Exercise 12.3

Answer 1E.

(A) $\left(\vec{a}.\vec{b}\right).\vec{c}$

This is a meaningless expression because $\vec{a}.\vec{b}$ is a scalar quantity and $\left(\vec{a}.\vec{b}\right).\vec{c}$ becomes the dot product of a scalar and a vector, which is not possible.

(B) $\left(\vec{a}.\vec{b}\right)\vec{c}$ (meaningful)

Since $\left(\vec{a}.\vec{b}\right)$ is a scalar quantity and a vector $\vec{c}$ can be multiplied by the scalar quantity.

(C) $\left|\vec{a}\right|\left(\vec{b}.\vec{c}\right)$ (meaningful)

Here $\left|\vec{a}\right|$ is magnitude of the vector $\vec{a}$, which is a scalar quantity and $\left(\vec{b}.\vec{c}\right)$ is also a scalar quantity. Two scalar quantities can be multiplied.

(D) $\vec{a}.\left(\vec{b}+\vec{c}\right)$ (meaningful)

Since $\left(\vec{b}+\vec{c}\right)$ is a vector and it is possible to find the dot product of two vectors.

(E) $\vec{a}.\vec{b}+\vec{c}$

This is a meaningless expression because $\vec{a}.\vec{b}$ is a scalar quantity and $\vec{a}.\vec{b}+\vec{c}$ becomes the addition of a scalar and a vector, which is impossible.

(F) $\left|\vec{a}\right|.\left(\vec{b}+\vec{c}\right)$

This is a meaningless expression because $\vec{b}+\vec{c}$ is a vector quantity and $\left|\vec{a}\right|$ is a scalar quantity. The dot product of a vector with a scalar quantity is meaningless.

Answer 2E.

If $\vec{a} = \langle a_1, a_2 \rangle$ and $\vec{b} = \langle b_1, b_2 \rangle$ then their dot product is the number $\vec{a} \cdot \vec{b}$ given by

$$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2$$

In this problem, we are given that $\vec{a} = \langle -2, 3 \rangle, \vec{b} = \langle 0.7, 1.2 \rangle$.

Using the formula above, their dot product would be

$$\langle -2, 3 \rangle \cdot \langle 0.7, 1.2 \rangle = (-2)(0.7) + 3(1.2)$$
$$= -1.4 + 3.6$$
$$= 2.2$$

Answer 3E.

If $\vec{a} = \langle a_1, a_2 \rangle$ and $\vec{b} = \langle b_1, b_2 \rangle$ then their dot product is the number $\vec{a} \cdot \vec{b}$ given by

$$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2$$

In this problem, we are given that $\vec{a} = \left\langle -2, \frac{1}{3} \right\rangle, \vec{b} = \langle -5, 12 \rangle$. Using the formula above, their dot product would be

$$\left\langle -2, \frac{1}{3} \right\rangle \cdot \langle -5, 12 \rangle = (-2)(-5) + \frac{1}{3}(12) = 10 + 4 = 14$$

Answer 4E.

540314-12.3-4E AID: 500 | 22/11/2015

Consider the following two vectors:

$\mathbf{a} = \langle 6, -2, 3 \rangle$, and

$\mathbf{b} = \langle 2, 5, -1 \rangle$.

The objective is to find the value of $\mathbf{a} \cdot \mathbf{b}$.

The dot product of $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$, and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$ is the number $\mathbf{a} \cdot \mathbf{b}$, defined by as follows:

$$\mathbf{a} \cdot \mathbf{b} = \langle a_1, a_2, a_3 \rangle \cdot \langle b_1, b_2, b_3 \rangle$$
$$= a_1 b_1 + a_2 b_2 + a_3 b_3 \text{. ......(1)}$$

Then, the dot product of $\mathbf{a} = \langle 6, -2, 3 \rangle$, and $\mathbf{b} = \langle 2, 5, -1 \rangle$ will be,

$$\mathbf{a} \cdot \mathbf{b} = \langle 6, -2, 3 \rangle \cdot \langle 2, 5, -1 \rangle$$
$$= 6(2) + (-2)(5) + 3(-1) \text{ Use (1)}$$
$$= 12 - 10 - 3$$
$$= -1.$$

Therefore, the dot product of $\mathbf{a} = \langle 6, -2, 3 \rangle$, and $\mathbf{b} = \langle 2, 5, -1 \rangle$ is $\mathbf{a} \cdot \mathbf{b} = \boxed{-1}$.

Answer 5E.

Consider the following vectors:

$$\mathbf{a} = \left\langle 4, 1, \frac{1}{4} \right\rangle$$

$$\mathbf{b} = \langle 6, -3, -8 \rangle$$

Find the dot product of the given vectors.

The dot product of two vectors $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$ is:

$$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 .$$

So, use the formula for $\mathbf{a} \cdot \mathbf{b}$ and find the dot product of the given vectors $\mathbf{a}$ and $\mathbf{b}$. This can be computed as follows:

$$\begin{aligned} \mathbf{a} \cdot \mathbf{b} &= \left\langle 4, 1, \frac{1}{4} \right\rangle \cdot \langle 6, -3, -8 \rangle \\ &= 4(6) + (-3)(1) + (-8)\left(\frac{1}{4}\right) \quad \text{Multiply the corresponding elements} \\ &= 24 - 3 - 2 \\ &= 19 \end{aligned}$$

Therefore, the dot product of the given vectors is,

$$\boxed{\mathbf{a} \cdot \mathbf{b} = 19}.$$

Answer 6E.

540314-12.3-6E AID: 500 | 22/11/2015

Consider the following two vectors:

$\mathbf{a} = \langle p, -p, 2p \rangle$, and

$\mathbf{b} = \langle 2q, q, -q \rangle$.

The objective is to find the value of $\mathbf{a} \cdot \mathbf{b}$.

The dot product of $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$, and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$ is the number $\mathbf{a} \cdot \mathbf{b}$, defined by as follows:

$\mathbf{a} \cdot \mathbf{b} = \langle a_1, a_2, a_3 \rangle \cdot \langle b_1, b_2, b_3 \rangle$

$= a_1 b_1 + a_2 b_2 + a_3 b_3$. ……(1)

Then, the dot product of $\mathbf{a} = \langle p, -p, 2p \rangle$, and $\mathbf{b} = \langle 2q, q, -q \rangle$ will be,

$\mathbf{a} \cdot \mathbf{b} = \langle p, -p, 2p \rangle \cdot \langle 2q, q, -q \rangle$

$= p(2q) + (-p)(q) + 2p(-q)$ Use (1)

$= 2pq - pq - 2pq$

$= -pq$.

Therefore, the dot product of $\mathbf{a} = \langle p, -p, 2p \rangle$, and $\mathbf{b} = \langle 2q, q, -q \rangle$ is $\mathbf{a} \cdot \mathbf{b} = \boxed{-pq}$.

Answer 7E.

540314-12.3-7E AID: 500 | 22/11/2015

Consider the following two vectors:

$\mathbf{a} = 2\mathbf{i} + \mathbf{j}$, and

$\mathbf{b} = \mathbf{i} - \mathbf{j} + \mathbf{k}$.

The objective is to find the value of $\mathbf{a} \cdot \mathbf{b}$.

The dot product of $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$, and $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$ is the number $\mathbf{a} \cdot \mathbf{b}$, defined by as follows:

$\mathbf{a} \cdot \mathbf{b} = (a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}) \cdot (b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k})$

$= a_1 b_1 + a_2 b_2 + a_3 b_3$. ……(1)

Then, the dot product of $\mathbf{a} = 2\mathbf{i} + \mathbf{j}$, and $\mathbf{b} = \mathbf{i} - \mathbf{j} + \mathbf{k}$ will be,

$\mathbf{a} \cdot \mathbf{b} = (2\mathbf{i} + \mathbf{j}) \cdot (\mathbf{i} - \mathbf{j} + \mathbf{k})$

$= (2\mathbf{i} + 1\mathbf{j} + 0\mathbf{k}) \cdot (1\mathbf{i} - 1\mathbf{j} + 1\mathbf{k})$

$= 2(1) + 1(-1) + 0(1)$ Use (1)

$= 2 - 1 + 0$

$= 1$.

Therefore, the dot product of $\mathbf{a} = 2\mathbf{i} + \mathbf{j}$, and $\mathbf{b} = \mathbf{i} - \mathbf{j} + \mathbf{k}$ is $\mathbf{a} \cdot \mathbf{b} = \boxed{1}$.

Answer 8E.

Consider the following two vectors:

$\mathbf{a} = 3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$, and $\mathbf{b} = 4\mathbf{i} + 5\mathbf{k}$.

The objective is to find the value of $\mathbf{a} \cdot \mathbf{b}$.

The dot product of $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$, and $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$ is the number $\mathbf{a} \cdot \mathbf{b}$, defined as follows:

$$\begin{aligned}\mathbf{a} \cdot \mathbf{b} &= (a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}) \cdot (b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}) \\ &= a_1b_1 + a_2b_2 + a_3b_3 \qquad \cdots\cdots(1)\end{aligned}$$

The dot product of $\mathbf{a} = 3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$, and $\mathbf{b} = 4\mathbf{i} + 5\mathbf{k}$ will be,

$$\begin{aligned}\mathbf{a} \cdot \mathbf{b} &= (3\mathbf{i} + 2\mathbf{j} - \mathbf{k}) \cdot (4\mathbf{i} + 5\mathbf{k}) \\ &= (3\mathbf{i} + 2\mathbf{j} - 1\mathbf{k}) \cdot (4\mathbf{i} + 0\mathbf{j} + 5\mathbf{k}) \\ &= 3(4) + 2(0) - 1(5) \qquad \text{Use equation (1).} \\ &= 12 + 0 - 5 \\ &= 7\end{aligned}$$

Therefore, the dot product of $\mathbf{a} = 3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$, and $\mathbf{b} = 4\mathbf{i} + 5\mathbf{k}$ is $\mathbf{a} \cdot \mathbf{b} = \boxed{7}$.

Answer 9E.

Consider the magnitudes,

$|\mathbf{a}| = 6$, $|\mathbf{b}| = 5$

Angle between the vectors is $2\pi/3$.

That is $\theta = 2\pi/3$

Write the angle formula between the vectors $\mathbf{a}$ and $\mathbf{b}$

$$\cos\theta = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$$

Use the given angle in the formula to find $\mathbf{a} \cdot \mathbf{b}$

$$\begin{aligned}\cos\frac{2\pi}{3} &= \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|} \\ -\frac{1}{2} &= \frac{\mathbf{a} \cdot \mathbf{b}}{(6)(5)} \qquad \left(\cos\frac{2\pi}{3} = -\frac{1}{2}\right) \\ -\frac{1}{2} &= \frac{\mathbf{a} \cdot \mathbf{b}}{30}\end{aligned}$$

$$\begin{aligned}\mathbf{a} \cdot \mathbf{b} &= -\frac{30}{2} \\ &= -15\end{aligned}$$

Therefore $\mathbf{a} \cdot \mathbf{b} = \boxed{-15}$

Answer 10E.

Given

$$|a| = 3,\ |b| = \sqrt{6}$$

Angle θ between vectors **a** and **b** is 45o.

To solve this, we use the formula from the definition of the dot product:

a ·**b** = $|a||b|\cos(\theta)$

Plugging the given values into the formula,

a ·**b** = (3) ($\sqrt{6}$) (cos(45o))

$$= 3\sqrt{6}\left(\frac{\sqrt{2}}{2}\right)$$

$$= 3\sqrt{3}$$

Answer 11E.

$\vec{u}$ is a unit vector ,then

$$|\vec{u}| = 1, |\vec{v}| = 1, |\vec{w}| = 1,$$

Angle between $\vec{u}$ and $\vec{v}$ is $\theta = 60^o$

Then $\vec{u}.\vec{v} = |\vec{u}||\vec{v}|.\cos\theta$

$$= 1.1\cos 60^o$$

$$= \boxed{\frac{1}{2}}$$

Angle between $\vec{u}$ and $\vec{w}$ is $\theta = 180 - 60^o$

$$= 120^o$$

Then

$$\vec{u}.\vec{w} = |\vec{u}||\vec{w}|\cos 120^o$$

$$= 1.1\left(-\frac{1}{2}\right).$$

$$= \boxed{-\frac{1}{2}}$$

Answer 12E.

Angle between $\vec{u}$ and $\vec{v}$ is $\frac{90^o}{2} = 45^o$

Then $\vec{u}.\vec{v} = |\vec{u}||\vec{v}|\cos 45^o$

$= 1.\frac{1}{\sqrt{2}}.\frac{1}{\sqrt{2}}$ (as $|\vec{v}| = \frac{1}{\sqrt{2}}$, half the length of diagonal of a unit square)

$= \boxed{\frac{1}{2}}$

Angle between $\vec{u}$ and $\vec{w}$ is 90^0, Then

$\vec{u}.\vec{w} = |\vec{u}||\vec{w}|\cos 90^o$

$= 1.1.0$

$= \boxed{0}$.

Answer 13E.

We know that $\hat{i}, \hat{j}$ *and* $\hat{k}$ are unit vectors along x-axis, y-axis and z-axis.

Thus these three are perpendicular to each other.

(a)

$\hat{i}.\hat{j} = |\hat{i}|.|\hat{j}|\cos 90^0$

$= 1.1.0$

$= 0$

$\hat{j}.\hat{k} = |\hat{j}||\hat{k}|\cos 90^o$

$= 1.1.0$

$= 0$

$\hat{k}.\hat{i} = |\hat{k}||\hat{i}|.\cos 90^o$

$= 1.1.0$

$= 0$

(b)

$\hat{i}.\hat{i} = |\hat{i}|.|\hat{i}|\cos 0^o$

$= 1.1.1$

$= 1$

$\hat{j}.\hat{j} = |\hat{j}||\hat{j}|\cos 0^o$

$= 1.1.1$

$= 1$

$\hat{k}.\hat{k} = |\hat{k}||\hat{k}|\cos 0^o$

$= 1.1.1$

$= 1.$

Answer 14E.

If a street vendor sells a hamburgers, b hot dogs and c soft drinks on a given day and if he charges \$2 for a hamburger, \$ 1.50 for a hot dog and \$1 for a soft drink, if

$$\vec{A} = \langle a, b, c \rangle \quad and \quad \vec{P} = \langle 2, 1.5, 1 \rangle$$

Then $\mathbf{A} \cdot \mathbf{P} = \langle a, b, c \rangle \cdot \langle 2, 1.5, 1 \rangle$

$= 2a + 1.5b + 1c$

$= 2a + 1.5b + c$

i.e $\mathbf{A} \cdot \mathbf{P}$ denotes the total amount which the street vendor earns in a day.

Answer 15E.

First find the values of $|\mathbf{a}|$ and $|\mathbf{b}|$.

Now, the value of $|\mathbf{a}|$, for the vector; $\mathbf{a} = \langle 4, 3 \rangle$, will be,

$|\mathbf{a}| = |\langle 4, 3 \rangle|$

$= \sqrt{4^2 + 3^2}$ Use $|\langle a_1, a_2 \rangle| = \sqrt{a_1^2 + a_2^2}$

$= \sqrt{16 + 9}$

$= \sqrt{25}$

$= 5$.

And, the value of $|\mathbf{b}|$, for the vector; $\mathbf{b} = \langle 2, -1 \rangle$, will be,

$|\mathbf{b}| = |\langle 2, -1 \rangle|$

$= \sqrt{2^2 + (-1)^2}$ Use $|\langle a_1, a_2 \rangle| = \sqrt{a_1^2 + a_2^2}$

$= \sqrt{4 + 1}$

$= \sqrt{5}$.

Therefore, the values of $|\mathbf{a}|$ and $|\mathbf{b}|$ for $\mathbf{a} = \langle 4, 3 \rangle$, and $\mathbf{b} = \langle 2, -1 \rangle$ are,

$|\mathbf{a}| = 5$ and $|\mathbf{b}| = \sqrt{5}$.

Next, find the dot product $\mathbf{a} \cdot \mathbf{b}$.

The dot product of $\mathbf{a} = \langle 4, 3 \rangle$, and $\mathbf{b} = \langle 2, -1 \rangle$ will be,

$\mathbf{a} \cdot \mathbf{b} = \langle 4, 3 \rangle \cdot \langle 2, -1 \rangle$

$= 4(2) + 3(-1)$ Use $\langle a_1, a_2 \rangle \cdot \langle b_1, b_2 \rangle = a_1 b_1 + a_2 b_2$

$= 8 - 3$

$= 5$.

Therefore, the dot product of $\mathbf{a} = \langle 4, 3 \rangle$, and $\mathbf{b} = \langle 2, -1 \rangle$ is $\mathbf{a} \cdot \mathbf{b} = 5$.

By above steps, the values of $|\mathbf{a}|$, $|\mathbf{b}|$, and $\mathbf{a} \cdot \mathbf{b}$ for $\mathbf{a} = \langle 4,3 \rangle$, and $\mathbf{b} = \langle 2,-1 \rangle$ are,

$|\mathbf{a}| = 5$, and

$|\mathbf{b}| = \sqrt{5}$, and

$\mathbf{a} \cdot \mathbf{b} = 5$.

Use formula (1), and find the angle *between* $\mathbf{a} = \langle 4,3 \rangle$, *and* $\mathbf{b} = \langle 2,-1 \rangle$ *as follows:*

$$\cos\theta = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|} \quad \textit{Use formula (1)}$$

$$\cos\theta = \frac{5}{5\sqrt{5}} \quad \textit{Substitute values}$$

$$\cos\theta = \frac{1}{\sqrt{5}}$$

$$\theta = \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$$

$$\approx 63^\circ.$$

Therefore, the angle between the vectors $\mathbf{a} = \langle 4,3 \rangle$, *and* $\mathbf{b} = \langle 2,-1 \rangle$ *is* $\boxed{\cos^{-1}\left(\frac{1}{\sqrt{5}}\right) \approx 63^\circ}$.

Answer 16E.

First find the values of $|\mathbf{a}|$ and $|\mathbf{b}|$.

