Waves

Fill in the Blanks

Q.1. A travelling wave has the frequency n and the particle displacement amplitude A. For the wave the particle velocity amplitude is ---------- and the particle acceleration amplitude is ----------. (1983 - 2 Marks)

Ans. A (2πv), A $(2\pi v)^2$

Solution. Since y = A sin (ωt – k x)

Displacement amplitude = A (Max displacement)

Particle velocity, v = dy/dt = A w cos (ωt – k x)

∴ Velocity amplitude = Aω =2πvA

Particle acceleration

$$\text{Acc} = \frac{dv}{dt} = -A\,\omega^2 \sin(\omega t - kx)$$

∴ Acceleration (Max acc) amplitude = $A\omega^2 = 4\pi^2 v^2 A$

Q.2. Sound waves of frequency 660 Hz fall normally on a perfectly reflecting wall. The shortest distance from the wall at which the air particles have maximum amplitude of vibration is ..... metres. (1984- 2 Marks)

Ans. 0.125 m

Solution. $c = v\lambda \qquad \therefore\ \lambda = \frac{c}{v} = \frac{330}{660} = 0.5\text{m}$

The rarefaction will be at a distance of

$$\frac{\lambda}{4} = \frac{0.5}{4} = 0.125\text{m}$$

Q.3. Two simple harmonic motions are represented by the equations

$y_1 = 10 \sin(3\pi t + \pi/4)$ and $y_2 = 5(\sin 3\pi t + \sqrt{3}\cos 3\pi t)$

Their amplitudes are in the ratio of .... (1986 - 2 Marks)

Ans. 1 : 1

Solution.

$y_1 = 10 \sin(3\pi t + \pi/4)$ ... (i)

$y_2 = 5 \sin 3\pi t + 5\sqrt{3} \cos 3\pi t$ ... (ii)

$\therefore \; y_2 = 5 \times 2 \left[\frac{1}{2}\sin 3\pi t + \frac{\sqrt{3}}{2}\cos 3\pi t\right] = 10 \sin(3\pi t + \pi/3)$

The ratio of amplitudes is 10 : 10 = 1 : 1

Q.4. In a sonometer wire, the tension is maintained by suspending a 50.7 kg mass from the free end of the wire. The suspended mass has a volume of 0.0075 m^3 . The fundamental frequency of vibration of the wire is 260 Hz. If the suspended mass is completely submerged in water, the fundamental frequency will become ....Hz. (1987 - 2 Marks)

Ans. 240 Hz

Solution.

$$\frac{\nu_1}{\nu_2} = \frac{\frac{1}{2\ell}\sqrt{\frac{50.7 \times 8}{m}}}{\frac{1}{2\ell}\sqrt{\frac{43.2 \times g}{m}}} \Rightarrow \nu_2 = \nu_1 \sqrt{\frac{43.2}{50.7}} = 260\sqrt{\frac{43.2}{50.7}} = 240 \text{Hz.}$$

Q.5. The amplitude of a wave disturbance propagating in the positive x-direction is given by $y = \frac{1}{(1+x)^2}$ at time t = 0 and by $y = \frac{1}{[1+(x-1)^2]}$ at t = 2 seconds, where x and y arein metres. The shape of the wave disturbance does not change during the propagation. The velocity of the wave is .... m/s. (1990 - 2 Marks)

Ans. 0.5 ms^{-1}

Solution.

As $y = \frac{1}{(1+x)^2}$

At t = 0 and x = 0, we get y = 1.

Also at t = 2 and x = 1, again y = 1

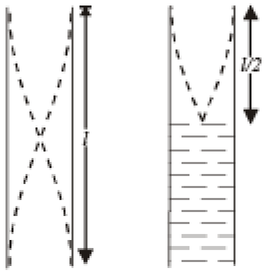
The wave pulse has travelled a distance of 1m in 2 sec.

$$\therefore \quad v = \frac{1}{2} = 0.5\,\text{ms}^{-1}$$

Q.6. A cylinder resonance tube open at both ends has fundamental frequency F in air. Half of the length of the tube is dipped vertically in water. The fundamental frequency to the air column now is .... (1992 - 1 Mark)

Ans. f

Solution.



In figure (i)

$$\frac{\lambda}{2} = \ell \quad \Rightarrow \lambda = 2\,\ell$$

$$\text{and} \quad f = \frac{c}{\lambda} = \frac{c}{2\ell}$$

In figure (ii)

$$\Rightarrow \quad \frac{\lambda'}{4} = \frac{\ell}{2} \Rightarrow \lambda' = 2\ell$$

$$\text{and} \quad f' = \frac{c}{\lambda'} = \frac{c}{2\ell} = f$$

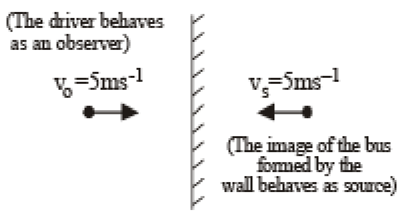
Q.7. A bus is moving towards a huge wall with a velocity of 5 ms^{-1}. The driver sounds a horn of frequency 200 Hz. The frequency of the beats heard by a passenger of the bus will be..... Hz (Speed of sound in air = 342 ms^{-1}) (1994 - 2 Marks)

Ans. 6 Hz

Solution. The first frequency that driver of bus hears is the original frequency of 200 Hz. The second frequency that driver hears is the frequency of sound reflected from the wall. The two frequencies of sound heard by driver is

(a) Original frequency (200 Hz.)

(b) Frequency of sound reflected from the wall (n')



The frequency of sound reflected from the wall

$$\nu' = \nu\left[\frac{v+v_0}{v-v_s}\right] \Rightarrow \nu' = 200\left[\frac{342+5}{342-5}\right] \approx 206\,\text{Hz.}$$

$$\therefore \quad \text{Frequency of beats} = \nu' - \nu = 6\,\text{Hz.}$$

True/False

Q.1. A man stands on the ground at a fixed distance from a siren which emits sound of fixed amplitude. The man hears the sound to be louder on a clear night than on a clear day. (1980)

Ans. F

Solution. The intensity of sound at a given point is the energy per second received by a unit area perpendicular to the direction of propagation.

$$I = \frac{1}{2}\rho V \omega^2 A^2$$

Also intensity varies as distance from the point source as

$$I \propto \frac{1}{r^2}$$

NOTE : None of the parameters are changing in case of a clear night or a clear day.

