

LIMITS AND CONTINUITY

SYNOPSIS

DEFINITIONS :

- If $f: A \rightarrow B$, A and B are sub-sets of R , then f is called a real function.
- The function $f: R^+ \rightarrow R$; $f(x) = \log x$ (or $\ln x$) is called the natural logarithmic function.
- $f: R \rightarrow R^+$, $f(x) = e^x$, is called the exponential function.
- $f: R \rightarrow R$, $f(x) = [x]$ (the greatest integer not exceeding x), is called the greatest integer function.

NEIGHBOURHOOD:

Let a and δ be two real numbers, then $(a-\delta, a+\delta)$ is called δ - neighbourhood of a . The interval $(a-\delta, a)$ is called the left δ - neighbourhood of ' a ' and the interval $(a, a+\delta)$ is called the right δ - neighbourhood of a . The deleted δ neighbourhood of ' a ' is denoted by $(a-\delta, a) \cup (a, a+\delta)$

LIMIT OF A FUNCTION:

Let f be a function defined in a deleted neighbourhood of a . For every positive real number ϵ , there exists another positive real number δ , such that $|f(x) - l| < \epsilon$ whenever $0 < |x - a| < \delta$. Then the function $f(x)$ is said to be tending to the limit l as x tends to a and it is denoted by

$$\lim_{x \rightarrow a} f(x) = l$$

RIGHT AND LEFT LIMITS:

Let f be defined in a right neighbourhood of a and ' l ' be any real number, if For every $\epsilon > 0$, there exists $\delta > 0$ such that $a < x < a + \delta \Rightarrow |f(x) - l| < \epsilon$, then the function $f(x)$ tends to l as x tends to ' a ' from right hand side and is denoted by

$$\lim_{x \rightarrow a^+} f(x) = l$$

Similarly if for every $\epsilon > 0$, there exists $\delta > 0$ such that $a - \delta < x < a \Rightarrow |f(x) - l| < \epsilon$, then $f(x)$ tends to l as x tends to ' a ' from left hand side and is denoted by $\lim_{x \rightarrow a^-} f(x) = l$

If the right and left limits of a function are equal, then the function is an existing limit function. If

$$\lim_{x \rightarrow a^+} f(x) = \lim_{x \rightarrow a^-} f(x), \text{ then } \lim_{x \rightarrow a} f(x) \text{ exists.}$$

- **INFINITE LIMITS:** Let f be a function defined in a deleted neighbourhood of ' a ', if For every $\epsilon > 0$, there exists a $\delta > 0$ such that $0 < |x - a| < \delta \Rightarrow f(x) > \epsilon$, then the function $f(x)$ tends to $+\infty$ as x tends to ' a ' and is written as $\lim_{x \rightarrow a} f(x) = \infty$

Similarly we define $\lim_{x \rightarrow a} f(x) = -\infty$

PROPERTIES OF LIMITS :

- $\lim_{x \rightarrow a} [f(x) \pm g(x)] = \lim_{x \rightarrow a} f(x) \pm \lim_{x \rightarrow a} g(x)$
- $\lim_{x \rightarrow a} f(x) \cdot g(x) = \lim_{x \rightarrow a} f(x) \cdot \lim_{x \rightarrow a} g(x)$
- $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)}$ where $\lim_{x \rightarrow a} g(x) \neq 0$
- $\lim_{x \rightarrow a} (C \cdot f(x)) = C \cdot \lim_{x \rightarrow a} f(x)$
- $\lim_{x \rightarrow a} [k \pm f(x)] = k \pm \lim_{x \rightarrow a} f(x)$
- $\lim_{x \rightarrow a} [f(x)]^n = \left[\lim_{x \rightarrow a} f(x) \right]^n$
- $\lim_{x \rightarrow a} |f(x)| = \left| \lim_{x \rightarrow a} f(x) \right|$
- $\lim_{x \rightarrow a} f[g(x)] = f \left[\lim_{x \rightarrow a} g(x) \right]$
- $\lim_{x \rightarrow a} [f(x)]^{g(x)} = \left[\lim_{x \rightarrow a} f(x) \right]^{\lim_{x \rightarrow a} g(x)}$

STANDARD FORMULAE

- For all real values of n , $\lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = n \cdot a^{n-1}$
- If $0 < |x| < \frac{\pi}{2}$ and x is measured in radians.

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \text{ and } \lim_{x \rightarrow 0} \frac{\tan x}{x} = 1$$

- If x is measured in degrees

$$\lim_{x \rightarrow 0} \frac{\sin x^0}{x^0} = 1 \text{ and } \lim_{x \rightarrow 0} \frac{\tan x^0}{x^0} = 1$$

- $\lim_{x \rightarrow \infty} \frac{\sin x}{x} = 0$ and $\lim_{x \rightarrow \infty} \frac{\cos x}{x} = 0$

- $\lim_{x \rightarrow 0} \frac{\sin^{-1} x}{x} = 1$ and $\lim_{x \rightarrow 0} \frac{\tan^{-1} x}{x} = 1$

$$\bullet \lim_{x \rightarrow 0} \frac{\sinh x}{x} = 1 \text{ and } \lim_{x \rightarrow 0} \frac{\tanh x}{x} = 1$$

$$\bullet \lim_{x \rightarrow 0} \frac{\sinh^{-1} x}{x} = 1 \text{ and } \lim_{x \rightarrow 0} \frac{\tanh^{-1} x}{x} = 1$$

$$\bullet \lim_{x \rightarrow 0} (1+x)^{\frac{1}{x}} = e$$

$$\bullet \lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = e$$

$$\bullet \lim_{x \rightarrow 0} \left(\frac{e^x - 1}{x}\right) = 1$$

$$\bullet \lim_{x \rightarrow 0} \left(\frac{a^x - 1}{x}\right) = \log_e a \quad (a > 0)$$

$$\bullet \lim_{x \rightarrow 0} \frac{a^x - b^x}{x} = \log_e \left(\frac{a}{b}\right)$$

$$\bullet \lim_{x \rightarrow 0} \frac{a^x - 1}{b^x - 1} = \log_b a$$

• INDETERMINATE FORMS

The values of $\frac{0}{0}, \frac{\infty}{\infty}, \infty - \infty, 0 \times \infty, 0^0, \infty^0, 1^\infty, 0^\infty$ etc. are indeterminate forms

• L'HOSPITAL'S RULE

If $\lim_{x \rightarrow a} \frac{f(x)}{g(x)}$ = indeterminate then

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$$

If $\lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$ = indeterminate then

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f^{(11)}(x)}{g^{(11)}(x)}$$

If $\lim_{x \rightarrow a} \frac{f^{(11)}(x)}{g^{(11)}(x)}$ = indeterminate, then proceed as above till a finite limit is obtained.

• CONTINUITY OF A FUNCTION:

• A function $f(x)$ is said to be continuous at $x=a$ if

$$\lim_{x \rightarrow a} f(x) = f(a)$$

• If $\lim_{x \rightarrow a} f(x) \neq f(a)$ then $f(x)$ is discontinuous function at $x=a$.

• If $\lim_{x \rightarrow a^+} f(x) = f(a)$ then $f(x)$ is right continuous function at $x=a$.

• If $\lim_{x \rightarrow a^-} f(x) = f(a)$ then $f(x)$ is left continuous function at $x=a$.

• If f is continuous at $x=a$ and g is continuous at $f(a)$, then $g \circ f$ is continuous at $x=a$.

• Every constant function is continuous on $\mathbb{R}$.

• The identity function is continuous on $\mathbb{R}$.

• Every polynomial function is continuous on $\mathbb{R}$.

• The functions $\sin x$, $\cos x$ are continuous on $\mathbb{R}$.

• The functions $\tan x$ and $\sec x$ are continuous on $\mathbb{R} - \left\{(2n+1) \cdot \frac{\pi}{2}\right\}$ where n is any integer

• The functions $\cot x$ and $\operatorname{cosec} x$ are continuous on $\mathbb{R} - \{n\pi\}$ where n is any integer

• The function $f(x) = |x|$ is continuous on $\mathbb{R}$.

• The functions $f(x) = e^x$ and $f(x) = a^x (a > 0)$ are continuous on $\mathbb{R}$.

• The function $f(x) = [x]$ is continuous at all non integral values and discontinuous at all integral values.

• IMPORTANT FORMULAE:

• Let $S = \{x, \sin x, \tan x, \sinh x, \tanh x, \sin^{-1} x, \tan^{-1} x, \sinh^{-1} x, \tanh^{-1} x\}$

$$\text{If } f(x), g(x) \in S \quad \text{then } \lim_{x \rightarrow 0} \frac{f(mx)}{g(nx)} = \frac{m}{n}$$

$$\text{Ex: } \lim_{x \rightarrow 0} \frac{\sin ax}{\tan bx} = \frac{a}{b}$$

• If $f_1(x), f_2(x), g_1(x), g_2(x) \in S$ then

$$\lim_{x \rightarrow 0} \frac{f_1(mx) \pm f_2(nx)}{g_1(px) \pm g_2(qx)} = \frac{m \pm n}{p \pm q}$$

$$\text{Ex: } \lim_{x \rightarrow 0} \frac{\sin 7x + \sin 5x}{\tan 5x - \tan 2x} = \frac{7+5}{5-2} = 4$$

• If $f_1(x), f_2(x), g_1(x), g_2(x) \in S$ and $m+n=p$

$$+q \text{ then } \lim_{x \rightarrow 0} \frac{f_1^m(ax) f_2^n(bx)}{g_1^p(cx) g_2^q(dx)} = \frac{a^m b^n}{c^p d^q}$$

$$\text{Ex: } \lim_{x \rightarrow 0} \frac{\sin^3 2x \tan^2 3x}{x \sin^4 4x} = \frac{2^3 \times 3^2}{4^4} = \frac{9}{32}$$

- If $g_1(x), g_2(x) \in S$ then

$$\lim_{x \rightarrow 0} \frac{1 - \cos ax}{g_1(cx) g_2(dx)} = \frac{a^2}{2cd}$$

$$\text{Ex: } \lim_{x \rightarrow 0} \frac{1 - \cos x}{x \sin 3x} = \frac{1}{6}$$

- If $g_1(x), g_2(x) \in S$ then

$$\lim_{x \rightarrow 0} \frac{1 - \cos^n(ax)}{g_1(cx) g_2(dx)} = \frac{na^2}{2cd}$$

$$\text{Ex: } \lim_{x \rightarrow 0} \frac{1 - \cos^3 2x}{\sin 5x \tan 7x} = \frac{3 \times 2^2}{2 \times 5 \times 7} = \frac{6}{35}$$

- If $g_1(x), g_2(x), \dots, g_{2n}(x) \in S$ then

$$\lim_{x \rightarrow 0} \frac{1 - \cos(ax^n)}{g_1(c_1x) g_2(c_2x) \dots g_{2n}(c_{2n}x)} = \frac{a^2}{2c_1c_2 \dots c_{2n}}$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos(2x^3)}{x \sin^2(2x) \tan^3(3x)} = \frac{4}{2 \times 2^2 \times 3^3} = \frac{1}{54}$$

- If $g_1(x), g_2(x) \in S$ then

$$\lim_{x \rightarrow 0} \frac{\cos ax - \cos bx}{f(cx) g(dx)} = \frac{b^2 - a^2}{2cd}$$

$$\text{Ex: } \lim_{x \rightarrow 0} \frac{\cos 3x - \cos 5x}{x^2} = \frac{25 - 9}{2} = 8$$

- If $g_1(x), g_2(x) \in S$ then

$$\lim_{x \rightarrow 0} \frac{\cos^n ax - \cos^n bx}{g_1(cx) g_2(dx)} = \frac{n(b^2 - a^2)}{2cd}$$

$$\text{Ex: } \lim_{x \rightarrow 0} \frac{\cos^3 3x - \cos^3 5x}{x^2} = \frac{3(25 - 9)}{2} = 24$$

- If $g_1(x), g_2(x), \dots, g_{2n}(x) \in S$ then

$$\lim_{x \rightarrow 0} \frac{\cos(ax^n) - \cos(bx^n)}{g_1(c_1x) g_2(c_2x) \dots g_{2n}(c_{2n}x)} = \frac{b^2 - a^2}{2c_1c_2 \dots c_{2n}}$$

$$\lim_{x \rightarrow 0} \frac{\cos(2x^3) - \cos(5x^3)}{x \sin^2(2x) \tan^3(3x)} = \frac{25 - 4}{2 \times 2^2 \times 3^3} = \frac{7}{18}$$

- If $g_1(x) \in S$ then

$$\lim_{x \rightarrow 0} \frac{\tan^n(ax) - \sin^n(ax)}{[g(x)]^{n+2}} = \frac{na^{n+2}}{2}$$

$$\text{Ex: } \lim_{x \rightarrow 0} \frac{\tan x - \sin x}{x^3} = \frac{1}{2}$$

- $\lim_{x \rightarrow 0} \frac{\sqrt{1+x^n} - \sqrt{1-x^n}}{x^n} = 1$

$$\text{Ex: } \lim_{x \rightarrow 0} \frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{x^2} = 1$$

- $\lim_{x \rightarrow 0} \frac{\sqrt[n]{a+x^m} - \sqrt[n]{a-x^m}}{x^m} = \frac{2}{n} a^{\frac{1}{n}-1}$

$$\text{Ex: } \lim_{x \rightarrow 0} \frac{\sqrt{a+x} - \sqrt{a-x}}{x} = a^{\frac{1}{2}-1} = \frac{1}{\sqrt{a}}$$

- If $g(x) \in S$ then $\lim_{x \rightarrow 0} \frac{\sqrt{a+x} - \sqrt{a-x}}{g(x)} = \frac{1}{\sqrt{a}}$

$$\text{Ex: } \lim_{x \rightarrow 0} \frac{\sqrt{3+x} - \sqrt{3-x}}{\sin x} = \frac{1}{\sqrt{3}}$$

- If $g(x) \in S$ then $\lim_{x \rightarrow 0} \frac{\sqrt{a+x} - \sqrt{a}}{g(x)} = \frac{1}{2\sqrt{a}}$

$$\text{Ex: } \lim_{x \rightarrow 0} \frac{\sqrt{2+x} - \sqrt{2}}{x} = \frac{1}{2\sqrt{2}}$$

- $\lim_{x \rightarrow 0} \frac{x \cdot a^{ax} - x}{1 - \cos(mx)} = \frac{2\alpha}{m^2} \log a$

$$\text{Ex: } \lim_{x \rightarrow 0} \frac{x \cdot 2^{3x} - x}{1 - \cos(3x)} = \frac{2 \times 3}{3^2} \log 2 = \frac{2}{3} \log 2$$

- $\lim_{x \rightarrow 0} (1 + ax)^{\frac{1}{x}} = e^a$

$$\text{Ex: } \lim_{x \rightarrow 0} (1 + 2x)^{\frac{1}{x}} = e^2$$

- $\lim_{x \rightarrow \infty} \left(1 + \frac{a}{x}\right)^x = e^a$

$$\text{Ex: } \lim_{x \rightarrow \infty} \left(1 + \frac{2}{x}\right)^x = e^2$$

• $\lim_{x \rightarrow \infty} \frac{a^{x+\alpha} + b}{a^{x+\beta} + c} = a^{\alpha-\beta}$

Ex: $\lim_{x \rightarrow \infty} \frac{2^{x+7} + 13}{2^{x+5} + 10} = 2^{7-5} = 4$

• $\lim_{x \rightarrow \infty} \left\{ \sqrt{x^2 + ax + b} - x \right\} = \frac{a}{2}$

Ex: $\lim_{x \rightarrow \infty} \left\{ \sqrt{x^2 - 2x + 5} - x \right\} = \frac{-2}{2} = -1$

LEVEL-I

1. $\lim_{x \rightarrow a} \frac{x^{\frac{5}{8}} - a^{\frac{5}{8}}}{x^{\frac{1}{3}} - a^{\frac{1}{3}}} =$

1) $\frac{15}{8} a^{\frac{7}{24}}$ 2) $\frac{15}{4} a^{\frac{7}{24}}$

3) $-\frac{15}{8} a^{\frac{7}{24}}$ 4) $\frac{15}{4} a^{-\frac{7}{24}}$

2. $\lim_{x \rightarrow 2} \frac{x^5 - 32}{x^3 - 8} =$

1) $\frac{3}{20}$ 2) $\frac{20}{3}$ 3) $\frac{10}{3}$ 4) $\frac{3}{10}$

3. $\lim_{x \rightarrow 1} \frac{x^{\frac{2}{3}} - 1}{x^{\frac{3}{4}} - 1} =$

1) $\frac{5}{9}$ 2) $\frac{9}{5}$ 3) $\frac{8}{9}$ 4) 0

4. $\lim_{x \rightarrow a} \frac{x^p - a^p}{x^q - a^q} =$

1) $\frac{p}{q} a^{p-q}$ 2) $pq a^{p-q}$ 3) $\frac{q}{p} a^{q-p}$ 4) a^{q-p}

5. $\lim_{x \rightarrow 2} \frac{7x^2 - 11x - 6}{3x^2 - x - 10} =$

1) $\frac{17}{11}$ 2) $\frac{11}{17}$ 3) $\frac{17}{14}$ 4) $-\frac{17}{11}$

6. $\lim_{x \rightarrow 3} \frac{2x^2 - 5x - 3}{x^2 - 4x + 3} =$

1) 7 2) 14 3) $\frac{2}{7}$ 4) $\frac{7}{2}$

7. $\lim_{x \rightarrow 3} \frac{x^3 - 6x - 9}{x^4 - 81} =$

1) $\frac{7}{36}$ 2) $\frac{36}{7}$ 3) $\frac{1}{36}$ 4) $-\frac{1}{36}$

8. $\lim_{x \rightarrow 1} \frac{(2x-3)(\sqrt{x}-1)}{2x^2 + x - 3} =$

1) $\frac{1}{10}$ 2) $-\frac{1}{10}$ 3) $\frac{2}{5}$ 4) $-\frac{2}{5}$

9. $\lim_{x \rightarrow 0} \frac{a^x - b^x}{x} =$

1) $\log\left(\frac{a}{b}\right)$ 2) $\log\left(\frac{b}{a}\right)$ 3) $\log(ab)$ 4) $\log_b a$

