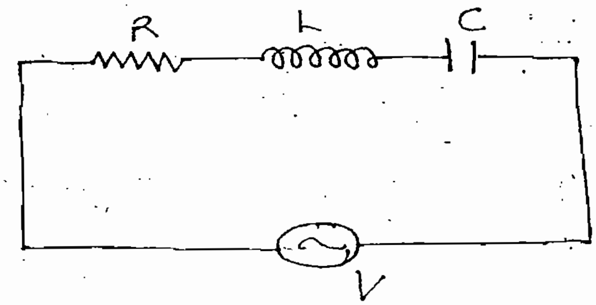
Lecture-6

By KVL

$$V = iR + L\frac{di}{dt} + \frac{1}{C}\int i\,dt$$



Diff w.r.t t

$$V' = R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{i}{C}$$

Dividing both sides of L

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = \frac{V'}{L}$$

$$\left(D^2 + \frac{R}{L}D + \frac{1}{LC}\right)i = 0$$

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

$\frac{d}{dt} = D$

$$2\xi\omega_n = \frac{R}{L}$$

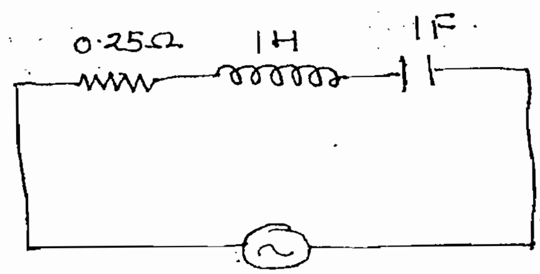
$$2\xi\frac{1}{\sqrt{LC}} = \frac{R}{L}$$

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Damping ratio $= \xi = \frac{R}{2}\sqrt{\frac{C}{L}}$

$$\boxed{\xi = \frac{1}{2Q}}$$

Ques:- Find f_0, Q, ξ, B.W, f_1, f_2, I at f_0



$V(t) = 10\sin\omega t$

Soln:- (i) $f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi}$

(ii) $Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{0.25}\sqrt{\frac{1}{1}} = 4$

(iii) $\delta = \frac{1}{2Q} = \frac{1}{8}$

(iv) $\underset{(f_2-f_1)}{BW} = \frac{f_0}{Q} = \frac{1/2\pi}{4} = \frac{1}{8\pi}$

(v) $f_2 - f_1 = \frac{1}{8\pi}$

$f_1 f_2 = f_0^2 = \left(\frac{1}{2\pi}\right)^2$

$(f_2+f_1)^2 - (f_2-f_1)^2 = 4f_1 f_2$

$f_1 =$

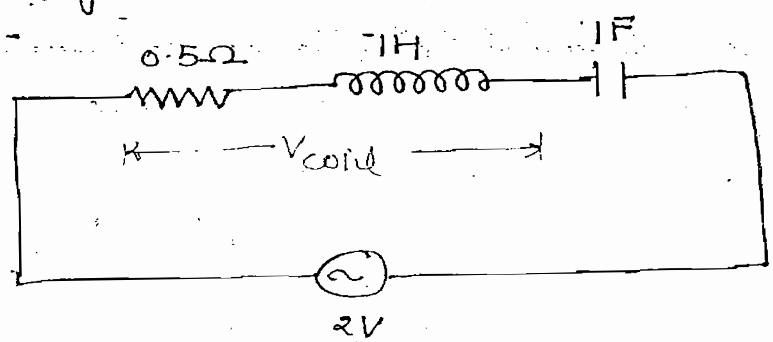
$f_2 =$

(vi) $I = \frac{V}{Z} = \frac{V}{R}$ (At Resonance)

$I = \frac{10/\sqrt{2}}{0.25}$

$I = \frac{40}{\sqrt{2}}$, Ans.

Ques:- Find voltage across the coil under resonance condition :-



Soln:- $V_R = V = 2$ [When nothing is given take it as RMS]

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$Q = \frac{1}{0.5}\sqrt{\frac{1}{1}} = 2$$

$$Q = \frac{V_L}{V}$$

$$\Rightarrow 2 = \frac{V_L}{2}$$

$$\Rightarrow V_L = 4$$

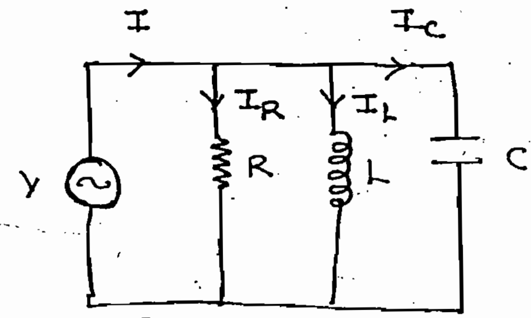
$$V_{coil} = \sqrt{V_R^2 + V_L^2}$$

$$\Rightarrow V_{coil} = \sqrt{2^2 + 4^2}$$

$$= \sqrt{20}$$, Ans.

Parallel Resonance :-

Case-(1) :-

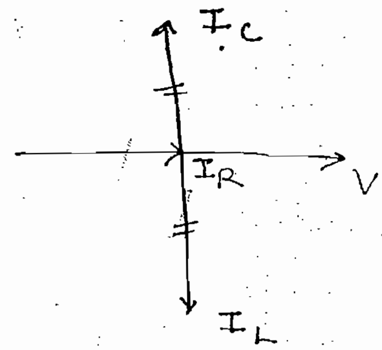


$$I_C = I_L$$

$$\frac{V}{X_C} = \frac{V}{X_L} \qquad X_L = X_L$$

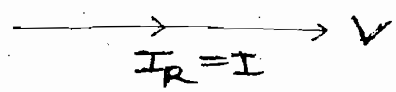
$$\Rightarrow \boxed{B_C = B_L}$$

$$\omega C = \frac{1}{\omega L}$$



$$\boxed{\begin{aligned} \omega_0 &= \frac{1}{\sqrt{LC}} \text{ rad/sec} \\ f_0 &= \frac{1}{2\pi\sqrt{LC}} \text{ Hz} \end{aligned}}$$

(I) $Y = G + j\underbrace{(B_C - B_L)}_{0}$



$$Y_{min} = G$$

(II) $Z_{max} = \dfrac{1}{Y_{min}}$

(III) $I_{min} = \dfrac{V}{Z_{max}}$

(IV) $\cos\theta = 1$

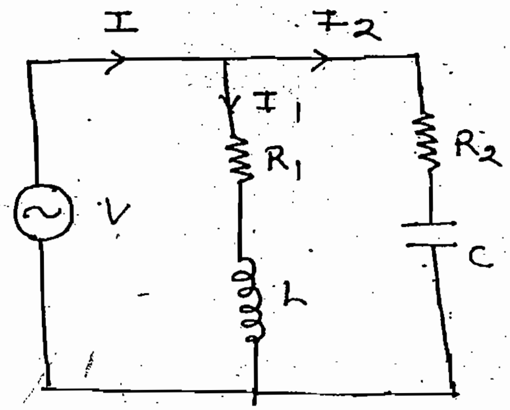
(V) $I_R = I$

(VI) Net Reactive current = 0

(VII) current in inductor or current in capacitor is greater than total current. This phenomenon is called as current magnification

viii) Parallel Resonance circuit is also called as Anti-Resonance circuit.