Therefore the intensity will remain the same.

Q.2. A plane wave of sound travelling in air is incident upon a plane water surface. The angle of incidence is 60°. Assuming snell's law to be valid for sound waves, it follows that the sound wave will be refracted into water away from the normal. (1984- 2 Marks)

Ans. T

Solution. Speed of sound waves in water is greater than in air.

Q.3. A source of sound with frequency 256 Hz is moving with a velocity V towards a wall and an observer is stationary between the source and the wall. When the observer is between the source and the wall he will hear beats (1985 - 3 Marks)

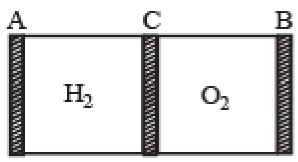
Ans. F

Solution. NOTE : If the sound reaches the observer after being reflected from a stationary surface and the medium is also stationary, the image of the source will become the source of reflected sound.

Thus in both the cases, one sound coming directly from the source and the other coming after reflection will have the same apparent frequency (Since velocity of source w.r.t. observer is same in both the cases). Therefore no beats will be heard.

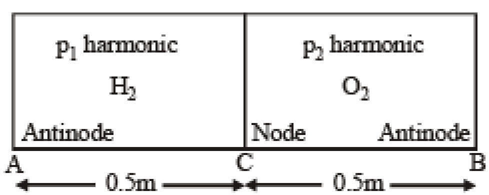
Subjective Questions

Q.1. AB is a cylinder of length 1m fitted with a thin flexible diaphragm C at the middle and other thin flexible diaphragms A and B at the ends. The portions AC and BC contain hydrogen and oxygen gases respectively. The diaphragms A and B are set into vibrations of same frequency. What is the minimum frequency of these vibrations for which diaphragm C is a node? (Under the conditions of experiment vH_2 = 1100 m/s, vO_2 = 300 m/s).



Ans. 1650 Hz

Solution. It is given that C acts as a node. This implies that at A and B antinodes are formed. Again it is given that the frequencies are same.



$$\Rightarrow \quad \frac{v_1}{4\ell} \times p_1 = \frac{v_2}{4\ell} \times p_2 \quad \text{or} \quad \frac{p_1}{p_2} = \frac{v_1}{v_2} = \frac{3}{11}$$

or, $11p_1 = 3p_2$

This means that the third harmonic in AC is equal to 11th harmonic in CB.

Now, the fundamental frequency in AC

$$= \frac{v_1}{4\ell} = \frac{1100}{4 \times 0.5} = 550\,\text{Hz}$$

and the fundamental frequency in CB

$$= \frac{v_2}{4\ell} = \frac{300}{4 \times 0.5} = 550\,\text{Hz}$$

∴Frequency in AC = 3 × 550 = 1650

Hz and frequency in CB = 11 × 150 = 1650 Hz

Q.2. A copper wire is held at the two ends by rigid supports. At 30°C, the wire is just taut, with negligible tension. Find the speed of transverse waves in this wire at 10°C.

Given : Young modulus of copper = 1.3×10^{11} N/m2 .

Coefficient of linear expansion of copper = $1.7 \times 10^{-5}\ {}^{o}C^{-1}$.

Density of copper = 9×10^{3} kg/m^{3}. (1979)

Ans. 70 m/s

Solution. Using the formula of the coefficient of linear expansion of wire, $\Delta\ell = \ell\alpha\Delta\theta$ we get

$F = YA\alpha\Delta\theta$

Speed of transverse wave is given by

$$v = \sqrt{\frac{F}{m}} \quad \left[\text{where } m = \text{mass per unit length} = \frac{Al\rho}{\ell} = A\rho\right]$$

$$= \sqrt{\frac{YA\alpha\Delta\theta}{A\rho}} = \sqrt{\frac{Y\alpha\Delta\theta}{\rho}}$$

$$= \sqrt{\frac{1.3\times10^{11}\times1.7\times10^{-5}\times20}{9\times10^{3}}} = 70 \text{ m/s}$$

Q.3. A tube of a certain diameter and of length 48 cm is open at both ends. Its fundamental frequency of resonance is found to be 320 Hz. The velocity of sound in air is 320 m/sec.

Estimate the diameter of the tube.

One end of the tube is now closed. Calculate the lowest frequency of resonance for the tube.

Ans. 3.33 cm; 163 Hz

Solution. Tube open at both ends :

$$(a)\quad \nu = \frac{v}{2(\ell + 0.6D)} \quad \therefore\ 320 = \frac{320}{2(0.48 + 0.6 \times D)}$$

$$0.48 + 0.6D = 0.5 \Rightarrow 0.6D = 0.02$$

$$\Rightarrow D = \frac{0.02}{60} \times 100 \text{ cm} = 3.33 \text{ cm}$$

Tube closed at one end :

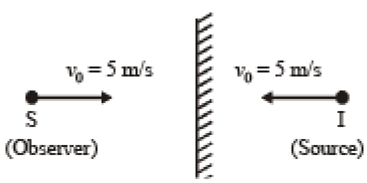
$$\nu = \frac{v}{4(\ell + 0.3D)} = \frac{320}{4(0.48 + 0.3 \times 0.033)}$$

$$\approx 163 \text{ Hz}$$

Q.4. A source of sound of frequency 256 Hz is moving rapidly towards wall with a velocity of 5 m/sec. How many beats per second will be heard if sound travels at a speed of 330 m/sec? (1981 - 4 Marks)

Ans. 8

Solution.



NOTE : If the sound reaches the observer after being reflected from a stationary surface and the medium is also stationary, the image of the source in the reflecting surface will become the source of the reflected sound.

$$\nu' = \nu\left[\frac{c - v_0}{c - v_s}\right]$$

v_0, v_s are + ve if they are directed from source to the observer and – ve if they are directed from observer to source.