10. $\lim_{x \rightarrow \sqrt{3}} \frac{x^2 - 3}{x^4 + x^2 + 1} =$

1) 0 2) -1 3) 1 4) 2

11. $\lim_{x \rightarrow \frac{1}{2}} \frac{8x^3 - 1}{6x^2 - 5x + 1} =$

1) 6 2) 3 3) $\frac{1}{3}$ 4) $\frac{1}{6}$

12. $\lim_{x \rightarrow 1} \frac{x^2 - \sqrt{x}}{\sqrt{x} - 1} =$

1) 1 2) 2 3) 3 4) 4

13. $\lim_{x \rightarrow 0} \frac{\log(1+ax) - \log(1+bx)}{x} =$

1) $a+b$ 2) $a-b$ 3) $-(a+b)$ 4) ab

14. $\lim_{x \rightarrow 0} \frac{e^x + \sin x - 1}{\log(1+x)} =$

1) 1 2) $\frac{1}{3}$ 3) $\frac{2}{3}$ 4) 2

15. $\lim_{x \rightarrow e} \frac{\log x - 1}{x - e} =$

1) 1 2) $\frac{1}{e}$ 3) e^2 4) e

16. $\lim_{x \rightarrow 0} \frac{e^{2x} - 1}{3x} =$

1) $\frac{2}{3}$ 2) 6 3) $\frac{3}{2}$ 4) $\frac{1}{6}$

17. $\lim_{x \rightarrow 0} \frac{a^x - 1}{b^x - 1} =$

- 1) $\log_b^a$ 2) $\log_a^b$ 3) $\log_c^{ab}$ 4) $\log_e\left(\frac{a}{b}\right)$

18. $\lim_{x \rightarrow 0} \frac{e^x - 1}{\sqrt{1+x} - 1} =$

- 1) 1 2) $\frac{1}{2}$ 3) $\frac{1}{3}$ 4) 2

19. $\lim_{x \rightarrow a} \frac{\log x - \log a}{\tan(x-a)} =$

- 1) $\frac{1}{a}$ 2) a 3) 0 4) $\frac{2}{a}$

20. $\lim_{x \rightarrow 0} \frac{5^x - 2^x}{\sin x} =$

- 1) $\log\left(\frac{5}{2}\right)$ 2) $\log\left(\frac{2}{5}\right)$
3) $\log 10$ 4) $\log 2$

21. $\lim_{x \rightarrow \frac{1}{2}} \left(\frac{8x-3}{2x-1} - \frac{4x^2+1}{4x^2-1} \right) =$

- 1) $\frac{7}{2}$ 2) $\frac{7}{3}$ 3) $\frac{5}{3}$ 4) $\frac{3}{5}$

22. $\lim_{x \rightarrow 0} \frac{e^x - e^{-x} - 2x}{x - \sin x} =$

- 1) 2 2) 3 3) 4 4) 5

23. If $f(x) = x^2 + x + 1$, then $\lim_{x \rightarrow 1} \frac{f(x) - f(1)}{x - 1} =$

- 1) 3 2) 0 3) -1 4) 2

24. $f(x) = x^2 - 3x + 5$, then $\lim_{x \rightarrow 2} \frac{f(x) - f(2)}{x - 2} =$

- 1) 0 2) 1 3) $\frac{1}{2}$ 4) 4

25. $\lim_{x \rightarrow 0} \frac{\sqrt[k]{1+x} - 1}{x}$ (K is a positive integer)

- 1) K 2) $-K$ 3) $\frac{1}{K}$ 4) $-\frac{1}{K}$

26. If $f(9) = 9$, $f'(9) = 4$, $\lim_{x \rightarrow 9} \frac{\sqrt{f(x)} - 3}{\sqrt{x} - 3} =$

- 1) 4 2) $\frac{1}{4}$ 3) $\frac{1}{2}$ 4) $-\frac{1}{2}$

27. If $f(a) = 2$, $f'(a) = 1$, $g(a) = -1$, $g'(a) = 2$, then

$\lim_{x \rightarrow a} \frac{g(x)f(a) - g(a)f(x)}{x - a} =$

- 1) $\frac{1}{5}$ 2) 5 3) $-\frac{1}{5}$ 4) -5

28. $\lim_{x \rightarrow \frac{1}{\sqrt{2}}} \frac{x - \cos(\sin^{-1} x)}{1 - \tan(\sin^{-1} x)} =$

- 1) $\frac{1}{\sqrt{2}}$ 2) $-\frac{1}{\sqrt{2}}$ 3) 0 4) 1

29. $\lim_{x \rightarrow 0} \frac{7^{3x} - 5^{4x}}{x} =$

- 1) $\log\left(\frac{125}{343}\right)$ 2) $\log\left(\frac{243}{125}\right)$
3) $\log\left(\frac{343}{625}\right)$ 4) $\log\left(\frac{625}{343}\right)$

30. $f(x) = \begin{cases} x & \text{If } x \leq 2 \\ x+5 & \text{If } 2 < x \leq 3 \\ x-7 & \text{If } x > 3 \end{cases}$ $\lim_{x \rightarrow 3+} f(x) =$

- 1) -2 2) -4 3) 0 4) 4

31. $\lim_{h \rightarrow 0} \frac{(2+h)\cos(2+h) - 2\cos 2}{h} =$

- 1) $\cos 2 - 2\sin 2$ 2) $\cos 2 + 2\sin 2$
3) $\sin 2 - 2\cos 2$ 4) $\sin 2 + 2\cos 2$

32. $\lim_{h \rightarrow 0} \frac{(a+h)^2 \sin(a+h) - a^2 \sin a}{h} =$

- 1) $a^2 \cos a - 2a \cos a$ 2) $a^2 \cos a + 2a \sin a$
3) $a^2 \sin a$ 4) $a \cos a - \sin a$

33. If $\lim_{x \rightarrow 0} \frac{\log(3+x) - \log(3-x)}{x} = K$, then the value of $K =$

- 1) $-\frac{2}{3}$ 2) 0 3) $-\frac{1}{3}$ 4) $\frac{2}{3}$

34. If $7 - \frac{x^2}{12} \leq f(x) \leq 7 + \frac{x^3}{5}$ for all $x \neq 0$, then

$$\lim_{x \rightarrow 0} f(x) =$$

- 1) 3 2) 5 3) 7 4) 9

35. If $\lim_{x \rightarrow 2} \frac{x^{n+1} - 2^{n+1}}{x - 2} = 192$ and $n \in N$, then $n =$

- 1) 4 2) 5 3) 3 4) 2

36. $\lim_{x \rightarrow 1} \frac{\left(\sum_{K=1}^{200} x^K \right) - 200}{x - 1} =$

- 1) 5050 2) 21000 3) 2010 4) 20100

37. $\lim_{x \rightarrow 0} \frac{(1+x)^2 - (1-x)^2}{(1+x)^3 - (1-x)^3} =$

- 1) 1 2) $\frac{1}{2}$ 3) $\frac{2}{3}$ 4) $\frac{3}{2}$

38. $\lim_{x \rightarrow 0} \frac{\sqrt[3]{8+x} - 2}{x} =$

- 1) $\frac{1}{10}$ 2) $\frac{1}{11}$ 3) $\frac{1}{12}$ 4) $\frac{1}{9}$

39. $\lim_{x \rightarrow 0} \frac{\sqrt[3]{1+x} - \sqrt[3]{1-x}}{x} =$

- 1) $\frac{1}{3}$ 2) $\frac{2}{3}$ 3) 0 4) 1

40. $\lim_{x \rightarrow 0} \frac{\sqrt{1+x+x^2} - 1}{x} =$

- 1) 1 2) $\frac{1}{2}$ 3) $\frac{1}{3}$ 4) 0

41. $\lim_{x \rightarrow 0} \frac{x}{\sqrt{4-x} - \sqrt{4+x}} =$

- 1) -2 2) $\frac{1}{2}$ 3) 2 4) 0

42. $\lim_{x \rightarrow -1} \frac{x+1}{\sqrt{x^2+3} - 2} =$

- 1) -2 2) $\frac{1}{2}$ 3) 2 4) 0

43. $\lim_{x \rightarrow 4} \frac{3 - \sqrt{5+x}}{x-4} =$

- 1) $-\frac{1}{5}$ 2) $\frac{1}{6}$ 3) $\frac{1}{5}$ 4) $-\frac{1}{6}$

44. $\lim_{x \rightarrow 0} \frac{x^2}{1 - \sqrt[3]{1+x^2}} =$

- 1) 1 2) $\frac{1}{2}$ 3) -3 4) $-\frac{1}{3}$

45. $\lim_{x \rightarrow 1} \frac{\sqrt[3]{x} - 1}{\sqrt{x} - 1} =$

- 1) $\frac{1}{3}$ 2) $\frac{2}{3}$ 3) 0 4) $-\frac{1}{3}$

46. $\lim_{x \rightarrow 2} \frac{x^5 \sqrt{x} - 32\sqrt{2}}{x^3 \sqrt{x} - 8\sqrt{2}} =$

- 1) $\frac{22}{7}$ 2) $\frac{41}{7}$ 3) $\frac{44}{7}$ 4) 2

47. $\lim_{x \rightarrow 0} \frac{\sqrt[5]{2+x} - \sqrt[5]{2-x}}{\sin hx} =$

- 1) 1 2) $\sqrt[5]{2}$ 3) $\frac{\sqrt{5}}{2}$ 4) $\frac{2^5}{5}$

48. $\lim_{x \rightarrow 0} \frac{\sqrt[3]{1+\sin x} - \sqrt[3]{1-\sin x}}{x} =$

- 1) 0 2) 1 3) $\frac{2}{3}$ 4) $\frac{3}{2}$

49. $\lim_{x \rightarrow 0} \frac{\sqrt{1+\tan x} - \sqrt{1-\tan x}}{\sin x} =$

- 1) 0 2) 1 3) 2 4) $\frac{1}{2}$

50. $\lim_{x \rightarrow 0} \frac{\sqrt{1+\sin x + \sin^2 x} - 1}{x} =$

- 1) 0 2) 1 3) 2 4) $\frac{1}{2}$

51. $\lim_{x \rightarrow 0} \frac{\sqrt{4+x} - \sqrt[3]{8+3x}}{x} =$

- 1) $-\frac{1}{2}$ 2) $\frac{1}{2}$ 3) -3 4) 0

52. $\lim_{x \rightarrow 0} \frac{x(1 - \sqrt{1 - x^2})}{\sqrt{1 - x^2}(\sin^{-1}(x))^3} =$
- 1) 1 2) $\frac{1}{2}$ 3) $-\frac{1}{2}$ 4) -1
53. $\lim_{x \rightarrow 1} \frac{\sqrt{x^2 - 1} + \sqrt{x - 1}}{\sqrt{x^2 - 1}} =$
- 1) $1 + \frac{1}{\sqrt{2}}$ 2) $1 - \frac{1}{\sqrt{2}}$
- 3) $-1 + \frac{1}{\sqrt{2}}$ 4) $-1 - \frac{1}{\sqrt{2}}$
54. $\lim_{x \rightarrow 0} \frac{\sin 3x \tan 4x}{x \sin 5x} =$
- 1) 1 2) $\frac{5}{12}$ 3) 0 4) $\frac{12}{5}$
55. $\lim_{x \rightarrow 0} \frac{\sin 2x \cdot \sin 3x}{3x \tan 4x} =$
- 1) $\frac{2}{3}$ 2) $\frac{3}{2}$ 3) 2 4) $\frac{1}{2}$
56. $\lim_{x \rightarrow 0} \frac{\sin 5x - \sin 3x}{x} =$
- 1) 2 2) 1 3) $\frac{1}{2}$ 4) $\frac{1}{3}$
57. $\lim_{x \rightarrow 0} \frac{1 - \cos ax}{x \sin 3x} =$
- 1) a^2 2) $\frac{a^2}{3}$ 3) $\frac{a^2}{6}$ 4) $-\frac{a^2}{3}$
58. $\lim_{x \rightarrow 0} \frac{1 - \cos mx}{1 - \cos nx} =$
- 1) m^2 2) n^2 3) $m^2 - n^2$ 4) $\frac{m^2}{n^2}$
59. $\lim_{x \rightarrow 0} \frac{x \sin 5x}{\sin^2 4x} =$
- 1) $\frac{5}{4}$ 2) $\frac{4}{5}$ 3) $\frac{5}{16}$ 4) 0

60. $\lim_{x \rightarrow 5} \frac{\sin^2(x - 5) \tan(x - 5)}{(x^2 - 25)(x - 5)} =$
- 1) 1 2) $\frac{1}{10}$ 3) 0 4) -6
61. $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\left(\frac{\pi}{2} - x\right) \sec x}{\cos ec x} =$
- 1) 1 2) 0 3) -1 4) $\frac{1}{\pi}$
62. $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\sec x \cdot \tan(4x - \pi)}{\sin(4x - \pi)} =$
- 1) $\sqrt{2}$ 2) $\frac{1}{\sqrt{2}}$ 3) $-\sqrt{2}$ 4) $\frac{-1}{\sqrt{2}}$
63. $\lim_{x \rightarrow \frac{\pi}{2}} \frac{1 - \sin x}{(\pi - 2x)^2} =$
- 1) $\frac{1}{3}$ 2) $\frac{1}{4}$ 3) $\frac{1}{6}$ 4) $\frac{1}{8}$
64. $\lim_{x \rightarrow \frac{\pi}{6}} \frac{\sin\left(x - \frac{\pi}{6}\right)}{\frac{\sqrt{3}}{2} - \cos x} =$
- 1) 1 2) $\frac{1}{3}$ 3) 2 4) 0
65. $\lim_{x \rightarrow 0} \frac{\sec 4x - \sec 2x}{\sec 3x - \sec x} =$
- 1) $\frac{3}{2}$ 2) $\frac{2}{3}$ 3) $\frac{1}{3}$ 4) $\frac{3}{4}$
66. $\lim_{x \rightarrow \alpha} \frac{\sin x - \sin \alpha}{\cos x - \cos \alpha} =$
- 1) $\tan \alpha$ 2) $\sin \alpha$ 3) $\cot \alpha$ 4) $-\cot \alpha$
67. $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x \log(1 + x)} =$
- 1) 1 2) 0 3) -1 4) $\frac{1}{2}$
68. $\lim_{x \rightarrow 0} \frac{\tan x - \sin x}{x^3} =$
- 1) 1 2) $\frac{1}{2}$ 3) $\frac{1}{3}$ 4) $\frac{1}{3}$

$$69. \lim_{x \rightarrow 0} \frac{\cos ax - \cos bx}{x^2} =$$

$$1) 0$$

$$2) 1$$

$$3) \frac{a^2 - b^2}{2}$$

$$4) \frac{b^2 - a^2}{2}$$

$$70. \lim_{x \rightarrow 0} \frac{\tan x^\circ}{x} =$$

$$1) 0$$

$$2) 1$$

$$3) \frac{\pi}{180}$$

$$4) \pi$$

$$71. \lim_{x \rightarrow 0} \frac{1 - \cos 2x^\circ}{x^2} =$$

$$1) 0$$

$$2) 1$$

$$3) \left(\frac{\pi}{180}\right)^2$$

$$4) 2 \cdot \left(\frac{\pi}{180}\right)^2$$

$$72. \lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos x}{\left(\frac{\pi}{2} - x\right) \sin x} =$$

$$1) 0$$

$$2) 1$$

$$3) -1$$

$$4) \frac{1}{2}$$

$$73. \lim_{x \rightarrow \frac{\pi}{2}} \frac{1 - \sin x}{x \cos x} =$$

$$1) 0$$

$$2) 1$$

$$3) 2$$

$$4) 4$$

$$74. \lim_{x \rightarrow 0} \frac{1 - \cos^3 x}{x \sin 2x} =$$

$$1) \frac{1}{2}$$

$$2) \frac{3}{2}$$

$$3) \frac{3}{4}$$

$$4) \frac{1}{4}$$

$$75. \lim_{x \rightarrow \frac{\pi}{2}} (\cos x + \cos 2x - \cos 3x) =$$

$$1) 1$$

$$2) 0$$

$$3) -1$$

$$4) 3$$

$$76. \lim_{x \rightarrow 0} \frac{\sin x \sin\left(\frac{\pi}{3} + x\right) \sin\left(\frac{\pi}{3} - x\right)}{x} =$$

$$1) \frac{3}{4}$$

$$2) \frac{1}{4}$$

$$3) \frac{4}{3}$$

$$4) 0$$

$$77. \lim_{x \rightarrow 0} \frac{e^{\alpha x} - e^{\beta x}}{\sin \alpha x - \sin \beta x} =$$

$$1) 0$$

$$2) \frac{1}{2}$$

$$3) \frac{1}{3}$$

$$4) 1$$

$$78. \lim_{x \rightarrow 0} \frac{e^{\tan x} - e^x}{\tan x - x} =$$

$$1) 1$$

$$2) e$$

$$3) \frac{1}{e}$$

$$4) 0$$

$$79. \lim_{x \rightarrow \frac{\pi}{2}} (\sec x - \tan x) =$$

$$1) 0$$

$$2) \frac{2}{\pi}$$

$$3) \frac{1}{\pi}$$

$$4) \pi$$

$$80. \lim_{x \rightarrow \frac{\pi}{2}} \frac{\operatorname{cosec} x - \cot x}{x} =$$

$$1) 1$$

$$2) \frac{1}{10}$$

$$3) \frac{1}{2}$$

$$4) \frac{1}{5}$$

$$81. \lim_{x \rightarrow 0} \left(\operatorname{cosec} x - \frac{1}{x} \right) =$$

$$1) 1$$

$$2) 0$$

$$3) \frac{1}{3}$$

$$4) 2$$

$$82. \lim_{x \rightarrow 0} \left(\frac{1}{\sin x} - \frac{1}{\tan x} \right) =$$

$$1) 0$$

$$2) \frac{1}{2}$$

$$3) \frac{1}{3}$$

$$4) -1$$

$$83. \lim_{x \rightarrow \frac{\pi}{2}} \left(\frac{\pi}{2} - x \right) \tan x =$$