Case-(II):-

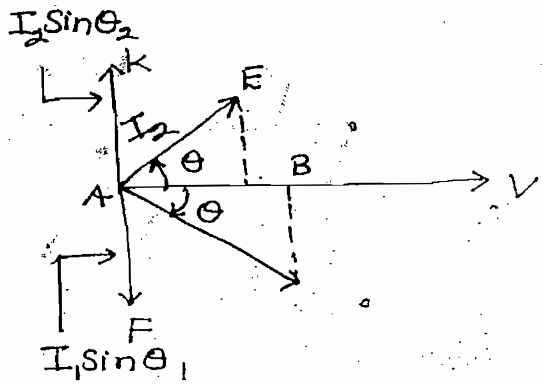


$AB = I_1 \cos\theta_1$

$AF = BC = I_1 \sin\theta_1$

$AD = I_2 \cos\theta_2$

$AK = DE = I_2 \sin\theta_2$

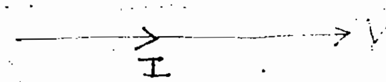


$B_L = B_C$

$$\frac{X_L}{R_1^2+X_L^2} = \frac{X_C}{R_2^2+X_C^2}$$

$$\frac{\omega L}{R_1^2+(\omega L)^2} = \frac{1/\omega C}{R_2^2+\left(\frac{1}{\omega C}\right)^2}$$

$$\boxed{f_0 = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{R_1^2-\frac{L}{C}}{R_2^2-L/C}}}$$



$I = I_1\cos\theta_1 + I_2\cos\theta_2$

$I = VY$

$$\Rightarrow I = V\left[(G_1+G_2) + j\underbrace{(B_C-B_L)}_{0}\right]$$

$$\Rightarrow \boxed{I = V\left[\frac{R_1}{R_1^2+X_L^2} + \frac{R_2}{R_2^2+X_C^2}\right]}$$

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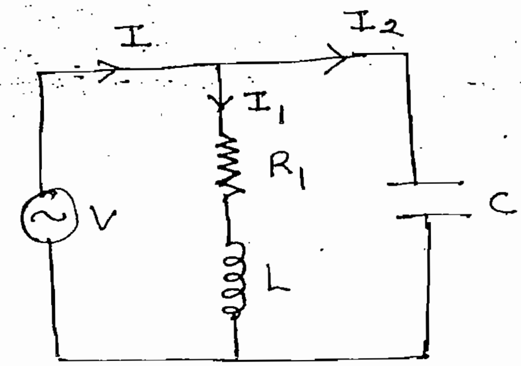
Case-(III):-



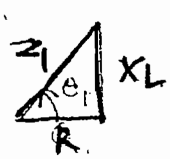
Tank ckt

$AB = I_1 \cos\theta_1$

$AF = BC = I_1 \sin\theta_1$

$\cos\theta_1 = \frac{R_1}{Z_1}$

$\sin\theta_1 = \frac{X_L}{Z_1}$



$$I_2 = I_1 \sin\theta_1$$

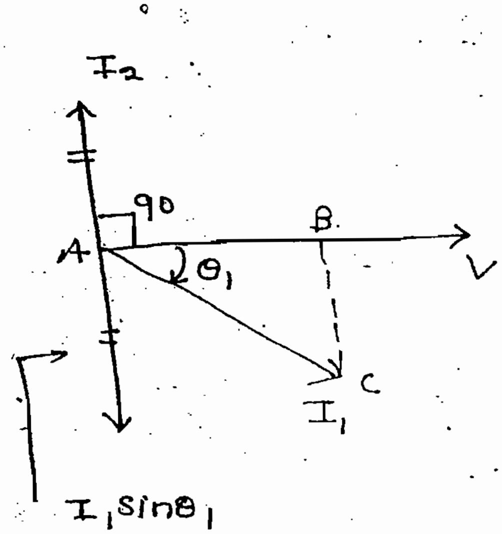
$$\frac{V}{X_C} = \frac{V}{Z_1}\,\frac{X_L}{Z_1}$$

$$\Rightarrow Z_1^2 = X_L X_C$$

$$\Rightarrow Z_1^2 = \frac{\omega L}{\omega C}$$

$$\Rightarrow Z_1^2 = \frac{L}{C}$$

$$\Rightarrow \boxed{Z_1 = \sqrt{\frac{L}{C}}}\ \Omega$$



$\rightarrow V$

$I = I_1 \cos\theta_1$

$\rightarrow$ $B_L = B_C$

$$Z_1^2 = R_1^2 + X_L^2$$

$$\frac{L}{C} = R_1^2 + (2\pi f_0 L)^2$$

$$\boxed{f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R_1^2}{L^2}}}$$

$\rightarrow$ $I = I_1 \cos\theta_1$

$$\Rightarrow I = \frac{V}{Z_1}\,\frac{R_1}{Z_1}$$

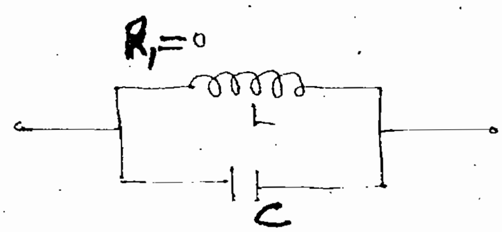
$$\Rightarrow I = \frac{VR_1}{Z_1^2} \quad \Rightarrow I = \frac{VR_1}{L/C}$$

$$\Rightarrow \boxed{I = \frac{V}{\frac{L}{R_1 C}}}$$

$$\Rightarrow \boxed{Z_{Dyn} = \frac{L}{R_1 C}}\ \Omega$$

Ideal Tank Circuit :-

$$\boxed{\begin{aligned} Z_{Dyn} &= \frac{L}{R_1 C} = \infty \\ f_0 &= \frac{1}{2\pi\sqrt{LC}} \end{aligned}}$$

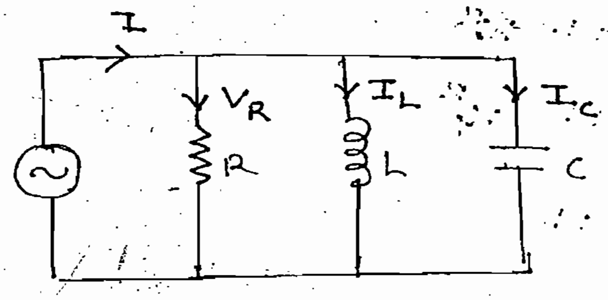


@p

Q - Factor :-

$$\boxed{Q = \frac{V_L \text{ or } V_C}{V}} \longrightarrow \text{Series}$$

$$\boxed{Q = \frac{I_L \text{ or } I_C}{I}} \longrightarrow \text{Parallel}$$



$$\boxed{Q = \frac{I_L}{I} = \frac{I_L}{I_R} = \frac{\text{Reactive component of current}}{\text{Active component of current}}}$$

This combination is valid for any combination of parallel circuit

$$\boxed{Q = \frac{I_L}{I_R} = \frac{V/X_L}{V/R} = \frac{R}{X_L} = \frac{R}{\omega L}}$$

$$\left(\omega = \frac{1}{\sqrt{LC}} \right)$$

$$\boxed{Q = \frac{1/X_L}{1/R} = \frac{B_L \text{ or } B_C}{G}}$$

$$(B_L = B_C)$$

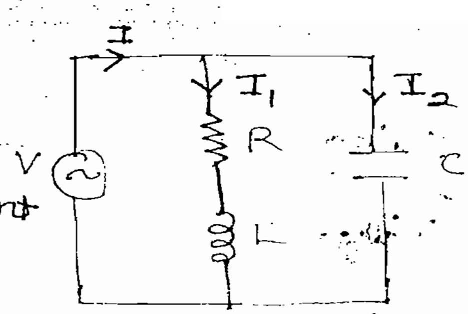
$$\boxed{Q = R\sqrt{\frac{C}{L}}}$$

$$\boxed{Q = \frac{I_C}{I} = \frac{I_C}{I_R} = \frac{V/X_C}{V/R} = \frac{R}{X_C} = R\omega C}$$

For $\boxed{Q > 1 \;,\; R > X_L \;,\; R > X_C}$

Tank Circuit :-

$$Q = \frac{\text{Reactive component of current}}{\text{Active comp. of current}}$$

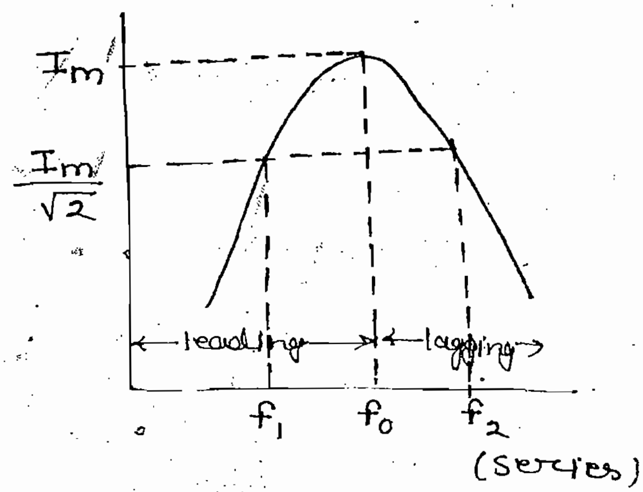


$$\boxed{Q = \frac{I_1 \sin\theta_1 \text{ or } I_2}{I}}$$

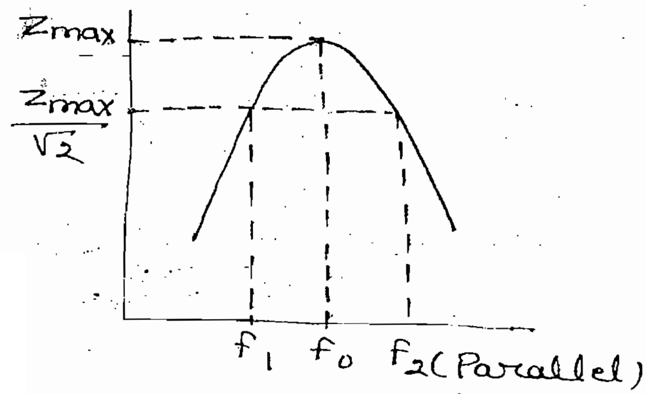
$$\boxed{Q = \frac{I_2}{I} = \frac{V/X_C}{\frac{V}{L/RC}} = \frac{\frac{1}{1/\omega C}}{\frac{1}{L/RC}} = \frac{\omega L}{R} = \frac{X_L}{R}}$$