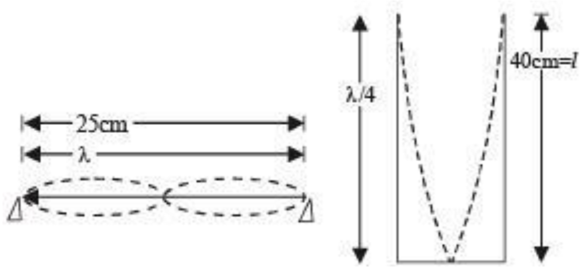
$$\nu' = 256\left[\frac{330 - (-5)}{330 - 5}\right] = 264 \text{ Hz}$$

∴ Beat frequency = 264 – 256 = 8

Q.5. A string 25 cm long and having a mass of 2.5 gm is under tension. A pipe closed at one end is 40 cm long. When the string is set vibrating in its first overtone and the air in the pipe in its fundamental frequency, 8 beats per second are heard. It is observed that decreasing the tension in the string decreases beat frequency. If the speed of sound in air is 320 m/s, find the tension in the string. (1982 - 7 Marks)

Ans. 27.04 N

Solution.



First Overtone

Mass of string per unit length $= \frac{2.5 \times 10^{-3}}{0.25} = 0.01$ kg/m

$\therefore$ Frequency, $\nu_s = \frac{1}{\lambda}\sqrt{\frac{T}{m}} = \frac{1}{0.25}\sqrt{\frac{T}{0.01}}$ ... (i)

Fundamental frequency

$\therefore \quad \frac{\lambda}{4} = 0.4 \Rightarrow \lambda = 1.6\,\text{m}$

$\therefore \quad \nu_T = \frac{c}{\lambda_T} = \frac{320}{1.6} = 200\,\text{Hz}$ ... (ii)

Given that 8 beats/second are heard. The beat frequency decreases with the decreasing tension. This means that beat frequency decreases with decreasing v_s. So beat frequency is given by the expression.

$v = v_s - v_T$

$$\therefore \quad 8 = \frac{1}{0.25}\sqrt{\frac{T}{0.01}} - 200 \quad \Rightarrow \quad T = 27.04\,N$$

Q.6. A uniform rope of length 12 m and mass 6 kg hangs vertically from a rigid support. A block of mass 2 kg is attached to the free end of the rope. A transverse pulse of wavelength 0.06 m is produced at the lower end of the rope. What is the wavelength of the pulse when it reaches the top of the rope? (1984 - 6 Marks)

Ans. 0.75 m/s

Solution. KEY CONCEPT : The velocity of wave on the string is given by the formula

$$v = \sqrt{\frac{T}{m}}$$

where T is the tension and m is the mass per unit length.

Since the tension in the string will increase as we move up the string (as the string has mass), therefore the velocity of wave will also increase. (m is the same as the rope is uniform)

$$\therefore \quad \frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}} = \sqrt{\frac{2 \times 9.8}{8 \times 9.8}} = \frac{1}{2} \quad \therefore \quad v_2 = 2v_1$$

Since frequency remains the same

$\therefore \lambda_2 = 2\lambda_1 = 2 \times 0.06 = 0.12$ m

Q.7. A steel wire of length 1 m, m ass 0.1 kg and uniform cross-sectional area 10^{-6} m^2 is rigidly fixed at both ends.

The temperature of the wire is lowered by 20°C. If transverse waves are set up by plucking the string in the middle, calculate the frequency of the fundamental mode of vibration.

Given for steel Y = 2×10^{11} N/m^2

$\alpha = 1.21 \times 10^{-5}$ per ^{o}C (1984 - 6 Marks)

Ans. 11 Hz

Solution. KEY CONCEPT : Using the formula of the coefficient of linear expansion,

$$\Delta\ell = \ell\alpha \times \Delta\theta$$

$$\text{Also,} \quad Y = \frac{\text{stress}}{\text{strain}} = \frac{T/A}{\Delta\ell/\ell} = \frac{T/A}{\alpha A\theta} \quad \therefore T = YA\,\alpha\,\Delta\theta$$

The frequency of the fundamental mode of vibration.

$$\nu = \frac{1}{2\ell}\sqrt{\frac{T}{m}} = \frac{1}{2\ell}\sqrt{\frac{YA\,\alpha\,\Delta\theta}{m}}$$

$$= \frac{1}{2\times 1}\sqrt{\frac{2\times 10^{11}\times 10^{-6}\times 1.21\times 10^{-5}\times 20}{0.1}} = 11\text{ Hz}$$

Q.8. The vibrations of a string of length 60 cm fixed at both ends are represented by the equation—

$$y = 4\sin\left(\frac{\pi x}{15}\right)\cos(96\,\pi t)$$

(1985 - 6 Marks)

Where x and y are in cm and t in seconds.

(i) What is the maximum displacement of a point at x = 5 cm?

(ii) Where are the nodes located along the string?

(iii) What is the velocity of the particle at x = 7.5 cm at t = 0.25 sec.?

(iv) Write down the equations of the component waves whose superposition gives the above wave

Ans. (i) 3.46 cm

(ii) 0, 15, 30

(iii) zero

(iv) $2\sin\left(96\pi t+\frac{\pi x}{15}\right)$ and $-2\sin\left(96\pi t-\frac{\pi x}{15}\right)$

Solution. (i) Here amplitude, A = 4 sin $\left(\frac{\pi x}{15}\right)$

At x = 5 m

$$A = 4\sin\left(\frac{\pi\times 5}{15}\right) = 4\times 0.866 = 3.46\,\text{cm}$$

(ii) Nodes are the position where A = 0

$$\therefore \quad \sin\left(\frac{\pi x}{15}\right) = 0 = \sin n\pi \quad \therefore \quad x = 15\,\text{n}$$

where n = 0, 1, 2 x = 15 cm, 30 cm, 60 cm, ...

