5. Arithmetic Progression

Exercise 5.1

1 A. Question

Find the first term a and the common difference d for the following arithmetic progressions -

6, 9, 12, 15, ……

Answer

Comparing the given arithmetic progression with general form,We get, a = 6and common difference = difference of two consecutive termi.e. 9 – 6 = 3, 12 – 9 = 3.

Hence, First term a = 6 and common arithmetic difference = 3.

1 B. Question

Find the first term a and the common difference d for the following arithmetic progressions -

-7, -9, -11, -13, ……

Answer

Comparing the given arithmetic progression with general form,We get, a = -7and common difference = difference of two consecutive termi.e. (–9) – (-7) = 2, (-11) – (-9) = -2.

Hence, First term a = -7 and common arithmetic difference = -2.

1 C. Question

Find the first term a and the common difference d for the following arithmetic progressions -

$$\frac{3}{2}, \frac{1}{2}, \frac{-1}{2}, \frac{-3}{2}, \ldots\ldots$$

Answer

Comparing the given arithmetic progression with general form,We get, a = $\frac{3}{2}$ and common difference = difference of two consecutive termi.e.

$$\frac{1}{2} - \frac{3}{2} = -\frac{2}{2} = -1, \left(-\frac{1}{2}\right) - \frac{1}{2} = \frac{-1-1}{2} = -\frac{2}{2} = -1$$

Hence, First term a = $\frac{3}{2}$ and common arithmetic difference = -1.

1 D. Question

Find the first term a and the common difference d for the following arithmetic progressions -

1, -2, -5, -8, ……

Answer

Comparing the given arithmetic progression with general form,We get, a = 1and common difference = difference of two consecutive termi.e. (−2) − (1) = -3, (-5) − (-2) = -3.

Hence, First term a = 1 and common arithmetic difference = -3.

1 E. Question

Find the first term a and the common difference d for the following arithmetic progressions -

$$-1, \frac{1}{4}, \frac{3}{2}, \ldots.$$

Answer

Comparing the given arithmetic progression with general form,We get, a = -1and common difference = difference of two consecutive termi.e.

$$\frac{1}{4} - (-1) = \frac{1}{4} + 1 = \frac{1+4}{4} = \frac{5}{4}, \frac{3}{2} - \frac{1}{4} = \frac{6}{4} - \frac{1}{4} = \frac{6-1}{4} = \frac{5}{4}$$

Hence, First term a = -1 and common arithmetic difference = $\frac{5}{4}$.

1 F. Question

Find the first term a and the common difference d for the following arithmetic progressions -

3, 1, -1, -3, ……

Answer

Comparing the given arithmetic progression with general form,We get, a = 3and common difference = difference of two consecutive termi.e. (1) − (3) = -2, (-1) − (1) = -2.

Hence, First term a = 3 and common arithmetic difference = -2.

1 G. Question

Find the first term a and the common difference d for the following arithmetic progressions -

3, -2, -7, -12, ……

Answer

Comparing the given arithmetic progression with general form,We get, a = 3and common difference = difference of two consecutive termi.e. (−2) − (3) = -5, (-7) − (-2) = -5.

Hence, First term a = 3 and common arithmetic difference = -5.

2 A. Question

If for any arithmetic progression first term a and the common difference d are graph as follows, then write first four terms of that progression-

a = -1, $d = \frac{1}{2}$

Answer

Let the first four terms of arithmetic progression be a_1, a_2, a_3, a_4Given, First term a = -1 and common difference d = $\frac{1}{2}$.⇒ a_1 = a = -1.⇒ $a_2 = a_1 + d = -1 + \frac{1}{2} = -\frac{2}{2} + \frac{1}{2} = \frac{-2+1}{2} = -\frac{1}{2}$.⇒ $a_3 = a_2 + d = -\frac{1}{2} + \frac{1}{2} = \frac{-1+1}{2} = 0$.⇒ $a_4 = a_3 + d = 0 + \frac{1}{2} = \frac{1}{2}$.

Hence, First four terms are $-1, -\frac{1}{2}, 0, \frac{1}{2}$.

2 B. Question

If for any arithmetic progression first term a and the common difference d are graph as follows, then write first four terms of that progression-

$a = \frac{1}{3}, \ d = \frac{4}{3}$

Answer

Let the first four terms of arithmetic progression be a_1, a_2, a_3, a_4Given, First term a = $\frac{1}{3}$ and common difference d = $\frac{4}{3}$.⇒ a_1 = a = $\frac{1}{3}$.⇒ $a_2 = a_1 + d = \frac{1}{3} + \frac{4}{3} = \frac{1+4}{3} = \frac{5}{3}$.⇒ $a_3 = a_2 + d = \frac{5}{3} + \frac{4}{3} = \frac{5+4}{3} = \frac{9}{3}$.⇒ $a_4 = a_3 + d = 3 + \frac{4}{3} = \frac{9}{3} + \frac{4}{3} = \frac{9+4}{3} = \frac{13}{3}$.

Hence, First four terms are $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3}$.

2 C. Question

If for any arithmetic progression first term a and the common difference d are graph as follows, then write first four terms of that progression-

a = 0.6, d = 1.1

Answer

Let the first four terms of arithmetic progression be a_1, a_2, a_3, a_4Given, First term a = 0.6 and common difference d =1.1 .⇒ $a_1 = a = 0.6$⇒ $a_2 = a_1 + d = 0.6 + 1.1 = 1.7$⇒ $a_3 = a_2 + d = 1.7 + 1.1 = 2.8$⇒ $a_4 = a_3 + d = 2.8 + 1.1 = 3.9$

Hence, First four terms are 0.6, 1.7, 2.8, 3.9.

2 D. Question

If for any arithmetic progression first term a and the common difference d are graph as follows, then write first four terms of that progression-

a = 4, d = -3

Answer

Let the first four terms of arithmetic progression be a_1, a_2, a_3, a_4Given, First term a = 4 and common difference d = -3.⇒ $a_1 = a = 4$⇒ $a_2 = a_1 + d = 4 + (-3) = 1$⇒ $a_3 = a_2 + d = 1 + (-3) = -2$⇒ $a_4 = a_3 + d = (-2) + (-3) = -5$

Hence, First four terms are 4, 1, -2, -5.

2 E. Question

If for any arithmetic progression first term a and the common difference d are graph as follows, then write first four terms of that progression-

a = 11, d = -4

Answer

Let the first four terms of arithmetic progression be a_1, a_2, a_3, a_4Given, First term a = 11 and common difference d = -4.⇒ $a_1 = a = 11$⇒ $a_2 = a_1 + d = 11 + (-4) = 7$⇒ $a_3 = a_2 + d = 7 + (-4) = 3$⇒ $a_4 = a_3 + d = 3 + (-4) = -1$

Hence, First four terms are 11, 7, 3, -1.

2 F. Question

If for any arithmetic progression first term a and the common difference d are graph as follows, then write first four terms of that progression-

a = -1.25, d = -0.25

Answer

Let the first four terms of arithmetic progression be a_1, a_2, a_3, a_4Given, First term a = -1.25 and common difference d = -0.25.⇒ a_1 = a = -1.25⇒ a_2 = a_1 + d = -1.25 + (-0.25) = -1.5⇒ a_3 = a_2 + d = -1.5 + (-0.25) = -1.75⇒ a_4 = a_3 + d = -1.75 + (-0.25) = -2.00

Hence, First four terms are -1.25, -1.5, -1.75, -2.00.

