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VARIOUS FORMS OF EQUATION OF A LINE:

HORIZONTAL AND VERTICAL LINES:

Art: Find the equation (eqn) of straight line parallel to Y-axis and at a distance 'a' from it.

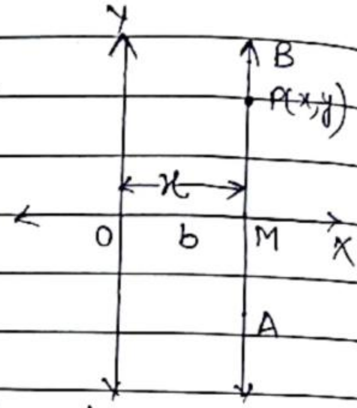
Proof: Let AB be a straight line ∥ to y-axis meeting x-axis in M. so that $OM = a$. ....(i)

Let $P(x,y)$ be any point on AB.

From P draw $PM \perp x$-axis

Then $OM = x$. ....(ii)

and $MP = y$.



from (i) and (ii) we get $\boxed{x = a}$ which is eqn of line ∥ to y-axis at a distance 'a' from it.

NOTE: The line $x = a$ lies to the right or left of y-axix according as 'a' is +ve or −ve resp.

If $a = 0$, then the line coincides with Y-axis and Eqn of Y-axis is $x = 0$

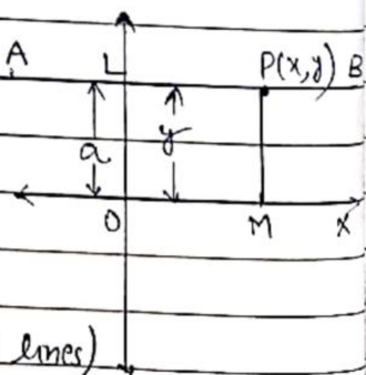
Art: Find the eqn. of st. line parallel to x-axis at a distance 'b' from it.

Proof: Let AB be a st. line ∥ to x-axis meeting y-axis in L, so that $OL = b$.

Let $P(x, y)$ be any point on AB.

from P, draw $PM \perp x$-axis

Then $OM = x$ and $MP = y$.



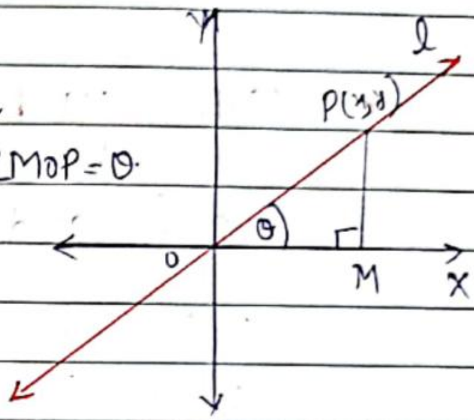
Now $MP = OL$ (⊥r. distance between ∥ lines)

∴ $y = b$ is eqn of line ∥ to x-axis at a distance b from it.

NOTE: - The line $y = b$ lies above or below the x-axis according as b is +ve or −ve resp.

If $b=0$, then the line coincides with x-axis and
$\therefore$ Equation of X-axis is $y=0$

* Equation of straight line passing through the origin and making an angle θ with x-axis.



Let $P(x,y)$ be any point on given line l which makes an angle of 'θ' with x-axis so that $\angle MOP=\theta$.

Draw $PM \perp$ x-axis

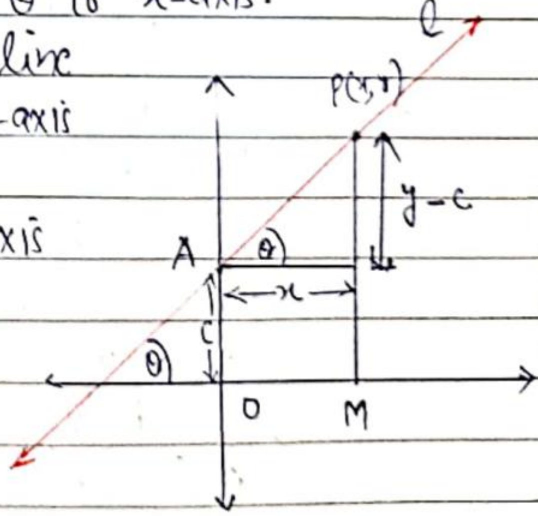
Then in rt $\angle$d Δ OMP.

$$\frac{MP}{OM} = \tan\theta$$

ie $$\frac{y}{x} = m$$

$\therefore$ $\boxed{y = mx}$ which is the required equation.

