Mathematics

(Chapter – 8) (Application of Integrals) (Exercise 8.1)
(Class – XII)

Question 1:

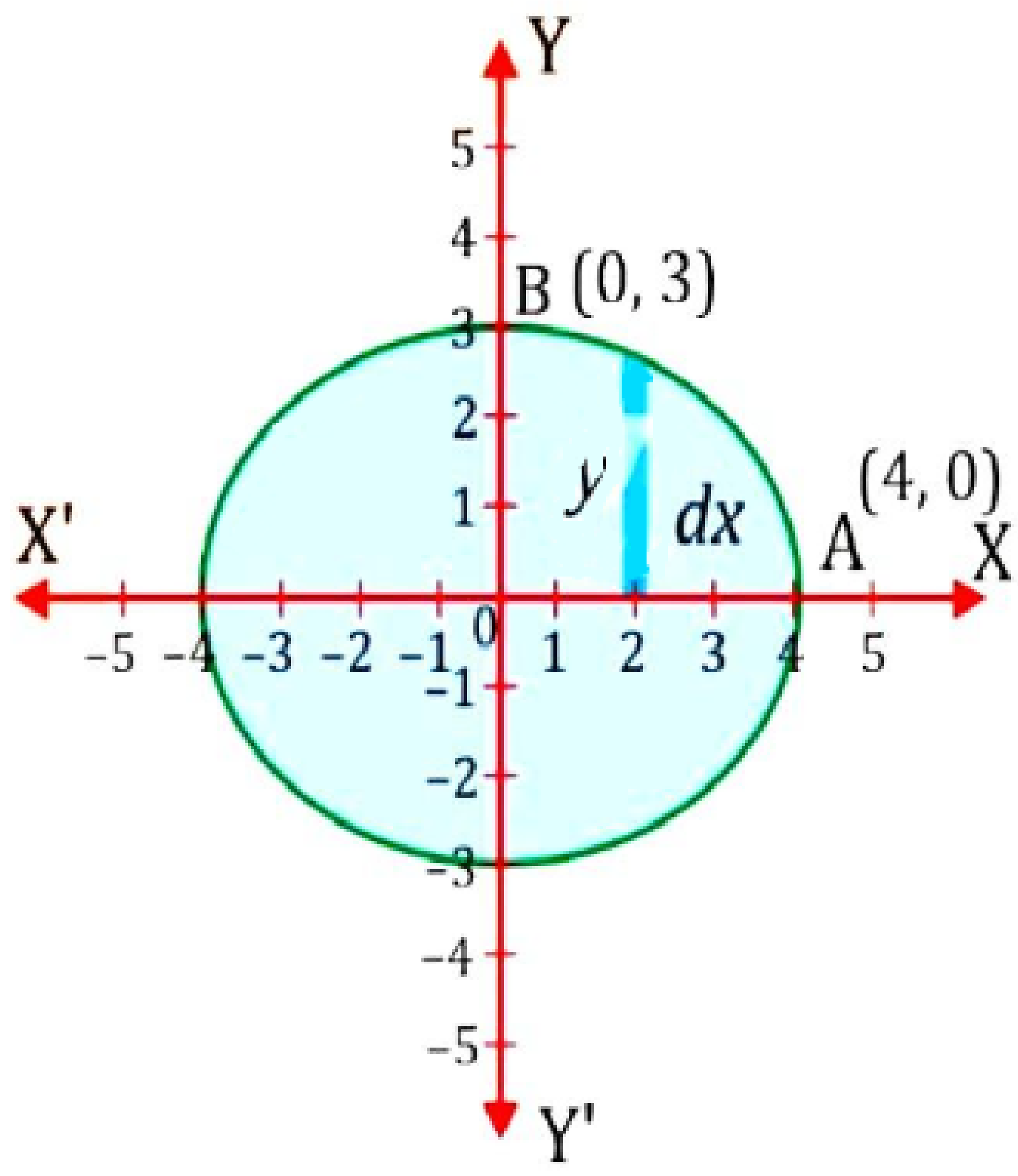
Find the area of the region bounded by the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$

Answer 1:

The given equation of the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$

It can be observed that the ellipse is symmetrical about x-axis and y-axis.

Area bounded by ellipse = 4 × Area of OAB



Area of ABCD $= \int_0^4 y dx$

$$= \int_0^4 3\sqrt{1-\frac{x^2}{16}dx} = \frac{3}{4}\int_0^4 \sqrt{16-x^2}\ dx$$

$$= \frac{3}{4}\left[\frac{x}{2}\sqrt{16-x^2}+\frac{16}{2}sin^{-1}\frac{x}{4}\right]_0^4$$