2 G. Question

If for any arithmetic progression first term a and the common difference d are graph as follows, then write first four terms of that progression-

a = 20, $d = \frac{-3}{4}$

Answer

Let the first four terms of arithmetic progression be a_1, a_2, a_3, a_4Given, First term a = 20 and common difference d = $-\frac{3}{4}$.⇒ a_1 = a = 20⇒ a_2 = a_1 + d = $20 - \frac{3}{4} = \frac{80}{4} - \frac{3}{4} = \frac{80-3}{4} = \frac{77}{4}$⇒ a_3 = a_2 + d = $\frac{77}{4} - \frac{3}{4} = \frac{77-3}{4} = \frac{74}{4}$⇒ a_4 = a_3 + d = $\frac{74}{4} - \frac{3}{4} = \frac{74-3}{4} = \frac{71}{4}$

Hence, First four terms are $20, \frac{77}{4}, \frac{74}{4}, \frac{71}{4}$.

3 A. Question

Check for an arithmetic progression in the following lists of numbers of any one of these as an arithmetic progression, then find its common difference and also write its next four terms—

$2, \frac{5}{2}, 3, \frac{7}{2}, \ldots.$

Answer

Let the arithmetic progression be a_1, a_2, a_3, a_4 ……. So here$a_2 - a_1 = \frac{5}{2} - 2 = \frac{5}{2} - \frac{4}{2} = \frac{5-4}{2} = \frac{1}{2}$$a_3 - a_2 = 3 - \frac{5}{2} = 3 - \frac{5}{2} = \frac{6}{2} - \frac{5}{2} = \frac{6-5}{2} = \frac{1}{2}$$a_4 - a_3 = \frac{7}{2} - 3 = \frac{7}{2} - \frac{6}{2} = \frac{7-6}{2} = \frac{1}{2}$For each term difference is same $\frac{1}{2}$. So this sequence is an arithmetic progression and its common difference is d = $\frac{1}{2}$.

Next four terms are :⇒ a_5 = a_4 + d = $\frac{7}{2} + \frac{1}{2} = \frac{7+1}{2} = \frac{8}{2} = 4$⇒ a_6 = a_5 + d

$= 4 + \frac{1}{2} = \frac{8}{2} + \frac{1}{2} = \frac{9}{2} \Rightarrow a_7 = a_6 + d = \frac{9}{2} + \frac{1}{2} = \frac{9+1}{2} = \frac{10}{2} = 5 \Rightarrow a_8 = a_7 + d = 5 + \frac{1}{2} = \frac{10}{2} + \frac{1}{2} = \frac{10+1}{2} = \frac{11}{2}$

Hence, Yes, $d = \frac{1}{2}$, Next four terms are $4, \frac{9}{2}, 5, \frac{11}{2}$.

3 B. Question

Check for an arithmetic progression in the following lists of numbers of any one of these as an arithmetic progression, then find its common difference and also write its next four terms—

$$\frac{-1}{2}, \frac{-1}{2}, \frac{-1}{2}, \frac{-1}{2}, \ldots\ldots$$

Answer

Let the arithmetic progression be a_1, a_2, a_3, a_4 ……. So here$a_2 - a_1 = -\frac{1}{2} - \left(-\frac{1}{2}\right) = -\frac{1}{2} + \frac{1}{2} = 0 a_3 - a_2 = -\frac{1}{2} - \left(-\frac{1}{2}\right) = -\frac{1}{2} + \frac{1}{2} = 0 a_4 - a_3 = -\frac{1}{2} - \left(-\frac{1}{2}\right) = -\frac{1}{2} + \frac{1}{2} = 0$

For each term difference is same 0. So this sequence is an arithmetic progression and its common difference is d = 0. Next four terms are :$\Rightarrow a_5 = a_4 + d = -\frac{1}{2} + 0 = -\frac{1}{2} \Rightarrow a_6 = a_5 + d = -\frac{1}{2} + 0 = -\frac{1}{2} \Rightarrow a_7 = a_6 + d = -\frac{1}{2} + 0 = -\frac{1}{2} \Rightarrow a_8 = a_7 + d = -\frac{1}{2} + 0 = -\frac{1}{2}$

Hence, Yes, d = 0, Next four terms are $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}$.

3 C. Question

Check for an arithmetic progression in the following lists of numbers of any one of these as an arithmetic progression, then find its common difference and also write its next four terms—

$a, a^2, a^3, a^4, \ldots..$

Answer

Let the arithmetic progression be a_1, a_2, a_3, a_4 ……. So here$a_2 - a_1 = a^2 - a = a(a - 1) a_3 - a_2 = a^3 - a^2 = a^2(a - 1) a_4 - a_3 = a^4 - a^3 = a^3(a - 1)$Here common difference in not same. So this sequence is not an arithmetic progression.

Hence, No.

3 D. Question

Check for an arithmetic progression in the following lists of numbers of any one of these as an arithmetic progression, then find its common difference and also write its next four terms—

$\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}, \ldots\ldots$

Answer

Let the arithmetic progression be a_1, a_2, a_3, a_4 ……. So here$a_2 - a_1 = \sqrt{6} - \sqrt{3} = \sqrt{3}(\sqrt{2}) - \sqrt{3} = \sqrt{3}(\sqrt{2} - 1)a_3 - a_2 = \sqrt{9} - \sqrt{6} = \sqrt{3}(\sqrt{3}) - \sqrt{3}(\sqrt{2}) = \sqrt{3}(\sqrt{3} - \sqrt{2})a_4 - a_3 = \sqrt{12} - \sqrt{9} = \sqrt{3}(\sqrt{4}) - \sqrt{3}(\sqrt{3}) = \sqrt{3}(\sqrt{4} - \sqrt{3}) = \sqrt{3}(2 - \sqrt{3})$

Here common difference in not same. So this sequence is not an arithmetic progression.

Hence, No.

3 E. Question

Check for an arithmetic progression in the following lists of numbers of any one of these as an arithmetic progression, then find its common difference and also write its next four terms—

$\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, \ldots\ldots$

Answer

Let the arithmetic progression be a_1, a_2, a_3, a_4 ……. So here$a_2 - a_1 = \sqrt{8} - \sqrt{2} = \sqrt{2}(\sqrt{4}) - \sqrt{2} = 2\sqrt{2} - \sqrt{2} = \sqrt{2}a_3 - a_2 = \sqrt{18} - \sqrt{8} = \sqrt{2}(\sqrt{9}) - \sqrt{2}(\sqrt{4}) = 3\sqrt{2} - 2\sqrt{2} = \sqrt{2}a_4 - a_3 = \sqrt{32} - \sqrt{18} = \sqrt{2}(\sqrt{16}) - \sqrt{2}(\sqrt{9}) = 4\sqrt{2} - 3\sqrt{2} = \sqrt{2}$

For each term difference is same $\sqrt{2}$. So this sequence is an arithmetic progression and its common difference is d =$\sqrt{2}$. Next four terms are :⇒ $a_5 = a_4 + d = \sqrt{32} + \sqrt{2} = 4\sqrt{2} + \sqrt{2} = 5\sqrt{2} = \sqrt{25}(\sqrt{2}) = \sqrt{50}$⇒ $a_6 = a_5 + d = \sqrt{50} + \sqrt{2} = 5\sqrt{2} + \sqrt{2} = 6\sqrt{2} = \sqrt{36}(\sqrt{2}) = \sqrt{72}$⇒ $a_7 = a_6 + d = \sqrt{72} + \sqrt{2} = 6\sqrt{2} + \sqrt{2} = 7\sqrt{2} = \sqrt{49}(\sqrt{2}) = \sqrt{98}$⇒ $a_8 = a_7 + d = \sqrt{98} + \sqrt{2} = 7\sqrt{2} + \sqrt{2} = 8\sqrt{2} = \sqrt{64}(\sqrt{2}) = \sqrt{128}$

Hence, Yes, d = $\sqrt{2}$, Next four terms are $\sqrt{50}$, $\sqrt{72}$, $\sqrt{98}$, $\sqrt{128}$.

