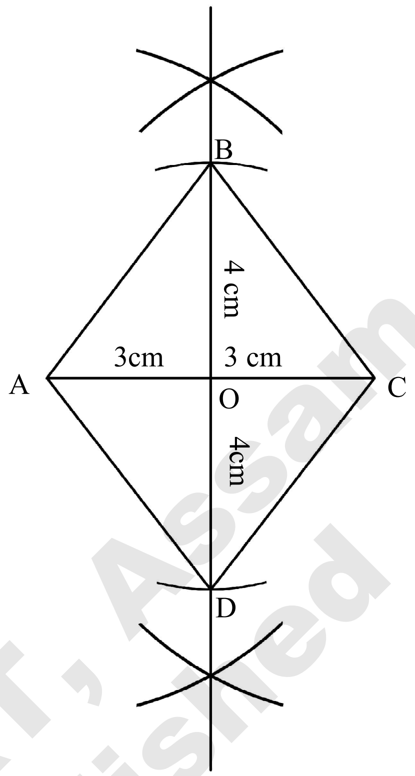


fig -xxii

Example (iv) : Construct a Rhombus ABCD where side AB=4 cm and diagonal AC= 6 cm.

Solution : All the four sides of a rhombus are equal, therefore (AB=BC=CD=DA)= 4cm

Steps of Construction :

1. Draw a line segment AC of length 6 cm.
2. Taking A and C as centres draw arcs of radius 4cm above and below the line AC. Let B be the point of intersection of the arcs above AC and D be the point of intersection below AC. Now Join AB, BC, CD, DA to complete the rhombus ABCD (fig-xxii)

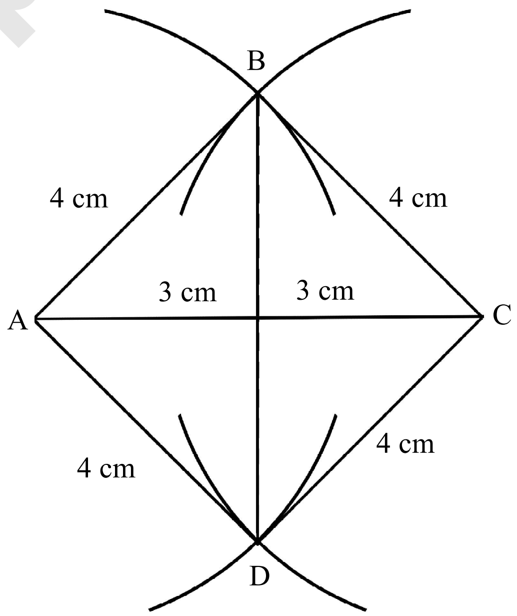


fig -xxiii

Exercise- 4.1

1. Construct the following quadrilaterals :

i) Quadrilateral ABCD where, AB = 4 cm, BC = 6 cm, CD = 5 cm, DA = 5.5 cm and diagonal AC = 7 cm.

ii) Quadrilateral ABCD where,AB = 4 cm, BC = 3 cm, DA = 2.8 cm, diagonal AC = 5 cm, diagonal BD = 4.5cm.

iii) Quadrilateral PQRS where, QR= 4.5 cm, PS = 5.5 cm, RS= 5 cm, diagonal PR = 5.5cm, diagonal QS = 7 cm.

iv) Parallelogram EFGH where, FG = 7 cm, GH = 5.5 cm and HF = 8.5 cm.

v) Rhombus DEFG where, DE = 5 cm and EG = 6.5 cm.

vi) Rhombus LMNO where, LN = 6cm, MO = 7cm

Till now we have discussed how to construct a quadrilateral when the measurement of five sides viz. (i)four sides, one diagonal (ii) three sides, two diagonals are known. Now we are going to consider angle measures along with sides for construction of quadrilaterals. For example, (i) four sides, one angle, (ii) three sides, two angles and (iii) two sides, three angles.

You have already learnt in class VI how to construct angles of measure 30^0, 45^0, 60^0, 90^0, 120^0, 150^0 105^0 etc. We need this concept for the following constructions.

4.1.3 When four sides and one angle is given

Example : Construct a quadrilateral ABCD, where AB = 4.5 cm, BC = 3.5 cm,CD = 4.8 cm,AD = 4 cm and $\angle B$=120^0

[As in earlier cases, here also we first make a rough sketch of ABCD with $\angle B$=120^0, AB = 4.5 cm, BC = 3.5 cm, AD = 4 cm and DC = 4.8 cm.]