$$= \frac{3}{4}\ \left[2\sqrt{16-16}+8\,sin^{-1}(1)-0-8\,sin^{-1}(0)\right]$$

$$= \frac{3}{4}\left[\frac{8\pi}{2}\right]$$

$$\Rightarrow \frac{3}{4}\left[\frac{8\pi}{2}\right] = 3\pi$$

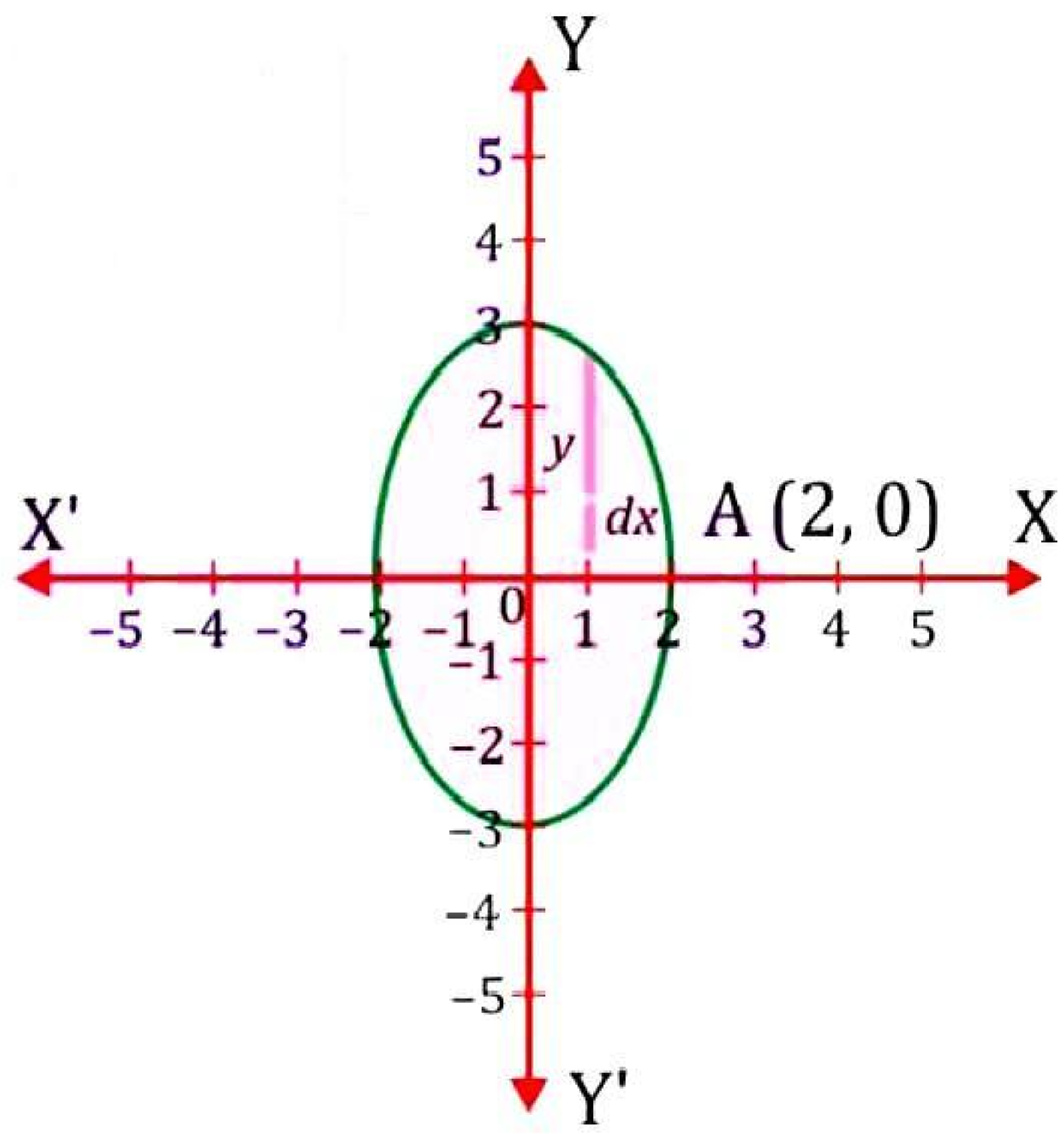
Therefore, area bounded by the ellipse = 4 × 3π = 12π units

Question 2:

Find the area of the region bounded by the ellipse $\frac{x^2}{4}+\frac{y^2}{9}=1$

Answer 2:

The given equation of the ellipse can be represented as $\frac{x^2}{4}+\frac{y^2}{9}=1$



$$\Rightarrow y = 3\sqrt{1-\frac{x^2}{4}dx}$$

$$= \frac{3}{2}\int_0^2 \sqrt{4-x^2}\ dx$$

$$= \frac{3}{2}\left[\frac{x}{2}\sqrt{4-x^2}+\frac{4}{2}sin^{-1}\frac{x}{2}\right]_0^2$$

$$= \frac{3}{4}\ \left[\frac{2\pi}{2}\right] = \frac{3\pi}{2}$$

Therefore, area bounded by the ellipse

$$= 4\times\frac{3\pi}{2} = 6\pi \text{ units}$$

Choose the correct answer in the following Exercises 3 and 4.

