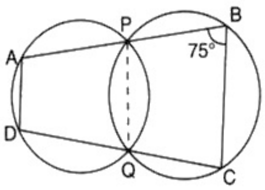
CBSE Test Paper 04

CH-10 Circles

1. The given figure shows two intersecting circles. If $\angle ABC = 75^o$, then the measure of $\angle PAD$ is



   a. 75^o

   b. 105^o

   c. 150^o

   d. 125^o

2. ABCD is a cyclic quadrilateral such that AB is a diameter of the circle circumscribing it and $\angle ADC = 140^o$, then $\angle BAC$ is equal to

   a. 40^o

   b. 30^o

   c. 75^o

   d. 50^o

3. Two circle are congruent if they have equal.

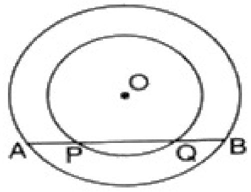
   a. diameter

   b. chord

   c. radii

   d. secant

4. If a straight line APQB is drawn to cut two concentric circles, then



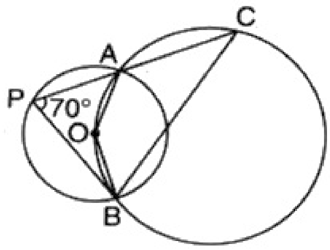
   a. AP $>$ BQ

   b. AP $<$ BQ

   c. AQ $>$ PB

   d. AP = BQ

5. The figure shows two circles which intersect at A and B. The centre of the smaller circle is O and it lies on the circumference of the larger circle. If $\angle APB = 70^o$, then the measure of $\angle ACB$ is



   a. 40^o

   b. 50^o

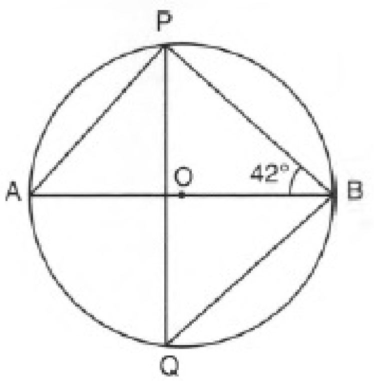
   c. 70^o

   d. 50^o

6. Fill in the blanks:

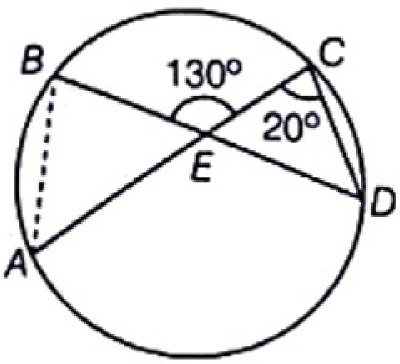
   The centre of a circle lies in ________ of the circle.

7. In the figure, find m∠PQB where O is the centre of the circle.

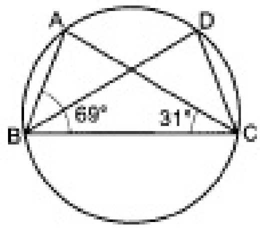


8. Two circles are drawn with sides AB, AC of a triangle ABC as diameters. The circles intersect at a point D. Prove that D lies on BC.

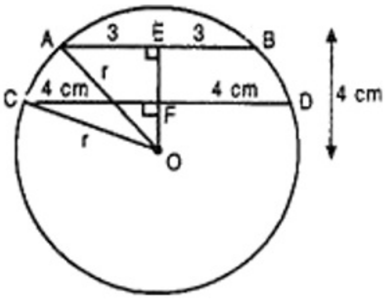
9. In the given figure, A, B, C, and D are four points on a circle. AC and BD intersect at point E such that ∠BEC =130° and ∠ECD = 20°. Find ∠BAC.



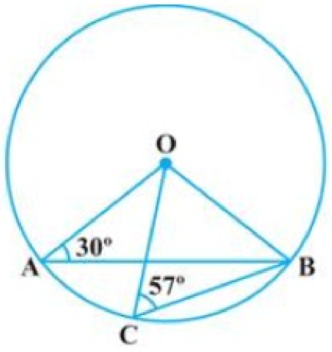
10. In given figure, ∠ABC = 69°, ∠ACB = 31°, find ∠BDC.