$$1) 1$$

$$2) \frac{1}{2}$$

$$3) -1$$

$$4) -\frac{1}{2}$$

$$84. \lim_{x \rightarrow 1} (1 - x) \tan\left(\frac{\pi x}{2}\right) =$$

$$1) \pi$$

$$2) 2\pi$$

$$3) \frac{\pi}{2}$$

$$4) \frac{2}{\pi}$$

$$85. \lim_{x \rightarrow \infty} x \cos\left(\frac{\pi}{8x}\right) \sin\left(\frac{\pi}{8x}\right) =$$

$$1) \pi$$

$$2) \frac{\pi}{2}$$

$$3) \frac{\pi}{8}$$

$$4) \frac{\pi}{4}$$

$$86. \lim_{x \rightarrow \frac{\pi}{2}} \left(2x \tan x - \frac{\pi}{\cos x} \right) =$$

$$1) 1$$

$$2) \frac{1}{2}$$

$$3) -1$$

$$4) -2$$

87. $\lim_{x \rightarrow \frac{\pi}{2}} \tan x =$
 1) 1 2) 0
 3) $\frac{1}{\pi}$ 4) doesn't exist
88. $\lim_{x \rightarrow 0} \frac{\sin^3 ax \tan^2 bx}{x \sin^4 cx} =$
 1) $\frac{ab}{c}$ 2) $\frac{a^2 b^3}{c^4}$ 3) $\frac{c}{ab}$ 4) $\frac{a^3 b^2}{c^4}$
89. $\lim_{x \rightarrow 0} \frac{3 \sin x - \sin 3x}{x^3} =$
 1) 4 2) -4 3) 8 4) -8
90. $\lim_{x \rightarrow 0} \frac{3 \tan x - \tan 3x}{2x^3} =$
 1) $\frac{1}{4}$ 2) $\frac{3}{4}$ 3) 4 4) -4
91. $\lim_{x \rightarrow 0} \frac{a \sin ax - b \sin bx}{\tan ax - \tan bx} =$
 1) $a^2 - b^2$ 2) 0 3) $a + b$ 4) $a - b$
92. $\lim_{x \rightarrow 0} \frac{3 \sin x^\circ - \sin 3x^\circ}{x^3} =$
 1) 0 2) 1
 3) $\left(\frac{\pi}{180}\right)^3$ 4) $4 \cdot \left(\frac{\pi}{180}\right)^3$
93. $\lim_{x \rightarrow 0} \frac{\sinh x}{x} =$
 1) 0 2) 3 3) 2 4) 1
94. $\lim_{x \rightarrow 0} \frac{\sin^{-1} x - \tan^{-1} x}{x^3} =$
 1) 2 2) 1 3) -1 4) $\frac{1}{2}$
95. $\lim_{x \rightarrow 0} \frac{\tan^{-1} ax}{x} =$
 1) 1 2) a 3) $\frac{1}{a}$ 4) $\frac{2}{a}$
96. $\lim_{x \rightarrow 0} \frac{\sin^{-1} ax - \sin^{-1} bx}{x} =$
 1) 0 2) 1 3) $a - b$ 4) $a + b$

97. $\lim_{x \rightarrow -1} \frac{\sqrt{\pi} - \sqrt{\cos^{-1} x}}{\sqrt{x+1}} =$
 1) $\frac{1}{\sqrt{2}}$ 2) $\frac{1}{\sqrt{2\pi}}$ 3) $\frac{1}{\sqrt{\pi}}$ 4) 1
98. $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\cos x - \sin x}{\left(\frac{\pi}{4} - x\right)(\cos x + \sin x)} =$
 1) 2 2) 1 3) 0 4) 3
99. $\lim_{x \rightarrow 0} \frac{(1 - \cos 2x) \sin 5x}{x^2 \sin 3x} =$
 1) $\frac{10}{3}$ 2) $\frac{3}{10}$ 3) $\frac{6}{5}$ 4) 56
100. $\lim_{x \rightarrow 0} \frac{\cos(2x^3) - 1}{\sin^6 2x} =$
 1) $\frac{1}{16}$ 3) $\frac{-1}{16}$ 3) $\frac{1}{32}$ 4) $\frac{-1}{32}$
101. $\lim_{x \rightarrow 0} \frac{x \tan 2x - 2x \tan x}{(1 - \cos 2x)^2} =$
 1) 2 2) -2 3) $\frac{1}{2}$ 4) $\frac{1}{2}$
102. $\lim_{x \rightarrow 0} \left(\frac{\sin 2x}{3x} + \frac{x \sin x^2}{\sin x^3} \right) =$
 1) $\frac{5}{3}$ 2) 1 3) $\frac{4}{3}$ 4) $\frac{2}{3}$
103. $\lim_{x \rightarrow 0} \frac{\sin(\pi \cos^2 x)}{x^2} =$
 1) $-\pi$ 2) π 3) $\frac{\pi}{2}$ 4) $\frac{-\pi}{2}$
104. $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\cot x - \cos x}{\left(\frac{\pi}{2} - x\right)^3} =$
 1) $\frac{-1}{2}$ 2) $\frac{1}{2}$ 3) 2 4) -2
105. $\lim_{x \rightarrow 0} \frac{\sin(x^g)}{x}$ ($g = \text{grades}$)
 1) $\frac{\pi}{180}$ 2) $\frac{\pi}{90}$ 3) $\frac{\pi}{100}$ 4) $\frac{\pi}{200}$

$$106. \lim_{x \rightarrow 0} \frac{3 \sin(x^g) - \sin(3x^g)}{x^3} =$$

$$1) \left(\frac{\pi}{200}\right)^3 \quad 2) 4\left(\frac{\pi}{200}\right)^3$$

$$3) \frac{\pi}{200} \quad 4) \frac{\pi}{100}$$

$$107. \lim_{x \rightarrow 0} \frac{\sin(x^n)}{(\sin x)^m} (m, n \in N) = 0 \text{ If}$$

$$1) m > n \quad 2) m = n \quad 3) n > m \quad 4) \text{Always}$$

$$108. \lim_{x \rightarrow 0} \frac{\sin x}{\sqrt{x^2}} =$$

$$1) 1 \quad 2) -1 \quad 3) 0 \quad 4) \text{doesn't exist}$$

$$109. \lim_{x \rightarrow 0} \frac{\sin x - x}{x^3} =$$

$$1) \frac{1}{3} \quad 2) \frac{-1}{3} \quad 3) \frac{1}{6} \quad 4) \frac{-1}{6}$$

$$110. \lim_{x \rightarrow 0} \frac{\sqrt{\cos x} - \sqrt[3]{\cos x}}{\sin^2 x} =$$

$$1) \frac{1}{3} \quad 2) -\frac{1}{3} \quad 3) \frac{1}{12} \quad 4) -\frac{1}{12}$$

$$111. \lim_{n \rightarrow \infty} \frac{1+2+3+4+\dots+n}{n^2} =$$

$$1) 0 \quad 2) \frac{1}{2} \quad 3) 0 \quad 4) -1$$

$$112. \lim_{n \rightarrow \infty} \frac{2^n - 1}{2^n + 1} =$$

$$1) 1 \quad 2) \frac{1}{2} \quad 3) -1 \quad 4) 0$$

$$113. \lim_{n \rightarrow \infty} \frac{2^{\frac{1}{n}} - 1}{2^{\frac{1}{n}} + 1} =$$

$$1) 1 \quad 2) \frac{1}{2} \quad 3) -1 \quad 4) 0$$

$$114. \lim_{n \rightarrow \infty} \frac{n!}{(n+1)! - n!} =$$

$$1) 1 \quad 2) 1/2 \quad 3) -1 \quad 4) 0$$

$$115. \lim_{n \rightarrow \infty} \frac{(n+2)! + (n+1)!}{(n+3)!} =$$

$$1) 1 \quad 2) 1/2 \quad 3) 0 \quad 4) 2$$

$$116. \lim_{n \rightarrow \infty} \frac{(n+2)! + (n+1)!}{(n+2)! - (n+1)!} =$$

$$1) 1 \quad 2) \frac{1}{2} \quad 3) -\frac{1}{2} \quad 4) \frac{1}{3}$$

$$117. \lim_{n \rightarrow \infty} \left(\frac{1+2+3+\dots+n}{n+2} - \frac{n}{2} \right) =$$

$$1) 1 \quad 2) \frac{1}{2} \quad 3) -\frac{1}{2} \quad 4) \frac{1}{3}$$

$$118. \lim_{x \rightarrow \infty} \frac{(x+1)^{10} + (x+2)^{10} + \dots + (x+100)^{10}}{x^{10} + 10^{10}} =$$

$$1) 10 \quad 2) 100 \quad 3) 1000 \quad 4) 1$$

$$119. \lim_{x \rightarrow \infty} \left(\frac{x^3}{2x^2 - 1} - \frac{x^2}{2x + 1} \right) =$$

$$1) 1 \quad 2) \frac{1}{2} \quad 3) \frac{1}{3} \quad 4) \frac{1}{4}$$

$$120. \lim_{x \rightarrow \infty} \frac{\sqrt{x^2 + 1} - \sqrt[3]{x^2 + 1}}{\sqrt[4]{x^4 + 1} - \sqrt{x^2 + 1}} =$$

$$1) 1 \quad 2) -1 \quad 3) 0 \quad 4) \infty$$

$$121. \lim_{n \rightarrow \infty} \frac{1+3+5+\dots+(2n-1)}{2+4+6+\dots+2n} =$$

$$1) 0 \quad 2) 1 \quad 3) -1 \quad 4) 5$$

$$122. \lim_{n \rightarrow \infty} \frac{2^{3n}}{3^{2n}} =$$

$$1) 0 \quad 2) 1 \quad 3) \frac{2}{3} \quad 4) \frac{3}{2}$$

$$123. \lim_{n \rightarrow \infty} \frac{3^{n+1} + 2^{n+2}}{3^{n-1} + 2^{n-2}} =$$

$$1) 4 \quad 2) \frac{5}{2} \quad 3) 9 \quad 4) 8$$

$$124. \lim_{n \rightarrow \infty} \frac{1^2 + 2^2 + 3^2 + \dots + n^2}{2n^3 + 3n^2 + 4n + 1} =$$

$$1) \frac{1}{3} \quad 2) \frac{1}{6} \quad 3) \frac{1}{10} \quad 4) \frac{1}{12}$$

$$125. \lim_{n \rightarrow \infty} \frac{1+2+3+\dots+n}{(n+2)(2n+3)} =$$

$$1) \frac{1}{4} \quad 2) 4 \quad 3) 0 \quad 4) \frac{12}{5}$$

126. $\lim_{x \rightarrow -\infty} \tan^{-1} x =$

1. 1 2. -1 3. 0 4. $\frac{1}{2}$

127. $\lim_{x \rightarrow \infty} \frac{x + \sin x}{x + \cos x} =$

1. 1 2. -1 3. $\frac{1}{3}$ 4. 0

128. For $a > 1$ then $\lim_{x \rightarrow \infty} \frac{a^x - a^{-x}}{a^x + a^{-x}} =$

1. 1 2. $\frac{1}{2}$ 3. $\frac{1}{3}$ 4. $\frac{1}{15}$

129. If $f(x) = \sqrt{\frac{x - \sin^2 x}{x + \cos x}}$ then $\lim_{x \rightarrow \infty} f(x) =$

1. $\frac{1}{2}$ 2. -1 3. 0 4. 1

130. $\lim_{x \rightarrow \infty} \frac{x - \log x}{x + \log x} =$

1. 1 2. -1 3. 0 4. 2

131. $\lim_{x \rightarrow \infty} \frac{[x]}{\log x} =$

1. 0 2. -1 3. ∞ 4. Does not exist

132. $\lim_{x \rightarrow \infty} \frac{12x^2 + 3x + 11}{3x^2 - 7x + 8} =$

1. 1 2. 0 3. $\frac{4}{3}$ 4. 4

133. $\lim_{x \rightarrow \infty} \left(\frac{x+1}{x+2} \right) \left(\frac{2x+3}{3x+4} \right) =$

1. $\frac{2}{3}$ 2. 0 3. $\frac{1}{3}$ 4. $\frac{1}{4}$

134. $\lim_{x \rightarrow \infty} \frac{x}{\sqrt{4x^2 + 1} - 1} =$

1. 1 2. -1 3. $\frac{1}{3}$ 4. $\frac{1}{2}$

135. $\lim_{n \rightarrow \infty} \frac{n^3 - 100n^2 + 1}{100n^2 + 15n} =$

1. 0 2. $\frac{1}{2}$ 3. ∞ 4. $-\infty$

136. $\lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^3 + 2n} - 1}{n + 2} =$

1. 0 2. 2 3. -1 4. 1

137. $\lim_{n \rightarrow \infty} \frac{1 + \frac{1}{2} + \frac{1}{4} + \dots + \frac{1}{2^n}}{1 + \frac{1}{3} + \frac{1}{9} + \dots + \frac{1}{3^n}} =$

1. $\frac{4}{3}$ 2. $\frac{3}{4}$ 3. $\frac{1}{2}$ 4. 0

138. $\lim_{n \rightarrow \infty} \left(\frac{1}{1.2} + \frac{1}{2.3} + \dots + \frac{1}{(n-1)n} \right) =$

1. 1 2. -1 3. $\frac{1}{2}$ 4. 0

139. $\lim_{n \rightarrow \infty} \left(\frac{1}{1.3} + \frac{1}{3.5} + \dots + \frac{1}{(2n-1)(2n+1)} \right) =$

1. 1 2. $\frac{1}{2}$ 3. $\frac{1}{3}$ 4. $\frac{1}{4}$

140. $\lim_{x \rightarrow \infty} \frac{8[X] + 3X}{3[X] - 2X} =$

1. 11 2. 8 3. 0 4. $\frac{1}{8}$

141. $\lim_{n \rightarrow \infty} \frac{\sqrt{3 + 4n^4}}{1 + 2 + 3 + \dots + n} =$

1. $2\sqrt{6}$ 2. $\frac{1}{4}$ 3. 4 4. 1

142. $\lim_{x \rightarrow \infty} \left(1 - \frac{1}{x} \right)^{2x} =$

1. $\frac{1}{e}$ 2. e^3 3. $\frac{1}{e^2}$ 4. e^2

143. $\lim_{n \rightarrow \infty} \left(\frac{n}{n+1} \right)^n =$

1. 1 2. $-\frac{1}{e}$ 3. $\frac{1}{e}$ 4. $1+e$

144. $\lim_{x \rightarrow \infty} \left(\frac{x+1}{x-2} \right)^{2x-1} =$

1. e^2 2. e^6 3. e^{-5} 4. e^{-6}

$$145. \lim_{x \rightarrow \infty} \left(\frac{3x-4}{3x+2} \right)^{\frac{x+1}{3}} =$$

1. $e^{-2/3}$ 2. $e^{3/2}$ 3. $e^{2/3}$ 4. e

$$146. \lim_{x \rightarrow \infty} \left(\frac{x^2+1}{x^2-1} \right)^{x^2} =$$

1. e 2. $\frac{1}{e}$ 3. e^2 4. e^{-2}

$$147. \lim_{x \rightarrow \infty} \left(1 + \frac{2}{x} \right)^{x+7} =$$

1. e 2. $\frac{1}{e}$ 3. e^2 4. e^{-2}

$$148. \lim_{y \rightarrow \infty} \left(1 + \frac{x}{y} \right)^{2y} =$$

1. e^{2x} 2. e^{-2x} 3. e^x 4. e^{-x}

$$149. \lim_{x \rightarrow \infty} 2x(\sqrt{x^2+1}-x) =$$

1. 1 2. $\frac{1}{2}$ 3. 0 4. -1

$$150. \lim_{x \rightarrow \infty} x^{3/2}(\sqrt{x^3+1}-\sqrt{x^3-1}) =$$

1. -1 2. 0 3. $\frac{1}{2}$ 4. 1

$$151. \lim_{x \rightarrow \infty} (\sqrt{x^2-2x-1}-\sqrt{x^2-7x+3}) =$$

1. $\frac{5}{2}$ 2. $\frac{2}{5}$ 3. $\frac{1}{5}$ 4. 0

$$152. \lim_{x \rightarrow \infty} (\sqrt{x^2+ax+a^2}-\sqrt{x^2+a^2}) =$$

1. 0 2. $\frac{a}{2}$ 3. $-\frac{a}{2}$ 4. a

$$153. \lim_{n \rightarrow \infty} \left(\frac{1}{1-n^2} + \frac{2}{1-n^2} + \dots + \frac{n}{1-n^2} \right) =$$

1. 1 2. $\frac{1}{3}$ 3. $-\frac{1}{2}$ 4. 2

$$154. \lim_{x \rightarrow \infty} \{x\sqrt{x^2+a^2}-\sqrt{x^4+a^4}\} =$$

1. a^2 2. $2a^2$ 3. $\frac{a^2}{2}$ 4. $-a^2$

$$155. \lim_{x \rightarrow \infty} x \cos \frac{\pi}{8x} \sin \frac{\pi}{8x} =$$

1. π 2. $\frac{\pi}{2}$ 3. $\frac{\pi}{8}$ 4. $\frac{\pi}{4}$

$$156. \lim_{x \rightarrow \infty} \left[\frac{(2+x)^{40}(4+x)^5}{(2-x)^{45}} \right] \text{ equals to}$$

- 1) -1 2) 1 3) 16 4) 32

$$157. \lim_{x \rightarrow \infty} \frac{x^n}{e^x} = 0 \text{ for}$$

- 1) $n = 0$ only
2) n is any whole number
3) $n = 2$ only 4) no value of n

$$158. \lim_{x \rightarrow \infty} 5^x \sin \left(\frac{a}{5^x} \right) =$$

- 1) 1 2) a 3) ∞ 4) not defined

$$159. \lim_{x \rightarrow \infty} \left(\frac{x-3}{x+2} \right)^x \text{ is equal to}$$