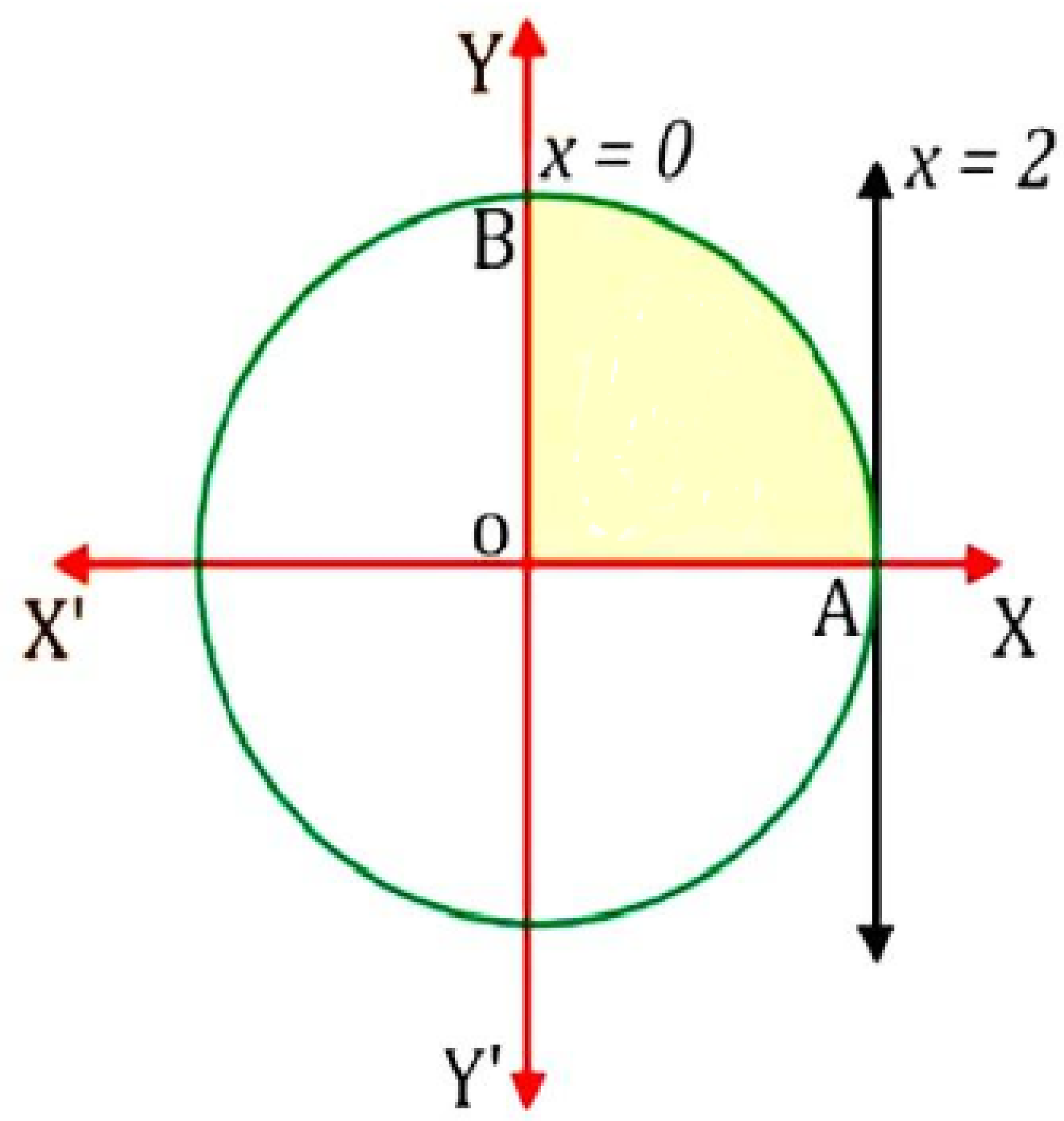
Question 3:

Area lying in the first quadrant and bounded by the circle $x^2+y^2 = 4$ and the lines $x = 0$ and $x = 2$ is

(A) π (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{4}$

Answer 3:

The area bounded by the circle and the lines, x = 0 and x = 2, in the first quadrant is represented as



Area of $OAB = \int_0^2 y\,dx$

$= \left[\int_0^2 \sqrt{4-x^2}\,dx\right]$

$= \left[\frac{x}{2}\sqrt{4-x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_0^2$

$= 2\left(\frac{\pi}{2}\right)$

$= \pi$ units

Thus, the correct answer is (A).

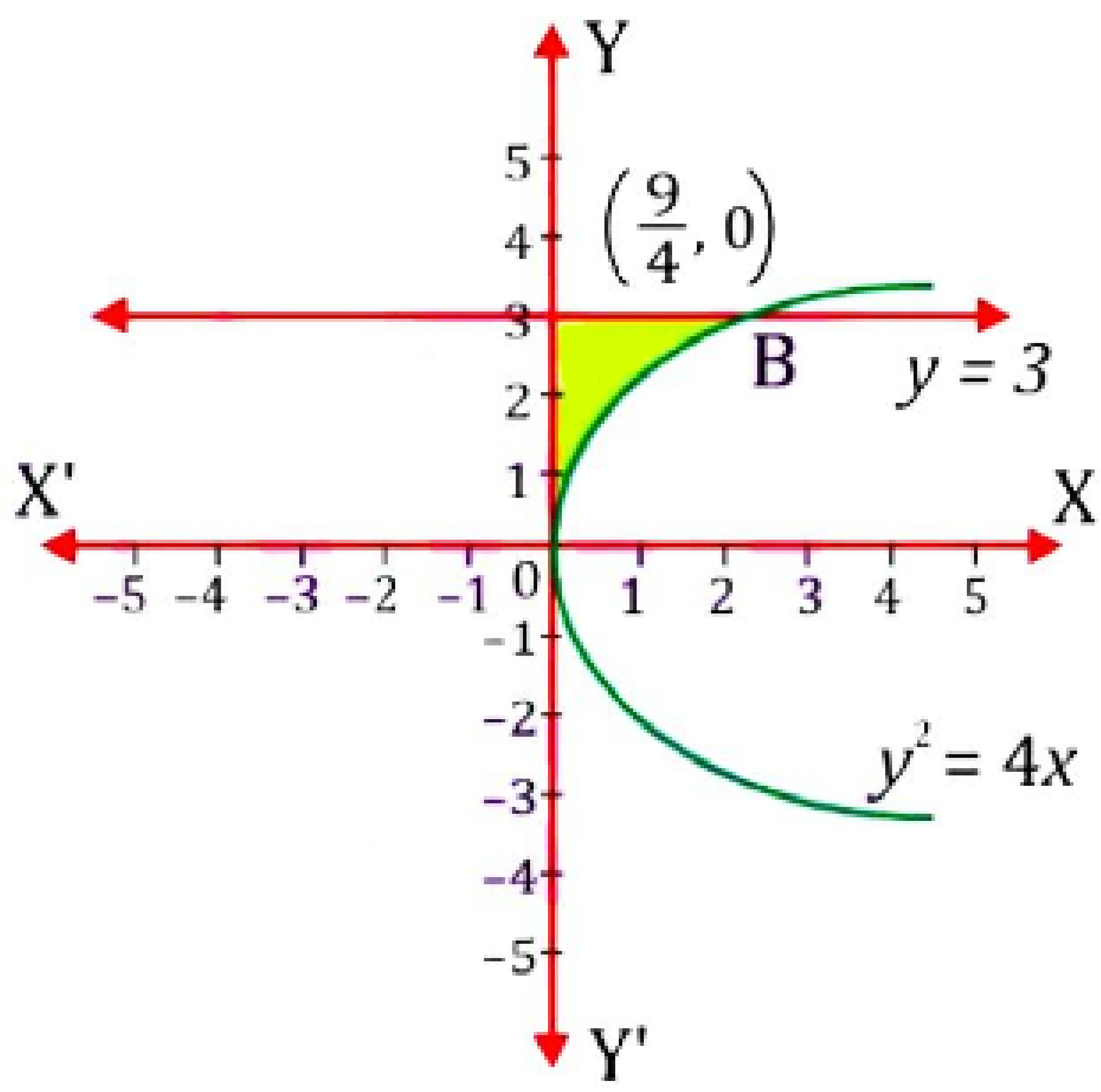
Question 4:

Area of the region bounded by the curve $y^2 = 4x$, y-axis and the line $y = 3$ is

(A) 2 (B) $\frac{9}{4}$ (C) $\frac{9}{3}$ (D) $\frac{9}{2}$

Answer 4:

The area bounded by the curve, $y^2 = 4x$, y-axis and $y = 3$ is represented as



Area $OAB = \int_0^3 x\,dy$

$= \int_0^3 \frac{y^2}{4}\,dy$

$= \frac{1}{4}\left[\frac{y^2}{4}\right]_0^3$

$= \frac{1}{12}(27)$

$= \frac{9}{4}$ units

Thus, the correct answer is (B).

Mathematics

(Chapter – 8) (Application of Integrals) (Miscellaneous Exercise)
(Class – XII)

Question 1:

Find the area under the given curves and given lines:

(i) $y = x^2$, $x = 1$, $x = 2$ and x–axis

(ii) $y = x^4$, $x = 1$, $x = 5$ and x-axis

Answer 1:

(i) The required area is represented by the shaded area ADCBA.

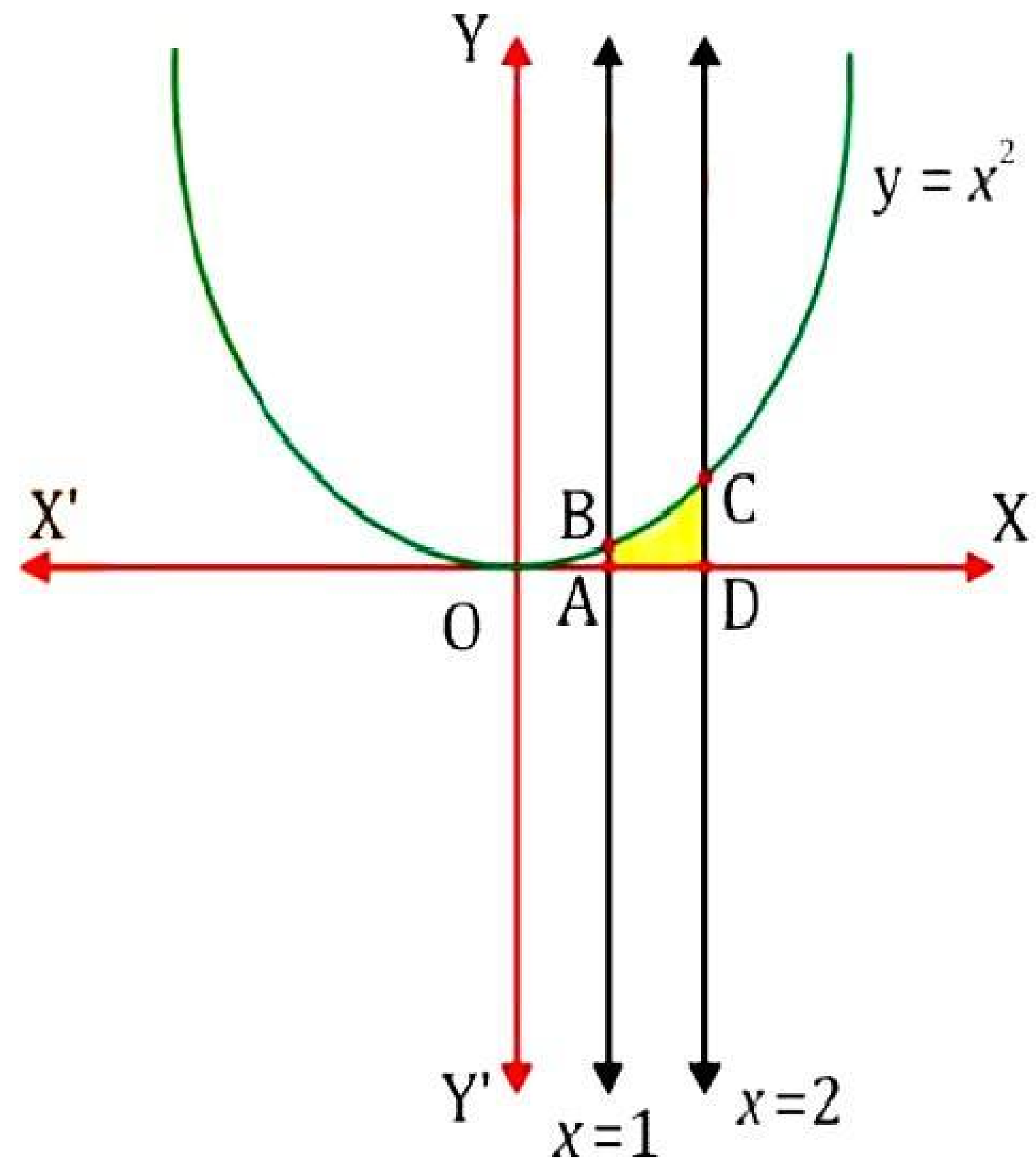
Area ADCBA $= \int_1^2 y\,dx$

$= \int_1^2 x^2\,dx$

$= \left[\frac{x^3}{3}\right]_1^2$

$= \frac{8}{3} - \frac{1}{3}$

$= \frac{7}{3}$ units



(ii) The required area is represented by the shaded area ADCBA.

