Lecture-2

Summary-3 :-

Col – Q

Gauss Law ← Mathematical → Ampere's Law

I – Amps

Cause
Source
Mathematical
Effects
Strength
Physical
Force b/w
Causes

Density – Col/m^2 → D

Colomb's Law ← Physical → Lorentz's Law

ϵ

Intensity
(V/m)

$\vec{E}$

$\vec{H}$ – amp/m
→ Intensity

μ

$\bar{B}$ → Weber/m^2
→ Density

Summary-4 :-

Divergence & stoke's theorem :-

→ $$\boxed{\oint D \cdot ds = Q = \int \rho_v \, dv = \int (\nabla \cdot D) \, dv}$$

This is gauss divergence theorem

→ $$\boxed{\oint H \cdot dl = I = \int J \cdot ds = \int (\nabla \times H) \cdot ds}$$

Stoke's theorem

Note :-

Wrong Notations :-

→ $\int H \cdot dl = \int (\nabla \times H) \cdot ds$ → $\int D \cdot ds = \int (\nabla \cdot D) dv$

→ $\oint H \cdot dl = \int (\nabla \cdot H) ds$ → $\oint D ds = \int (\nabla \times D) dv$

→ $\oint H \cdot dl = \oint (\nabla \times H) \cdot ds$

Note :-

Maxwell equations having two formats, 2 field, 2 (·) dot, 2 (x) cross, 2 line, 2 surface, 4 eqn

Summary 5 :-

Maxwell's Equation

Integral form		Point form
1. $\oint D \cdot ds = Q$	[Gauss Law]	1. $\nabla \cdot D = \rho_v$
2. $\oint E \cdot dl = 0$	[Irrotational Vector]	2. $\nabla \times E = 0$
3. $\oint B \cdot ds = 0$	[Solenoidal]	3. $\nabla \cdot B = 0$
4. $\oint H \cdot dl = I$	[Ampere's Law]	4. $\nabla \times H = J$

Note :-

→ $\nabla \times E = 0.$ (Always)

curl is generally applied with intensity vector

$$\nabla \times \left(\frac{D}{\epsilon}\right) = 0$$

If ϵ = constant i.e. medium is homogeneous & isotropic

$$\Rightarrow \frac{1}{\epsilon} \nabla \times D = 0 \quad \Rightarrow \boxed{\nabla \times D = 0}$$

→ Acc. to stoke's theorem (For integral form)

$$\oint E \cdot dl = \int (\nabla \times E) \cdot ds = 0$$

Note :-

i) $\nabla \cdot B = 0$ (Always)

Divergence is generally applied with density vector

$$\nabla \cdot (\mu H) = 0$$

If μ is constant i.e. medium is homogeneous and isotropic

$$\mu (\nabla \cdot H) = 0$$

$$\nabla \cdot H = 0 \longrightarrow \text{Also correct}$$

For integral form, apply divergence theorem

$$\oint B \cdot ds = \int (\nabla \cdot B)\, dv = 0$$

(ii) $\oint E \cdot dl = 0 \longrightarrow$ KVL

(iii) $\oint B \cdot ds = 0 \longrightarrow$ KCL

Coordinate Systems

It is a way of adressing/locating points in a 3d-space from a known reference

Reference :-

3 infinite mutually orthogonal planes

$\rightarrow$ XY, YZ & ZX planes

Cartesian Coordinate System :-

eg:- 1. Uniform plane waves

2. Rectangular wave guides

3. Capacitor plates

$\rightarrow$ Parameters (x, y, z)

$\rightarrow$ Unit vector (a_x, a_y, a_z)

Reference :-

| infinite line

axial — z-axis

Cylinderical coordinate system

eg:- (I) Line charges

(II) I carrying waves wires

(III) cylindrical waveguide

Parameter → ρ, ϕ, z

Unit vectors → a_ρ, a_ϕ, a_z

Reference :-

1 single point

point — origin

Spherical coordinate systems

eg:-(I) point charges

(II) antennas

Parameter → r, θ, ϕ

Unit vectors → a_r, a_θ, a_ϕ

Relation b/w coordinate systems :-

$x \rightarrow y \rightarrow z$

$\rho \rightarrow \phi \rightarrow z$

$r \rightarrow \theta \rightarrow \phi$

Note:-

All the three co-ordinates systems are unit

- orthogonal
- orthonormal
- right handed system

Orthogonal :-

The (·) dot product of any two unit vectors of the same coordinate system is always zero

