4 CHAPTER Modern Maths

PERMUTATION AND COMBINATION

1990

1. There are six boxes numbered 1, 2, 3, 4, 5, 6. Each box is to be filled up either with a white ball or a black ball in such a manner that at least one box contains a black ball and all the boxes containing black balls are consecutively numbered. The total number of ways in which this can be done equals.

   (a) 15 (b) 21
   (c) 63 (d) 64

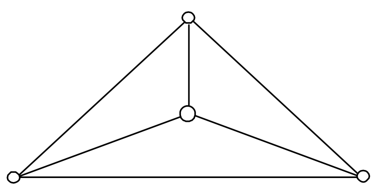
1991

2. How many 3-digit even numbers can you form such that if one of the digits is 5 then the following digit must be 7?

   (a) 5 (b) 405
   (c) 365 (d) 495

1993

3. Four cities are connected by a road network as shown in the figure. In how many ways can you start from any city and come back to it without travelling on the same road more than once?



   (a) 8 (b) 12
   (c) 16 (d) 20

4. A five digit number is formed using digits 1, 3, 5, 7 and 9 without repeating any one of them. What is the sum of all such possible numbers?

   (a) 6666600 (b) 6666660
   (c) 6666666 (d) None of these

5. Consider the five points comprising of the vertices of a square and the intersection point of its diagonals. How many triangles can be formed using these points?

   (a) 4 (b) 6
   (c) 8 (d) 10

1995

6. Boxes numbered 1, 2, 3, 4 and 5 are kept in a row, and they are to be filled with either a red or a blue ball, such that no two adjacent boxes can be filled with blue balls. Then how many different arrangements are possible, given that all balls of a given colour are exactly identical in all respects?

   (a) 8 (b) 10
   (c) 15 (d) 22

7. A, B, C and D are four towns, any three of which are non-collinear. Then the number of ways to construct three roads each joining a pair of towns so that the roads do not form a triangle is

   (a) 7 (b) 8
   (c) 9 (d) 24

1996

8. A man has 9 friends: 4 boys and 5 girls. In how many ways can he invite them, if there have to be exactly 3 girls in the invitees?

   (a) 320 (b) 160
   (c) 80 (d) 200

Direction for Question 9: The question is followed by two statements, I and II. Mark the answer as

(a) if the question cannot be answered even with the help of both the statements taken together.

(b) if the question can be answered by any one of the two statements.

(c) if each statement alone is sufficient to answer the question, but not the other one (e.g. statement I alone is required to answer the question, but not statement II and vice versa).

(d) if both statements I and II together are needed to answer the question.

9. How many different triangles can be formed?

   I. There are 16 coplanar, straight lines.

   II. No two lines are parallel.

1997

10. In how many ways can eight directors, the vice chairman and chairman of a firm be seated at a round table, if the chairman has to sit between the vice chairman and a director?

   (a) $9! \times 2$ (b) $2 \times 8!$
   (c) $2 \times 7!$ (d) None of these

1998

11. How many numbers can be formed from 1, 2, 3, 4, 5, without repetition, when the digit at the unit's place must be greater than that in the ten's place?

(a) 54 (b) 60
(c) 17 (d) 2 × 4!

1999

12. Ten points are marked on a straight-line and 11 points are marked on another straight-line. How many triangles can be constructed with vertices from among the above points?

(a) 495 (b) 550
(c) 1045 (d) 2475

13. For a scholarship, at the most n candidates out of 2n + 1 can be selected. If the number of different ways of selection of at least one candidate is 63, the maximum number of candidates that can be selected for the scholarship is

(a) 3 (b) 4
(c) 6 (d) 5

2000

14. One red flag, three white flags and two blue flags are arranged in a line such that:

I. No two adjacent flags are of the same colour
II. The flags at the two ends of the line are of different colours

In how many different ways can the flags be arranged?

(a) 6 (b) 4
(c) 10 (d) 2

15. Sam has forgotten his friend's seven-digit telephone number. He remembers the following: the first three digits are either 635 or 674, the number is odd, and the number 9 appears once. If Sam were to use a trial and error process to reach his friend, what is the minimum number of trials he has to make before he can be certain to succeed?

(a) 10,000 (b) 2,430
(c) 3,402 (d) 3,006

16. ABCDEFGH is a regular octagon. A and E are opposite vertices of the octagon. A frog starts jumping from vertex to vertex, beginning from A. From any vertex of the octagon except E, it may jump to either of the two adjacent vertices. When it reaches E, the frog stops and stays there. Let a_n be the number of distinct paths of exactly n jumps ending in E. Then what is the value of a_{2n-1}?

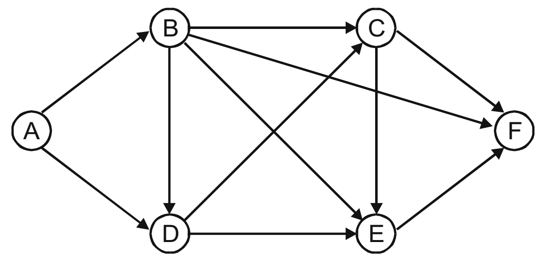
(a) 0 (b) 4
(c) 2n – 1 (d) Cannot be determined

2001

17. Ashish is given Rs. 158 in one-rupee denominations. He has been asked to allocate them into a number of bags such that any amount required between Re 1 and Rs. 158 can be given by handing out a certain number of bags without opening them. What is the minimum number of bags required?

(a) 11 (b) 12
(c) 13 (d) None of these

18. The figure below shows the network connecting cities A, B, C, D, E and F. The arrows indicate permissible direction of travel. What is the number of distinct paths from A to F?



(a) 9 (b) 10
(c) 11 (d) None of these

19. Let n be the number of different five-digit numbers, divisible by 4 with the digits 1, 2, 3, 4, 5 and 6, no digit being repeated in the numbers. What is the value of n?

(a) 144 (b) 168
(c) 192 (d) None of these

2002

20. If there are 10 positive real numbers $n_1 < n_2 < n_3 ... < n_{10}$, how many triplets of these numbers (n_1, n_2, n_3), (n_2, n_3, n_4), ... can be generated such that in each triplet the first number is always less than the second number, and the second number is always less than the third number?

(a) 45 (b) 90
(c) 120 (d) 180

21. Ten straight lines, no two of which are parallel and no three of which pass through any common point, are drawn on a plane. The total number of regions (including finite and infinite regions) into which the plane would be divided by the lines is

(a) 56
(b) 255
(c) 1024
(d) not unique

Directions for Questions 22 and 23: Answer the questions based on the following information.

Each of the 11 letters A, H, I, M, O, T, U, V, W, X and Z appears same when looked at in a mirror. They are called symmetric letters. Other letters in the alphabet are asymmetric letters.

22. How many four-letter computer passwords can be formed using only the symmetric letters (no repetition allowed)?

(a) 7,920 (b) 330
(c) 14,640 (d) 4,19,430

23. How many three-letter computer passwords can be formed (no repetition allowed)with at least one symmetric letter?

(a) 990 (b) 2,730
(c) 12,870 (d) 15,600

24. In how many ways is it possible to choose a white square and a black square on a chessboard so that the squares must not lie in the same row or column?

(a) 56 (b) 896
(c) 60 (d) 768

25. How many numbers greater than 0 and less than a million can be formed with the digits 0, 7 and 8?

(a) 486 (b) 1,084
(c) 728 (d) None of these

2003(R)

26. There are 12 towns grouped into four zones with three towns per zone. It is intended to connect the towns with a telephone lines such that every two towns are connected with three direct lines if they belong to the same zone, and with only one direct line otherwise. How many direct telephone lines are required?

(a) 72 (b) 90
(c) 96 (d) 144

27. An intelligence agency forms a code of two distinct digits selected from 0, 1, 2, …, 9 such that the first digit of the code is non-zero. The code, handwritten on a slip, can however potentially create confusion when read upside down — for example, the code 91 may appear as 16. How many codes are there for which no such confusion can arise?

(a) 80 (b) 78
(c) 71 (d) 69

Directions for Questions 28 and 29: Answer the questions on the basis of the information given below.

A string of three English letters is formed as per the following rules:

I. The first letter is any vowel.
II. The second letter is m, n or p.
III. If the second letter is m, then the third letter is any vowel which is different from the first letter.
IV. If the second letter is n, then the third letter is e or u.
V. If the second letter is p, then the third letter is the same as the first letter.

28. How many strings of letters can possibly be formed using the above rules?

(a) 40 (b) 45
(c) 30 (d) 35

29. How many strings of letters can possibly be formed using the above rules such that the third letter of the string is e?

(a) 8 (b) 9
(c) 10 (d) 11

2003(L)

30. How many three digit positive integers, with digits x, y and z in the hundred's, ten's and unit's place respectively, exist such that $x < y$, $z < y$ and $x \neq 0$?

(a) 245 (b) 285
(c) 240 (d) 320

31. There are 6 boxes numbered 1,2,… 6. Each box is to be filled up either with a red or a green ball in such a way that at least 1 box contains a green ball and the boxes containing green balls are consecutively numbered. The total number of ways in which this can be done is

(a) 5 (b) 21
(c) 33 (d) 60

32. A graph may be defined as a set of points connected by lines called edges. Every edge connects a pair of points. Thus, a triangle is a graph with 3 edges and 3 points. The degree of a point is the number of edges connected to it. For example, a triangle is a graph with three points of degree 2 each. Consider a graph with 12 points. It is possible to reach any point from any point through a sequence of edges. The number of edges, e, in the graph must satisfy the condition

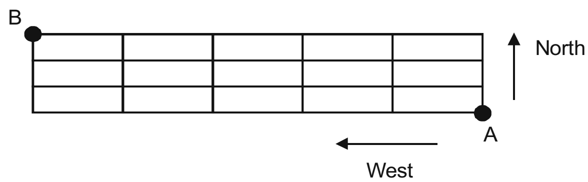
(a) $11 \le e \le 66$ (b) $10 \le e \le 66$
(c) $11 \le e \le 65$ (d) $0 \le e \le 11$

2004

33. N persons stand on the circumference of a circle at distinct points. Each possible pair of persons, not standing next to each other, sings a two-minute song one pair after the other. If the total time taken for singing is 28 minutes, what is N?

(a) 5 (b) 7
(c) 9 (d) None of the above

34. In the adjoining figure, the lines represent one-way roads allowing travel only northwards or only westwards. Along how many distinct routes can a car reach point B from point A?



(a) 15 (b) 56
(c) 120 (d) 336

35. A new flag is to be designed with six vertical stripes using some or all of the colours yellow, green, blue and red. Then, the number of ways this can be done so that no two adjacent stripes have the same colour is

(a) 12 × 81 (b) 16 × 192
(c) 20 × 125 (d) 24 × 216

2005

36. In a chess competition involving some boys and girls of a school, every student had to play exactly one game with every other student. It was found that in 45 games both the players were girls, and in 190 games both were boys. The number of games in which one player was a boy and the other was a girl is

(a) 200 (b) 216
(c) 235 (d) 256

37. Let S be the set of five-digit numbers formed by digits 1, 2, 3, 4 and 5, using each digit exactly once such that exactly two odd position are occupied by odd digits. What is the sum of the digits in the rightmost position of the numbers in S?

(a) 228 (b) 216
(c) 294 (d) 192

38. Three Englishmen and three Frenchmen work for the same company. Each of them knows a secret not known to others. They need to exchange these secrets over person-to-person phone calls so that eventually each person knows all six secrets. None of the Frenchmen knows English, and only one Englishman knows French. What is the minimum number of phone calls needed for the above purpose?