Area ADCBA $= \int_1^5 x^4 dx$

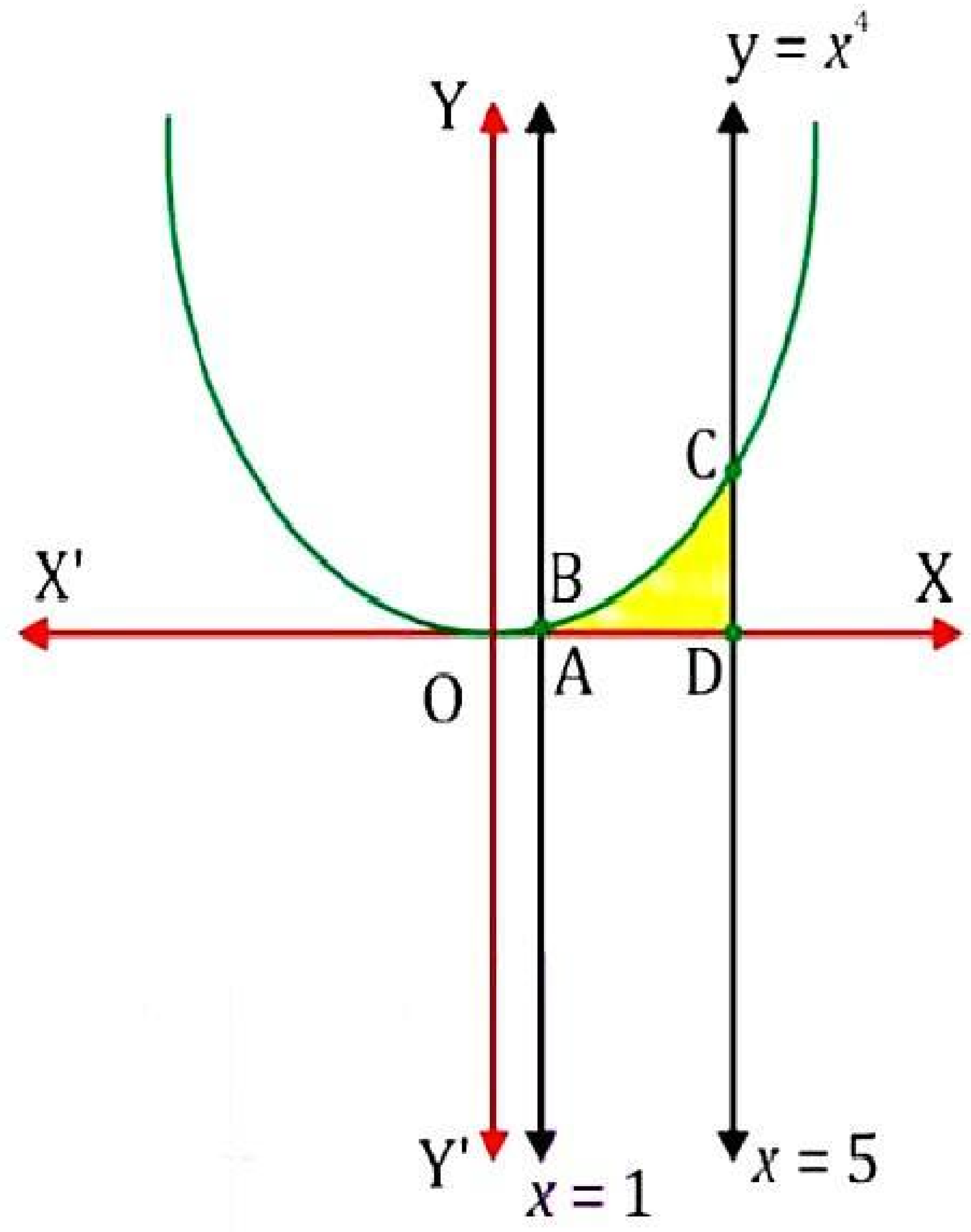
$= \left[\frac{x^5}{5}\right]_1^5$

$= \frac{(5)^5}{5} - \frac{1}{5}$

$= (5)^4 - \frac{1}{5}$

$= 625 - \frac{1}{5}$

$= 624.8$ units



Question 2:

Sketch the graph of $y = |x+3|$ and evaluate $\int_{-6}^{0} |x+3|\,dx$.

Answer 2:

The given equation is $y = |x+3|$.