* SLOPE - INTERCEPT FORM : Equation of st. line which cuts off a given intercept 'c' on y-axis and is inclined at a given angle θ to x-axis.



Let l be the given line which makes angle θ with x-axis and cuts off y-axis at $(0,c)$

ie makes intercept 'c' at y-axis

Then from fig

In rt $\angle$d Δ ALP

$\angle PAL = \theta$, $AL = x$

$PL = y-c$

Now In rt $\angle$dΔALP.

$$\frac{PL}{AL} = \tan\theta$$

ie $$\frac{y-c}{x} = m$$

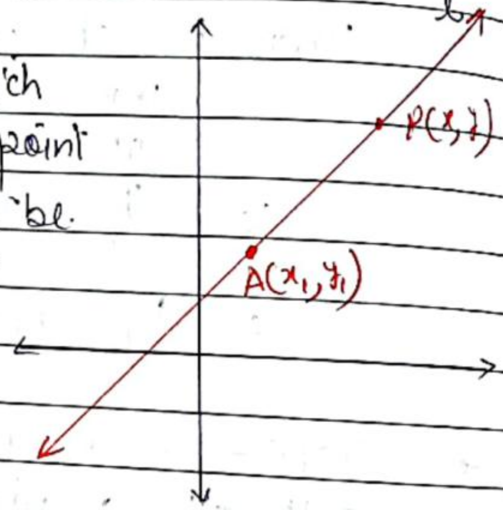
$$\Rightarrow y - c = mx$$

or $\boxed{y = mx + c}$ which is required eqn.

$\therefore$ Slope-Intercept form of line is $y = mx + c$
Where m is slope of line and c is y-intercept of line.

* ONE POINT FORM OR POINT-SLOPE FORM OF LINE:
Equation of line which passes through a given point and having slope m.

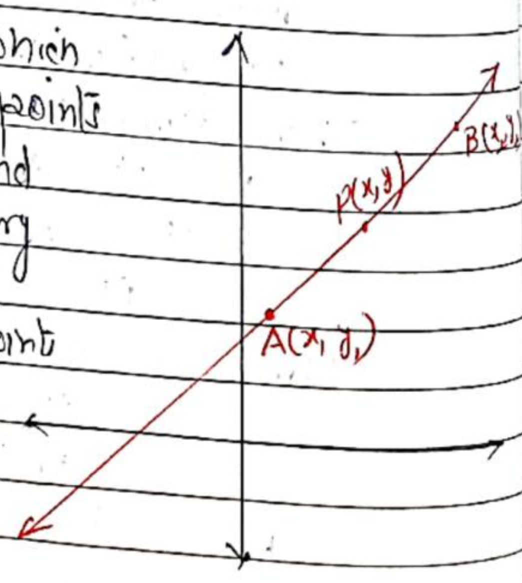
Let l be the given line which passes through given fixed point $A(x_1, y_1)$ and Let $P(x, y)$ be any arbitrary point on 'l'
Then by definition



$$\text{slope } m = \frac{y - y_1}{x - x_1}$$

$\Rightarrow \boxed{y - y_1 = m(x - x_1)}$ which is the required eqn.

* TWO-POINT FORM: Equation of straight line passing through Two-given points.
Let l be the given line which passes through two given points $A(x_1, y_1)$ and $B(x_2, y_2)$ and let $P(x, y)$ be any arbitrary point on l.



$\therefore$ A, B, P are collinear point

$\therefore$ Slope of AP = Slope of AB

$$\Rightarrow \frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$$

$$\Rightarrow y-y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1).$$

Thus Equation of line passing through two given points (x_1, y_1) and (x_2, y_2) is given by.

