CBSE Test Paper 05

Chapter 9 Differential Equations

1. Solution of $\frac{dy}{dx} = \sec y$ is
   a. $x = \sin 2y + C$
   b. $x = \sin y + C$
   c. None of these
   d. $x = cos\, y + C$
2. General solution of $\frac{dy}{dx} + 3y = e^{-2x}$ is
   a. $y = e^{-3x} - Ce^{-3x}$
   b. $y = e^{-2x} + Ce^{-3x}$
   c. $y = e^{-2x} - Ce^{-5x}$
   d. $y = e^{-2x} - Ce^{-3x}$
3. General solution of $\left(e^x + e^{-x}\right) dy - \left(e^x - e^{-x}\right) dx = 0$.
   a. $y = \left(e^x + e^{-x}\right) + C$
   b. $y = log\left(e^{2x} + e^{-x}\right) + C$
   c. $y = log\left|\left(e^x + e^{-x}\right)\right| + C$
   d. $y = log\left(e^{-2x} + e^{-x}\right) + C$
4. Differential equation of the family of circles touching the y-axis at origin is
   a. $2y\prime - x^2 = y^2$
   b. $2xyy\prime + x^2 = y^2$
   c. $2yy\prime + x^2 = y^2$
   d. $2xyy\prime - x^2 = y^2$
5. Determine order and degree(if defined) of$\left(\frac{d^2y}{dx^2}\right)^2 + \cos\left(\frac{dy}{dx}\right) = 0$.
   a. 1, degree undefined
   b. 3,1
   c. 2, degree undefined

d. 0, degree undefined

6. General solution of the differential equation of the type $\frac{dy}{dx} + p_1 x = Q_1$ is given by ________.

7. The general solution of the differential equation $\frac{dy}{dx} + \frac{y}{x} = 1$ is ________.

8. $\frac{dy}{dx} + \frac{y}{x \log x} = \frac{1}{x}$ is an equation of the type ________.

9. Find the order and degree of $\left(\frac{ds}{dt}\right)^4 + 3s\frac{d^2 s}{dt^2} = 0.$

10. Verify that the given function is a solution of the corresponding diff eq. y = cosx + c; y^1 + sinx = 0. **(1)**

11. Find order and degree of $\left(\frac{d^2 y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right) + 1 = 0.$

12. Form the differential equation of the family of hyper bolas having foci on x-axis and center at origin.

13. Verify that the function is a solution of the corresponding diff eq.
$y = x^2 + 2x + c; y' - 2x - 2 = 0$

14. Find the solution of $\frac{dy}{dx} = 2^{y-x}.$

15. Solve, x cos $\left(\frac{y}{x}\right) \frac{dy}{dx} = y\cos\left(\frac{y}{x}\right) + x.$

16. State the type of the differential equation for the equation.
$xdy - ydx = \sqrt{x^2 + y^2}dx$ and solve it.

17. Solve the differential equation, $e^x \tan y dx + (1 - e^x) \sec^2 y\, dy = 0.$

18. Show that the differential equation $\frac{dy}{dx} = \frac{y^2}{xy - x^2}$ is homogeneous and also solve it.

CBSE Test Paper 05
Chapter 9 Differential Equations

Solution

1. b. $x = \sin y + C$

   Explanation: $cosydy = dx$

   $\int cosydy = \int dx$

   $siny + c = x$

2. b. $y = e^{-2x} + Ce^{-3x}$

   Explanation: $\frac{dy}{dx} + 3y = e^{-2x} \Rightarrow P = 3, Q = e^{-2x}$

   $\Rightarrow I.F. = e^{\int 3.dx} = e^{3x}$

   $\Rightarrow y.e^{3x} = \int e^{-2x}e^{3x}dx \Rightarrow y.e^{3x} = \int e^{x}dx \Rightarrow y.e^{3x} = e^{x} + C$

   $\Rightarrow y = e^{-2x} + Ce^{-3x}$

3. c. $y = log|(e^{x} + e^{-x})| + C$

   Explanation: $(e^{x} + e^{-x})dy = (e^{x} - e^{-x})dx$

   $\int dy = \int \frac{(e^{x}-e^{-x})}{(e^{x}+e^{-x})}dx$ Since $\int \frac{f'(x)}{f(x)}dx = lnf(x) + c$

   $y = \log|(e^{x} + e^{-x})| + C$

4. b. $2xyy\prime + x^2 = y^2$

   Explanation:

   the eqaution of the circle is $(x-a)^2 + y^2 = a^2$ equation (1) where a is radius of circle

   $2(x-a) + 2yy' = 0$

   $(x-a) + yy' = 0$

   $(x-a) = -yy'$

   Putting in equation 1 we get

   $(-yy')^2 + y^2 = (x + yy')^2 \implies y^2 = x^2 + 2xyy'$

5. b. 2, degree undefined

   Explanation: order = 2, degree not defined, because the function $\frac{dy}{dx}$ present in angle of cosine function.