- 1) e 2) e^{-1} 3) e^{-5} 4) e^5

$$160. \lim_{x \rightarrow \infty} (\sqrt{x^2+1}-\sqrt{x^2-1}) =$$

- 1) 0 2) 1 3) 2 4) -1

$$161. \lim_{x \rightarrow 0} x \cdot \cos \frac{1}{x} =$$

1. 1 2. -1 3. $\frac{1}{2}$ 4. 0

$$162. \lim_{x \rightarrow \infty} \frac{x}{\sqrt{ax^2+b}-\sqrt{cx^2+d}} =$$

- 1) $\frac{1}{\sqrt{a}+\sqrt{c}}$ 2) $\frac{1}{\sqrt{c}-\sqrt{a}}$
3) $\frac{1}{\sqrt{a}-\sqrt{c}}$ 4) $\frac{1}{a-c}$

$$163. \lim_{x \rightarrow \infty} \frac{\sqrt[7]{x^7-1} + \sqrt[5]{x^5+2} + \sqrt[9]{x^9-2}}{\sqrt[6]{x^6+1} + \sqrt[5]{x^5+1} - \sqrt[4]{x^4+4}} =$$

- 1) 3 2) -3 3) $1/3$ 4) $-1/3$

$$164. a, b, c, d \text{ are +ve real numbers}$$

$$\lim_{n \rightarrow \infty} \left[1 + \frac{1}{a+bn} \right]^{c+dn} =$$

- 1) e^{db} 2) e^{d-b} 3) $e^{d/b}$ 4) $e^d \cdot e^b$

165. $f(x) = x^2$, $\phi(a) = b^2$, $f'(a) = n\phi'(a)$ and

$$\Phi'(a) \neq 0 \text{ then } \lim_{x \rightarrow a} \left(\frac{\sqrt{f(x)} - a}{\sqrt{\phi(x)} - b} \right)$$

1) $\frac{b}{a}$ 2) $\frac{nb}{a}$ 3) $\frac{a}{b}$ 4) $\frac{na}{b}$

166. $\lim_{x \rightarrow 0} \frac{\sin(\pi \cos^2 x)}{x^2} =$

1) $-\pi$ 2) π 3) $\pi/2$ 4) 1

167. $\lim_{x \rightarrow 0} \frac{\log_e^{(1+x)}}{3^x - 1}$

1) $\log_e^3$ 2) 0 3) 1 4) $\log_e^3$

168. If $\lim_{x \rightarrow 5} \frac{x^k - 5^k}{x - 5} = 500$, then the positive integral value of k is

1) 3 2) 4 3) 5 4) 6

169. $\lim_{n \rightarrow \infty} \left[\cos\left(\frac{x}{n}\right) \right]^n =$

1) e 2) $\frac{1}{e}$ 3) 1 4) 2

170. $\lim_{x \rightarrow 0} (1 + 2x)^{\frac{1}{3x}} =$

1) e^2 2) $e^{\frac{3}{2}}$ 3) $e^{\frac{2}{3}}$ 4) e^{-2}

171. $\lim_{x \rightarrow 0} (1 - 3x)^{\frac{1}{5x}} =$

1) $e^{-\frac{3}{5}}$ 2) $e^{\frac{5}{3}}$ 3) $e^{-\frac{5}{3}}$ 4) e^{-1}

172. $\lim_{x \rightarrow \frac{\pi}{2}} (1 + 3 \cos x)^{\sec x} =$

1) e^2 2) e^3 3) e^{-2} 4) e^{-3}

173. $\lim_{x \rightarrow \pi} (1 - 4 \tan x)^{\cot x} =$

1) e 2) e^4 3) e^{-1} 4) e^{-4}

174. $\lim_{x \rightarrow 1} = (2 - x)^{\tan\left(\frac{\pi x}{2}\right)}$

1) $e^{\frac{1}{\pi}}$ 2) $e^{\frac{2}{\pi}}$ 3) $-e^{\frac{2}{\pi}}$ 4) e

175. $\lim_{x \rightarrow \infty} (1 + e^{-x})^{e^x} =$

1) e 2) $\frac{1}{e}$ 3) $-e$ 4) 0

176. $\lim_{x \rightarrow 0} (\sin x + \cos x)^{\frac{1}{x}} =$

1) $-e$ 2) e^{-1} 3) e 4) $\frac{1}{e}$

177. $\lim_{x \rightarrow 0} (1 + \sin x)^{\frac{1}{x}} =$

1) e 2) e^2 3) e^3 4) $\frac{1}{e}$

178. $\lim_{x \rightarrow \infty} (1 + 3 \tan^2 x)^{\cot^2 x} =$

1) e 2) e^2 3) e^3 4) 1

179. $\lim_{x \rightarrow \infty} (1 + 2 \sin^2 x)^{\sec^2 x} =$

1) e 2) e^2 3) e^3 4) 1

180. $\lim_{x \rightarrow 1} x^{\left(\frac{1}{1-x^2}\right)} =$

1) $e^{-\frac{1}{2}}$ 2) $e^{\frac{2}{3}}$ 3) $e^{-\frac{3}{2}}$ 4) e

181. $\lim_{x \rightarrow 0+} (\sin x)^{\tan x} =$

1) e 2) e^2 3) -1 4) 1

182. $\lim_{x \rightarrow \frac{\pi}{4}} (\tan x)^{\tan 2x} =$

1) e 2) $\frac{1}{e}$ 3) $\frac{1}{e^2}$ 4) 2

183. $\lim_{x \rightarrow 1} (1 - x^2)^{\frac{1}{\log(1-x)}} =$

1) e 2) $\frac{1}{e}$ 3) $\frac{1}{e^2}$ 4) e^2

184. $\lim_{x \rightarrow \infty} (1 - x^2)^{e^{-x}} =$

1) e 2) $-e$ 3) 1 4) 0

185. The value of $\lim_{x \rightarrow 0} \left(\frac{1 + 5x^2}{1 + 3x^2} \right)^{1/x^2} =$

1) e 2) e^2 3) $1/e$ 4) 0

186. $\lim_{x \rightarrow \infty} \left[\frac{x^2 \sin\left(\frac{1}{x}\right) - x}{1 - |x|} \right] =$

1) 0 2) 1 3) -1 4) 2

187. $\lim_{x \rightarrow \infty} 2^{-x} \sin(2^x) =$

1) 1 2) 0
3) 2 4) does not exist

188. $\lim_{x \rightarrow \infty} e^x \sin\left(\frac{d}{e^x}\right) =$
 1) 0 2) d
 3) 2 4) does not exist
189. $\lim_{n \rightarrow \infty} (\pi n)^{2/n} =$
 1) 0 2) 1 3) 2 4) 3
190. $\lim_{n \rightarrow \infty} \left(\frac{e^n}{\pi}\right)^{1/n} =$
 1) 0 2) 1 3) e^2 4) e
191. $\lim_{n \rightarrow \infty} \frac{(1+2+3+\dots+n \text{ terms})(1^2+2^2+\dots+n \text{ terms})}{n(1^3+2^3+\dots+n \text{ terms})} =$
 1) $\frac{3}{2}$ 2) $\frac{2}{3}$ 3) 1 4) 0
192. $\lim_{x \rightarrow 0} \frac{\sin x}{|x|} =$
 1) 0 2) -1
 3) 1 4) does not exist
193. $\lim_{n \rightarrow \infty} \left(1 + \sin\left(\frac{a}{n}\right)\right)^n =$
 1) e 2) e^a 3) a^e 4) ae
194. $\lim_{x \rightarrow a} \frac{x-a}{|x-a|} =$
 1) 0 2) 1
 3) -1 4) does not exist
195. $\lim_{x \rightarrow \infty} \frac{(2+x)^{20}(4+x)^3}{(2-x)^{23}} =$
 1) -1 2) 1 3) 16 4) 32
196. $\lim_{x \rightarrow 0} \frac{(1+x)^{\frac{1}{m}} - (1-x)^{\frac{1}{m}}}{(1+x)^{\frac{1}{n}} - (1-x)^{\frac{1}{n}}} =$
 1) 1 2) $\frac{m+n}{m-n}$ 3) $\frac{n}{m}$ 4) $\frac{m}{n}$
197. The function $f(x) = \frac{x \tan 2x}{\sin 3x \sin 5x}$ for $x \neq 0$,
 $= k$ for $x=0$, is continuous at $x=0$, then $f(0)$
 1. $\frac{2}{13}$ 2. $\frac{2}{17}$ 3. $\frac{2}{11}$ 4. $\frac{2}{15}$

198. If $f(x) = \frac{1 - \cos ax}{1 - \cos bx}$ for $x \neq 0$, is continuous at $x=0$
 then $f(0) =$
 1. $\frac{a^2}{2}$ 2. $\frac{a}{b^2}$ 3. $\frac{a}{b}$ 4. $\frac{a^2}{b^2}$
199. The function $f(x) = [x]$ at $x=5$, is
 1. Left continuous 2. Right continuous
 3. continuous
 4. cannot be determined
200. The discontinuous points of $f(x) = \frac{1}{\log |x|}$ are
 1. 0, ± 2 2. 1, ± 2 3. 0, ± 1 4. 0, ± 3
201. The function defined by $f(x) = x \sin \frac{1}{x}$ for $x \neq 0$
 $= 0$ for $x=0$ is at $x=0$
 1. continuous 2. right continuous
 3. left continuous 4. can not be determined
202. The function $f(x) = (1+x)^{\frac{5}{x}}$ for $x \neq 0$, $= e^5$ for
 $x=0$ ----- at $x=0$
 1. continuous 2. right continuous
 3. left continuous 4. can not be determined
203. The value of $f(0)$ so that the function
 $f(x) = \frac{2x - \sin^{-1} x}{2x + \tan^{-1} x}$ is continuous at $x=0$, is
 1. 2 2. $\frac{1}{3}$ 3. $\frac{2}{3}$ 4. $-\frac{1}{3}$
204. If the function $f(x) = \frac{\sin^2 ax}{x^2}$ for $x \neq 0$
 $= 1$ for $x=0$ is continuous at $x=0$ then $a =$
 1. ± 1 2. 0 3. $\pm \frac{1}{2}$ 4. $\pm \frac{1}{3}$
205. If the function $f(x) = \frac{e^{x^2} - \cos x}{x^2}$ for $x \neq 0$ is
 continuous at $x=0$ then $f(0) =$
 1) $\frac{1}{2}$ 2. $\frac{3}{2}$ 3. 2 4. $\frac{1}{3}$
206. If the function $f(x) = (1+3x)^{\frac{1}{x}}$ for $x \neq 0$ is
 continuous at $x=0$ then $f(0) =$
 1. 3 2. $\frac{1}{e}$ 3. e^2 4. e^3

207. The interval on which $f(x) = \sqrt{1-x^2}$ is continuous in
 1. $(0, \infty)$ 2. $(1, \infty)$ 3. $[-1, 1]$ 4. $(-\infty, -1)$
208. If the function $f(x) = \frac{\log(1+ax) - \log(1-bx)}{x}$ for $x \neq 0$ is continuous at $x=0$ then $f(0) =$
 1. $a-b$ 2. $a+b$
 3. $\log a + \log b$ 4. $\log a - \log b$
209. A point of discontinuity of $f(x) = \tan x$ is
 1. $x=0$ 2. $x = \frac{\pi}{4}$ 3. $x = \frac{\pi}{3}$ 4. $x = \frac{\pi}{2}$
210. The function $f(x) = \frac{1 + \sin x - \cos x}{1 - \sin x - \cos x}$ is not defined at $x=0$. The value of $f(0)$ so that $f(x)$ is continuous at $x=0$ is
 1. 1 2. -1 3. 0 4. 2
211. The function $f(x) = \frac{\cos x - \sin x}{\cos 2x}$ is not defined at $x = \frac{\pi}{4}$. The value of $f\left(\frac{\pi}{4}\right)$ so that $f(x)$ is continuous at $x = \frac{\pi}{4}$ is
 1. $\frac{1}{\sqrt{2}}$ 2. $\sqrt{2}$ 3. $-\sqrt{2}$ 4. 1
212. The value of $f(0)$ for $f(x) = (1 + \tan^2 \sqrt{x})^{\frac{1}{2x}}$ so that $f(x)$ is continuous everywhere is
 1. e 2. $\frac{1}{2}$ 3. $e^{\frac{1}{2}}$ 4. 0
213. If the function $f(x) = \frac{2^{x+2} - 16}{4^x - 16}$ for $x \neq 2 = A$ for $x = 2$ is continuous at $x = 2$, then $A =$
 1. 2 2. $1/2$ 3. $1/4$ 4. 0
214. If $f(x) = \begin{cases} 1+x & x \leq 1 \\ 3-ax^2 & x > 1 \end{cases}$ is continuous at $x = 1$ then a is ____ ($a > 0$)
 1. 1 2. 2 3. -1 4. -2
215. The value of $f(0)$ for the function $f(x) = \frac{2 - \sqrt{x+4}}{\sin 2x}$ is continuous at $x = 0$ is
 1. $\frac{1}{8}$ 2. $\frac{1}{4}$ 3. $-\frac{1}{8}$ 4. $-\frac{1}{4}$

216. The value of $f(0)$ for the function $f(x) = \frac{e^x - e^{-x}}{x}$ so that it is continuous everywhere is
 1. 1 2. $\frac{1}{2}$ 3. 2 4. 0
217. If $f(x) = \begin{cases} (\cos x)^{\frac{1}{x}} & x \neq 0 \\ k & x = 0 \end{cases}$ and if $f(x)$ is continuous at $x = 0$ then $k =$
 1. 1 2. 0 3. -1 4. e
218. The integer x for which $\lim_{x \rightarrow 0} \frac{(\cos x - 1)(\cos x - e^x)}{x^3}$ is a finite non-zero number is
 1. 1 2. $\frac{1}{4}$ 3. $\frac{1}{2}$ 4. $-\frac{1}{2}$
219. If $f(x) = \frac{\sqrt{1+px} - \sqrt{1-px}}{x}$, $-1 \leq x < 0$
 $= \frac{2x+1}{x-2}$, $0 \leq x \leq 1$ is continuous in the interval $[-1, 1]$ then $p =$
 1. -1 2. $-\frac{1}{2}$ 3. $\frac{1}{2}$ 4. 1
220. At $x=0$ if $f(x) = (1+x)^{\cot x}$ is continuous then $f(0) =$
 1. 0 2. 1 3. e 4. e^{-1}
221. Let $f(x) = \frac{(e^{kx} - 1) \cdot \sin kx}{x^2}$, for $x \neq 0 = 4$, for $x = 0$ is continuous at $x = 0$ then $k =$
 1. ± 1 2. ± 2 3. 0 4. ± 3
222. If the function defined by $f(x) = \frac{\sin 3(x-p)}{\sin 2(x-p)}$ for $x \neq p$ is continuous at $x = p$ then $f(p) =$
 1. $\frac{3}{2}$ 2. $\frac{2}{3}$ 3. 6 4. $\frac{1}{6}$
223. Let $f(x) = \frac{3x + 4 \tan x}{x}$, for $x \neq 0 = 7$, for $x = 0$ then $f(x)$ is
 1. continuous at $x = 0$
 2. not continuous at $x = 0$
 3. not determined at $x = 0$
 4. $\lim_{x \rightarrow 0} f(x) = 8$

224. The function $f(x) = \frac{|x|}{x}$ at $x=0$ is
 1. left continuous 2. right continuous
 3. continuous 4. Discontinuous
225. A point of discontinuity of $f(x) = [x]$ is
 1) $\frac{1}{2}$ 2) $-\frac{1}{2}$ 3) 1 4) $\frac{2}{3}$
226. The function $f(x) = \begin{cases} 5 & \text{when } 0 < x < 1 \\ 8 & \text{when } 1 < x < 2 \\ 15 & \text{when } 2 < x \leq 3 \end{cases}$ is
 discontinuous
 1) at $x = 1, 2$ 2) at $x = 4$
 3) at $x = 0$ 4) at $x = 3$
227. If the function $f(x) = \begin{cases} \frac{1-\cos 4x}{x^2} & \text{if } x < 0 \\ a & \text{if } x = 0 \\ \frac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4} & \text{if } x > 0 \end{cases}$ is
 continuous at $x = 0$ then $a =$
 1) 8 2) $\frac{1}{8}$ 3) -8 4) 0
228. The value of $f(0)$ so that the function $f(x) = \frac{\log\left(1+\frac{x}{a}\right) - \log\left(1-\frac{x}{b}\right)}{x}$ ($x \neq 0$) is continuous
 at $x = 0$ is
 1) $\frac{a+b}{ab}$ 2) $\frac{a-b}{ab}$ 3) $\frac{ab}{a+b}$ 4) $\frac{ab}{a-b}$
229. If $f(x) = \begin{cases} 5x-4 & 0 < x \leq 1 \\ 4x^2+3bx & 1 < x < 2 \end{cases}$ and if
 $f(x)$ is continuous in the interval $(0,2)$ then $b =$
 1) -1 2) 0 3) 1 4) $\frac{13}{3}$
230. If the function $f(x) = \begin{cases} \frac{k \cos x}{\pi - 2x}, & x \neq \frac{\pi}{2} \\ 3 & \text{at } x = \frac{\pi}{2} \end{cases}$
 is continuous at $x = \frac{\pi}{2}$ then $k =$
 1) 2 2) 4 3) 6 4) 8