For $Q > 1$, $\boxed{X_L > R}$

$$\boxed{B \cdot W = f_2 - f_1}$$

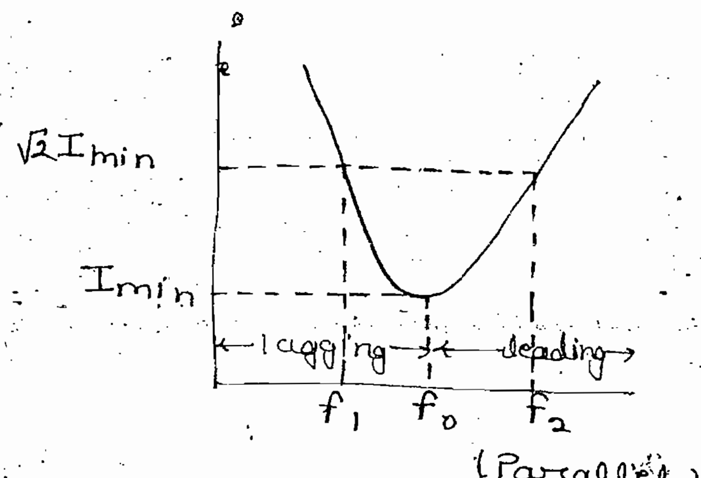


$$\boxed{B \cdot W = f_2 - f_1}$$



$$\boxed{B \cdot W = f_2 - f_1}$$

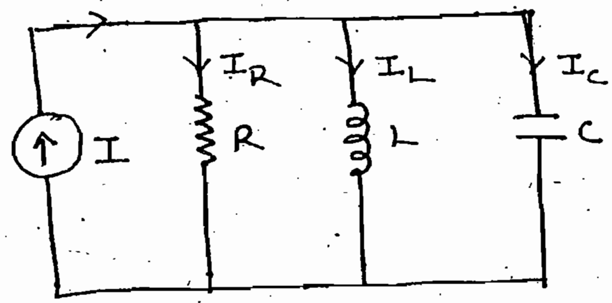
$$\boxed{\begin{aligned} B_L &= \frac{1}{2\pi f L} \\ B_C &= 2\pi f C \end{aligned}}$$



For parallel circuit we concentrate on the value of B_L and B_C

By KCL

$$I = \frac{V}{R} + C\frac{dV}{dt} + \frac{1}{L}\int V\,dt$$



Diff. w.r.t t

$$I' = \frac{1}{R}\frac{dV}{dt} + C\frac{d^2V}{dt^2} + \frac{V}{L}$$

Dividing both sides by C, we get

$$\frac{d^2V}{dt^2} + \frac{1}{RC}\frac{dV}{dt} + \frac{V}{LC} = \frac{I'}{C}$$

$$D = \frac{d}{dt}$$

$$\left(D^2 + \frac{1}{RC}D + \frac{1}{LC}\right)V = 0$$

Compare with $s^2 + 2\delta\omega_n s + \omega_n^2 = 0$ we get

$$2\delta\omega_n = \frac{1}{RC}, \quad \omega_n^2 = \frac{1}{LC}$$

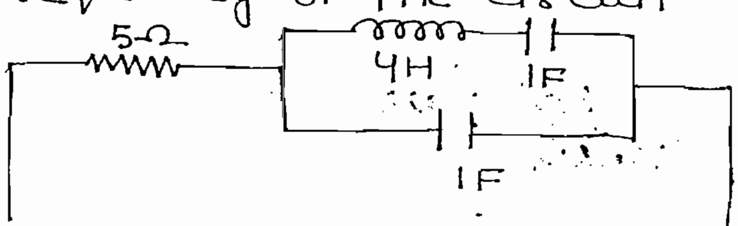
$$2\delta\frac{1}{\sqrt{LC}} = \frac{1}{RC}, \quad \omega_n = \frac{1}{\sqrt{LC}}$$

$$\boxed{Q = R\sqrt{\frac{C}{L}}}$$

$$\boxed{\text{Damping Ratio} = \delta = \frac{1}{2R}\sqrt{\frac{L}{C}}}$$

$$\boxed{\delta = \frac{1}{2Q}}$$

Ques:- Find resonant frequency of the circuit shown



Soln:- At resonance in any of the case imag. part = 0

Note:-

To find resonant frequency for any combination of the network

(I) Find Z_{eq}

(II) Equate imag. part of $Z = 0$

$$Z_1 = j(X_L - X_C) = j\left(\omega L - \frac{1}{\omega C}\right) = j\left(4\omega - \frac{1}{\omega}\right)$$

$$Z_2 = -jX_C = -\frac{j}{\omega C} = -\frac{j}{\omega}$$

$$Z'_{eq} = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

$$\Rightarrow Z'_{eq} = \frac{j\left(4\omega - \frac{1}{\omega}\right)\left(-j/\omega\right)}{j\left(4\omega - \frac{1}{\omega}\right) - j/\omega}$$

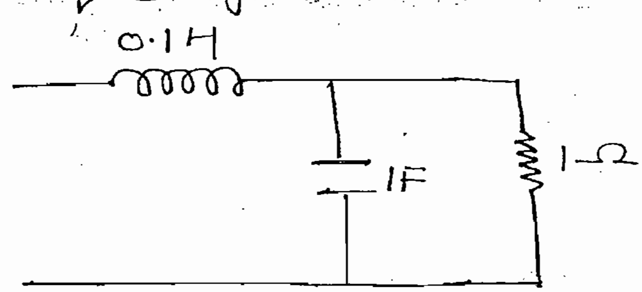
$$\Rightarrow Z'_{eq} = \frac{\left(4\omega - 1/\omega\right)\left(+1/\omega\right)}{j\left(4\omega - \frac{1}{\omega}\right) - j/\omega} \times \frac{j}{j}$$

$$\text{Im } Z'_{eq} = 0$$

$$\left(4\omega_0 - \frac{1}{\omega}\right)\frac{1}{\omega} = 0$$

$$\Rightarrow \boxed{\omega_0 = 0.5 \text{ rad/sec}}$$

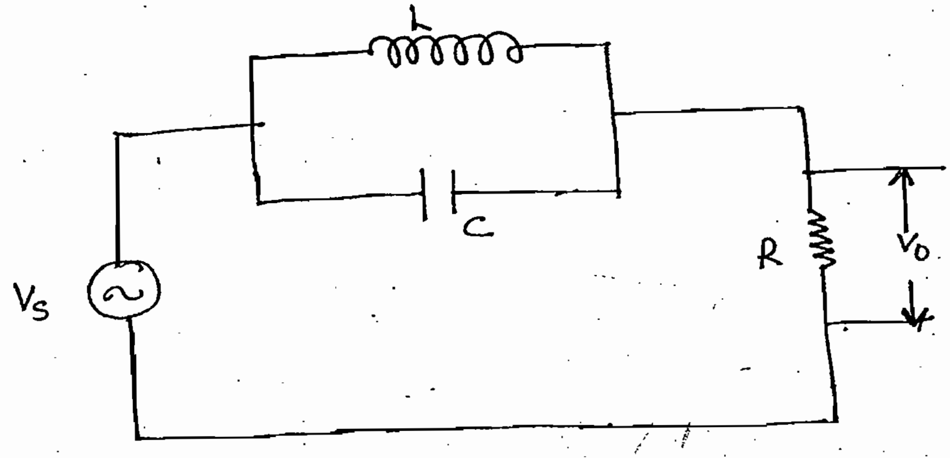
Ques:- Find resonant frequency of the circuit shown