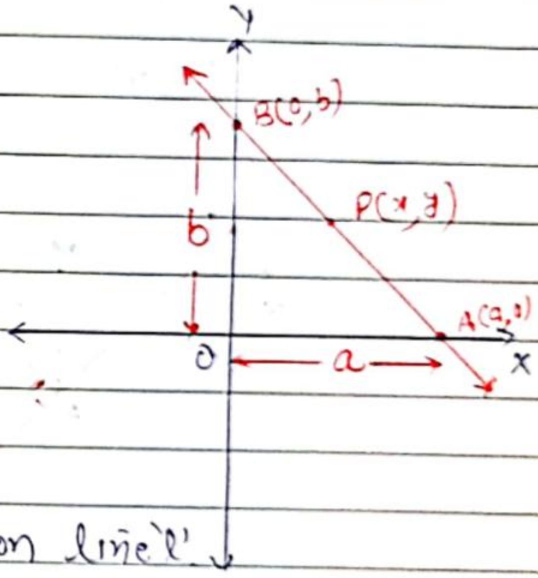
$$y-y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

* INTERCEPT FORM:

Equation of line which makes intercept on axes.

Let the given line makes x-intercept 'a' and y-intercept 'b' on axes.

$\therefore$ Line l meet x-axis at $A(a,0)$ and y-axis at $B(0,b)$.



Let $P(x,y)$ be any point on line 'l'.

Then by Two point form.

$$y-0 = \frac{b-0}{0-a}(x-a)$$

or $-ay = b(x-a)$.