231. Let $f(x) = 3x - 4$ for $0 \leq x \leq 2$
 $= 2x + \lambda$ for $2 < x \leq 3$
 If $f(x)$ is continuous at $x = 2$, then λ is
 1) -1 2) 0 3) -2 4) 2
232. The function $f(x) = \frac{|x-3|}{x-3}$ at $x=3$, is
 1. Left continuous 2. Right continuous
 3. continuous 4. discontinuous
233. The function $f(x) = \frac{\sin 2x \cdot \sin 3x}{x^2}$ for $x \neq 0$,
 $f(0) = 6$ at $x = 0$, is
 1. continuous 2. discontinuous
 3. left continuous 4. right continuous
234. The function $f(x) = \frac{\cos 3x - \cos 4x}{x \sin 2x}$ for $x \neq 0$,
 $f(0) = \frac{7}{4}$ at $x=0$, is
 1. Continuous 2. discontinuous
 3. left continuous 4. right continuous
235. The function $f(x) = \frac{3 \sin x - \sin 3x}{x^3}$ for $x \neq 0$,
 $f(0) = 1$ at $x=0$, is
 1. Continuous 2. discontinuous
 3. left continuous 4. right continuous
236. If $f(x) = \frac{1 - \cos ax}{1 - \cos bx}$ for $x \neq 0$, is continuous at $x=0$
 then $f(0) =$
 1. $\frac{a^2}{2}$ 2. $\frac{a}{b^2}$
 3. $\frac{a}{b}$ 4. $\frac{a^2}{b^2}$
237. If $f(x) = \frac{x^3 + 8}{x^5 + 32}$, $x \neq -2$ and $f(-2) = k$ is
 continuous at $x = -2$ then $k =$
 1) $\frac{1}{4}$ 2) $\frac{3}{20}$ 3) $\frac{3}{5}$ 4) $\frac{4}{5}$
238. $f(x) = [x]$ is continuous
 1) R 2) Z 3) N 4) R - Z
239. $f(x) = \frac{e^{1/x^2}}{e^{1/x^2} - 1}$, $x \neq 0$ $f(0) = 1$, then f at $x = 0$
 1) discontinuous 2) left continuous
 3) right continuous 4) both 2 and 3

240. $f(x) = x \left[3 - \log \left(\frac{\sin x}{x} \right) \right] - 2$ to be continuous at $x=0$, then $f(0) =$
 1) 0 2) 2 3) -2 4) 3

241. A function $f(x)$ is defined as

$$f(x) = \begin{cases} ax - b & x \leq 1 \\ 3x, & 1 < x < 2 \\ bx^2 - a & x \geq 2 \end{cases} \text{ is continuous at } x$$

$=1, 2$ then

- 1) $a = 5, b = 2$ 2) $a = 6, b = 3$
 3) $a = 7, b = 4$ 4) $a = 8, b = 5$

242. If $f(x) = \begin{cases} (1 + |\sin x|)^{\frac{a}{|\sin x|}} & -\frac{\pi}{6} < x < 0 \\ b & x = 0 \\ e^{\frac{\tan 2x}{\tan 3x}} & 0 < x < \frac{\pi}{6} \end{cases}$ is

continuous at $x=0$ then

- 1) $a = e^{2/3}, b = 2/3$ 2) $a = 2/3, b = e^{2/3}$
 3) $a = 1/3, b = e^{1/3}$ 4) $a = e^{1/3}, b = e^{1/3}$

243. If $f(x) = \frac{x(e^{1/x} - e^{-1/x})}{e^{1/x} + e^{-1/x}}, x \neq 0$ is continuous at $x=0$, then $f(0) =$
 1) 1 2) 2 3) 0 4) 3

244. $f(x) = \begin{cases} 2x-1 & \text{if } x > 2 \\ k & \text{if } x = 2 \\ x^2-1 & \text{if } x < 2 \end{cases}$ is continuous at

$x=2$ then $k =$

- 1) 1 2) 2 3) 3 4) 4

245. $f(x) = \begin{cases} \frac{x^3 + x^2 - 16x + 20}{(x-2)^2} & (x \neq 2) \\ k & \text{if } x = 2 \end{cases}$ if $x \neq 2$ $f(x)$ is

continuous at $x=2$ then

- 1) $k = 3$ 2) $k = 5$
 3) $k = 7$ 4) $k = 9$

246. $f(x) = |x| + |x-1|$ is continuous at
 1) 0 only 2) 0, 1 only
 3) every where 4) no where

247. $f(x) = [x] + |x-1|$ is

- 1) continuous at $x=0, 1$
 2) continuous at $x=0$ only
 3) continuous at $x=1$ only
 4) discontinuous at $x=0, 1$

248. $f(x) = \frac{P + q^{\frac{1}{x}}}{r + s^{\frac{1}{x}}}, s > 1, q > 1, r \neq 0, f(0) = 1$ is

left continuous at $x=0$ then

- 1) $p=0$ 2) $p=r$ 3) $p=q$ 4) $p \neq q$

249. The set of points of discontinuity of the

$$f(x) = \frac{1}{x^2 + x + 1}$$

- 1) ϕ 2) $\mathbb{R}$ 3) $\{0\}$ 4) $\mathbb{R}^-$

250. Set of points of discontinuity of $\frac{x^2}{|x|} =$

- 1) $\{0\}$ 2) $\mathbb{R}$ 3) $\mathbb{R}^+$ 4) $\mathbb{Z}$

251. $f(x) = \begin{cases} -2 \sin x & \text{if } x \leq -\frac{\pi}{2} \\ a \sin x + b & \text{if } -\frac{\pi}{2} < x < \frac{\pi}{2} \\ \cos x & \text{if } x \geq \frac{\pi}{2} \end{cases}$ and

$f(x)$ is continuous at every where then $(a, b) =$

- 1) (1, 1) 2) (-1, 1) 3) (1, -1) 4) (-1, -1)

252. Let $f'(x)$ be continuous at $x=0$ and $f'(0)=4$,

then $\lim_{x \rightarrow 0} \frac{2f(x) - 3f(2x) + f(4x)}{x^2} =$

- 1) 11 2) 2 3) 12 4) 10

253. If $f'(2)=2, f''(2)=1$ then $\lim_{x \rightarrow 2} \frac{2x^2 - 4f'(x)}{x-2} =$

- 1) 4 2) 0 3) 2 4) ∞

254. $\lim_{x \rightarrow 0} \left[\frac{\sin(2x)}{3x} + \frac{x \sin(x^2)}{\sin(x^3)} \right] =$

- 1) $\frac{5}{3}$ 2) 1 3) $\frac{4}{3}$ 4) $\frac{2}{3}$

255. $\lim_{n \rightarrow \infty} \frac{n^k \sin^2(n!)}{n+2}, 0 < k < 1$

- 1) ∞ 2) 1 3) 0 4) 2

KEY

1) 1	2) 2	3) 3	4) 1
5) 1	6) 4	7) 1	8) 2
9) 1	10) 1	11) 1	12) 3
13) 2	14) 4	15) 2	16) 1
17) 1	18) 4	19) 1	20) 1
21) 1	22) 1	23) 1	24) 2
25) 3	26) 1	27) 2	28) 2
29) 3	30) 2	31) 1	32) 2
33) 4	34) 3	35) 2	36) 4
37) 3	38) 3	39) 2	40) 2
41) 1	42) 1	43) 4	44) 3
45) 2	46) 3	47) 4	48) 3
49) 2	50) 4	51) 4	52) 2
53) 1	54) 4	55) 4	56) 1
57) 3	58) 4	59) 3	60) 3
61) 1	62) 1	63) 4	64) 2
65) 1	66) 4	67) 4	68) 2
69) 3	70) 3	71) 4	72) 2
73) 1	74) 3	75) 3	76) 1
77) 4	78) 1	79) 1	80) 1
81) 2	82) 1	83) 1	84) 4
85) 3	86) 4	87) 4	88) 1
89) 1	90) 4	91) 3	92) 4
93) 4	94) 4	95) 2	96) 3
97) 2	98) 2	99) 1	100) 4
101) 2	102) 1	103) 2	104) 2
105) 4	106) 2	107) 3	108) 4
109) 4	110) 4	111) 2	112) 1
113) 4	114) 4	115) 3	116) 1
117) 3	118) 2	119) 4	120) 1
121) 2	122) 1	123) 3	124) 2
125) 1	126) 2	127) 1	128) 1
129) 4	130) 1	131) 1	132) 4
133) 1	134) 4	135) 3	136) 4
137) 1	138) 1	139) 2	140) 1
141) 3	142) 3	143) 3	144) 2
145) 1	146) 3	147) 3	148) 1
149) 1	150) 4	151) 1	152) 2
153) 2	154) 3	155) 3	156) 1
157) 2	158) 2	159) 3	160) 1

161) 4	162) 3	163) 1	164) 3
165) 2	166) 2	167) 4	168) 2
169) 3	170) 3	171) 1	172) 2
173) 4	174) 2	175) 1	176) 3
177) 1	178) 3	179) 2	180) 1
181) 4	182) 2	183) 1	184) 3
185) 3	186) 1	187) 2	188) 2
189) 2	190) 4	191) 3	192) 4
193) 2	194) 4	195) 1	196) 3
197) 4	198) 4	199) 2	200) 3
201) 1	202) 1	203) 2	204) 1
205) 2	206) 4	207) 3	208) 2
209) 4	210) 1	211) 2	212) 3
213) 2	214) 1	215) 3	216) 3
217) 1	218) 3	219) 2	220) 3
221) 2	222) 1	223) 1	224) 4
225) 3	226) 1	227) 1	228) 1
229) 1	230) 3	231) 3	232) 4
233) 1	234) 1	235) 2	236) 4
237) 2	238) 4	239) 4	240) 3
241) 1	242) 2	243) 3	244) 3
245) 3	246) 3	247) 4	248) 2
249) 1	250) 1	251) 2	252) 3
253) 1	254) 4	255) 3	

HINTS

19. Use L - Hospital rule

$$46. \lim_{x \rightarrow 2} \frac{x^{11/2} - 2^{11/2}}{x^{7/2} - 2^{7/2}}$$

48. Use L - Hospital rule

$$57. \lim_{x \rightarrow 0} \frac{2 \sin^2 \frac{ax}{2}}{x \sin 3x} = \frac{2x \frac{a^2}{4}}{3}$$

68. Take tan x common from the Nr.

$$79. \lim_{x \rightarrow 0} \frac{e^x [e^{\tan x} - 1]}{\tan x - x}$$

$$118. \lim_{x \rightarrow 0} \frac{x^{10} \left[\left(1 + \frac{1}{x}\right)^{10} + \left(1 + \frac{2}{x}\right)^{10} + \dots + \left(1 + \frac{100}{x}\right)^{10} \right]}{x^{10} \left[1 + \frac{10^{10}}{x^{10}} \right]}$$

$$137. S_{\infty} = \frac{a}{1-r}$$

$$144. \lim_{x \rightarrow a} f(x) = 1 \text{ and } \lim_{x \rightarrow a} g(x) = \infty \text{ then}$$

$$\lim_{x \rightarrow a} (f(x))^{g(x)} = e^{\lim_{x \rightarrow a} (f(x)-1)g(x)}$$

196. Use L.Hospital rule

228. Use L - Hospital rule

$$243. \lim_{x \rightarrow 0} e^{-\frac{1}{x}} = 0$$

252. Use L - Hospital rule

253. Use L - Hospital rule

LEVEL - II

$$1. \lim_{x \rightarrow a} \frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} =$$

$$1) \frac{2}{\sqrt{3}} \quad 2) -\frac{1}{\sqrt{3}} \quad 3) \frac{2}{3\sqrt{3}} \quad 4) \frac{1}{\sqrt{3}}$$

$$2. \lim_{x \rightarrow 0} \frac{(1+x)^3 - 1 - 3x}{(1+x)^2 - 1 - 2x} =$$

$$1) 1 \quad 2) 3 \quad 3) 5 \quad 4) -2$$

$$3. \lim_{x \rightarrow 2} \frac{x^2 - 4}{\sqrt{x+2} - \sqrt{3x-2}} =$$

$$1) 8 \quad 2) \frac{1}{8} \quad 3) \frac{1}{3} \quad 4) -8$$

$$4. \lim_{x \rightarrow 0} \frac{\sqrt{1+x} - \sqrt{1+x^2}}{\sqrt{1-x^2} - \sqrt{1-x}} =$$

$$1) 1 \quad 2) -1 \quad 3) 0 \quad 4) 2$$

$$5. \lim_{x \rightarrow 1} \left(\frac{1}{1-x} - \frac{3}{1-x^3} \right) =$$

$$1) 1 \quad 2) -1 \quad 3) \frac{1}{2} \quad 4) -\frac{1}{2}$$

$$6. \lim_{x \rightarrow 1} \frac{x^x - x}{1-x+\log x} =$$

$$1) -\frac{2}{3} \quad 2) -1 \quad 3) -2 \quad 4) \frac{1}{4}$$

$$7. \lim_{x \rightarrow 0} \frac{9^x - 2 \cdot 6^x + 4^x}{\sin^2 x} =$$

$$1) \left(\log \frac{3}{2} \right)^2 \quad 2) \left(\log \frac{2}{3} \right)^3$$

$$3) \left(\log \frac{3}{2} \right) \quad 4) \left(\log \frac{2}{3} \right)$$

$$8. \lim_{x \rightarrow 0} \frac{27^x - 9^x - 3^x + 1}{\sqrt{2} - \sqrt{1+\cos x}} =$$

$$1) 0 \quad 2) 8\sqrt{2} (\log 3)^2$$

$$3) 8 (\log 3)^2 \quad 4) 1$$

$$9. \lim_{x \rightarrow -2} \frac{x^4 + 2x^3 + 3x^2 + 5x - 2}{x^5 + 3x^4 + 2x^3 + 3x^2 + 7x + 2} =$$

$$1) 0 \quad 2) 1 \quad 3) -1 \quad 4) -5$$

$$10. \text{ If } \lim_{x \rightarrow 5} f(x) = 4 \text{ then } \lim_{y \rightarrow 1/3} f(15y) =$$

$$1) 1 \quad 2) \frac{1}{3} \quad 3) \frac{4}{3} \quad 4) 4$$

$$11. \lim_{x \rightarrow 4^+} \frac{x^2 - 7x + 12}{x - [x]} =$$

$$1) 1 \quad 2) 0$$

$$3) 2 \quad 4) \text{ does not exist}$$

$$12. \lim_{x \rightarrow 0} \frac{e^x + \log\left(\frac{1-x}{e}\right)}{\tan x - x} =$$

$$1) 1/3 \quad 2) -1/2 \quad 3) 2 \quad 4) 1$$

$$13. \lim_{x \rightarrow 0} \frac{\sin 2x + 2\sin^2 x - 2\sin x}{\cos x - \cos^2 x} =$$

$$1) 0 \quad 2) 1 \quad 3) \frac{2}{3} \quad 4) 4$$

$$14. \lim_{x \rightarrow 0} \frac{3\sin x - \sin 3x}{x \cdot \tan^2 2x} =$$

$$1) 0 \quad 2) 1 \quad 3) 2 \quad 4) 4$$

$$15. \lim_{x \rightarrow \frac{1}{\sqrt{2}}} \frac{x - \cos(\sin^{-1} x)}{1 - \tan(\sin^{-1} x)} =$$

$$1) \frac{1}{2} \quad 2) e^2 \quad 3) -\frac{1}{\sqrt{2}} \quad 4) 1$$

$$16. \lim_{x \rightarrow 0} \frac{\sin x - x + \frac{x^3}{6}}{x^5} =$$

1) $\frac{1}{140}$ 2) $\frac{1}{5}$ 3) $\frac{1}{60}$ 4) $\frac{1}{120}$

$$17. \lim_{x \rightarrow 0} \frac{\sec x - 1}{x^2 (\sec x + 1)^2} =$$

1) $\frac{1}{8}$ 2) $\frac{1}{4}$ 3) 2 4) 0

$$18. \lim_{x \rightarrow 0} \frac{m \sin mx - n \sin nx}{\tan mx + \tan nx} =$$

1) m-n 2) m+n 3) $m^2 - n^2$ 4) 0

$$19. \lim_{x \rightarrow 1} \frac{1 + \cos \pi x}{\tan^2 \pi x} =$$

1) 1 2) $\frac{1}{2}$ 3) $\frac{1}{3}$ 4) -1

$$20. \lim_{x \rightarrow \frac{\pi}{4}} \frac{\sqrt{2} - \cos x - \sin x}{(4x - \pi)^2} =$$

1) $\frac{1}{16\sqrt{2}}$ 2) $\frac{1}{32\sqrt{2}}$ 3) $\frac{1}{16}$ 4) $\frac{1}{8}$

$$21. \lim_{x \rightarrow 0} \frac{\tan^4 x - \sin^4 x}{x^6} =$$

1) 2 2) $\frac{1}{6}$ 3) $\frac{2}{3}$ 4) $\frac{3}{2}$

$$22. \lim_{x \rightarrow 0} \frac{x(1 - \sqrt{1 - x^2})}{\sqrt{1 - x^2} \cdot (\sin^{-1} x)^3} =$$

1) 1 2) -1 3) $\frac{1}{2}$ 4) 0

$$23. \lim_{x \rightarrow 0} \frac{8}{x^8} \left(1 - \cos \frac{x^2}{2} - \cos \frac{x^2}{4} + \cos \frac{x^2}{2} \cdot \cos \frac{x^2}{4} \right) =$$

1) $\frac{1}{16}$ 2) $\frac{1}{15}$ 3) $\frac{1}{32}$ 4) 1

$$24. \text{ If } f(x) = \frac{\sin[x]}{[x]}, [x] \neq 0 = 0 \text{ if } [x] = 0 \text{ then}$$

$$\lim_{x \rightarrow 0} f(x) =$$

1) 0 2) $\sin 1$
3) 1 4) does not exist

$$25. \lim_{x \rightarrow 0} \left\{ \frac{\int_0^x \sin^2 x \, dx}{x^3} \right\} =$$

1) $\frac{1}{2}$ 2) $\frac{1}{3}$ 3) $\frac{1}{4}$ 4) $\frac{2}{3}$

$$26. \lim_{x \rightarrow -1^+} \frac{\sqrt{\pi} - \sqrt{\cos^{-1} x}}{\sqrt{x+1}} =$$

1) $\frac{1}{\sqrt{2\pi}}$ 2) $\frac{1}{2\sqrt{\pi}}$ 3) $\frac{1}{\pi\sqrt{2}}$ 4) $\frac{2}{\sqrt{\pi}}$

$$27. \lim_{x \rightarrow \frac{\pi}{4}} \frac{\sqrt{2} \cos x - 1}{\cot x - 1} =$$

1) 1 2) $\frac{1}{2}$ 3) $\frac{1}{\sqrt{2}}$ 4) $\frac{1}{2\sqrt{2}}$

$$28. \text{ Let } \alpha \text{ and } \beta \text{ be the roots of } ax^2 + bx + c = 0,$$

$$\text{then } \lim_{x \rightarrow \alpha} \frac{1 - \cos(ax^2 + bx + c)}{(x - \alpha)^2} =$$