3 F. Question

Check for an arithmetic progression in the following lists of numbers of any one of these as an arithmetic progression, then find its common difference and also write its next four terms— a, 2a, 3a, 4a ….

Answer

Let the arithmetic progression be a_1, a_2, a_3, a_4 ……. So here$a_2 - a_1 = 2a - a = aa_3 - a_2 = 3a - 2a = aa_4 - a_3 = 4a - 3a = a$For each term difference is same i.e., a. So this sequence is anarithmetic progression and its common difference is

d = a. Next four terms are :⇒ $a_5 = a_4 + d = 4a + a = 5a$⇒ $a_6 = a_5 + d = 5a + a = 6a$⇒ $a_7 = a_6 + d = 6a + a = 7a$⇒ $a_8 = a_7 + d = 7a + a = 8a$

Hence, Yes, d = a, Next four terms are 5a, 6a, 7a, 8a.

3 G. Question

Check for an arithmetic progression in the following lists of numbers of any one of these as an arithmetic progression, then find its common difference and also write its next four terms—

0.2, 0.22, 0.222, ….

Answer

Let the arithmetic progression be a_1, a_2, a_3, a_4 ……. So here$a_2 - a_1 = 0.22 - 0.2 = 0.02a_3 - a_2 = 0.222 - 0.22 = 0.002$

For each term common difference is not same. So this sequence is not an arithmetic progression.

Hence, No

3 H. Question

Check for an arithmetic progression in the following lists of numbers of any one of these as an arithmetic progression, then find its common difference and also write its next four terms—

$$3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2}, \ldots..$$

Answer

Let the arithmetic progression be a_1, a_2, a_3, a_4 ……. So here$a_2 - a_1 = (3+\sqrt{2}) - (3) = \sqrt{2}a_3 - a_2 = (3+2\sqrt{2}) - (3+\sqrt{2}) = \sqrt{2}a_4 - a_3 = (3+3\sqrt{2}) - (3+2\sqrt{2}) = \sqrt{2}$

For each term difference is same $\sqrt{2}$. So this sequence is an arithmetic progression and its common difference is d =$\sqrt{2}$. Next four terms are :⇒ $a_5 = a_4 + d = 3+3\sqrt{2} + \sqrt{2} = 3+4\sqrt{2}$⇒ $a_6 = a_5 + d = 3+4\sqrt{2} + \sqrt{2} = 3+5\sqrt{2}$ ⇒ $a_7 = a_6 + d = 3+5\sqrt{2} + \sqrt{2} = 3+6\sqrt{2}$⇒ $a_8 = a_7 + d = 3+6\sqrt{2} + \sqrt{2} = 3+7\sqrt{2}$

Hence, Yes, d = $\sqrt{2}$, Next four terms are $3+4\sqrt{2}$, $3+5\sqrt{2}$, $3+6\sqrt{2}$, $3+7\sqrt{2}$.

Exercise 5.2

1 A. Question

Find out

10^{th} term of arithmetic progression 2, 7, 12, ….

Answer

The given arithmetic progression is 2, 7, 12, ...Its first term a = 2Common difference = d = 7 – 2 = 5.So n^{th} term a_n of the given arithmetic progression is given bya_n = a + (n -1)d

Where,

a = First term of APd = Common difference of APand no of terms is 'n'

Thus 10^{th} term a_{10} = 2 + (10 - 1) × 5.⇒ a_{10} = 2 + 9 × 5 = 2 + 45 = 47

Hence, 10^{th} term is 47.

1 B. Question

Find out

18^{th} term of arithmetic progression $\sqrt{2}, 3\sqrt{2}, 5\sqrt{2}, ...$

Answer

The given arithmetic progression is √2, 3√2, 5√2, ...Its first term a = √2Common difference = d = 3√2 – √2 = 2√2.So n^{th} term a_n of the given arithmetic progression is given bya_n = a + (n -1)d

Where,

a = First term of APd = Common difference of APand no of terms is 'n'

Thus 18^{th} term a_{18} = √2 + (18 - 1) × 2√2.⇒ a_{18} = √2 + 17 × 2√2 = √2 + 34√2 = 35√2

Hence, 18^{th} term is 35√2.

1 C. Question

Find out

24^{th} term of arithmetic progression 9, 13, 17, 21, ...

Answer

The given arithmetic progression is 9, 13, 17, 21, ...Its first term a = 9Common difference = d = 13 – 9 = 4.So n^{th} term a_n of the given arithmetic progression is given bya_n = a + (n -1)d

Where,

a = First term of APd = Common difference of APand no of terms is 'n'

Thus 24^{th} term a_{24} = 9 + (24 - 1) × 4.⇒ a_{24} = 9 + 23 × 4 = 9 + 92 = 101

Hence, 24^{th} term is 101.

2 A. Question

Solve :

Which term of the arithmetic progression 21, 18, 15,... is -81 ?

Answer

Let the n^{th} term of the progression be -81.First term a = 21 and common difference = 18 – 21 = -3Since, $a_n = a + (n-1)d$∴ -81 = 21 + (n – 1) × (-3)⇒ -81 = 21 -3n -3(-1)⇒ -81 = 21 -3n + 3⇒ -81- 24 = -3n⇒ -105 = -3n

⇒ n = 35

Hence, -81 is the 35^{th} term of the progression.

2 B. Question

Solve :

Which term of the arithmetic progression 84, 80, 76 ... is zero ?

Answer

Let the n^{th} term of the progression be 0.First term a = 84 and common difference = 80 – 84 = -4Since, $a_n = a + (n-1)d$

Where,

a = First term of APd = Common difference of APand no of terms is 'n'

∴ 0 = 84 + (n – 1) × (-4)⇒ 0 = 84 -4n -4(-1)⇒ 0 = 84 -4n + 4⇒ -88 = -4n ⇒ n = 22

Hence, Zero is the 22^{nd} term of the progression.

2 C. Question

Solve :

Is 301 any term of the sequence of numbers 5, 11, 17, 23... ?

Answer

The arithmetic progression is 5, 11, 17, 23, ...Here, First term a = 5 and common difference = 11 – 5 = 6Let us assume that n^{th} term a_n of the progression is 301.Since, $a_n = a + (n-1)d$

Where,

a = First term of APd = Common difference of APand no of terms is 'n'

∴ 301 = 5 + (n – 1) × (6)⇒ 301 = 5 + 6n -6⇒ 302 = 6n⇒ n = 50.33Here n is a fraction, But any term n should be an integer.∴ 301 is not any term of the

arithmetic progression.