(a) 5 (b) 10
(c) 9 (d) 15

2006

39. There are 6 tasks and 6 persons. Task 1 cannot be assigned either to person 1 or to person 2; task 2 must be assigned to either person 3 or person 4. Every person is to be assigned one task. In how many ways can the assignment be done?

(a) 144 (b) 180
(c) 192 (d) 360
(e) 716

2007

Directions for Questions 40 and 41: Answer the following questions based on the information given below.

Let S be the set of all pairs (i, j)where, $1 \le i < j \le n$ and $n \ge 4$. Any two distinct members of S are called "friends" if they have one constituent of the pairs in common and "enemies" otherwise. For example, if n = 4, then S = {(1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4)}. Here, (1, 2) and (1, 3) are friends, (1, 2) and (2, 3) are also friends, but (1, 4) and (2, 3) are enemies.

40. For general 'n', how many enemies will each member of S have?

(a) $n - 3$ (b) $\frac{1}{2}\left(n^2 - 3n - 2\right)$
(c) $2n - 7$ (d) $\frac{1}{2}\left(n^2 - 5n + 6\right)$
(e) $\frac{1}{2}\left(n^2 - 7n + 14\right)$

41. For general 'n', consider any two members of S that are friends. How many other members of S will be common friends of both these members?

(a) $\frac{1}{2}\left(n^2 - 5n + 8\right)$ (b) $2n - 6$
(c) $\frac{1}{2}n\left(n - 3\right)$ (d) $n - 2$
(e) $\frac{1}{2}\left(n^2 - 7n + 16\right)$

42. In a tournament, there are n teams $T_1, T_2, ..., T_n$, with n > 5. Each team consists of 'k' players, k > 3. The following pairs of teams have one player in common:

T_1 & T_2, T_2 & T_3, ..., T_{n-1} & T_n, and T_n & T_1

No other pair of teams has any player in common. How many players are participating in the tournament, considering all the 'n' teams together?

(a) n(k – 1) (b) k(n – 1)
(c) n(k – 2) (d) k(n – 2)
(e) (n – 1)(k – 1)

2008

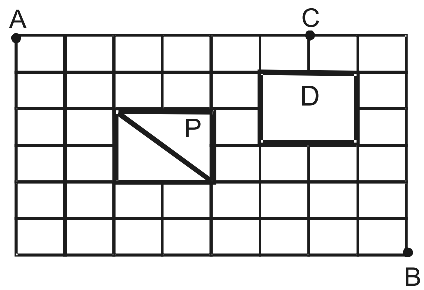
43. How many integers, greater than 999 but not greater than 4000, can be formed with the digits 0, 1, 2, 3 and 4, if repetition of digits is allowed?

(a) 499
(b) 500
(c) 375
(d) 376
(e) 501

2008

Directions for Questions 44 and 45:

The figure below shows the plan of a town. The streets are at right angles to each other. A rectangular park (P) is situated inside the town with a diagonal road running through it. There is also a prohibited region (D) in the town.



44. Neelam rides her bicycle from her house at A to her office at B, taking the shortest path. Then the number of possible shortest paths that she can choose is

(a) 60 (b) 75
(c) 45 (d) 90
(e) 72

45. Neelam rides her bicycle from her house at A to her club at C, via B taking the shortest path. Then the number of possible shortest paths that she can choose is

(a) 1170 (b) 630
(c) 792 (d) 1200
(e) 936

46. What is the number of distinct terms in the expansion of $(a + b + c)^{20}$?

(a) 231 (b) 253
(c) 242 (d) 210
(e) 228

MEMORY BASED QUESTIONS

2010

47. In how many ways can 18 identical balls be distributed among 3 identical boxes?

(a) 25 (b) 210
(c) 105 (d) 37

2011

48. There are four lines in a plane and exactly two of them are parallel. If P is the number of points of intersection of these lines, then how many different values of P are possible?

(a) 2 (b) 3
(c) 5 (d) 6

2012

49. All the two-digit natural numbers whose unit digit is greater than their ten's digit are selected. If all these numbers are written one after the other in a series, how many digits are there in the resulting number?

(a) 90 (b) 72
(c) 36 (d) 54

2013

50. How many triangles can be drawn by joining any three vertices of a pentagon?

(a) 8 (b) 9
(c) 11 (d) 10

51. In how many ways can 18 identical candies be distributed among 8 children such that the number of candies received by each child is a prime number?

(a) 4 (b) 8
(c) 28 (d) 12

2014

52. There are exactly sixty chairs around a circular table. There are some people sitting on these chairs in such a way that the next person to be seated around the table will have to sit next to someone. What is the least possible number of people sitting around the table currently?

(a) 10 (b) 20
(c) 30 (d) 40

53. Find the number of ways in which a batsman can score 100 runs by scoring runs in 2's, 4's and 6's, such that he hits at least one double, one boundary and one six.

(a) 184 (b) 185
(c) 192 (d) 208

54. Amar, Akbar and Antony are three students in a class of 9 students. A class photo is taken. The number of ways in which it can be taken such that no two of Amar, Akbar and Antony are sitting together is:

(a) 151200 (b) 120960
(c) 181440 (d) 241920

2015

55. x is the smallest positive integer such that when it is divided by 7, 8 and 9 leaves remainder as 4, 5 and 6 respectively. Find the remainder when $x^3 + 2x^2 - x - 3$ is divided by 132.

(a) 49 (b) 76
(c) 94 (d) 15

56. If we arrange the letters of the word 'KAKA' in all possible ways, what is the probability that vowels will not be together in an arrangement?

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$

(c) $\frac{1}{2}$ (d) $\frac{5}{6}$

2016

57. A man, having $2 in his pocket, goes to play his favourite game at a casino. If he wins he gets $2 whereas if he loses he gets nothing. He plays the game multiple times and pays $1 for each game as the entry fee. He does not lose more than once and leaves the casino as soon as he has $4 in his pocket. How many different Win-Loss sequences are possible for him?

(a) 4 (b) 3

(c) 8 (d) 5

58. India fielded 'n' (> 3) bowlers in a test match, and they operated in pairs. If a particular bowler did not bowl in pair with at least two other bowlers in the team, then at most how many bowlers could have bowled in pair with every other bowler in the team?

(a) n – 3 (b) n – 1

(c) n – 2 (d) None of these

2017

59. In how many ways can 8 identical pens be distributed among Amal, Bimal, and Kamal so that Amal gets at least 1 pen, Bimal gets at least 2 pens, and Kamal gets at least 3 pens?

60. How many four digit numbers, which are divisible by 6, can be formed using the digits 0, 2, 3, 4, 6, such that no digit is used more than once and 0 does not occur in the left-most position?

2018 Slot 1

61. How many numbers with two or more digits can be formed with the digits 1,2,3,4,5,6,7,8,9, so that in every such number, each digit is used at most once and the digits appear in the ascending order?

2018 Slot 2

62. In a tournament, there are 43 junior level and 51 senior level participants. Each pair of juniors play one match. Each pair of seniors play one match. There is no junior versus senior match. The number of girl versus girl matches in junior level is 153, while the number of boy versus boy matches in senior level is 276. The number of matches a boy plays against a girl is

63. How many two-digit numbers, with a non-zero digit in the units place, are there which are more than thrice the number formed by interchanging the positions of its digits?

(a) 7 (b) 6

(c) 5 (d) 8

PROBABILITY

1991

Direction for Question 1: Choose the best answer choice from those provided

1. A player rolls a die and receives the same number of rupees as the number of dots on the face that turns up. What should the player pay for each roll if he wants to make a profit of one rupee per throw of the die in the long run?

(a) Rs. 2.50 (b) Rs. 2

(c) Rs.3.50 (d) Rs. 4

1993

2. A box contains 6 red balls, 7 green balls and 5 blue balls. Each ball is of a different size. The probability that the red ball selected is the smallest red ball, is

(a) 1/18 (b) 1/3

(c) 1/6 (d) 2/3

1994

Directions for Question 3: Data is provided followed by two statements – I and II – both resulting in a value, say I and II. As your answer,

Mark (a) if I > II.

Mark (b) if I < II.

Mark (c) if I = II.

Mark (d) if nothing can be said.

3. I. The probability of encountering 54 Sundays in a leap year.

II. The probability of encountering 53 Sundays in a non-leap year.

1995

4. If a 4 digit number is formed with digits 1, 2, 3 and 5. What is the probability that the number is divisible by 25, if repetition of digits is not allowed?

(a) $\frac{1}{12}$

(b) $\frac{1}{4}$

(c) $\frac{1}{6}$

(d) None of these

MEMORY BASED QUESTIONS

2009

5. How many arrangements of the letters of the word CATASTROPHE are there in which both the 'A's appear before both the 'T's?

(a) $\frac{11!}{4!}$ (b) $\frac{11!}{4!(2!)^2}$

(c) $\frac{11!}{6}$ (d) $\frac{11!(2!)^2}{4!}$

2010

6. There are two boxes – I and II. Each contains balls of two colours – White and Black. A ball is selected at random. It is known that $P(W \cap I) = 0.3$ and $P(B'|II) = 0.8$, where W, B, I and II represent the events White ball selected, Black ball selected, ball selected from Box I and ball selected from Box II respectively. Find P(I|B).

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$

(c) $\frac{1}{2}$ (d) $\frac{3}{4}$

2011

7. Two six-faced unbiased dice are thrown simultaneously. What is the probability that the sum of the numbers that appear on the two dice is a prime number?

(a) $\frac{7}{12}$ (b) $\frac{5}{12}$

(c) $\frac{1}{3}$ (d) $\frac{1}{2}$

2012

8. What is the probability that the product of two integers chosen at random has the same unit digit as the two integers?

(a) $\frac{3}{10}$ (b) $\frac{1}{25}$

(c) $\frac{4}{15}$ (d) $\frac{7}{15}$

2013

9. The coefficient of $a^{12}b^8$ in the expansion of $(a^2 + b)^{13}$ is

(a) $\frac{13!}{12!\,6!}$ (b) $-\frac{13!}{6!\,8!}$

(c) $\frac{13!}{6!\,8!}$ (d) None of these

2014

10. If p is the probability of head turning up in the toss of a coin (not necessarily fair) and q is the probability of a tail turning up. Find the minimum possible value of

$X = pq + \left(\frac{1}{p}\right)\left(\frac{1}{q}\right).$

(a) 4.25 (b) $2\sqrt{5}$

(c) 2 (d) None of these

2015

11. Three persons - A, B and C - are playing the game of death. 3 bullets are placed randomly in a revolver having 6 chambers. Each one has to shoot himself by pulling the trigger once after which the revolver passes to the next person. This process continues till two of them are dead and the survivor of the game becomes the winner. What is the probability that B is the winner if A starts the game and A, B and C take turns in that order.

(a) 0.33 (b) 0.3

(c) 0.25 (d) None of these

2016

12. A cube is painted with red colour and then cut into 64 small identical cubes. If two cubes are picked randomly from the heap of 64 cubes, what is the probability that both of them have exactly two faces painted red?

(a) $\frac{23}{168}$ (b) $\frac{47}{84}$

(c) $\frac{1}{4}$ (d) $\frac{3}{8}$

SET THEORY

1991

Direction for Question 1: The question is followed by two statements. As the answer,

Mark (a) If the question can be answered with the help of statement I alone,

Mark (b) If the question can be answered with the help of statement II alone,

Mark (c) If both the statement I and statement II are needed to answer the question, and

Mark (d) If the question cannot be answered even with the help of both the statements.

