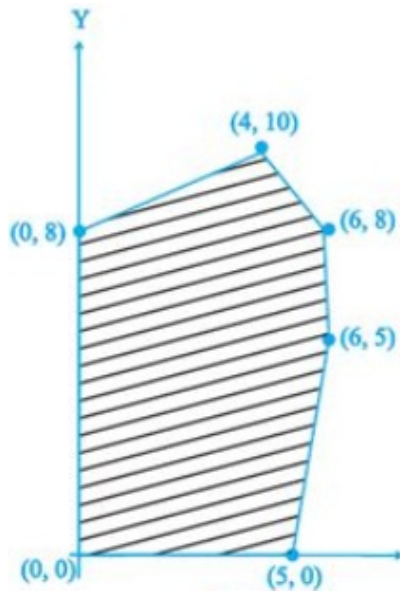


**CBSE Test Paper 01**  
**Chapter 12 Linear Programming**

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1. Maximise  $Z = 3x + 4y$  subject to the constraints:  $x + y \leq 4$ ,  $x \geq 0$ ,  $y \geq 0$ .
  - a. Maximum  $Z = 16$  at  $(0, 4)$
  - b. Maximum  $Z = 19$  at  $(1, 5)$
  - c. Maximum  $Z = 18$  at  $(1, 4)$
  - d. Maximum  $Z = 17$  at  $(0, 5)$
2. A linear programming problem is one that is concerned with
  - a. finding the optimal value (maximum or minimum) of a linear function of several variables
  - b. finding the limiting values of a linear function of several variables
  - c. finding the lower limit of a linear function of several variables
  - d. finding the upper limits of a linear function of several variables
3. Maximise  $Z = x + y$  subject to  $x + 4y \leq 8$ ,  $2x + 3y \leq 12$ ,  $3x + y \leq 9$ ,  $x \geq 0$ ,  $y \geq 0$ .
  - a.  $2\frac{10}{11}$
  - b.  $3\frac{19}{31}$
  - c.  $3\frac{10}{11}$
  - d.  $3\frac{9}{11}$
4. A farmer mixes two brands P and Q of cattle feed. Brand P, costing Rs 250 per bag, contains 3 units of nutritional element A, 2.5 units of element B and 2 units of element C. Brand Q costing Rs 200 per bag contains 1.5 units of nutritional element A, 11.25 units of element B, and 3 units of element C. The minimum requirements of nutrients A, B and C are 18 units, 45 units and 24 units respectively. Determine the number of bags of each brand which should be mixed in order to produce a mixture having a minimum cost per bag? What is the minimum cost of the mixture per bag?
  - a. 5 bags of brand P and 6 bags of brand Q; Minimum cost of the mixture = Rs2250
  - b. 3 bags of brand P and 6 bags of brand Q; Minimum cost of the mixture = Rs 1950
  - c. 6 bags of brand P and 6 bags of brand Q; Minimum cost of the mixture = Rs2350
  - d. 4 bags of brand P and 6 bags of brand Q; Minimum cost of the mixture = Rs 2150
5. The feasible solution for an LPP is shown in Figure. Let  $Z = 3x - 4y$  be the objective function. Minimum of  $Z$  occurs at



- $(0, 8)$
  - $(0, 0)$
  - $(5, 0)$
  - $(4, 10)$
- A corner point of a feasible region is a point in the region which is the \_\_\_\_\_ of two boundary lines.
  - In a L.P.P, the linear inequalities or restrictions on the variables are called \_\_\_\_\_.
  - In a LPP if the objective function  $Z = ax + by$  has the same maximum value on two corner points of the feasible region, then every point on the line segment joining these two points give the same \_\_\_\_\_ value.
  - A small firm manufactures necklaces and bracelets. The total number of necklaces and bracelets that it can handle per day is at most 24. It takes one hour to make a bracelet and half an hour to make a necklace. The maximum number of hours available per day is 16. If the profit on a necklace is Rs 100 and that on a bracelet is Rs 300. Formulate on L.P.P. for finding how many of each should be produced daily to maximise the profit? It is being given that at least one of each must be produced.
  - In order to supplement daily diet, a person wishes to take some X and some wishes Y tablets. The contents of iron, calcium and vitamins in X and Y (in milligram per tablet) are given as below:

Tablets	Iron	Calcium	Vitamin
X	6	3	2
Y	2	3	4

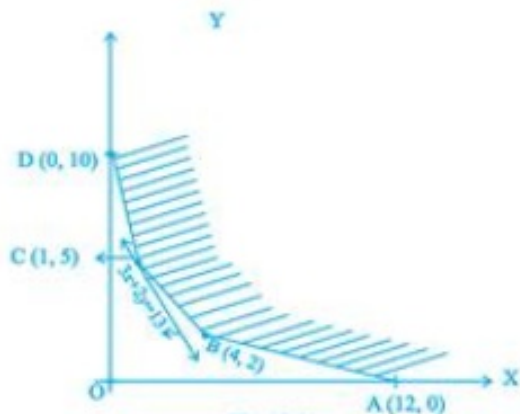
The person needs at least 18 milligram of iron, 21 milligram of calcium and 16 milligram of vitamins. The price of each tablet of X and Y is Rs 2 and Re 1 respectively. How many tablets of each should the person take in order to satisfy the above requirement at the minimum cost?

11. A factory owner purchases two types of machine A and B for his factory. The requirements and the limitations for the machines are as follows,

Machine	Area Occupied	Labour force	Daily
		on each machine	output (in units)
A	1000 m <sup>2</sup>	12 men	60
B	1200 m <sup>2</sup>	8 men	40

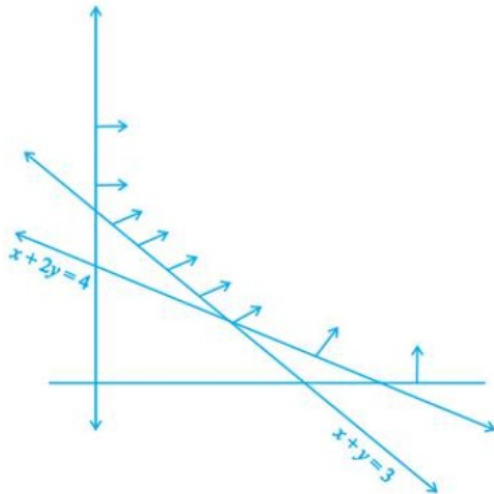
He has maximum area 9000 m<sup>2</sup> available and 72 skilled labours who can operate both the machines. How many machines of each type should he buy to maximize the daily out put?

12. Determine the minimum value of  $Z = 3x + 2y$  (if any), if the feasible region for an LPP is shown in Fig.



13. Minimise  $Z = 13x - 15y$ , subject to the constraints:  
 $x + y \leq 7, 2x - 3y + 6 \geq 0, x \geq 0, y \geq 0$ .
14. A manufacturer of electronic circuits has a stock of 200 resistors, 120 transistors and 150 capacitors and is required to produce two types of circuits A and B. Type A requires 20 resistors, 10 transistors and 10 capacitors. Type B requires 10 resistors, 20 transistors and 30 capacitors. If the profit on type A circuit is Rs 50 and that on type B circuit is Rs 60, formulate this problem as a LPP so that the manufacturer can maximise his profit.
15. The feasible region for a LPP is shown in Fig. 12.10. Evaluate  $Z = 4x + y$  at each of the

corner points of this region. Find the minimum value of  $Z$ , if it exists.



16. There are two types of fertilizers  $F_1$  and  $F_2$ .  $F_1$  consists of 10% nitrogen and 6% phosphoric acid and  $F_2$  consists of 5% nitrogen and 10% phosphoric acid. After testing the soil conditions, a farmer finds that she needs atleast 14kg of nitrogen and 14kg of phosphoric acid for her crop. If  $F_1$  costs Rs 6/kg and  $F_2$  costs Rs 5/kg, determine requirements are met at a minimum cost. What is the minimum cost?
17. A factory makes tennis rackets and cricket bats. A tennis racket takes 1.5 hours of machine time and 3 hours of craftman's time in its making while a cricket bat takes 3 hour of machine time an 1 hour of craftman's time. In a day, the factory has the availability of not more than 42 hours of machine time and 24 hours of craftsman's time.
  - i. What number of rackets and bats must be made if the factory is t work at full capacity?
  - ii. If the profit on a racket and on a bat is Rs 20 and Rs 10 respectively, find the maximum profit of the factory when it works at full capacity.
18. Two tailors A and B earn Rs. 150 and Rs. 200 per day, respectively. A can stitch 6 shirts and 4 pants per day, while B can stitch 10 shirts and 4 pants per day. How many days shall each work, if it is desired to produce at least 60 shirts and 32 pants at a minimum labour cost? Make it as an LPP and solve the problem graphically.