Hence, 301 is not any term of the progression.

2 D. Question

Solve :

Is -150 a term of the arithmetic progression 11, 8, 5, 2, …. ?

Answer

The arithmetic progression is 11, 8, 5, 2, ….Here, First term a = 11 and common difference = 8 – 11 = -3Let us assume that n^{th} term a_n of the progression is -150.Since, $a_n = a + (n -1)d$

Where,

a = First term of APd = Common difference of APand no of terms is 'n'

$\therefore$ -150 = 11 + (n – 1) × (-3)$\Rightarrow$ -150 = 11 - 3n + 3$\Rightarrow$ -150 -14 = -3n$\Rightarrow$ -164 = -3n$\Rightarrow$ n = 54.667Here n is a fraction, But any term n should be an integer.$\therefore$ -150 is not any term of the arithmetic progression.

Hence, -150 is not any term of the progression.

3. Question

The sixth term and the 17^{th} term of the arithmetic progression are 19 and 41 respectively. Then find the 40^{th} term.

Answer

Let first term be a and common difference be d.We know that, $a_n = a + (n - 1)d$ $\therefore a_6 = a + (6 - 1)d \Rightarrow 19 = a + 5d$ …(i)and $a_{17} = a + (17 - 1)d \Rightarrow 41 = a + 16d$ … (ii)

On solving eq. (i) and (ii), we get,a = 9 and d = 2

$\Rightarrow a_{40} = a + (40 - 1)d \Rightarrow a_{40} = 9 + (40 - 1) \times 2 \Rightarrow a_{40} = 9 + 39 \times 2 = 9 + 78 = 87$

Hence, 40^{th} term is 87.

4. Question

The third and ninth terms of an arithmetic progression are 4 and -8 respectively. Which term of this will be zero?

Answer

Let first term be a and common difference be d.We know that, $a_n = a + (n - 1)d$

Where,

a = First term of APd = Common difference of APand no of terms is ‘n’

$\therefore a_3 = a + (3 - 1)d \Rightarrow 4 = a + 2d$ ...(i)and $a_9 = a + (9 - 1)d \Rightarrow -8 = a + 8d$ ...(ii)

On solving eq. (i) and (ii), we get,a = 8 and d = -2

Let the n^{th} term be zero.$\Rightarrow a_n = a + (n - 1)d \Rightarrow 0 = 8 + (n - 1) \times (-2) \Rightarrow 0 = 8 - 2n$ $-2(-1) \Rightarrow 0 = 8 - 2n + 2 \Rightarrow -10 = -2n \Rightarrow n = 5$

Hence, 5^{th} will be zero.

5. Question

The third term of an arithmetic progression is 16 and the 7^{th} term is 12 more than the 5^{th} term. Find the arithmetic progression.

Answer

Let first term be a and common difference be d.We know that, $a_n = a + (n - 1)d$

Where,

a = First term of APd = Common difference of APand no of terms is ‘n’

$\therefore a_3 = a + (3 - 1)d \Rightarrow 16 = a + 2d$ ...(i)and $a_7 = a + (7 - 1)d \Rightarrow a_7 = a + 6d$ ...(ii)and $a_5 = a + (5 - 1)d \Rightarrow a_5 = a + 4d$ ...(iii)

Given, $a_7 = a_5 + 12 \Rightarrow a + 6d = a + 4d + 12$ (From (ii) and (iii))

$\Rightarrow 6d - 4d = 12 \Rightarrow 2d = 12 \Rightarrow d = 6$Substituting the value of d in eq. (i), we get,$16 = a + 2(6) \Rightarrow a = 16 - 12 = 4$

Hence first term a = 4 and common difference d = 6$\therefore a_1 = 4 \Rightarrow a_2 = a_1 + d = 4 + 6 = 10 \Rightarrow a_3 = a_2 + d = 10 + 6 = 16 \Rightarrow a_4 = a_3 + d = 16 + 6 = 22$

Hence, The arithmetic progression is 4, 10, 16, 2.

6. Question

How many three digit numbers are divisible by 7?

Answer

Smallest 3-digit number that is divisible by 7 is 105 and greatest 3-digit number that is divisible by 7 is 994.Let the no. of three digits numbers that are divisible by 7 be n.

Thus, $a_n = 994$, first term a = 105 and common difference d = 7.

Since, $a_n = a + (n - 1)d$

Where,

a = First term of APd = Common difference of APand no of terms is ‘n’

$\Rightarrow 994 = 105 + (n - 1) \times 7 \Rightarrow 994 - 105 = 7 \times (n - 1) \Rightarrow 889 = 7 \times (n - 1) \Rightarrow 127 = n - 1 \Rightarrow n = 128$

Hence, Total 3-digit numbers that are divisible by 7 are 128.

7. Question

Find the 11^{th} term from the last term of the arithmetic progression 10, 7, 4, …., -62.

Answer

Here last term of the progression l = -62.First term a = 10 and common difference = 7 – 10 = -3We know, In an AP, the nth term from last is

$a_n = l - (n - 1)d$

where a_n is nth term from last, l is last term and ‘d’ is common difference.

Thus 11^{th} term from the end= $l - (11 - 1)d = -62 - (10) \times (-3) = -62 + 30 = -32$

Hence, 11^{th} term from the last is -32.

8. Question

Find the 8^{th} term from end of the arithmetic progression 1, 4, 7, 10, …., 88.

Answer

Here last term of the progression l = 88.First term a = 1 and common difference = 4 – 1 = 3We know, In an AP, the nth term from last is

$a_n = l - (n - 1)d$

where a_n is nth term from last, l is last term and ‘d’ is common difference.

Thus 8^{th} term from the end= $l - (8 - 1)d = 88 - (7) \times (3) = 88 - 21 = 67$

Hence, 8^{th} term from the last is 67.

9. Question

There are 60 terms in an arithmetic progression. If its first and last term are respectively 7 and 125, then find its 32^{th} term.

Answer

Given, a = 7 and $a_{60} = 125$.$\therefore a_n = a + (n - 1)d$

Where,

a = First term of APd = Common difference of APand no of terms is ‘n’

$\Rightarrow a_{60} = 7 + (60 - 1)d \Rightarrow 125 = 7 + (59)d \Rightarrow 118 = 59 \times d \Rightarrow d = 2$

Now 32^{th} term, $a_{32} = a + (32 - 1)d$

$= 7 + 31 \times 2 = 7 + 62 = 69$

Hence, 32^{th} term is 69.

10. Question

Four numbers are an arithmetic progression. If the sum of the numbers is 50 and the greatest number is four times the smallest number, then find the numbers.

Answer

Let the first number be a and common difference be d, Then arithmetic progression be a, a+d, a+2d, a+3d.

Given, a + a+d + a+2d + a+3d = 50$\Rightarrow$ 4a + 6d = 50 ...(i)Also, a + 3d = 4(a)$\Rightarrow$ 3d = 3a$\Rightarrow$ a = d ...(ii)

Solving eq. (i) and (ii), we get,a = 5 and d = 5.