1. How many people (from the group surveyed) read both Indian Express and Times of India?

I. Out of total of 200 readers, 100 read Indian Express, 120 read Times of India and 50 read Hindu.

II. Out of a total of 200 readers, 100 read Indian Express, 120 read Times of India and 50 read neither.

2. Fifty college teachers are surveyed as to their possession of colour TV, VCR and tape recorder. Of them, 22 own colour TV, 15 own VCR and 14 own tape recorders. Nine of these college teachers own exactly two items out of colour TV, VCR and tape recorder; and, one college teacher owns all three. How many of the 50 teachers own none of the three, colour TV, VCR or tape recorder?

(a) 4 (b) 9
(c) 10 (d) 11

Directions for Questions 3 and 4: Answer the questions on the basis of the information given below.

There were a hundred schools in a town. Of these, the number of schools having a play – ground was 30, and these schools had neither a library nor a laboratory. The number of schools having a laboratory alone was twice the number of those having a library only. The number of schools having a laboratory as well as a library was one fourth the number of those having a laboratory alone. The number of schools having either a laboratory or a library or both was 35.

3. How many schools had none of the three viz., laboratory, library or play – ground?

(a) 20 (b) 5
(c) 30 (d) 35

4. What was the ratio of schools having laboratory to those having library?

(a) 1 : 2 (b) 5 : 3
(c) 2 : 1 (d) 2 : 3

5. There are 3 clubs A, B & C in a town with 40, 50 & 60 members respectively. While 10 people are members of all 3 clubs, 70 are members in only one club. How many belong to exactly two clubs?

(a) 20 (b) 25
(c) 50 (d) 70

1993

Directions for Questions 6 and 7 : Use the following information.

Eighty five children went to an amusement park where they could ride on the merry – go round, roller coaster, and Ferris wheel. It was known that 20 of them took all three rides, and 55 of them took at least two of the three rides. Each ride cost Re.1, and the total receipt of the amusement park was Rs.145.

6. How many children did not try any of the rides?

(a) 5 (b) 10
(c) 15 (d) 20

7. How many children took exactly one ride?

(a) 5 (b) 10
(c) 15 (d) 20

8. Out of 100 families in the neighbourhood, 45 own radios, 75 have TVs, 25 have VCRs. Only 10 families have all three and each VCR owner also has a TV. If 25 families have radio only, how many have only TV?

(a) 30 (b) 35
(c) 40 (d) 45

1994

Directions for Questions 9 to 11: Answer the questions based on the following information.

Ghoshbabu is staying at Ghosh Housing Society, Aghosh Colony, Dighospur , Calcutta. In Ghosh Housing Society 6 persons read daily Ganashakti and 4 read Anand Bazar Patrika; in his colony there is no person who reads both. Total number of persons who read these two newspapers in Aghosh Colony and Dighospur is 52 and 200 respectively. Number of persons who read Ganashakti in Aghosh Colony and Dighospur is 33 and 121 respectively; while the persons who read Anand Bazar Patrika in Aghosh Colony and Dighospur are 32 and 117 respectively.

9. Number of persons in Dighospur who read only Ganashakti is

(a) 121 (b) 83
(c) 79 (d) 127

10. Number of persons in Aghosh Colony who read both of these newspapers is

(a) 13 (b) 20
(c) 19 (d) 14

11. Number of persons in Aghosh Colony who read only one paper

(a) 29 (b) 19
(c) 39 (d) 20

1996

12. In a locality, two-thirds of the people have cable TV, one-fifth have VCR, and one-tenth have both. What is the fraction of people having either cable -TV or VCR?

(a) $\frac{19}{30}$ (b) $\frac{2}{3}$
(c) $\frac{17}{30}$ (d) $\frac{23}{30}$

1997

Directions for Questions 13 to 15: Answer the questions based on the following information.

A survey of 200 people in a community who watched at least one of the three channels — BBC, CNN and DD — showed that 80% of the people watched DD, 22% watched BBC and 15% watched CNN.

13. What is the maximum percentage of people who can watch all the three channels?

(a) 12.5% (b) 8.5%
(c) 15% (d) Data insufficient

14. If 5% of people watched DD and CNN, 10% watched DD and BBC, then what percentage of people watched BBC and CNN only?

(a) 2% (b) 5%
(c) 8.5% (d) Cannot be determined

15. Referring to the previous question, what percentage of people watched all the three channels?

(a) 3.5% (b) 0%
(c) 8.5% (d) Cannot be determined

1999

16. In a survey of political preferences, 78% of those asked were in favour of at least one of the proposals: I, II and III. 50% of those asked favoured proposal I, 30% favoured proposal II and 20% favoured proposal III. If 5% of those asked favoured all three of the proposals, what percentage of those asked favoured more than one of the three proposals?

(a) 10 (b) 12
(c) 17 (d) 22

Directions for Question 17 and 18: The questions is followed by two statements, I and II. Answer the question using the following instructions.

Mark the answer as

(a) if the question can be answered by one of the statements alone, but cannot be answered by using the other statement alone.
(b) if the question can be answered by using either statement alone.
(c) if the question can be answered by using both the statements together, but cannot be answered by using either statement alone.
(d) if the question cannot be answered even by using both statements together.

2000

17. How many people are watching TV programme P?

I. Number of people watching TV programme Q is 1,000 and number of people watching both the programmes P and Q, is 100.
II. Number of people watching either P or Q or both is 1,500.

2002

18. People in a club either speak French or Russian or both. Find the number of people in a club who speak only French.

A. There are 300 people in the club and the number of people who speak both French and Russian is 196.
B. The number of people who speak only Russian is 58.

2003(R)

19. Consider the sets $T_n = \{n, n+1, n+2, n+3, n+4\}$, where n = 1, 2, 3,..., 96. How many of these sets contain 6 or any integral multiple thereof (i.e. any one of the numbers 6, 12, 18, ...)?

(a) 80 (b) 81
(c) 82 (d) 83

20. A survey on a sample of 25 new cars being sold at a local auto dealer was conducted to see which of the three popular options — air conditioning, radio and power windows were already installed.

Following were the observation of the survey:

I. 15 had air conditioning
II. 2 had air conditioning and power windows but no radios
III. 12 had radio
IV. 6 had air conditioning and radio but no power windows
V. 11 had power windows
VI. 4 had radio and power windows
VII. 3 had all three options

What is the number of cars that had none of the options?

(a) 4 (b) 3
(c) 1 (d) 2

21. Seventy percent of the employees in a multinational corporation have VCD players, 75% have microwave ovens, 80% have ACS and 85% have washing machines. At least what percentage of employees has all four gadgets?

(a) 15 (b) 5
(c) 10 (d) Cannot be determined

2003(L)

Directions for Questions 22 and 23: Answer the questions on the basis of the information given below.

New Age Consultants have three consultants Gyani, Medha and Buddhi. The sum of the number of projects handled by Gyani and Buddhi individually is equal to the number of projects in which Medha is involved. All three consultants are involved together in 6 projects. Gyani works with Medha in 14 projects. Buddhi has 2 projects with Medha but without Gyani, and 3 projects with Gyani but without Medha. The total number of projects for New Age Consultants is one less than twice the number of projects in which more than one consultant is involved.

22. What is the number of projects in which Gyani alone is involved?
(a) Uniquely equal to zero.
(b) Uniquely equal to 1.
(c) Uniquely equal to 4.
(d) Cannot be determined uniquely.

23. What is the number of projects in which Medha alone is involved?
(a) Uniquely equal to zero.
(b) Uniquely equal to 1.
(c) Uniquely equal to 4.
(d) Cannot be determined uniquely.

2005

Directions for Questions 24 to 27: Answer the questions on the basis of the information given below:

Help Distress (HD)is an NGO involved in providing assistance to people suffering from natural disasters. Currently, it has 37 volunteers. They are involved in three projects: Tsunami Relief (TR)in Tamil Nadu, Flood Relief (FR in Maharashtra, and Earthquake Relief (ER) in Gujarat. Each volunteer working with Help Distress has to be involved in at least one relief work project.

- A maximum number of volunteers are involved in the FR project. Among them, the number of volunteers involved in FR project alone is equal to the volunteers having additional involvement in the ER project.
- The number of volunteers involved in the ER project alone is double the number of volunteers involved in all the three projects.
- 17 volunteers are involved in the TR project.
- The number of volunteers involved in the TR project alone is one less than the number of volunteers involved in ER project alone.
- Ten volunteers involved in the TR project are also involved in at least one more project.

24. Based on the information given above, the minimum number of volunteers involved in both FR and TR projects, but not in the ER project is
(a) 1 (b) 3
(c) 4 (d) 5

25. Which of the following additional information would enable to find the exact number of volunteers involved in various projects?
(a) Twenty volunteers are involved in FR.
(b) Four volunteers are involved in all the three projects.
(c) Twenty three volunteers are involved in exactly one project.
(d) No need for any additional information.

26. After some time, the volunteers who were involved in all the three projects were asked to withdraw from one project. As a result, one of the volunteers opted out of the TR project, and one opted out of the ER project, while the remaining ones involved in all the three projects opted out of the FR project. Which of the following statements, then, necessarily follows?
(a) The lowest number of volunteers is now in TR project.
(b) More volunteers are now in FR project as compared to ER project.
(c) More volunteers are now in TR project as compared to ER project.
(d) None of the above.

27. After the withdrawal of volunteers, as indicated in Question 25, some new volunteers joined the NGO. Each one of them was allotted only one project in a manner such that, the number of volunteers working in one project alone for each of the three projects became identical. At that point, it was also found that the number of volunteers involved in FR and ER projects was the same as the number of volunteers involved in TR and ER projects. Which of the projects now has the highest number of volunteers?
(a) ER (b) FR
(c) TR (d) Cannot be determined

2006

28. A survey was conducted of 100 people to find out whether they had read recent issues of Golmal, a monthly magazine. The summarized information regarding readership in 3 months is given below:

Only September: 18;
September but not August: 23;
September and July: 8;September: 28;
July: 48;
July and August: 10;
None of the three months: 24.

What is the number of surveyed people who have read exactly two consecutive issues (out of the three)?
(a) 7 (b) 9
(c) 12 (d) 14
(e) 17

MEMORY BASED QUESTIONS

2011

29. Two sets A and B are given below.

$A = \{2^0, 2^1, 2^2, 2^3, 2^4\}$
$B = \{3^0, 3^1, 3^2, 3^3, 3^4\}$

How many different proper fractions can be made by picking the numerator from one of the sets and the denominator from the other set?
(a) 24 (b) 20
(c) 12 (d) None of these

2013

30. Let S = {1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12}. The number of subsets of S comprising composite number(s) only and that of those comprising prime number(s) only are N_1 and N_2 respectively. What is the absolute difference between N_1 and N_2?

(a) 0 (b) 32
(c) 48 (d) 24

2015

31. All reputed B-schools place their students. One-sixth of those B-schools that place their students are reputed and one-fourth of all B-schools that are recognised, place their students. There are exactly 6 reputed B-schools that are recognised too and there are 39 B -schools that are recognised but do not place their students. If there is a total of 78 B-schools that place their students, then how many of these B-schools are neither recognised nor reputed but place their students?

32. A = {1, 4, 7, 10, ..., 20th term}
B = {9, 16, 23, 30, ..., 20th term}
If $S = A \cup B$, how many elements are there in S?