1) $\frac{a^2(\alpha - \beta)^2}{2}$ 2) $\frac{a^2}{2(\alpha - \beta)^2}$

3) $\frac{a^2}{(\alpha - \beta)^2}$ 4) $-\frac{a^2}{2(\alpha - \beta)^2}$

$$29. \text{ If } f(x) = \frac{\tan x}{\sqrt{1 + \tan^2 x}}, \quad x \rightarrow (\pi/2)^- \quad f(x) = a \text{ and}$$

$$x \rightarrow (\pi/2)^+ \quad f(x) = b \text{ then}$$

1) a = b 2) a = 1 + b
3) a + b = 0 4) a + b = 2

$$30. \lim_{x \rightarrow 0} \frac{\sqrt{\frac{1}{2}(1 - \cos x)}}{x} =$$

1) 1 2) -1
3) 0 4) does not exist

$$31. \lim_{x \rightarrow 0} \frac{1 - \cos 2x^3}{x^2 \cdot \tan^2 3x \sin^2 4x} =$$

1) $\frac{1}{72}$ 2) $\frac{1}{48}$ 3) $\frac{1}{24}$ 4) 1

$$32. \lim_{n \rightarrow \infty} \frac{(2n+1)^4 - (n-1)^4}{(2n+1)^4 + (n-1)^4} =$$

- 1) $\frac{15}{17}$ 2) $\frac{17}{15}$ 3) 0 4) ∞

$$33. \lim_{n \rightarrow \infty} \frac{(\sqrt{n^2+1}+n)^2}{\sqrt[3]{n^6+1}} =$$

- 1) 1 2) $\frac{1}{2}$ 3) $\frac{1}{3}$ 4) 4

$$34. \lim_{n \rightarrow \infty} \frac{2 \cdot 3^{n+1} - 3 \cdot 5^{n+1}}{2 \cdot 3^n + 3 \cdot 5^n} =$$

- 1) 5 2) $\frac{1}{5}$ 3) -5 4) 0

$$35. \lim_{n \rightarrow \infty} \frac{1^3 + 2^3 + 3^3 + \dots + n^3}{3n^4 + 5n^3 + 6} =$$

- 1) $\frac{1}{3}$ 2) $\frac{1}{5}$ 3) $\frac{1}{6}$ 4) $\frac{1}{12}$

$$36. \lim_{x \rightarrow \infty} x[\log(x+a) - \log x] =$$

- 1) e^2 2) e^a 3) a 4) $\frac{1}{a}$

$$37. \lim_{x \rightarrow \infty} (\cos \sqrt{x+1} - \cos \sqrt{x}) =$$

- 1) 1 2) -1 3) $\frac{1}{3}$ 4) 0

$$38. \lim_{x \rightarrow \infty} \frac{3\sqrt{x} + 5\sin^2 x - 10 \cdot \log x}{5\sqrt{x} + 7\cos^2 x + 100 \cdot \log x} =$$

- 1) $\frac{3}{5}$ 2) $\frac{5}{3}$ 3) 15 4) $\frac{1}{15}$

$$39. \lim_{x \rightarrow 0} (\cos x)^{\frac{1}{x^2}} =$$

- 1) $\frac{-1}{2\sqrt{e}}$ 2) ∞ 3) $\sqrt{e}$ 4) $\frac{1}{\sqrt{e}}$

$$40. \lim_{x \rightarrow a} \left(\frac{\sin x}{\sin a} \right)^{\frac{1}{x-a}} =$$

- 1) $e^{\cos a}$ 2) $e^{\cot a}$ 3) $e^{\sin a}$ 4) $e^{\tan a}$

$$41. \lim_{x \rightarrow 0} \left(\frac{a^x + b^x}{2} \right)^{\frac{1}{x}} =$$

- 1) ab 2) $\frac{1}{ab}$ 3) $a^2 b^2$ 4) $\sqrt{ab}$

$$42. \lim_{x \rightarrow 0} (\cos x)^{\frac{1}{\sin x}} =$$

- 1) 1 2) -1 3) 0 4) 3

$$43. \text{ If } a = \lim_{x \rightarrow \infty} \left(\sqrt{x} + \sqrt{x-1} - \sqrt{x} \right) \text{ and}$$

$$b = \lim_{x \rightarrow \infty} \left(x - \sqrt{x+x^2} \right) \text{ then}$$

- 1) $a + b = 1$ 2) $a + b = 0$
3) $a = b$ 4) $a + b = 2$

$$44. \text{ If } f(x) = \frac{4-7x}{7x+4}, \quad \lim_{x \rightarrow 0} f(x) = l \text{ and}$$

$$\lim_{x \rightarrow \infty} f(x) = m \text{ the quadratic equation having}$$

$$\text{roots as } \frac{1}{l} \text{ and } \frac{1}{m} \text{ is}$$

- 1) $x^2 - 1 = 0$ 2) $x^2 + 1 = 0$
3) $\frac{1}{2}$ 4) $x^3 - 1 = 0$

$$45. \lim_{x \rightarrow \infty} \frac{x^4 \cdot \sin\left(\frac{1}{x}\right) + x^2}{1 + |x|^3} =$$

- 1) 0 2) 1
3) -1 4) does not exist

$$46. \lim_{x \rightarrow \infty} x \left(a^{\frac{1}{x}} - b^{\frac{1}{x}} \right) =$$

- 1) 1 2) $\log_e a/b$ 3) $\log_e b/c$ 4) 0

$$47. \lim_{x \rightarrow \infty} \left(\frac{x^2 + 5x + 3}{x^2 + x + 2} \right)^x$$

- 1) e^4 2) e^3 3) e^2 4) 2^4

$$48. \text{ If } 0 < x < y \text{ then } \lim_{n \rightarrow \infty} \left(y^n + x^n \right)^{\frac{1}{n}} =$$

- 1) 1 2) x 3) y 4) e

$$49. \lim_{n \rightarrow \infty} n \cos\left(\frac{\pi}{n}\right) \sin\left(\frac{2\pi}{3n}\right) =$$

- 1) $\frac{2\pi}{3}$ 2) 1 3) π 4) 0

50. $\lim_{n \rightarrow \infty} \frac{1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + \dots + n \cdot n!}{(n+1)!} =$
 1) -1 2) 1 3) 1/2 4) 2
51. $\lim_{x \rightarrow 0} \frac{(4^x - 1)^3}{\sin\left(\frac{x}{4}\right) \log\left(1 + \frac{x^2}{3}\right)} =$
 1) $3(\log 4)^3$ 2) $4(\log 4)^3$
 3) $12(\log 4)^3$ 4) $15(\log 4)^3$
52. $\lim_{x \rightarrow 0} \left(1 + \tan^2 \sqrt{x}\right)^{\frac{1}{2x}} =$
 1) e 2) $\frac{1}{2}$
 3) $e^{\frac{1}{2}}$ 4) 0
53. $\lim_{x \rightarrow 0} \left\{ \tan\left(\frac{\pi}{4} + x\right) \right\}^{\frac{1}{x}} =$
 1) e 2) e^2 3) e^{-1} 4) e^{-2}
54. $\lim_{n \rightarrow \infty} \frac{1 + 3 + 6 + \dots + \frac{n(n+1)}{2}}{n^3} =$
 1) $\frac{1}{6}$ 2) $\frac{1}{3}$ 3) $\frac{1}{4}$ 4) $\frac{1}{2}$
55. $\lim_{n \rightarrow \infty} \frac{n(1^2 + 2^2 + 3^2 + \dots + n^2)^5}{(1^3 + 2^3 + 3^3 + \dots + n^3)^4} =$
 1) $\frac{256}{243}$ 2) $\frac{4^4}{6^5}$ 3) $\frac{243}{256}$ 4) ∞
56. $\lim_{n \rightarrow \infty} \frac{n + (-1)^n}{n - (-1)^n} =$
 1) 0 2) -1 3) 1 4) $\frac{1}{2}$
57. $\lim_{n \rightarrow \infty} \frac{{}^n P_n}{{}^{n+1} P_{n+1} - {}^n P_n} =$
 1) 1 2) -1 3) 0 4) ∞
58. If $\lim_{x \rightarrow \infty} \left(\frac{x+h}{x-h}\right)^x = 4$, then h =
 1) $\log_e 2$ 2) $\log_{10} 2$ 3) $\log_2 e$ 4) $\log_2 10$

59. The value of $f(0)$ so that the function $f(x) = \frac{\sqrt{1+x} - \sqrt[3]{1+x}}{x}$ becomes continuous is equal to
 1) $\frac{1}{6}$ 2) $\frac{1}{4}$ 3) 2 4) $\frac{1}{3}$
60. If $f(x) = \frac{1}{3^x + 1}$ then at $x=0$, $f(x)$ is
 1) continuous 2) discontinuous
 3) not determined 4) $\lim_{x \rightarrow 0} f(x) = 2$
61. The function $f(x) = \frac{1 - \sin x}{(\pi - 2x)^2}$ for $x \neq \frac{\pi}{2}$
 $=k$ for $x = \frac{\pi}{2}$ is continuous at $x = \frac{\pi}{2}$ then k is equal to
 1) $\frac{1}{8}$ 2) 4 3) 3 4) 1
62. $\lim_{x \rightarrow 0} \frac{\sin 2x + a \sin x}{x^3}$ exists and finite then a =
 1) 2 2) -2 3) $\frac{2}{3}$ 4) $-\frac{2}{3}$
63. $f(x) = \begin{cases} \frac{(x + bx^2)^{\frac{1}{2}} - x^{\frac{1}{2}}}{bx^{\frac{3}{2}}} & , x > 0 \\ C & , x = 0 \\ \frac{\sin(a+1)x + \sin x}{x} & , x < 0 \end{cases}$
 is continuous at $x=0$ then
 1) $a = \frac{-3}{2}, b = 0, c = \frac{1}{2}$
 2) $a = \frac{-3}{2}, b \neq 0, c = \frac{1}{2}$
 3) $a = \frac{3}{2}, b \neq 0, c = \frac{1}{2}$
 4) $a = \frac{3}{2}, b \neq 0, c = -\frac{1}{2}$

64. $\lim_{x \rightarrow 0} \frac{(1+a^3)+8e^{1/x}}{1+(1-b^3)e^{1/x}} = 2$ then
 1) $a=1, b=(-3)^{1/3}$ 2) $a=1, b=3^{1/3}$
 3) $a=-1, b=(-3)^{1/3}$ 4) $a=1, b=0$
65. $\lim_{x \rightarrow 0} \frac{\log(1+x+x^2)+\log(1-x+x^2)}{\sec x - \cos x} =$
 1) 1 2) -1 3) 0 4) ∞
66. $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\sqrt{1-\sqrt{\sin 2x}}}{\pi-4x} =$
 1) -1/4 2) 1/2
 3) 1/4 4) does not exist
67. $\lim_{x \rightarrow \infty} \frac{2\sqrt{x}+3\sqrt[3]{x}+5\sqrt[5]{x}}{\sqrt{3x-2}+\sqrt[3]{2x-3}} =$
 1) $\frac{2}{\sqrt{3}}$ 2) $\sqrt{3}$ 3) $\frac{1}{\sqrt{3}}$ 4) $\frac{\sqrt{3}}{2}$
68. If α is a repeated root of $ax^2+bx+c=0$ then
 $\lim_{x \rightarrow \alpha} \frac{\tan(ax^2+bx+c)}{(x-\alpha)^2} =$
 1) a 2) b 3) c 4) 0
69. $\lim_{x \rightarrow a} \frac{a^x - x^a}{x^x - a^a} = -1$ then $a =$
 1) 1 2) $\log a$ 3) $-\log a$ 4) 2
70. If $f'(0)=3$, $\lim_{x \rightarrow 0} \frac{x^2}{f(x^2)-6f(4x^2)+5f(7x^2)} =$
 1) $\frac{1}{36}$ 2) $-\frac{1}{36}$ 3) $\frac{1}{34}$ 4) $\frac{1}{106}$
71. $\lim_{x \rightarrow 1} \frac{Px^2+Ex+r}{x-1} = 0$, then
 1) $P=E=r$ 2) $P=r=-\frac{E}{2}$
 3) $2P=2r=E$ 4) $P=r=E$
72. If $\lim_{x \rightarrow \infty} \left[1+x+\frac{f(x)}{x}\right]^{\frac{1}{x}} = e^3$ then the value of the function $f(x)$ may be
 1) $\frac{x^2}{2}$ 2) x^2 3) $2x^2$ 4) $3x^2$

73. $f(x) = \min\{x, x^2\} \forall x \in R$. Then $f(x)$ is
 1) discontinuous at 0 2) discontinuous at 1
 3) continuous on R 4) continuous at 0, 1
74. If $f(x) = \frac{\sin 3(x-p)}{\sin 2(x-p)}$, $x \neq p$ is continuous at $x=p$ then $f(p) =$
 1) 1 2) 0 3) $\frac{2}{3}$ 4) $\frac{3}{2}$
75. If $f(x) = \frac{|x|}{[x]}$ and $x \notin [0,1]$ then
 $\lim_{x \rightarrow 2+} \frac{f(x)-f(2)}{x-2} =$
 1) $\frac{1}{4}$ 2) $\frac{1}{2}$ 3) 1 4) 0
76. If $f(x)$ is differentiable function and where $f'(1)=4$, $f'(2)=6$, $f'(c)$ means the derivative of f at c then
 $\lim_{h \rightarrow 0} \frac{f(2+2h+h^2)-f(2)}{f(1+h-h^2)-f(1)} =$
 1) does not exist 2) -3
 3) 3 4) $\frac{3}{2}$
77. $\lim_{x \rightarrow 0} \frac{1-\cos(1-\cos x)}{x^4} =$
 1) $\frac{1}{8}$ 2) $\frac{1}{2}$ 3) $\frac{1}{4}$ 4) 1
78. $\lim_{x \rightarrow 0} \frac{\sin^{-1} x + 3x}{\tan x + 2\sin\left(\frac{1}{2}\sin^{-1} x\right)\left(3-4\sin^2\left(\frac{1}{2}\sin^{-1} x\right)\right)} =$
 1) -1 2) 1 3) 0 4) 2
79. $\lim_{x \rightarrow 2} \frac{\sqrt{x+7}-3\sqrt{2x-3}}{\sqrt[3]{x+6}-2\sqrt[3]{3x-5}} =$
 1) $\frac{34}{23}$ 2) $\frac{23}{17}$ 3) $\frac{7}{23}$ 4) $\frac{23}{7}$

80. $\lim_{n \rightarrow \infty} \frac{(2n-1)(3n+5)}{(n-1)(3n+1)(3^n+2^n)} =$
 1) 2 2) ∞ 3) 0 4) 1
81. The value of P for which the function $f(x) =$

$$\frac{(4^x - 1)^3}{\sin \frac{x}{p} \cdot \log \left(1 + \frac{x^2}{3} \right)} \text{ for } x \neq 0$$

 $= 12(\log 4)^3$ for $x = 0$
 is continuous at $x = 0$, is
 1) 4 2) 2 3) 3 4) 1
82. The value of $f(0)$ so that the function $f(x)$
 $= \frac{1 - \cos(1 - \cos x)}{x^4}$ is continuous everywhere is
 1) $\frac{1}{8}$ 2) $\frac{1}{2}$ 3) $\frac{1}{4}$ 4) $\frac{1}{3}$

KEY

- | | | | | |
|-------|-------|-------|-------|-------|
| 1) 3 | 2) 2 | 3) 4 | 4) 1 | 5) 2 |
| 6) 3 | 7) 1 | 8) 2 | 9) 4 | 10) 4 |
| 11) 1 | 12) 2 | 13) 4 | 14) 2 | 15) 3 |
| 16) 4 | 17) 1 | 18) 1 | 19) 2 | 20) 1 |
| 21) 1 | 22) 3 | 23) 3 | 24) 4 | 25) 2 |
| 26) 1 | 27) 2 | 28) 1 | 29) 3 | 30) 4 |
| 31) 1 | 32) 1 | 33) 4 | 34) 3 | 35) 3 |
| 36) 3 | 37) 4 | 38) 1 | 39) 4 | 40) 2 |
| 41) 4 | 42) 2 | 43) 2 | 44) 1 | 45) 2 |
| 46) 2 | 47) 1 | 48) 3 | 49) 1 | 50) 1 |
| 51) 3 | 52) 3 | 53) 2 | 54) 1 | 55) 1 |
| 56) 3 | 57) 3 | 58) 1 | 59) 1 | 60) 2 |
| 61) 1 | 62) 2 | 63) 2 | 64) 1 | 65) 1 |
| 66) 4 | 67) 1 | 68) 1 | 69) 1 | 70) 1 |
| 71) 2 | 72) 3 | 73) 3 | 74) 4 | 75) 2 |
| 76) 3 | 77) 1 | 78) 2 | 79) 1 | 80) 3 |
| 81) 1 | 82) 1 | | | |

HINTS

23. Put $\frac{x^2}{4} = a$

21. $\lim_{x \rightarrow 0} \frac{\tan^4 x \left(\frac{1 - \cos^4 x}{x^2} \right)}{x^4}$

24. $\lim_{x \rightarrow 0^+} \frac{\sin[x]}{[x]} = 1; \quad \lim_{x \rightarrow 0^-} \frac{\sin[x]}{[x]} = \sin 1$

25. $\frac{d}{dx} \left(\int_{\psi(x)}^{\varphi(x)} f(t) dt \right) = f(\varphi(x))\varphi'(x) - f(\psi(x))\psi'(x)$

28. $\lim_{x \rightarrow \alpha} \frac{1 - \cos[a(x - \alpha)(x - \beta)]}{(x - \alpha)^2}$ and apply

$$\frac{1 - \cos ax}{x^2} = \frac{a^2}{2}$$

34. Divide Nr. and Dr. by 5^n

66. $\lim_{x \rightarrow 0} \frac{|\sin x|}{x}$ does not exist

72. $\lim_{x \rightarrow \infty} [1 + x(1 + k)]^{1/x} = e^{k+1}$

LEVEL - III

1. $\lim_{n \rightarrow \infty} \frac{1}{n^4} \sum_{r=1}^n r(r+2)(r+4) =$

1) $\frac{3}{4}$ 2) 0 3) $\frac{1}{8}$ 4) $\frac{1}{4}$

2. $\lim_{x \rightarrow 0} \frac{e^{x^3} - 1 - x^3}{\sin^6(2x)} =$

1) $\frac{1}{128}$ 2) $\frac{2}{127}$ 3) $\frac{1}{126}$ 4) $\frac{1}{125}$

3. $\lim_{n \rightarrow \infty} \log \left(\prod_{r=1}^n \frac{1}{5^{2^r}} \right) =$