Now, the value of $|\mathbf{a}|$, for the vector; $\mathbf{a} = \langle -2,5 \rangle$, will be,

$$|\mathbf{a}| = |\langle -2,5 \rangle|$$

$$= \sqrt{(-2)^2 + 5^2} \quad \text{Use } |\langle a_1, a_2 \rangle| = \sqrt{a_1^2 + a_2^2}$$

$$= \sqrt{4+25}$$

$$= \sqrt{29}.$$

And, the value of $|\mathbf{b}|$, for the vector; $\mathbf{b} = \langle 5,12 \rangle$, will be,

$$|\mathbf{b}| = |\langle 5,12 \rangle|$$

$$= \sqrt{5^2 + 12^2} \quad \text{Use } |\langle a_1, a_2 \rangle| = \sqrt{a_1^2 + a_2^2}$$

$$= \sqrt{25+144}$$

$$= \sqrt{169}$$

$$= 13.$$

Therefore, the values of $|\mathbf{a}|$ and $|\mathbf{b}|$ for $\mathbf{a} = \langle -2,5 \rangle$, and $\mathbf{b} = \langle 5,12 \rangle$ are,

$|\mathbf{a}| = \sqrt{29}$ and $|\mathbf{b}| = 13$.

Next, find the dot product $\mathbf{a} \cdot \mathbf{b}$.

The dot product of $\mathbf{a} = \langle -2, 5 \rangle$, and $\mathbf{b} = \langle 5, 12 \rangle$ will be,

$$\mathbf{a} \cdot \mathbf{b} = \langle -2, 5 \rangle \cdot \langle 5, 12 \rangle$$

$= -2(5) + 5(12)$ Use $\langle a_1, a_2 \rangle \cdot \langle b_1, b_2 \rangle = a_1 b_1 + a_2 b_2$

$$= -10 + 60$$

$$= 50.$$

Therefore, the dot product of $\mathbf{a} = \langle -2, 5 \rangle$, and $\mathbf{b} = \langle 5, 12 \rangle$ is $\mathbf{a} \cdot \mathbf{b} = 50$.

By above steps, the values of $|\mathbf{a}|$, $|\mathbf{b}|$, and $\mathbf{a} \cdot \mathbf{b}$ for $\mathbf{a} = \langle -2, 5 \rangle$, and $\mathbf{b} = \langle 5, 12 \rangle$ are,

$|\mathbf{a}| = \sqrt{29}$, and

$|\mathbf{b}| = 13$, and

$\mathbf{a} \cdot \mathbf{b} = 50$.

Use formula (1), and find the angle *between* $\mathbf{a} = \langle -2, 5 \rangle$, *and* $\mathbf{b} = \langle 5, 12 \rangle$ *as follows:*

$\cos\theta = \dfrac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$ *Use formula (1)*

$\cos\theta = \dfrac{50}{13\sqrt{29}}$ *Substitute values*

$$\theta = \cos^{-1}\left(\frac{50}{13\sqrt{29}}\right)$$

$$\approx 44^\circ.$$

Hence, the angle between the vectors $\mathbf{a} = \langle -2, 5 \rangle$, *and* $\mathbf{b} = \langle 5, 12 \rangle$ *is* $\boxed{\cos^{-1}\left(\frac{50}{13\sqrt{29}}\right) \approx 44^\circ}$.

Answer 17E.

Consider the vectors,

$\mathbf{a}=\langle 3,-1,5\rangle$, and $\mathbf{b}=\langle -2,4,3\rangle$

Write the angle formula between the vectors $\mathbf{a}$ and $\mathbf{b}$

$$\cos\theta=\frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$$

Calculate $|\mathbf{a}|$, $|\mathbf{b}|$

$$\begin{aligned}|\mathbf{a}|&=\sqrt{a^2{}_1+a^2{}_2+a^2{}_3}\\&=\sqrt{(3)^2+(-1)^2+(5)^2}\\&=\sqrt{9+1+25}\\&=\sqrt{35}\end{aligned}$$

$$\begin{aligned}|\mathbf{b}|&=\sqrt{(-2)^2+(4)^2+(3)^2}\\&=\sqrt{4+16+9}\\&=\sqrt{29}\end{aligned}$$

Calculate $\mathbf{a}\cdot\mathbf{b}$

$$\begin{aligned}\mathbf{a}\cdot\mathbf{b}&=a_1b_1+a_2b_2+a_3b_3\\&=(3)(-2)+(-1)(4)+(5)(3)\\&=-6-4+15\\&=5\end{aligned}$$

Supply the values and calculate the angle between the vectors.

$$\begin{aligned}\cos\theta&=\frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}||\mathbf{b}|}\\&=\frac{5}{\sqrt{35}\sqrt{29}}\\&=\frac{5}{\sqrt{1015}}\end{aligned}$$

Solve for θ

$$\begin{aligned}\theta&=\cos^{-1}\left(\frac{5}{\sqrt{1015}}\right)\\&\approx 81^\circ\end{aligned}$$

Therefore angle between the vectors is $\theta\approx\boxed{81^\circ}$

Answer 18E.

Consider the vectors,

$\mathbf{a}=\langle 4,0,2\rangle$, and $\mathbf{b}=\langle 2,-1,0\rangle$

The objective is to find the angle between the vectors.

Write the angle formula between the vectors $\mathbf{a}$ and $\mathbf{b}$

Calculate $|\mathbf{a}|$, $|\mathbf{b}|$

$$|\mathbf{a}| = \sqrt{a^2{}_1 + a^2{}_2 + a^2{}_3}$$
$$= \sqrt{(4)^2 + (0)^2 + (2)^2}$$
$$= \sqrt{16+0+4}$$
$$= \sqrt{20}$$
$$= 2\sqrt{5}$$

$$|\mathbf{b}| = \sqrt{(2)^2 + (-1)^2 + (0)^2}$$
$$= \sqrt{4+1+0}$$
$$= \sqrt{5}$$

Calculate $\mathbf{a}\cdot\mathbf{b}$

$$\mathbf{a}\cdot\mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$$
$$= (4)(2) + (0)(-1) + (2)(0)$$
$$= 8+0+0$$
$$= 8$$

Supply the values and calculate the angle between the vectors.

$$\cos\theta = \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$$
$$= \frac{8}{\left(2\sqrt{5}\right)\left(\sqrt{5}\right)}$$
$$= \frac{8}{10}$$
$$= \frac{4}{5}$$

Solve for θ

$$\theta = \cos^{-1}\left(\frac{4}{5}\right)$$
$$\approx 37^\circ$$

Therefore, angle between the vectors is $\theta \approx \boxed{37^\circ}$.

Answer 19E.

First find the values of $|\mathbf{a}|$ and $|\mathbf{b}|$.

Now, the value of $|\mathbf{a}|$, for the vector; $\mathbf{a} = 4\mathbf{i} - 3\mathbf{j} + \mathbf{k}$, will be,

$$|\mathbf{a}| = |4\mathbf{i} - 3\mathbf{j} + \mathbf{k}|$$

$= \sqrt{4^2 + (-3)^2 + 1^2}$ Use $|a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$

$$= \sqrt{16 + 9 + 1}$$

$$= \sqrt{26}.$$

And, the value of $|\mathbf{b}|$, for the vector; $\mathbf{b} = 2\mathbf{i} - \mathbf{k}$, will be,

$$|\mathbf{b}| = |2\mathbf{i} - \mathbf{k}|$$

$= \sqrt{2^2 + 0^2 + (-1)^2}$ Use $|a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$

$$= \sqrt{4 + 0 + 1}$$

$$= \sqrt{5}.$$

Therefore, the values of $|\mathbf{a}|$ and $|\mathbf{b}|$ for $\mathbf{a} = 4\mathbf{i} - 3\mathbf{j} + \mathbf{k}$, and $\mathbf{b} = 2\mathbf{i} - \mathbf{k}$ are,

$|\mathbf{a}| = \sqrt{26}$ and $|\mathbf{b}| = \sqrt{5}$.

Next, find the dot product $\mathbf{a} \cdot \mathbf{b}$.

The dot product of $\mathbf{a} = 4\mathbf{i} - 3\mathbf{j} + \mathbf{k}$, and $\mathbf{b} = 2\mathbf{i} - \mathbf{k}$ will be,

$$\mathbf{a} \cdot \mathbf{b} = (4\mathbf{i} - 3\mathbf{j} + \mathbf{k}) \cdot (2\mathbf{i} - \mathbf{k})$$

$$= 4(2) - 3(0) + 1(-1)$$

Apply $(a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}) \cdot (b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}) = a_1b_1 + a_2b_2 + a_3b_3$

$$= 8 - 0 - 1$$

$$= 7.$$

Therefore, the dot product is $\mathbf{a} \cdot \mathbf{b} = 7$.

By above steps, the values of $|\mathbf{a}|$, $|\mathbf{b}|$, and $\mathbf{a} \cdot \mathbf{b}$ for $\mathbf{a} = 4\mathbf{i} - 3\mathbf{j} + \mathbf{k}$, and $\mathbf{b} = 2\mathbf{i} - \mathbf{k}$ are,

$|\mathbf{a}| = \sqrt{26}$, and

$|\mathbf{b}| = \sqrt{5}$, and

$\mathbf{a} \cdot \mathbf{b} = 7$.

Use formula (1), and find the angle *between* $\mathbf{a} = 4\mathbf{i} - 3\mathbf{j} + \mathbf{k}$, *and* $\mathbf{b} = 2\mathbf{i} - \mathbf{k}$ *as follows:*

$$\cos\theta = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$$ *Use formula (1)*

$$\cos\theta = \frac{7}{\sqrt{26} \cdot \sqrt{5}}$$ *Substitute values*

$$\cos\theta = \frac{7}{\sqrt{26 \cdot 5}}$$

$$\cos\theta = \frac{7}{\sqrt{130}}$$

$$\theta = \cos^{-1}\left(\frac{7}{\sqrt{130}}\right)$$

$$\approx 52^\circ.$$

Therefore, the angle between the vectors $\mathbf{a} = 4\mathbf{i} - 3\mathbf{j} + \mathbf{k}$, *and* $\mathbf{b} = 2\mathbf{i} - \mathbf{k}$ *is,*

$$\boxed{\cos^{-1}\left(\frac{7}{\sqrt{130}}\right) \approx 52^\circ}.$$

Answer 20E.

Consider the following vectors:

$\mathbf{a} = \mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$ and $\mathbf{b} = 4\mathbf{i} - 3\mathbf{k}$

Find the angle between the vectors $\mathbf{a} = \mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$ and $\mathbf{b} = 4\mathbf{i} - 3\mathbf{k}$.

Use the formula for finding the angle between two vectors:

The angle θ between two vectors $\mathbf{a}$ and $\mathbf{b}$ is $\cos\theta = \dfrac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}| \cdot |\mathbf{b}|}$

As the vectors $\mathbf{a} = \mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$ and $\mathbf{b} = 4\mathbf{i} - 3\mathbf{k}$

So, solve as follows:

$$\begin{aligned}|\mathbf{a}| &= \sqrt{1^2 + 2^2 + (-2)^2} \\ &= \sqrt{1+4+4} \\ &= \sqrt{9} \\ &= 3\end{aligned}$$

And,

$$\begin{aligned}|\mathbf{b}| &= \sqrt{4^2 + (-3)^2} \\ &= \sqrt{16+9} \\ &= \sqrt{25} \\ &= 5\end{aligned}$$

The dot product between the vectors $\mathbf{a} = \mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$ and $\mathbf{b} = 4\mathbf{i} - 3\mathbf{k}$ is calculated as follows:

$$\begin{aligned}\mathbf{a} \cdot \mathbf{b} &= (\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}) \cdot (4\mathbf{i} - 3\mathbf{k}) \\ &= 1(4) + 2(0) + (-2)(-3) \\ &= 4 + 0 + 6 \\ &= 10\end{aligned}$$

Plugin $|\mathbf{a}| = 3$, $|\mathbf{b}| = 5$, and $\mathbf{a} \cdot \mathbf{b} = 10$ in the angle formula $\cos\theta = \dfrac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}| \cdot |\mathbf{b}|}$

$$\begin{aligned}\cos\theta &= \frac{10}{3 \cdot 5} \\ &= \frac{10}{15} \\ &= \frac{2}{3}\end{aligned}$$

So, the angle between the vectors $\mathbf{a}$ and $\mathbf{b}$ is given below:

$$\begin{aligned}\theta &= \cos^{-1}\left(\frac{2}{3}\right) \\ &= 48.19^\circ \qquad \text{Using calculator.}\end{aligned}$$

Therefore, the angle between the vectors $\mathbf{a} = \mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$ and $\mathbf{b} = 4\mathbf{i} - 3\mathbf{k}$ is $\boxed{\theta = 48.19^\circ}$.

Answer 21E.

Find the angle ‘ p ’ between the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$.

The angle *between the two vectors* $\mathbf{a}$ *and* $\mathbf{b}$ *is defined as follows:*

$$\cos\theta = \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}||\mathbf{b}|} \quad \text{......(1)}$$

Hence, the angle p between the vectors $\overrightarrow{PQ}$ *and* $\overrightarrow{PR}$ *is defined as follows:*

$$\cos p = \frac{\overrightarrow{PQ}\cdot\overrightarrow{PR}}{\left|\overrightarrow{PQ}\right|\left|\overrightarrow{PR}\right|} \quad \text{......(i)}$$

Find the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ by using the following fact:

For the points $A(x_1, y_1)$ and $B(x_2, y_2)$, the vector $\overrightarrow{AB}$ will be,

$$\overrightarrow{AB} = \langle x_2 - x_1, y_2 - y_1\rangle \quad \text{......(2)}$$

Then, for the points $P(2,0)$ and $Q(0,3)$, the vector $\overrightarrow{PQ}$ will be,

$\overrightarrow{PQ} = \langle 0-2, 3-0\rangle$ Substitute $(x_1, y_1) = (2,0)$, $(x_2, y_2) = (0,3)$ in (2)

$= \langle -2, 3\rangle$.

And, for the points $P(2,0)$ and $R(3,4)$, the vector $\overrightarrow{PR}$ will be,

$\overrightarrow{PR} = \langle 3-2, 4-0\rangle$ Substitute $(x_1, y_1) = (2,0)$, $(x_2, y_2) = (3,4)$ in (2)

$= \langle 1, 4\rangle$.

Substitute $\overrightarrow{PQ} = \langle -2,3\rangle$, and $\overrightarrow{PR} = \langle 1,4\rangle$ into (i), and find the value of p as follows:

$$\cos p = \frac{\overrightarrow{PQ}\cdot\overrightarrow{PR}}{\left|\overrightarrow{PQ}\right|\left|\overrightarrow{PR}\right|}$$ Write (i)

$$\cos p = \frac{\langle -2,3\rangle\cdot\langle 1,4\rangle}{\left|\langle -2,3\rangle\right|\left|\langle 1,4\rangle\right|}$$ Substitute values

$$= \frac{-2(1)+3(4)}{\sqrt{(-2)^2+3^2}\cdot\sqrt{1^2+4^2}}$$

Use $\langle a_1,a_2\rangle\cdot\langle b_1,b_2\rangle = a_1b_1 + a_2b_2$ and $\left|\langle a_1,a_2\rangle\right| = \sqrt{a_1^2+a_2^2}$

$$= \frac{-2+12}{\sqrt{4+9}\cdot\sqrt{1+16}}$$

$$= \frac{10}{\sqrt{13}\cdot\sqrt{17}}$$

$$= \frac{10}{\sqrt{221}}.$$

Hence, for $\cos p = \frac{10}{\sqrt{221}}$, the value of p will be,

$$p = \cos^{-1}\left(\frac{10}{\sqrt{221}}\right)$$

$$\approx 48^\circ.$$

Therefore, the angle between the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ is $p \approx \boxed{48^\circ}$.

Find the angle ' q ' **between the vectors** $\overrightarrow{QP}$ **and** $\overrightarrow{QR}$.

By (1), the angle q between the vectors $\overrightarrow{QP}$ and $\overrightarrow{QR}$ is defined as follows:

$$\cos q = \frac{\overrightarrow{QP}\cdot\overrightarrow{QR}}{\left|\overrightarrow{QP}\right|\left|\overrightarrow{QR}\right|} \quad \text{.......(ii)}$$

Find the vectors $\overrightarrow{QP}$ and $\overrightarrow{QR}$ by using (2):

Then, for the points $Q(0,3)$ and $P(2,0)$, the vector $\overrightarrow{QP}$ will be,

$\overrightarrow{QP} = \langle 2-0, 0-3\rangle$ Substitute $(x_1,y_1)=(0,3)$, $(x_2,y_2)=(2,0)$ in (2)

$= \langle 2,-3\rangle$.

And, for the points $Q(0,3)$ and $R(3,4)$, the vector $\overrightarrow{QR}$ will be,

$\overrightarrow{QR} = \langle 3-0, 4-3\rangle$ Substitute $(x_1,y_1)=(0,3)$, $(x_2,y_2)=(3,4)$ in (2)

$= \langle 3,1\rangle$.

Substitute $\overrightarrow{QP} = \langle 2,-3\rangle$, and $\overrightarrow{QR} = \langle 3,1\rangle$ into (ii), and find the value of *q* as follows:

$$\cos q = \frac{\overrightarrow{QP}\cdot\overrightarrow{QR}}{|\overrightarrow{QP}||\overrightarrow{QR}|}$$ Write (ii)

$$\cos q = \frac{\langle 2,-3\rangle\cdot\langle 3,1\rangle}{|\langle 2,-3\rangle||\langle 3,1\rangle|}$$ Substitute values

$$= \frac{2(3)-3(1)}{\sqrt{2^2+(-3)^2}\cdot\sqrt{3^2+1^2}}$$

Use $\langle a_1,a_2\rangle\cdot\langle b_1,b_2\rangle = a_1b_1+a_2b_2$ and $|\langle a_1,a_2\rangle| = \sqrt{a_1^2+a_2^2}$

$$= \frac{6-3}{\sqrt{4+9}\cdot\sqrt{9+1}}$$

$$= \frac{3}{\sqrt{13}\cdot\sqrt{10}}$$

$$= \frac{3}{\sqrt{130}}.$$

Hence, for $\cos q = \frac{3}{\sqrt{130}}$, the value of *q* will be,

$$q = \cos^{-1}\left(\frac{3}{\sqrt{130}}\right)$$

$$\approx 75^\circ.$$

Therefore, the angle between the vectors $\overrightarrow{QP}$ and $\overrightarrow{QR}$ is $q \approx \boxed{75^\circ}$.

Find the angle '*r*' between the vectors $\overrightarrow{RP}$ and $\overrightarrow{RQ}$.