(iii) $y = 4\sin\left(\frac{\pi x}{15}\right)\cos(96\pi t)$

$$v = \frac{dy}{dt} = 4\sin\left(\frac{\pi x}{15}\right)\,[-96\pi\sin(96\,\pi t)]$$

At x = 7.5 cm, t = 0.25 cm

$$v = 4\sin\left(\frac{\pi\times 7.5}{15}\right)\,[-96\pi\sin(96\,\pi\times 0.25)]$$

$$= 4\sin\left(\frac{\pi}{2}\right)[-96\,\pi\sin(24\pi)] = 0$$

(iv) $y = 4\sin\left(\frac{\pi x}{15}\right)\cos[96\,\pi t]$

$$= 2\left[2\sin\left(\frac{\pi x}{15}\right)\cos(96\pi t)\right]$$

$$= 2\left[\sin\left(96\,\pi t+\frac{\pi x}{15}\right)-\sin\left(96\,\pi t-\frac{\pi x}{15}\right)\right]$$

$$= 2\sin\left(96\,\pi t+\frac{\pi x}{15}\right)-2\sin\left(96\,\pi t-\frac{\pi x}{15}\right)$$

$= y_1 + y_2$

where $y_1 = 2\sin\left(96\pi t + \frac{\pi x}{15}\right)$

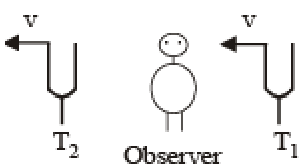
and $y_2 = -2\sin\left(96\pi t - \frac{\pi x}{15}\right)$

Q.9. Two tuning forks with natural frequencies of 340 Hz each move relative to a stationary observer. One fork moves away from the observer, while the other moves towards him at the same speed. The observer hears beats of frequency 3 Hz.

Find the speed of the tuning fork. (1986 - 8 Marks)

Ans. 1.5 m/s

Solution. The apparent frequency from tuning fork T_1 as heard by the observer will be



$$\nu_1 = \frac{c}{c-v} \times \nu \qquad \text{...(i)}$$

where c = velocity of sound

v = velocity of turning fork The apparent frequency from tuning fork T2 as heard by the observer will be

$$\nu_2 = \frac{c}{c+v} \times \nu \qquad \text{...(ii)}$$

Given $\nu_1 - \nu_2 = 3$

$$\therefore \quad c \times \nu \left[\frac{1}{c-v} - \frac{1}{c+v}\right] = 3 \text{ or, } 3 = \frac{c \times \nu \times 2v}{c^2 - v^2}$$

$$\text{Since, } v << c \qquad \therefore 3 = \frac{c \times \nu \times 2v}{c^2}$$

$\therefore \quad v = \frac{3 \times 340 \times 340}{340 \times 340 \times 2} = 1.5\,m/s$

Q.10. The following equations represent transverse waves :

$z_1 = A \cos(kx - \omega t)$; (1987 - 7 Marks)

$z_2 = A \cos(kx + \omega t)$; $z_3 = A \cos(ky - \omega t)$

Identify the combination (s) of the waves which will produce (i) standing wave (s), (ii) a wave travelling in the directon making an angle of 45° degrees with the positive x and positive y axes. In each case, find the positions at which the resultant intensity is always zero.

Ans.

(i) z_1 **and** z_2; $\frac{(2n+1)\pi}{2K}$ where n = 0, 1, 2, ...

(ii) z_1 **and** z_3; $\frac{(2n+1)\pi}{K}$

Solution. (i) KEY CONCEPT : When two progressive waves having same amplitude and period, but travelling in opposite direction with same velocity superimpose, we get standing waves.

The following two equations qualify the above criteria and hence produce standing wave

$z_1 = A \cos(k x - \omega t)$

$z_2 = A \cos(k x + \omega t)$

The resultant wave is given by $z = z_1 + z_2$

$\Rightarrow z = A \cos(k x - \omega t) + A \cos(k x + \omega t)$

$= 2A \cos k x \cos \omega t$

The resultant intensity will be zero when

$2A \cos k x = 0$

$$\Rightarrow \quad \cos k x = \cos \frac{(2n+1)}{2}\pi$$

$$\Rightarrow \quad k x = \frac{2n+1}{2}\pi \quad \Rightarrow x = \frac{(2n+1)\pi}{2k}$$

where n = 0, 1, 2, ...

(ii) The transverse waves

$z_1 = A \cos (k x - \omega t)$

$z_3 = A \cos (k y - \omega t)$

Combine to produce a wave travelling in the direction making an angle of 45° with the positive x and positive y axes.

The resultant wave is given by $z = z_1 + z_3$

$z = A \cos (k x - \omega t) + A \cos (ky - \omega t)$

$$\Rightarrow \quad z = 2A \cos \frac{(x-y)}{2} \cos\left[\frac{k(x+y) - 2\omega t}{2}\right]$$

The resultant intensity will be zero when

$$2A \cos \frac{k(x-y)}{2} = 0 \quad \Rightarrow \quad \cos \frac{k(x-y)}{2} = 0$$

$$\Rightarrow \quad \frac{k(x-y)}{2} = \frac{2n+1}{2}\pi \quad \Rightarrow \quad (x-y) = \frac{(2n+1)}{k}\pi$$

Q.11. A train approaching a hill at a speed of 40 km/hr sounds a whistle of frequency 580 Hz when it is at a distance of 1 km from a hill. A wind with a speed of 40 km/hr is blowing in the direction of motion of the train Find (1988 - 5 Marks)

(i) the Frequency of the whistle as heard by an observer on the hill,

(ii) the distance from the hill at which the echo from the hill is heard by the

driver and its frequency.

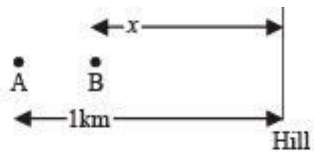
(Velocity of sound in air =1,200 km/hr)

Ans. (i) 599 Hz

(ii) 0.935 km, 621 Hz

Solution. (i) The frequency of the whistle as heard by observer on the hill

$$n' = n\left[\frac{v+v_m}{v+v_m-v_s}\right]$$



$$= 580\left[\frac{1200+40}{1200+40-40}\right] = 599 \text{ Hz}$$

(ii) Let echo from the hill is heard by the driver at B which is at a distance x from the hill.