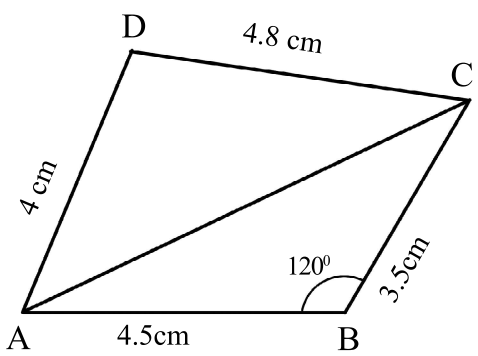


fig -xxiv

Construction :

Step 1 : Draw AB = 4.5 cm (with scale)

Step 2 : Construct $\angle$B = 120^0 with AB as its one side.

Step 3: Taking B as centre draw an arc of radius 3.5cm. cutting the other side of $\angle B$ at C.

Step 4: Draw an arc of radius 4cm, taking A as centre.

Step 5:Draw an arc of radius 4.8cm with C as centre which interesects with the arc of step 4.

Step 6 **:** The point of intersection so obtained is D (since AD=4cm and CD= 4.8, D being the common point)

Step 7 **:** Join AD and CD to complete ABCD (fig-xxv)

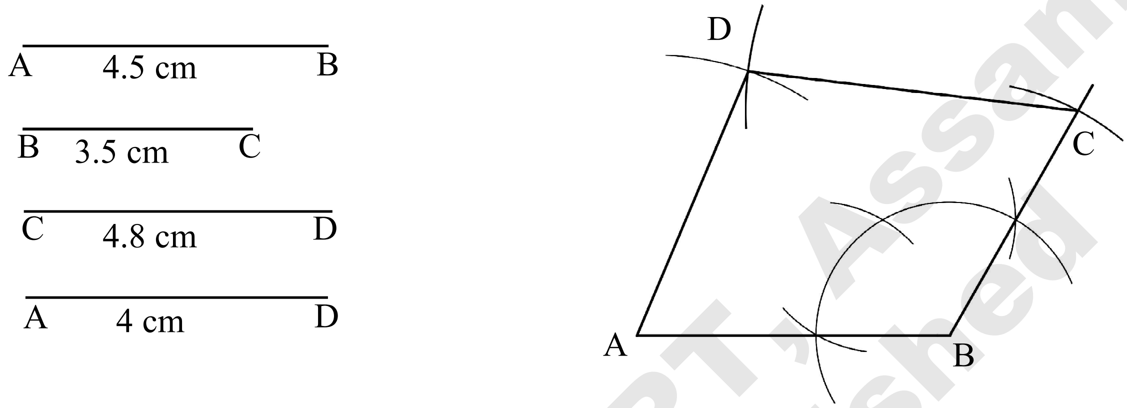


fig-xxv

4.1.4 Given two adjacent sides and three angles to contstruct the quadrilateral

Example : Construct a quadrilateral ABCD, where $\angle A = 60^0$, $\angle B = 105^0$, $\angle C = 105^0$ and AB = 6 cm, BC = 4.5 cm.
(Draw a rough sketch first)

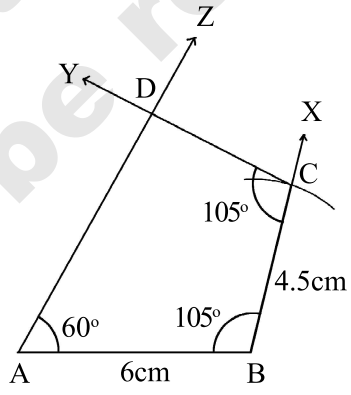


fig- xxvi)

Solution :

Step 1 : With the help of a scale draw AB=6cm

Step 2 : Construct $\angle A = 60^0$

Step 3 : draw $\angle B = 105^0$ (by protractor or any other way)

Step 4 : With B as centre draw an arc of radius 4.5 cm which interesects the ray $\overrightarrow{BX}$ at C (since BC=4.5 cm)

Step 5 : Draw $\angle C = 105^0$ and mark $\overrightarrow{CY}$ ray.

Step 6 : The point of intersection of the two rays $\overrightarrow{AZ}$ and $\overrightarrow{CY}$ is the fourth vertex D of the quadrilateral. Thus ABCD is the required quadrilateral (fig. xxvi)

4.1.5 Given three sides and two included angles to construct of the quadrilateral

Example : Construct a quadrilateral PQRS where, PQ = 3.5 cm, QR= 3 cm, RS = 4 cm, $\angle Q = 75^0$ and $\angle R = 120^0$

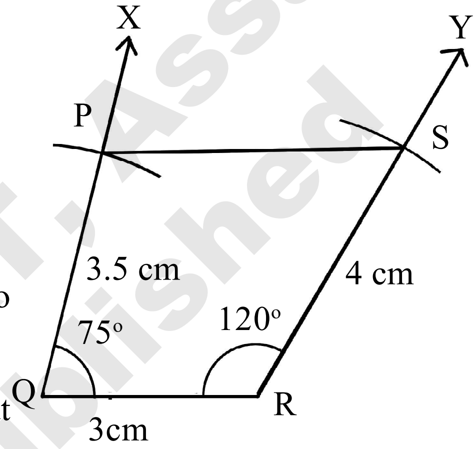


fig -xxvii

Solution :

Step 1: Draw the line segment QR = 3 cm

Step 2 : Draw $\angle Q = 75^0$ (by protractor or any other way)

Step 3 : Draw $\angle R = 120^0$

Step 4 : Draw an arc of radius 3.5 cm with Q as centre to cut $\overrightarrow{QX}$ at P.