$\Rightarrow -ay = bx - ab.$

$\Rightarrow ay + bx = ab$

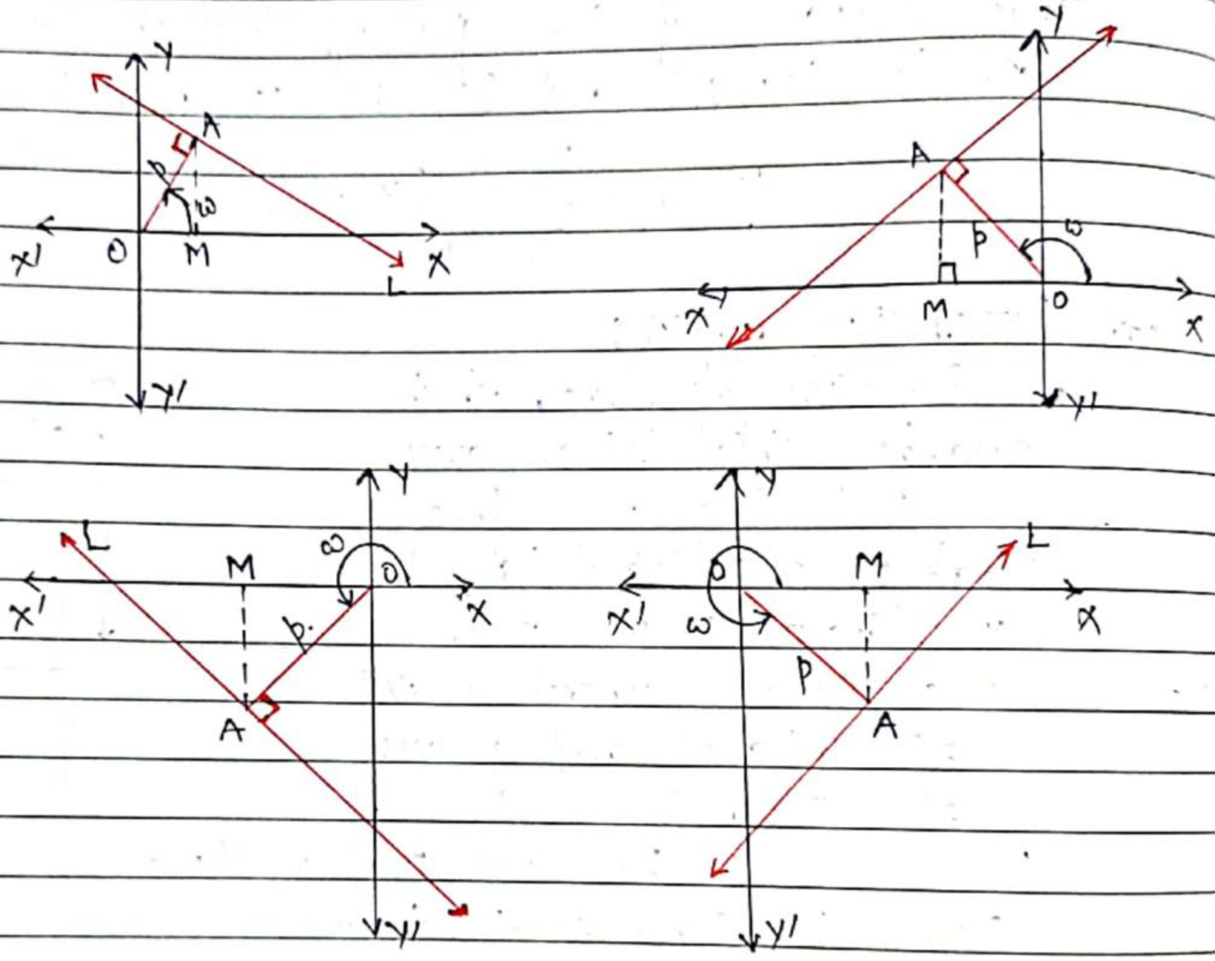
Dividing both sides by ab

$$\frac{ay}{ab} + \frac{bx}{ab} = \frac{ab}{ab}$$

$$\Rightarrow \frac{y}{b} + \frac{x}{a} = 1 \quad \text{or} \quad \boxed{\frac{x}{a} + \frac{y}{b} = 1}$$

which is required equation.

* **NORMAL FORM:** To find the equation of line in terms of p, the length of perpendicular from origin upon it aw. 'ω' is the angle which this perpendicular makes with x-axis.



Let OA = p be the length of perpendicular from origin on the given line l (perpendicular distance of line from origin)
and let OA makes angle $\angle XOA = \omega$ with +ve direction of x-axis.
The possible positions of line L, in Cartesian plane are shown in fig.
Draw AM ⊥ x-axis
Using Trigonometry we find that, In each case

$OM = p\cos\omega$

$MA = p\sin\omega$.

So that the Coordinates of point A are $a(p\cos\omega, p\sin\omega)$

Also Line L is perpendicular to OA

$$\therefore \text{Slope of line } L = -\frac{1}{\text{Slope of } OA} = -\frac{1}{\tan\omega}$$

$$= -\frac{1}{\frac{\sin\omega}{\cos\omega}} = -\frac{\cos\omega}{\sin\omega}$$

The equation of line passing through $A(p\cos\omega, p\sin\omega)$ having slope $m = \frac{-\cos\omega}{\sin\omega}$ will be

$$y - p\sin\omega = \frac{-\cos\omega}{\sin\omega}(x - p\cos\omega) \quad [\text{Point- slope form}]$$

$$\Rightarrow y\sin\omega - p\sin^2\omega = -x\cos\omega + p\cos^2\omega$$

$$\Rightarrow x\cos\omega + y\sin\omega = p(\sin^2\omega + \cos^2\omega)$$

$$\Rightarrow x\cos\omega + y\sin\omega = p. \quad [\because \sin^2\omega + \cos^2\omega = 1]$$

Thus, the equation of line having normal distance p from the origin and angle ω which the normal makes with +ve direction of x-axis is given by

$$x\cos\omega + y\sin\omega = p.$$

EXERCISE 10.2

Q.No1: Write equation of x-axis and y-axis

Sol. Since every point on x-axis has y-coordinate as 0.

$\therefore$ Equation of x-axis is $y = 0$

Similarly, every point on y-axis has x-coordinate as 0

$\therefore$ Equation of y-axis is $x = 0$

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Q No 2 Passing through point $(-4,3)$ with slope $\frac{1}{2}$.

Sol. Using one-point form. required eqn of line will be

$$y-3 = \frac{1}{2}(x-(-4))$$

i.e $2y-6 = x+4$

$\Rightarrow$ $x-2y+10=0$

Q No 3 Passing through $(0,0)$ with slope m.

Sol. Using one point form eqn of required line will be.

$$y-0 = m(x-0)$$

$$y = mx.$$

Q No 4: Passing through $(2, 2\sqrt{3})$ and inclined with x-axis at an angle of $75°$.

Soln: Angle of inclination $= 75°$

$\therefore$ Slope $m = \tan 75° = \tan(45°+30°)$

$$= \frac{\tan 45° + \tan 30°}{1 - \tan 45° \tan 30°}$$

$$= \frac{1+\frac{1}{\sqrt{3}}}{1 - 1\times\frac{1}{\sqrt{3}}} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

$\therefore$ Required equation will be.

$$y - 2\sqrt{3} = \frac{\sqrt{3}+1}{\sqrt{3}-1}(x-2)$$

$$(y-2\sqrt{3})(\sqrt{3}-1) = (\sqrt{3}+1)(x-2)$$

$\Rightarrow$ $(\sqrt{3}y - y) - 2\sqrt{3}(\sqrt{3}-1) = (\sqrt{3}+1)x - 2(\sqrt{3}+1)$