The time taken by the driver to reach from A to B

$$t_1 = \frac{1-x}{40} \qquad \text{... (i)}$$

The time taken by the echo to reach from hill

$$t_2 = t_{AH} + t_{HB}$$

$$t_2 = \frac{1}{(1200+40)} + \frac{x}{(1200-40)} \qquad \text{... (ii)}$$

where t_{AH} = time taken by sound from A to H with velocity (1200 + 40)

t_{HB} = time taken by sound from H to B with velocity 1200 – 40

From (i) and (ii)

$$t_1 = t_2 \Rightarrow \frac{1-x}{40} = \frac{1}{1200+40} + \frac{x}{1200-40}$$

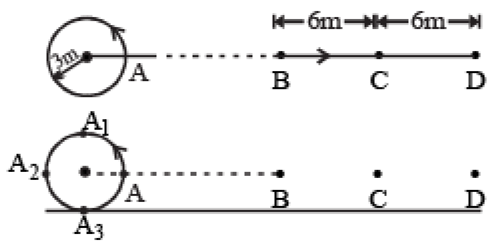
⇒ x = 0.935 km

The frequency of echo as heard by the driver can be calculated by considering that the source is the acoustic image.

$$n'' = n\left[\frac{(v - v_m) + v_s}{(v - v_m) - v_0}\right]$$

$$= 580\left[\frac{(1200 - 40) + 40}{(1200 - 40) - 40}\right] = 621\,\text{Hz}$$

Q.12. A source of sound is moving along a circular orbit of radius 3 metres with an angular velocity of 10 rad/s. A sound detector located far away from the source is executing linear simple harmonic motion along the line BD with an amplitude BC = CD = 6 metres. The frequency of oscillation of the detector is 5/π per second. The source is at the point A whenthe detector is at the point B. If the source emits a continous sound wave of frequency 340 Hz, find the maximum and the minimum frequencies recorded by the detector. (1990 - 7 Mark)

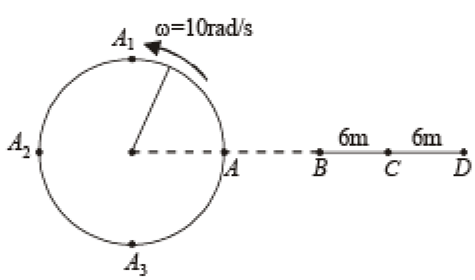


Ans. 438.7 Hz, 257.3 Hz

Solution. The angular frequency of the detector = 2πv

$$= 2\pi \times \frac{5}{\pi} = 10\,\text{rad/s}$$

The angular frequency of the detector matches with that of the source.



⇒ When the detector is at C moving towards D, the source is at A_1 moving leftwards. It is in this situation that the frequency heard is minimum

$$v' = v\left[\frac{v - v_0}{v + v_s}\right] = 340 \times \frac{(340-60)}{(340+30)} = 257.3\,\text{Hz}$$

Again when the detector is at C moving towards B, the source is at A_3 moving rightward. It is in this situation that the frequency heard is maximum.

$$v'' = v\left[\frac{v + v_0}{v - v_s}\right] = 340 \times \frac{(340+60)}{(340-30)} = 438.7\,\text{Hz}$$

Q.13. The displacement of the medium in a sound wave is given by the equation $y_1 = A \cos (ax + bt)$ where A, a and b are positive constants. The wave is reflected by an obstacle situated at x = 0. The intensity of the reflected wave is 0.64 times that of the incident wave. (1991 - 4 × 2 Marks)

(a) What are the wavelength and frequency of incident wave?

(b) Write the equation for the reflected wave.

(c) In the resultant wave formed after reflection, find the maximum and minimum values of the particle speeds in the medium.

(d) Express the resultant wave as a superposition of a standing wave and a travelling wave. What are the positions of the antinodes of the standing wave ?

What is the direction of propagation of travelling wave?

Ans. **(a)** $\frac{2\pi}{a}, \frac{b}{2\pi}$

(b) $y = -0.8A \cos(ax - bt)$

(c) 1.8 Ab, 0

(d) $y = -1.6 A \sin ax \sin bt + 0.2 A \cos(ax + bt)$ $\left[n + \frac{(-1)^n}{2}\right]\frac{\pi}{a}$, –X direction

Solution. (a) KEY CONCEPT : Use the equation of a plan e progressive wave which is as follows.

$$y = A\cos\left(\frac{2\pi}{\lambda}x + 2\pi\nu t\right)$$

The given equation is

$y_1 = A \cos(ax + bt)$

On comparing, we get $\frac{2\pi}{\lambda} = a \Rightarrow \lambda = \frac{2\pi}{a}$

Also, $2\pi v = b$

$$\Rightarrow \nu = \frac{b}{2\pi}$$

(b) Since the wave is reflected by an obstacle, it will suffer a phase difference of p. The intensity of the reflected wave is 0.64 times of the incident wave.

Intensity of original wave $I \propto A^2$

Intensity of reflected wave $I' = 0.64\, I$

$\Rightarrow I' \propto A'^2 \Rightarrow 0.64\, I \propto A'^2$

$\Rightarrow 0.64\, A^2 \propto A'^2 \Rightarrow A' \propto 0.8A$

So the equation of resultant wave becomes

$y^2 = 0.8A \cos(ax - bt + \pi) = -0.8\, A \cos(ax - bt)$

(c) KEY CONCEPT : The resultant wave equation can be found by superposition principle

$y = y_1 + y_2$

$= A \cos(ax + bt) + [-0.8\, A \cos(ax - bt)]$

The particle velocity can be found by differentiating the above equation

$$v = \frac{dy}{dt} = -Ab\sin(ax + bt) - 0.8\,Ab\sin(ax - bt)$$

$= – Ab [\sin (ax + bt) + 0.8 \sin (ax – bt)]$

$= – Ab [\sin ax \cos bt + \cos ax \sin bt + 0.8 \sin ax \cos bt – 0.8 \cos ax \sin bt]$