Hence the numbers are :$a_1 = 5a_2 = a + d = 5 + 5 = 10a_3 = a + 2d = 5 + 2(5) = 5 + 10 = 15a_4 = a + 3d = 5 + 3(5) = 5 + 15 = 20$.

Hence, The numbers are 5, 10, 15, 20.

Exercise 5.3

1 A. Question

Find the sum of the following arithmetic progressions:

1, 3, 5, 7, ... upto 12 terms

Answer

The arithmetic progression 1, 3, 5, 7, ... is given.Here first term is 1 and common difference is 3 – 1 = 2.Since the sum of n terms is$S_n = \frac{n}{2}[2a + (n-1)d]$

Where,

a = First term of APd = Common difference of APand no of terms is 'n'

So sum of first 12 terms $S_{12} = \frac{12}{2}[2 \times 1 + (12-1) \times 2 = 6[2 + 11 \times 2] = 6[2 + 22] = 6 \times 24 = 144$

Hence, Sum of first 12 terms is 144.

1 B. Question

Find the sum of the following arithmetic progressions:

8, 3, -2, … upto 22 terms.

Answer

The arithmetic progression 8, 3, -2, …. is given.Here first term is 8 and common difference is 3 – 8 = -5.Since the sum of n terms is$S_n = \frac{n}{2}[2a + (n-1)d]$

Where,

a = First term of APd = Common difference of APand no of terms is 'n'

So sum of first 22 terms $S_{22} = \frac{22}{2}[2 \times 8 + (22-1) \times (-5)]$= 11[16 + 21×(-5)]= 11[16 - 105]= 11 × (-89)= -979

Hence, Sum of first 22 terms is -979.

1 C. Question

Find the sum of the following arithmetic progressions:

$\frac{1}{15}, \frac{1}{12}, \frac{1}{10}, \ldots\ldots$ … upto 11 terms.

Answer

The arithmetic progression $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}, \ldots$ is given.Here first term is $\frac{1}{15}$ and common difference is $\frac{1}{12} - \frac{1}{15} = \frac{5-4}{60} = \frac{1}{60}$.Since the sum of n terms is$S_n = \frac{n}{2}[2a + (n-1)d]$

Where,

a = First term of APd = Common difference of APand no of terms is 'n'

So sum of first 11 terms $S_{11} = \frac{11}{2}[2 \times \frac{1}{15} + (11-1) \times \frac{1}{60}]$

$= \frac{11}{2}[\frac{2}{15} + (10) \times \frac{1}{60}] = \frac{11}{2}[\frac{2}{15} + \frac{1}{6}] = \frac{11}{2}[\frac{4}{30} + \frac{5}{30}]$

$= \frac{11}{2}[\frac{9}{30}] = = \frac{11}{2}[\frac{3}{10}] = \frac{33}{20}$

Hence, Sum of first 11 terms is $\frac{33}{20}$.

2 A. Question

Find the sum of the following:

3 + 11 + 19 + … + 803

Answer

The arithmetic progression 3, 11, 19, …., 803 is given.Here first term is 3 and common difference is 11 – 3 = 8.Last term l = 803

Let no. of terms be n.So, $a_n = a + (n - 1)d$

Where,

a = First term of APd = Common difference of APand no of terms is 'n'

⇒ 803 = 3 + (n - 1) × 8⇒ 800 = (n - 1) × 8⇒ 100 = n – 1⇒ n = 101

Since the sum of n terms is$S_n = \frac{n}{2}[2a + (n-1)d]$

Where,

a = First term of APd = Common difference of APand no of terms is 'n'

So sum of 101 terms $S_{101} = \frac{101}{2}[2 \times 3 + (101-1) \times (8)]$= 50.5[6 + 100×(8)]= 50.5[6 + 800]= 50.5 × (806)= 40703

Hence, Sum is 40703.

2 B. Question

Find the sum of the following:

$$7 + 10\frac{1}{2} + 14 + \ldots\ldots + 84$$

Answer

The arithmetic progression $7, 10\frac{1}{2}, 14, \ldots, 84$ is given.Here first term is 7 and common difference is $10\frac{1}{2} - 7 = \frac{21}{2} - 7 = \frac{7}{2}$Last term l = 84

Let no. of terms be n.Since, $a_n = a + (n - 1)d$

Where,

a = First term of APd = Common difference of APand no of terms is 'n'

$\Rightarrow 84 = 7 + (n-1) \times \frac{7}{2} \Rightarrow 77 = (n-1) \times \frac{7}{2}$⇒ 22 = n – 1⇒ n = 23

Since the sum of n terms is$S_n = \frac{n}{2}[2a + (n-1)d]$ So sum of 23 terms S_{23} $= \frac{23}{2}[2 \times 7 + (23-1) \times \frac{7}{2}]$= 11.5 [14 + 22 × 3.5]= 11.5[14 + 77]= 11.5 × (91)= 1046.5

Hence, Sum is 1046.5 =$1046\frac{1}{2}$.

3 A. Question

Find the number of terms:

How many terms of the arithmetic progression 9, 17, 25, … be taken so that their sum is 636?

Answer

Here first term a = 9 and common difference d = 17 – 9 = 8.Also, S_n = 636Since the sum of n terms is$S_n = \frac{n}{2}[2a + (n-1)d]$

Where,

a = First term of APd = Common difference of APand no of terms is 'n'

$636 = \frac{n}{2}[2 \times 9 + (n-1) \times 8]636 = \frac{n}{2}[18 + (n-1) \times 8]$

$636 = \frac{n}{2} \times 2[9 + (n-1) \times 4]$636 = n[9 + (n - 1)4]636 = 9n + 4n(n - 1)636 = 9n + $4n^2$ – 4n636 = $4n^2$ + 5n$4n^2$ + 5n – 636 = 0⇒ n = 12, n = -13.25 (which is not applicable ∵ n can neither be negative nor be in decimal)∴ n = 12

Hence, 12 terms of the arithmetic progression must be taken to get sum as 636.

3 B. Question

Find the number of terms:

How many terms of the arithmetic progression 63, 60, 57, … be taken so that their sum is 693?

Answer

Here first term a = 63 and common difference d = 60 – 63 = -3.Also, S_n = 693Since the sum of n terms is$S_n = \frac{n}{2}[2a + (n-1)d]$

Where,

a = First term of APd = Common difference of APand no of terms is 'n'

$693 = \frac{n}{2}[2 \times 63 + (n-1) \times (-3)] 693 = \frac{n}{2}[126 + (n-1) \times (-3)]$

693 × 2 = n[126 + (n - 1)(-3)]1386 = 126n - 3n(n - 1)1386 = 126n - $3n^2$ + 3n1386 = -$3n^2$ + 129n3n^2 - 129n + 1386 = 0n^2 - 43n + 462 = 0⇒ n = 21, n = 22∴ n = 21 or n = 22

Hence, Either 21 or 22 terms of the arithmetic progression must be taken to get sum as 693.