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**CBSE Test Paper 01**  
**Chapter 12 Linear Programming**

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**Solution**

1. a. Maximum  $Z = 16$  at  $(0, 4)$

**Explanation:** Objective function is  $Z = 3x + 4y$  .....(1).

The given constraints are :  $x + y \leq 4$ ,  $x \geq 0$ ,  $y \geq 0$ .

The corner points obtained by constructing the line  $x + y = 4$ , are  $(0,0)$ ,  $(0,4)$  and  $(4,0)$ .

Corner points	$Z = 3x + 4y$
O ( 0 , 0 )	$Z = 3(0) + 4(0) = 0$
A ( 4 , 0 )	$Z = 3(4) + 4 (0) = 12$
B ( 0 , 4 )	$Z = 3(0) + 4 ( 4) = 16$ ...( Max. )

therefore  $Z = 16$  is maximum at  $( 0 , 4 )$ .

2. a. finding the optimal value (maximum or minimum) of a linear function of several variables

**Explanation:** A linear programming problem is one that is concerned with finding the optimal value (maximum or minimum) of a linear function of several variables.

3. c.  $3\frac{10}{11}$

**Explanation:** Here , Maximise  $Z = x + y$  subject to  $x + 4y \leq 8$ ,  $2x + 3y \leq 12$ ,  $3x + y \leq 9$ ,  $x \geq 0$ ,  $y \geq 0$ .

Corner points	$Z = x + y$
P( 0 , 0 )	0
Q(3 , 0)	3
R( 0 , 2 )	2
S(28/11 , 15/11 )	43/11.(Max.)

Hence the maximum value is  $43/11 = 3\frac{10}{11}$

4. b. 3 bags of brand P and 6 bags of brand Q; Minimum cost of the mixture = Rs 1950

**Explanation:** Let number of bags of cattle feed of brand P = x

And number of bags of cattle feed of brand Q = y

Therefore, the above L.P.P. is given as :

Minimise,  $Z = 250x + 200y$ , subject to the constraints :  $3x + 1.5y \geq 80$ ,  $2.5x + 11.25y \geq 45$ ,  $2x + 3y \geq 24$ ,  $x, y \geq 0$ ,

Corner points	$Z = 250x + 200y$
C( 0,12 )	2400
B (18,0)	4500
D(3,6 )	1950 (Min.)
A(9,2)	2650

Here  $Z = 1950$  is minimum.

i.e. 3 bags of brand P and 6 bags of brand Q; Minimum cost of the mixture = Rs 1950.

5. a. (0, 8)

**Explanation:**

Corner points	$Z = 3x - 4y$
(0, 0)	0
(5,0)	15
(6,8)	-14
(6 ,5)	-2
(4,10)	-28
(0,8)	-32.....(Min.)

The minimum value occurs at (0,8)

6. Intersection

7. Linear constraints

8. maximum

9. Let number of necklaces and bracelets produced by firm per day be x and y, respectively. Given that the maximum number of both necklaces and bracelets that firm can handle per day is almost 24.

which means that the firm can produce maximum 24 items which includes both necklaces and bracelets per day. Hence the inequality related to the number constraint is given as

$$\therefore x + y \leq 24$$

Also given that It takes one hour per day to make a bracelet and half an hour per day to make a necklace and maximum number of hours available per day is 16.

Hence the inequality representing the hour constraint is given as

$$\therefore x + \frac{1}{2}y \leq 16 \Rightarrow \text{( when multiplying throughout the inequality by 2 we get )}$$

$$2x + y \leq 32$$

Also the non negative constraints which restricts the feasible region of the problem within the first quadrant is given as  $x \geq 0, y \geq 0$ , since the given situations are real world connected and cannot have the solution as negative which means that the values of the variables  $x$  and  $y$  are non negative.