By (1), the angle *r* between the vectors $\overrightarrow{RP}$ and $\overrightarrow{RQ}$ is defined as follows:

$$\cos r = \frac{\overrightarrow{RP}\cdot\overrightarrow{RQ}}{|\overrightarrow{RP}||\overrightarrow{RQ}|}. \quad \text{.......(iii)}$$

Find the vectors $\overrightarrow{RP}$ and $\overrightarrow{RQ}$ by using (2):

Then, for the points $R(3,4)$ and $P(2,0)$, the vector $\overrightarrow{RP}$ will be,

$\overrightarrow{RP} = \langle 2-3, 0-4\rangle$ Substitute $(x_1,y_1)=(3,4)$, $(x_2,y_2)=(2,0)$ in (2)

$= \langle -1,-4\rangle$.

And, for the points $R(3,4)$ and $Q(0,3)$, the vector $\overrightarrow{RQ}$ will be,

$\overrightarrow{RQ} = \langle 0-3, 3-4\rangle$ Substitute $(x_1,y_1)=(3,4)$, $(x_2,y_2)=(0,3)$ in (2)

$= \langle -3,-1\rangle$.

Substitute $\overrightarrow{RP} = \langle -1,-4\rangle$, and $\overrightarrow{RQ} = \langle -3,-1\rangle$ into (iii), and find the value of *q* as follows:

$$\cos r = \frac{\overrightarrow{RP}\cdot\overrightarrow{RQ}}{|\overrightarrow{RP}||\overrightarrow{RQ}|} \quad \text{Write (iii)}$$

$$\cos r = \frac{\langle -1,-4\rangle\cdot\langle -3,-1\rangle}{|\langle -1,-4\rangle||\langle -3,-1\rangle|} \quad \text{Substitute values}$$

$$= \frac{-1(-3)-4(-1)}{\sqrt{(-1)^2+(-4)^2}\cdot\sqrt{(-3)^2+(-1)^2}}$$

Use $\langle a_1,a_2\rangle\cdot\langle b_1,b_2\rangle = a_1b_1 + a_2b_2$ and $|\langle a_1,a_2\rangle| = \sqrt{a_1^2+a_2^2}$

$$= \frac{3+4}{\sqrt{1+16}\cdot\sqrt{9+1}}$$

$$= \frac{7}{\sqrt{17}\cdot\sqrt{10}}$$

$$= \frac{7}{\sqrt{170}}.$$

Hence, for $\cos r = \frac{7}{\sqrt{170}}$, the value of *r* will be,

$$r = \cos^{-1}\left(\frac{7}{\sqrt{170}}\right)$$

$$\approx 57^\circ.$$

Therefore, the angle between the vectors $\overrightarrow{RP}$ and $\overrightarrow{RQ}$ is $r \approx \boxed{57^\circ}$.

Therefore, the three angles of the triangle are $\boxed{48^\circ, 75^\circ, 57^\circ}$.

Answer 22E.

The vertices of the triangle is given as $A(1,0,-1), B(3,-2,0)$, and $C(1,3,3)$.

The objective is to find the angles of the triangle *ABC* with the above vertices.

The angles of the triangle are,

$\angle A =$ Angle between the vectors $\overrightarrow{AB}, \overrightarrow{AC}$

$\angle B =$ Angle between the vectors $\overrightarrow{BC}, \overrightarrow{BA}$

$\angle C =$ Angle between the vectors $\overrightarrow{CA}, \overrightarrow{CB}$.

Recollect that, the angle between two vectors **a** and **b** is given by,

$$\cos\theta = \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}||\mathbf{b}|}.$$

Find the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

$$\overrightarrow{AB} = \langle 2, -2, 1 \rangle$$
$$\overrightarrow{AC} = \langle 0, 3, 4 \rangle$$

Then,

$$\begin{aligned} \overrightarrow{AB} \cdot \overrightarrow{AC} &= \langle 2, -2, 1 \rangle \cdot \langle 0, 3, 4 \rangle \\ &= 0 - 6 + 4 \\ &= -2 \end{aligned}$$

Also,

$$\begin{aligned} \left| \overrightarrow{AB} \right| &= \sqrt{4 + 4 + 1} \\ &= 3 \end{aligned}$$

$$\begin{aligned} \left| \overrightarrow{AC} \right| &= \sqrt{0 + 9 + 16} \\ &= 5 \end{aligned}$$

Therefore, the angle *A* is,

$$\begin{aligned} \angle A &= \cos^{-1}\left(\frac{\overrightarrow{AB} \bullet \overrightarrow{AC}}{\left|\overrightarrow{AB}\right|\left|\overrightarrow{AC}\right|} \right) \\ &= \cos^{-1}\left(\frac{-2}{3 \cdot 5} \right) \\ &= \cos^{-1}\left(\frac{-2}{15} \right) \\ &\approx 98^\circ \end{aligned}$$

Find the vectors $\overrightarrow{BC}$ and $\overrightarrow{BA}$.

$$\overrightarrow{BC} = \langle -2,5,3 \rangle$$
$$\overrightarrow{BA} = \langle -2,2,-1 \rangle$$

Then,

$$\begin{aligned}\overrightarrow{BC} \cdot \overrightarrow{BA} &= \langle -2,5,3 \rangle \cdot \langle -2,2,-1 \rangle \\ &= 4+10-3 \\ &= 11\end{aligned}$$

And,

$$\begin{aligned}\left|\overrightarrow{BC}\right| &= \sqrt{4+25+9} \\ &= \sqrt{38}\end{aligned}$$

$$\begin{aligned}\left|\overrightarrow{BA}\right| &= \sqrt{4+4+1} \\ &= 3\end{aligned}$$

Therefore, the angle *B* is,

$$\begin{aligned}\angle B &= \cos^{-1}\left(\frac{\overrightarrow{BC} \cdot \overrightarrow{BA}}{\left|\overrightarrow{BC}\right|\left|\overrightarrow{BA}\right|}\right) \\ &= \cos^{-1}\left(\frac{11}{3\sqrt{38}}\right) \\ &\approx 54^\circ\end{aligned}$$

In a triangle ΔABC,

$$\begin{aligned}\angle A + \angle B + \angle C &= 180^\circ \\ 98^\circ + 54^\circ + \angle C &= 180^\circ \\ \angle C &= 180^\circ - (98^\circ + 54^\circ) \\ \angle C &= 28^\circ\end{aligned}$$

Thus, the angles of the triangle are $\boxed{\angle A = 98^\circ, \angle B = 54^\circ \text{ and } \angle C = 28^\circ}$.

Answer 23E.

(A)

$$\vec{a} = \langle -5,3,7\rangle, \vec{b} = \langle 6,-8,2\rangle$$

Then vectors $\vec{a}$ and $\vec{b}$ are not parallel because, their corresponding components are not equal.

Now

$$\vec{a}.\vec{b} = (-5)(6)+(3)(-8)+7(2)$$
$$= -30-24+14$$
$$= -40 \neq 0$$

The vectors $\vec{a}$ and $\vec{b}$ are not perpendicular.

(B)

$$\vec{a} = \langle 4,6\rangle, \vec{b} = \langle -3,2\rangle$$
$$\vec{a}.\vec{b} = 4(-3)+(6)(2)$$
$$= -12+12$$
$$= 0$$

The vectors $\vec{a}$ and $\vec{b}$ are orthogonal.

(C)

$$\vec{a} = -\hat{i}+2\hat{j}+5\hat{k}, \qquad \vec{b} = 3\hat{i}+4\hat{j}-\hat{k}$$
$$\vec{a}.\vec{b} = (-1)(3)+(2)(4)+(5)(-1)$$
$$= -3+8-5$$
$$= 0$$

The vectors $\vec{a}$ and $\vec{b}$ are orthogonal.

(D)

$$\vec{a} = 2i+6j-4k, \qquad \vec{b} = -3i-9j+6k$$
$$\vec{a} = 2(i+3j-2k) \qquad \vec{b} = -3(i+3j-2k)$$

Because vectors $\frac{\vec{a}}{2}$ and $\frac{\vec{b}}{-3}$ are equal,

The vectors $\vec{a}$ and $\vec{b}$ are parallel.

Answer 24E.

(A)

$$\vec{u} = \langle -3,9,6\rangle, \quad \vec{v} = \langle 4,-12,-8\rangle$$
$$\vec{u} = -3\langle 1,-3,-2\rangle, \qquad \vec{v} = 4\langle 1,-3,-2\rangle$$

Because the vectors $\frac{\vec{u}}{-3}$ and $\frac{\vec{v}}{4}$ and equal,

Then $\vec{u}$ and $\vec{v}$ vectors are parallel.

(B)

$$\vec{u} = i - j + 2k, \quad \vec{v} = 2i - j + k$$

$$\vec{u}.\vec{v} = (1)(2) + (-1) + (2)(1)$$
$$= 2 + 1 + 2$$
$$= 5 \neq 0$$

Thus the vectors $\vec{u}$ and $\vec{v}$ are neither perpendicular nor parallel.

(C)

$$\vec{u} = \langle a, b, c \rangle, \quad \vec{v} = \langle -b, a, o \rangle$$

$$\vec{u}.\vec{v} = a(-b) + b(a) + c(o)$$
$$= -ab + ab$$
$$= 0$$

Thus $\vec{u}$ and $\vec{v}$ vectors are orthogonal.

Answer 25E.

$$P(1, -3, -2), \quad Q(2, 0, -4), \quad R(6, -2, -5)$$

$$\overrightarrow{PQ} = \langle (2-1), 0+3, -4+2 \rangle$$
$$= \langle 1, 3, -2 \rangle$$

$$\overrightarrow{QR} = \langle 6-2, -2-0, -5+4 \rangle$$
$$= \langle 4, -2, -1 \rangle$$

$$\overrightarrow{RP} = \langle 1-6, -3+2, -2+5 \rangle$$
$$= \langle -5, -1, 3 \rangle$$

Now

$$\overrightarrow{PQ}.\overrightarrow{QR} = 1(4) + 3(-2) + (-2)(-1)$$
$$= 4 - 6 + 2$$
$$= 0$$

Because two sides of a triangle PQR gives dot product zero. Therefore these two sides are orthogonal to each other. Hence the triangle with vertices P, Q and R is right angled.

Answer 26E.

We know that the angle between two vectors is given by

$$\cos\theta = \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}||\mathbf{b}|}.$$

Let us find the magnitude of $\mathbf{a}\cdot\mathbf{b}$.

$$\begin{aligned}\mathbf{a}\cdot\mathbf{b} &= (2)(1)+(1)(x)+(-1)(0)\\ &= 2+x\end{aligned}$$

Find $|\mathbf{a}|$.

$$\begin{aligned}|\mathbf{a}| &= \sqrt{2^2+1^2+(-1)^2}\\ &= \sqrt{4+1+1}\\ &= \sqrt{6}\end{aligned}$$

Similarly, we get $|\mathbf{b}|$ as $\sqrt{1+x^2}$.

Substitute the known values in $\cos\theta = \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$.

$$\begin{aligned}\cos 45^\circ &= \frac{2+x}{(\sqrt{6})(\sqrt{1+x^2})}\\ \frac{1}{\sqrt{2}} &= \frac{2+x}{\sqrt{6+6x^2}}\end{aligned}$$

Solve for x.

$$\begin{aligned}\frac{1}{2} &= \frac{(2+x)^2}{6+6x^2}\\ 3+3x^2 &= 4+4x+x^2\\ 2x^2-4x-1 &= 0\\ x &= 1\pm\frac{\sqrt{6}}{2}\end{aligned}$$

We thus get x as approximately $\boxed{1\pm\frac{\sqrt{6}}{2}}$.

Answer 27E.

Consider a vector $a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ ---(1)

This vector is a unit vector, then its magnitude is unity

That is , $\sqrt{a_1^{\,2}+a_2^{\,2}+a_3^{\,2}} = 1$

Squaring, $a_1^{\,2}+a_2^{\,2}+a_3^{\,2} = 1$ ---(2)

The vector (1) is orthogonal to $\hat{i}+\hat{j}$, we get

$$\left(a_1\hat{i}+a_2\hat{j}+a_3\hat{k}\right).\left(\hat{i}+\hat{j}+o\hat{k}\right)=0$$
$$\Rightarrow a_1+a_2=0$$
$$\Rightarrow a_2=-a_1$$

Also the vector (1) is orthogonal to $\hat{i}+\hat{k}$, we get

$$\left(a_1\hat{i}+a_2\hat{j}+a_3\hat{k}\right).\left(\hat{i}+o\hat{j}+\hat{k}\right)=0$$
$$\Rightarrow a_1+a_3=0$$
$$\Rightarrow a_3=-a_1$$

Place these values of a_2 and a_3 in equation (2),

We get

$$a_1^{\,2}+a_1^{\,2}+a_1^{\,2}=1$$
$$\text{or, } 3a_1^{\,2}=1$$
$$\text{or, } a_1^{\,2}=\frac{1}{3}$$
$$\Rightarrow a_1=\pm\frac{1}{\sqrt{3}}$$

When $a_1=+\frac{1}{\sqrt{3}}$, *Then* $a_2=-\frac{1}{\sqrt{3}}$ *and* $a_3=-\frac{1}{\sqrt{3}}$

When $a_1=-\frac{1}{\sqrt{3}}$, *Then* $a_2=\frac{1}{\sqrt{3}}$ *and* $a_3=\frac{1}{\sqrt{3}}$

Therefore the required unit vectors are

$$\boxed{\left(\hat{i}-\hat{j}-\hat{k}\right)\frac{1}{\sqrt{3}}} \text{ or } \boxed{\left(-\hat{i}+\hat{j}+\hat{k}\right)\frac{1}{\sqrt{3}}}.$$

Answer 28E.

Let $\vec{a}=\langle a_1,a_2\rangle$ be the unit vector that makes an angle of 60^o with $\vec{v}=\langle 3,4\rangle$

Since $\vec{a}$ is a unit vector, then $\left|\vec{a}\right|=1$

$$i.e.\quad \sqrt{a_1^{\,2}+a_2^{\,2}}=1$$
$$i.e.\quad a_1^{\,2}+a_2^{\,2}=1$$

Now $\vec{v}.\vec{a} = |\vec{v}||\vec{a}|\cos 60^\circ$

i.e. $\langle 3,4\rangle.\langle a_1, a_2\rangle = \sqrt{3^2+4^2}, (1).\frac{1}{2}$

i.e. $3a_1 + 4a_2 = \frac{5}{2}$

i.e. $6a_1 + 8a_2 = 5$

$\Rightarrow a_1 = \frac{5-8a_2}{6}$

Then $a_1 = \frac{5}{6} - \frac{4}{3}a_2$

When $a_2 = \frac{4+3\sqrt{3}}{10}$ *Then* $a_1 = \frac{3-4\sqrt{3}}{10}$

And when $a_2 = \frac{4-3\sqrt{3}}{10}$ *then* $a_1 = \frac{3+4\sqrt{3}}{10}$

Then the two unit vectors are:

$$\left\langle \frac{3-4\sqrt{3}}{10}, \frac{4+3\sqrt{3}}{10} \right\rangle \quad \textit{and} \quad \left\langle \frac{3+4\sqrt{3}}{10}, \frac{4-3\sqrt{3}}{10} \right\rangle$$

Or $\boxed{\langle -0.392, 0.919\rangle}$ *and* $\boxed{\langle 0.992, -0.119\rangle}$

Since $a_1^2 + a_2^2 = 1$

$$\Rightarrow \left(\frac{5-8a_2}{6}\right)^2 + a_2^2 = 1$$

i.e. $25 + 64a_2^2 - 80a_2 + 36a_2^2 = 36$

i.e. $100a_2^2 - 80a_2 - 11 = 0$

i.e. $a_2 = \frac{80 \pm \sqrt{6400+4400}}{200}$

i.e. $a_2 = \frac{80 \pm \sqrt{10800}}{200}$

$$= \frac{80 \pm 60\sqrt{3}}{200} = \frac{4 \pm 3\sqrt{3}}{10}$$

Answer 29E.

First, find the slopes of the above lines by following definitions:

An equation of the line with slope *m* and *y*-intercept *b* is,

$y = mx + b$. ......(3)

Rewrite the equation (1) by solving it for *y* in terms of *x*:

$2x - y = 3$, or

$y = 2x - 3$.

Compare this equation with equation (3), the slope of the line, $2x - y = 3$, is $m = 2$.

So, by above fact, a vector parallel to the line, $2x - y = 3$, is,

$\mathbf{a} = \langle 1, 2 \rangle$. ......(i)

Rewrite the equation (2) by solving it for *y* in terms of *x*:

$3x + y = 7$, or

$y = -3x + 7$.

Compare this equation with equation (3), the slope of the line, $3x + y = 7$, is $m = -3$.

So, by above fact, a vector parallel to the line, $3x + y = 7$, is,

$\mathbf{b} = \langle 1, -3 \rangle$. ......(ii)

Now, the objective is to find the acute angle between the vectors $\mathbf{a} = \langle 1, 2 \rangle$ and $\mathbf{b} = \langle 1, -3 \rangle$.

The cosine of the angle *between the two vectors* $\mathbf{a}$ *and* $\mathbf{b}$ *is defined as follows:*

$\cos\theta = \dfrac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$. ......*(4)*

Find the values of $|\mathbf{a}|$ *for the vector;* $\mathbf{a} = \langle 1, 2 \rangle$, *as follows:*

$|\mathbf{a}| = |\langle 1, 2 \rangle|$

$= \sqrt{1^2 + 2^2}$ Use $|\langle a_1, a_2 \rangle| = \sqrt{a_1^2 + a_2^2}$

$= \sqrt{5}$.

Find the values of $|\mathbf{b}|$ *for the vector;* $\mathbf{b} = \langle 1, -3 \rangle$, *as follows:*

$|\mathbf{b}| = |\langle 1, -3 \rangle|$

$= \sqrt{1^2 + (-3)^2}$ Use $|\langle a_1, a_2 \rangle| = \sqrt{a_1^2 + a_2^2}$

$= \sqrt{10}$.

Next, find the dot product $\mathbf{a} \cdot \mathbf{b}$ *for the vector,* $\mathbf{a} = \langle 1, 2 \rangle$ *and* $\mathbf{b} = \langle 1, -3 \rangle$, *as follows:*

$\mathbf{a} \cdot \mathbf{b} = \langle 1, 2 \rangle \cdot \langle 1, -3 \rangle$

$= 1(1) + 2(-3)$ Use $\langle a_1, a_2 \rangle \cdot \langle b_1, b_2 \rangle = a_1 b_1 + a_2 b_2$

$= -5$.