1) $\log_e 5$ 2) $\log_5 e$ 3) 0 4) $2 \log_e 5$

4. $\lim_{n \rightarrow \infty} \left[\frac{7}{10} + \frac{29}{10^2} + \frac{133}{10^3} + \dots + \frac{5^n + 2^n}{10^n} \right] =$

1) $\frac{3}{4}$ 2) 2 3) $\frac{5}{4}$ 4) $\frac{1}{2}$

5. $\lim_{x \rightarrow 0} \frac{(1+x)^{1/x} - e}{x} =$

1) 1 2) $e/2$ 3) $-e/2$ 4) $2/e$

6. Suppose $f(n+1) = \frac{1}{2} \left\{ f(n) + \frac{9}{f(n)} \right\}, n \in N$. If

$f(n) > 0, \forall n \in N$, then $\lim_{n \rightarrow \infty} (f(n)) =$

- 1) 3^{-1} 2) -3^{-1} 3) 3 4) -3

7. If $|x| < 1$, then $\lim_{n \rightarrow \infty} (1+x)(1+x^2)(1+x^4) \dots (1+x^{2^n}) =$

- 1) $\frac{1}{x}$ 2) $\frac{1}{1+x}$ 3) $\frac{1}{1-x}$ 4) $\frac{1}{x-1}$

8. If $a_1 = 1$ and $a_{n+1} = \frac{4+3a_n}{3+2a_n}$, then $\lim_{n \rightarrow \infty} a_n =$

- 1) 0 2) -2 3) $\sqrt{2}$ 4) $-\sqrt{2}$

9. $\lim_{x \rightarrow \infty} \left(\frac{x^2 + 2x - 1}{2x^2 - 3x - 2} \right)^{\frac{2x+1}{2x-1}} =$

- 1) $-\frac{1}{2}$ 2) $\frac{1}{2}$ 3) $e^{-\frac{1}{2}}$ 4)

10. $\lim_{n \rightarrow \infty} \frac{\sin\left(\frac{a}{2^n}\right)}{\sin\left(\frac{b}{2^{n+1}}\right)} =$

- 1) $\frac{a}{2b}$ 2) $\frac{2a}{b}$ 3) 1 4) 0

11. $\lim_{x \rightarrow 0} \left[\frac{\log \sec \left[\frac{x}{2} \right] \cos x}{\log \sec x \cos \left(\frac{x}{2} \right)} \right] =$

- 1) 14 2) 15 3) 16 4) 17

12. $\lim_{x \rightarrow \frac{\pi}{2}} \frac{1 - (\sin x)^{\sin x}}{\cos^2 x} =$

- 1) 2 2) 1 3) $\frac{1}{2}$ 4) $\frac{1}{4}$

13. The set of points of discontinuity of

$$f(x) = \lim_{x \rightarrow \infty} \frac{(2 \sin x)^{2n}}{3^n - (2 \cos x)^{2n}} \text{ is}$$

1) $\{n\pi / n \in \mathbb{Z}\}$ 2) $\left\{ \frac{n\pi \pm \frac{\pi}{6}}{n \in \mathbb{Z}} \right\}$

3) $\left\{ \frac{n\pi \pm \frac{\pi}{2}}{n \in \mathbb{Z}} \right\}$ 4) $\left\{ \frac{n\pi \pm \frac{\pi}{3}}{n \in \mathbb{Z}} \right\}$

14. $\lim_{x \rightarrow 0} \left[\frac{x - 1 + \sin x}{x} \right]^{\frac{1}{x}} =$

- 1) 1 2) 0 3) $e^{-1/2}$ 4) $e^{1/2}$

15. $\lim_{x \rightarrow 0} \frac{x \sin(\sin x)}{1 - \cos x} =$

- 1) 1 2) 2 3) 4 4) 0

16. If $[x]$ denotes the greatest integer less than or equal to x then

$$\lim_{n \rightarrow \infty} \frac{1}{n^3} \left\{ [1^2 x] + [2^2 x] + [3^2 x] + \dots + [n^2 x] \right\} =$$

- 1) $\frac{x}{2}$ 2) $\frac{x}{3}$ 3) $\frac{x}{6}$ 4) 0

17. $\lim_{x \rightarrow \infty} \frac{[x] + [2x] + \dots + [nx]}{n^2} =$

- 1) $x/2$ 2) $x/3$
3) x 4) 0

18. $\lim_{x \rightarrow 0} \frac{\tan \left[-\pi^2 \right] x^2 - \tan \left[-\pi^2 \right] x^2}{\sin^2 x}$ where $[.]$ is the greatest integer function

- 1) $\tan 10 - 10$ 2) $\tan^2 10 - 20$
3) $\tan 10$ 4) 0

19. $\lim_{x \rightarrow -1} \frac{\cos 2 - \cos x}{x^2 - |x|} =$

- 1) $2 \sin 2$ 2) $\sin 2$
3) $2 \cos 2$ 4) does not exist

20. $\lim_{n \rightarrow \infty} \sqrt[2]{\left(1 - \cos \frac{1}{n} \right) \sqrt{\left(1 - \cos \frac{1}{n} \right) \sqrt{\left(1 - \cos \frac{1}{n} \right) \dots \infty}}} =$

- 1) $1/4$ 2) $1/3$ 4) $1/2$ 4) 1

21. $\lim_{x \rightarrow 1} \left[\frac{x^3 + 2x^2 + x + 1}{x^2 + 2x + 3} \right]^{\frac{1 - \cos(x-1)}{(x-1)^2}} =$
- 1) e 2) $e^{1/2}$ 3) 1 4) $\left(\frac{5}{6}\right)^{1/2}$
22. The value of $\lim_{x \rightarrow \infty} \cos\left(\frac{x}{2}\right) \cos\left(\frac{x}{4}\right) \cos\left(\frac{x}{8}\right) \dots \cos\left(\frac{x}{2^n}\right) =$
- 1) 1 2) $\frac{\sin x}{x}$ 3) $\frac{x}{\sin x}$ 4) 2
23. $\lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)^{\frac{\sin x}{x - \sin x}} =$
- 1) e 2) e^2
3) e^3 4) $1/e$
24. $\lim_{x \rightarrow \infty} (1 + e^{-x})^{e^x} =$
- 1) e 2) e^2
3) e^3 4) $1/e$
25. $\lim_{x \rightarrow \infty} \left\{ \sqrt{x^2 - \sqrt{x^4 + 1}} - \sqrt{2x} \right\}$
- 1) 0 2) 1 3) 2 4) 4
26. $\lim_{x \rightarrow \infty} \frac{1 - 2 + 3 - 4 + 5 - 6 + \dots - 2n}{\sqrt{n^2 + 1} + \sqrt{4n^2 - 1}} =$
- 1) $\frac{1}{3}$ 2) $-\frac{1}{3}$ 3) $-\frac{1}{5}$ 4) $\frac{1}{5}$
27. $\lim_{n \rightarrow \infty} \left\{ \log_{n-1}^n \cdot \log_n^{n+1} \cdot \log_{n+1}^{n+2} \dots \log_{n^k-1}^{n^k} \right\} =$
- 1) 2n 2) n 3) k 4) 2k
28. $f(x) = \begin{vmatrix} 2 \cos x & x & 1 \\ 2 \sin x & x^2 & 2x \\ \tan x & x & 1 \end{vmatrix}$ then $\lim_{x \rightarrow 0} \frac{f'(x)}{x} =$
- 1) 1 2) -1 3) 2 4) -2
29. $f(x) = \begin{vmatrix} \cos x & 1 & 0 \\ 1 & 2 \cos x & 1 \\ 0 & 1 & 2 \cos x \end{vmatrix}$ then $\lim_{x \rightarrow 0} f(x) =$
- 1) 0 2) 1 3) 2 4) 3

30. If $a = \min \{x^2 + 4x + 5, x \in R\}$ and $b = \lim_{\theta \rightarrow 0} \frac{1 - \cos 2\theta}{\theta^2}$ then the value of $\sum_{r=0}^n a^r b^{n-r} =$
- 1) $\frac{2^{n+1} - 1}{4 \cdot 2^n} \cdot 2$ 2) $2^{n+1} - 1$
3) $\frac{2^{n+1} - 1}{3 \cdot 2^n}$ 4) $2^n - 1$
31. $\lim_{x \rightarrow 1} (\log_2 2x)^{\log_x 5} =$
- 1) $e^{\log_2 2}$ 2) $e^{\log_2 5}$ 3) e^5 4) e^2
32. If $f(x) = \frac{\sqrt{a^2 - ax + x^2} - \sqrt{a^2 + ax + x^2}}{\sqrt{a+x} - \sqrt{a-x}}$ is continuous at $x = 0$ then $f(0) =$
- 1) $\sqrt{a}$ 2) $-\sqrt{a}$ 3) $a\sqrt{a}$ 4) $-a\sqrt{a}$
33. If $f(x) = \begin{cases} x^\alpha \cos(1/x), & x \neq 0 \\ 0, & x = 0 \end{cases}$ is continuous at $x = 0$ then
- 1) $\alpha < 0$ 2) $\alpha > 0$ 3) $\alpha = 0$ 4) $\alpha \geq 0$
34. The graph of $y = f(x)$ has unique tangent at the point $(a, 0)$ through which the graph passes. Then $\lim_{x \rightarrow a} \frac{\log[1 + 6f(x)]}{3f(x)} =$
- 1) 0 2) 1 3) 2 4) ∞
35. $\lim_{x \rightarrow 0} \frac{\sin(1+x) - \sin(1-x)}{x} =$
- 1) 2 2) $2 \cos 1$ 3) $\cos 2$ 4) 0

KEY

- 1) 4 2) 1 3) 1 4) 3 5) 3
6) 3 7) 3 8) 3 9) 2 10) 2
11) 3 12) 3 13) 2 14) 3 15) 2
16) 2 17) 1 18) 1 19) 1 20) 4
21) 4 22) 2 23) 4 24) 1 25) 1
26) 2 27) 3 28) 4 29) 2 30) 2
31) 2 32) 2 33) 2 34) 3 35) 2

HINTS

1. $\lim_{n \rightarrow \infty} \frac{\sum n^m}{n^{m+1}} = \frac{1}{m+1}$
3. $\lim_{x \rightarrow 0} \log \left(5^{\frac{1}{2}} \times 5^{\frac{1}{4}} \times \dots \right) = \log_e 5$
8. If $a_n = 1$, $\lim_{n \rightarrow \infty} (a_n + 1) = 1$, $\lim_{n \rightarrow \infty} a_n = 1$
15. Use formula $\lim_{x \rightarrow 0} \frac{1 - \cos ax}{x^2} = \frac{a^2}{2}$

LEVEL-IV

1. Assertion : If $|x| < 1$ then

$$\lim_{n \rightarrow \infty} (1+x)(1+x^2)(1+x^4) \dots (1+x^{2n}) = \frac{1}{1-x}$$

Reason : $(1+x)(1+x^2) \dots = \frac{1-x^{2n+1}}{1-x}$

 - 1) Both A and R are true and R is the correct explanation of A
 - 2) Both A and R are true and R is not the correct explanation of A
 - 3) A is true but R is false
 - 4) R is true but A is false
2. Assertion : $\lim_{x \rightarrow \infty} \left(\frac{x^2 + 5x + 3}{x^2 + x + 2} \right)^x = e^4$

Reason : $\lim_{x \rightarrow \infty} (1+x)^{1/x} = e$

 - 1) Both A and R are true and R is the correct explanation of A
 - 2) Both A and R are true and R is not the correct explanation of A
 - 3) A is true but R is false
 - 4) R is true but A is false
3. Assertion : $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$ is continuous at $x = 0$

Reason : Both

$$h(x) = x^2, g(x) = \begin{cases} \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases} \text{ are}$$

continuous at $x = 0$

- 1) Both A and R are true and R is the correct explanation of A
 - 2) Both A and R are true and R is not the correct explanation of A
 - 3) A is true but R is false 4) R is true but A is false
4. Assertion : $f(x) = \frac{\sin \{ [x] \pi \}}{1+x^2}$ is continuous on R.

Reason : Every constant function is continuous on R

 - 1) Both A and R are true and R is the correct explanation of A
 - 2) Both A and R are true and R is not the correct explanation of A
 - 3) A is true but R is false
 - 4) R is true but A is false
 5. Assertion : $f(x) = x \left(\frac{1+e^{1/x}}{1-e^{1/x}} \right) (x \neq 0)$, $f(0) = 0$ is continuous at $x = 0$.

Reason : A function is said to be continuous at 'a' if both limits are exists and equal to $f(a)$.

 - 1) Both A and R are true and R is the correct explanation of A
 - 2) Both A and R are true and R is not the correct explanation of A
 - 3) A is true but R is false
 - 4) R is true but A is false
 6. List - I List - II

a) $\lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)^{\frac{1}{x^2}}$ b) $\lim_{x \rightarrow 0} \left(\frac{\tan x}{x} \right)^{\frac{1}{x^2}}$ c) $\lim_{x \rightarrow 0} (\cos x)^{\frac{1}{\tan x}}$ d) $\lim_{x \rightarrow 0} \left(2 - \frac{x}{a} \right)^{\tan\left(\frac{\pi x}{2a}\right)}$	1) $e^{\frac{-1}{6}}$ 2) $e^{\frac{1}{3}}$ 3) $e^{\frac{2}{\pi}}$ 4) 1
---	--

Then the correct match for List - I from List - II

	a	b	c	d
1)	1	4	3	2
2)	1	2	3	4
3)	2	1	4	3
4)	1	2	4	3

7. List - I

List - II

a) $f(x) = [x]$ is continuous on

1) R

b) $f(x) = |x|$ is continuous on

2) R - Z

c) $f(x) = \frac{|x-2|}{x-2}$ is continuous on

3) $R - \{2\}$

Then the correct match for List - I from List - II

	a	b	c
1)	1	2	3
2)	1	3	2
3)	2	1	3
4)	2	3	1

8. If m, n are +ve integers and $a_0, b_0 \neq 0$ are non-zero real numbers

$$\lim_{n \rightarrow \infty} \frac{a_0 x^m + a_1 x^{m-1} + \dots + a_{m-1} x + a_m}{b_0 x^n + b_1 x^{n-1} + \dots + b_{n-1} x + b_n}$$

List - I

List - II

a) $m = n$

1) $\frac{a_0}{b_0}$

b) $m < n$

2) ∞

c) $m > n$

3) 0

Then the correct match for List - I from List - II

	a	b	c
1)	1	2	3
2)	1	3	2
3)	2	1	3
4)	1	3	3

9. Match the following :

List - I

List - II

a) The no. of points of discontinuity of the function

1) 0

$$f(x) = \frac{1}{\log |x|} \text{ is}$$

b) If $f(x) = (x+1)^{\operatorname{cosec} x}$

is continuous

2) 3

at $x=0$ then $f(0) =$

$$c) f(x) = \frac{\cos x - \sin x}{\cos 2x},$$

$$x \neq \frac{\pi}{4} \text{ If is}$$

3) 1

continuous on $\left[0, \frac{\pi}{4}\right]$

$$\text{then } f\left(\frac{\pi}{4}\right) =$$

$$d) \text{ If } f(x) = \frac{\sqrt{1+\sin x} - \sqrt{1-\sin x}}{x}$$

is continuous

4) e

everywhere

$$\text{then } f(0) =$$

5) $\frac{1}{\sqrt{2}}$

Then the correct match for List - I from List - II

	a	b	c	d
1)	1	2	3	4
2)	4	3	1	2
3)	2	4	5	3
4)	2	4	1	5

10. Match the following :

$$A) \lim_{x \rightarrow \infty} \left(\frac{a^{1/x} + b^{1/x} + c^{1/x}}{3} \right)^{1/x} =$$

1) 1/2

$$B) \lim_{x \rightarrow 5} \frac{x^2 - 9x + 20}{x - [x]} =$$

2) -1

$$C) \lim_{x \rightarrow 0} \frac{1 - e^{1/x^2}}{1 + e^{1/x^2}} =$$

3) does not exist

$$D) \lim_{n \rightarrow \infty} \frac{2 \cdot 3^n + 5 \cdot 2^n}{4 \cdot 3^n - 7 \cdot 2^n} =$$

4) $(abc)^{1/3}$

	A	B	C	D
1)	1	2	3	4
2)	4	3	3	1
3)	4	3	2	1
4)	4	3	1	2

11. I: $\lim_{n \rightarrow \infty} \frac{n!}{(n+1)! - n!} = 0$
- II: $\lim_{x \rightarrow 0} \sqrt{\frac{x - \sin^2 x}{x + \cos x}} = 1$
- 1) Only I is true 2) Only II is true
3) Both I and II are true
4) Neither I nor II is true
12. I: If $a > 0$ then $\lim_{x \rightarrow \infty} \frac{[ax+b]}{x} = a$
- II: $\lim_{x \rightarrow \frac{\pi}{2}} [\sin x] = 0$
- 1) Only I is true 2) Only II is true
3) Both I and II are true
4) Neither I nor II is true
13. I: $\lim_{n \rightarrow \infty} \frac{1+3+6+\dots+\frac{n(n+1)}{2}}{n^3} = \frac{1}{6}$
- II: $\lim_{n \rightarrow \infty} \frac{1.1! + 2.2! + 3.3! + \dots + n.n!}{(n+1)!} = 1$
- 1) Only I is true 2) Only II is true
3) Both I and II are true
4) Neither I nor II is true
14. I: $\lim_{x \rightarrow 0} \left(\frac{\tan[e^2]x^2 - \tan[-e^2]x^2}{\sin^2 x} \right) = 15$
- II: $\lim_{x \rightarrow 0} \left(\frac{1^x + 2^x + 3^x + \dots + n^x}{n} \right)^{\frac{1}{x}} = (n!)^{1/n}$
- 1) Only I is true 2) Only II is true
3) Both I and II are true
4) Neither I nor II is true
15. I: $\lim_{\theta \rightarrow 0} \left(\left[\frac{n \sin \theta}{\theta} \right] + \left[\frac{n \tan \theta}{\theta} \right] \right) = \text{even integer}$
- where $[.]$ denotes the greatest integer function.
- II: $\lim_{x \rightarrow 0} \frac{ae^x - b}{x} = 2$ then $a=2, b=2$
- 1) Only I is true
2) Only II is true
3) Both I and II are true
4) Neither I nor II is true