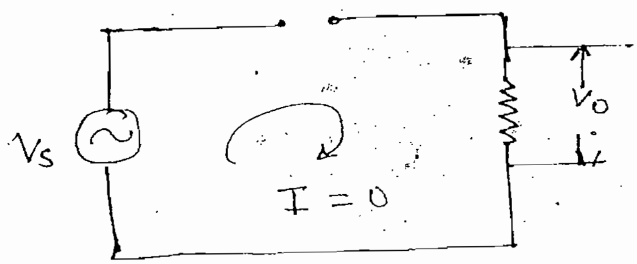
Ans:- $\omega_0 = 3$ rad/sec

ωω

Ques 1- Find V_0 under resonance condition



Soln- Ideal tank circuit, $Z_{Dyn} = \infty$

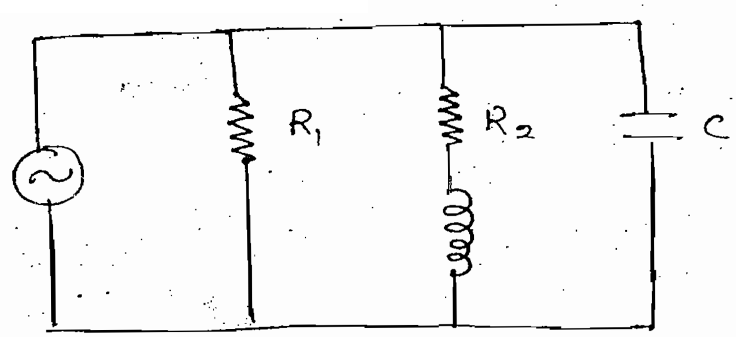


$V_0 = 0$, Ans

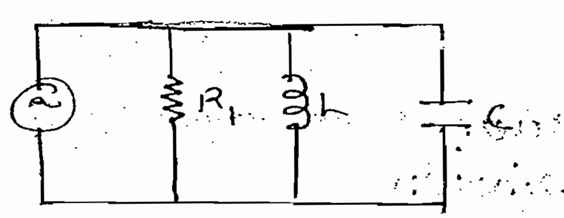
Note:-

$R_1 \uparrow$ $Q \uparrow$

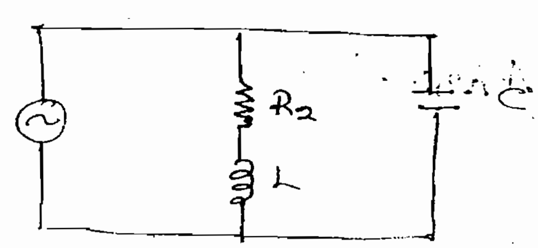
$R_2 \uparrow$ $Q \downarrow$



$Q = \dfrac{R_1}{\omega L}$ ←



$Q = \dfrac{\omega L}{R_2}$ ←



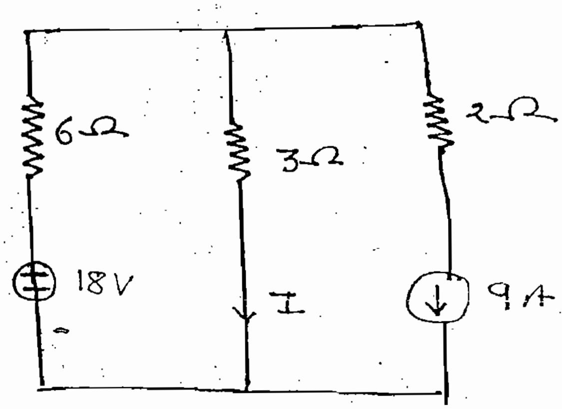
THEOREMS :—

When the N/w is having more no. of nodes and more no. of meshes, the response in any one of the branches can be easily obtained by using theorem

Superposition theorem :-

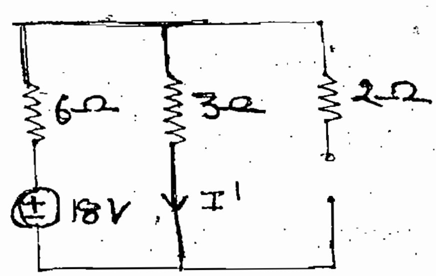
In any linear bidirectional circuit having more than one independent source the response in any of the branches is equal to algebraic sum of the responses caused by individual sources while rest of the sources are replaced by its internal resistances

Ques:- Find the value of I by using superposition theorem



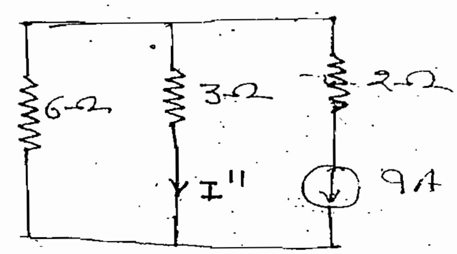
Soln:- Case-(I):-

Due to 18V



$$I' = \frac{18}{6+3} = 2A$$

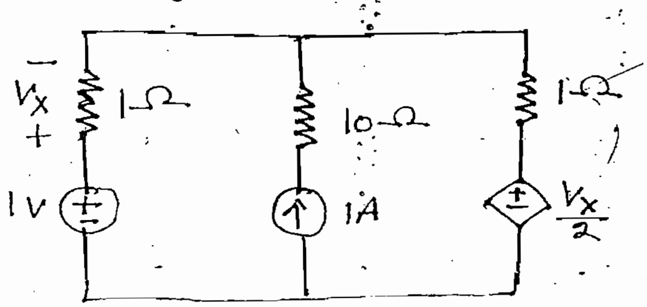
Case-(II):-



$$I'' = -9\,\frac{6}{6+3} = -6$$

$$I = I' + I'' = 2-6 = -4 \text{, Ans}$$

Ques:- Find V_x by using superposition theorem



Note :-

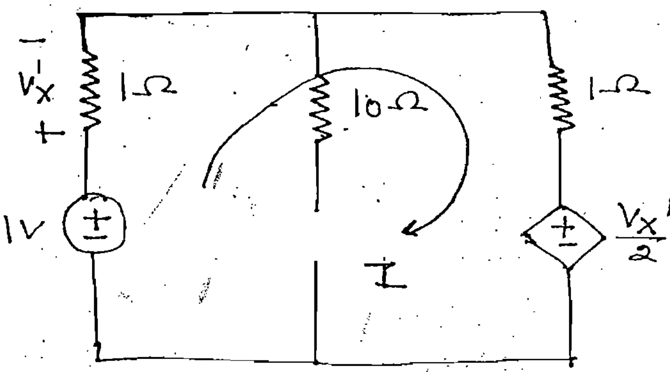
In the above circuit while applying superposition theorem dependent sources neither replaced by open circuit nor S.C & it remains as original ckt.

Soln :- Case-(1) :-

$$I = \frac{1 - \frac{V_x'}{2}}{1+1}$$

$$V_x' = 1 \times I = I$$

$$V_x' = \frac{2}{5}$$



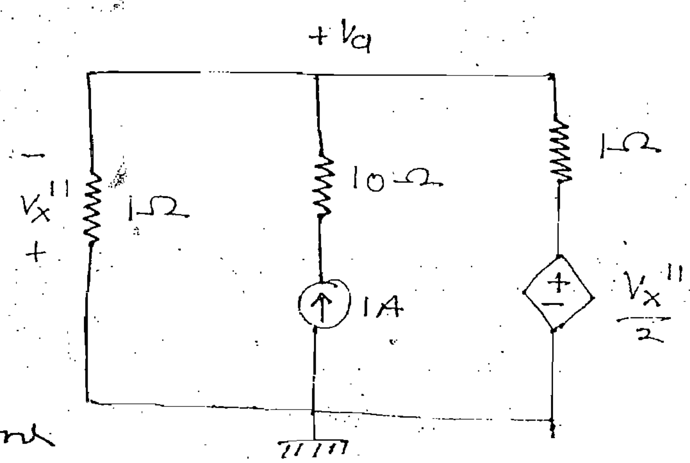
Case-(II) :-

$$V_a = -V_x''$$

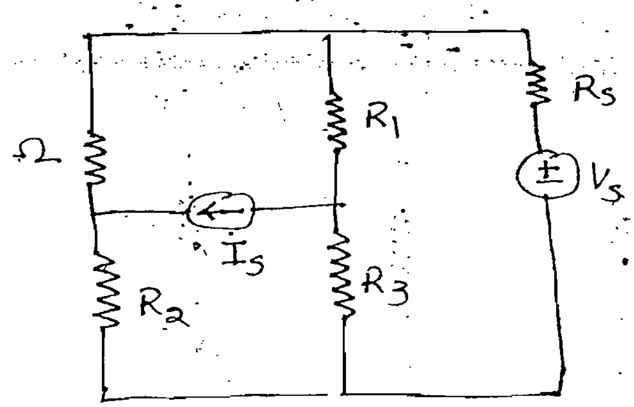
$$\frac{V_a}{1} + \frac{V_a - \frac{V_x''}{2}}{1} = 1$$

$$V_x'' = -\frac{2}{5}$$

$$V_x = V_x' + V_x'' = 0, \text{ Ans}$$



Ques :- In the circuit shown power dissipation in 1Ω resistor is 576 W when voltage source is acting along and power dissipation in 1Ω is resistor is 1W when current source is acting along. Find total power dissipation in 1 ohm resistor