By above steps, the values of $|\mathbf{a}|$, $|\mathbf{b}|$, and $\mathbf{a}\bullet\mathbf{b}$ for $\mathbf{a}=\langle 1,2\rangle$ and $\mathbf{b}=\langle 1,-3\rangle$ are,

$|\mathbf{a}|=\sqrt{5}$, $|\mathbf{b}|=\sqrt{10}$ and $\mathbf{a}\bullet\mathbf{b}=-5$.

Use formula (4), and find the angle *between* $\mathbf{a}=\langle 1,2\rangle$ *and* $\mathbf{b}=\langle 1,-3\rangle$ *as follows:*

$$\cos\theta=\frac{\mathbf{a}\bullet\mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$$ *Use formula (4)*

$$\cos\theta=\frac{-5}{\sqrt{5}\cdot\sqrt{10}}$$ *Substitute values*

$$=\frac{-5}{\sqrt{5\cdot 10}}$$
$$=\frac{-5}{5\sqrt{2}}$$
$$=\frac{-1}{\sqrt{2}}.$$

Thus, for $\cos\theta=\frac{-1}{\sqrt{2}}$*, the value of* θ *will be,*

$$\theta=135^\circ.$$

Hence, the angle between the lines $2x-y=3$ *and* $3x+y=7$ *is* $\theta=135^\circ$*. And the acute angle between the lines will be,*

$$180^\circ-135^\circ=45^\circ.$$

Therefore, the acute angle between the lines $2x-y=3$ *and* $3x+y=7$ *is* $\boxed{45^\circ}$.

Answer 30E.

First, find the slopes of the above lines by following definitions:

An equation of the line with slope *m* and *y*-intercept *b* is,

$y = mx + b$. ......(3)

Rewrite the equation (1) by solving it for *y* in terms of *x*:

$x + 2y = 7$, or

$y = -\frac{1}{2}x + \frac{7}{2}$.

Compare this equation with equation (3), the slope of the line, $x + 2y = 7$, is $m = -\frac{1}{2}$.

So, by above fact, a vector parallel to the line, $x + 2y = 7$, is,

$\mathbf{a} = \left\langle 1, -\frac{1}{2} \right\rangle$. ......(i)

Rewrite the equation (2) by solving it for *y* in terms of *x*:

$5x - y = 2$, or

$y = 5x - 2$.

Compare this equation with equation (3), the slope of the line, $5x - y = 2$, is $m = 5$.

So, by above fact, a vector parallel to the line, $5x - y = 2$, is,

$\mathbf{b} = \langle 1, 5 \rangle$. ......(ii)

Now, the objective is to find the angle between the vectors $\mathbf{a}=\left\langle 1,-\frac{1}{2}\right\rangle$ and $\mathbf{b}=\langle 1,5\rangle$.

The cosine of the angle *between the two vectors* $\mathbf{a}$ *and* $\mathbf{b}$ *is defined as follows:*

$$\cos\theta=\frac{\mathbf{a}\bullet\mathbf{b}}{|\mathbf{a}||\mathbf{b}|} \quad \text{.......(4)}$$

Find the values of $|\mathbf{a}|$ *as follows:*

$$|\mathbf{a}|=\left|\left\langle 1,-\frac{1}{2}\right\rangle\right|$$

$$=\sqrt{1^2+\left(-\frac{1}{2}\right)^2} \quad \text{Use } |\langle a_1,a_2\rangle|=\sqrt{a_1^2+a_2^2}$$

$$=\frac{\sqrt{5}}{2}.$$

Find the values of $|\mathbf{b}|$ *as follows:*

$$|\mathbf{b}|=|\langle 1,5\rangle|$$

$$=\sqrt{1^2+5^2} \quad \text{Use } |\langle a_1,a_2\rangle|=\sqrt{a_1^2+a_2^2}$$

$$=\sqrt{26}.$$

Next, find the dot product $\mathbf{a}\bullet\mathbf{b}$ *for the vector,* $\mathbf{a}$ *and* $\mathbf{b}$*, as follows:*

$$\mathbf{a}\bullet\mathbf{b}=\left\langle 1,-\frac{1}{2}\right\rangle\bullet\langle 1,5\rangle$$

$$=1(1)-\frac{1}{2}(5) \quad \text{Use } \langle a_1,a_2\rangle\bullet\langle b_1,b_2\rangle=a_1b_1+a_2b_2$$

$$=-\frac{3}{2}.$$

By above steps, the values of $|\mathbf{a}|$, $|\mathbf{b}|$, and $\mathbf{a}\bullet\mathbf{b}$ are,

$|\mathbf{a}| = \frac{\sqrt{5}}{2}$, $|\mathbf{b}| = \sqrt{26}$ and $\mathbf{a}\bullet\mathbf{b} = -\frac{3}{2}$.

Use formula (4), and find the angle *between* $\mathbf{a} = \left\langle 1, -\frac{1}{2} \right\rangle$ *and* $\mathbf{b} = \langle 1, 5 \rangle$ *as follows:*

$$\cos\theta = \frac{\mathbf{a}\bullet\mathbf{b}}{|\mathbf{a}||\mathbf{b}|} \quad \text{Use formula (4)}$$

$$\cos\theta = \frac{-\frac{3}{2}}{\frac{\sqrt{5}}{2}\cdot\sqrt{26}} \quad \text{Substitute values}$$

$$= \frac{-3}{\sqrt{5}\cdot\sqrt{26}}$$

$$= \frac{-3}{\sqrt{130}}$$

$$\theta = \cos^{-1}\left(\frac{-3}{\sqrt{130}}\right)$$

$$\approx 105^\circ.$$

Hence, the angle between the lines $x + 2y = 7$ and $5x - y = 2$ is $\theta \approx 105^\circ$.

Then, the acute angle between the lines will be,

$$180^\circ - 105^\circ = 75^\circ.$$

Therefore, the acute angle between the lines $x + 2y = 7$ and $5x - y = 2$ is about $\boxed{75^\circ}$.

Answer 31E.

Consider the curves

$y = x^2, y = x^3.$

Need to find the acute angles between the curves at their points of intersection.

Since the angle between two curves is given by the angle between their tangent lines at the point of intersection.

Find the point of intersection of the two curves by equating the two curves.

$$\begin{aligned} x^2 &= x^3 \\ x^2 - x^3 &= 0 \\ x^2(1-x) &= 0 \\ x &= 0, 1 \end{aligned}$$

Thus the points of intersection of the two curves are $(0,0)$ and $(1,1)$.

Find the tangent lines to the curves.

Slope of the curve $y = x^2$ is

$$\begin{aligned} m &= \frac{d}{dx}(x^2) \\ &= 2x \end{aligned}$$

At the point $(0,0)$

$$\begin{aligned} m &= 2(0) \\ &= 0 \end{aligned}$$

Then tangent line of the curve at the point $(0,0)$ is

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 0 &= 0(x - 0) \\ y &= 0 \end{aligned}$$

At the point $(x, y) = (1,1)$, slope of the curve $y = x^2$ is

$$\begin{aligned} m &= 2x \\ &= 2(1) \\ &= 2 \end{aligned}$$

Then tangent line of the curve $y = x^2$ at the point $(1,1)$ is

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 1 &= 2(x - 1) \\ y &= 2x - 1 \end{aligned}$$

Slope of the curve $y = x^3$ is

$$m = \frac{d}{dx}(x^3)$$
$$= 3x^2$$

At the point $(0,0)$

$$m = 3x^2$$
$$= 3(0)^2$$
$$= 0$$

Then tangent line of the curve $y = x^3$ at the point $(0,0)$ is

$$y - y_1 = m(x - x_1)$$
$$y - 0 = 0(x - 0)$$
$$y = 0$$

At the point $(x, y) = (1,1)$, slope of the curve $y = x^3$ is

$$m = 3x^2$$
$$= 3(1)^2$$
$$= 3$$

Then tangent line of the curve $y = x^3$ at the point $(1,1)$ is

$$y - y_1 = m(x - x_1)$$
$$y - 1 = 3(x - 1)$$
$$y = 3x - 2$$

At the point $(0,0)$:

The tangent line to the curve $y = x^2$ is $y = 0.$

The tangent line to the curve $y = x^3$ is $y = 0.$

The direction of the vectors representing the tangent lines at (0, 0) are $\langle 0, 1 \rangle$ and $\langle 0, 1 \rangle$.

The angle between two vectors **a** and **b** is

$$\cos\theta = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|}.$$

Substitute the known values in the equation and simplify.

$$\begin{aligned}\cos\theta &= \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}||\mathbf{b}|}\\ &= \frac{\langle 0,1\rangle\cdot\langle 0,1\rangle}{|\langle 0,1\rangle||\langle 0,1\rangle|}\\ &= \frac{(0)(0)+(1)(1)}{\left(\sqrt{0^2+1^2}\right)\left(\sqrt{0^2+1^2}\right)}\\ &= \frac{0+1}{(1)(1)}\end{aligned}$$

$$= 1$$

And

$$\begin{aligned}\cos\theta &= 1\\ \cos\theta &= \cos 0^\circ\\ \theta &= 0^\circ\end{aligned}$$

The acute angle between the curves at (0, 0) is $\boxed{0^\circ}$.

At the point $(1,1)$:

The tangent line to the curve $y = x^2$ is $y = 2x - 1.$

The tangent line to the curve $y = x^3$ is $y = 3x - 2.$

The direction of the vectors representing the tangent lines at $(1,1)$ are $\langle 2, -1 \rangle$ and $\langle 3, -1 \rangle$.

The angle between two vectors $\langle 2, -1 \rangle$ and $\langle 3, -1 \rangle$ is

$$\begin{aligned}
\cos\theta &= \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|} \\
&= \frac{\langle 2,-1 \rangle \cdot \langle 3,-1 \rangle}{|\langle 2,-1 \rangle||\langle 3,-1 \rangle|} \\
&= \frac{(2)(3)+(-1)(-1)}{\left(\sqrt{2^2+(-1)^2}\right)\left(\sqrt{3^2+(-1)^2}\right)} \\
&= \frac{6+1}{\left(\sqrt{4+1}\right)\left(\sqrt{9+1}\right)} \\
&= \frac{7}{\sqrt{5}\sqrt{10}} \\
&= \frac{7}{\sqrt{50}}
\end{aligned}$$

And

$$\begin{aligned}
\cos\theta &= \frac{7}{\sqrt{50}} \\
\theta &= \cos^{-1}\left(\frac{7}{\sqrt{50}}\right) \\
&\approx 8.1^\circ
\end{aligned}$$

Therefore, the acute angle between the two given curves at (1, 1) is about $\boxed{8.1^\circ}$.

Answer 32E.

Find the acute angles between the curves at their points of intersection.

The given curves are $y=\sin x, y=\cos x, 0\le x\le\frac{\pi}{2}.$

The angle between two curves is given by the angle between their tangent lines at the point of intersection.

Start by finding the point of intersection of the two curves.

$$\sin x = \cos x$$
$$\Rightarrow x = \frac{\pi}{4}$$

Thus, the point of intersection of the curves as $\left(\frac{\pi}{4},\frac{1}{\sqrt{2}}\right).$

Now, find the tangent to the curves.

Slope of the tangent to the curve $y=\sin x$ at $x=\frac{\pi}{4}$ is obtained by substituting $\frac{\pi}{4}$ for *x* in the derivative of the function $y=\sin x.$

So the slope of the tangent for the curve $y=\sin x$ at $x=\frac{\pi}{4}$ is $\cos\left(\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}}.$

Equation to the line passing through the point $\left(\frac{\pi}{4},\frac{1}{\sqrt{2}}\right)$ with slope $\frac{1}{\sqrt{2}}$ is obtained by

$$y-\frac{1}{\sqrt{2}}=\frac{1}{\sqrt{2}}\left(x-\frac{\pi}{4}\right)$$
$$\Rightarrow y=\frac{1}{\sqrt{2}}x+\left(\frac{1}{\sqrt{2}}-\frac{\pi}{4}\right)$$
$$\Rightarrow 4\sqrt{2}y=4x+\left(4-\pi\sqrt{2}\right)$$
$$\Rightarrow 4x-4\sqrt{2}y+\left(4-\pi\sqrt{2}\right)=0$$

Hence the tangent is in the direction of the vector $\left\langle 4,-4\sqrt{2}\right\rangle.$

Slope of the tangent to the curve $y = \cos x$ at $x = \frac{\pi}{4}$ is obtained by substituting $\frac{\pi}{4}$ for x in the derivative of the function $y = \cos x$.

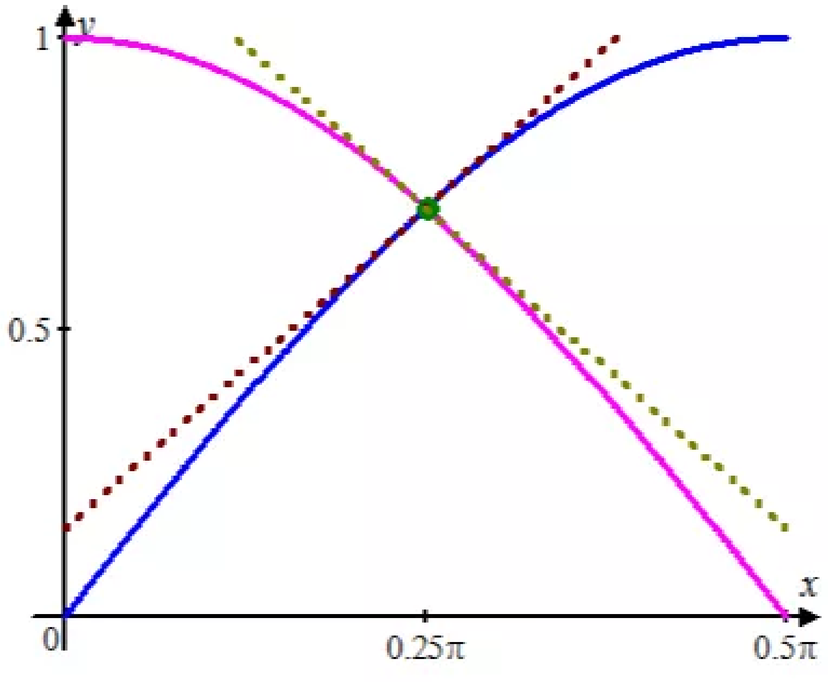
So the slope of the tangent for the curve $y = \cos x$ at $x = \frac{\pi}{4}$ is $-\sin\left(\frac{\pi}{4}\right) = \frac{-1}{\sqrt{2}}$.

Equation to the line passing through the point $\left(\frac{\pi}{4}, \frac{1}{\sqrt{2}}\right)$ with slope $\frac{-1}{\sqrt{2}}$ is obtained by

$$y - \frac{1}{\sqrt{2}} = \frac{-1}{\sqrt{2}}\left(x - \frac{\pi}{4}\right)$$
$$\Rightarrow y = \frac{-1}{\sqrt{2}}x + \left(\frac{1}{\sqrt{2}} - \frac{\pi}{4}\right)$$
$$\Rightarrow 4\sqrt{2}y = -4x + \left(4 - \pi\sqrt{2}\right)$$
$$\Rightarrow 4x + 4\sqrt{2}y - \left(4 - \pi\sqrt{2}\right) = 0$$

Hence the tangent is in the direction of the vector $\langle 4, 4\sqrt{2} \rangle$.

The following diagram shows the tangents and the point of intersection of the given curves.



Angle between the tangents is same as the angle between the vectors which are in the directions of the tangents.

Find the angle between the vectors $\langle 4, -4\sqrt{2} \rangle$ and $\langle 4, 4\sqrt{2} \rangle$.

Where the angle between the vectors be θ.

The angle between the vectors $\langle 4,-4\sqrt{2}\rangle$ and $\langle 4,4\sqrt{2}\rangle$ is obtained by

$$\cos\theta = \left|\frac{\langle 4,-4\sqrt{2}\rangle\cdot\langle 4,4\sqrt{2}\rangle}{\left|\langle 4,-4\sqrt{2}\rangle\right|\left|\langle 4,4\sqrt{2}\rangle\right|}\right|$$

$$= \left|\frac{4\cdot 4+\left(-4\sqrt{2}\right)4\sqrt{2}}{\sqrt{4^2+\left(-4\sqrt{2}\right)^2}\sqrt{4^2+\left(4\sqrt{2}\right)^2}}\right|$$

$$= \left|\frac{16-32}{\sqrt{16+32}\sqrt{16+32}}\right|$$

$$= \frac{|-16|}{48}$$

$$= \frac{16}{48}$$

$$= \frac{1}{3}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{1}{3}\right) \approx \boxed{70.5^\circ}$$ Use calculator to find the exact value.

Therefore the acute angle between the curves $y=\sin x, y=\cos x$ in the interval $0 \le x \le \frac{\pi}{2}$ is $\boxed{70.5^\circ}$.

Answer 33E.

First, find the values of $|\mathbf{a}|$ for the vector $\mathbf{a}=\langle 2,1,2\rangle$ as follows:

$$|\mathbf{a}| = |\langle 2,1,2\rangle|$$

$$= \sqrt{2^2+1^2+2^2}$$ Apply $|\langle a_1,a_2,a_3\rangle| = \sqrt{a_1^2+a_2^2+a_3^2}$

$$= \sqrt{4+1+4}$$

$$= \sqrt{9}$$

$$= 3.$$

Use formula (1), and find the direction cosines for $\mathbf{a} = \langle 2,1,2 \rangle$ as follows:

$$\cos\alpha = \frac{a_1}{|\mathbf{a}|}, \quad \cos\beta = \frac{a_2}{|\mathbf{a}|}, \quad \cos\gamma = \frac{a_3}{|\mathbf{a}|}$$

$$\cos\alpha = \frac{2}{3}, \quad \cos\beta = \frac{1}{3}, \quad \cos\gamma = \frac{2}{3}$$. Use $\langle a_1, a_2, a_3 \rangle = \langle 2,1,2 \rangle$, $|\mathbf{a}| = 3$

Therefore, the direction cosines for $\mathbf{a} = \langle 2,1,2 \rangle$ are $\boxed{\frac{2}{3}, \frac{1}{3}, \frac{2}{3}}$.