(a) 37 (b) 38
(c) 39 (d) 35

2018 Slot 1

33. Each of 74 students in a class studies at least one of the three subjects H, E and P. Ten students study all three subjects, while twenty study H and E, but not P. Every student who studies P also studies H or E or both. If the number of students studying H equals that studying E, then the number of students studying H is

34. If among 200 students, 105 like pizza and 134 like burger, then the number of students who like only burger can possibly be

(a) 26 (b) 96
(c) 23 (d) 93

2018 Slot 2

35. For two sets A and B, let $A \Delta B$ denote the set of elements which belong to A or B but not both. If P = {1, 2, 3, 4}, Q = {2, 3, 5, 6,}, R = {1, 3, 7, 8, 9}, S = {2, 4, 9, 10}, then the number of elements in $(P \Delta Q) \Delta (R \Delta S)$ is

(a) 7 (b) 8
(c) 9 (d) 6

ANSWERS

Permutation and Combination

1. (b)	**2.** (a)	**3.** (b)	**4.** (a)	**5.** (c)	**6.** (*)	**7.** (d)	**8.** (b)	**9.** (a)	**10.** (b)
11. (b)	**12.** (c)	**13.** (a)	**14.** (a)	**15.** (c)	**16.** (a)	**17.** (d)	**18.** (b)	**19.** (c)	**20.** (c)
21. (a)	**22.** (a)	**23.** (c)	**24.** (d)	**25.** (c)	**26.** (b)	**27.** (c)	**28.** (d)	**29.** (c)	**30.** (c)
31. (b)	**32.** (a)	**33.** (b)	**34.** (b)	**35.** (a)	**36.** (a)	**37.** (b)	**38.** (c)	**39.** (a)	**40.** (d)
41. (d)	**42.** (a)	**43.** (d)	**44.** (d)	**45.** (a)	**46.** (a)	**47.** (d)	**48.** (a)	**49.** (b)	**50.** (d)
51. (c)	**52.** (b)	**53.** (a)	**54.** (a)	**55.** (d)	**56.** (c)	**57.** (b)	**58.** (a)	**59.** 6	**60.** 50
61. 502	**62.** (1098)	**63.** (b)							

Probability

1. (a)	**2.** (c)	**3.** (b)	**4.** (a)	**5.** (a)	**6.** (a)	**7.** (b)	**8.** (b)	**9.** (d)	**10.** (a)
11. (b)	**12.** (a)								

Set Theory

1. (b)	**2.** (c)	**3.** (d)	**4.** (b)	**5.** (b)	**6.** (c)	**7.** (c)	**8.** (c)	**9.** (b)	**10.** (a)
11. (c)	**12.** (b)	**13.** (c)	**14.** (a)	**15.** (d)	**16.** (c)	**17.** (c)	**18.** (c)	**19.** (a)	**20.** (d)
21. (c)	**22.** (d)	**23.** (b)	**24.** (c)	**25.** (a)	**26.** (b)	**27.** (d)	**28.** (b)	**29.** (a)	**30.** (c)
31. 58	**32.** (a)	**33.** (52)	**34.** (d)	**35.** (a)					

EXPLANATIONS

Permutation and combination

1. b If there is only one box containing black ball, the boxes can be filled in 6 ways.

If there are two boxes containing black ball, the boxes can be filled in 5 ways. (The two black balls can be in either of the boxes (1,2), (2,3), (3,4), (4,5) or (5,6)).

If there are 3 boxes containing black ball the boxes can be filled in 4 ways viz.(123), (234), (345), (456).

Similarly if there are 4 boxes, it can be done in 3 ways viz.(1234), (2345), (3456), if there are 5 boxes it can be done in 2 ways viz.(12345), (23456) and all 6 boxes can have a black ball only in 1 way.

Hence, total number of ways = 6 + 5 + 4 + 3 + 2 + 1 = 21.

2. a Three digit number such that 7 follows 5 could be of the form 57_ or _57.

Since, the number is an even number

Therefore possible numbers are 570,572,574,576 or 578.

Hence, 5 such numbers are possible.

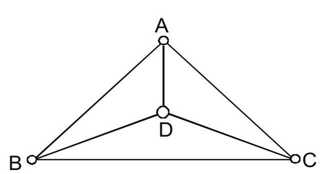
3. b It can be seen that every city is connected to all the other 3 cities.

If we start from city A, there are 3 ways in which we can proceed, viz. AB, AD or AC.

Once we are at any of these cities, each one of them is connected to the other 3 cities. But since we cannot go back to city A, there are only 2 ways in which we can proceed from here.

If we are at B, we can take either paths BD or BC.

From this point, we have a choice of going directly to A (thus skipping 4th city) or go to 4th city and come back to A. Eg. If we are at D, we can either take DA or DCA. So there are 2 more ways to go from here. Hence, required number of ways = 3 x 2 x 2 = 12.



4. a If we assume that any digit is in a fixed position, then the remaining four digits can be arranged in 4! = 24 ways. So each of the 5 digits will appear in each of the five places 24 times. So the sum of the digits in each position is 24(1 + 3 + 5 + 7 + 9) = 600.

Hence, the sum of all such numbers will be 600(1 + 10 + 100 + 1000 + 10000) = 6666600.

5. c We can form a triangle with any 3 points which are not collinear. 3 points out of 5 can be chosen in $^5C_3$ = 10 ways. But of these, the three points lying on the two diagonals will be collinear. So 10 – 2 = 8 triangles can be formed.

6. * There cannot be four or more blue balls.

Case 1: If there are three blue balls, then they can be only in box 1, 3 and 5.

Case 2: If there are two blue balls, then total number of cases = $^5C_2$ = 10

But in 4 cases the blue ball will be in adjacent boxes. These cases are when blue balls in boxes 1 and 2 or 2 and 3 or 3 and 4 or 4 and 5.

Therefore, total number of cases when there are two blue balls = 10 – 4 = 6

Case 3: If there is one blue ball, then total number of cases = $^5C_1$ = 5

Case 4: If there is no blue ball, then total number of cases = $^5C_5$ = 1

Hence, total number of cases = 1 + 6 + 5 + 1 = 13.

*** The correct answer is not available in the given options.**

7. d Let us choose a town, say A.

If I were to consider this as the base town and construct two roads such that I connect any pair of towns, I get the following combinations:

1. AB – BC, 2. AB – BD, 3. AC – CB, 4. AC – CD, 5. AD – DB and 6. AD – DC.

From any of these combinations, if I were to construct a road such that it again comes back to A, then it would form a triangle.

To avoid a triangle, the third road that I construct should not be connected to A but to the third town.

Hence, the combination would be:

1. AB – BC – CD, 2. AB – BD – DC, 3. AC – CB – BD, 4. AC – CD – DB, 5. AD – DB – BC and 6. AD – DC – CB.

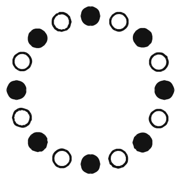
Thus, from each town, we can construct 6 such combinations.

Hence, total number of combinations that we can have from four towns = (6 × 4) = 24.

8. b Out of the 5 girls, 3 girls can be invited in $^5C_3$ ways. Nothing is mentioned about the number of boys that he has to invite. He can invite one, two, three, four or even no boys. Each boy can be invited or not. He can invite them in 2^4 ways. Thus, the total number of ways is $^5C_3 \times (2)^4 = 10 \times 16 = 160$.

9. a The question cannot be answered until and unless number of concurrent lines are known.

10. b



If we consider the Chairman and the vice chairman as one set, we can see that this set can fit 8 slots in between the 8 directors. Hence, this can be done in 8! ways. Between themselves, the chairman and the vice chairman can be arranged in 2 ways. Hence, the required answer = 2 x 8!.

11. b — — — $\underset{\text{Ten's place}}{\underline{\quad t \quad}}$ $\underset{\text{Unit's place}}{\underline{\quad u \quad}}$

$u > t$

Case (i): t = 1

u can be 2, 3, 4, 5 $\Rightarrow$ 4 possibilities

So total possible number that can be formed = 4 × 3 × 2 × 1

Case (ii): t = 2

u can be 3, 4, 5 $\Rightarrow$ 3 possibilities

So total possible number that can be formed = 3 × 3 × 2 × 1

Case (iii): t = 3

u can be 4, 5 $\Rightarrow$ 2 possibilities

So total possible number that can be formed = 2 × 3 × 2 × 1

Case (iv): t = 4

u can be 5 $\Rightarrow$ 1 possibility

So total possible number that can be formed = 1 × 3 × 2 × 1

Hence, total possible number = (4 + 3 + 2 + 1)

$\underline{|3}$ = 60

12. c The answer is $^{10}C_2 \times 11 + {}^{11}C_2 \times 10 = 45 \times 11 + 55 \times 10 = 1045$.

13. a At least 1 and at most n are to be selected

$\Rightarrow {}^{2n+1}C_1 + {}^{2n+1}C_2 + \ldots + {}^{2n+1}C_n = 63$

$\Rightarrow \frac{1}{2}\left(2^{2n+1} - 2\right) = 63$

$\Rightarrow n = 3$

14. a The possibilities are W@W@W@ or @W@W@W, where 2 blue and 1 red flag occupy the space marked as @. Hence, the total permutation is $2 \times \frac{3!}{2!}$ = 6.

15. c There are two possible cases. The number 9 comes at the end, or it comes at position 4, 5, or 6.

For the first case, the number would look like:

635 - - - 9 or 674 - - - 9

In both these cases, the blanks can be occupied by any of the available 9 digits (0, 1, 2, ..., 8).

Thus, total possible numbers would be

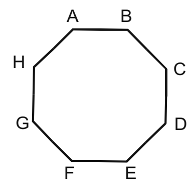
2 × (9 × 9 × 9) = 1458.

For the second case, the number 9 can occupy any of the given position 4, 5, or 6, and there shall be an odd number at position 7.

Thus, the total number of ways shall be 2[3(9 × 9 × 4)] = 1944.

Hence, answer is 3402.

16. a



In order to reach E from A, it can walk clockwise as well as anticlockwise. In all cases, it will have to take odd number of jumps from one vertex to another. But the sum will be even. In simple case, if n = 4, then a_n = 2.

For a_{2n-1} = 7 (odd), we cannot reach the point E.

17. d Number of one-rupee coins = 158.

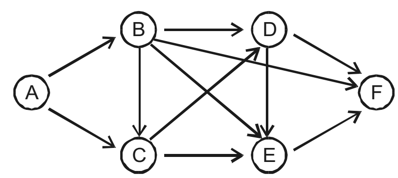
Possible arrangements of coins are listed as

1, 2, 4, 8, 16, 32, 64 and 31.

$\therefore$ Number of arrangements = 8.

So the least number of bags required = 8.

18. b



The number of distinct routes from A to F are listed below.

(1) ABDF	(2) ACEF	(3) ABF
(4) ABEF	(5) ACDF	(6) ABCDEF
(7) ACDEF	(8) ABDEF	(9) ABCDF
(10) ABCEF		

There are 10 way to reach F from A.

19. c The last two digits can be 12, 16, 24, 32, 36, 52, 56, and 64, i.e. 8 possibilities.

Remaining digits can be chosen in $^4P_3 = 24$ ways.

Hence, total number of such five-digit numbers = 24 × 8 = 192.

20. c Total possible arrangements = 10 × 9 × 8

Now 3 numbers can be arranged among themselves in 3! ways = 6 ways

Given condition is satisfied by only 1 out of 6 ways.