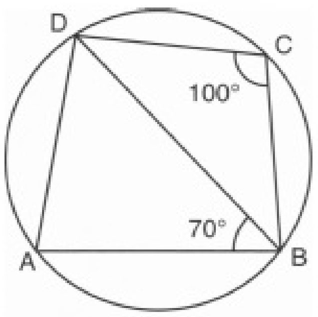
11. The lengths of two parallel chords of a circle are 6 cm and 8 cm. If the smaller chord is at the distance of 4 cm from the centre, what is the distance of the other chord form the centre?



12. ∠OAB = 30º and ∠OCB = 57º. Find ∠BOC and ∠AOC.



13. In Fig., ABCD is a cyclic quadrilateral. If ∠BCD = 100° and ∠ABD = 70°, find ∠ADB.



14. If two intersecting chords of a circle make equal angles with the diameter passing through their point of intersection, prove that the chords are equal.

15. PQ and RQ are chords of a circle equidistant from the centre. Prove that the diameter passing through Q bisects $\angle PQR$ and $\angle PSR$.

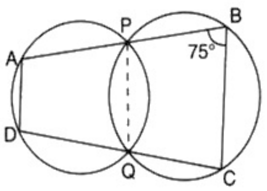
CBSE Test Paper 04

CH-10 Circles

Solution

1. (b) 105^o

   Explanation:

   Since $\angle PQC + \angle PBC = 180^0$ (Opposite angles of a cyclic quadrilateral)
   $\angle PQC = 180^0 - 75^0 = 105^0$
   Again, $\angle DQP + \angle PQC = 180^0$ (Linear Pair)
   so, $\angle DQP = 75^0$
   Also, $\angle PAD + \angle DQP = 180^0$ (Opposite angles of a cyclic quadrilateral)
   $\angle PAD = 105^0$

2. (d) 50^o

   Explanation:

   In the given quadrilateral,

   $\angle ABC + \angle ADC = 180^0$

   $140^0 + \angle ABC = 180^0$

   $\angle ABC = 40^0$

   Since, AB is diameter so ABCD lies in semi-circle.

   thus, $\angle BCA = 90^0$

   In triangle, ABC,

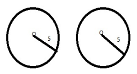
   $\angle A + \angle B + \angle C = 180^0$

   $\angle BAC = 180^0 - 40^0 - 90^0 = 180^0 - 130^0 = 50^0$

$\angle BAC = 50^0$

3. (c) radii

   Explanation:

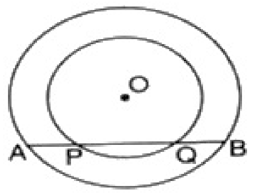
   

   As per theorem,

   two circles are congruent only if they have same radii.

4. (d) AP = BQ

   Explanation:

   Let OD is perpendicular to AB. Then AD = DB.

   Also DP = DQ

   Therefore, AP = AD - PD

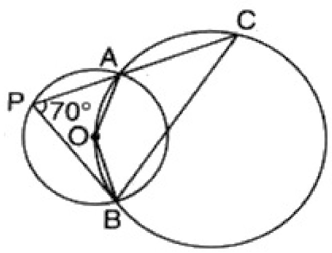
   = BD - DQ

   = BQ

   Hence, AP = BQ

5. (a) 40^o

   Explanation:

   Since, AB is a chord and makes $\angle APB = 70^0$ at the circumference, so $\angle AOB = 140^0$

   Now, as AOBC is a cyclic quadrilateral then, sum of opposite angles must be 180^0.

   $\angle$AOB + $\angle$ACB = 180^0

   $\Rightarrow$ 140^0 + $\angle$ACB = 180^0

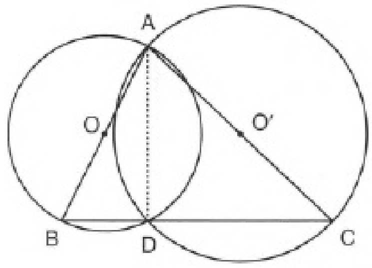
$\Rightarrow \angle ACB = 40^{o}$

6. interior

7. Since angle in a semi-circle is a right angle.

   $\therefore \angle APB = 90^{o}$
   In $\triangle APB$, we have
   $\angle PAB + \angle PBA + \angle APB = 180°$
   $\Rightarrow \angle PAB + 42° + 90° = 180°$
   $\Rightarrow \angle PAB = 48^{o}$
   Consider arc BP. We find that $\angle PAB$ and $\angle PQB$ are angles in the same segment of a circle.
   $\therefore \angle PQB = \angle PAB$ [$\because$ Angles in the same segment are equal]
   $\Rightarrow \angle PQB = 48^{o}$