$\rightarrow a_x \cdot a_y = a_y \cdot a_z = a_z \cdot a_x = 0$

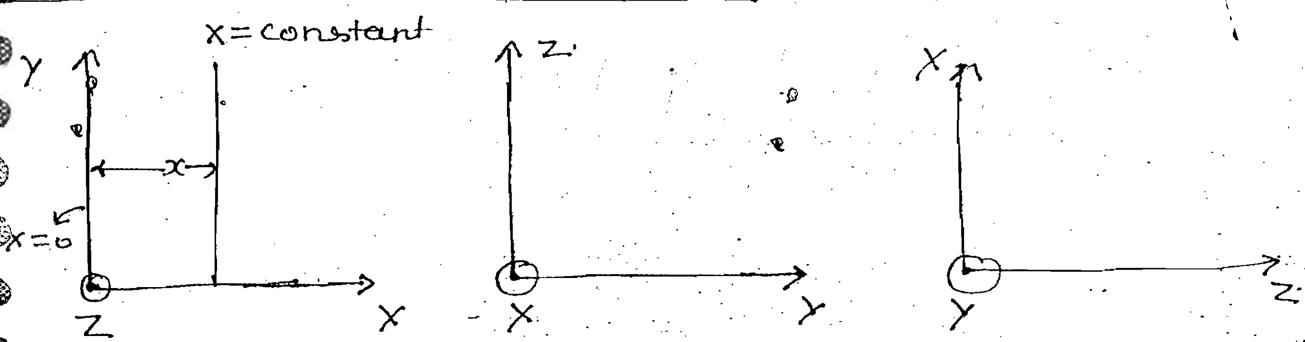
$\rightarrow a_x \cdot a_x = a_y \cdot a_y = a_z \cdot a_z = 1$

Orthonormal :-

The (x) cross product of any two unit vectors of the same co-ordinate system is always the third unit vector

$$a_x \times a_y = a_z$$
$$a_y \times a_z = a_x$$
$$a_z \times a_x = a_y$$

Cartesian Coordinate Systems!-



X $\rightarrow$ It is the shortest distance or perpendicular distance of the point from YZ plane

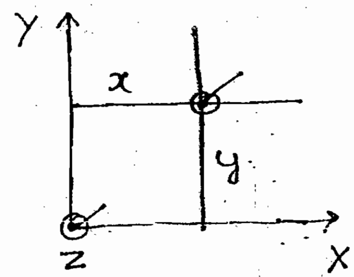
Note:-

The locus of all the points with x = constant is an infinite plane parallel to YZ plane

Range of X $\rightarrow$ $(-\infty, \infty)$

Note:-

The locus of all the points with X, Y = constant is an infinite line parallel to z-axis

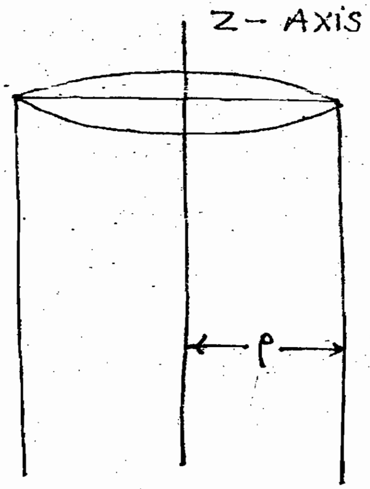


Cylindrical Coordinate System :-

ρ :$\rightarrow$ It is the shortest distance / perpendicular distance / radial ~~ay~~ distance of the point from reference line

Note:-

The locus of all the points with ρ = constant. is a concentric cylinder around a reference line