4 A. Question

Find the sum of the first 25 terms of the following series whose n^{th} term is given:

$a_n = 3 + 4n$

Answer

General n^{th} term a_n is given as:$a_n = a + (n - 1)d$ ... (i)

Where,

a = First term of APd = Common difference of APand no of terms is 'n'

and $a_n = 3 + 4n \Rightarrow a_n = (7-4) + 4n \Rightarrow a_n = 7 + 4n - 4 \Rightarrow a_n = 7 + 4(n - 1) \Rightarrow a_n = 7 + (n - 1) \times 4$ ...(ii)

On comparing eq. (i) and (ii), we get,a = 7 and d = 4

Here first term is 7 and common difference is 4.Since the sum of n terms is$S_n = \frac{n}{2}[2a + (n-1)d]$

Where,

a = First term of APd = Common difference of APand no of terms is 'n'

So sum of first 25 terms $S_{25} = \frac{25}{2}[2 \times 7 + (25-1) \times 4$= 12.5 × [14 + 24×4]= 12.5 × [14 + 96]= 12.5 × 110= 1375

Hence, Sum of first 25 terms is 1375.

4 B. Question

Find the sum of the first 25 terms of the following series whose n^{th} term is given:

$a_n = 7 - 3n$

Answer

General n^{th} term a_n is given as:$a_n = a + (n - 1)d$ ... (i)

Where,

a = First term of APd = Common difference of APand no of terms is ‘n’

and $a_n = 7 - 3n \Rightarrow a_n = (4 + 3) - 3n \Rightarrow a_n = 4 - 3n + 3 \Rightarrow a_n = 4 + (-3) \times (n - 1) \Rightarrow a_n = 4 + (n - 1) \times (-3)$ ...(ii)

On comparing eq. (i) and (ii), we get,a = 4 and d = -3

Here first term is 4 and common difference is -3.Since the sum of n terms is$S_n = \frac{n}{2}[2a + (n - 1)d]$

Where,

a = First term of APd = Common difference of APand no of terms is ‘n’

So sum of first 25 terms $S_{25} = \frac{25}{2}[2 \times 4 + (25 - 1) \times (-3)$= 12.5 × [8 + 24×(-3)]= 12.5 × [8 - 72]= 12.5 × (- 64)= -800

Hence, Sum of first 25 terms is -800.

5. Question

Find the sum of the first 51 terms of an arithmetic progression in which second and third terms are respectively 14 and 18.

Answer

Common difference = 18 – 14 = 4∴ a = a_2 – d = 14 – 4 = 10First term a = 10 and common difference d = 4

Since the sum of n terms is$S_n = \frac{n}{2}[2a + (n - 1)d]$

Where,

a = First term of APd = Common difference of APand no of terms is ‘n’

So sum of first 51 terms $S_{51} = \frac{51}{2}[2 \times 10 + (51 - 1) \times (4)$= 25.5 × [20 + 50×4]= 25.5 × [20 + 200]= 25.5 × 220= 5610

Hence, Sum of first 51 terms is 5610.

6. Question

The first and the last terms of an arithmetic progression are 17 and 350 respectively. If the common difference is 9 then what is the number of terms in the arithmetic progression and what is their sum?

Answer

Here first term is 17 and common difference is 9Last term l = 350

Let no. of terms be n.Since, $a_n = a + (n - 1)d$

Where,

a = First term of APd = Common difference of APand no of terms is 'n'

$\Rightarrow 350 = 17 + (n - 1) \times 9 \Rightarrow 333 = (n - 1) \times 9 \Rightarrow 37 = n - 1 \Rightarrow n = 38$

Since the sum of n terms is $S_n = \frac{n}{2}[2a + (n - 1)d]$ So sum of 38 terms $S_{38} = \frac{38}{2}[2 \times 17 + (38 - 1) \times 9] = 19\ [34 + 37 \times 9] = 19 \times 367 = 6973$

Hence, Number of terms is 38 and Sum is 6973.

7. Question

Find the sum of all the odd numbers divisible by 3 between 1 and 1000.

Answer

Starting from 1 the first odd number divisible by 3 is 3 then second odd number divisible by 3 is 9.Greatest odd number till 1000 i.e. divisible by 3 is 999Thus arithmetic progression is 3, 9, 15, ..., 999Here first term a = 3 and last term l = 999and common difference = 9 – 3 = 6.

Let number of all odd number divisible by 3 be n.Since, $a_n = a + (n - 1)d$

Where,

a = First term of APd = Common difference of APand no of terms is 'n'

$\Rightarrow 999 = 3 + (n - 1) \times 6 \Rightarrow 996 = (n - 1) \times 6 \Rightarrow 166 = n - 1 \Rightarrow n = 167$

Since the sum of n terms is $S_n = \frac{n}{2}[2a + (n - 1)d]$ So sum of 167 terms $S_{167} = \frac{167}{2}[2 \times 3 + (167 - 1) \times 6] = 83.5 \times [6 + 166 \times 6] = n = 83.5 \times 1002 = 83667$

Hence, Sum is 83667.

8. Question

The first term of an arithmetic progression is 8, n^{th} term is 33 and sum of first n term is 123. Then find the n and common difference d.

Answer

Here first term a = 8, n^{th} term = 33 and $s_n = 123$General n^{th} term a_n is given as:$a_n = a + (n - 1)d$

Where,

a = First term of APd = Common difference of APand no of terms is ‘n’

33 = 8 + (n -1) × d25 = (n - 1) × d … (i)

Since the sum of n terms is$S_n = \frac{n}{2}[2a + (n-1)d]$

Where,

a = First term of APd = Common difference of APand no of terms is ‘n’

So sum of first 25 terms $S_{25}123 = \frac{n}{2}[2 \times 8 + (n-1) \times (d)]$246 = n [16 + (n - 1) × d]246 – 16 n = (n) × (n - 1) × d … (ii)

On dividing eq. (ii) from (i), we get,$\frac{246 - 16n}{25} = \frac{n \times (n-1) \times d}{(n-1) \times d} \Rightarrow$

$\frac{246 - 16n}{25} = \frac{n}{1} \Rightarrow$ (246 – 16n) = 25n⇒ 25n + 16n = 246⇒ 41n = 246⇒ n = 6

Substituting the value of n in eq. (i), we get,d = 5

Hence, Value of n is 6 and common difference is 5.

9. Question

A sum of 280 is to be used to give four prizes. If after first prize, each prize is 20 less than the preceding prize, then find the value of each of the prizes.

Answer

Given,S_n = 280, n = 4 and common difference is -20.

Let the first term be a.

Since the sum of n terms is$S_n = \frac{n}{2}[2a + (n-1)d]$

Where,

a = First term of APd = Common difference of APand no of terms is ‘n’

So sum of 4 terms $S_4280 = \frac{4}{2}[2 \times a + (4-1) \times (-20)]$280 = 2× [2a + 3 × (-20)]140 = 2a – 602a = 140 + 602a = 200a = 100

a_1 = a = 100a_2 = a_1 + d = 100 - 20 = 80a_3 = a_2 + d = 80 - 20 = 60a_4 = a_3 + d = 60 - 20 = 40

Hence, The value of prizes are 100, 80, 60 and 40.

10. Question

A manufacture of TV sets produces 600 TV sets in the third year and 700 TV sets in seventh year. Assuming that the production increases, uniformly by a

fixed number every year, find(i) the production in the 1^{st} year(ii) the production in the 10^{th} year(iii) the total production in first 7 years.