Soln:-

$I = \pm I' \pm I''$ ✓

$V = \pm V' \pm V''$ ✓

$P = \pm P' \pm P''$ ✗

$P = I^2 R$ $\qquad\qquad P' = I'^2 R$

$$I' = \sqrt{\frac{P'}{R}}$$

$$I'' = \sqrt{\frac{P''}{R}}$$

$$I = \pm I' \pm I''$$

$$\Rightarrow I = \pm \sqrt{\frac{P'}{R}} \pm \sqrt{\frac{P''}{R}}$$

$$P = I^2 R$$

$$\Rightarrow P = \left(\pm \sqrt{\frac{P'}{R}} \pm \sqrt{\frac{P''}{R}} \right)^2 R$$

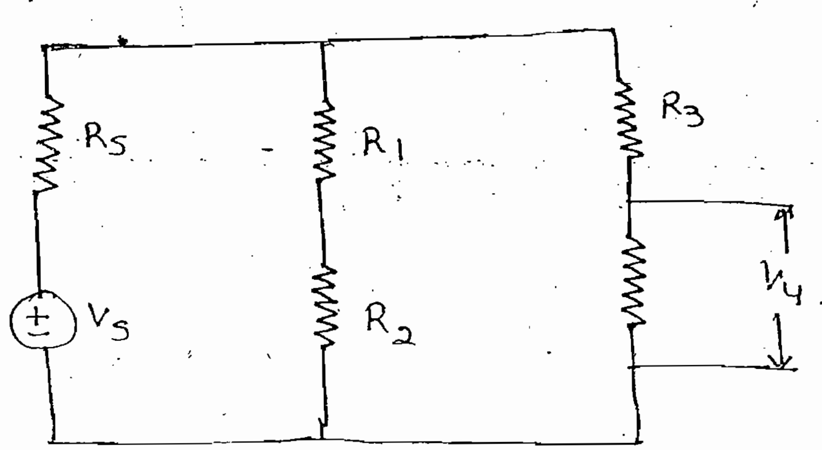
$$\Rightarrow ** \boxed{P = \left(\pm \sqrt{P'} \pm \sqrt{P''} \right)^2}$$

$$P = \left(+\sqrt{P'} - \sqrt{P''} \right)^2$$

$$= \left(+\sqrt{576} - \sqrt{1} \right)^2 = 529\ W, \text{ Ans.}$$

Ques:- In the circuit shown if the source voltage is increased by 10% then find variation of the power in the R_4 resistor.

Ans:-



Soln:—

$$P_4 = \frac{V_4^2}{R_4}$$

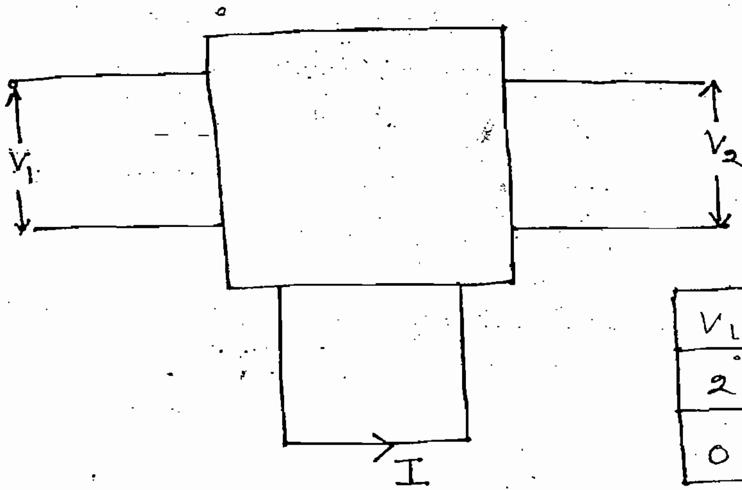
$$P_4' = \frac{(1.1V_4)^2}{R_4} = 1.21\frac{V_4^2}{R_4} = 1.21\,P_4$$

Inc by 21%.

Note:—

When the N/w is having linear bidirectional element if excitation is particularly constant K the response of each element also multiplied by constant K (Homogenity Principle)

Ques:- In the circuit shown find I when $V_1 = 10V$ & $V_2 = -12V$



V_1	V_2	I
2	0	3A
0	3V	-4A

Soln:— $V_1 = 2$ $\quad I = 3A$

$V_1 = 10$ then $I' = 5\times3 = 15A$
(2x5)
5times ↑

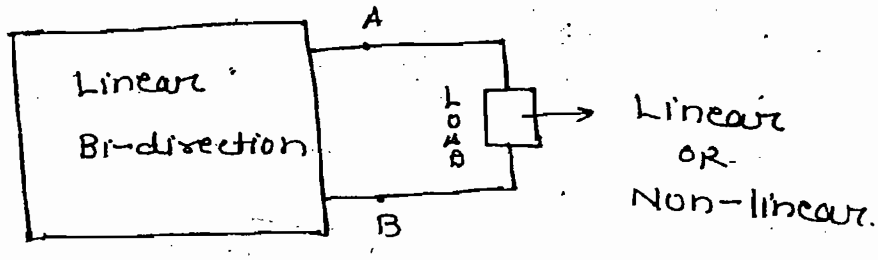
$V_2 = 3V$ $\quad I = -4$

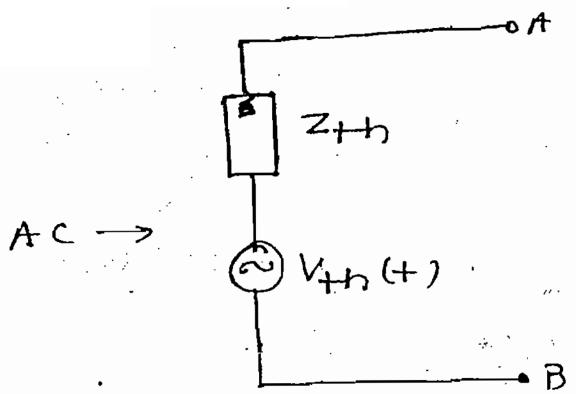
$V_2 = -12V$ then $I'' = (-4)(-4)$
(-4x3) $\quad = 16$