Use formula (2), and find the direction angles for $\mathbf{a} = \langle 2,1,2 \rangle$ as follows:

$$\alpha = \cos^{-1}\frac{a_1}{|\mathbf{a}|}, \quad \beta = \cos^{-1}\frac{a_2}{|\mathbf{a}|}, \quad \gamma = \cos^{-1}\frac{a_3}{|\mathbf{a}|}$$

$$\alpha = \cos^{-1}\frac{2}{3}, \quad \beta = \cos^{-1}\frac{1}{3}, \quad \gamma = \cos^{-1}\frac{2}{3}$$ Use $\langle a_1, a_2, a_3 \rangle = \langle 2,1,2 \rangle$, $|\mathbf{a}| = 3$

$$\alpha \approx 48^\circ, \qquad \beta \approx 71^\circ, \qquad \gamma \approx 48^\circ.$$

Therefore, the direction angles for $\mathbf{a} = \langle 2,1,2 \rangle$ are $\boxed{48^\circ, 71^\circ, 48^\circ}$.

Answer 34E.

First, find the values of $|\mathbf{a}|$ for the vector $\mathbf{a} = \langle 6,3,-2 \rangle$ as follows:

$$|\mathbf{a}| = |\langle 6,3,-2 \rangle|$$

$$= \sqrt{6^2 + 3^2 + (-2)^2}$$ Apply $|\langle a_1, a_2, a_3 \rangle| = \sqrt{a_1^2 + a_2^2 + a_3^2}$

$$= \sqrt{36+9+4}$$

$$= \sqrt{49}$$

$$= 7.$$

Use formula (1), and find the direction cosines for $\mathbf{a} = \langle 6,3,-2 \rangle$ as follows:

$$\cos\alpha = \frac{a_1}{|\mathbf{a}|}, \quad \cos\beta = \frac{a_2}{|\mathbf{a}|}, \quad \cos\gamma = \frac{a_3}{|\mathbf{a}|}$$

$$\cos\alpha = \frac{6}{7}, \quad \cos\beta = \frac{3}{7}, \quad \cos\gamma = \frac{-2}{7}$$. Use $\langle a_1, a_2, a_3 \rangle = \langle 6,3,-2 \rangle$, $|\mathbf{a}| = 7$

Therefore, the direction cosines for $\mathbf{a} = \langle 6,3,-2 \rangle$ are $\boxed{\frac{6}{7}, \frac{3}{7}, -\frac{2}{7}}$.

Use formula (2), and find the direction angles for $\mathbf{a}=\langle 6,3,-2\rangle$ as follows:

$$\alpha=\cos^{-1}\frac{a_1}{|\mathbf{a}|},\quad \beta=\cos^{-1}\frac{a_2}{|\mathbf{a}|},\quad \gamma=\cos^{-1}\frac{a_3}{|\mathbf{a}|}$$

$$\alpha=\cos^{-1}\frac{6}{7},\quad \beta=\cos^{-1}\frac{3}{7},\quad \gamma=\cos^{-1}\frac{-2}{7}$$ Use $\langle a_1,a_2,a_3\rangle=\langle 6,3,-2\rangle$, $|\mathbf{a}|=7$

$$\alpha\approx 31^\circ,\quad \beta\approx 65^\circ,\quad \gamma\approx 107^\circ.$$

Therefore, the direction angles for $\mathbf{a}=\langle 6,3,-2\rangle$ are $\boxed{31^\circ,65^\circ,107^\circ}$.

Answer 35E.

First, find the values of $|\mathbf{a}|$ for the vector $\mathbf{a}=\mathbf{i}-2\mathbf{j}-3\mathbf{k}$ as follows:

$$|\mathbf{a}|=|\mathbf{i}-2\mathbf{j}-3\mathbf{k}|$$

$$=\sqrt{1^2+(-2)^2+(-3)^2}$$ Apply $|a_1\mathbf{i}+a_2\mathbf{j}+a_3\mathbf{k}|=\sqrt{a_1^2+a_2^2+a_3^2}$

$$=\sqrt{1+4+9}$$

$$=\sqrt{14}.$$

Use formula (1), and find the direction cosines for $\mathbf{a}=\mathbf{i}-2\mathbf{j}-3\mathbf{k}$ as follows:

$$\cos\alpha=\frac{a_1}{|\mathbf{a}|},\quad \cos\beta=\frac{a_2}{|\mathbf{a}|},\quad \cos\gamma=\frac{a_3}{|\mathbf{a}|}$$

$$\cos\alpha=\frac{1}{\sqrt{14}},\quad \cos\beta=\frac{-2}{\sqrt{14}},\quad \cos\gamma=\frac{-3}{\sqrt{14}}.$$

Use $a_1\mathbf{i}+a_2\mathbf{j}+a_3\mathbf{k}=\mathbf{i}-2\mathbf{j}-3\mathbf{k}$, and $|\mathbf{a}|=\sqrt{14}$

Therefore, the direction cosines for $\mathbf{a}=\mathbf{i}-2\mathbf{j}-3\mathbf{k}$ are $\boxed{\frac{1}{\sqrt{14}},-\frac{2}{\sqrt{14}},-\frac{3}{\sqrt{14}}}$.

Use formula (2), and find the direction angles for $\mathbf{a}=\mathbf{i}-2\mathbf{j}-3\mathbf{k}$ as follows:

$$\alpha=\cos^{-1}\frac{a_1}{|\mathbf{a}|},\qquad \beta=\cos^{-1}\frac{a_2}{|\mathbf{a}|},\qquad \gamma=\cos^{-1}\frac{a_3}{|\mathbf{a}|}$$

$$\alpha=\cos^{-1}\frac{1}{\sqrt{14}},\quad \beta=\cos^{-1}\frac{-2}{\sqrt{14}},\quad \gamma=\cos^{-1}\frac{-3}{\sqrt{14}}$$

Use $a_1\mathbf{i}+a_2\mathbf{j}+a_3\mathbf{k}=\mathbf{i}-2\mathbf{j}-3\mathbf{k}$, and $|\mathbf{a}|=\sqrt{14}$

$$\alpha\approx 74^\circ,\qquad \beta\approx 122^\circ,\qquad \gamma\approx 143^\circ.$$

Therefore, the direction angles for $\mathbf{a}=\mathbf{i}-2\mathbf{j}-3\mathbf{k}$ are $\boxed{74^\circ,122^\circ,143^\circ}$.

Answer 36E.

First, find the values of $|\mathbf{a}|$ for the vector $\mathbf{a}=\frac{1}{2}\mathbf{i}+\mathbf{j}+\mathbf{k}$ as follows:

$$|\mathbf{a}|=\left|\frac{1}{2}\mathbf{i}+\mathbf{j}+\mathbf{k}\right|$$

$$=\sqrt{\left(\frac{1}{2}\right)^2+1^2+1^2}$$ Apply $|a_1\mathbf{i}+a_2\mathbf{j}+a_3\mathbf{k}|=\sqrt{a_1^2+a_2^2+a_3^2}$

$$=\sqrt{\frac{1}{4}+1+1}$$

$$=\sqrt{\frac{9}{4}}$$

$$=\frac{3}{2}.$$

Use formula (1), and find the direction cosines for $\mathbf{a}=\frac{1}{2}\mathbf{i}+\mathbf{j}+\mathbf{k}$ as follows:

$$\cos\alpha=\frac{a_1}{|\mathbf{a}|},\qquad \cos\beta=\frac{a_2}{|\mathbf{a}|},\qquad \cos\gamma=\frac{a_3}{|\mathbf{a}|}$$

$$\cos\alpha=\frac{1/2}{3/2},\quad \cos\beta=\frac{1}{3/2},\quad \cos\gamma=\frac{1}{3/2}$$

Use $a_1\mathbf{i}+a_2\mathbf{j}+a_3\mathbf{k}=\frac{1}{2}\mathbf{i}+\mathbf{j}+\mathbf{k}$, and $|\mathbf{a}|=\frac{3}{2}$

$$\cos\alpha=\frac{1}{3},\qquad \cos\beta=\frac{2}{3},\qquad \cos\gamma=\frac{2}{3}.$$

Therefore, the direction cosines for $\mathbf{a}=\frac{1}{2}\mathbf{i}+\mathbf{j}+\mathbf{k}$ are $\boxed{\frac{1}{3},\frac{2}{3},\frac{2}{3}}$.

Use formula (2), and find the direction angles for $\mathbf{a} = \frac{1}{2}\mathbf{i} + \mathbf{j} + \mathbf{k}$ as follows:

$$\alpha = \cos^{-1}\frac{a_1}{|\mathbf{a}|}, \quad \beta = \cos^{-1}\frac{a_2}{|\mathbf{a}|}, \quad \gamma = \cos^{-1}\frac{a_3}{|\mathbf{a}|}$$

$$\alpha = \cos^{-1}\frac{1}{3}, \quad \beta = \cos^{-1}\frac{2}{3}, \quad \gamma = \cos^{-1}\frac{2}{3}$$

Use $a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k} = \frac{1}{2}\mathbf{i} + \mathbf{j} + \mathbf{k}$, and $|\mathbf{a}| = \frac{3}{2}$

$$\alpha \approx 71^\circ, \quad \beta \approx 48^\circ, \quad \gamma \approx 48^\circ.$$

Therefore, the direction angles for $\mathbf{a} = \frac{1}{2}\mathbf{i} + \mathbf{j} + \mathbf{k}$ are $\boxed{71^\circ, 48^\circ, 48^\circ}$.

Answer 37E.

Find the values of $|\mathbf{a}|$ for the vector; $\mathbf{a} = \langle -5, 12 \rangle$, as follows:

$$|\mathbf{a}| = |\langle -5, 12 \rangle|$$

$= \sqrt{(-5)^2 + 12^2}$ Use $|\langle a_1, a_2 \rangle| = \sqrt{a_1^2 + a_2^2}$

$$= \sqrt{25 + 144}$$

$$= \sqrt{169}$$

$$= 13.$$

Next, find the dot product $\mathbf{a} \cdot \mathbf{b}$ for the vector, $\mathbf{a} = \langle -5, 12 \rangle$, and $\mathbf{b} = \langle 4, 6 \rangle$, as follows:

$$\mathbf{a} \cdot \mathbf{b} = \langle -5, 12 \rangle \cdot \langle 4, 6 \rangle$$

$= -5(4) + 12(6)$ Use $\langle a_1, a_2 \rangle \cdot \langle b_1, b_2 \rangle = a_1 b_1 + a_2 b_2$

$$= -20 + 72$$

$$= 52.$$

By above steps, the values of $|\mathbf{a}|$, and $\mathbf{a} \cdot \mathbf{b}$ for $\mathbf{a} = \langle -5, 12 \rangle$, and $\mathbf{b} = \langle 4, 6 \rangle$ are,

$|\mathbf{a}| = 13$, and $\mathbf{a} \cdot \mathbf{b} = 52$.

Use formula (1), find the scalar projection of $\mathbf{b}$ onto $\mathbf{a}$ as follows:

$$\text{comp}_{\mathbf{a}}\mathbf{b} = \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|}$$ Write (1)

$$\text{comp}_{\mathbf{a}}\mathbf{b} = \frac{52}{13}$$ Substitute $|\mathbf{a}| = 13$, and $\mathbf{a}\cdot\mathbf{b} = 52$

$$\text{comp}_{\mathbf{a}}\mathbf{b} = 4.$$

Therefore, the scalar projection of $\mathbf{b}$ onto $\mathbf{a}$ is $\boxed{\text{comp}_{\mathbf{a}}\mathbf{b} = 4}$.

Use formula (2), find the vector projection of $\mathbf{b}$ onto $\mathbf{a}$ as follows:

$$\text{proj}_{\mathbf{a}}\mathbf{b} = \left(\frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|}\right)\frac{\mathbf{a}}{|\mathbf{a}|}$$ Write (2)

$$\text{proj}_{\mathbf{a}}\mathbf{b} = \left(\frac{52}{13}\right)\frac{\langle -5,12\rangle}{13}$$ Substitute $|\mathbf{a}| = 13$, $\mathbf{a}\cdot\mathbf{b} = 52$ and $\mathbf{a} = \langle -5,12\rangle$

$$= (4)\frac{\langle -5,12\rangle}{13}$$

$$= \frac{4}{13}\langle -5,12\rangle$$

$$= \left\langle -\frac{4}{13}\times 5, \frac{4}{13}\times 12\right\rangle$$

$$= \left\langle -\frac{20}{13}, \frac{48}{13}\right\rangle.$$

Therefore, the vector projection of $\mathbf{b}$ onto $\mathbf{a}$ is $\boxed{\text{proj}_{\mathbf{a}}\mathbf{b} = \left\langle -\frac{20}{13}, \frac{48}{13}\right\rangle}$.

Answer 38E.

$$\vec{a} = \langle c,c,c\rangle, \quad c > 0$$

$$|\vec{a}| = \sqrt{c^2 + c^2 + c^2}$$

$$= \sqrt{3c^2} = \sqrt{3}c$$

The direction cosines are

$$\cos\alpha = \frac{c}{\sqrt{3}c} = \frac{1}{\sqrt{3}},$$

$$\cos\beta = \frac{c}{\sqrt{3}c} = \frac{1}{\sqrt{3}}$$

And $$\cos\gamma = \frac{c}{\sqrt{3}c} = \frac{1}{\sqrt{3}}$$

The direction angles are

$$\alpha = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right) = \cos^{-1}(0.577) = 55^o$$

$$\beta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right) = \cos^{-1}(0.577) = 55^o$$

$$\gamma = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right) = \cos^{-1}(0.577) = 55^o$$

Answer 39E.

If $\alpha = \pi/4$, $\quad \cos\alpha = \cos\pi/4 = \frac{1}{\sqrt{2}}$

$\beta = \pi/3$, $\quad \cos\beta = \cos\pi/3 = \frac{1}{2}$

We know that

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

Then $\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2 + \cos^2\gamma = 1$

$$\Rightarrow \frac{1}{2} + \frac{1}{4} + \cos^2\gamma = 1$$

$$\Rightarrow \cos^2\gamma = 1 - \frac{3}{4}$$

$$\Rightarrow \cos^2\gamma = \frac{1}{4}$$

$$\Rightarrow \cos\gamma = +\frac{1}{2} = \cos\pi/3 \quad or \quad \cos\frac{2\pi}{3}$$

$$\Rightarrow \boxed{\gamma = \pi/3 \ or \ 2\pi/3.}$$

Answer 40E.

Find the values of $|\mathbf{a}|$ for the vector; $\mathbf{a} = \langle 1, 4 \rangle$, as follows:

$$|\mathbf{a}| = |\langle 1,4 \rangle|$$
$$= \sqrt{1^2 + 4^2} \quad \text{Use } |\langle a_1, a_2 \rangle| = \sqrt{a_1^2 + a_2^2}$$
$$= \sqrt{1+16}$$
$$= \sqrt{17}.$$

Next, find the dot product $\mathbf{a} \cdot \mathbf{b}$ for the vector, $\mathbf{a} = \langle 1, 4 \rangle$, and $\mathbf{b} = \langle 2, 3 \rangle$, as follows:

$$\mathbf{a} \cdot \mathbf{b} = \langle 1,4 \rangle \cdot \langle 2,3 \rangle$$
$$= 1(2) + 4(3) \quad \text{Use } \langle a_1, a_2 \rangle \cdot \langle b_1, b_2 \rangle = a_1 b_1 + a_2 b_2$$
$$= 2 + 12$$
$$= 14.$$

By above steps, the values of $|\mathbf{a}|$, and $\mathbf{a} \cdot \mathbf{b}$ for $\mathbf{a} = \langle 1, 4 \rangle$, and $\mathbf{b} = \langle 2, 3 \rangle$ are,

$|\mathbf{a}| = \sqrt{17}$, and $\mathbf{a} \cdot \mathbf{b} = 14$.

Use formula (1), find the scalar projection of $\mathbf{b}$ onto $\mathbf{a}$ as follows:

$$\text{comp}_{\mathbf{a}} \mathbf{b} = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|} \quad \text{Write (1)}$$

$$\text{comp}_{\mathbf{a}} \mathbf{b} = \frac{14}{\sqrt{17}}. \quad \text{Substitute } |\mathbf{a}| = \sqrt{17}, \text{ and } \mathbf{a} \cdot \mathbf{b} = 14$$

Therefore, the scalar projection of $\mathbf{b}$ onto $\mathbf{a}$ is $\boxed{\text{comp}_{\mathbf{a}} \mathbf{b} = \frac{14}{\sqrt{17}}}$.

Use formula (2), find the vector projection of $\mathbf{b}$ onto $\mathbf{a}$ as follows:

$$\text{proj}_{\mathbf{a}}\mathbf{b} = \left(\frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|}\right)\frac{\mathbf{a}}{|\mathbf{a}|}$$ Write (2)

$$\text{proj}_{\mathbf{a}}\mathbf{b} = \left(\frac{14}{\sqrt{17}}\right)\frac{\langle 1,4\rangle}{\sqrt{17}}$$ Substitute $|\mathbf{a}| = \sqrt{17}$, $\mathbf{a}\cdot\mathbf{b} = 14$ and $\mathbf{a} = \langle 1,4\rangle$

$$= \frac{14}{17}\langle 1,4\rangle$$

$$= \left\langle \frac{14}{17}\times 1, \frac{14}{17}\times 4\right\rangle$$

$$= \left\langle \frac{14}{17}, \frac{56}{17}\right\rangle.$$

Therefore, the vector projection of $\mathbf{b}$ onto $\mathbf{a}$ is $\boxed{\text{proj}_{\mathbf{a}}\mathbf{b} = \left\langle \frac{14}{17}, \frac{56}{17}\right\rangle}$.

Answer 42E.

Consider the vectors

$$\mathbf{a} = \langle -2,3,-6\rangle$$
$$\mathbf{b} = \langle 5,-1,4\rangle$$

The objective is to find the scalar and vector projections of **b** onto **a**.

The formula for scalar projection of **b** onto **a** is $\text{comp}_{\mathbf{a}}\mathbf{b} = \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|}$.

The formula for vector projection of **b** onto **a** is $\text{proj}_{\mathbf{a}}\mathbf{b} = \left(\frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|^2}\right)\mathbf{a}$.