$\Rightarrow$ $(\sqrt{3}-1)y - 6 + 2\sqrt{3} = (\sqrt{3}+1)x - 2\sqrt{3} - 2$

$\Rightarrow$ $(\sqrt{3}+1)x - (\sqrt{3}-1)y = -6 + 2\sqrt{2} + 2\sqrt{3} + 2$

$\Rightarrow$ $(\sqrt{3}+1)x - (\sqrt{3}-1)y = 4\sqrt{3} - 4$

$\Rightarrow$ $(\sqrt{3}+1)x - (\sqrt{3}-1)y = 4(\sqrt{3}-1)$

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QNo5: Intersecting the x-axis at a distance of 3 units to the left of origin with slope -2.

Soln: Since line meets x-axis at 3 units to the left of origin $\therefore$ Line meets x-axis at $(-3,0)$ and slope of line $m = -2$.

$\therefore$ Using Point-slope form required eqn of line will be

$$y - 0 = -2(x-(-3))$$

$$y = -2x - 6$$

$$\Rightarrow 2x + y + 6 = 0$$

QNo6: Intersecting y-axis at a distance of 2 units above the origin and making an angle of 30° with +ve direction of x-axis.

Soln: Since line intersect y-axis at 2 units above origin.

$\therefore$ Line passes through the point $(0,2)$

Slope of line $= \tan 30° = \frac{1}{\sqrt{3}}$

$\therefore$ Using Point-Slope form, required equation will be

$$y - 2 = \frac{1}{\sqrt{3}}(x - 0)$$

$$\Rightarrow \sqrt{3}(y-2) = x$$

$$\Rightarrow x - \sqrt{3}y + 2\sqrt{3} = 0.$$

QNo.7: Passing through points $(-1,1)$ and $(2,-4)$

Soln: Using Two-Point form.

$$y - 1 = \frac{-4-1}{2-(-1)}(x-(-1))$$

$$\Rightarrow y - 1 = \frac{-5}{3}(x+1)$$

$$\Rightarrow 3(y-1) = -5(x+1)$$

$$\Rightarrow 3y - 3 = -5x - 5$$

$$\Rightarrow 5x + 3y + 2 = 0$$

Q.No.8. Perpendicular distance from origin is 5 units and the angle made by the perpendicular with the +ve x-axis is 30°.

Soln: Here perpendicular distance of line from origin

$p = 5$

and. $\omega = 30°$

∴ Using Normal form of equation, the required eqn will be

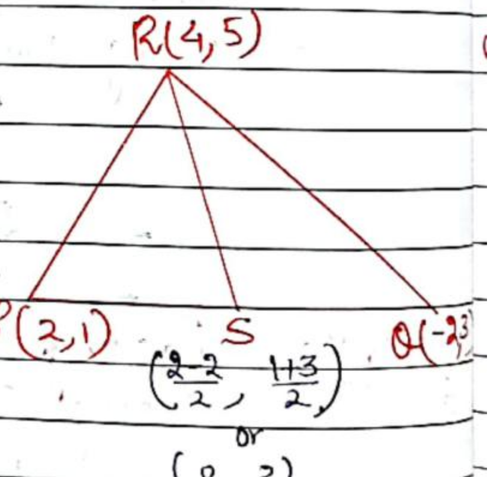
$$x\cos 30° + y\sin 30° = 5$$

ie $$x \cdot \frac{\sqrt{3}}{2} + y \cdot \frac{1}{2} = 5$$

$$\Rightarrow \sqrt{3}x + y = 10$$

Q.No.9: The vertices of $\triangle PQR$ are $P(2,1)$; $Q(-2,3)$ and $R(4,5)$. Find the equation of median through the vertex R.