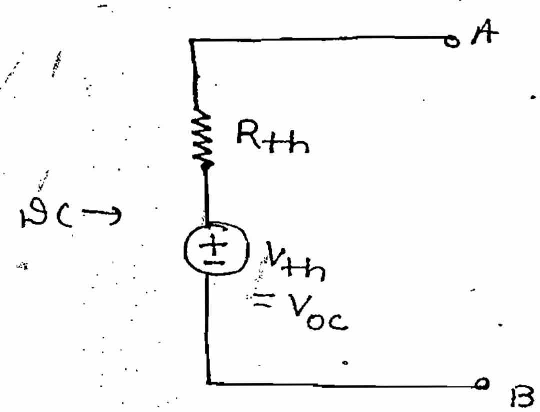
$$I = I' + I''$$

$$I = 15+16 = 31\,A$$, Ans

Thevenins Theorem :-

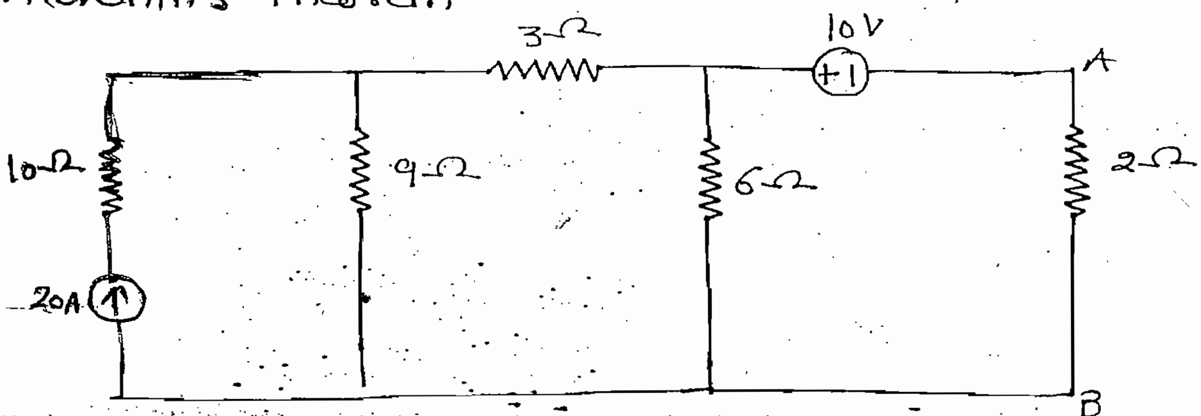






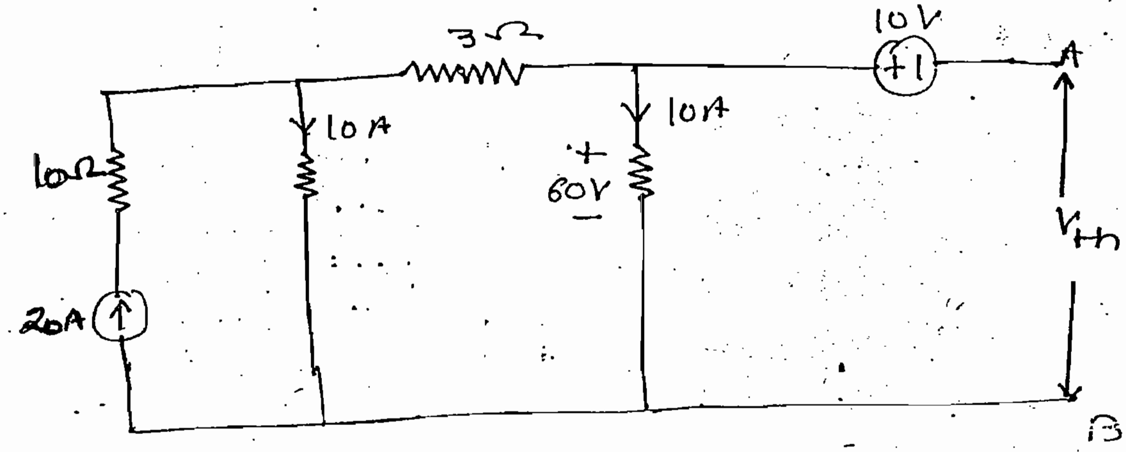
When N/w is having linear bidirectional elements and more no. of active and passive elements, it can be replaced by single equivalent circuit consisting of equivalent voltage source (V_{th}) in series with equivalent resistance (R_{th})

Ques:- Find current in the 2Ω resistor by using thevenin's theorem



Soln:- Case-(1) → (V_{th}) :-

Disconnect the load resistor and o.c voltage across the load terminals

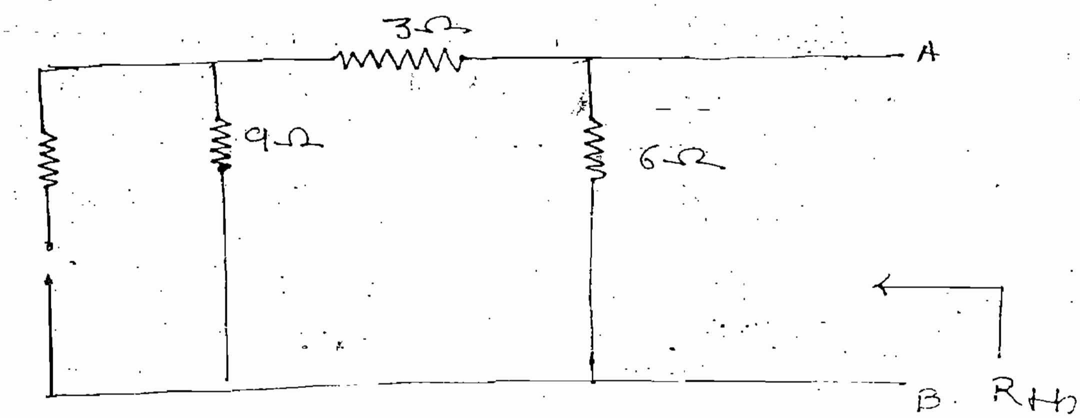


$$-60 + 10 + V_{th} = 0$$

$$\Rightarrow \quad V_{th} = 50\,V$$

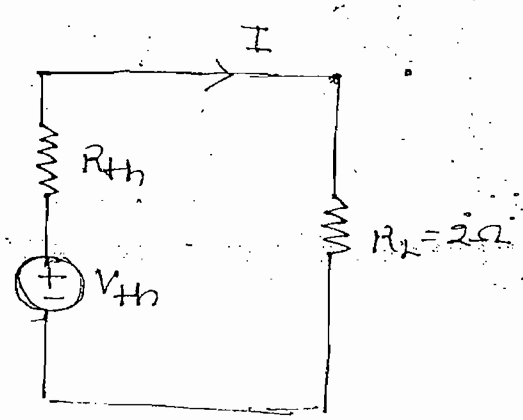
Case-(ii) $\longrightarrow$ (R_{th}) :-

Deactivate all the independent sources and find eq. resistance w.r.t. load terminals



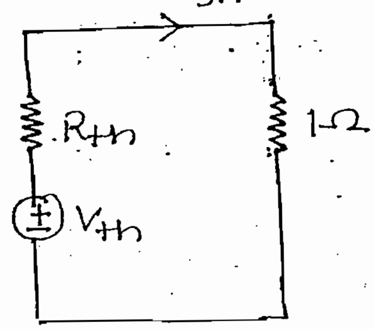
$$R_{th} = \frac{12 \times 6}{12+6} = 4\,\Omega$$

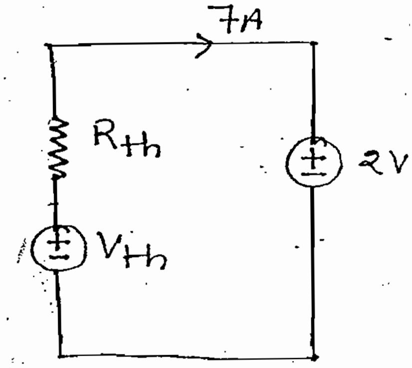
$$I = \frac{V_{th}}{R_{th} + R_L}$$



Ques:- When a battery charger connected to 1Ω resistor, current in the resistor is 5A. When same battery charger is connected for charging of ideal 2V battery at 7A rate. Find V_{th} & R_{th}

Soln:-



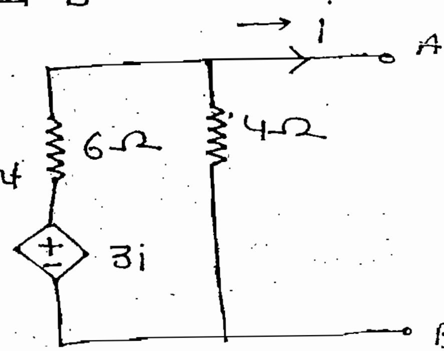


$$5 = \frac{V_{th}}{R_{th}+1} \quad -(I)$$

$$7 = \frac{V_{th}-2}{R_{th}} \quad -(II)$$

$\Rightarrow V_{th} = 12.5$ and $R_{th} = 1.5$

Ques:- Find V_{th} w.r.t A and B



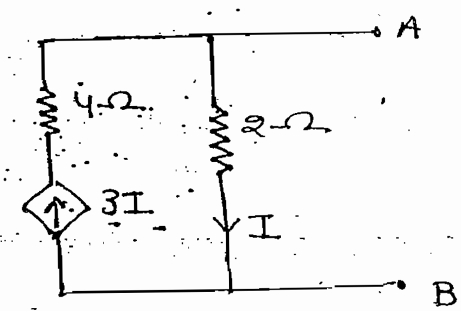
Note:-

In above N/w no independent source is present Therefore $V_{th} = 0$

Ques:- Find V_{th} w.r.t A and B.

