# Comparing Quantities

## Variations

If two quantities are related with each other in such a way that change in one quantity will produce the corresponding change in the other quantity then they are said to be in variations. The variation may be that if we increase or decrease the one quantity then other quantity may also increase or decrease and vice-versa. If increase in one quantity results in the corresponding increase or decrease in other quantity then it is called direct variation and if increase in one quantity will result in to decrease in other quantity or vice-versa then it is called indirect variation.

For example increase in the cost with the increase in quantity is a direct variation whereas decrease in the time taken for a work with increase in the number of workers is an inverse variation.

## Direct variation

Two quantities are said to varies directly if increase in one quantity will results the increase in other or decrease in one quantity will results the decrease in other quantity In other words if two quantities are in direct variation, then they are said to be directly proportional to each other.

**Following are some examples of direct variations**:

- The cost of articles varies directly as the number of articles increases.
- The distance covered by a moving object varies directly as its speed increases or decreases. (It means if speed increases then the more distance covered in the same time).
- The work done varies directly as the number of men increases.
- The work done varies directly as the working time increases.

- **Example:**

A car travels 225 km with 15 litre of petrol. How many litre of petrol are needed to travel 135 km?

(a) 6 litres (b) 9 litres
(c) 10 litres (d) 12 litres
(e) None of these

Answer (b)

**Explanation:** Petrol required to cover a distance of 225 km = 15 litres

Petrol required to cover a distance of 1 km $= \frac{15}{225}$ litres

Petrol required to cover a distance of 135 km $= \frac{15}{225} \times 135 = 9$ litres

## Inverse variation

Two quantities are said to be in inverse variation if increase in one quantity results in decrease in the other quantity and vice versa.

Following are some examples of inverse variations:

The time taken to finish a piece of work varies inversely as the number of men at work varies, (more men take less time to finish the job and less men will take more time)

The speed varies inversely to the time taken to cover a given distance (more is the speed less is the time taken to cover a distance.

The number of hours it takes for a block of ice to melt varies inversely to the temperature.

- **Example:**

A certain project can be completed by 5 workers in 24 days. How many workers are needed to finish the project in 15 days?

(a) 5 (b) 6
(c) 8 (d) 10
(e) None of these

Answer (c)

**Explanation:** Total work done by 5 workers in 24 days $= 5 \times 24 = 120$

Numbers of workers needed to finish the job in 15 days $= \frac{120}{15} = 8$

## Constant of Variation

If x varies inversely to y i.e. $x \propto \frac{1}{y}$ then $y = \frac{k}{x}$, where $k$ is $a$ real number. The constant k is called the constant of variation.

## Time and Work

In our daily life we come across many problems which are based on time and work. The term time and work are interrelated with each other. Time and work are directly proportional to each other. The amount of work done increases with the time and the amount of work left decreases with the number of labourers or workers. If the number of workers increases then the time taken to complete the work will decrease. Thus the number of workers and time are inversely proportional to each other. We normally solve the problems related to time and work using unitary method.

## Working Efficiency

Acting efficiency of a worker is defined as the work done by an individual in one day. This efficiency is inversely proportional to the number of days to complete a work. In other words we can say that a person who takes more time to complete a work is less efficient than a man who takes less time to complete that task. Following are some important points which are very useful in solving the problems based on time and work.

If $x$ can do a work in $n$ days, then one day's work of $x$ is $\frac{1}{n}$.

Total work done by a group of workers = number of days required to complete the work x total number of workers.

If $x_1$ persons can do $w_1$ work in $n_1$ days and $x_2$ persons can do $w_2$ work in $n_2$ days, then $\frac{x_1 n_1}{x_2 n_2} = \frac{w_1}{w_2}$

If A and B together can do a piece of work in m days, B and C together in n days and C and A in p days then A, B and C together will do the same work in $\left(\frac{2mnp}{mn + np + mp}\right)$ days.

- **Example:**

A is thrice as good as B and therefore is able to finish a work in 80 days less than B. Find the number of days required to complete the work if they are working together.

(a) 20 days (b) 30 days
(c) 32 days (d) 40 days
(e) None of these
Answer (b)

**Explanation:** The percentage of students that passed at least one subject is:
P (passed English) + P (passed math) $-$P (passed both) $-80\% + 80\% - 75\% - 90\%$
Note that we need to subtract those that passed both (where the circles overlap above) to avoid double counting.
The percentage of students that failed both subjects is:
$100 - P$ (passed at least one subject) $-100\% - 90\% = 10\%$ We know that 60 students failed both. If we let the total number of students be T, this implies:
T' * (10/100) =60
Solving for T, we get T = 600 students.

## Percentage

The word percent means per hundred. It can be defined as the fraction whose denominator is 100, then the numerator of the fraction is called percent. It is denoted by the symbol '%'. If we have to find the percentage of any number we usually find the quantity per hundred of that number.