Let  $z$  be the objective function which represents the total maximum profit. Hence the equation of the profit function  $Z$  is given as

$\therefore z = 100x + 300y$ , which is to be maximised  
subject to the constraints,

$$x + y \leq 24$$

$$2x + y \leq 32$$

$$x, y \geq 0$$

10. Let the person takes  $x$  units of tablet X and  $y$  unit of tablet Y.

So, from the given information, we have

$$6x + 2y \geq 18 \Rightarrow 3x + y \geq 9 \dots (i)$$

$$3x + 3y \geq 21 \Rightarrow x + y \geq 7 \dots (ii)$$

$$\text{And } 2x + 4y \geq 16 \Rightarrow x + 2y \geq 8 \dots (iii)$$

$$\text{Also, we know that here, } x \geq 0, y \geq 0 \dots (iv)$$

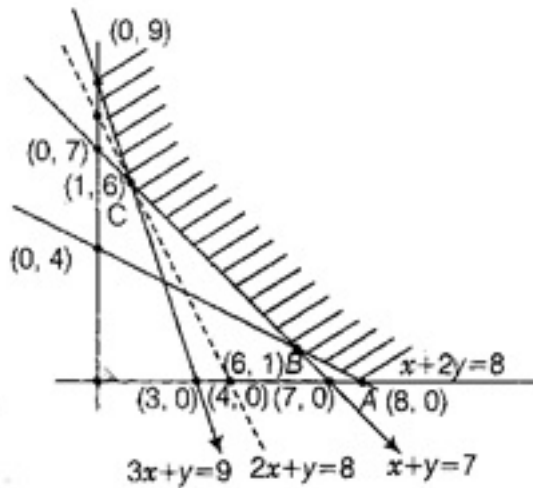
The price of each tablet of X and Y is Rs 2 and Rs 1, respectively.

So, the corresponding LPP is minimise  $Z = 2x + y$  subject to

$$3x + y \geq 9, x + y \geq 7, x + y \geq 7, x + 2y \geq 8, x \geq 0, y \geq 0$$

From the shaded graph, we see that for the shown unbounded region, we have coordinates of corner points A, B, C and D as (8,0), (6,1), (1,6) and (0,9) respectively.

[On solving  $x + 2y = 8$  and  $x + y = 7$  we get  $x = 6, y = 1$  and on solving  $3x + y = 9$  and  $x + y = 7$ , we get  $x = 1, y = 6$ ]



Corner Points	Values of $Z = 2x + y$
(8, 0)	16
(6, 1)	13
(1, 6)	8 (minimum)
(0, 9)	9

Thus, we see that 8 is the minimum value of  $Z$  at the corner point (1, 6). Here we see that the feasible region is unbounded. Therefore, 8 may or may not be the minimum value of  $Z$ . To decide this issue, we graph the inequality

$$2x + y < 8 \dots (v)$$

And check whether the resulting open half has points in common with feasible region or not. If it has common point, then 8 will not be the minimum value of  $Z$ , otherwise 8 will be the minimum value of  $Z$ .

Thus, from the graph it is clear that, it has no common point.

Therefore,  $Z = 2x + y$  has 8 as minimum value subject to the given constraint.

Hence, the person should take 1 unit of X tablet and 6 unit of Y tablets to satisfy the given requirements and at the minimum cost of Rs 8.

11. Let  $x$  machines of type A and  $y$  machines of type B be bought and let  $Z$  be the daily output.

$$\Rightarrow Z = 60x + 40y \dots (i)$$

$$\text{Maximum area available} = 9000 \text{ m}^2$$

$$\Rightarrow 1000x + 1200y \leq 9000 \dots (ii)$$

or



$$5x + 6y \leq 45$$

Maximum labour available = 72 men

$$\Rightarrow 12x + 8y \leq 72$$

$3x + 2y < 18$ ... (iii) Now, the mathematical formulation of given L.P.P. is as follows

Maximize  $Z = 60x + 40y$  Subject to constraints  $5x + 6y < 45$

$$3x + 2y < 18$$

s.t.  $x > 0, y > 0$

We draw the line  $5x + 6y = 45$  and  $3x + 2y = 18$  and shaded the feasible region w.r.t. the constraint sign.

We observed that the feasible region is bounded and corner points are

$O(0, 0), A(6, 0)$

$B\left(\frac{9}{4}, \frac{45}{8}\right)$  and  $C\left(0, \frac{15}{2}\right)$

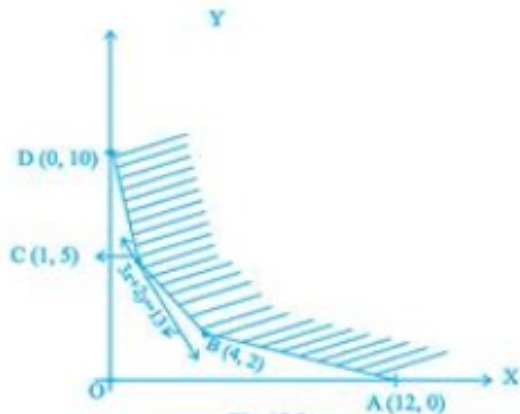
Evaluating the  $Z$  at each corner point, we have