Now the scalar projection of **b** onto **a** is

$$\begin{aligned}
\text{comp}_{\mathbf{a}}\mathbf{b} &= \frac{\langle -2,3,-6\rangle \cdot \langle 5,-1,4\rangle}{|\langle -2,3,-6\rangle|} \\
&= \frac{(-2)(5)+(3)(-1)+(-6)(4)}{\sqrt{(-2)^2+(3)^2+(-6)^2}} \\
&= \frac{-10-3-24}{\sqrt{4+9+36}} \\
&= -\frac{37}{\sqrt{49}} \\
&= -\frac{37}{7}
\end{aligned}$$

Hence, the scalar projection of **b** onto **a** is $\boxed{\text{comp}_{\mathbf{a}}\mathbf{b} = -\frac{37}{7}}$.

The projection of **b** onto **a** is $\text{proj}_{\mathbf{a}}\mathbf{b} = \left(\frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|^2}\right)\mathbf{a}.$

Therefore,

$$\begin{aligned}
\text{proj}_{\mathbf{a}}\mathbf{b} &= \left[\frac{\langle -2,3,-6\rangle \cdot \langle 5,-1,4\rangle}{|\langle -2,3,-6\rangle|^2}\right]\langle -2,3,-6\rangle \\
&= \left[\frac{(-2)(5)+(3)(-1)+(-6)(4)}{\left(\sqrt{(-2)^2+(3)^2+(-6)^2}\right)^2}\right]\langle -2,3,-6\rangle \\
&= \left[\frac{-10-3-24}{49}\right]\langle -2,3,-6\rangle \\
&= \left(-\frac{37}{49}\right)\langle -2,3,-6\rangle \\
&= \left\langle \frac{74}{49}, -\frac{111}{49}, \frac{222}{49}\right\rangle
\end{aligned}$$

Hence, the vector projection of **b** onto **a** is $\boxed{\text{proj}_{\mathbf{a}}\mathbf{b} = \left\langle \frac{74}{49}, -\frac{111}{49}, \frac{222}{49}\right\rangle}$.

Answer 43E.

Given that a = 2i - j + 4**k**

b = j + 1/2**k**

Scalar projection of **b** onto **a** = compab = $\frac{a \cdot b}{|a|}$

Vector projection of **b** onto **a** = projab = $\frac{a \cdot b}{|a|^2} \cdot a$

$$a \cdot b = (2 \times 0) + (-1 \times 1) + \left(4 \times \frac{1}{2}\right) = 1$$

$$|a| = \sqrt{2^2 + -1^2 + 4^2} = \sqrt{21}$$

$$|b| = \sqrt{1^2 + (1/2)^2} = \sqrt{\frac{5}{4}}$$

compab $= \frac{1}{\sqrt{21}}$

projab $= \frac{1}{21} \cdot <2, -1, 4> = <\frac{2}{21}, \frac{-1}{21}, \frac{4}{21}>$

Answer 44E.

Consider the following vectors:

$\mathbf{a} = \mathbf{i} + \mathbf{j} + \mathbf{k}, \ \ \mathbf{b} = \mathbf{i} - \mathbf{j} + \mathbf{k}$

Use the following details to solve:

Scalar projection of $\mathbf{b}$ onto $\mathbf{a}$???????????? $\text{comp}_{\mathbf{a}}\mathbf{b} = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|}$?

Vector projection of $\mathbf{b}$ onto $\mathbf{a}$???????

$$\text{Proj}_{\mathbf{a}}\mathbf{b} = \left(\frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|}\right)\frac{\mathbf{a}}{|\mathbf{a}|} \ ?$$

$$= \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|^2}\mathbf{a}$$

The vectors are represented as follows:

$\mathbf{a} = \mathbf{i} + \mathbf{j} + \mathbf{k}, \quad \mathbf{b} = \mathbf{i} - \mathbf{j} + \mathbf{k}$

$$\begin{aligned} \mathbf{a} &= \mathbf{i} + \mathbf{j} + \mathbf{k} & \mathbf{b} &= \mathbf{i} - \mathbf{j} + \mathbf{k} \\ &= \langle 1,1,1 \rangle & &= \langle 1,-1,1 \rangle \end{aligned}$$

$$\begin{aligned} |\mathbf{a}| &= \sqrt{1^2 + 1^2 + 1^2} \\ &= \sqrt{1+1+1} \\ &= \sqrt{3} \end{aligned}$$

The scalar projection of $\mathbf{b}$ onto $\mathbf{a}$ is calculated as follows:

$$\begin{aligned} \text{comp}_{\mathbf{a}}\mathbf{b} &= \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|} \\ &= \frac{(1)(1)+(1)(-1)+(1)(1)}{\sqrt{3}} \\ &= \frac{1-1+1}{\sqrt{3}} \\ &= \boxed{\frac{1}{\sqrt{3}}} \end{aligned}$$

The vector projection is the scalar projection times the unit vector in the direction of $\mathbf{a}$. This is calculated as follows:

$$\begin{aligned} \text{Proj}_{\mathbf{a}}\mathbf{b} &= \left(\frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|}\right)\frac{\mathbf{a}}{|\mathbf{a}|} \\ &= \left(\frac{1}{\sqrt{3}}\right)\frac{\mathbf{a}}{\sqrt{3}} \\ &= \frac{\mathbf{a}}{\left(\sqrt{3}\right)^2} \end{aligned}$$

$$= \frac{\mathbf{a}}{3}$$

$$= \boxed{\left\langle \frac{1}{3}, \frac{1}{3}, \frac{1}{3} \right\rangle}$$

Answer 45E.

It is given that

$\text{Orth}_{\vec{a}}\vec{b} = \vec{b} - \text{proj}_{\vec{a}}\vec{b}$

We require to prove that $\text{Orth}_{\vec{a}}\vec{b}$ is orthogonal to $\vec{a}$

We recall the following:

(1) Two vectors $\mathbf{a}$ and $\mathbf{b}$ are orthogonal if and only if $\mathbf{a}\cdot\mathbf{b} = 0$

(2) $\text{proj}_{\vec{a}}\vec{b} = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|^2}\vec{a}$

(3) $\vec{a}\cdot(\vec{b}+\vec{c}) = \vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}$

Consider $\vec{a}\cdot(\text{orth}_{\vec{a}}\vec{b})$

$$= \vec{a}\cdot\left(\vec{b} - \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|^2}\vec{a}\right) \qquad \left\{\text{as } \text{proj}_{\vec{a}}\vec{b} = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|^2}\vec{a}\right.$$

$$= \vec{a}\cdot\vec{b} - \frac{(\vec{a}\cdot\vec{b})(\vec{a}\cdot\vec{a})}{|\vec{a}|^2} \qquad \left(\vec{a}\cdot(\vec{b}+\vec{c}) = \vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}\right)$$

$$= \vec{a}\cdot\vec{b} - \frac{(\vec{a}\cdot\vec{b})|\vec{a}|^2}{|\vec{a}|^2} \qquad \left\{\text{as } \vec{a}\cdot\vec{a} = |\vec{a}|^2\right.$$

$$= \vec{a}\cdot\vec{b} - \vec{a}\cdot\vec{b}$$
$$= 0$$

Answer 46E.

We know that $\text{orth}_{a}b = b - \text{proj}_{a}b$, where $\text{proj}_{\mathbf{a}}\mathbf{b} = \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|^2}\mathbf{a}$.

It is given that $\mathbf{a} = \langle 1, 4\rangle$ and $\mathbf{b} = \langle 2, 3\rangle$.

Find $\mathbf{a}\cdot\mathbf{b}$.

$$\begin{aligned}\mathbf{a}\cdot\mathbf{b} &= (1)(2)+(4)(3)\\ &= 2+12\\ &= 14\end{aligned}$$

Now, find $|\mathbf{a}|$.

$$\begin{aligned}|\mathbf{a}| &= \sqrt{1+16}\\ &= \sqrt{17}\end{aligned}$$

We thus get $|\mathbf{a}|^2$ as 17.

Substitute the known values in $\text{proj}_{\mathbf{a}}\mathbf{b} = \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|^2}\mathbf{a}$

And simplify.

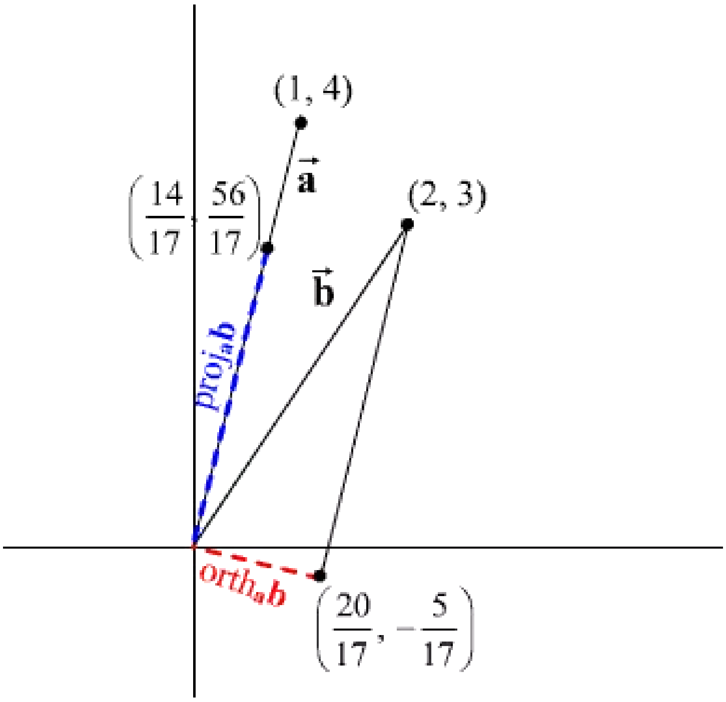
$$\begin{aligned}\text{proj}_{\mathbf{a}}\mathbf{b} &= \frac{14}{17}(\mathbf{i}+4\mathbf{j})\\ &= \frac{14}{17}\mathbf{i} + \frac{56}{17}\mathbf{j}\end{aligned}$$

Replace $\text{proj}_{\mathbf{a}}\mathbf{b}$ with $\frac{14}{17}\mathbf{i}+\frac{56}{17}\mathbf{j}$ and b with $2\mathrm{i}+3\mathrm{j}$ in $\text{orth}_{\text{a}}\text{b} = \text{b} - \text{proj}_{\text{a}}\text{b}$.

$$\begin{aligned}\text{orth}_{\mathbf{a}}\mathbf{b} &= (2\mathbf{i}+3\mathbf{j}) - \left(\frac{14}{17}\mathbf{i}+\frac{56}{17}\mathbf{j}\right) \\ &= \frac{20}{17}\mathbf{i} - \frac{5}{17}\mathbf{j}\end{aligned}$$

Thus, we get the vector orthogonal projection as $\frac{20}{17}\mathbf{i}-\frac{5}{17}\mathbf{j}$.

Now, let us sketch a, b, $\text{proj}_{\text{a}}\text{b}$, and $\text{orth}_{\text{a}}\text{b}$.



Hi sir,

I am sending the text file to through the attachment and modifications
In text file 4757, all chapters contains true or false questions, also answers for those questions given in text book but as per our guide lines i mentioned all those question in text file.

Answer 47E.

$$\vec{a} = \langle 3, 0, -1\rangle$$

$$comp_{\vec{a}}\vec{b} = \frac{\vec{a}.\vec{b}}{|\vec{a}|} = 2(given)$$

Let $\vec{b} = \langle b_1, b_2, b_3\rangle$

Then $\frac{\langle 3,0,-1\rangle.\langle b_1,b_2,b_3\rangle}{\sqrt{9+0+1}} = 2$

i.e. $\frac{3b_1+0-b_3}{\sqrt{10}} = 2$

i.e. $3b_1 - b_3 = 2\sqrt{10}$

Take $b_1 = s \quad \in R$

Then $b_3 = 3s - 2\sqrt{10}$

Then $\vec{b}$ is any vector of the form

$$\boxed{\left\langle s, t, 3s - 2\sqrt{10} \right\rangle} \qquad \textit{where} \;\; s, t \in R$$

Answer 48E.

Consider the non-zero vectors **a** and **b**.

a).

The objective is to find the condition for $\text{comp}_{\mathbf{a}}\mathbf{b} = \text{comp}_{\mathbf{b}}\mathbf{a}$.

The formula for scalar projection of **b** onto **a** is $\text{comp}_{\mathbf{a}}\mathbf{b} = \dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|}$.

Suppose, $\text{comp}_{\mathbf{a}}\mathbf{b} = \text{comp}_{\mathbf{b}}\mathbf{a}$.

$$\frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|} = \frac{\mathbf{b}\cdot\mathbf{a}}{|\mathbf{b}|} \qquad \left(\text{Since } \text{comp}_{\mathbf{a}}\mathbf{b} = \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|}.\right)$$

$$|\mathbf{b}|(\mathbf{a}\cdot\mathbf{b}) = |\mathbf{a}|(\mathbf{b}\cdot\mathbf{a})$$

$$|\mathbf{b}| = |\mathbf{a}| \qquad (\text{Since } \mathbf{a}\cdot\mathbf{b} = \mathbf{b}\cdot\mathbf{a})$$

Therefore, $\text{comp}_{\mathbf{a}}\mathbf{b} = \text{comp}_{\mathbf{b}}\mathbf{a}$, if $\boxed{|\mathbf{b}| = |\mathbf{a}|}$.

---

b).

Here the objective is to find the condition for $\text{proj}_{\mathbf{a}}\mathbf{b} = \text{proj}_{\mathbf{b}}\mathbf{a}$.

The formula for vector projection of **b** onto **a** is $\text{proj}_{\mathbf{a}}\mathbf{b} = \left(\dfrac{\mathbf{a}\bullet\mathbf{b}}{|\mathbf{a}|^2}\right)\mathbf{a}$.

Suppose, $\text{proj}_{\mathbf{a}}\mathbf{b} = \text{proj}_{\mathbf{b}}\mathbf{a}$.

$$\frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|^2}\mathbf{a} = \frac{\mathbf{b}\cdot\mathbf{a}}{|\mathbf{b}|^2}\mathbf{b}$$

$$\frac{\mathbf{a}}{|\mathbf{a}|^2} = \frac{\mathbf{b}}{|\mathbf{b}|^2} \qquad (\text{Since } \mathbf{a}\cdot\mathbf{b} = \mathbf{b}\cdot\mathbf{a})$$

Therefore, $\text{proj}_{\mathbf{a}}\mathbf{b} = \text{proj}_{\mathbf{b}}\mathbf{a}$, if $\boxed{\dfrac{\mathbf{a}}{|\mathbf{a}|^2} = \dfrac{\mathbf{b}}{|\mathbf{b}|^2}}$.

Answer 49E.

Given F=8i-6j+9k which moves an object from the point (0, 10, 8) to (6, 12, 20)

we need to solve for work.W=Fd

distance between the points = (6, 2, 12)

$W = F \cdot d$

= 6(8)+2(-6)+12(9)

= 48-12+108

= 144J

Answer 50E.

Given $\theta = 30$, T=1500. we are suppose to find how much work is done by the truck pulling the car 1km= 1000m

w = Fd cos θ

w = 1500N(1000m)cos(30)

= 1299038.106J

Answer 51E.

Given a 30 pound force acting at an angle of 40 degrees above the horizontal moves the sled 80ft. we are suppose to find the work done by the force.

W=Fd cos θ

W= 30(80)cos(40)

= 1838.51 ft-lb

Answer 52E.

A boat sails south with the help of a wind blowing in the direction $S36^0E$ with magnitude 400 lb and the wind as the boat moves 120 ft.

Recollect that:

If θ is the angle between the vectors $\mathbf{a}$ and ,then $\mathbf{a}\cdot\mathbf{b}=|\mathbf{a}||\mathbf{b}|\cos\theta$

The wind moves towards vector $\mathbf{F}$ and the boat moves towards vector $\mathbf{D}$

A wind blowing in the direction $S36^0E$

That is $\theta = 36^0$

To find the work of wind is a W

$$\begin{aligned} W &= |\mathbf{F}|\cdot|\mathbf{D}|\cos\theta \\ &= (400)(120)\cos 36^0 \\ &= 48000\left(\cos 36^0\right) \\ &= \boxed{38{,}832.81573\ \text{ft,lb}} \end{aligned}$$

Answer 53E.

Consider the equation of the line is $ax+by+c=0$ and the point is $P(x_1, y_1)$.

By using the scalar projection, we have to show that the distance between the given line and the point is $\dfrac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$.

The formula for scalar projection of **b** onto **a** is $\text{comp}_{\mathbf{a}}\mathbf{b} = \dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|}$.

First we must create a vector like follows that is perpendicular to the line.

Namely, $\mathbf{n} = \langle a, b\rangle$ that lies on the following points $(a_1, b_1), (a_2, b_2)$

Such that we have the following

$$\mathbf{n}\cdot\mathbf{u} = aa_2 - aa_1 + bb_2 - bb_1 = 0$$

From the equation of a line we know we have the following

$$-c = aa_1 + bb_1$$
$$-c = aa_2 + bb_2$$

Then the distance from the first point P_1 to the line is the absolute value of the scalar projection of $\vec{P_1P_2}$ onto $\mathbf{n}$

$$\text{comp}_{\mathbf{n}}\left(\vec{P_1P_2}\right) = \left|\frac{\mathbf{n}\cdot\langle x_2 - x_1, y_2 - y_1\rangle}{|\mathbf{n}|}\right|$$
$$= \left|\frac{a(x_2 - x_1) + b(y_2 - y_1)}{\sqrt{a^2 + b^2}}\right|$$
$$= \left|\frac{ax_2 - ax_1 + by_2 - by_1}{\sqrt{a^2 + b^2}}\right|$$

Continuation to the above steps

$$= \left|\frac{-ax_1 - by_1 + ax_2 + by_2}{\sqrt{a^2 + b^2}}\right|$$
$$= \left|\frac{-ax_1 - by_1 - c}{\sqrt{a^2 + b^2}}\right| \qquad (\text{Since } ax_2 + by_2 = -c)$$
$$= \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \qquad (\text{Since } |-a| = a)$$

Hence the distance from a line to a point is $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$.

Now to find the distance from the point $(-2,3)$ to the line $3x - 4y + 5 = 0$.

The distance from a line to a point is $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$.