(a) 0 (b) 4 (c) 6

(d) None

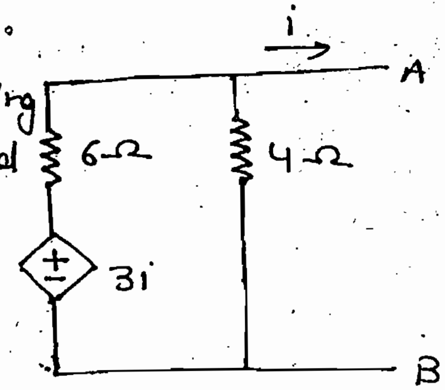


Note:-

In the above ckt, it is not possible to find o.c Voltage since it is not satisfying KCL

Ques:- Find R_{th} w.r.t A and B

Note:-
In above N/w while finding R_{th} dependent source is replaced by neither OC nor short ckt and it remains same as original ckt.



Soln:-

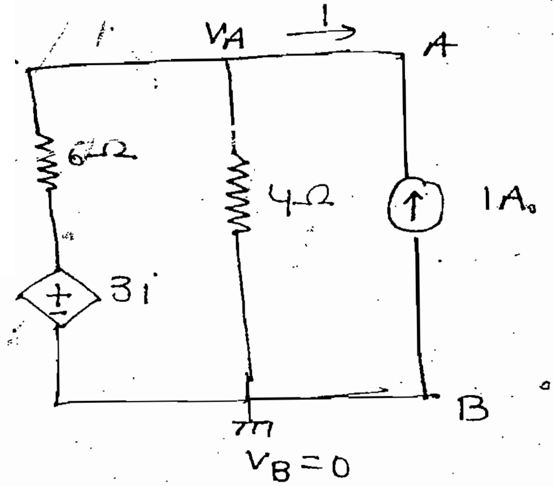
$$\frac{V_A - 3i}{6} + \frac{V_A}{4} = 1$$

$$\Rightarrow \quad i = -1$$

$$V_A = 1.2$$

$$V_{AB} = V_A - V_B$$
$$= 1.2 - 0 = 1.2$$

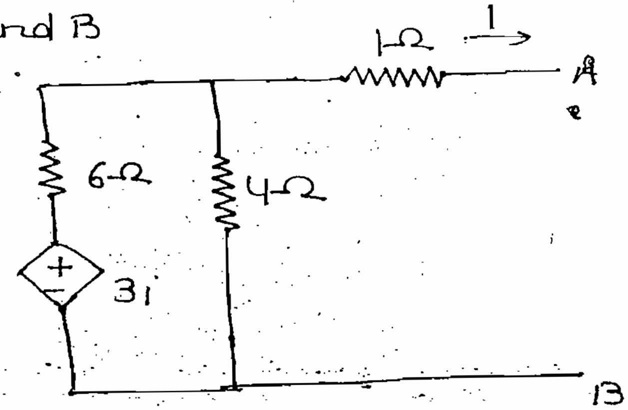
$$R_{th} = \frac{V_{AB}}{I_s} = \frac{1.2}{1} = 1.2\,\Omega \text{ , Ans}$$



In parallel consider current source for simple calculation.

Ques:- Find R_{th} w.r.t A and B

Soln:- In series consider voltage source

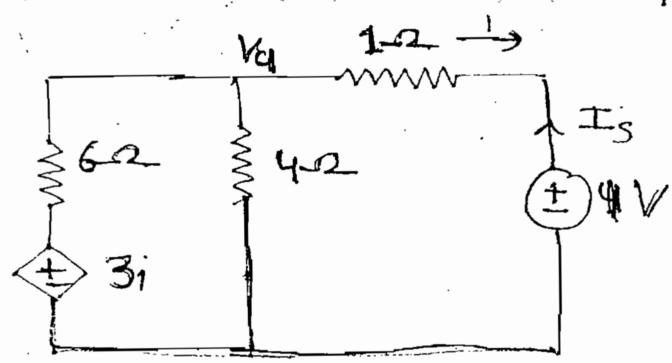


$$\frac{V_1 - 3i}{6} + \frac{V_1}{4} + \frac{V_1 - 1}{1} = 0$$

$$i = \frac{V_1 - 1}{1} \Rightarrow \cancel{V_1 = \frac{6}{11}}$$

$$= -\frac{5}{11}$$

$$\& \quad V_1 = \frac{6}{11}$$

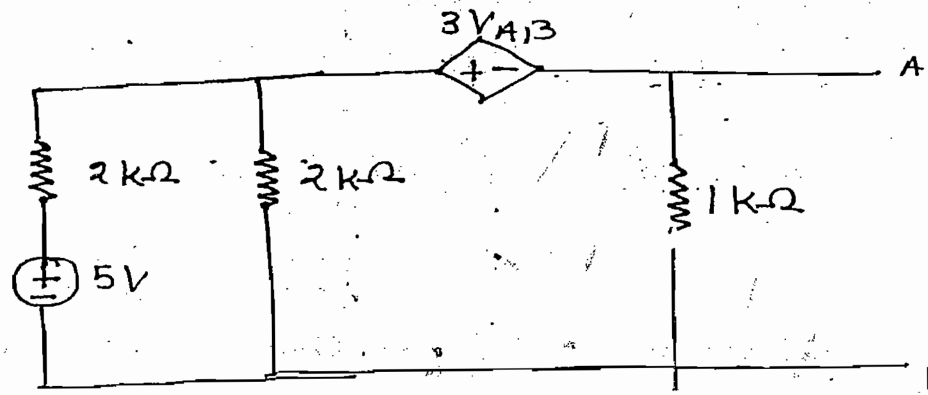


$$I_s = -i = -\left(-\frac{5}{11}\right) = \frac{5}{11}$$

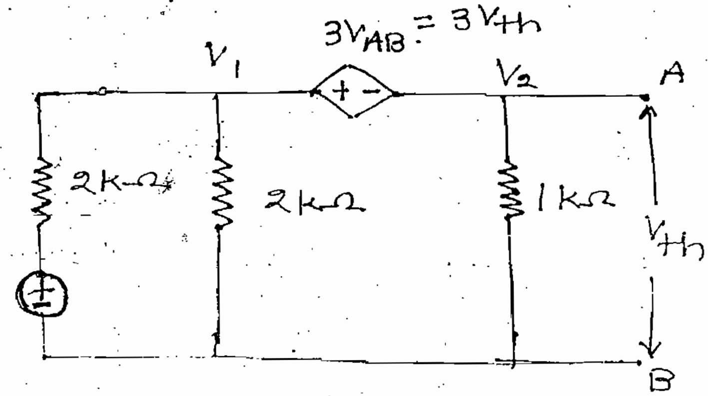
$$R_{th} = \frac{V_s}{I_s} = \frac{1}{5/11} = \frac{11}{5}\,\Omega \quad \text{, Ans}$$

$$= 2.2\,\Omega \quad \text{, Ans.}$$

Ques:— Find V_{th} and R_{th} w.r.t A and B



Soln:— Case-(I):—



$$\frac{V_1 - 5}{2\times10^3} + \frac{V_1}{2\times10^3} + \frac{V_2}{1\times10^3} = 0 \quad \text{—(I)}$$