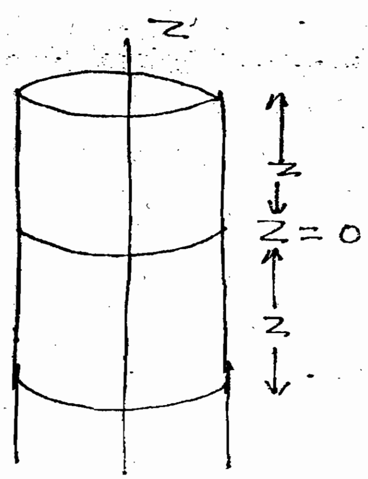


Range $\rightarrow$ $[0, \infty)$

Z :$\rightarrow$

It is the height of the point along the reference line

Range :- $(-\infty, \infty)$

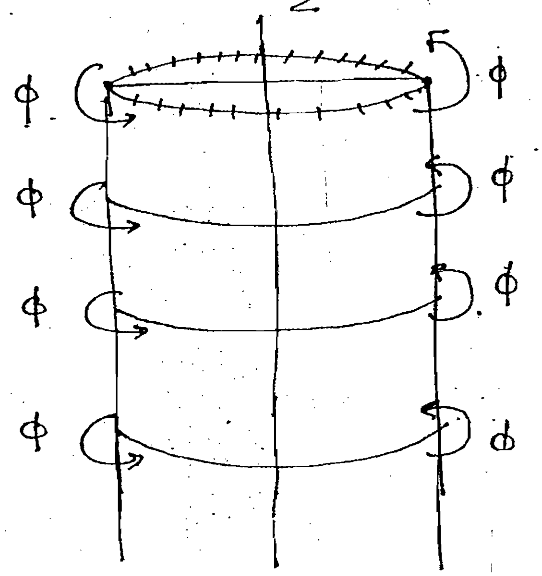


Note :-

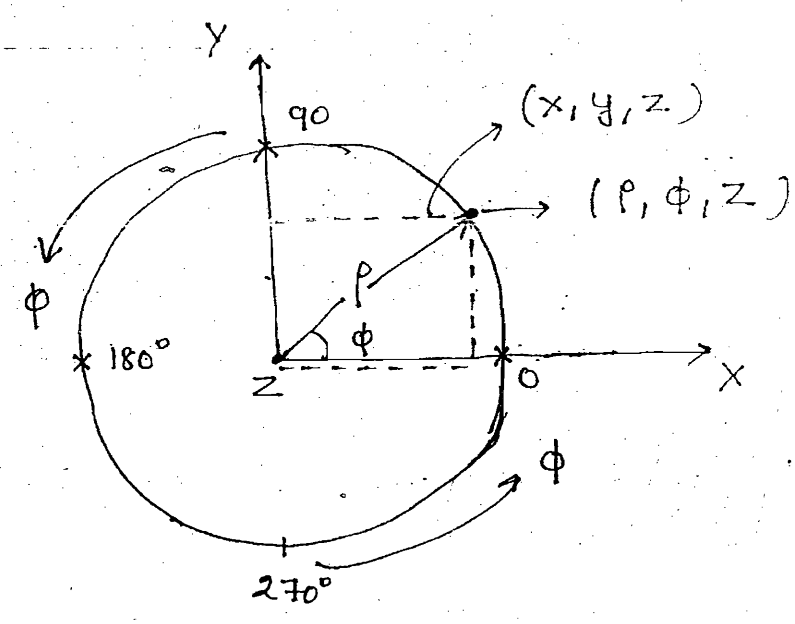
The locus of all the points with ρ, z = constant is a cocentric circle around the line