Now we can plug in for the following point and line $3x - 4y = 5, (-2,3)$

Therefore,

$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}} = \frac{|3(-2) + (-4)3 + 5|}{\sqrt{3^2 + (-4)^2}}$$
$$= \frac{|-6 - 12 + 5|}{\sqrt{9 + 16}}$$
$$= \frac{|-13|}{\sqrt{25}}$$
$$= \frac{13}{5} \qquad (\text{Since } |-a| = a)$$

Therefore, the distance from the point $(-2,3)$ to the line $3x - 4y + 5 = 0$ is $\boxed{\frac{13}{5}}$.

Answer 54E.

$$\vec{r} = \langle x, y, z\rangle, \quad \vec{a} = \langle a_1, a_2, a_3\rangle, \vec{b} = \langle b_1, b_2, b_3\rangle$$

$$\vec{r} - \vec{a} = \langle x - a_1, y - a_2, z - a_3\rangle$$

$$\vec{r} - \vec{b} = \langle x - b_1, y - b_2, z - b_3\rangle$$

Now

$$(\vec{r} - \vec{a}).(\vec{r} - \vec{b}) = (x - a_1)(x - b_1) + (y - a_2)(y - b_2) + (z - a_3)(z - b_3)$$

$$= x^2 - (a_1 + b_1)x + a_1b_1 + y^2 - (a_2 + b_2)y + a_2b_2 + z^2 - (a_3 + b_3)z + a_3b_3$$

$\because (\vec{r} - \vec{a})(\vec{r} - \vec{b}) = 0$ Then

$$\left(x^2 - (a_1 + b_1)x + \left(\frac{a_1 + b_1}{2}\right)^2\right) + \left(y^2 - (a_2 + b_2)y + \left(\frac{a_2 + b_2}{2}\right)^2\right) + \left(z^2 - (a_3 + b_3)z + \left(\frac{a_3 + b_3}{2}\right)^2\right)$$

$$= -a_1b_1 - a_2b_2 - a_3b_3 + \frac{(a_1 + b_1)^2}{4} + \frac{(a_2 + b_2)^2}{4} + \frac{(a_3 + b_3)^2}{4}$$

or, $$\left[x - \left(\frac{a_1 + b_1}{2}\right)\right]^2 + \left[y - \frac{(a_2 + b_2)}{2}\right]^2 + \left[z - \left(\frac{a_3 + b_3}{2}\right)\right]^2$$

$$= \frac{\left[(a_1 + b_1)^2 - 4a_1b_1\right] + \left[(a_2 + b_2)^2 - 4a_2b_2\right] + \left[(a_3 + b_3)^2 - 4a_3b_3\right]}{4}$$

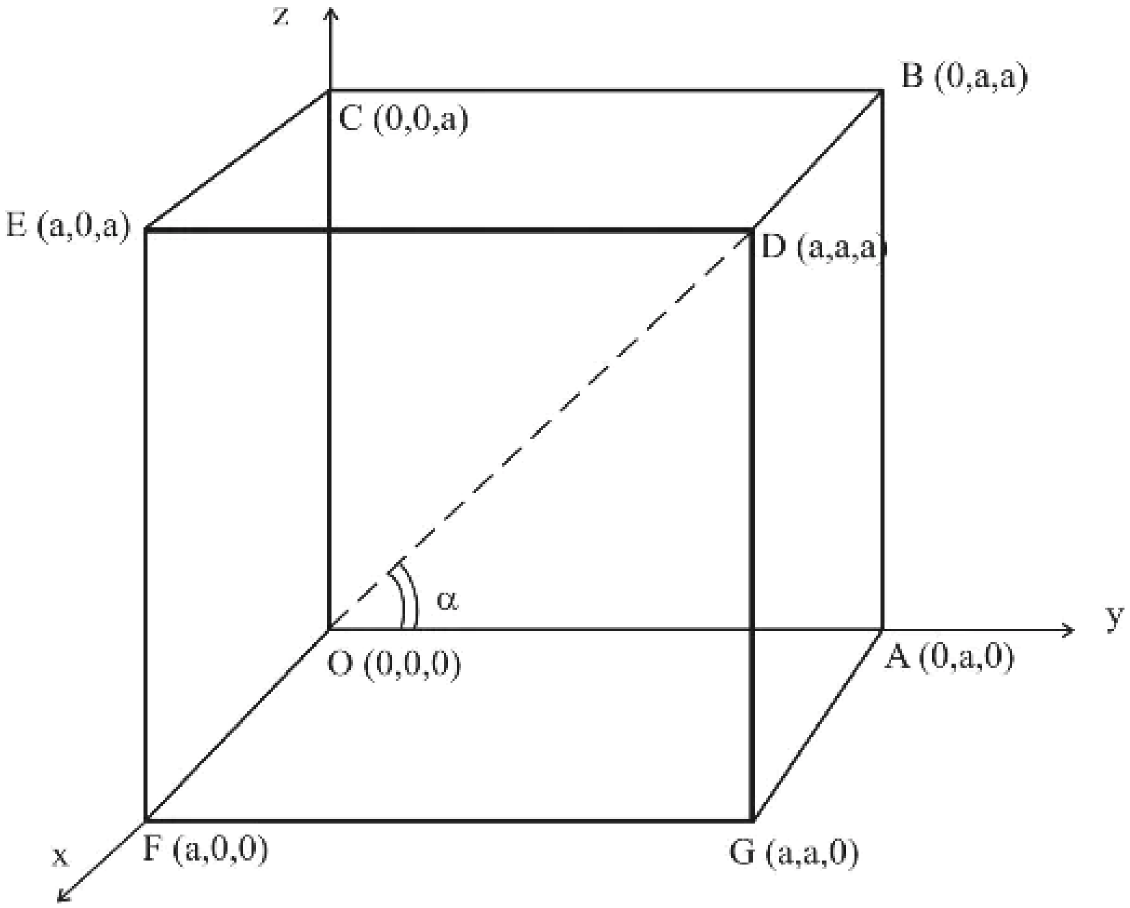
$$= \frac{(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2}{4}$$

This is the equation of sphere of centre

$$\boxed{\left(\frac{(a_1 + b_1)}{2}, \frac{(a_2 + b_2)}{2}, \frac{(a_3 + b_3)}{2}\right)}$$ and

Radius $= \boxed{\frac{1}{2}\sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2}}$

Answer 55E.



Let a be the side of the cube OABCDEFG.

Then the diagonal $\overrightarrow{OD} = \langle a, a, a \rangle$

Let us consider the edge A.

Then $\overrightarrow{OA} = (0, a, 0)$

Let α be the angle between the diagonal OD and the edge OA.

Then

$$\overrightarrow{OA}.\overrightarrow{OD} = \left|\overrightarrow{OA}\right|.\left|\overrightarrow{OD}\right| \cos\alpha,$$

by the definition of the scalar product of the vectors

Therefore,

$$(o, a, o).\langle a, a, a \rangle = \sqrt{o^2 + a^2 + o^2}.\sqrt{a^2 + a^2 + a^2}.\cos\alpha$$

$$\text{or, } o + a^2 + o = a.\sqrt{3a^2}\cos\alpha$$

$$\text{or, } a^2 = a.\sqrt{3}a\cos\alpha$$

$$\text{or, } a^2 = \sqrt{3}.a^2\cos\alpha$$

$$\therefore 1 = \sqrt{3}\cos\alpha$$

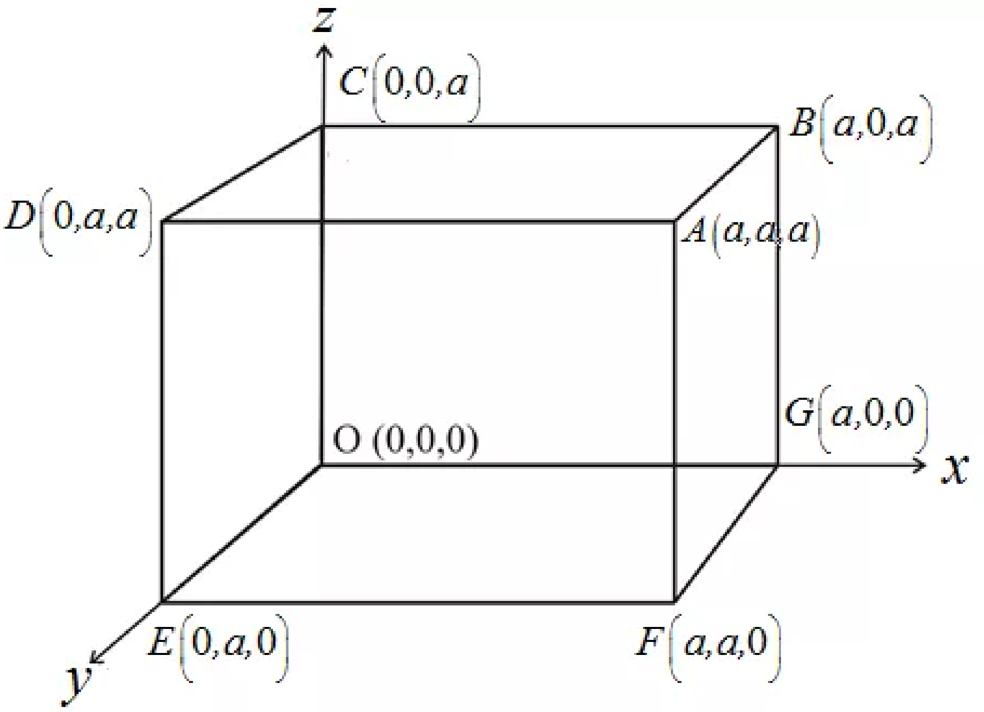
Or $\cos\alpha = \dfrac{1}{\sqrt{3}}$

Or $\boxed{\alpha = \cos^{-1}\dfrac{1}{\sqrt{3}}}$

In a similar manner we can take other edges and find the angle between the diagonal OD and them. Here, as calculated the angle between the diagonal and the edge OA is

$$\boxed{\cos^{-1}\frac{1}{\sqrt{3}} \approx 55^o}$$

Answer 56E.



Find the angle between a diagonal of a cube and a diagonal of one of its faces.

Let a be the side of the cube.

The diagonal $\overrightarrow{OA} = \langle a,a,a \rangle$ and diagonal $\overrightarrow{OB} = \langle a,0,a \rangle$ of face OABC.

Let θ be the angle between the vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$, then

$$\overrightarrow{OA} \cdot \overrightarrow{OB} = \left|\overrightarrow{OA}\right|\left|\overrightarrow{OB}\right| \cos\theta$$

$$\langle a,a,a \rangle \cdot \langle a,0,a \rangle = \sqrt{a^2+a^2+a^2} \cdot \sqrt{a^2+0+a^2} \cos\theta$$

$$a^2+0+a^2 = \sqrt{3a^2} \cdot \sqrt{2a^2} \cdot \cos\theta$$

$$2a^2 = a^2 \cdot \sqrt{6} \cdot \cos\theta$$

$$\frac{2}{\sqrt{6}} = \cos\theta$$

$$\cos\theta = \frac{2}{\sqrt{6}}$$

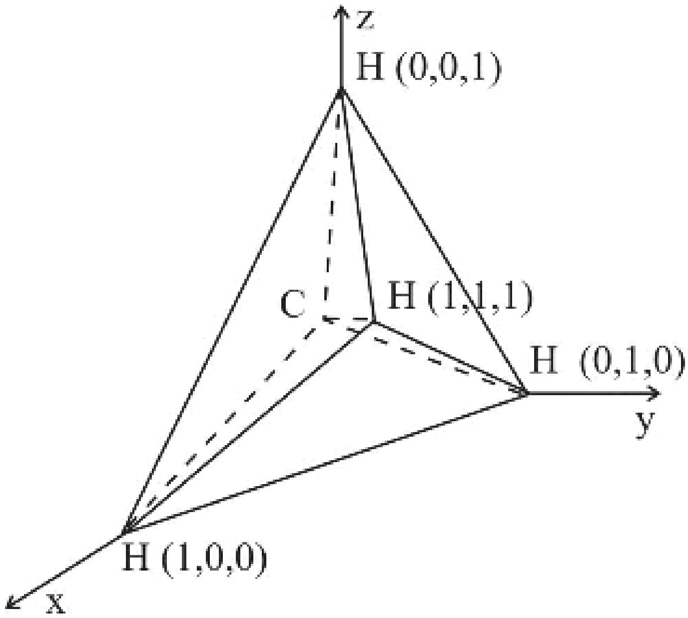
$$\theta = \cos^{-1}\left(\frac{2}{\sqrt{6}}\right)$$

$$\approx 35.3^0$$

Similarly, angle between the diagonal of one of its faces is 35.3^0.

Hence, the angle between the diagonal of a cube and a diagonal of one of its faces is $\boxed{\theta \approx 35.3^0}$.

Answer 57E.



A molecule of methane CH_4 is structured with four hydrogen atoms at the vertices of a regular tetrahedron and the carbon atom at the centroid.
Let the vertices of tetrahedron (i.e. let the hydrogen atoms be at :)
$(1,0,0).(0,1,0),(0,0,1)$ and (1, 1, 1). Then the centroid is $\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)$ *i.e* the carbon atom is at $\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)$

Consider any H-C-H bond

Let us take $H(0,0,1)-C\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)-H(1,1,1)$ bond

And let θ be the bond angle.

Then $\overrightarrow{CH}_{(1,1,1)}=\left\langle\frac{1}{2},\frac{1}{2},\frac{1}{2}\right\rangle$

Taking the dot product,

$$\left\langle-\frac{1}{2},-\frac{1}{2},\frac{1}{2}\right\rangle\cdot\left\langle\frac{1}{2},\frac{1}{2},\frac{1}{2}\right\rangle=\sqrt{\frac{1}{4}+\frac{1}{4}+\frac{1}{4}}\sqrt{\frac{1}{4}+\frac{1}{4}+\frac{1}{4}}\cos\theta$$

i.e. $\frac{-1}{4}\frac{-1}{4}\frac{+1}{4}=\sqrt{\frac{3}{4}}\cdot\sqrt{\frac{3}{4}}\cdot\cos\theta$

i.e. $\frac{-1}{4}=\frac{3}{4}\cos\theta$

i.e. $\cos\theta=\frac{-1}{3}$

i.e. $\theta=\cos^{-1}\left(\frac{-1}{3}\right)$

i.e. $\theta=109.5^{o}$

Similarly we find that for all the bounds, the bond angle is $\boxed{109.5^{o}}$

Hence Proved.

Answer 58E.

Let θ_1 be the angle between $\vec{a}$ and $\vec{c}$ and θ_2 be the angle between $\vec{b}$ and $\vec{c}$

It is given that $\vec{a}, \vec{b}, \vec{c}$ are non-zero vectors

and $\quad \vec{c} = |\vec{a}|\vec{b} + |\vec{b}|\vec{a} \qquad ---(1)$

Taking dot product with $\vec{a}$ on both sides,

$$\vec{a}.\vec{c} = |\vec{a}|\vec{a}.\vec{b} + |\vec{b}|\vec{a}.\vec{a}$$

i.e. $\quad |\vec{a}||\vec{c}|\cos\theta_1 = |\vec{a}|\vec{a}.\vec{b} + |\vec{b}||\vec{a}|^2$

i.e. $\quad \cos\theta_1 = \dfrac{\vec{a}.\vec{b}}{|\vec{c}|} + \dfrac{|\vec{a}||\vec{b}|}{|\vec{c}|} \qquad ---(2) \quad \left\{as\ |\vec{a}| \neq 0\right\}$

Now taking dot product with $\vec{b}$ on both sides of Eq. .(1)

$$\vec{b}.\vec{c} = |\vec{a}|\vec{b}.\vec{b} + |\vec{b}|\vec{b}.\vec{a}$$

i.e. $\quad |\vec{b}||\vec{c}|\cos\theta_2 = |\vec{a}||\vec{b}|^2 + |\vec{b}|\vec{a}.\vec{b} \qquad \left(as\ \vec{a}.\vec{b} = \vec{b}.\vec{a}\right)$

i.e. $\quad \cos\theta_2 = \dfrac{|\vec{a}||\vec{b}|}{|\vec{c}|} + \dfrac{\vec{a}.\vec{b}}{|\vec{c}|} \qquad ---(3) \quad \left\{as\ |\vec{b}| \neq 0\right\}$

From (2) and (3) we find that

$$\cos\theta_1 = \cos\theta_2$$

i.e. $\quad \theta_1 = \theta_2$

i.e. $\quad$ angle between $\vec{a}$ and $\vec{c}$ = angle between $\vec{b}$ and $\vec{c}$

i.e. $\quad \vec{c}$ bisects the angle between $\vec{a}$ and $\vec{b}$.

Hence Proved.

Answer 59E.