16. Arrange the following limits in ascending order

- a) $\lim_{x \rightarrow 0} \frac{\sin x^0}{x}$
- b) $\lim_{x \rightarrow 0} \frac{3 \sin x^0 - \sin 3x^0}{x^3}$
- c) $\lim_{x \rightarrow 0} \frac{\tan x^0}{2x}$
- d) $\lim_{x \rightarrow 0} \frac{(\sec x^0 + 1)(\sec x^0 - 1)}{x^2}$

1) b,d,c,a 2) b,c,a,d 3) b,c,d,a 4) b,a,d,c

17. Arrange the following limits in the ascending order.

- a) $\lim_{x \rightarrow 0} \frac{\tan^4 x - \sin^4 x}{x^6}$
- b) $\lim_{x \rightarrow 0} \frac{\tan^8 x - \sin^8 x}{x^5 \tan x^5 x}$
- c) $\lim_{x \rightarrow 0} \frac{\tan^3 x - \sin^3 x}{x \sin^4 x}$
- d) $\lim_{x \rightarrow 0} \frac{\tan^5 x - \sin^5 x}{x^2 \cdot \sinh^3 x \cdot \tan^2 x}$

1) a,b,a,d 2) c,a,d,b 3) a,b,d,c 4) b,a,c,d

18. Arrange the following limits in the ascending order

- a) $\lim_{x \rightarrow 0} \left(\frac{1^x + 2^x + 4^x}{3} \right)^{\frac{1}{x}}$
- b) $\lim_{x \rightarrow 0} \left(\frac{2^x + 3^x + 8^x + 27^x}{4} \right)^{\frac{1}{x}}$
- c) $\lim_{x \rightarrow 0} \left(\frac{1^x + \left(\frac{1}{2}\right)^x + \left(\frac{1}{4}\right)^x}{3} \right)^{\frac{1}{x}}$

- d) $\lim_{x \rightarrow 0} \left(\frac{\left(\frac{1}{2}\right)^x + \left(\frac{1}{3}\right)^x + \left(\frac{1}{8}\right)^x + \left(\frac{1}{27}\right)^x}{4} \right)^{\frac{1}{x}}$

1) d,c,a,b 2) d,a,b,c 3) a,c,b,d 4) a,b,c,d

19. Arrange the following limits in the ascending order

a) $\lim_{x \rightarrow \infty} \left(\frac{1+x}{2+x} \right)^{x+2}$ b) $\lim_{x \rightarrow 0} (1+2x)^{3/x}$

c) $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{2\theta}$ d) $\lim_{x \rightarrow 0} \frac{\log(1+x)}{x}$

- 1) a,b,c,d 2) a, c, d, b 3) a, d, c, b 4) c,d, a, b

20. Arrange the following limits in the descending order.

a) $\lim_{x \rightarrow 3^-} \frac{3-x}{|x-3|}$

b) $\lim_{x \rightarrow 0} \frac{(1+x)^{\frac{1}{8}} - (1-x)^{\frac{1}{8}}}{x}$

c) $\lim_{x \rightarrow 0} \frac{\tan x - \sin x}{x^3}$

d) $\lim_{x \rightarrow \infty} \left\{ \sqrt{x^2 + 6x + 7} + x \right\}$

- 1) b,c,d,a 2) d,a,c,b 3) d,c, b, a 4) b,c, a, d

KEY

- 1) 1 2) 3 3) 3 4) 1 5) 1
6) 4 7) 3 8) 2 9) 3 10) 3
11) 3 12) 3 13) 3 14) 3 15) 2
16) 1 17) 2 18) 1 19) 2 20) 2

LEVEL-V

- I. A function 'f' is said to be continuous at a point $x = a$ if $\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x) = f(a)$. It is said to be discontinuous at $x = a$ if

$\lim_{x \rightarrow a} f(x) \neq f(a)$. Given $f(x) =$

$$\begin{cases} \left(\frac{3}{x^2} \right) \sin 2x^2 & \text{if } x < 0 \\ \frac{x^2 + 2x + c}{1 - 3x^2} & \text{if } x \geq 0, x \neq \frac{1}{\sqrt{3}} \\ 0 & \text{if } x = \frac{1}{\sqrt{3}} \end{cases}$$

1. $\lim_{x \rightarrow 0^-} f(x) =$

- 1) 2 2) 4 3) 6 4) 8

2. $\lim_{x \rightarrow 0^+} f(x) =$

1) $\frac{c}{4}$ 2) $2c$

3) $\frac{c}{3}$ 4) c

3. If f is to be continuous at $x=0$ then the value of c is

- 1) 2 2) 4 3) 6 4) 8

- II. For all real values of u and v a function 'f' is defined as $2f(u) \cos v = f(u+v) + f(u-v)$

and also $f(0) = \lim_{x \rightarrow 0} \left\{ \frac{1 + \tan x}{1 + \sin x} \right\}^{\cos ec x}$ and

$f\left(\frac{\pi}{2}\right) = m$ where m is the no. of points at

which the function $g(x) = [x]$ is not continuous in $[2, 3)$ then

1. $f(x) + f(-x) =$

- 1) $\cos x$ 2) $\sin x$
3) $2\cos x$ 4) $2\sin x$

2. $f(\pi + x) + f(-x) =$

- 1) 0 2) 1
3) $2\cos x$ 4) non of these

3. $f(\pi - x) + f(x) =$

- 1) $2\sin x$ 2) $\sin x$
3) $\cos x$ 4) $2\cos x$

KEY

- I. 1. 3 2. 4 3. 3
II. 1. 3 2. 4 3. 1

PREVIOUS EAMCET QUESTIONS 2005

1. $\lim_{x \rightarrow 0} x^2 \sin \frac{\pi}{x} =$

- 1) 1 2) 0
3) does not exist 4) ∞

$$f(x) = \begin{cases} \frac{x-2}{x^2-3x+2} & \text{if } x \in R - \{1, 2\} \\ 2 & \text{if } x = 1 \\ 1 & \text{if } x = 2 \end{cases}$$

then $\lim_{x \rightarrow 2} \frac{f(x) - f(2)}{x - 2} =$

- 1) 0 2) -1 3) 1 4) $-\frac{1}{2}$

3. If $f: R \rightarrow R$ is defined by

$$f(x) = \begin{cases} \frac{x+2}{x^2+3x+2} & \text{if } x \in R - \{1, -2\} \\ -1 & \text{if } x = -2 \\ 0 & \text{if } x = -1 \end{cases}$$

then f is continuous on the set :

- 1) R 2) $R - \{-2\}$
3) $R - \{-1\}$ 4) $R - \{-1, -2\}$

2004

4. $\lim_{n \rightarrow \infty} \sum_{h=1}^n (h^2 x) =$

- 1) x 2) $\frac{x}{2}$ 3) $\frac{x}{3}$ 4) $\frac{x}{4}$

5. If $f(x) = \begin{cases} [x] & \text{if } -3 < x \leq -1 \\ |x| & \text{if } -1 < x < 1 \\ 1[-x] & \text{if } 1 \leq x < 3 \end{cases}$

Then $\{x / f(x) \geq 0\} =$

- 1) $(-1, 3)$ 2) $[-1, 3]$ 3) $(-1, 3]$ 4) $\{-1, 3\}$

2003

6. $\lim_{x \rightarrow \frac{\pi}{6}} \frac{3 \sin x - \sqrt{3} \cos x}{6x - \pi} =$

- 1) $\sqrt{3}$ 2) $\frac{1}{\sqrt{3}}$ 3) $-\sqrt{3}$ 4) $-\frac{1}{\sqrt{3}}$

7. If $a > 0$ and $\lim_{x \rightarrow a} \frac{a^x - x^a}{x^x - a^a} = -1$ then $a =$
1) 0 2) 1 3) e 4) $2e$

8. If $f(x) = \begin{cases} \frac{\sqrt{1+kx} - \sqrt{1-kx}}{x} & \text{for } 1n < 0 \leq \\ 2x^2 + 3x - 2 & \text{for } 0 \leq x \leq 1 \end{cases}$

is continuous at $x = 0$ then $k =$

- 1) -4 2) -3 3) -2 4) -1

2002

9. $\lim_{x \rightarrow 0} \frac{4^x - 9^x}{x(4^x + 9^x)} =$

- 1) $\log\left(\frac{3}{2}\right)$ 2) $\log\left(\frac{2}{3}\right)$
3) $\log\left(\frac{4}{3}\right)$ 4) $\log 2$

10. The quadratic equation whose roots l, m where

$$l = \lim_{\theta \rightarrow 0} \frac{3 \sin \theta - 4 \sin^2 \theta}{\theta}, \quad m = \lim_{\theta \rightarrow 0} \frac{2 \tan \theta}{1 - \tan^2 \theta} \text{ is}$$

- 1) $x^2 - 5x + 6 = 0$ 2) $x^2 + 5x + 6 = 0$
3) $x^2 - x + 6 = 0$ 4) $x^2 - x - 6 = 0$

11. The set of all discontinuous of $f(x) = x - [x]$ is

- 1) Set of all integers
2) Set of all irrational numbers
3) Set of all real numbers
4) Set of fractional vlaues

12. If $f(x) = \begin{cases} a^2 \cos^2 x + b^2 \sin^2 x, & x \leq 0 \\ e^{ax+b}, & x > 0 \end{cases}$

at $x = 0$ $f(x)$ is continuous then

- 1) $2 \log|a| = b$ 2) $2 \log|b| = e$
3) $\log a = 2 \log|b|$ 4) $a = b$

2001

13. If $f(x) = \begin{cases} 2x + b (x < \alpha) \\ x + d (x \geq \alpha) \end{cases}$ is such that

$$\lim_{x \rightarrow \alpha} f(x) = l, \text{ then } l =$$

- 1) $2d - b$ 2) $2b - d$ 3) $2d + b$ 4) $b - 2d$

14. $\lim_{x \rightarrow \infty} \left(\frac{x+a}{x+b} \right)^{x+b}$

- 1) 1 2) e^{b-a} 3) e^{a-b} 4) e^b

15. $\lim_{x \rightarrow 0} \frac{x \cdot 10^x - x}{1 - \cos x}$
 1) $\log 10$ 2) $2\log 10$ 3) $3\log 10$ 4) $4\log 10$
16. If $f(x) = \frac{x^2 - 10x + 25}{x^2 - 7x + 10}$ for $x \neq 5$ and f is continuous at $x=5$, then $f(5) =$
 1) 0 2) 5 3) 10 4) 25
2000
17. $\lim_{\theta \rightarrow \frac{\pi}{2}} \frac{1 - \sin \theta}{\cos \theta \left(\frac{\pi}{2} - \theta \right)} =$
 1) 1 2) -1 3) $-\frac{1}{2}$ 4) $\frac{1}{2}$
18. $\lim_{x \rightarrow 0} \frac{\log_e (1+x)}{3^x - 1} =$
 1) $\log_e 3$ 2) 0 3) 1 4) $\log_3 e$
19. If the function $f(x) = \begin{cases} \frac{\sin 3x}{x} & (x \neq 0) \\ \frac{k}{2} & (x = 0) \end{cases}$ is continuous at $x=0$, then $k =$
 1) 3 2) 6 3) 9 4) 12
1999
20. $\lim_{x \rightarrow 0} \frac{2x + 7 \sin x}{4x + 3 \cos x} =$
 1) 1 2) -1 3) $\frac{1}{2}$ 4) $-\frac{1}{2}$
1998
21. $\lim_{x \rightarrow 0} \log \left| \frac{\log(1+x)}{x} \right| =$
 1) 0 2) 1 3) e 4) $1/e$
22. $\lim_{x \rightarrow \infty} \left(\frac{a^{1/x} + b^{1/x} + c^{1/x}}{3} \right)^x$, where a, b, c are real and non-zero =
 1) 0 2) $(abc)^{1/3}$ 3) $(abc)^{-1/3}$ 4) 1
23. $\lim_{x \rightarrow \infty} (\sin \sqrt{x+1} - \sin \sqrt{x}) =$
 1) 2 2) -2 3) 0 4) 1

24. $\lim_{x \rightarrow 0} \left(\frac{1}{\sin^2 x} - \frac{1}{\sinh^2 x} \right) =$
 1) $\frac{2}{3}$ 2) 0 3) $\frac{1}{3}$ 4) $-\frac{2}{3}$
25. If $f(x) = \frac{\sin x}{x}$, $x \neq 0$ is to be continuous at $x=0$ then $f(0) =$
 1) 0 2) 1 3) -1 4) 2
1997
26. $\lim_{x \rightarrow \infty} \frac{3x^2 + 1}{2x^2 + 1} =$
 1) $\frac{3}{2}$ 2) $\frac{2}{3}$ 3) $-\frac{3}{2}$ 4) $-\frac{2}{3}$
27. Given that the function f is defined by
 $f(x) = \begin{cases} 2x-1, & x > 2 \\ k, & x = 2 \\ x^2-1, & x < 2 \end{cases}$ is continuous. Then $k =$
 1) 2 2) 3 3) 4 4) -3
1996
28. $\lim_{x \rightarrow 0} \frac{x^2}{\int_0^x \tan^{-1} x dx} =$
 1) 2 2) 1 3) 3 4) -1
29. $\lim_{n \rightarrow \infty} \frac{3 \cdot 2^{n+1} - 4 \cdot 5^{n+1}}{5 \cdot 2^n + 7 \cdot 5^n} =$
 1) $-\frac{20}{7}$ 2) $+\frac{20}{7}$ 3) $\frac{10}{7}$ 4) $-\frac{10}{7}$
30. If $f(x) = x^{\frac{1}{x-1}}$ for $x \neq 1$ and f is continuous at $x=1$ then $f(1) =$
 1) e 2) e^{-1} 3) e^{-2} 4) e^2
1995
31. $\lim_{x \rightarrow 0} \frac{x}{\sqrt{x+4} - 2} =$
 1) 4 2) $\sqrt{2}$ 3) $2\sqrt{2}$ 4) $1/\sqrt{2}$
32. $\lim_{x \rightarrow 0} \left(\frac{\int_0^x \sin^3 x \cos x dx}{x^4} \right) =$
 1) 0.25 2) 2.5 3) 5.2 4) 0.52

33. If $f(x) = \frac{\tan x}{x}, x \neq 0$
 $= 1, x = 0$ } then $f(x)$ at $x=0$ is
- 1) differentiable 2) Continuous
 3) Discontinuous 4) None

1994

34. $f(x) = \frac{x^2}{|x|}, x \neq 0; f(0) = 0$ then
- 1) $f(x)$ is discontinuous everywhere
 2) $f(x)$ is continuous everywhere
 3) $f(x)$ exists in $(-1, 1)$
 4) $f(x)$ exists in $(-2, 2)$

35. $\lim_{x \rightarrow 0} \frac{e^{\tan x} - e^x}{\tan x - x} =$
- 1) 1 2) e 3) e^{-1} 4) 0

36. $\lim_{x \rightarrow 1+} \frac{\sqrt{x-1} + \sqrt{x-1}}{\sqrt{x^2-1}} =$
- 1) $\frac{1}{2}$ 2) $\sqrt{2}$ 3) 1 4) $\frac{1}{\sqrt{2}}$

1993

37. $\lim_{x \rightarrow 0} \frac{a^x - b^x}{x} =$
- 1) $xa^{x-1} - xb^{x-1}$ 2) $\log a/b$ 3) $\log b/a$ 4) $\log ab$

38. $\lim_{x \rightarrow \infty} \left(\frac{x}{1+x} \right)^x =$
- 1) 1 2) e 3) $1/e$ 4) ∞

39. $\lim_{n \rightarrow \infty} \frac{1}{n^3} \left\{ 1 + 3 + 6 + \dots + \frac{n(n+1)}{2} \right\} =$
- 1) 0 2) 2 3) $\frac{1}{6}$ 4) $\frac{1}{3}$

1992

40. The function $f(x) = \frac{x}{1+|x|}$ is differentiable at
- 1) every where 2) except at $x = \pm 1$
 3) except at $x = 0$ 4) except at $x = 0$ or ± 1
41. If a, b, c, d are positive real numbers then

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{a+bn} \right)^{c+dn} =$$

- 1) $e^{d/b}$ 2) $e^{c/a}$ 3) $e^{c+d/a+b}$ 4) $e^{a+b/c+d}$

42. If $f(x) = \frac{3x+4\tan x}{x}, x \neq 0$ is continuous at $x=0$
 then $f(0) =$
- 1) 5 2) 6 3) 7 4) 8

43. $\lim_{x \rightarrow 0} \frac{x - \sin x}{x + \cos^2 x} =$
- 1) 0 2) 1 3) -1 4) 2

1990

44. $\lim_{n \rightarrow \infty} \frac{\sin n\theta}{\sqrt{n}} =$
- 1) 0 2) ∞ 3) 1 4) 2

KEY

- 1) 2 2) 2 3) 3 4) 3 5) 1
 6) 2 7) 1 8) 3 9) 2 10) 1
 11) 1 12) 1 13) 1 14) 3 15) 2
 16) 1 17) 4 18) 4 19) 2 20) 3
 21) 1 22) 2 23) 3 24) 1 25) 2
 26) 1 27) 2 28) 1 29) 1 30) 1
 31) 1 32) 1 33) 2 34) 2 35) 1
 36) 2 37) 2 38) 3 39) 3 40) 1
 41) 1 42) 3 43) 1 44) 1