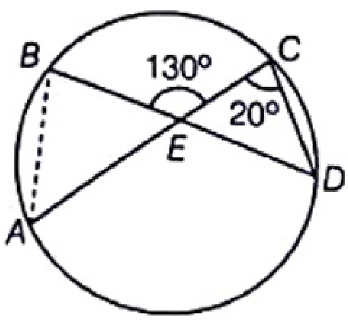
8. Join AD. Since angle in a semi-circle is a right angle.
   $\therefore \angle ADB = 90°$ and $\angle ADC = 90°$



   $\Rightarrow \angle ADB + \angle ADC = 90° + 90°$
   $\Rightarrow \angle ADB + \angle ADC = 180°$
   $\Rightarrow$ BDC is a straight line $\Rightarrow$ D lies on BC.

9. Given: $\angle BEC = 130°$ and $\angle ECD = 20°$.

   To Find : $\angle BAC$



Solution:Since the exterior angle of a triangle is equal to the sum of the interior opposite angles,

$\therefore \angle$BEC = $\angle$ECD + $\angle$CDE

$\Rightarrow$ 130° = 20°+ $\angle$CDE

$\Rightarrow$ $\angle$CDE = 130° - 20°= 110°

$\angle$BDC = 110°

Now, $\angle$BAC = $\angle$BDC (Angles in the same segment)

$\text{Hence } \angle \text{BAC} = 110^\circ$

10. From the given figure, in $\triangle$ABC, we can write

$\angle$ABC + $\angle$ACB + $\angle$BAC = 180^0 (by angle sum property)

$69^0 + 31^0$ + $\angle$BAC = 180^0

$\Rightarrow$ $\angle$BAC = 180° -100° = 80°

$\angle$BDC = $\angle$BAC (Angles in the same segment)

$\therefore \angle$BDC = 80°

11. Let AB = 6 cm and CD = 8 cm are the chords of circle with centre O.

Join OA and OC.

Since perpendicular from the centre of the circle to the chord bisects the chord.

$\therefore$ AE = EB = $\frac{1}{2}$ AB = $\frac{1}{2} \times 6$ = 3 cm

And CF = FD = $\frac{1}{2}$ CD = $\frac{1}{2} \times 8$ = 4 cm

Perpendicular distance of chord AB from the centre O is OE.

$\therefore$ OE = 4 cm

Now in right angled triangle AOE,

$OA^2 = AE^2 + OE^2$ [Using Pythagoras theorem]

$\Rightarrow r^2 = 3^2 + 4^2$

$\Rightarrow r^2 = 9 + 16 = 25$

$\Rightarrow$ r = 5 cm

Perpendicular distance of chord CD from the center O is OF.

Now in right angled triangle OFC,

$OC^2 = CF^2 + OF^2$ [Using Pythagoras theorem]

$\Rightarrow r^2 = 4^2 + OF^2$

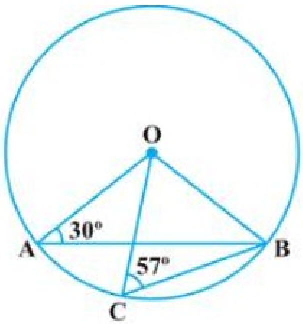
$\Rightarrow 5^2 = 16 + OF^2$

$\Rightarrow$ OF^2 = 25 - 16

$\Rightarrow$ OF^2 = 9

$\Rightarrow$OF = 3cm

Hence distance of other chord from the centre is 3 cm.

12. 

In ΔOBC we have

OB = OC [Radii of the same circle]

$\therefore \angle OCB = \angle OBC = 57^{\circ}$

Now, in ΔBOC , we have

$\angle OCB + \angle OBC + \angle BOC = 180^{\circ}$

$57^{\circ} + 57^{\circ} + \angle \mathrm{BOC} = 180^{\circ}$

$\Rightarrow \angle BOC = 180^{\circ} - 114^{\circ} = 66^{\circ}$

Again, in ΔAOB , we have

OB = OA [Radii of the same circle]

$\angle OAB = \angle OBA = 30$

$\angle OAB + \angle OBA + \angle AOB = 180^{\circ}$

$\Rightarrow 30^{\circ} + 30^{\circ} + (\angle AOC + \angle BOC) = 180^{\circ}$

$\Rightarrow 60^{\circ} + \angle AOC + 66^{\circ} = 180^{\circ}$

$\Rightarrow \angle AOC = 180^{\circ} - 126^{\circ} = 54^{\circ}$

Hence, $\angle BOC = 66^\circ$ and $\angle AOC = 54^\circ$

13. We have, $\angle$BCD = 100^o and $\angle$ABD = 70^o

$\therefore \angle$ DAB + $\angle$BCD = 180^o [Opposite angles of cyclic quad.]