6. $xe^{\int p_1 dy} = \int \left(Q_1 \times e^{\int p_1 dy}\right) dy + c$
7. $xy = \frac{x^2}{2} + c$

8. $\frac{dy}{dx} + py = Q$
9. Order = 2

   Degree = 1
10. y = cos x + c

    y^1 = -sin x

    y^1 + sin x = 0
11. order = 2

    degree = not defined because it is not a polynomial in $\frac{dy}{dx}$
12. The equation of family of hyperbola is , $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ ...(1)

    Differentiating w.r.t x

    $\frac{2x}{a^2} - \frac{2y}{b^2}y' = 0$

    $\Rightarrow \frac{x}{a^2} - \frac{y}{b^2}y' = 0$

    $\Rightarrow \frac{b^2}{a^2} = \frac{y}{x}y'$

    Again differentiating

    $\Rightarrow 0 = \frac{y}{x}y'' + y'\left(\frac{xy'-y}{x^2}\right)$

    $\Rightarrow 0 = \frac{xyy''+x(y')^2-yy'}{x^2}$

    $\Rightarrow xyy'' + x(y')^2 - yy' = 0$

    $\Rightarrow xyy'' + x(y')^2 = yy'$
13. $y = x^2 + 2x + c$

    $\Rightarrow y' = 2x + 2$

    $\Rightarrow y' - 2x - 2 = 0$

    Hence proved.
14. $\frac{dy}{dx} = 2^y.2^{-x}$

    $2^{-y}dy = 2^{-x}dx$

    $\frac{2^{-y}}{-log2} = \frac{2^{-x}}{-log2} + c$

    $2^{-y} = 2^{-x}$ - c log 2

    $2^{-y} = 2^{-x}$ + c' [Putting c' = -c log 2]
15. $x\cos\left(\frac{y}{x}\right)\frac{dy}{dx} = y\cos\left(\frac{y}{x}\right) + x$

    $\frac{dy}{dx} = \frac{y\cos\left(\frac{y}{x}\right)+x}{x\cos\left(\frac{y}{x}\right)}$ ...(i) let y = vx

$\frac{dy}{dx} = v.1 + x.\frac{dv}{dx}$ ...(ii)

Put $\frac{dy}{dx}$ in eq ...(i)

$v + x\frac{dv}{dx} = \frac{vx\cos v + x}{x\cos v}$

$v + x\frac{dv}{dx} = \frac{v\cos v + 1}{\cos v}$

$x\frac{dv}{dx} = \frac{v\cos v - 1}{\cos v} - v$

$x\frac{dv}{dx} = \frac{v\cos v + 1 - v\cos v}{\cos v}$

$x\frac{dv}{dx} = \frac{1}{\cos v}$

$\int \cos v dv = \int \frac{dx}{x}$

$\sin v = \log x + \log c$sin v = log x + log c

$\sin v = \log|cx|$ sin v = log | cx | $[\because y = vx]$[y = vx]

$\sin\left(\frac{y}{x}\right) = \log|cx|$

16. Given equation can be written as $xdy = \left(\sqrt{x^2 + y^2} + y\right)dx$ i.e.,

$\frac{dy}{dx} = \frac{\sqrt{x^2+y^2}+y}{x}$ ....(1)

Clearly, RHS of (1) is a homogeneous function of degree zero. Therefore, the given equation is a homogeneous differential equation. Substituting y = vx, we get form (1)

$v + x\frac{dv}{dx} = \frac{\sqrt{x^2+v^2x^2}+vx}{x}$ $i.e.$ $v + x\frac{dv}{dx} = \sqrt{1+v^2} + v$

$x\frac{dv}{dx} = \sqrt{1+v^2} \Rightarrow \frac{dv}{\sqrt{1+v^2}} = \frac{dx}{x}$ ....(2)

Integrating both sides of (2), we get

$\log\left(v + \sqrt{1+v^2}\right) = \log x + \log c \Rightarrow v + \sqrt{1+v^2} = cx$

$\Rightarrow \frac{y}{x} + \sqrt{1 + \frac{y^2}{x^2}} = cx \Rightarrow y + \sqrt{x^2+y^2} = cx^2$

17. $e^x \tan y dx + (1 - e^x)\sec^2 y\, dy = 0$

$\Rightarrow e^x \tan y dx = -(1 - e^x)\sec^2 y\, dy$

$\Rightarrow \int \frac{e^x}{1-e^x}dx = -\int \frac{\sec^2 y}{\tan y}dy$

$\Rightarrow -\log(1 - e^x) = -\log\tan y + \log c$

$\Rightarrow \log\left(\frac{\tan y}{1-e^x}\right) = \log c$

$\Rightarrow \frac{\tan y}{1-e^x} = c$

$\Rightarrow \tan y = c(1 - e^x)$

18. According to the question ,

Given differential equation is

$\frac{dy}{dx} = \frac{y^2}{xy-x^2}$ ...(i)

Let $F(x,y) = \frac{y^2}{xy-x^2}$

Now, on replacing x by λx and y by λy , we get

$F(\lambda x, \lambda y) = \frac{\lambda^2 y^2}{\lambda^2 (xy-x^2)} = \lambda^0 \frac{y^2}{xy-x^2} = \lambda^0 F(x,y)$

Thus, the given differential equation is a homogeneous differential equation.

Now, to solve it, put y = vx

$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

From Eq. (i), we get

$v + x\frac{dv}{dx} = \frac{v^2 x^2}{vx^2 - x^2} = \frac{v^2}{v-1}$

$\Rightarrow \quad x\frac{dv}{dx} = \frac{v^2}{v-1} - v = \frac{v^2 - v^2 + v}{v-1}$

$\Rightarrow \quad x\frac{dv}{dx} = \frac{v}{v-1} \Rightarrow \frac{v-1}{v}dv = \frac{dx}{x}$

On integrating both sides, we get

$\int \left(1 - \frac{1}{v}\right) dv = \int \frac{dx}{x}$

$\Rightarrow \quad v - \log|v| = \log|x| + C$

$\Rightarrow \quad \frac{y}{x} - \log\left|\frac{y}{x}\right| = \log|x| + C \left[\text{ put } v = \frac{y}{x}\right]$

$\Rightarrow \quad \frac{y}{x} - \log|y| + \log|x| = \log|x| + C \ \left[\because \log\left(\frac{m}{n}\right) = \log m - \log n\right]$

$\therefore \quad \frac{y}{x} - \log|y| = C$

which is the required solution.