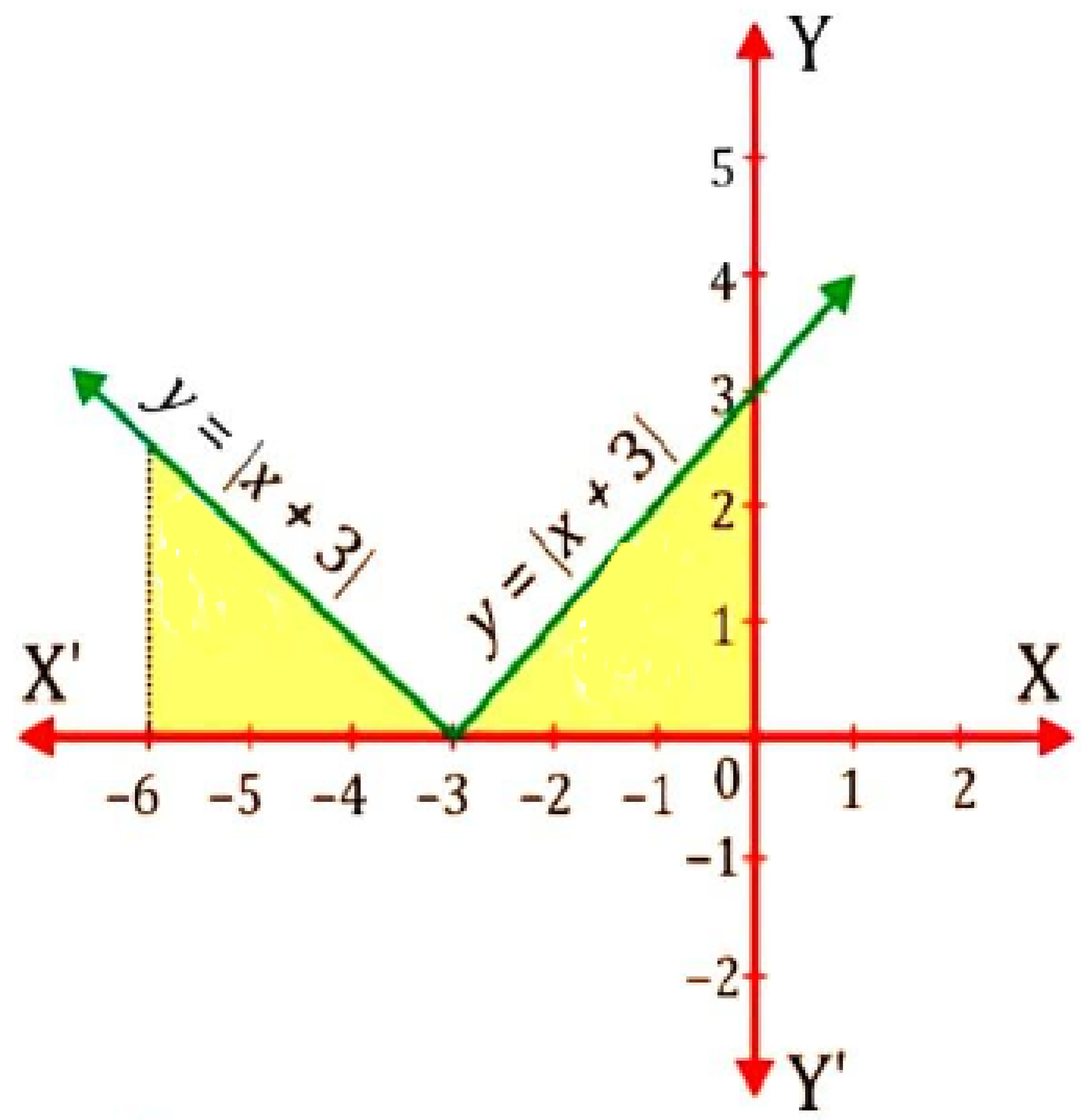
The corresponding values of x and y are given in the following table:

x	-6	-5	-4	-3	-2	-1	0
y	3	2	1	0	1	2	3

On plotting these points, we obtain the graph of $y = |x+3|$ as follows:

It is known that:

$(x+3) \le x \le -3$ and $(x+3) \ge 0$ for $-3 \le x \le 0$.



$$\int_{-6}^{0} |x+3| dx = -\int_{-6}^{-3} (x+3)dx + \int_{-3}^{0} (x+3)dx$$

$$= \left[\frac{x^2}{2} + 3x\right]_{-6}^{-3} - \left[\frac{x^2}{2} + 3x\right]_{-3}^{0}$$

$$= \left[\left(\frac{(-3)^2}{2} + 3(-3)\right) - \left(\frac{(-6)^2}{2} + 3(-6)\right)\right] - \left[0 - \left(\frac{(-3)^2}{2} + 3(-3)\right)\right]$$

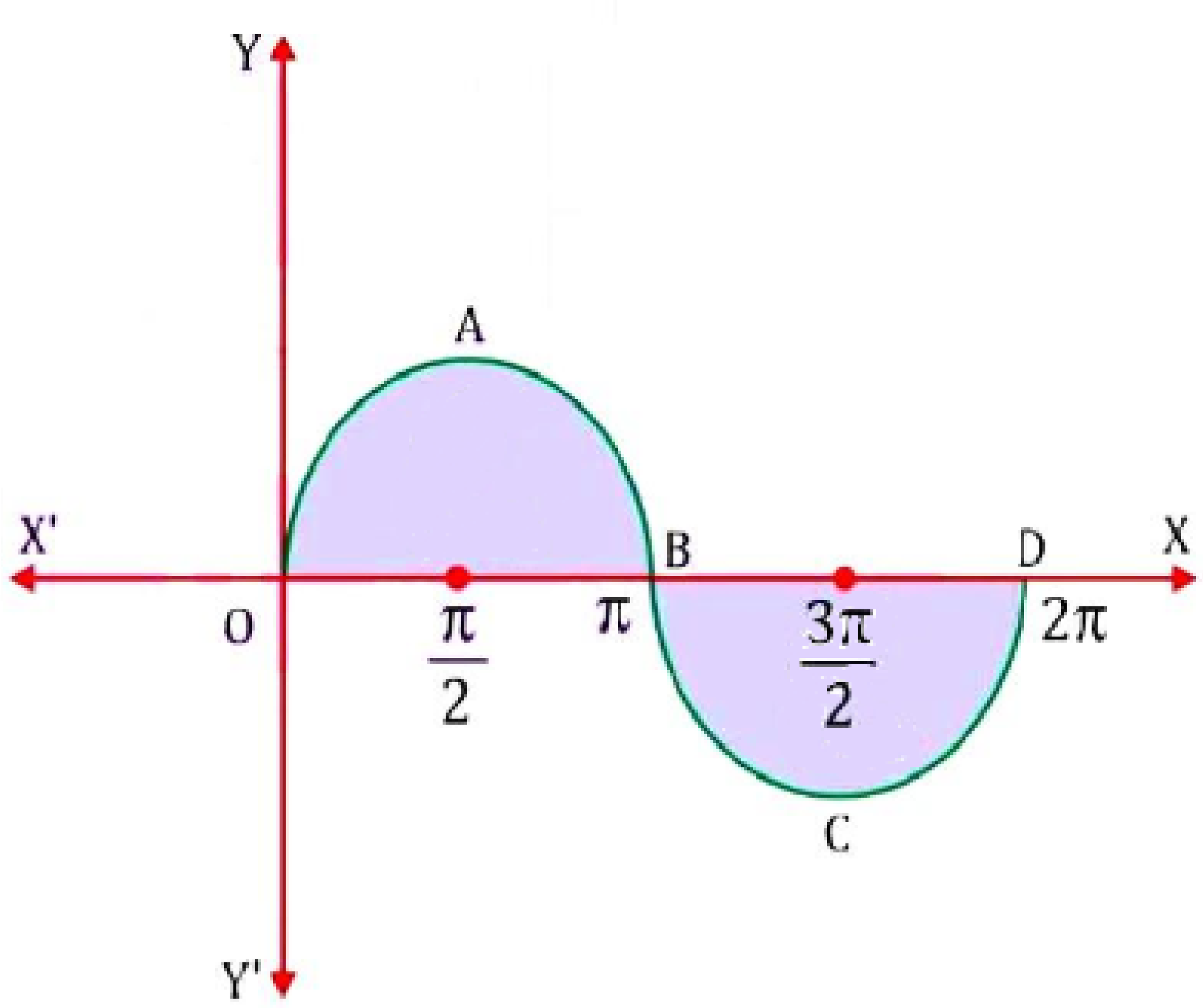
$$= \left[\frac{9}{2}\right] - \left[-\frac{9}{2}\right] = 9$$