Corner Points	$Z = 60x + 40y$
$O(0, 0)$	$Z = 0$
$A(6, 0)$	$Z = 360$ Maximum
$B\left(\frac{9}{4}, \frac{45}{8}\right)$	$Z = 360$ Maximum
$C\left(0, \frac{15}{2}\right)$	$Z = 300$

Hence, either 6 machine of type A and no machine of type B or 2 machine of type A and 6 machine of type B be used to have maximum output.

12. The feasible region (R) is unbounded. Therefore, a minimum of  $Z$  may or may not exist. If it exists, it will be at the corner point Fig.

Corner Point	Value of $Z$
A (12, 0)	$3(12) + 2(0) = 36$
B(4, 2)	$3(4) + 2(2) = 16$
C(1, 5)	$3(1) + 2(5) = 13$ (smallest)
D(0, 10)	$3(0) + 2(10) = 20$



Let us graph  $3x + 2y < 13$ . We see that the open half plane determined by  $3x + 2y < 13$  and  $R$  do not have a common point. So, the smallest value 13 is the minimum value of  $Z$ .

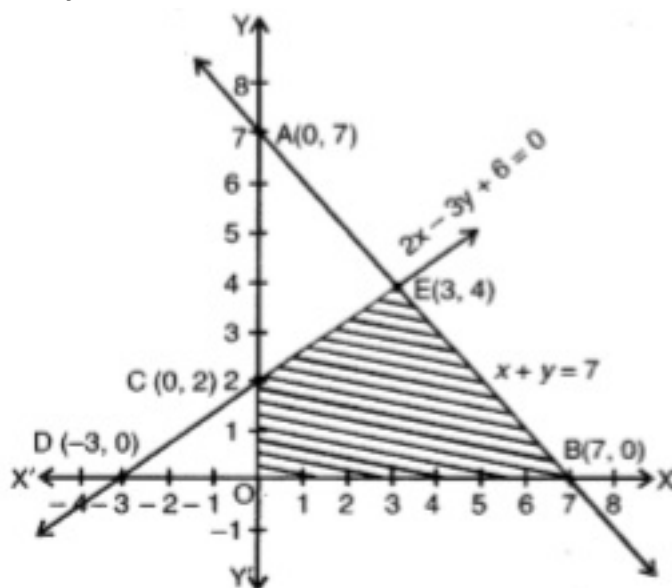
13. Consider  $x + y = 7$

When  $x = 0$ , then  $y = 7$  and

when  $y = 0$ , then  $x = 7$

So,  $A(0, 7)$  and  $B(7, 0)$  are the points on line

$x + y = 7$



Consider  $2x - 3y + 6 = 0$

When  $x = 0$ , then  $y = 2$  and when  $y = 0$ , then  $x = -3$ , So  $C(0, 2)$  and  $D(-3, 0)$  are the points on line  $2x - 3y + 6 = 0$

Also, we have  $x > 0$  and  $y > 0$ .

The feasible region  $OBEC$  is bounded, so, minimum value will obtain at a corner point of this feasible region.

Corner points are  $O(0, 0)$ ,  $B(7, 0)$ ,  $E(3, 4)$  and  $C(0, 2)$

$$Z = 13x - 15y$$

At  $O(0, 0)$ ,  $Z = 0$

At  $B(7, 0)$ ,  $Z = 13(7) - 15(0) = 91$

At  $E(3, 4)$ ,  $Z = 13(3) - 15(4) = -21$

At  $C(0, 2)$ ,  $Z = 13(0) - 15(2)$   
 $= -30$  (minimum)

Hence, the minimum value is -30 at the point (0, 2).

14. Let the manufacturer produce  $x$  units of type A circuit and  $y$  units of type B circuits.

From the given information, we have following corresponding constraint table.

	Type A(x)	Type B(y)	Maximum stock
Resistors	20	10	200
Transistors	10	20	120
Capacitors	10	30	150
Profit	Rs 50	Rs 60	

Thus, we see that total profit  $Z = 50x + 60y$  (in Rs).

Now, we have the following mathematical model for the given problem.

Maximise  $Z = 50x + 60y$  ..... (i)

Subject to the constraints.