$\Rightarrow \angle$DAB + 100^o = 180^o

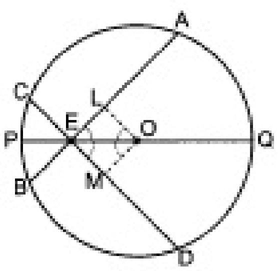
$\Rightarrow \angle$DAB + 180^o - 100^o = 80^o

In $\triangle$DAB, by angle sum property,

$\angle$ADB + $\angle$DAB + $\angle$ABD = 180^o

$\Rightarrow \angle$ADB + 80^o + 70^o = 180^o

$\Rightarrow \angle$ADB = 180^o - 80^o - 70^o = 30^o

14. 

Given: AB and CD are two chords of a circle with centre O, intersecting at point E. PQ is a diameter through E, such that $\angle$AEQ = $\angle$DEQ.

To prove: AB = CD

Construction: Draw OL $\perp$ AB and OM $\perp$ CD

Proof: $\angle$LOE + $\angle$LEO + $\angle$OLE = 180° (Angle sum property of a triangle)

$\Rightarrow \angle$LOE + $\angle$LEO + 90° = 180°

$\angle$LOE + $\angle$LEO =90° ....(i)

Similarly, $\angle$MOE + $\angle$MEO + $\angle$OME = 180°

$\Rightarrow\angle$MOE + $\angle$MEO + 90° = 180°

$\angle$MOE + $\angle$MEO = 90° . ...(ii)

From (i) and (ii) we get

$\angle$LOE + $\angle$LEO = $\angle$MOE + $\angle$MEO ....(iii)

Also, $\angle$LEO = $\angle$MEO (Given) ...(iv)

From (iii) and (iv) we obtain

$\angle$LOE = $\angle$MOE

Now in triangles OLE and OME

$\angle$LEO = $\angle$MEO (Given)

$\therefore$ $\angle$LOE = $\angle$MOE (Proved above)

EO = EO (Common)

$\therefore$ by ASA congruence criterion we have:

$\triangle$OLE $\cong$ $\triangle$ OME

$\therefore$ OL = OM (by CPCT)

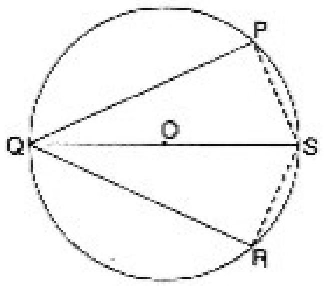
Thus, chords AB and CD are equidistant from the centre O of the circle. Since, chords of a circle which are equidistant from the centre are equal.

$\therefore$ AB = CD

15. Given: PQ and RQ are two chords of a circle equidistant from the centre.

To prove: The diameter QS passing through Q bisects $\angle \mathrm{PQR}$ and $\angle \mathrm{PQR}$

Construction : Join PS and RS.



Proof : Chords PQ and RQ are equidistant from the centre.

$\therefore PQ = RQ$ | $\because$ chords of a circle equidistant from the centre are equal

Also $\angle QPS = \angle QRS = 90^\circ$ | $\because$ An angle in a semi-circle is a right angle

$\therefore$ $\triangle$s PQS and RQS are right $\triangle s$

Now in right $\triangle s$ PQS and RQS

PQ = RQ |Proved above

Hyp. QS = Hyp.QS |Common

$\therefore \triangle PQS \cong \triangle RQS$ |R.H.S. Axiom

$\therefore \angle PQS = \angle RQS$|c.p.c.t

and $\angle PSQ = \angle RSQ$ |c.p.c.t

i.e. The diameter QS passing through Q bisects $\angle \mathrm{PQR}$ and $\angle \mathrm{PSR}$ Proved.