ϕ :-

It is the orientation of the point around the reference line



Range :- $[0, 2\pi)$



Point Transformation :-

Cartesian $\longleftrightarrow$ cylindrical

$$x = \rho \cos\phi$$

$$y = \rho \sin\phi \qquad \rho = \sqrt{x^2 + y^2}$$

$$z = z \qquad \phi = \tan^{-1}(y/x)$$

Spherical Coordinate Systems :-

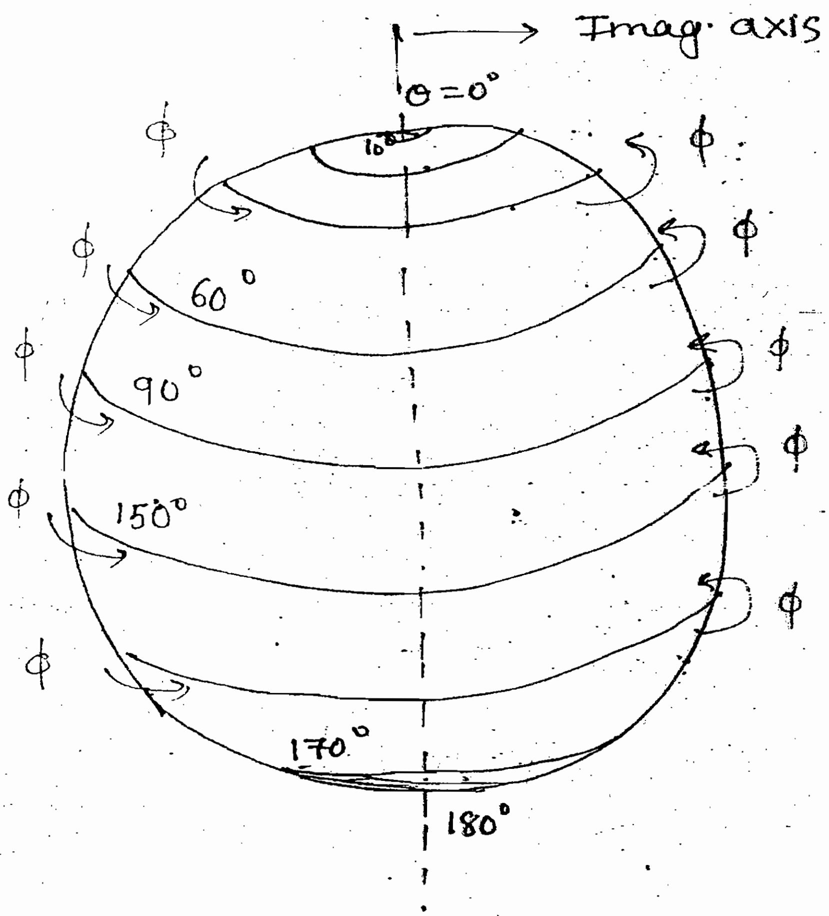
r :→

It is the radial distance or shortest distance of the point from reference point.

Note :-

The locus of the all the points with r is constant ~~as~~ is concentric sphere around the origin

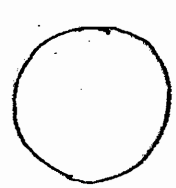
Range → $[0, \infty)$



θ :-

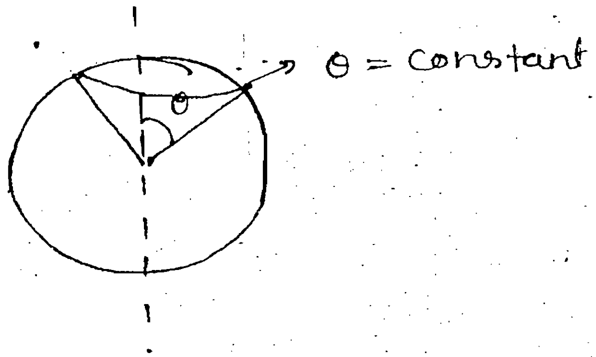
It is the angle of cone whose base is a circle on the r = constant sphere

Range → $[0, \pi]$



Identification of θ constant circle :-

Consider an imag. axis through the centre of the sphere and take a radial segment inclined by θ with imaginary axis. Rotate the radial segment which results in a cone whose base is a circle on the $r =$ constant sphere