Answer

Given, $a_3 = 600$ and $a_7 = 700$

Let the first term be a and common difference be d.Since, nth term an AP is given by :$a_n = a + (n - 1)d$

Where,

a = First term of APd = Common difference of APand no of terms is 'n'

$\Rightarrow a_3 = a + (3 - 1)d \Rightarrow 600 = a + 2d$ ... (i)

$\Rightarrow a_7 = a + (7 - 1)d \Rightarrow 700 = a + 6d$ ...(ii)

On solving eq. (i) and (ii), we get :a = 550 and d =25

$\therefore$ production in first year a_1 = a 550and production in tenth year $a_{10} = a + (10 - 1)d = 550 + 9 \times 25 = 550 + 225 = 775 \therefore a_{10} = 775$

Since the sum of n terms is$S_n = \frac{n}{2}[2a + (n-1)d]$ Total production in first seven year $S_7 S_7 = \frac{7}{2}[2a + (7-1)d] S_7 = 3.5 \times [(2 \times 550) + (6 \times 25)] S_7 = 3.5 \times [1100 + 150] S_7 = 3.5 \times 1250 S_7 = 4375$

Hence, (i) the production in the 1^{st} year is 550(ii) the production in the 10^{th} year is 775(iii) the total production in first 7 years is 4375.

Miscellaneous Exercise 5

1. Question

The common difference of two arithmetic progression is the same. The first term of one of them is 8 and of the second is 3. Then difference of their 30^{th} term is :

A. 11

B. 3

C. 8

D. 5

Answer

Since the common difference of both the term is same.$\therefore$ each term will change by same number.$\Rightarrow$ Difference of their 30^{th} term is same as difference of their 1^{st} term.$\Rightarrow$ Difference of 30^{th} term = Difference of 1^{st} term = 8 – 3 = 5.

2. Question

If 18, a, b, -3 are in the arithmetic progression then a + b =

A. 19

B. 7

C. 11

D. 15

Answer

Since they are in arithmetic progression their common difference is same.⇒ a – 18 = b – a = -3 – b⇒ a – 18 = -3 – b⇒ a + b = 18 – 3⇒ a + b = 15.

3. Question

If the 7^{th} and the 13^{th} terms of an arithmetic progression are 34 and 64 respectively, then its 18^{th} term is :

A. 89

B. 88

C. 87

D. 90

Answer

Let the first term be a and common difference be d.Since, nth term an is given by :$a_n = a + (n - 1)d$

Where,

a = First term of APd = Common difference of APand no of terms is 'n'

$\Rightarrow a_7 = a + (7 - 1)d \Rightarrow 34 = a + 6d$ … (i)

$\Rightarrow a_{13} = a + (13 - 1)d \Rightarrow 64 = a + 12d$ …(ii)

On solving eq. (i) and (ii), we get :a = 4 and d = 5

$\Rightarrow a_{18} = a + (18 - 1)d \Rightarrow a_{18} = 4 + 17 \times 5 \Rightarrow a_{18} = 4 + 85 \Rightarrow a_{18} = 89$

4. Question

If an arithmetic progression's first term is 2 and common difference 8 and sum of n terms is 90 then value of n will be:

A. 3

B. 4

C. 5

D. 6

Answer

Given, first term a = 2 and common difference d = 8 and S_n = 90.

Since the sum of n terms is$S_n = \frac{n}{2}[2a + (n-1)d]$

Where,

a = First term of APd = Common difference of APand no of terms is 'n'

$90 = \frac{n}{2}[2 \times 2 + (n-1) \times 8] \Rightarrow$ 180 = n [4 + 8(n - 1)]⇒ 180 = 4n + 8n(n – 1)⇒ 180 = 4n + $8n^2$ – 8n⇒ 180 = $8n^2$ – 4n⇒ 45 = $2n^2$ – n⇒ $2n^2$ – n – 45 = 0⇒ n = 5, n = -4.5Since, n is a number of terms, it cannot be negative.∴ n = 5.

5. Question

If the sum of n terms of an arithmetic progression in $3n^2$ + 5n, then which term of its is 164:

A. 12^{th}

B. 15^{th}

C. 27^{th}

D. 20^{th}

Answer

Since the sum of n terms is$S_n = \frac{n}{2}[2a + (n-1)d]$ ...(i)

Where,

a = First term of APd = Common difference of APand no of terms is 'n'

Also $S_n = 3n^2 + 5n \Rightarrow S_n = n(3n + 5) \Rightarrow S_n = n(3n + 8 - 3) \Rightarrow S_n = n[8 + 3(n-1)] \Rightarrow S_n = n[8 + (n - 1)3] \Rightarrow S_n = \frac{n}{2}[2 \times 8 + (n-1)6]$ ...(ii)

On comparing eq. (i) and (ii), we get :a = 8 and d = 6

Since, n^{th} term a_n is given by :

$a_n = a + (n - 1)d$

Where,

a = First term of APd = Common difference of APand no of terms is 'n'

$a_n = 8 + (n - 1)6 \Rightarrow 164 = 8 + (n - 1) \times 6 \Rightarrow 156 = (n - 1) \times 6 \Rightarrow (n - 1) = 26 \Rightarrow n = 27$.

6. Question

If the sum of the first n terms of the arithmetic progression is S_n and $S_{2n} = 3S_n$, then $S_{3n} : S_n$ will be :

A. 10

B. 11

C. 6

D. 4

Answer

Since the sum of n terms is $S_n = \frac{n}{2}[2a + (n-1)d]$

Where,

a = First term of APd = Common difference of APand no of terms is 'n'

$\Rightarrow S_{2n} = 3S_n \Rightarrow \frac{2n}{2}[2a + (2n-1)d] = 3 \times \frac{n}{2}[2a + (n-1)d] \Rightarrow 2[2a + (2n - 1)d] = 3[2a + (n - 1)d]$

$\Rightarrow 4a + (2n - 1)2d] = 6a + (n - 1)3d \Rightarrow 4nd - 2d = 2a + 3nd - 3d \Rightarrow nd + d = 2a \Rightarrow (n + 1)d = 2a$

$S_{3n}:S_n = \frac{\frac{3n}{2}[2a+(3n-1)d]}{\frac{n}{2}[2a+(n-1)d]} = \frac{3[2a+(3n-1)d]}{[2a+(n-1)d]} = \frac{3[(n+1)d+(3n-1)d]}{[(n+1)d+(n-1)d]} = \frac{3[(n+1)+(3n-1)]}{[(n+1)+(n-1)]} = \frac{3(4n)}{2n} = 6 S_{3n} : S_n = 6:1$

7. Question

The first term and the last term of an arithmetic progression are 1 and 11 respectively. If the sum of its terms is 36, then its number of terms will be:

A. 5

B. 6

C. 9

D. 11

Answer

Given, first term a = 1, last term l = 11 and S_n = 36.Since, n^{th} term a_n is given by :a_n = a + (n - 1)d

Where,

a = First term of APd = Common difference of APand no of terms is 'n'

⇒ 11 = 1 + (n - 1)d⇒ 10 = (n -1)d ... (i)

Since the sum of n terms is$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow 36 = \frac{n}{2}[2(1) + (n-1)]d \Rightarrow$ 72 = n[2 + (n-1)d] ...(ii)

From eq. (i) and (ii), we get,⇒ 72 = n[2 + 10]⇒ 72 = 12n⇒ n = 6

8. Question

Write 5^{th} term from end of the arithmetic progression 3, 5, 7, 9, ..., 201.