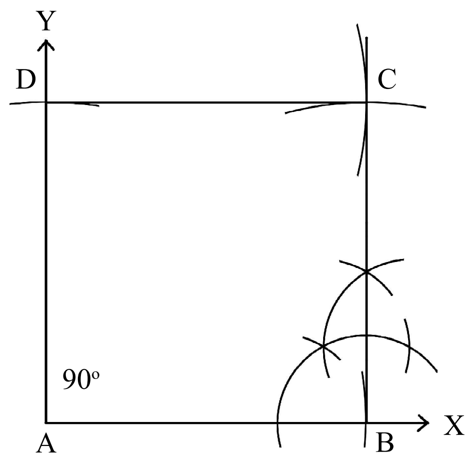
Step 5 : Draw an arc of radius 4 cm with centre R to cut $\overrightarrow{RY}$ at S. (Since RS = 4 cm)

Step 6 : Complete the quadrilateral of PQRS by joining PS. (fig-xxvii)

4.1.6 Construction of quadrilateral using their special properties.

We have already considered construction of Rhombus and parallelogram by using their respective properties. Similarly we are going to construct a square using its own property.

Example : Construct a square of side 5 cm.



Step 1: Draw a line segment AX of length slightly greater than 5 cm. Draw an arc of radius 5 cm with A as a centre to cut $\overrightarrow{AX}$ at B. AB is a side of the square.

Step 2: Draw an angle of 90^0 at A using a protractor or compass. Draw an arc of radius 5 cm with A as centre. The arc cuts AY at D. So AD=5cm.

Step 3: Taking D and B as centres draw two arcs of radius equal to 5 cm to intersect at C. Join DC and BC and complete the square ABCD. (AB=AD = BC =CD = 5cm)

Activity: (i) Try to construct the square using other methods.
(ii) Construct a rectangle with adjacent sides of length 4 cm and 5 cm respectively.

Group Activity :

Make groups of 4-5 students each and discuss the following.

* Try to construct a quadrilateral whose one side is 7 cm and four angles are 75^0, 85^0, 110^0 and 90^0 respectively. Can it be drawn?

* Can a kite ABCD be constructed where AD=4 cm, AC=8cm and CD=6 cm? (use property of kite)

* A quadrilateral cannot be constructed if its 4 angles and one side are given. Justify.

Note : Drawing of rough sketch is not mandatory. It has been done only to get an idea about the final drawing. After adequate practice you will no more need rough drawing

Exercise 4.2

1. Draw a quadrilateral ABCD where AB = 6cm, BC = 7cm, CD = 6.5cm, DA = 5.5cm and $\angle B = 105^0$
2. Draw a quadrilateral ABCD with AB = 5cm, BC = 4 cm, CD = 3.5 cm, DA = 4.5 cm and $\angle C = 75^0$
3. Draw a quadrilateral ABCD where AB = 4cm, BC = 7cm, $\angle A = 105^0$, $\angle B = 75^0$ and $\angle C = 120^0$
4. Draw a quadrilateral EFGH, where, EF = 5cm, FG = 7.5cm, $\angle E = 90^0$, $\angle G = 105^0$ and $\angle H = 80^0$
5. Draw a parallelogram PQRS where PQ = 6cm, QR = 7 cm and $\angle S = 85^0$
6. Draw a rectangle LMNO where LM = 6cm and MN = 4 cm
7. Draw a quadrilateral PQRS where PQ = 6 cm, QR = 7cm, RS = 7.5 cm, $\angle Q = 105^0$ and $\angle R = 80^0$
8. Draw a quadrilateral ABCD where AB = 4.5 cm, BC = 5.5cm, CD = 5 cm, $\angle B = 68^0$, and $\angle C = 90^0$
9. Draw a rectangle having adjacent sides of lengths 5 cm and 7 cm respectively.
(Protractor can be used where necessary)

What we have learnt

In order to construct a quadrilateral uniquely, at least five measures are needed

- measures of four sides and one diagonal are given.
- measures of three sides and two diagonals are given.
- measures of four sides and one angle are given.
- measures of two adjascent sides and three angles are given.
- measures of three sides and two included angles are given.
- its special properties are known.

❑❑❑

- Mathematics is the most beautiful and most powerful creation of the human spirit.

  - Stefan Banach
- Mathematics is the abstract key which turms the look of the physical universe.

  - John Polkinghome
- Do Mathematics (you) can Mathematics.

  - K.D. Krove