$v = – Ab [1.8 \sin ax \cos bt + 0.2 \cos ax \sin bt]$

The maximum velocity will occur when sin ax = 1 and cos bt = 1 under these condition cos ax = 0 and sin bt = 0

$\therefore \quad |v_{max}| = 1.8\,Ab$

Also, $|v_{min}| = 0$

(d) $y = [A \cos (ax + bt)] – [0.8 A \cos (ax – bt)]$

$= [0.8 A \cos (ax + bt) + 0.2 A \cos (ax + bt)] – [0.8 A \cos (ax – bt)]$

$= [0.8 A \cos (ax + bt) – 0.8 A \cos (ax – bt)] + 0.2 A \cos (ax + bt)]$

$$= 0.8\,A\left[-2\ \sin\left\{\frac{(ax+bt)+(ax-bt)}{2}\right\}\quad \sin\left\{\frac{(ax+bt)-(ax-bt)}{2}\right\}\right]\ 0.2\,A\cos(ax+bt)]$$

$\Rightarrow$ y = – 1.6 A sin ax sin bt + 0.2 A cos (ax + bt) where (– 1.6 A sin ax sin bt) is the equation of a standing wave and 0.2 A cos (ax + bt) is the equation of travelling wave.

The wave is travelling in –x direction.

NOTE : Antinodes of the standing waves are the positions where the amplitude is maximum,

$$\text{i.e.,}\quad \sin ax = 1 = \sin\left[n\pi + (-1)^n\frac{\pi}{2}\right]$$

$$\Rightarrow\quad x = \left[n + \frac{(-1)^n}{2}\right]\frac{\pi}{a}$$

Q.14. Two radio stations broadcast their programmes at the same amplitude A and at slightly different frequencies ω1 and ω2 respectively, where ω1 - ω2 = 10^3 Hz A detector receives the signals from the two stations simultaneously.

It can only detect signals of intensity $\geq$ 2 A^2. (1993 - 4 Marks)

(i) Find the time interval between successive maxima of the intensity of the signal received by the detector.

(ii) Find the time for which the detector remains idle in each cycle of the intensity of the signal.

Ans. (i) $\frac{2\pi}{10^3}$ sec. (ii) $\frac{\pi}{2}\times 10^{-3}$ sec

Solution. Let the two radio waves be represented by the equations

$y_1 = A \sin 2\pi v_1 t$

$y_2 = A \sin 2\pi v_2 t$

The equation of resultant wave according to superposition principle

$y = y_1 + y_2 = A \sin 2\pi v_1 t + A \sin 2\pi v_2 t$

$= A [\sin 2 \pi v_1 t + \sin 2\pi v_2 t]$

$$= A \times 2 \sin \frac{(2\pi v_1 + 2\pi v_2)t}{2} \cos \frac{(2\pi v_1 + 2\pi v_2)t}{2}$$
$$= 2A \sin \pi (v_1 + v_2)\, t \cos \pi (v_1 - v_2)\, t$$

where the amplitude $A' = 2A \cos \pi (v_1 - v_2) t$

Now, intensity $\propto$ (Amplitude)^2

$\Rightarrow I \propto A'^2$

$I \propto 4A^2 \cos^2\pi (v_1 - v_2) t$

The intensity will be maximum when

$\cos^2\pi (v_1 - v_2) t = 1$

or, $\cos \pi (v_1 - v_2) t = 1$

or, $\pi (v_1 - v_2) t = n\pi$

$$\Rightarrow \quad \frac{(\omega_1-\omega_2)}{2}t = n\pi \quad \text{or,} \quad t = \frac{2n\pi}{\omega_1-\omega_2}$$

∴ Time interval between two maxima

$$\text{or,} \quad \frac{2n\pi}{\omega_1-\omega_2} - \frac{2(n-1)\pi}{\omega_1-\omega_2} \quad \text{or,} \quad \frac{2\pi}{\omega_1-\omega_2} = \frac{2\pi}{10^3}\text{sec}$$

Time interval between two successive maximas is $2\pi \times 10^{-3}$ sec

(ii) For the detector to sense the radio waves, the resultant intensity $\geq 2A^2$

$$\therefore \quad \text{Resultant amplitude} \geq \sqrt{2}A$$

$$\text{or,} \quad 2A\cos\pi(\nu_1-\nu_2)t \geq \sqrt{2A}$$

$$\text{or,} \quad \cos\pi(\nu_1-\nu_2)t \geq \frac{1}{\sqrt{2}} \quad \text{or,} \quad \cos\left[\frac{(\omega_1-\omega_2)t}{2}\right] \geq \frac{1}{\sqrt{2}}$$

The detector lies idle when the values of cos $\left[\frac{(\omega_1-\omega_2)t}{2}\right]$ is between 0 and 1/√2

$$\therefore \quad t_1 = \frac{\pi}{\omega_1-\omega_2} \text{ and } t_2 = \frac{\pi}{2(\omega_1-\omega_2)}$$

$$\therefore \quad \text{The time gap } = t_1 - t_2$$

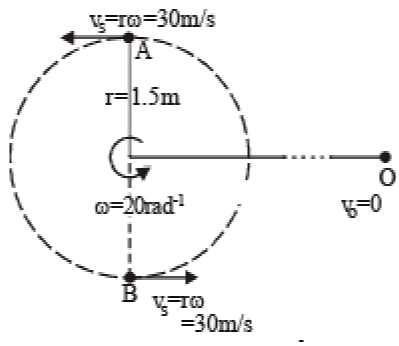
$$= \frac{\pi}{\omega_1-\omega_2} - \frac{\pi}{2(\omega_1-\omega_2)} = \frac{\pi}{2(\omega_1-\omega_2)}$$

$$= \frac{\pi}{2} \times 10^{-3} \text{ sec}$$

Q.15. A whistle emitting a sound of frequency 440 Hz is tied to a string of 1.5m length and rotated with an angular velocity of 20 rad s^{-1} in the horizontal plane. Calculate the range of frequencies heard by an observer stationed at a large distance from the whistle. (1996 - 3 Marks)