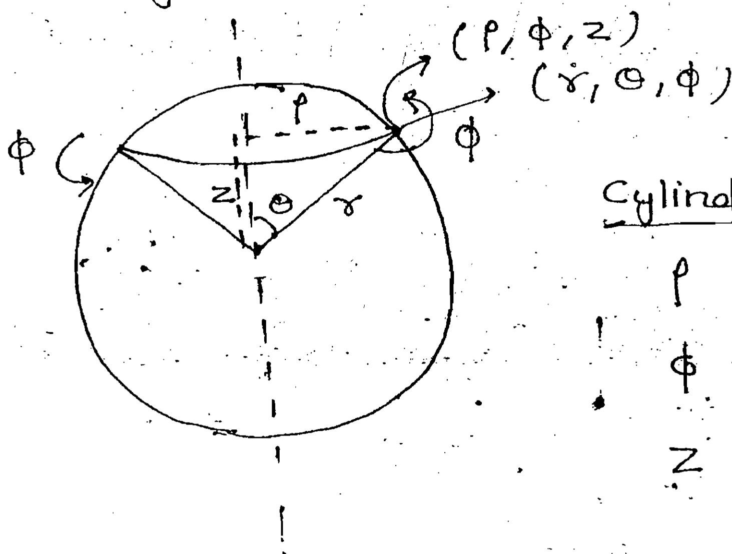
ϕ :-

It is the orientation angle around the imaginary axis

Range of $\phi \rightarrow [0, 2\pi]$

Point transformation :-

If z-axis of cylindrical coordinates coincide with imaginary axis of spherical coordinates then ϕ is same in both the coordinate system



Cylindrical $\longleftrightarrow$ Spherical

$$\rho = r\sin\theta$$

$$\phi = \phi$$

$$z = r\cos\theta$$

Point Transformation :-

Cartesian $\longleftrightarrow$ Spherical

$$x = r\sin\theta\cos\phi$$
$$y = r\sin\theta\cdot\sin\phi$$
$$z = r\cos\theta \qquad r = \sqrt{x^2+y^2+z^2}$$

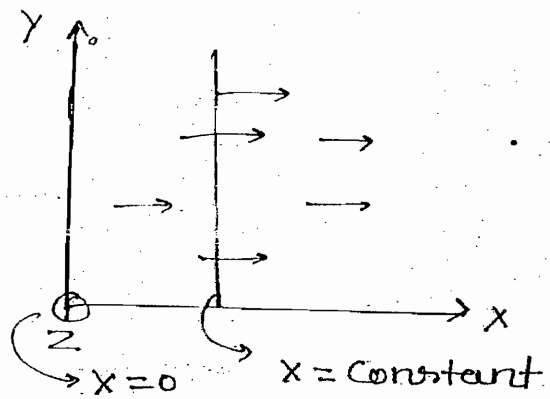
Unit Vectors & Orthogonality in coordinate system :-