$20x + 10y \leq 200$  [resistor constraint]

$\Rightarrow 2x + y \leq 20$ ..... (ii)

And  $10x + 20y \leq 120$  [transistor constraint]

$\Rightarrow x + 2y \leq 12$  .... (iii)

And  $10x + 30y \leq 150$  [capacitor constraint]

$\Rightarrow x + 3y \leq 15$ ..... (iv)

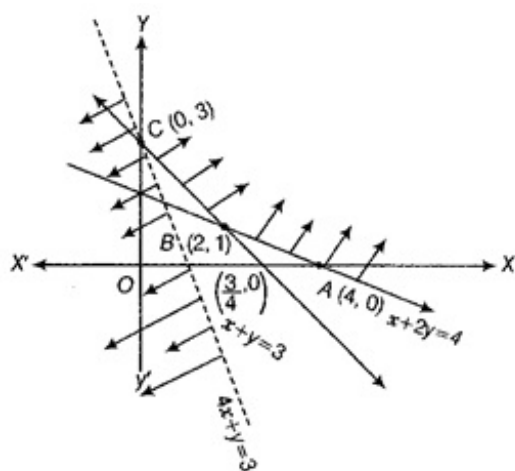
And  $x \geq 0, y \geq 0$  .... (v)  $x \geq 0, y \geq 0$  ....(v) [non-negative constraint]

So, Maximise  $Z = 50x + 60y$ , subject to  $2x + y \leq 20, x + 2y \leq 12, x + 3y \leq 15,$   
 $x \geq 0, y \geq 0$

15. From the shaded region, it is clear that feasible region is unbounded with the corner points A(4, 0), B(2, 1) and C(0, 3).

Also, we have  $Z = 4x + y$ .

[Since,  $x + 2y = 4$  and  $x + y = 3 \Rightarrow y = 1$  and  $x = 2$ ]

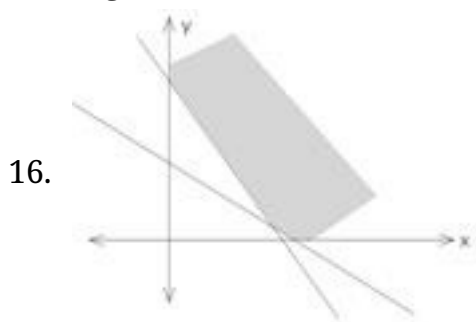


Corner Points	Corresponding value of Z
(4, 0)	16
(2, 1)	9
(0, 3)	3 (minimum)

Now, we see that 3 is the smallest value of Z at the corner point (0, 3). Note that here we see that the region is unbounded, therefore 3 may or may not be the minimum value of Z.

To decide this issue, we graph the inequality  $4x + y < 3$  and check whether the resulting open half plane has no point in common with feasible region otherwise, Z has no minimum value.

From the shown graph above, it is clear that there is no point in common with feasible region and hence Z has minimum value of 3 at (0, 3).



Let the farmer use  $x$  Kg of  $F_1$  and  $y$  Kg of  $F_2$ .

$$Z = 6x + 5y$$

$$\frac{10x}{100} + \frac{5y}{100} \geq 14$$

$$\frac{6x}{100} + \frac{10y}{100} \geq 14$$

$$x \geq 0, y \geq 0$$

On solving these equations we get  $x = 100$  and  $y = 80$

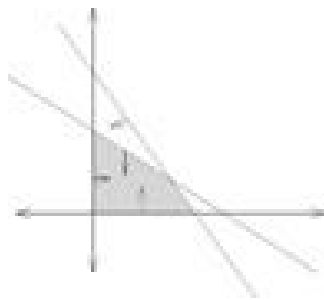
Minimum cost

$$z = 6 \times 100 + 5 \times 80 = 600 + 400 = 1000$$

Thus, minimum cost = Rs 1000.

Hence, 100 kg of fertilizer  $F_1$  and 80 kg of fertilizer  $F_2$  should be used so that nutrient requirements are met at minimum cost of Rs 1000.