Soln: Let S be the mid point of PQ



The coordinates of S will be using mid point formula.

$$S\left(\frac{2-2}{2}, \frac{1+3}{2}\right)$$

or $S(0, 2)$

Now Equation of median through $R(4,5)$ and $S(0,2)$ will be

$$y - 5 = \frac{2-5}{0-4}(x-4) \quad \text{[Two point form]}$$

$$\Rightarrow y - 5 = \frac{-3}{-4}(x-4)$$

$$\Rightarrow y - 5 = \frac{3}{4}(x-4)$$

$$\Rightarrow 4y - 20 = 3x - 12$$

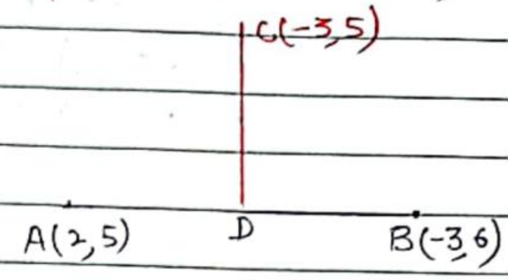
$$\Rightarrow 3x - 4y + 8 = 0$$

Q No 10 Find the eqn. of line passing through $(-3,5)$ and perpendicular to line through the pts. $(2,5)$ and $(-3,6)$

Sol: Let $A(2,5)$; $B(-3,6)$

Then slope of line AB

$$= \frac{6-5}{-3-2} = -\frac{1}{5}$$



$\therefore$ Slope of line CD which is perpendicular to $AB = m = +5$

$\therefore$ Eqn of reqd line (l) passing through C and $\perp$r to AB will be

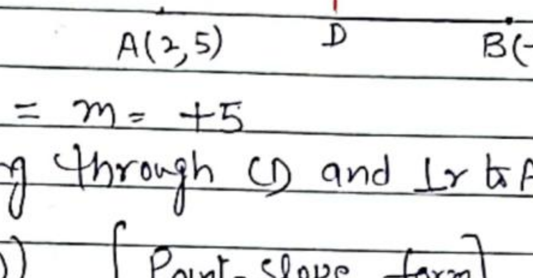
$$y-5 = 5(x-(-3)) \quad \text{[Point-slope form]}$$

$$\Rightarrow y-5 = 5x+15$$

$$\Rightarrow 5x - y + 20 = 0$$

Q No 11 A line $\perp$ to the line segment joining the pts. $(1,0)$ and $(2,3)$ divides it in the ratio $1:n$. find the eqn of line.

Soln: Let CD be the line $\perp$ to the line segment joining $A(1,0)$ and $B(2,3)$ such that D divides AB in the ratio $1:n$.



$\therefore$ By Section Formula

$$D \leftrightarrow D\left(\frac{n+2}{n+1}, \frac{3}{n+1}\right)$$

Now slope of $AB = \frac{3-0}{2-1} = \frac{3}{1} = 3$.

$\therefore$ Slope of $CD = -\frac{1}{3}$ [-ve reciprocal Since $AB \perp CD$]

$\therefore$ Eq. of CD is

$$y - \frac{3}{n+1} = -\frac{1}{3}\left(x - \frac{n+2}{n+1}\right)$$

$$\Rightarrow y - \frac{3}{n+1} = -\frac{1}{3}x + \frac{n+2}{3(n+1)}$$

$$\Rightarrow 3(n+1)y - 9 = -(n+1)x + (n+2)$$

$$\text{or} \quad (n+1)x + 3(n+1)y = n+11$$

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Q No. 12 Find the eqn of line that cuts off equal intercepts on the coordinate axes and passes through pts (2,3)

Soln: Let the reqd line makes equal intercepts on axes equal to 'a'.

$\therefore$ By Intercept form eqn of line will be

$$\frac{x}{a} + \frac{y}{a} = 1 \quad \text{—} (1)$$

Since (1) passes through (2,3)

$$\therefore \quad \frac{2}{a} + \frac{3}{a} = 1$$

$$\Rightarrow \quad \frac{5}{a} = 1 \Rightarrow a = 5$$

$\therefore$ Reqd eqn is $\frac{x}{5} + \frac{y}{5} = 1$ or $x + y = 5$.

Q No. 13 Find the eqn. of line passing through the pt (2,3) and cutting off intercepts on axes whose sum is 9

Sol: Let the reqd line makes x-intercept = a

ATQ $\therefore$ y-intercept = $9 - a$

Now eqn of line using intercept form

$$\frac{x}{a} + \frac{y}{9-a} = 1$$

$$\Rightarrow \quad x(9-a) + y \times a = a(9-a) \quad \text{—} (1)$$

Since (1) passes through pt. (2,2)

$$\therefore \quad 2(9-a) + 2a = a(9-a)$$

$$\Rightarrow \quad 18 - \cancel{2a} + \cancel{2a} = 9a - a^2$$

$$\Rightarrow \quad a^2 - 9a + 18 = 0$$

$$\Rightarrow \quad a = \frac{9 \pm \sqrt{81-72}}{2} = \frac{9 \pm 3}{2} = 6, 3$$