Question 3:

Find the area bounded by the curve y = sin x between x = 0 and $x = 2\pi$.

Answer 3:

The graph of y = sin x can be drawn as shown in figure.



Required area = Area OABO + Area BCDB

$$= \int_{0}^{\pi} \sin x \, dx + \left|\int_{\pi}^{2\pi} \sin x \, dx\right|$$

$$= [-\cos x]_0^{\pi} + |[-\cos x]_{\pi}^{2\pi}|$$

$$= [-\cos \pi + \cos 0] + |(-\cos 2\pi + \cos \pi)|$$

$$= 1 + 1 + |(-1 - 1)|$$

$$= 2 + |-2| = 2 + 2$$

= 4 units

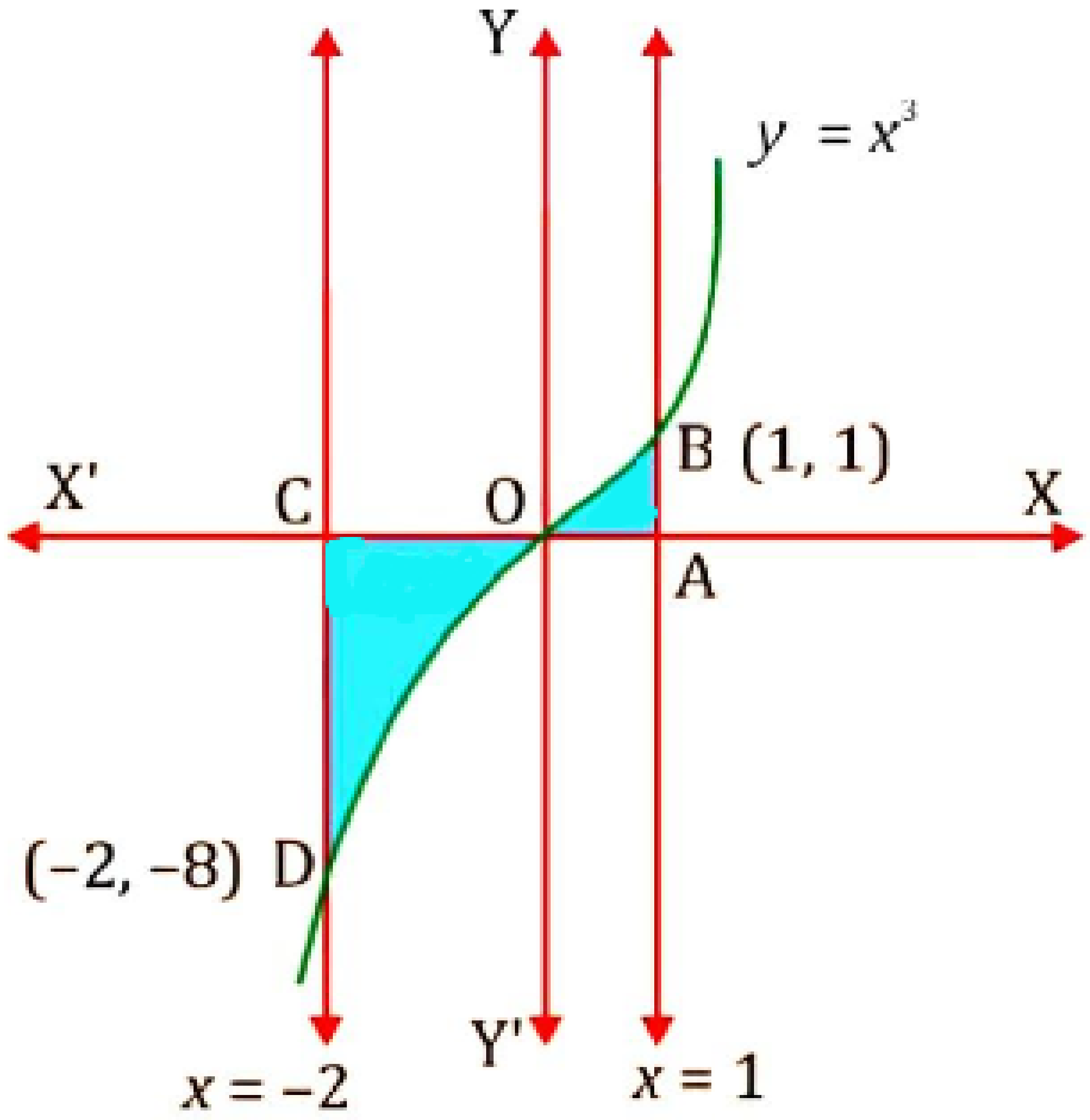
Choose the correct answer in the following Exercises from 4 to 5.