Let $\vec{a} = \langle a_1, a_2, a_3 \rangle$

$\vec{b} = \langle b_1, b_2, b_3 \rangle$

Property 2: $\vec{a}.\vec{b} = \vec{b}.\vec{a}$

Consider $\vec{a}.\vec{b}$

$= \langle a_1, a_2, a_3 \rangle . \langle b_1, b_2, b_3 \rangle$

$= a_1 b_1 + a_2 b_2 + a_3 b_3 \qquad \{by\ def.\}$

$= b_1 a_1 + b_2 a_2 + b_3 a_3$

$(by\ commutative\ property\ in\ algebea)$

$= \langle b_1, b_2, b_3 \rangle . \langle a_1, a_2, a_3 \rangle$

$= \vec{b}.\vec{a}$

Perperty 4: $\left(c\vec{a}\right).\vec{b} = c\left(\vec{a}.\vec{b}\right) = \vec{a}.\left(c\vec{b}\right)$, (c a scalar)

Consider $c\vec{a}.\vec{b}$

$= \left(c\langle a_1, a_2, a_3 \rangle\right) . \langle b_1, b_2, b_3 \rangle$

$= \langle ca_1, ca_2, ca_3 \rangle . \langle b_1, b_2, b_3 \rangle$

$= (ca_1)b_1 + (ca_2)b_2 + (ca_3)b_3$

$= c(a_1 b_1) + c(a_2 b_2) + c(a_3 b_3) \qquad ---(3)$

$= c(a_1 b_1 + a_2 b_2 + a_3 b_3)$

$= c\left(\langle a_1, a_2, a_3 \rangle . \langle b_1, b_2, b_3 \rangle\right)$

$= c\left(\vec{a}.\vec{b}\right)$

Now from equation (3),

$\left(c\vec{a}\right).\vec{b} = c(a_1 b_1) + c(a_2 b_2) + c(a_3 b_3)$

$= a_1(cb_1) + a_2(cb_2) + a_3(cb_3)$

$= \langle a_1, a_2, a_3 \rangle . \langle cb_1, cb_2, cb_3 \rangle$

$= \vec{a}.\left(c\vec{b}\right)$

Hence $\left(c\vec{a}.\vec{b}\right) = c\left(\vec{a}.\vec{b}\right) = \vec{a}.\left(c\vec{b}\right)$

Proeerty 5: $\vec{0}.\vec{a} = 0$

We know $\vec{0} = \langle o, o, o \rangle$

Consider $\vec{0}.\vec{a}$

$= \langle 0, 0, 0 \rangle . \langle a_1, a_2, a_3 \rangle$

$= (0)a_1 + (0)a_2 + (0)a_3$

$= 0 + 0 + 0$

$= 0$

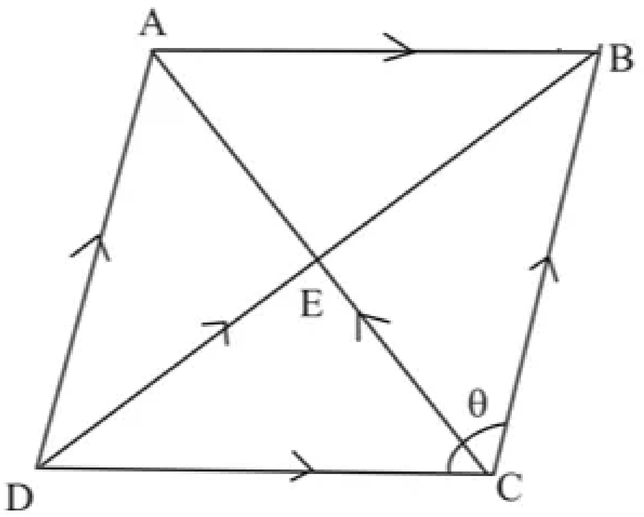
Hence $\vec{0}.\vec{a} = 0$

Answer 60E.

All the sides of quadrilateral are equal in length and opposite sides are parallel.

The objective is to prove that diagonals are perpendicular.

Sketch the following diagram:



Let ABCD be the quadrilateral with diagonals AC and BD. Let θ be the angle between CD and CB. It is given that AB = BC = CD =DA = a (say) and $\mathbf{AD} \| \mathbf{BC}$, $\mathbf{AB} \| \mathbf{DC}.$

In $\Delta DCB,$ by the triangle law,

$\overrightarrow{DB} = \overrightarrow{DC} + \overrightarrow{CB}$ ......(1)

And in $\Delta CAB,$ by triangle law,

$\overrightarrow{CA} = \overrightarrow{CB} - \overrightarrow{AB}$ ......(2)

Take dot product of equations (1) and (2) ,

$$\overrightarrow{DB} \bullet \overrightarrow{CA} = \left(\overrightarrow{DC} + \overrightarrow{CB}\right) \bullet \left(\overrightarrow{CB} - \overrightarrow{AB}\right)$$

$$= \overrightarrow{DC} \bullet \overrightarrow{CB} - \overrightarrow{DC} \bullet \overrightarrow{AB} + \overrightarrow{CB} \bullet \overrightarrow{CB} - \overrightarrow{CB} \bullet \overrightarrow{AB}$$

$$= -\overrightarrow{CD} \bullet \overrightarrow{CB} - \overrightarrow{DC} \bullet \overrightarrow{AB} + \overrightarrow{CB} \bullet \overrightarrow{CB} + \overrightarrow{CB} \bullet \overrightarrow{BA}$$

$$= -\left|\overrightarrow{CD}\right|\left|\overrightarrow{CB}\right|\cos\theta - \left|\overrightarrow{DC}\right|\left|\overrightarrow{AB}\right|\cos 0^o + \left|\overrightarrow{CB}\right|\left|\overrightarrow{CB}\right|\cos 0^o + \left|\overrightarrow{CB}\right|\left|\overrightarrow{BA}\right|\cos\theta$$

$$= -a^2\cos\theta - a^2 + a^2 + a^2\cos\theta$$

= 0

That is $\overrightarrow{DB} \bullet \overrightarrow{CA} = 0$

The angle between $\overrightarrow{DB}, \overrightarrow{CA}$ is 90^o .

That is $\overrightarrow{DB}$ *and* $\overrightarrow{CA}$ are perpendicular

Therefore, the diagonals of the quadrilateral with allthe sides equal in length and opposite sides parallel, are perpendicular

Answer 61E.

Consider $\left|\vec{a}.\vec{b}\right|$

$= \left|\left|\vec{a}\right|\left|\vec{b}\right|\cos\theta\right|$

,(since we know that if θ is the angle between the vectors $\vec{a}$ and $\vec{b}$,then $\vec{a}.\vec{b} = \left|\vec{a}\right|\left|\vec{b}\right|\cos\theta$)

where θ is the angle between $\vec{a}$ and $\vec{b}$.

Then $\left|\vec{a}.\vec{b}\right| = \left|\left|\vec{a}\right|\left|\vec{b}\right|\cos\theta\right|$

$= \left|\vec{a}\right|\left|\vec{b}\right|\left|\cos\theta\right|$

Since $-1 \le \cos\theta \le 1$

or, $\left|\cos\theta\right| \le 1$

Therefore, $\left|\vec{a}\right|\left|\vec{b}\right|\left|\cos\theta\right| \le \left|\vec{a}\right|\left|\vec{b}\right|$

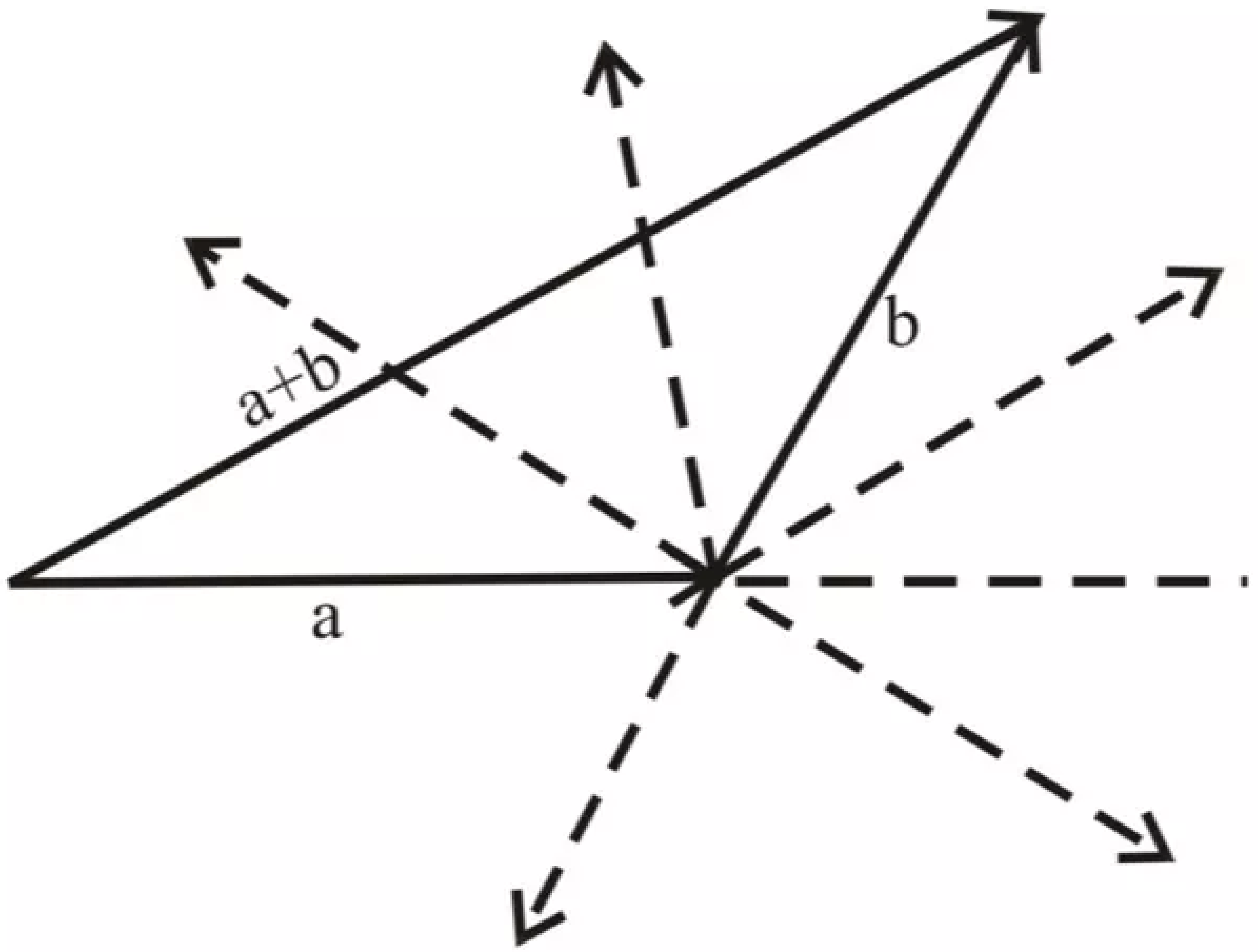
Hence,

$\left|\vec{a}.\vec{b}\right| \le \left|\vec{a}\right|\left|\vec{b}\right|$

Answer 62E.

(A)

Consider the following figure:



Consider $\vec{a}$ and $\vec{b}$ are two sides of a triangle.

Here, the tip of $\vec{a}$ is at the tail of $\vec{b}$.

Then, $\vec{a}+\vec{b}$ is the length of the third side of this triangle.

Hence, the triangle inequality states that the length of the third side is less than the sum of the lengths of the other two sides.

(B)

Prove the Triangle inequality as follows:

$$\left|\vec{a}+\vec{b}\right|^2=\left(\vec{a}+\vec{b}\right)\cdot\left(\vec{a}+\vec{b}\right)$$

$$=\vec{a}\cdot\vec{a}+\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{a}+\vec{b}\cdot\vec{b}$$

$$=\left|\vec{a}\right|^2+\vec{a}.\vec{b}+\vec{a}\cdot\vec{b}+\left|\vec{b}\right|^2 \qquad \left[\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{a}\right]$$

$$=\left|\vec{a}\right|^2+2\vec{a}\cdot\vec{b}+\left|\vec{b}\right|^2$$

$$\le\left|\vec{a}\right|^2+2\left|\vec{a}\cdot\vec{b}\right|+\left|\vec{b}\right|^2 \qquad \left[\vec{a}\cdot\vec{b}\le\left|\vec{a}\cdot\vec{b}\right|\right]$$

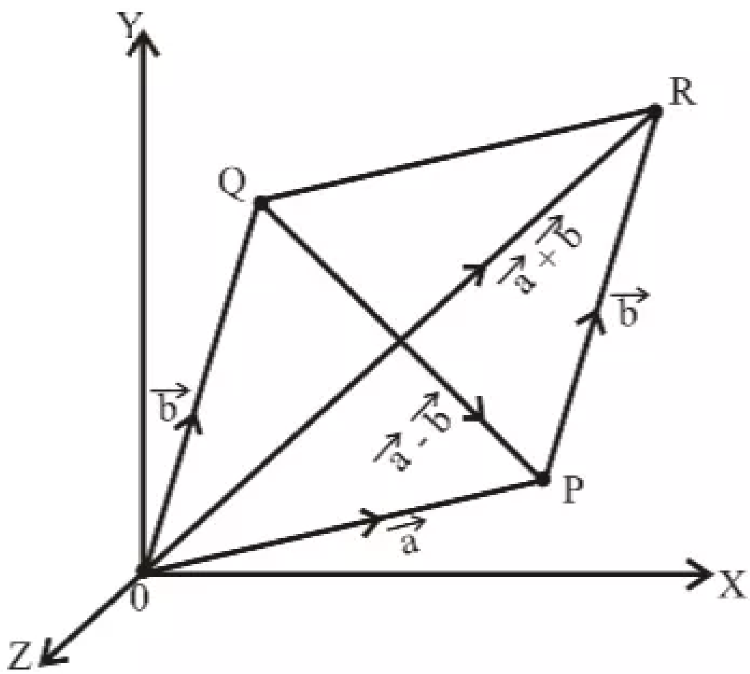
$$\le\left|\vec{a}\right|^2+2\left|\vec{a}\right|\left|\vec{b}\right|+\left|\vec{b}\right|^2 \qquad \left[\left|\vec{a}\cdot\vec{b}\right|\le\left|\vec{a}\right|\left|\vec{b}\right|\right]$$

$$=\left(\left|\vec{a}\right|+\left|\vec{b}\right|\right)^2$$

Therefore, prove the inequality $\left|\vec{a}+\vec{b}\right|\le\left|\vec{a}\right|+\left|\vec{b}\right|$

Answer 63E.

(A)



(A):

Let P and Q be the points whose position vectors are $\vec{a}$ and $\vec{b}$ respectively complete the parallelogram OPRQ.

Then we have

$$|\vec{a}| = OP \qquad |\vec{b}| = OQ$$

$$|\vec{a}+\vec{b}| = OR \qquad |\vec{a}-\vec{b}| = QP$$

From the parallelogram law,

$$|a+b|^2 + |a-b|^2 = 2|a|^2 + 2|b|^2$$

i.e. $$OR^2 + QP^2 = 2(OP)^2 + 2(OQ)^2$$

Thus the parallelogram law tells that the sum of squares of the lengths of the diagonals of a parallelogram is equal to the sum of squares of sides.

(B):

We have $$|\vec{a}+\vec{b}|^2 = (\vec{a}+\vec{b})\cdot(\vec{a}+\vec{b})$$

$$|\vec{a}-\vec{b}|^2 = (\vec{a}-\vec{b})\cdot(\vec{a}-\vec{b})$$

Now, $$|\vec{a}+\vec{b}|^2 + |\vec{a}-\vec{b}| = (\vec{a}+\vec{b})\cdot(\vec{a}+\vec{b}) + (\vec{a}-\vec{b})\cdot(\vec{a}-\vec{b})$$

$$= \vec{a}\cdot\vec{a} + \vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{a} + \vec{b}\cdot\vec{b} + \vec{a}\cdot\vec{a} - \vec{a}\cdot\vec{b} - \vec{b}\cdot\vec{a} + \vec{b}\cdot\vec{b}$$

$$= 2\vec{a}\cdot\vec{a} + 2\vec{b}\cdot\vec{b}$$

$$= 2|\vec{a}|^2 + 2|\vec{b}|^2$$

Hence,

$$\boxed{|\vec{a}+\vec{b}|^2 + |\vec{a}-\vec{b}|^2 = 2|\vec{a}|^2 + 2|\vec{b}|^2}$$

Answer 64E.

Suppose the two vectors $(\mathbf{u}+\mathbf{v})$ and $(\mathbf{u}-\mathbf{v})$ are orthogonal.

The objective is to show that if $(\mathbf{u}+\mathbf{v})$ and $(\mathbf{u}-\mathbf{v})$ are orthogonal then the vectors $\mathbf{u}$ and $\mathbf{v}$ must have the same length.

When the two vectors dot product is zero then the two vectors are orthogonal to each other.

If the two vectors $\mathbf{a}$ and $\mathbf{b}$ are orthogonal then $\mathbf{a} \cdot \mathbf{b} = \mathbf{0}$

Take the two vectors as $\mathbf{a} = \mathbf{u} + \mathbf{v}$, $\mathbf{b} = \mathbf{u} - \mathbf{v}$

$\mathbf{u} = \langle \mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3 \rangle$ and $\mathbf{v} = \langle \mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3 \rangle$

$$\mathbf{a} \cdot \mathbf{b} = 0$$

$$\langle \mathbf{u}_1 + \mathbf{v}_1, \mathbf{u}_2 + \mathbf{v}_2, \mathbf{u}_3 + \mathbf{v}_3 \rangle \cdot \langle \mathbf{u}_1 - \mathbf{v}_1, \mathbf{u}_2 - \mathbf{v}_2, \mathbf{u}_3 - \mathbf{v}_3 \rangle = 0$$

$$\left(\mathbf{u}_1^2 - \mathbf{v}_1^2\right) + \left(\mathbf{u}_2^2 - \mathbf{v}_2^2\right) + \left(\mathbf{u}_3^2 - \mathbf{v}_3^2\right) = 0$$

If $\mathbf{u}$ and $\mathbf{v}$ are to have equal lengths then their magnitudes must be equal.

$$|\mathbf{u}| = |\mathbf{v}|$$

$$\sqrt{\mathbf{u}_1^2 + \mathbf{u}_2^2 + \mathbf{u}_3^2} = \sqrt{\mathbf{v}_1^2 + \mathbf{v}_2^2 + \mathbf{v}_3^2}$$

Squaring on both sides,

$$\mathbf{u}_1^2 + \mathbf{u}_2^2 + \mathbf{u}_3^2 = \mathbf{v}_1^2 + \mathbf{v}_2^2 + \mathbf{v}_3^2$$

$$\left(\mathbf{u}_1^2 + \mathbf{u}_2^2 + \mathbf{u}_3^2\right) - \left(\mathbf{v}_1^2 + \mathbf{v}_2^2 + \mathbf{v}_3^2\right) = 0$$

$$\left(\mathbf{u}_1^2 - \mathbf{v}_1^2\right) + \left(\mathbf{u}_2^2 - \mathbf{v}_2^2\right) + \left(\mathbf{u}_3^2 - \mathbf{v}_3^2\right) = 0$$

$$\langle \mathbf{u}_1 + \mathbf{v}_1, \mathbf{u}_2 + \mathbf{v}_2, \mathbf{u}_3 + \mathbf{v}_3 \rangle \cdot \langle \mathbf{u}_1 - \mathbf{v}_1, \mathbf{u}_2 - \mathbf{v}_2, \mathbf{u}_3 - \mathbf{v}_3 \rangle = 0$$

$$\mathbf{a} \cdot \mathbf{b} = \mathbf{0}$$

$$(\mathbf{u} + \mathbf{v}) \cdot (\mathbf{u} - \mathbf{v}) = 0$$

Hence, $(\mathbf{u} + \mathbf{v})$ and $(\mathbf{u} - \mathbf{v})$ are orthogonal then the vectors $\mathbf{u}$ and $\mathbf{v}$ must have the same length.