Answer

Here last term of the progression l = 201.First term a = 3 and common difference = 5 – 3 = 2We know, In an AP, the nth term from last is

a_n = l – (n – 1)d

where a_n is nth term from last, l is last term and 'd' is common difference.

Thus 5^{th} term from the end= l – (5 -1)d= 201 – (4) × (2)= 201 - 8= 193

Hence, 5^{th} term from the last is 193.

9. Question

If three consecutive terms of an arithmetic progression are $\frac{4}{5}$, a, 2, then write the value of a.

Answer

We know that if three terms A, B and C are in AP then2B = A + C

$\Rightarrow 2a = \frac{4}{5} + 2 \Rightarrow 2a = \frac{4}{5} + \frac{10}{5} \Rightarrow 2a = \frac{14}{5} \Rightarrow a = \frac{7}{5}$

Hence, Value of a is $\frac{7}{5}$.

10. Question

Find the sum of first 1000 positive integers.

Answer

Required arithmetic progression is 1, 2, 3, …, 1000Here, first term a= 1 and common difference d = 1.

Since the sum of n terms is$S_n = \frac{n}{2}[2a + (n-1)d]$

Where,

a = First term of APd = Common difference of APand no of terms is ‘n’

$S_{1000} = \frac{1000}{2}[2 \times 1 + (1000-1) \times 1]$= 500 [2 + 999]

= 500 × 1001= 500500

Hence, Sum of first 1000 positive integers is 500500.

11. Question

Is 299 any term in the sequence of numbers 5, 11, 17, 23, …?

Answer

The arithmetic progression is 5, 11, 17, 23, ….Here, First term a = 5 and common difference = 11 – 5 = 6Let us assume that n^{th} term a_n of the progression is 301.we know, a_n = a + (n -1)d

Where,

a = First term of APd = Common difference of APand no of terms is ‘n’

∴ 299 = 5 + (n – 1) × (6)⇒ 299 = 5 + 6n -6⇒ 300 = 6n⇒ n = 5

Hence, Yes, 299 is the 5^{th} term of the progression.

12. Question

Which term of the arithmetic progression $19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, \ldots$ is the first negative term ?

Answer

Here, first term a = 20 and common difference d $= 19\frac{1}{4} - 20 = \frac{77}{4} - \frac{80}{4} = -\frac{3}{4}$We know that general term a_n = a + (n - 1)d

Where,

a = First term of APd = Common difference of APand no of terms is ‘n’

Since the common difference is negative we get the first negative term when -

(n - 1)d > a$\Rightarrow -(n-1)\times\left(-\frac{3}{4}\right) > 20 \Rightarrow (n-1)\times\left(\frac{3}{4}\right) > 20$

$\Rightarrow (n-1) > 20\times\frac{4}{3}$⇒ (n - 1) > 26.66⇒ n > 26.66 + 1⇒ n > 27.66⇒ n = 28

Hence, 28^{th} term of the arithmetic progression is first negative term.

13. Question

Four numbers are in arithmetic progression. If their sum is 20 and the sum of their squares is 120, then find the numbers.

Answer

Let the first term be a – 3d and common difference be 2d.Thus arithmetic progression will be a - 3d, a - d, a + d, a+3d.

Given, Sum of terms = 20⇒ (a - 3d) + (a - d) + (a + d) + (a + 3d) = 20⇒ 4a = 20⇒ a = 5 ...(i)

Also,

Sum of their square = 120

$(a-3d)^2 + (a-d)^2 + (a+d)^2 + (a+3d)^2 = 120$

$\Rightarrow (5-3d)^2 + (5-d)^2 + (5+d)^2 + (5+3d)^2 = 120$

$\Rightarrow (25+9d^2-30d) + (25+d^2-10d) + (25+d^2+10d) + (25+9d^2+30d) = 120$

$\Rightarrow 100 + 20d^2 = 120$

$\Rightarrow 20d^2 = 20$

$\Rightarrow d^2 = 1$

$\Rightarrow d = \pm 1$

If d = 1

First term, a – 3d = 5 – 3 = 2 and common difference, 2d = 2 and four terms will be 2, 4, 6, 8

If d = -1

First term, a – 3d = 5 –(– 3) = 8 and common difference, 2d = -2 and four terms will be 8, 6, 4 and 2.

14. Question

If the sum of a terms of an arithmetic progression is $\frac{3n^2}{2}+\frac{5n}{2}$, then find its 25^{th} term.

Answer

Since the sum of n terms is$S_n = \frac{n}{2}[\,2a+(n-1)d]$ ...(i)

Where,

a = First term of APd = Common difference of APand no of terms is 'n'

Also $S_n = \frac{3n^2}{2}+\frac{5n}{2} \Rightarrow S_n = \frac{n}{2}[\,3n+5] \Rightarrow S_n = \frac{n}{2}[\,3n+8-3\,] \Rightarrow S_n = \frac{n}{2}[\,8+3(n-1)] \Rightarrow S_n = \frac{n}{2}[\,2(4)+(n-1)3]$ ...(ii)

On comparing eq. (i) and (ii), we get:a = 4 and d = 3

Also, n^{th} term a_n is given by:a_n = a + (n – 1)d

For given AP, we have

a_n = 4 + (n - 1)6$\Rightarrow a_{25}$ = 4 + (25 - 1) × 3$\Rightarrow a_{25}$ = 4 + 24 × 3$\Rightarrow a_{25}$ = 4 + 72$\Rightarrow a_{25}$ = 76

Hence, 25^{th} term is 76.

15. Question

The houses of a row are numbered consecutively from 1 to 49. Show that there is a value of x such that the sum of the numbers of the houses preceding the house numbered x is equal to the sum of the numbers of the houses following it. Find the values of x.

Answer

Clearly, No of houses are in AP and

Given,

First term, a = 1

common difference d = 1

According to given statement: $S_{1 \text{ to } x-1} = S_{(x+1) \text{ to } 49} \Rightarrow S_{x+1 \text{ to } 49} = S_{49} - S_x$ and $S_{1 \text{ to } x-1} = S_{x-1}$

Since the sum of n terms is $S_n = \frac{n}{2}[\,2a+(n-1)d]$

Where,

a = First term of APd = Common difference of APand no of terms is ‘n’

Applying given condition:

$S_{1 \text{ to } x-1} = S_{(x+1) \text{ to } 49} \Rightarrow S_{x-1} = S_{49} - S_x \Rightarrow S_{x-1} = S_{49} - S_x$

$$\Rightarrow \frac{x-1}{2}[2(1) + ((x-1)-1)(1)] =$$

$$\frac{49}{2}[2(1) + (49-1)(1)] - \frac{x}{2}[2(1) + (x-1(1))]$$

$$\Rightarrow \frac{(x-1)}{2}[2 + (x-2)] = \frac{49}{2}[2 + 48] - \frac{x}{2}[2 + (x-1)] \Rightarrow (x-1)(x) = 49(50) - x(x+1) \Rightarrow x^2 - x = 49(50) - x^2 - x \Rightarrow 2x^2 = 49 \times 50 \Rightarrow x^2 = 49 \times 25 \Rightarrow x = 35 \text{ and } x = -35$$

Here since the x is a number between 1 to 49, ∴ x = 35.

Hence, The value of x is 35.