Hence, the required number of arrangements

$= \frac{10 \times 9 \times 8}{6} = 120$

Alternate solution:

$^{10}C_3 = 120$

Any three numbers selected out of 10 numbers will have only one possible arrangement.

21. a Number of regions = $\frac{n(n+1)}{2} + 1$, where n = Number of lines, i.e. for 0 line we have region = 1.

For 1 line we have region = 2.

It can be shown as:

Number of lines	0	1	2	3	4	5	…	10
Number of regions	1	2	4	7	11	16	…	56

Therefore, for n = 10, it is $\frac{10 \times 11}{2} + 1 = 56$

22. a Total number of passwords with atleast 1 symmetric letter = 11 × 10 × 9 × 8 = 7920

23. c Total number of passwords with atleast 1 symmetric letter

= Total number of passwords using all letters – Total number of passwords using no symmetric letters

= (26 × 25 × 24)– (15 × 14 × 13)

= 12870

24. d A black square can be chosen in 32 ways. Once a black square is there, you cannot choose the 8 white squares in its row or column. So the number of white squares available = 24

Number of ways = 32 × 24 = 768

25. c Number of ways for single digit = 2

2 digits = 2 × 3 = 6

3 digits = 2 × 3 × 3 = 18

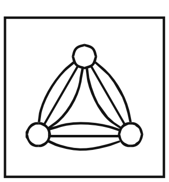
4 digits = 2 × 3 × 3 × 3 = 54

5 digits = 2 × 3 × 3 × 3 × 3 = 162

6 digits = 2 × 3 × 3 × 3 × 3 × 3 = 486

Total number of ways = 728

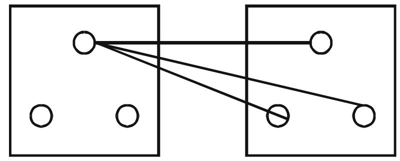
26. b Consider first zone. The number of telephone lines can be shown a follows.



= 9 lines

Therefore, total number of lines required for internal connections in each zone = 9 × 4 = 36 lines.

Now consider the connection between any two zones.



Each town in first zone can be connected to three towns in the second zone.

Therefore, the lines required = 3 × 3 = 9

Therefore, total number of lines required for connecting towns of different zones = $^4C_2$ × 9 = 6 × 9 = 54

Therefore, total number of lines in all = 54 + 36 = 90

27. c Total codes which can be formed = 9 × 9 = 81.

(Distinct digit codes)

The digits which can confuse are 1, 6, 8, 9, from these digit we can form the codes = 4 × 3 = 12

Out of these 12 codes two numbers 69 and 96 will not create confusion.

Therefore, (12 – 2)= 10 codes will create a confusion.

Therefore, total codes without confusion= 81 – 10 = 71.

28. d **Case I:** ___ m ___

First place can be selected in five ways and hence the third in four ways.

$\therefore 5 \times 4 = 20$ ways

Case II: __ n ___

First place can be selected in 5 ways and third in 2 ways.

$\therefore 5 \times 2 = 10$ ways

Case III: ___ p ___

First place can be selected in 5 ways and last letter will be same, i.e. one way.

$\therefore 5 \times 1 = 5$ way

$\therefore$ Total ways = 20 + 10 + 5 = 35 ways.

29. c As third letter is e, it can be selected in one way only.

Case I: 4 m e $\Rightarrow$ 4 ways

Case II: 5 n e $\Rightarrow$ 5 ways

Case III: e p e $\Rightarrow$ 1 ways

$\Rightarrow$ 10 ways

30. c If y = 2 (it cannot be 0 or 1), then x can take 1 value and z can take 2 values.

Thus with y = 2, a total of 1 × 2 = 2 numbers can be formed. With y = 3, 2 × 3 = 6 numbers can be formed. Similarly checking for all values of y from 2 to 9 and adding up we get the answer as 240.

31. b GRRRRR, RGRRRR, RRGRRR, RRRGRR, RRRRGR, RRRRRG

GGRRRR, RGGRRR, RRGGRR, RRRGGR, RRRRGG

GGGRRR, RGGGRR, RRGGGR, RRRGGG

GGGGRR, RGGGGR, RRGGGG

GGGGGR, RGGGGG

GGGGGG

Hence 21 ways.

32. a The least number of edges will be when one point is connected to each of the other 11 points, giving a total of 11 lines. One can move from any point to any other point via the common point.

The maximum edges will be when a line exists between any two points. Two points can be selected from 12 points in $^{12}C_2$ i.e. 66 lines.

33. b Each person will form a pair with all other persons except the two beside him. Hence he will form (n – 3) pairs.

If we consider each person, total pairs = n (n – 3) but here each pair is counted twice.

Hence actual number of pairs $= \dfrac{n(n-3)}{2}$

They will sing for $\dfrac{n(n-3)}{2} \times 2 = n(n-3)$ min

Hence n(n -3)

$= 28 \Rightarrow n^2 - 3n - 28 = 0 \Rightarrow n = 7$ or -4

Discarding the negative value: n = 7

34. b From A to B, there are 8 on-way roads out of which 3 roads are in Northwards and 5 roads are Westwards.

Therefore number of distinct routes is = $\dfrac{8!}{5!3!} = 56$

35. a The first strip can be of any of the four colours, The 2nd can be of any colour except that of the first (i.e. 3). Similarly, each subsequent strip can be of any colour except that of the preceding strip (=3)

Hence number of ways = 4 × 3^5 = 12 × 81

36. a Let there be m boys and n girls

$^nC_2 = 45 = \dfrac{n(n-1)}{2} \Rightarrow n(n-1) = 90 \Rightarrow n = 10$

$^mC_2 = 190 \Rightarrow \dfrac{m(m-1)}{2} = 190$

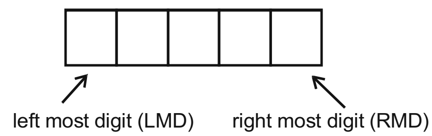
$\Rightarrow m(m-1) = 380 \Rightarrow m = 20$

Number of games between one boy and one girl

= $^{10}C_1 \times {}^{20}C_1 = 10 \times 20 = 200$

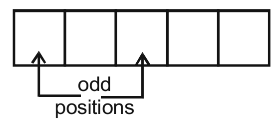
Hence, option (a)is the correct answer.

37. b

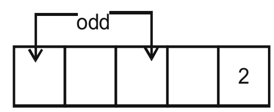


odd positions can be counted in 2 ways.

(i) Counting from the LMD-end:



We have 1, 2, 3, 4 & 5 to be filled in these blocks. Odd nos. (1, 3, 5)to be filled in at odd positions. Other places are to be filled by even numbers (2 or 4) Let's count, how many such numbers are there with 2 at the unit's digit



Odd numbers can be filled in $^3P_2$ = 6 way.

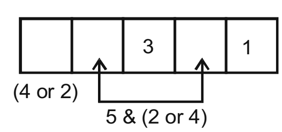
The remaining two places are to be filled by 2 numbers (one odd number left out of 1, 3, 5 & one even i.e. 4) in = 2 ways.

So, there are 6 × 2 = 12 number with 2 at the rightmost place. Similarly; there are 12 such numbers with 4 at the rightmost digits.

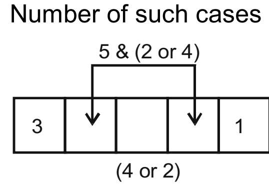
The sum of rightmost digits in all such number = 12(2 + 4) = 72

(ii) Now counting from the RMD-end.

Let's place 1 at the units place and check, how many numbers are possible with (1, 3) at the odd positions:



Number of such cases = 2 × 2 = 4 ways.



Here again number of ways = 2 × 2 = 4 ways.

So, there are 4 + 4 = 8 numbers, in which (1, 3) are at odd positions. Similarly there are 8 numbers in which (1, 5)are at odd positions. So, in all there are 16 numbers where 1 is at unit's place. Similarly there are 16 numbers with 3 at unit's place and 16 more with 5 at unit's place.

Summing up all the odd unit's digits = 16(1 + 3 + 5) = 144

From (i) and (ii)we can now, sum up all (even or odd)numbers at units place = 72 +144 = 216.

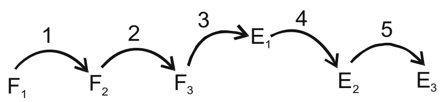
Hence answer is (b).

38. c Frenchmen : F_1, F_2, F_3

Englishmen: E_1, E_2, E_3

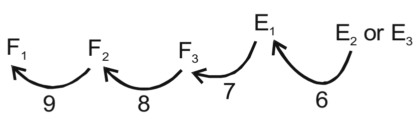
Let E_1 knows French

I round of calls:



Persons	Secrets know after I-round
F_1	F_1, F_2
F_2	F_1, F_2, F_3
F_3	F_1, F_2, F_3, F_4
E_1	F_1, F_2, F_3, E_2
E_2	$F_1, F_2, F_3, E_1, E_2, E_3$) All known
E_3	$F_1, F_2, F_3, E_1, E_2, E_3$) All know

II round calls



In the 6th call, E_1 knows all the secrets. Similarly, after 9th call, everybody know all the secrets.

39. a Task 2 can only be given to two persons i.e. (3 and 4)

$\therefore$ Number of ways = 2 ways

First task can be done in 3 ways by 3 persons.

Third task can be done by 4 persons.

$\therefore$ 4 ways similarly for fourth, five and six tasks, number of ways is 3, 2 and 1 respectively.

$\therefore$ Total number of ways = 144 ways

40. d The number of members in the set S = ${}^nC_2$, where n is greater than = 4

Each member of S has two distinct numbers.

Let us say (1, 2)is one of the members of S.

To find the number of enemies, each member of S will have to be equal to ${}^{n-2}C_2 = \frac{n^2 - 5n + 6}{2}$.

Alternative Method:

Number of enemies for this member is 6, i.e. (3, 4), (3, 5), (3, 6), (4, 5)(4, 6)and (5, 6).

Checking by options, this is only satisfied by $\frac{n^2 - 5n + 6}{2}$

Hence, $\frac{n^2 - 5n + 6}{2}$ is the correct choice.

41. d Considering any two members of S, that are friends there will be 1 number of the pairs that will be common.

The common element of these pairs will have n – 3 pairs, with the remaining n – 3 elements. There will be one more member made up of the remaining two constituent elements which are not same. In total there are n – 3 + 1 = n – 2 other members of S that are common friends of the chosen two pairs or numbers.

Alternative Method:

For n = 6 lets consider the members (1, 2) and (1, 3)

Friends of the member (1, 2)in the set S are (1, 4), (1, 5), (1, 6), (2, 3), (2, 4), (2, 5), (2, 6).

Friends of the member (1, 3) in the set S (1, 4), (1, 5), (1, 6), (2, 3), (3, 4), (3, 5), (3, 6).

The number of members of S that are common friends to the above member are 4, i.e. (1, 4), (1, 5), (1, 6), (2, 3).

So the answer is n – 2.

42. a In each team, T_j there are two players, one it shares with T_{j-1} and other with T_{j-1}. Other (k – 2)players team T_j shares with no other team. So, total players which play for only one team = (k – 2)n.

One player is common in T_1 and T_2, one in T_2 and T_3 and so on.

Number of such players = number of pairs = n

So, total players = (k – 2)n + n = n(k – 1)

43. d In other words we need to find the total number of 4-digit numbers not more than 4000 using the digits 0, 1, 2, 3 and 4.