Ans. 403.3 Hz to 484 Hz

Solution. The whistle which is emitting sound is being rotated in a circle.



r = 1.5 m (given); ω = 20 rads–1 (given)

We know that

$v = r\omega = 1.5 \times 20 = 30\ ms^{-1}$

When the source is instantaneously at the position A, then the frequency heard by the observer will be

$$\nu' = \nu\left[\frac{v}{v - v_s}\right] = 440\left[\frac{330}{330-30}\right] = 484\,Hz$$

When the source is instantaneously at the position B, then the frequency heard by the observer will be

$$\nu'' = \nu\left[\frac{v}{v + v_s}\right] = 440\left[\frac{330}{330+30}\right] = 403.3\,Hz$$

Hence the range of frequencies heard by the observer is 403.3 Hz to 484 Hz.

Q.16. A band playing music at a frequency f is moving towards a wall at a speed v_b. A motorist is following the band with a speed v_m. If v is the speed of sound, obtain an expression for the beat frequency heard by the motorist. (1997 – 5 Marks)

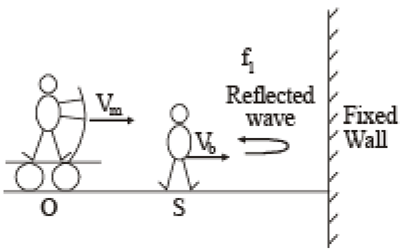
Ans. $\frac{(v+v_m)\times 2v_b f}{v^2 - v_b^2}$

Solution. KEY CONCEPT : Motorist will listen two sound waves.

One directly from the sound source and other reflected from the fixed wall. Let the apparent frequencies of these two waves as received by motorist are f 'and f " respectively.

For Direct Sound : V_m will be positive as it moves towards the source and tries to increase the apparent frequency. V_b will be taken positive as it move away from the observer and hence tries to decrease the apparent frequency value.

$$f' = \frac{v+v_m}{v+v_b} f \qquad \text{...(1)}$$



For reflected sound :

For sound waves moving towards stationary observer (i.e. wall), frequency of sound as heard by wall

$$f_1 = \frac{v}{v-v_b} f$$

After reflection of sound waves having frequency f1 fixed wall acts as a stationary source of frequency f1 for the moving observer i.e. motorist. As direction of motion of motorist is opposite to direction of sound waves, hence frequency f " of reflected sound waves as received by the motorist is

$$f'' = \frac{v+v_m}{v} f_1 = \frac{v+v_m}{v-v_b} f \qquad \text{...(2)}$$

Hence, beat frequency as heard by the motorist

$$\Delta f = f'' - f' = \left(\frac{v+v_m}{v-v_b} - \frac{v+v_m}{v+v_b}\right) f$$

$$\text{or,} \quad \Delta f = \frac{2v_b(v+v_m)f}{v^2 - v_b^{\,2}}$$

Q.17. The air column in a pipe closed at one end is made to vibrate in its second overtone by a tuning fork of frequency 440 Hz. The speed of sound in air is 330 m s^{-1}. End corrections may be neglected. Let P_0 denote the mean pressure at any point in the pipe, and ΔP_0 the maximum amplitude of pressure variation.

(a) Find the length L of the air column. (1998 - 8 Marks)

(b) What is the amplitude of pressure variation at the middle of the column? (c) What are the maximum and minimum pressures at the open end of the pipe?

(d) What are the maximum and minimum pressures at the closed end of the pipe?

Ans. (a) $\frac{15}{16}$m (b) $\frac{\Delta P_0}{\sqrt{2}}$

(c) equal to mean pressure

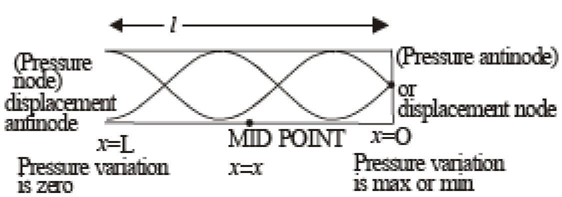
(d) $P_0 + \Delta P_0$, $P_0 - \Delta P_0$

Solution. (a) For second overtone as shown,

$$\frac{5\lambda}{4} = \ell \quad \therefore \quad \lambda = \frac{4\ell}{5}$$

Also, $v = \nu\lambda$

$$\Rightarrow \quad 330 = 440 \times \frac{4\ell}{5} \quad \Rightarrow \quad \ell = \frac{15}{16} \text{m.}$$



(b) KEY CONCEPT : At any position x, the pressure is given by

$\Delta P = \Delta P_0 \cos kx \cos \omega t$

Here amplitude $A = \Delta P_0 \cos kx = \Delta P_0 \cos \frac{2\pi}{\lambda} x$

For $x = \frac{15}{2 \times 16} = \frac{15}{32}$ m (mid point)

$$\text{Amplitude} = \Delta P_0 \cos \left[\frac{2\pi}{(330/440)} \times \frac{15}{32} \right] = \frac{\Delta P_0}{\sqrt{2}}$$

(c) At open end of pipe, pressure is always same i.e. equal to mean pressure

$\therefore \Delta P = 0, P_{max} = P_{min} = P_0$

(d) At the closed end :

Maximum Pressure = $P_0 + \Delta P_0$

Minimum Pressure = $P_0 - \Delta P_0$

Q.18. A long wire PQR is made by joining two wires PQ and QR of equal radii PQ has length 4.8 m and mass 0.06 kg. QR has length 2.56 m and mass 0.2 kg. The wire PQR is under a tension of 80 N. A sinusoidal wave-pulse of amplitude 3.5 cm is sent along the wire PQ from the end P. No power is dissipated during the propagation of the wave-pulse.

Calculate. (1999 - 10 Marks)

(a) the time taken by the wave-pulse to reach the other end R of the wire, and
(b) the amplitude of the reflected and tran smitted wave-pulses after the incident wave-pulse crosses the joint Q.