Question 4:

Area bounded by the curve $y = x^3$, the x-axis and the ordinates $x = -2$ and $x = 1$ is

(A) -9 (B) $-\frac{15}{4}$ (C) $\frac{15}{4}$ (D) $\frac{17}{4}$

Answer 4:



$$\text{required area} = \int_{-2}^{0} y\,dx + \int_{0}^{1} y\,dx$$

$$= \int_{-2}^{0} x^3 dx + \int_{0}^{1} x^3 dx$$

$$= \left[\frac{x^4}{4}\right]_{-2}^{0} + \left[\frac{x^4}{4}\right]_{0}^{1}$$

$$= \left[\frac{(-2)^4}{4} + \frac{1}{4}\right]$$

$$= \left(4 + \frac{1}{4}\right) = \frac{17}{4}$$

Correct answer is D

Question 5:

The area bounded by the curve $y = x|x|$, x-axis and the ordinates $x = -1$ and $x = 1$ is given by [Hint: $y = x^2$ if $x > 0$ and $y = -x^2$ if $x < 0$]

(A) 0 (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{4}{3}$

Answer 5:

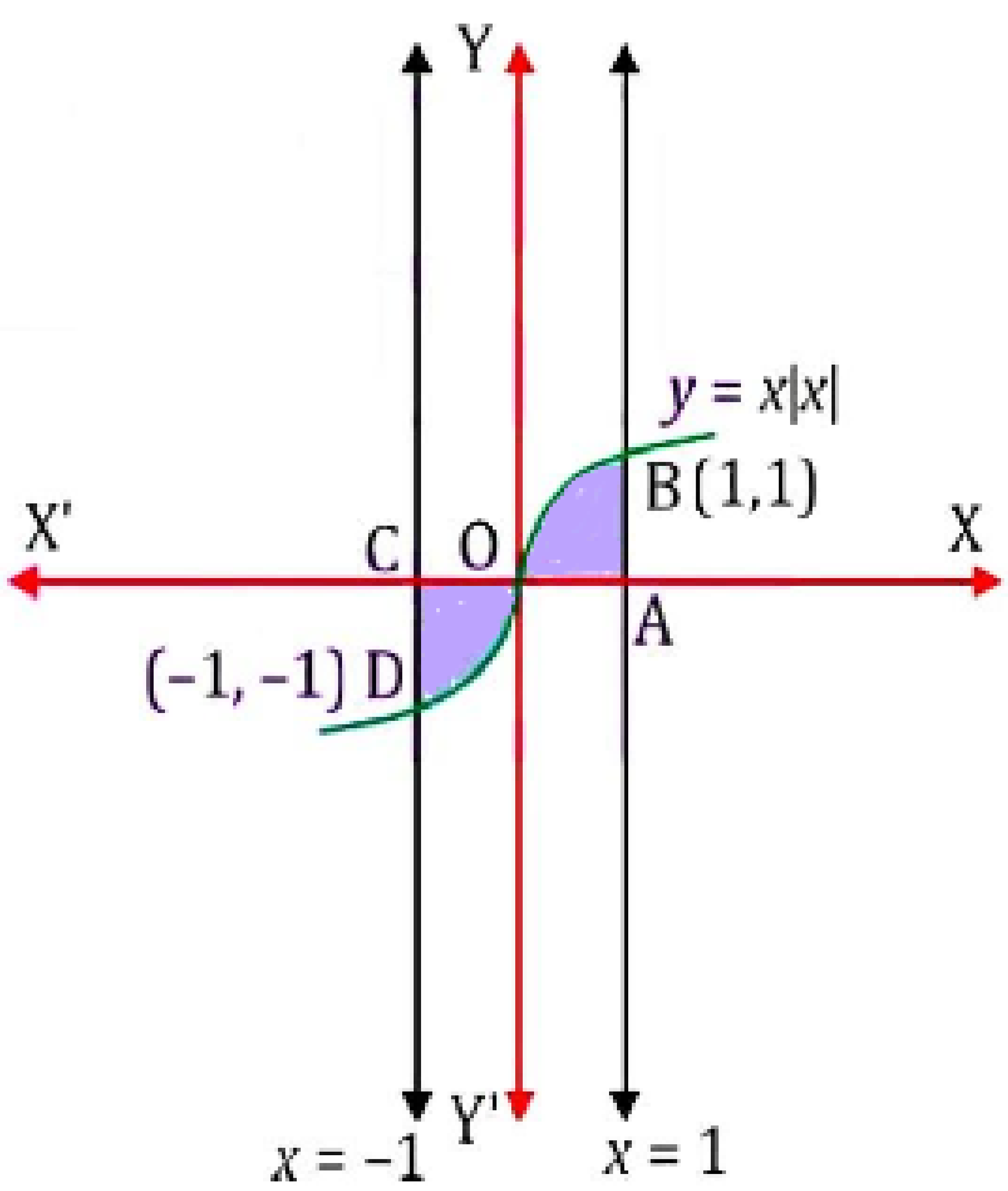
Required area $= \int_{-1}^{1} y\,dx$

$$= \int_{-1}^{1} x|x|\,dx$$

$$= \int_{-1}^{1} x^2 dx$$

$$= \left[\frac{x^3}{3}\right]_{-1}^{0} + \left[\frac{x^3}{3}\right]_{0}^{1}$$

$$= \frac{2}{3} \text{ units}$$



Thus, the correct answer is (C).