The digit at the thousands place can be selected in 3 ways.

The digits at the hundreds place can be selected in 5 ways.

The digits at the tens place can be selected in 5 ways.

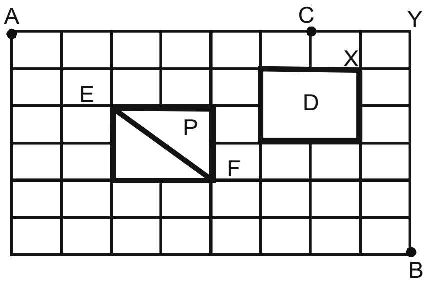
The digits at the units place can be selected in 5 ways.

Therefore, the total number of 4-digit numbers less than 4000 is equal to

3 × 5 × 5 × 5 = 375.

Therefore, the total number of 4-digit numbers not more than 4000 is equal to 375 + 1 = 376.

44. d



For the shortest route, Neelam follows the following path: $A \to E \to F \to B$

Number of ways to reach from A to E: $\frac{(2+2)!}{2! \times 2!} = 6$

Number of ways to reach from E to F: 1

Number of ways to reach from F to B: $\frac{(4+2)!}{4! \times 2!} = 15$

$\Rightarrow$ Total number of possible shortest paths

= 6 × 1 × 15 = 90

45. a Neelam has to reach C via B.

From A to B, the number of paths are 90, as found in question 44.

From B to C, Neelam follows the route:

Case I: $B \to X \to C$

OR **Case II:** $B \to Y \to C$.

Case I: $B \to X \to C$

Number of ways to reach from B to X: $\frac{(5+1)!}{5! \times 1!} = 6$

Number of ways to reach from X to C : 2

So, total number of paths are 6 × 2 = 12 ways.

Case II: $B \to Y \to C$**:**

There is just one way.

Therefore, from B to C, there are 6 × 2 + 1 = 13 ways

$\therefore$ Total number of ways of reaching from A to C, via B = 90 × 13 = 1170.

46. a Number of terms in the given expansion is nothing but the non-negative integral solutions of the equation a + b + c = 20.

Total number of non-negative integral solutions $= {}^{20+3-1}C_{3-1} = {}^{22}C_2 = 231$

Alternative Method:

$$(a+b+c)^{20} = \{(a+b)+c\}^{20}$$

$$= {}^{20}C_0(a+b)^{20}.C^0 + {}^{20}C_1(a+b)^{19}.C^1 + ... {}^{20}C_{20}(a+b)^0.C^{20}$$

Number of terms = 21 + 20 + 19 + ….. + 1 = 231

47. d (i) Let the box with the smallest number of balls does not contain any ball. Then 18 balls can go into 2 identical boxes in 10 ways (0, 18), (1, 17).... (9, 9).

(ii) Let the box with the smallest number of balls contains 1 ball. Then 17 balls can go into 2 identical boxes in 8 ways (1, 16), (2, 15).... (8, 9).

(iii) Let the box with the smallest number of balls contains 2 balls. Then 16 balls can go into 2 identical boxes in 7 ways (2, 14), (3, 13).... (8, 8).

(iv) Let the box with the smallest number of balls contains 3 balls. Then 15 balls can go into 2 identical boxes in 5 ways (3, 12), (4, 11).... (7, 8).

(v) Let the box with the smallest number of balls contains 4 balls. Then 14 balls can go into 2 identical boxes in 4 ways (4, 10), (5, 9).... (7, 7).

(vi) Let the box with the smallest number of balls contains 5 balls. Then 13 balls can go into 2 identical boxes in 2 ways (5, 8) and (6, 7).

(vii) Let the box with the smallest number of balls contains 6 balls. Then 12 balls can go into 2 identical boxes in just 1 way (6, 6).

The number of possible ways

= 10 + 8 + 7 + 5 + 4 + 2 + 1

= 37

Alternate Method:

Case I:

All the boxes contain an equal number of balls.

There is only one possible case i.e. 6, 6 and 6.

Case II:

Exactly two boxes contain an equal number of balls.

There are 9 possible cases i.e. (0, 0, 18), (1, 1, 16), (2, 2, 14), (3, 3, 12), (4, 4, 10), (5, 5, 8), (7, 7, 4), (8, 8, 2) and (9, 9, 0).

For each of these cases 3 combinations were possible had the boxes been non-identical.

Case III:

Each box contains a different number of balls.

Let the number of cases be x.

For each of these cases 6 combinations were possible had the boxes been non-identical.

$\therefore \quad 1 + 9 \times 3 + 6x = {}^{18+3-1}C_{3-1}$

$\Rightarrow \quad 28 + 6x = \frac{20 \times 19}{2} = 190$

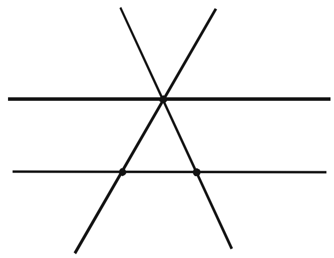
$\Rightarrow \quad x = 27$

So the required number of ways

= 27 + 9 + 1 = 37

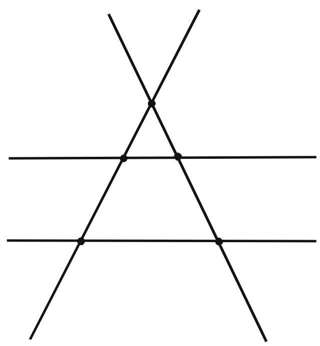
48. a Only two cases are possible:

Case I:



Points of intersection = 3

Case II:



Points of intersection = 5

Hence, possible values of P are 3 and 5.

49. b It is equivalent to finding how many such 2-digit numbers are there.

In such numbers we cannot have 0 or 1 in unit's place.

When we have 2 in unit's place, we have 1 such number, 12.

When we have 3 in unit's place, we have 2 such numbers, 13 and 23.

.

.

.

When we have 9 in unit's place, we have 8 such numbers.

So number of such numbers is (1 + 2 + 3 + ... + 8)

= 36.

Hence the resulting number has 72 digits.

50. d $\therefore$ No three points corresponding to the five vertices of a pentagon are collinear, by joining any of the three vertices of a pentagon, we get a triangle.

Hence, the number of triangles that can be drawn = $^5C_3$ = 10.

51. c Since the number of candies received by each child is a prime number, each child must get at least 2 candies.

Once each child has received 2 candies, the remaining 2 candies should be distributed in such a manner that the number of candies with any child after the distribution remains a prime number.

The above condition can realise only if the remaining 2 candies are given to exactly two children in such a way that both the children receive one candy each.

Hence, the number of ways of distribution = $^8C_2$ = 28.

52. b If there are 60 chairs around a circular table, consider a scenario wherein there are two chairs vacant between every two consecutive people. Thus, there will be exactly 20 people sitting and exactly 40 vacant seats between them and in such a scenario, next person coming to sit will have to sit next to someone.

53. a Let the batsman scored a 2's, b 4's and c 6's.

$\Rightarrow$ 2a + 4b + 6c = 100

$\Rightarrow$ a + 2b + 3c = 50. … (i)

When c = 1, (i) becomes a + 2b = 47

$\Rightarrow$ a = 47 – 2b … (ii)

Since a $\geq$ 1 and b $\geq$ 1, the number of solutions of (ii) is 23.

When c = 2, (i) becomes a + 2b = 44

$\Rightarrow$ a = 44 – 2b … (iii)

Since a $\geq$ 1 and b $\geq$ 1, the number of solutions of (iii) is 21.

When c = 3, (i) becomes a + 2b = 41

$\Rightarrow$ a = 41 – 2b … (iv)

Since a $\geq$ 1 and b $\geq$ 1, the number of solutions of (iv) is 20.

When c = 4, (i) becomes a + 2b = 38

$\Rightarrow$ a = 38 – 2b … (v)

Since a $\geq$ 1 and b $\geq$ 1, the number of solutions of (v) is 18.

Thus, we see a pattern emerging.

$\therefore$ The total number of ways

= 23 + 21 + 20 + 18 +…+ 3 + 2

= 184.

54. a First let the 6 other students be seated in 6 chairs.

The number of spaces between the 6 students = 7.

$\therefore$ Amar, Akbar and Anthony can be seated in the 7 places in $^7C_3$ ways.

Thus, the number of ways in the class photo can be taken such that no two of Amar, Akbar and Anthony are sitting together is

= 7C3 × 3! × 6!

= 151200.

55. d x = L.C.M. of (7, 8, 9) – 3 = 504 – 3 = 501

$x^3 + 2x^2 + x - 3 = (x - 1)(x + 1)(x + 2) - 1$

$= 500 \times 502 \times 503 - 1$

Remainder when $500 \times 502 \times 503 - 1$ is divided by:

11 = 4

3 = 0

4 = 3

Required remainder = least possible number which when divided by 11, 3 and 4 leavs remainder 4, 0 and 3 respectively

Such least no. is 15.

56. c Letters of the word 'KAKA' can be arranged in

$\left(\frac{4!}{2! \times 2!}\right) = 6$ ways

If we consider the cases where vowels are together, then considering two A's as single entity,

we have $\left(\frac{3!}{2!}\right) = 3$ arrangements.

So, there must be (6 – 3) = 3 arrangements, where vowels are not together.

Therefore, the required probability $= \frac{3}{6} = \frac{1}{2}$.

57. b Let the number of games won and those lost by the man be N_w and N_l respectively.

Since he gains \$2, $N_w - N_l = 2$.

Also, he does not lose more than once.

The possible cases are (W $\rightarrow$ Win and L $\rightarrow$ Loss):

Case (i): $N_w = 2$ and $N_l = 0$.

The only possible sequence is WW.

Case (ii): $N_w = 3$ and $N_l = 1$.

Possible sequences: LWWW and WLWW.

Hence, the total number of possible Win-Loss sequences = 1 + 2 = 3.

58. a Let the bowlers be represented by $B_1, B_2, B_3, \ldots, B_n$.

If a particular bowler B_1 (say) did not bowl in pair with two other bowlers, B_2 and B_3 (say), then B_2 and B_3 also did not bowl in pair with every bowler in the team. Therefore, at least three bowlers must not have bowled in pair with every other bowler in the team.

Hence, the maximum number of bowlers who could have bowled in pair with every other bowler in the team is n – 3.

59. 6 Amal Bimal Kamal

≥ 1 ≥ 2 ≥ 3

	A	B	K
Pens	3	2	3
	1	4	3
	1	2	5
	2	3	3
	1	3	4
	2	2	4

Total cases = 6

60. 50 For divisibility by 6, last digit should be even and sum of digits is divisible by 3.

Case I :

2, 3, 4, 6

$3 \times 2 \times 1 \times 3$ = 18 ways

Case II:

(i) 2, 3, 4, 0(when last digit is zero)

$3 \times 2 \times 1 \times 1$ = 6 ways

(ii) $2 \times 2 \times 1 \times 2$ = 8 ways

Total 8 + 6 = 14 ways

Case III:

0, 2, 4, 6

$3 \times 3 \times 2 \times 1$ = 18 ways

So, total possible ways = 50.

61. 502 Digits available = 1, 2, 3, 4, 5, 6, 7, 8, 9

The numbers will be ${}^9C_2 + {}^9C_3 + \ldots\ldots {}^9C_9$.

$= 2^9 - {}^9C_0 - {}^9C_1$.

= 512 – 1 – 9 = 502.