When $a = 6$, $9 - a = 3$	When $a = 3$ $9-a$.
$\therefore$ eqn Line will be	eqn of Line will be
$\frac{x}{6} + \frac{y}{3} = 1$	$\frac{x}{3} + \frac{y}{6} = 1$
or $\cancel{x}$ $2y = 6$	or $2x + y = 6$

Q.No.14: Find the eqn of line through the point $(0,2)$ making an angle $\frac{2\pi}{3}$ with the +ve x-axis. Also find the eqn of line parallel to it crossing the y-axis at a distance of 2 units below the origin.

Soln.: Let m be the slope of line through $(0,2)$ and making an angle $\frac{2\pi}{3}$ with x-axis

$$\therefore\ m = \tan\frac{2\pi}{3} = \tan\left(\pi - \frac{\pi}{3}\right) = -\tan\pi/3 = -\sqrt{3}.$$

$\therefore$ Equation of line is

$$y - 2 = -\sqrt{3}(x - 0) \quad [\because \text{Point-Slope form}]$$

ie $$\sqrt{3}x + y - 2 = 0$$

Now The equation of line through $(0,-2)$ parallel to line with slope $m = -\sqrt{3}$ is

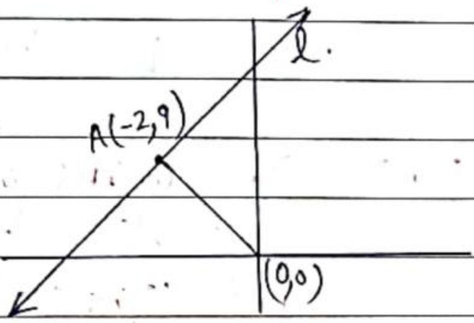
$$y - (-2) = -\sqrt{3}(x - 0) \quad [\because \parallel \text{ lines have same slope}]$$

$$\Rightarrow \sqrt{3}x + y + 2 = 0.$$

Q.No.15: The perpendicular to the line from origin meets it at the point $(-2,9)$. Find the equation of line.

Soln: Let l be the reqd line.

and Let $A(-2,9)$ be the foot of $\perp r$ from O to line l.



Then slope of OA

$$= \frac{9-0}{-2-0} = \frac{9}{-2}$$

Since line is $\perp r$ to OA.

$\therefore$ Slope of reqd line l will be $m = +\frac{2}{9}$

$\therefore$ Equation of reqd line passing through $(-2,9)$ will be

$$y - 9 = \frac{2}{9}(x - (-2))$$

$$9y - 81 = 2x + 4$$

$$\Rightarrow 2x - 9y + 85 = 0$$

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Q No-16: The length L (in cms) of a copper rod is a linear fn. of its Celsius temp. C. In an experiment if L = 124.942 when C = 20 and L = 125.134 when C = 110. Express L in terms of C.

Soln:

When $C = 20$; $L = 124.942$

When $C = 110$; $L = 125.134$

∴ Equation of line passing through $(20, 124.942)$ and $(110, 125.134)$ will be.

$$L - 124.942 = \frac{125.134 - 124.942}{110 - 20}(C - 20)$$

[Two point form]

$$\Rightarrow 90(L - 124.942) = 0.192(C - 20) \quad \text{—— (1)}$$

$$\Rightarrow 90L - 11244.78 = 0.192C - 3.84$$

$$\Rightarrow 90L = 0.192C + 11240.94$$

$$\Rightarrow L = \frac{0.192C}{90} + 124.856$$

OR, Simply from (1):

$$L = \frac{0.192(C-20)}{90} + 124.942$$

Q No.17: The owner of milk store find that he can sell 980 litres of milk each week at Rs 14/litre and 1220 litres of milk each week at Rs 16/litre. Assuming a linear relationship between selling price and demand, how many litres could he sell weekly at Rs 17/litre?

Soln: Let y litre of milk is sold at Rs x/litre

x and y have linear relationship

Now Let x_1 = Rs 14/litre $y_1 = 980$

x_2 = ~~1220 litre~~ 16/litre $y_2 = 1220$

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Then By Two point form, equation of line will be.

$$y - 980 = \frac{1220-980}{16-14}(x-14)$$

ie $y - 980 = \frac{240}{2}(x-14)$

ie $y - 980 = 120(x-14)$

ie $y - 980 = 120x - 120 \times 14$

or So $y = 980 + 120(x-14)$.