17. Let the number of Tennis rackets and the number of cricket bats to be made in a day be  $x$  and  $y$  respectively.



$$Z = x + y$$

$$\text{and also } P = 20x + 10y$$

$$\frac{3}{2}x + 3y \leq 42$$

$$\Rightarrow x + 2y \leq 28$$

$$3x + y \leq 24$$

$$x \geq 0, y \geq 0$$

Solving,  $x + 2y = 28$  and  $3x + y = 24$ , we get,  $x = 4, y = 12$

i. Maximum  $Z = 16$  at  $x = 4, y = 12$

ii.  $P = 20 \times 4 + 10 \times 12$   
 $= 200$

18. Let tailors A and B work for  $x$  and  $y$  days, respectively.

The given data can be written in tabular form as follows:

	Tailor A	Tailor B	Required
Shirts per day	6	10	at least 60
Pants per day	4	4	at least 32
Wages per day	150	200	

$\therefore$  The required linear programming problem is to minimize the wages per day. Let  $Z$  represent the objective function which represent the sum of the wages. Hence the

equation of the objective function is given as  $(Z) = 150x + 200y$

Subject to constraints

$6x + 10y \geq 60$  ( constraints for tailor A) ( dividing throughout by 2 we get)

$$\Rightarrow 3x + 5y \geq 30$$

$4x + 4y \geq 32$  ( constraints for tailor B) ( dividing throughout by 4 we get)

$$\Rightarrow x + y \geq 8$$

and  $x \geq 0, y \geq 0$  ( non negative constraints ,which will restrict the solution of the given inequalities in the first quadrant only)

On considering the inequalities as equations, we get

$$3x + 5y = 30 \dots(i)$$

$$x + y = 8 \dots(ii)$$

Table for line  $3x + 5y = 30$  is

x	0	10
y	6	0

So, it passes through the points with coordinates (0, 6) and (10, 0).

On replacing the coordinates of the origin O (0, 0) in  $3x + 5y \geq 30$  we get

$$0 \geq 30 \text{ [which is false]}$$

So, the half plane for the inequality of the line ( i) is away from the origin, which means that the origin is not a point in the feasible region.

Again, table for line ( ii)  $x + y = 8$  is given below.

x	0	8
y	8	0

So, it passes through the points with coordinates (0, 8) and (8, 0).

On replacing the origin O (0, 0) in  $x + y \geq 8$  , we get

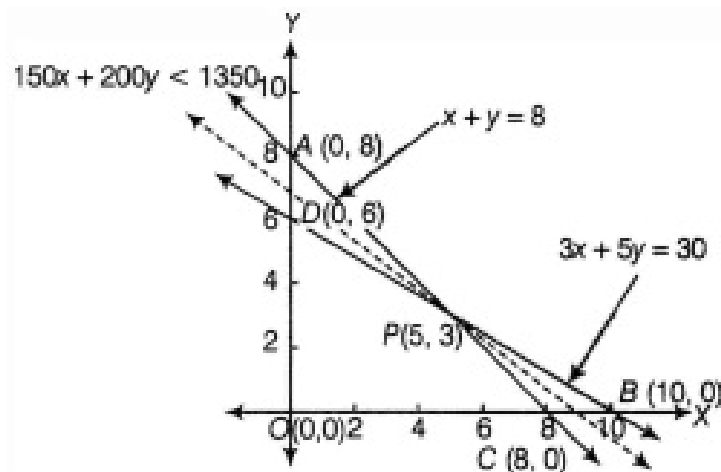
$$0 \geq 8 \text{ (which is false)}$$

So, the half plane for the inequation of the line ( ii) is away from origin, which means that the point O( 0,0) is not a point in the feasible region of the inequality of the line (ii).

On solving Eqs. (i) and (ii), we get

$$x = 5 \text{ and } y = 3$$

so, the point of intersection is P(5, 3).



from the above graph, APB is the feasible region and it is unbounded. The corner points are A(0, 8), P(5, 3) and B(10, 0).

The values of Z at corner points are as follows:

Corner points	Value of $Z = 150x + 200y$
A(0, 8)	$Z = 150(0) + 200(8) = 1600$
P(5, 3)	$Z = 150(5) + 200(3) = 1350$ (minimum)
B(10, 0)	$Z = 150(10) + 200(0) = 1500$

from table, the minimum value of Z is 1350 at P( 5, 3).

As the feasible region is unbounded, therefore 1350 may or may not be the minimum value of Z. For this, we draw a graph of the inequality  $150x + 200y < 1350$  with dotted lines whose feasible region is towards the origin. Hence the feasible region has no point common with the feasible region of the inequality  $150x + 200y < 1350$ .

Hence, The minimum labour cost is Rs. 1350, when tailor A works for 5 days and tailor B works for 3 days.