62. 1098 Let the number of girls in junior section be g, thus ${}^gC_2 = 153$

$\Rightarrow \frac{g(g-1)}{2} = 153$

$\Rightarrow$ g = 18

Therefore, number of girls in junior section = 18 and that of boys = 25

Again, let the number of boys in senior section be b

$\Rightarrow \frac{b(b-1)}{2} = 276$

$\Rightarrow$ b = 24

Therefore, number of boys in senior section = 24 and that of girls = 26

Hence, the number of matches played between boys and girls = 25 × 18 + 24 × 27 = 1098.

63. b Given that: 10a + b > 3(10b + a)

$\Rightarrow$ 7a > 29b

For b = 1 we get a = 5, 6, 7, 8 and 9

For b = 2 we get a = 9

Hence, required answer will be 6.

Probability

1. a Since in the long run the probability of each number appearing is the same, we can say in 'n' throws one can get 1, 2, 3, 4, 5 and 6, $\frac{n}{6}$ times each.

Hence he would earn

$\frac{(1+2+3+4+5+6)n}{6} = \text{Rs.}\frac{7n}{2}$.

In order to make a profit of 1 Re. per throw he has to totally earn a profit of Rs.n. Hence his cost for the n throws should be $\frac{7n}{2} - n$.

So his cost per throw should be

$\left(\frac{7}{2} - 1\right) = \frac{5}{2} = \text{Rs.}2.50$.

2. c Since there are 6 red balls and all six of them are of different sizes, probability of choosing the smallest among them is $\frac{1}{6}$.

3. b 53 Sundays can occur in a non-leap year, if 1st January is either a Saturday or a Sunday. But 54 Sundays can never occur.

Hence, I < II.

4. a Total number of four-digit numbers that can be formed = 4!.

If the number is divisible by 25, then the last two digits are 25.

So the first two digits can be arranged in 2! ways.

Hence, required probability = $\frac{2!}{4!} = \frac{1}{12}$.

5. a The total number of arrangements possible $= \frac{11!}{(2!)^2}$

The total arrangements of 2 'A's and 2 'T's $= \frac{4!}{(2!)^2} = 6$

Out of these 6 possible arrangements only 1 arrangement AATT is acceptable with the given condition.

So for every 6 arrangements of the letters of the word CATASTROPHE, only 1 would satisfy the given condition.

Hence, the required arrangements $= \frac{11!}{(2!)^2.6} = \frac{11!}{4!}$

6. a $P(I) = P(II) = \frac{1}{2}$

$\therefore P(B \cap I) = P(I) - P(W \cap I)$

$\Rightarrow P(B \cap I) = \frac{1}{2} - \frac{3}{10} = \frac{1}{5}$ ...(i)

Also, $P(B' | II) = \frac{P(B' \cap II)}{P(II)}$

$\Rightarrow P(B' \cap II) = \frac{8}{10} \times \frac{1}{2} = \frac{2}{5}$

$\because P(B' \cap II) = P(W \cap II)$

$\Rightarrow P(W \cap II) = \frac{2}{5}$

$\therefore P(B \cap II) = P(II) - P(W \cap II) = \frac{1}{2} - \frac{2}{5} = \frac{1}{10}$

Now, $P(I | B) = \frac{P(I \cap B)}{P(B)}$

As P(B) = $P(B \cap I) + P(B \cap II)$, therefore,

from (i) and (ii) $P(B) = \frac{1}{5} + \frac{1}{10} = \frac{3}{10}$

$\Rightarrow P(I | B) = \frac{1}{5} \div \frac{3}{10} = \frac{2}{3}$.

7. b The total number of possibe outcomes = 6 × 6 = 36.

The favourable values of the sum are 2, 3, 5, 7 and 11.

Favourable cases when the sum is:

I) 2 = (1, 1)

II) 3 = (1, 2) and (2, 1)

III) 5 = (1, 4), (4, 1), (2, 3) and (3, 2)

IV) 7 = (1, 6), (6, 1), (2, 5), (5, 2), (3, 4) and (4, 3)

V) 11 = (5, 6) and (6, 5)

The total number of favourable cases = 15.

Required probability $= \frac{15}{36} = \frac{5}{12}$.

8. b An integer can end with any of the ten digits (0, 1, 2 .... 9) out of which if it ends with one of the four (0, 1, 5, 6), the required condition will be satisfied.

The probability of an integer ending with 0 or 1 or 5 or 6 is 4/10 = 2/5

Now the probability of 2nd integer also ending with the digit that has come in unit's place of the first integer is 1/10

$\therefore$ The required probability = (2/5) × (1/10) = 1/25

9. d The expression $a^{12}b^8$ can be rewritten as $(a^2)^6 b^8$.

We can observe that the sum of the powers of a^2 and b in $(a^2)^6 b^8$ is (6 + 8) i.e.14. But in the expansion of $(a^2 + b)^{13}$ the sum of the powers of a^2 and b must be 13 in each of the terms. Hence, the given term does not exist in the expansion i.e. the required coefficient is zero.

10. a p + q = 1, i.e. q = 1 – p (0 = p, q = 1)

Now when the sum of two variables is a constant then their multiplication is the maximum when they are equal. So, pq will be maximum and $\left(\frac{1}{p}\right)\left(\frac{1}{q}\right)$ the minimum when p = q = $\frac{1}{2}$.

Thus, the minimum value of X = 0.25 + 4 = 4.25.

11. b Total number of the cases = $^6C_3$ = 20.

The favourable cases for B surviving are:

(B : Bullet; N : No Bullet)

BNBBNN or BNBNBN or BNBNNB or BNNNBB or NNBBNB or NNBBBN.

Hence probability = $\frac{6}{20}$ = 0.3

12. a The number of ways of picking two small cubes

$$= {}^{64}C_2 = 32 \times 63$$

The number of small cubes with exactly two faces painted red

= 2 × 12 = 24 (Since two such cubes will be obtained from each edge of the large cube.)

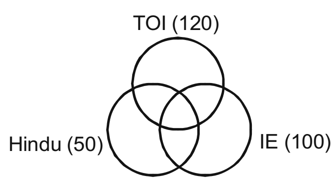
The number of ways of picking two such cubes

$$= {}^{24}C_2 = 23 \times 12$$

So the required probability $= \frac{23 \times 12}{63 \times 32} = \frac{23}{168}$.

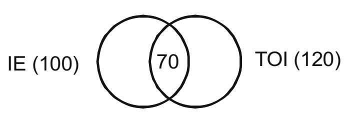
Set Theory

1. b From the statement I, we have the following Venn diagram :



Using this we cannot find the answer.

From the statement II however we can find the answer, as we get the following Venn diagram.



2. c We know that x + y + z = T and x + 2y + 3z = R_T, where

x = number of members belonging to exactly 1 set

y = number of members belonging to exactly 2 sets = 9

z = number of members belonging to exactly 3 sets = 1

T = Total number of members

R_T = Repeated total of all the members

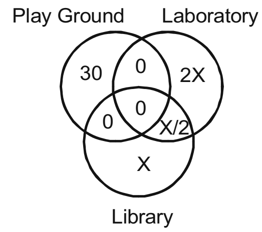
= (22 + 15 + 14) = 51

R_T = T + y + 2z

Thus we have two equations and two unknowns. Solving this we get T = 40.

In other words, the number of teachers owing at least 1 out of the three items = 40. Hence the number of teachers owing none = 50 – 40 = 10.

For questions 3 and 4:



It is given that X + 2X + X/2 = 35

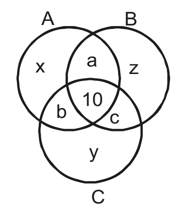
Therefore, X =10.

3. d Total number of schools that had at least one of the three = 30 + 10 + 20 + 5 = 65. Hence the number of schools having none of them = 100 – 65 = 35.

4. b Number of schools having library = 15. And number of schools having laboratory = 25.

Hence the ratio = 25 : 15 = 5 : 3.

5. b



x + a + b = 40 – 10 = 30 ... (i)

y + b + c = 50 – 10 = 40 ... (ii)

z + a + c = 60 – 10 = 50 ... (iii)

From (i), (ii) and (iii)

a + b + c = 25.

For questions 6 and 7:

Let x, y and z be the number of children who took 1 rides, 2 rides and 3 rides respectively.

Since z = 20 and y + z = 55, y = 35.

Then, total number of rides = x + 2y + 3z = 145

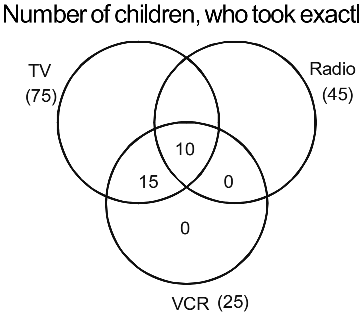
$\Rightarrow$ x + 2 × 35 + 3 × 20 = 145 $\Rightarrow$ x = 15

6. c Number of children, who did not try any of the rides

= 85 – (x + y + z)= 85 – (15 + 35 + 20) = 15

7. c Number of children, who took exactly one ride = x = 15

8. c

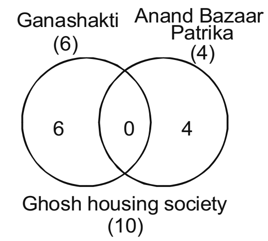


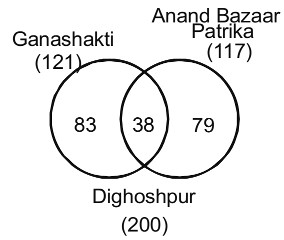
Since each VCR owner also has a TV, therefore, 15 families own both TV and VCR but not Radio.

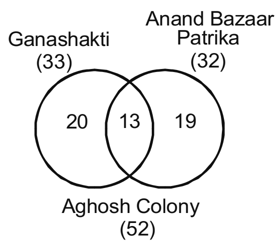
Since 25 families have radio only, therefore, 10 families own both TV and Radio but not VCR.

Hence, number of families having only TV = 75 – 10 –10 – 15 = 40.

For questions 9 to 11: The data can be represented in the following Venn diagrams.







9. b Number of persons in Dighoshpur who read only Ganashakti = 83.

10. a Number of persons in Aghosh Colony who read both the newspapers = 13.

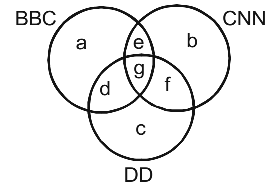
11. c Number of persons in Aghosh Colony who read only 1 newspaper = 20+19 = 39.

12. b We are supposed to find out what fraction of the population has exactly one among the two (since either cable TV or VCR indicates they do not have both). Now $\frac{2}{3}$ of the people have cable TV, of whom $\frac{1}{10}$ of people also have VCR.

Hence, fraction of population having only cable TV $= \left(\frac{2}{3} - \frac{1}{10}\right) = \frac{17}{30}$. Also $\frac{1}{5}$ of the people have VCR, of whom $\frac{1}{10}$ of people also have cable TV.

Hence, fraction of people having only VCR $= \left(\frac{1}{5} - \frac{1}{10}\right) = \frac{1}{10}$. The total fraction of the people who either have cable TV or VCR $= \left(\frac{17}{30} + \frac{1}{10}\right) = \frac{2}{3}$.