Unit vector of Parameter :-

It has unit magnitude & a direction in which the parameter increases

I) Cartesian coordinate system :-

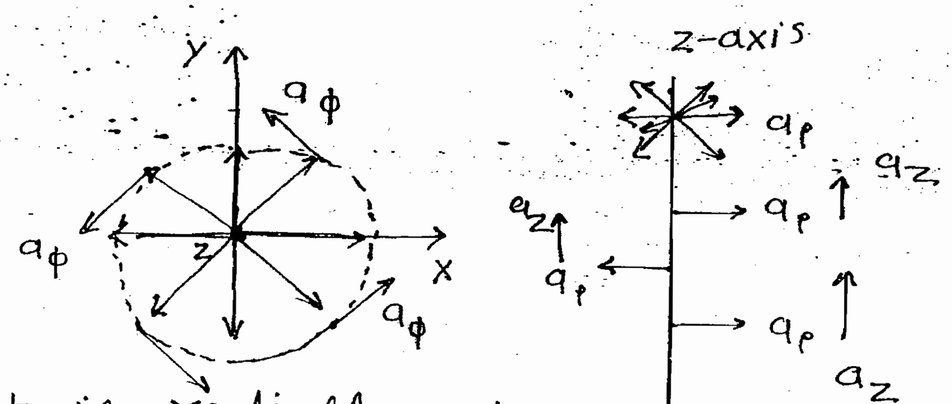
a_x :-



It is always normal to YZ plane or any plane which has X = constant

II) Cylindrical coordinate system :-

a_ρ :-



It is radially outward from the reference line

a_ϕ :-

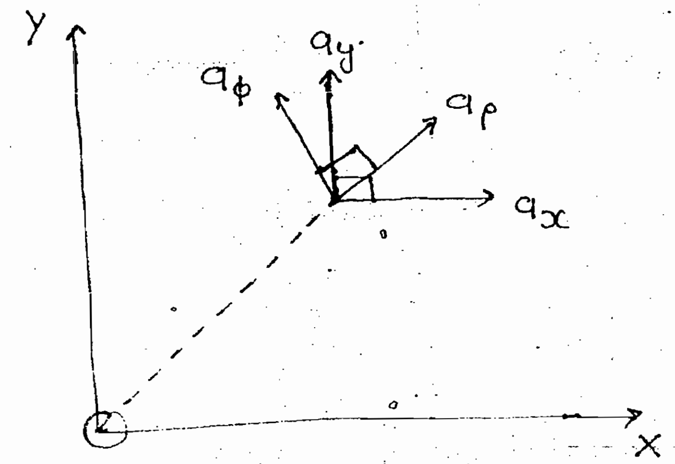
It is tangentially around the reference line

** $\boxed{a_\rho \perp a_\phi \perp a_z}$

a_z :-

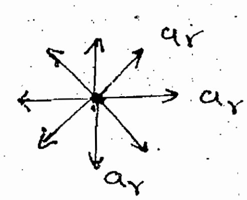
It is linearly along the reference line

Summary :-



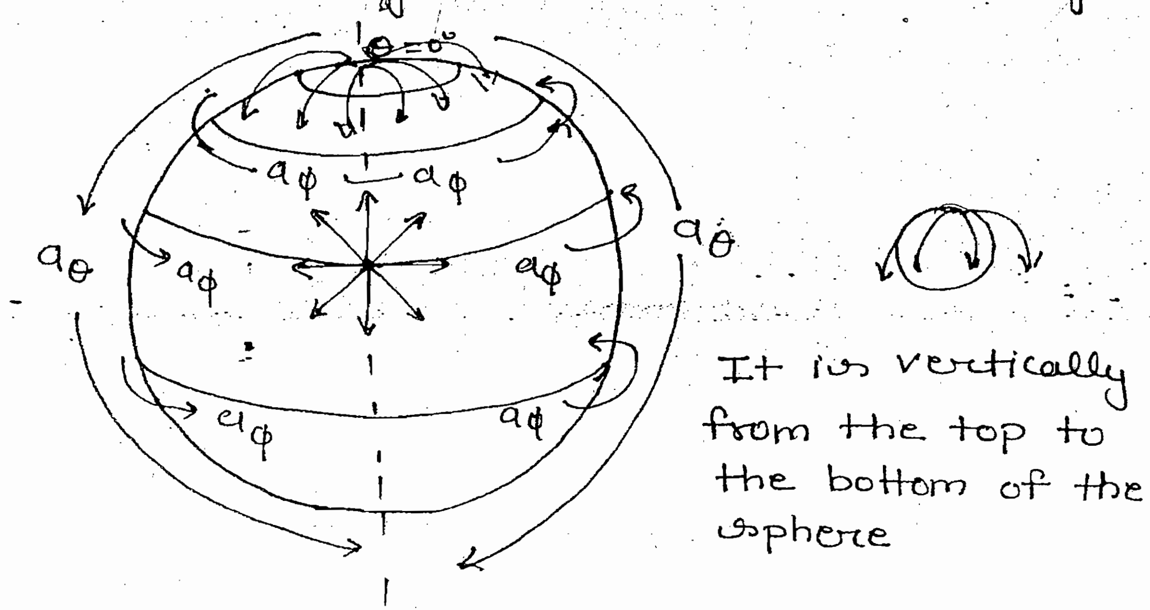
Spherical Coordinate System :-

a_r :-



It is radially outward from the origin

a_θ :-

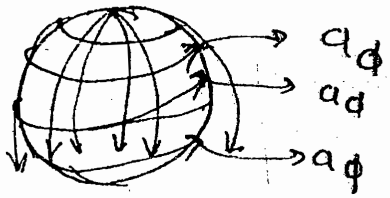


It is vertically from the top to the bottom of the sphere

It is horizontally around the imaginary axis.

ϕ = constant , $\theta \uparrow$

ϕ = constant ; $\theta \uparrow$



$$\boxed{a_r \perp a_\theta + a_\phi}$$