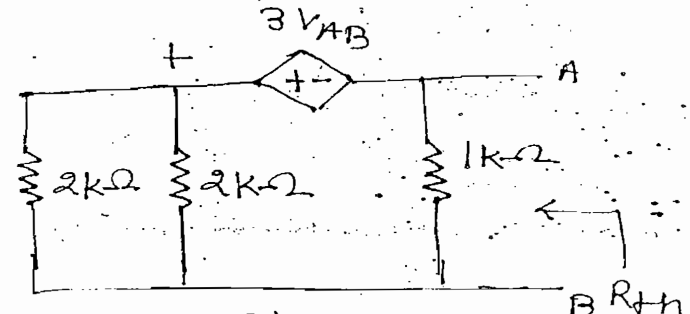
$$V_1 - V_2 = 3V_{th} \quad \text{—(II)}$$

$$V_2 = V_{th}$$

$$V_1 = 4V_{th}$$

$$V_{th} = 0.5\,V$$

Case-(II):— For R_{th}



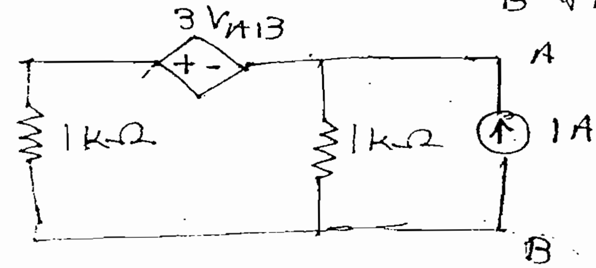
$$\frac{V_A + 3V_{AB}}{10^3} + \frac{V_A}{1\times10^3} = 1$$

$$V_{AB} = V_A - V_B = V_A$$

$$V_{AB} = 200$$

$$R_{th} = \frac{V_{AB}}{I_s} = \frac{200}{1} = 200\,\Omega$$

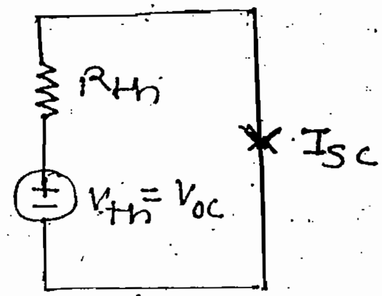
And



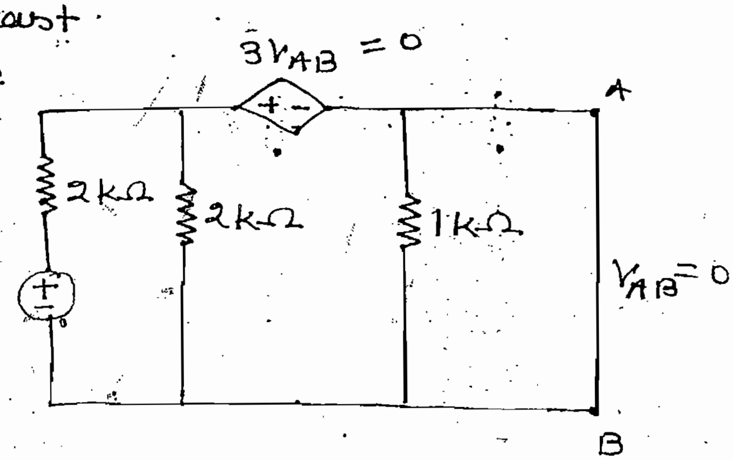
Verification :-

$$I_{sc} = \frac{V_{oc}}{R_{th}}$$

$$R_{th} = \frac{V_{oc}}{I_{sc}}$$



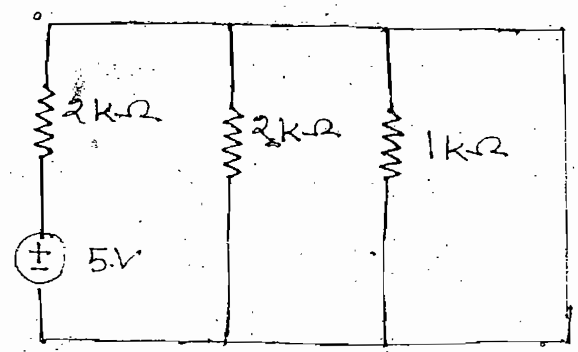
For this method atleast one independent source should be present



$$I_{sc} = \frac{5}{2\times10^3}$$

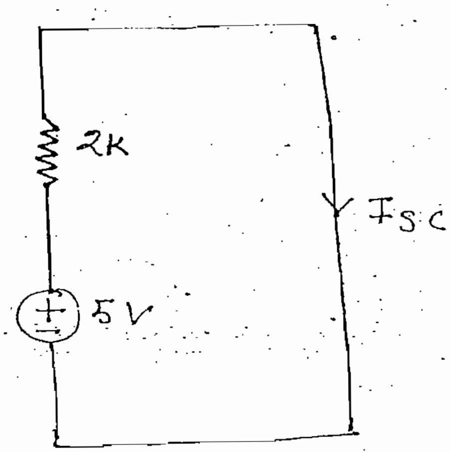
$$R_{th} = \frac{V_{oc}}{I_{sc}} = \frac{0.5}{5/2\times10^3}$$

$$= 200\,\Omega$$

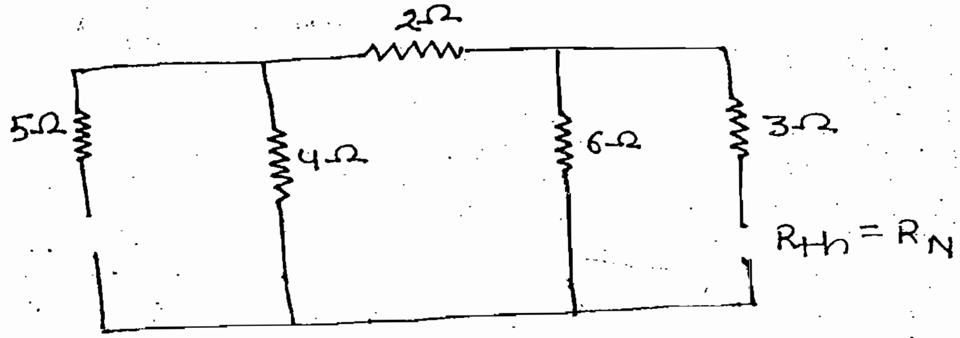


Note :-

The above method of R_{th} calculation can be done provided original N/w should consist of atleast one independent source.



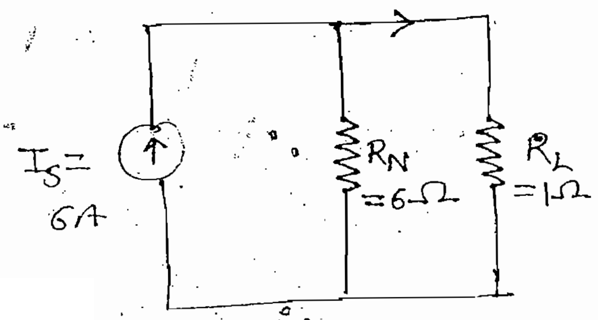
Case – 2 (R_{th}):-



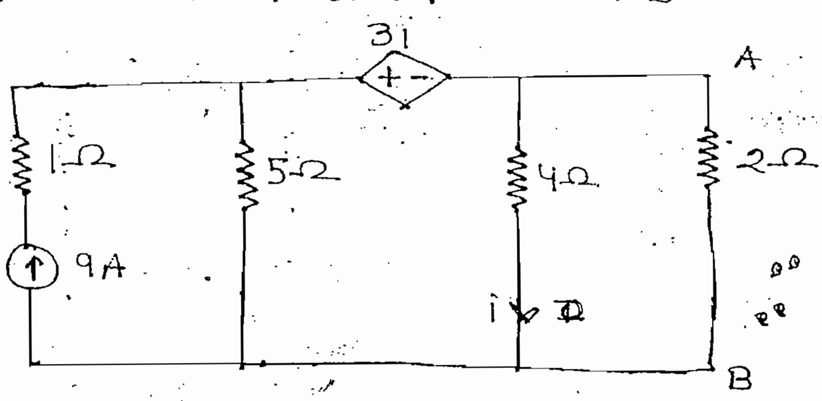
$$R_{th} = R_N = 6\Omega$$

$$I_L = 6 \times \frac{6}{6+1}$$

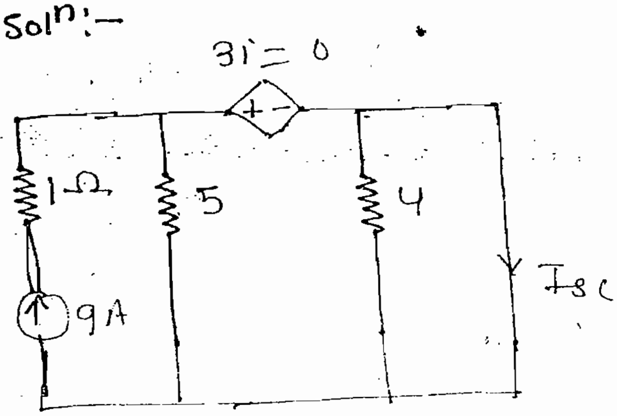
$$= \frac{36}{7} A \text{ , Ans}$$



Ques:- Find S.C current w.r.t A and B



Soln:-



$\Leftarrow$