For questions 13 to 15:



Here, $a + b + c + d + e + f + g = 200$ ...(i)

80% of the people watch DD implies

$c + d + f + g = 160$ ...(ii)

22% of the people watch BBC implies

$a + d + e + g = 44$ ...(iii)

15% of the people watch CNN implies

$b + e + f + g = 30$ ...(iv)

(ii) + (iii) + (iv) gives

$a + b + c + 2(d + e + f) + 3g = 234$

Subtracting (i) from this equation,

$d + e + f + 2g = 34$ ...(v)

13. c To maximize g, in equation (v), we put $d = e = f = 0$

$\therefore$ Maximum value of g $= \frac{34}{2} = 17$

$\therefore$ Required percentage $= \frac{17}{200} \times 100 = 8.5\%$

14. a 5% of people watch DD and CNN implies

$f + g = 10$ ...(vi)

10% of people watch DD and BBC implies

$d + g = 20$ ...(vii)

(v) – (vi) – (vii) gives

$e = 4$

$\therefore$ Required percentage $= \frac{4}{200} \times 100 = 2\%$

15. d From equation (v), we have

$(d + 4 + f) + 2g = 34$

$\Rightarrow (d + f) + 2g = 30$

Since we cannot find the values of d and f, the value of g cannot be ascertained.

16. c Let ‘a’ be the percentage of people who favoured exactly one proposal, ‘b’ be the percentage of people who favoured exactly by two proposals and ‘c’ be the percentage of people who favoured exactly three proposals.

$a + b + c = 78$ ... (i)

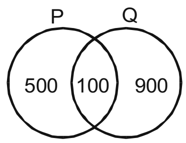
$a + 2b + 3c = 100$ ... (ii)

(ii) – (i) implies $b + 2c = 22$

Since $c = 5$, $b = 12$

Required percentage $= b + c = 12 + 5 = 17\%$.

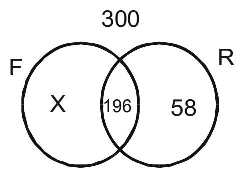
17. c The Venn diagram arrived at from both I and II clearly indicates that 500 people are watching programme P.



18. c From statement A, we cannot find anything.

From B alone we cannot find.

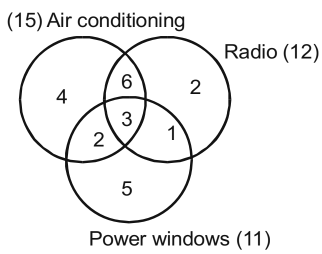
From A and B,



x + 196 + 58 = 300. Thus, x can be found.

19. a By observing, we see 6 will appear in 5 sets T_2, T_3, T_4, T_5 and T_6. Similarly, 12 will also appear in 5 sets and these sets will be distinct from the sets in which 6 appears, i.e. T_8, T_9, T_{10}, T_{11} and T_{12}. Thus, each multiple of 6 will appear in 5 distinct sets. Till T_{96}, there will be 16 multiplies of 6. These 16 multiples of 6 will appear in 16 × 5 = 80 sets.

20. d



Total = 4 + 6 + 2 + 2 + 3 + 1 + 5 = 23

∴ Cars having none of the option = 25 – 23 = 2.

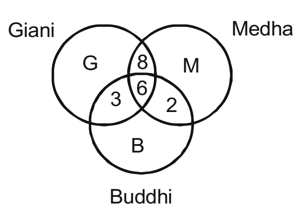
21. c VCD = 70%, Microwave = 75%, ACS = 80%, Washing M/c = 85%

Least percentage of employees having both VCD and Microwave = 70 + 75 – 100 = 45%

Least percentage having all three – VCD, Microwave, ACS = 45 + 80 – 100 = 25%

Least percentage having all four = 25% + 85% – 100% = 10%

For question 22 and 23: As per the given data, we get the following:



22. d Putting the value of M in either equation, we get G + B = 17.

Hence, neither of two can be uniquely determined.

23. b G + B = M + 16

Also, M + B + G + 19 = (2 × 19) – 1

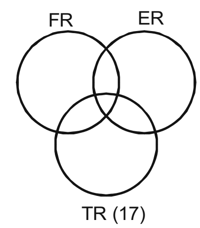
i.e. (G + B) = 18 – M

Thus, M + 16 = 18 – M

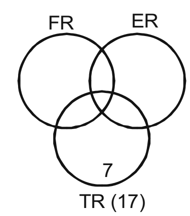
i.e. M = 1

Questions 24 to 27:

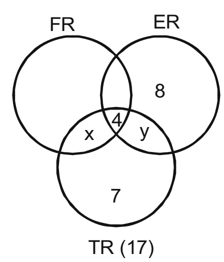
- 17 in TR



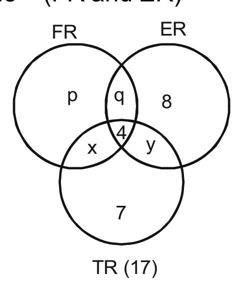
- 10 in TR also in at least one more ⇒ 7 in TR alone



- TR alone = one less than ER alone ⇒ ER alone = 8
- ER alone = double of all 3 ⇒ In all three = $\frac{8}{2} = 4$



- FR alone = (FR and ER)



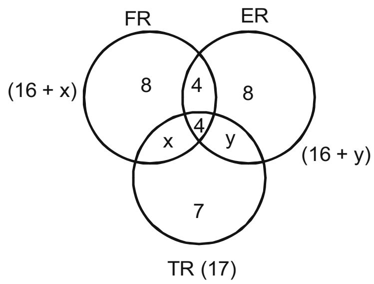
⇒ p = q + 4 ...(1)

Total = 37

[7 + 8 + p + (x + y + q) + 4] = 37 [p + q = 12]

⇒ p – a = 4

⇒ p = 8 and q = 4



Now, total number of FR is maximum

$\Rightarrow$ 8 + 4 + 4 + x > 8 + 4 + 4 + y

$\Rightarrow$ x > y and x + y = 6

as n(TR) = 17 = x + y + 4 + 7

$\Rightarrow$ x = {4, 5, 6}

y = {0, 1, 2}

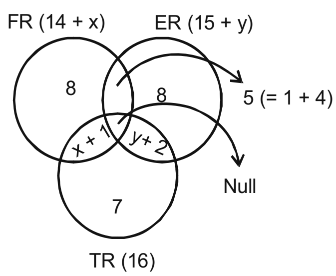
24. c Both FR and TR but not ER = x

Minimum x = 4

25. a Option (b) and option (c)are superfluous. They are not required.

Option (a), if given, would tell us the value of x = 4 and hence y = 2.

26. b Out of 4 who are in all three projects, 2 move out of FR and one-one move out of ER and TR.



Minimum in FR = 14 + x = 14 + 4 = 18

Maximum in ER = 15 + y = 15 + 2 = 17

$$\left\{\begin{array}{c} \text{As} \\ x = \{4, 5, 6\} \\ y = \{0, 1, 2\} \end{array}\right\}$$

Hence, option (b).

27. d FR and ER = 5

ER and TR = y + 2

$\Rightarrow$ 5 = y + 2

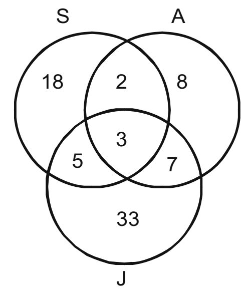
$\Rightarrow$ y = 3;

which is not a possible value as y is 0, 1, or 2 only.

$\Rightarrow$ option (d)

Inconsistent data.

28. b



So, total people reading the newspaper in consecutive months i.e. July and August and August and Sept. is 2 + 7 = 9 people.

29. a A = {1, 2, 4, 8, 16}

B = {1, 3, 9, 27, 81}

The number of possible proper fractions when denominator is equal to 2, 4, 8 and 16 are 1, 2, 2 and 3 respectively.

The number of possible proper fractions when denominator is equal to 3, 9, 27 and 81 are 2, 4, 5 and 5 respectively.

Since no two of these fractions can be equal, the answer = 24.

Alternate method: All the combinations can result in a proper fraction except when 1 is chosen from both the sets.

So the answer = 5 × 5 – 1 = 25 – 1 = 24.

30. c The given set S has 6 composite and 4 prime numbers.

The number of subsets of S comprising composite numbers only = $2^6 - 1$

The number of subsets of S comprising prime numbers only = $2^4 - 1$

Hence, the required difference = $(2^6 - 1) - (2^4 - 1) = 48$.

31. 58 There are a total of 78 B-schools that place their students

$\therefore$ No. of B-schools which are reputed and place their students $= \frac{1}{6} \times 78 = 13$

Let No. of B-schools that are recognised = x

$\therefore$ No. of recognised B-schools that place their students $= \frac{1}{4}x$

$\therefore$ No. of recognised B-schools that do not place their students $= \frac{3x}{4}$

$\therefore \frac{3x}{4} = 39 \Rightarrow x = 52$

Out of 13 reputed B-schools, 6 are recognised too

∴ Number of B-schools that are either recognised and place their students or reputed and place their students = 13 + 13 – 6 = 20

∴ Number of B-school that are neither reputed nor recognised but place their students = 78 – 20 = 58.

32. a A = {1, 4, 7, 10, …, 20 terms}

B = {9, 16, 23, 30, ..., 20 terms}

We first need to find $n(A \cap B)$.

The n^{th} term of A = 1 + (n – 1)3 = 3n – 2.

The m^{th} term of B = 9 + (m – 1)7 = 7m + 2.

$n(A \cap B)$ will be equal to the number of solutions of 3n – 2 = 7m + 2, where n and m are natural numbers not more than 20.

$$\Rightarrow \quad n = \frac{7m+4}{3}$$

If m = 2, n = 6;

m = 5, n = 13;

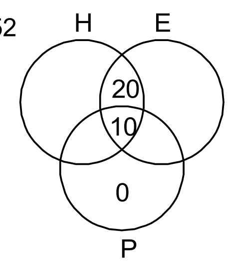
m = 8, n = 20

$\Rightarrow$ $n(A \cap B) = 3$

Hence, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 20 + 20 - 3 = 37.$

33. 52



Number of students studying H equals that studying E = z + 30

As total number of students = 74

∴ z + z + 30 = 74

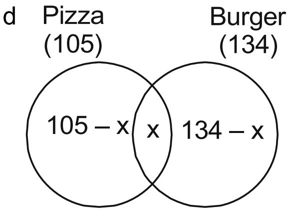
2z + 30 = 74

2z = 44

z = 22

So, the number of students studying H is z + 30 = 22 + 30 = 52.

34. d



Total students = 200

So, the number of students who like only burger

= 134 – x

first, check option 1,

134 – x = 26

x = 108 which is greater than the number of students who like Pizza. So, 26 cannot be the answer.

Similarly, 134 – x = 96

x = 38

So, the number of students who like only Pizza

= 105 – x = 105 – 38 = 67

So, the total number of students

= 67 + 38 + 96

= 201, which is wrong.

Hence, option 2 cannot be the answer.

Now, put option 3,

134 – x = 23

x = 111, which is again greater than the number of students who like Pizza, hence, option (3) cannot be the answer,

If 134 – x = 93

$\Rightarrow$ x = 41

∴ Number of students who like Pizza only

= 105 – 41 = 64

So, total number of these students who like one or more items = 93 + 41 + 64 = 198

Hence, 93 can be the possible answer.

35. a $(P \Delta Q) = (1, 4, 5, 6)$ and $(R \Delta S) = (1, 2, 3, 4, 7, 8, 10)$

$\Rightarrow (P \Delta Q) \Delta (R \Delta S) = (2, 3, 5, 6, 7, 8, 10)$

Hence, required number of elements = 7.