## Ratio as a Percentage

Percent can also be expressed as the ratio with its second term being 100 and the first term is equal to the given percent. In order to convert the given ratio into a percent, we have to convert the given ratio first into the fraction and then multiply the fraction by 100. Conversely if we have to convert the given percent into ratio, we first convert the percent into the fraction and then reduce it to the lowest term.
For example,

$$x : y = \left(\frac{x}{y} \times 100\right)\% \text{ and } x\% = \frac{x}{100} = x : 100$$

- **Example:**

In an examination 80% of the students passed in science, 85% in Mathematics and B 75% in both the subjects. If 60 students failed in both the subjects then the total number of students is:
(a) 200 (b) 400
(c) 600 (d) 800
(e) None of these
Answer (c)

**Explanation:** Let number of students be 100. Then,
Number of students who passed in Mathematics only $= (85 - 75) = 10$
Number of students passing in at least one subject or both
$= 5 + 10 + 75 = 90$
Number of failed students = 10
Total number of students $= \frac{100 \times 60}{10} = 600$

**Decimals as a Percentage**

Percent can also be expressed in the form of decimal. In order to convert the given fraction into the decimal we divide the numerator with 100 and get the required decimal form or by simply putting the decimals two digit to the left of the numerator.

- **Example:**

Basic salary of a worker is increased by 20%. What amount does worker get if his last salary was Rs. 8800?
(a) Rs. 15780 (b) Rs. 11450
(c) Rs. 10560 (d) Rs. 10300
(e) None of these
Answer (c)

**Explanation:** Increased in salary =20% of Rs. 8800
New salary = Rs. 8800 + Rs. 1760 = Rs. 10560

**Profit, Loss and Discount**

The term profit and loss are related to the business and marketing. If a merchant purchases the goods at a certain rate and sells it at the rate higher than the purchase price then he said to have earn profit and if he sells at the price less than the purchase price then he said to have loss. In this chapter, apart from profit and loss we will also discuss about the Value Added Tax or simply VAT. the another term we will use in this chapter is discount.

**Discount**

Discount is the amount reduced on the marked price of the article by the shopkeeper. The rate of discount is the rate at which the amount is reduced on the marked price. The marked price of the article is the price which is mentioned on the article or on the tag of the article. There is a difference between the marked price and cost price of the article. If MP > CP, then shopkeeper will have profit on that particular article on the other hand if MP < CP, then the shopkeeper will have loss on the article.

**Value Added Tax**

The tax we pay on the goods we purchase is called as value added tax or VAT. The tax we pay as a VAT is the nominal amount on the goods which goes to government funds and used by the government for providing the various facilities to the public such as road, electricity, water and many other facilities.

**Some Important Relations**

- Profit = Selling Price - Cost Price
- Profit % $= \frac{Profit}{Cost\ price} \times 100$
- Loss = Cost Price - Selling Price
- $Loss\% = \frac{Loss}{Cost\ Price} \times 100$
- Selling price = $\left[1 + \frac{Profit\%}{100}\right] \times Cost\ Price$
- Selling price = $\left[1 - \frac{Loss\%}{100}\right] \times Cost\ price$
- Selling price = $Marked\ price - Discount$

**Note:** In the cases where S.P. of two articles is same and one is sold at a loss of k% and other is sold at a profit of k%, the transactions always suffer a loss and such.

Loss% is equal to: $\left(\frac{common\ loss\ or\ gain\%}{10}\right)^2$ or $\left(\frac{k}{10}\right)^2\%$

**Example:**

What price should a shopkeeper mark on an article, costing him Rs.306 to 20%, after allowing a discount of 15%?

(a) Rs. 448 (b) Rs. 432
(c) Rs. 308 (d) Rs. 324
(e) None of these

Answer (b)

**Explanation:** Selling price of the article $= Rs.\ 306 \times 1.2$

Let marked price of the article be Rs. x, then

$$x \times \frac{85}{100} = 306 \times 1.2 \Rightarrow x = \frac{306 \times 120}{85} \Rightarrow x = 432$$

**Simple Interest**

If the interest reckoned is uniform then it is called as the simple interest. It is the easiest type of interest to calculate and understand. Formula used for calculate simple interest is as follows:

$S.I = \frac{P \times R \times T}{100}$, where P= Principle

R = Rate of Interest and T = Time Period

**Example:**

The simple interest on Rs. 2400 is Rs.60 more than the interest on Rs. 2000 for 3 years, the rate of interest per annum will be:

(a) 5% (b) 6%
(c) 7% (d) 8%
(e) None of these

Answer (a)

**Explanation:** Let r be the rate of interest per annum. Then,

$$\frac{2400 \times r \times 3}{100} - \frac{2000 \times r \times 3}{100} = 60$$
$$\Rightarrow 72r - 60r = 60 \Rightarrow 12r = 60 \Rightarrow r = 5\%$$

## Compound Interest

Formula used for calculating compound interest is as follows:

$$C.I = P\left(1 + \frac{R}{100}\right)^n - 1$$

Where, P = Principal, R = Rate of Interest, n = Time Period

- **Example:**

A sum of money placed at compound interest becomes 3 times in 4 years. The some money will amount to 27 times in:

(a) 10 years (b) 12 years
(c) 14 years (d) 15 years
(e) None of these

Answer (b)

**Explanation:** $3P = P\left(1 + \frac{r}{100}\right)^4 \Rightarrow \left(1 + \frac{r}{100}\right)^4 = 3$ (i)

and, $27P = P\left(1 + \frac{r}{100}\right)^n \Rightarrow \left(1 + \frac{r}{100}\right)^n = 27 = 3^3$ (ii)

from both equations, we get

$$\left(1 + \frac{r}{100}\right)^n = \left(1 + \frac{r}{100}\right)^{12} \Rightarrow n = 12 \text{ years.}$$