Long Answer Type Questions

[4 MARKS]

Que 1. Using factor theorem, factorise the polynomial $\boldsymbol{x^3 + x^2 - 4x - 4}$.

Sol. $Let\ p(x) = x^3 + x^2 - 4x - 4$.

The constant term in $p(x)$ is equal to -4 and factors of -4 are $\pm 1, \pm 2$,

Putting $x\ x = -1\ in\ p(x), we\ have$

$$p(-1) = (-1)^3 + (-1)^2 - 4 \times (-1) - 4$$

$$= -1 + 1 + 4 - 4 = 0$$

$\therefore$ $(x + 1)$ is a factor of $p(x)$

Putting $x\ \ x = 2\ in\ p(x)$, we have

$$p(2) = 2^3 + 2^2 - 4 \times 2 - 4$$

$$= -8 + 4 + 8 - 4$$

$$p(-2) = 0$$

$\therefore$ $(x + 2)$ is a factor of $p(x)$.

As $p(x)$ is a polynomial of degree 3, so it cannot have more than three linear factors.

$$\therefore \quad p(x) = k(x + 1)(x + 2)(x - 2)$$

$$x^2 + x^2 - 4x - 4 = 1(x + 1)(x + 2)(x - 2)$$

$$= (x + 1)(x + 2)(x - 2)$$

Que 2. Factorise: $\boldsymbol{x^8 - y^8}$.

Sol. $x^8 - y^8 = (x^4)^2 - (y^4)^2$

$$= (x^4 + y^4)(x^4 - y^4) \quad [Using\ a^2 - b^2 = (a + b)(a - b)]$$

$$= (x^4 + y^4)[(x^2)^2 - (y^2)^2]$$

$$= (x^4 + y^4)(x^2 + y^2)(x^2 - y^2)$$

$$= (x^4 + y^4)(x^2 + y^2)(x + y)(x - y)$$

Que 3. Factorise: $\boldsymbol{x^3 + 13x^2 + 32x + 20}$.

Sol. Let $p(x) = x^3 + 13x^2 + 32x + 20$

The constant term in is equal to 20 and the factors of 20 are $\pm 1, \pm 2, \pm 4, \pm 5, \pm 10$.

Putting $x = -2\ in\ p(x)$, we have

$$p(-2) = (-2)^3 + 13(-2)^2 + 32(-2) + 20$$

$$= -8 + 52 - 64 + 20 = -72 + 72 = 0$$

$$p(-2) = 0$$

As $p(-2) = 0, so (x + 2)$ is a factor of $p(x)$. Now, divide $p(x)$ by $(x + 2)$

$$x^2 + 11x + 10$$

$$x + 2 \overline{) x^3 + 13x^2 + 32x + 20}$$

$$\underline{{}_-x^3 \pm 2x^2}$$

$$11x^2 + 32x + 20$$

$$\underline{{}_-11x^2 \pm 22x}$$

$$10x + 20$$

$$\underline{{}_-10x \pm 20}$$

$$\underline{0}$$

$$\therefore\ p(x) = (x + 2)\ (x^2 + 11x + 10)$$

$$= (x + 2)\ [x^2 + 10x + x + 10]$$

$= (x + 2)[x(x + 10) + 1(x + 10)] = (x + 2)[(x + 10)(x + 1)]$

$= (x + 1)(x + 2)(x + 10)$

Que 4. Factorise $2x^2 - 3x^2 - 17x + 30$.

Sol. Let, $p(x) = 2x^3 - 3x^2 - 17x + 30$.

$\therefore \quad p(2) = 2 \times 2^3 - 3 \times 2^2 - 17 \times 2 + 30 = 16 - 12 - 34 + 30$

$p(2) = 46 - 46 = 0$

As $p(2) = 0$, Therefore (x – 2) is a factor of p(x)

Let us divide $p(x)\ by\ (x - 2)$ by long division method as given below:

$$
\begin{array}{r|l}
 & 2x^2 + x - 15 \\
\hline
x - 2 & 2x^3 - 3x^2 - 17x + 30 \\
 & \underline{{}_{-}2x^3 \mp 4x^2} \\
 & \qquad x^2 - 17x + 30 \\
 & \qquad \underline{{}_{-}x^2 \mp 2x} \\
 & \qquad\quad -15x + 30 \\
 & \qquad\quad \underline{\mp 15x \pm 30} \\
 & \qquad\qquad 0
\end{array}
$$