When $x =$ Rs 17 / litre

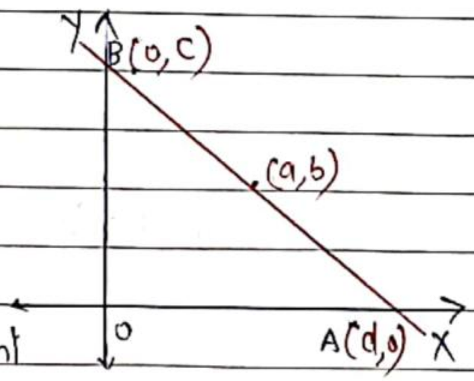
$$y = 980 + 120(17-14)$$
$$= 980 + 120 \times 3$$
$$= 980 + 360 = 1340$$

Hence 1340 litres milk may be sold at Rs 17/ litre.

Q No 18. P(a,b) is the mid point of a line segment between axes. Show that the equation of line is $\frac{x}{a} + \frac{y}{b} = 2$

Soln: Let the required line makes intercepts d, c respectively on x-axis and y-axis

∴ The line meets the axes A (d,0) and B(0,c) as shown.



Now since P(a,b) is midpoint of AB.

$$\therefore \frac{0+d}{2} = a \quad \text{and} \quad \frac{c+0}{2} = b$$

$$\Rightarrow d = 2a \quad \text{and} \quad c = 2b.$$

∴ By Intercept form.

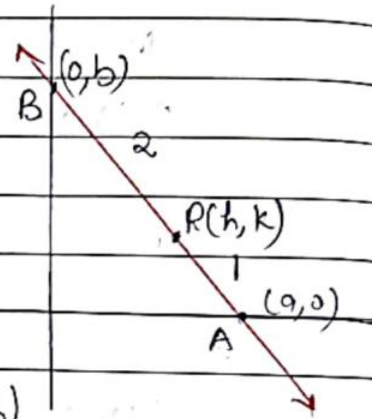
The equation of line will be.

$$\frac{x}{2a} + \frac{y}{2b} = 1$$

or $\frac{1}{2}\left[\frac{x}{a} + \frac{y}{b}\right] = 1$ or $\frac{x}{a} + \frac{y}{b} = 2.$

QNo. 19: Point $R(h,k)$ divides a line segment between the axes in the ratio 1:2. Find equation of line.

Soln: Let the line make intercept a, b on the axes



$\therefore$ Equation of line is

$$\frac{x}{a} + \frac{y}{b} = 1 \quad \text{—(1)}$$

Now the line meet x-axis in $A(a,0)$ and y-axis in $B(0,b)$

$\because$ $R(h,k)$ divides AB in ratio 1:2

$\therefore$ By Section formula:

$$h = \frac{1(0)+2(a)}{1+2} = \frac{2a}{3}$$

$$\Rightarrow a = \frac{3h}{2}.$$

and $k = \frac{1(b)+2(0)}{1+2} = \frac{b}{3}$

$$\Rightarrow b = 3k.$$

$\therefore$ (1) becomes

$$\frac{x}{\frac{3h}{2}} + \frac{y}{3k} = 1$$

or $\frac{2x}{3h} + \frac{y}{3k} = 1$

or $2kx + hy = 3hk$. which is required equation.

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Q.No 20: By Using the concept of equation of a line prove that the three points $(3,0)$ $(-2,-2)$ and $(8,2)$ are Collinear.

Sol.: Three points will be collinear if they lie on same line, i.e. If all points satisfy the equation of line.
Now Equation of line through $(3,0)$ and $(-2,-2)$ is

$$y-0 = \frac{-2-0}{-2-3}(x-3)$$

$$\Rightarrow \quad y = \frac{-2}{-5}(x-3)$$

$$\Rightarrow \quad 5y = 2x-6$$

$$\Rightarrow \quad 2x-5y-6=0 \quad \text{—} (1)$$

Substituting the third point in (1)

$$\text{LHS} \quad 2(8)-5(2)-6$$
$$= 16-10-6$$
$$= 0$$
$$= \text{RHS}$$

∴ Third points also lies on (1)

∴ All the points are collinear.

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