$\therefore\ p(x) = 2x^3 - 3x^2 - 17x + 30 = (x-2)(2x^2 + x - 15)$

$$= (x-2)(2x^2 + 6x - 5x - 15) = (x-2)[2x(x+3) - 5(x+3)]$$

$$= (x-2)[(x+3)(2x-5)] = (x-2)(x+3)(2x-5)$$

Que 5. If both $(x-2)$ and $\left(x - \frac{1}{2}\right)$ are factors of $px^2 + 5x + r$, Show that p = r.

Sol. $Let\ f(x) = px^2 + 5x + r,$

As $(x - 2$ is a factor of $f(x), So\ f(2) = 0$

$$p \times 2^2 + 5 \times 2 + r = 0$$

$$\Rightarrow\ 4p + 10 + r = 0 \qquad \ldots\ldots\ldots.(i)$$

Also $\left(x - \frac{1}{2}\right)$ is a factor of $f(x), so\ f(\frac{1}{2})$ = 0

$$p\left(\frac{1}{2}\right)^2 + 5.\frac{1}{2} + r = 0$$

$$\Rightarrow \frac{p}{4} + \frac{5}{2} + r = 0 \qquad \Rightarrow p + 10 + 4r = 0$$

From equations (i) and (ii), we have

$$4p + 10 + r = p + 10 + 4r$$

$$4p - p = 10 + 4r - 10 - r$$

$$\Rightarrow\ 3p = 3r \quad \Rightarrow\ p = r$$

Que 6. Without actual division, prove that $2x^4 + x^3 - 14x^2 - 19x - 6$ is exactly divisible by $x^2 + 3x + 2$.

Sol. Let $p(x) = 2x^4 + x^3 - 14x^2 - 19x - 6\ and\ q(x) = x^2 + 3x + 2$

Then, $q(x) = x^2 + 3x + 2 = x^2 + 2x + x + 2$

$= x(x+2) + 1(x+2) = (x+2)(x+1)$

Now, $p(-1) = 2(-1)^4 + (-1)^3 - 14(-1)^2 - 19(-1) - 6$

$= 2 - 1 - 14 + 19 - 6 = 21 - 21$

$p(-1) = 0$

And, $p(-2) = 2(-2)^4 + (-2)^3 - 14(-2)^2 - 19(-2) - 6$

$= 32 - 8 - 56 + 38 - 6 = 70 - 70$

$p(-2) = 0$

$\Rightarrow (x+1)\ and\ (x+2)$ are the factors of $p(x), so\ p(x)$ is divisible by $(x+1)$ and $(x+2)$.

Hence, $p(x)$ is divisible by $(x+1)(x+2) = x^2 + 3x + 2$.

Que 7. Find the value of $\frac{1}{27}r^3 - s^3 + 125t^3 + 5rst, when\ s = \frac{r}{3} + 5t$.

Sol. $\frac{1}{27}r^3 - s^3 + 125t^3 + 5rst$

$= \frac{1}{3^3}r^3 + (-s)^3 + 5^3t^3 + 5rst = \left(\frac{r}{3}\right)^3 + (-s)^3 + (5t)^3 - 3\left(\frac{r}{3}\right)(-s)(5t)$

$= \left(\frac{r}{3} + (-s) + 5t\right)\left[\left(\frac{r}{3}\right)^2 + (-s)^2 + (5t)^2 - \frac{r}{3}.(-s) - (-s)(5t) - \frac{r}{3}(5t)\right]$

$= \left(\frac{r}{3} - s + 5t\right)\left(\frac{r2}{9} + s^2 + 25t^2 + \frac{rs}{3} + 5st - \frac{5rt}{3}\right)$

Now, $s = \frac{r}{3} + 5t$ (Given) $\Rightarrow \frac{r}{3} - s + 5t = 0$

$\therefore \frac{1}{27}r^3 - s^3 + 125t^3 + 5rst = 0 \times \left(\frac{r^2}{9} + s^2 + 25t^2 + \frac{rs}{3} + 5st - \frac{5rt}{3}\right) = 0$