Ans. (a) 0.14s

(b) 2.0 cm, 1.5 cm

Solution. Mass per unit length of PQ

$m_1 = \frac{0.06}{4.8}$ kg/m

Mass per unit length of QR, $m_2 = \frac{0.2}{2.56}$ kg / m

Velocity of wave in PQ is

$$v_1 = \sqrt{\frac{T}{m_1}} = \sqrt{\frac{80}{0.06/4.8}} = 80\,ms^{-1} [\because T = 80\text{ N given}]$$

Velocity of wave in QR is

$$v_2 = \sqrt{\frac{T}{m_2}} = \sqrt{\frac{80}{0.2/2.56}} = 32\text{ m/s}$$

∴ Time taken for the wave to reach from P to R

$$= t_{PQ} + t_{QR}$$
$$= \frac{4.8}{80} + \frac{2.56}{32} = 0.14\text{ s}$$

(b) When the wave which initiates from P reaches Q (a denser medium) then it is partly reflected and partly transmitted.

In this case the amplitude of reflected wave

$$A_r = \left(\frac{v_2 - v_1}{v_2 + v_1}\right) A_i \qquad \text{...(i)}$$

where A_i = amplitude of incident wave.

Also amplitude of transmitted wave is

$$A_t = \left(\frac{2v_2}{v_1 + v_2}\right) A_i \qquad \text{...(ii)}$$

From (i), (ii)

Therefore, A_t = 2 cm and A_r = – 1.5 cm.

Q.19. A 3.6 m lon g vertical pipe resonates with a source of frequency 212.5 Hz when water level is at certain height in the pipe. Find the height of water level (from the bottom of the pipe) at which resonance occurs. Neglect end correction.

Now, the pipe is filled to a height H (≈ 3.6 m). A small hole is drilled very close to its bottom and water is allowed to leak.

Obtain an expression for the rate of fall of water level in the pipe as a function of H. If the radii of the pipe and the hole are 2×10^{-2} m and 1×10^{-3} m respectively, calculate the time interval between the occurance of first two resonances.
Speed of sound in air is 340 m/s and g = 10 m/s^2. (2000 - 10 Marks)

Ans. $\frac{-dH}{dt} = (1.11 \times 10^{-2})\sqrt{H}$, 43 sec.

Solution. Speed of sound, v = 340 m/s.

Let ℓ_0 be the length of air column corresponding to the fundamental frequency. Then

$$\frac{v}{4\ell_0} = 212.5$$

or $$\ell_0 = \frac{v}{4(212.5)} = \frac{340}{4(212.5)} = 0.4\,m.$$

NOTE : In closed pipe only odd harmonics are obtained.

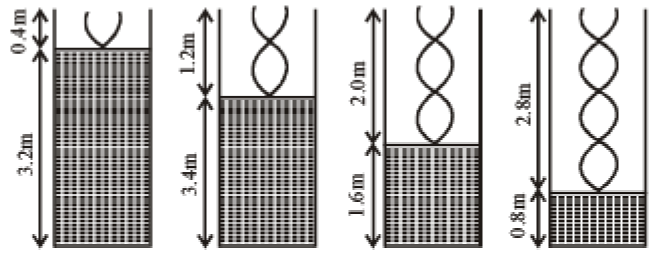
Now, let ℓ_1, ℓ_2, ℓ_3 ,ℓ_4, etc. be the lengths corresponding to the 3rd harmonic, 5th harmonic, 7th harmonic etc. Then

$$3\left(\frac{v}{4\ell_1}\right) = 212.5 \Rightarrow \ell_1 = 1.2\,m;$$

$$5\left(\frac{v}{4\ell_2}\right) = 212.5 \Rightarrow \ell_2 = 2.0\,m$$

$$7\left(\frac{v}{4\ell_3}\right) = 212.5 \Rightarrow \ell_3 = 2.8\,m;$$

$$9\left(\frac{v}{4\ell_4}\right) = 212.5 \Rightarrow \ell_4 = 3.6\,m$$



or heights of water level are (3.6 – 0.4) m, (3.6 – 1.2) m, (3.6 – 2.0) m and (3.6 – 2.8) m.

Therefore heights of water level are 3.2 m, 2.4 m, 1.6 m and 0.8 m.

Let A and a be the area of cross-sections of the pipe and hole respectively. Then

$A = \pi (2 \times 10^{-2})2 = 1.26 \times 10^{-3}\ m^{-2}$ and $a = \pi (10^{-3})^2 = 3.14 \times 10^{-6}\ m^2$

Velocity of efflux, $v = \sqrt{2gH}$

Continuity equation at 1 and 2 gives,

$$a\sqrt{2gH} = A\left(\frac{-dH}{dt}\right)$$

Therefore, rate of fall of water level in the pipe,

$$\left(\frac{-dH}{dt}\right) = \frac{a}{A}\sqrt{2gH}$$

Substituting the values, we get

$$\frac{-dH}{dt} = \frac{3.14 \times 10^{-6}}{1.26 \times 10^{-3}}\sqrt{2 \times 10 \times H}$$

$$\Rightarrow \frac{-dH}{dt} = (1.11 \times 10^{-2})\sqrt{H}$$

Between first two resonances, the water level falls from 3.2 m to 2.4 m.

$$\therefore \quad \frac{dH}{\sqrt{H}} = -1.11 \times 10^{-2} dt$$

$$\Rightarrow \quad \int_{3.2}^{2.4} \frac{dH}{\sqrt{H}} = -(1.11 \times 10^{-2}) \int_0^t dt$$

$$\Rightarrow \quad 2[\sqrt{2.4} - \sqrt{3.2}] = -(1.1 \times 10^{-2}) t$$

$$\Rightarrow \quad t \approx 43 \text{second}$$

Q.20. A boat is traveling in a river with a speed 10 m/s along the stream flowing with a speed 2 m/s. From this boat, a sound transmitter is lowered into the river through a rigid support.

The wavelength of the sound emitted from the transmitter inside the water is 14.45 mm. Assume that attenuation of sound in water and air is negligible.

(a) What will be the frequency detected by a receiver kept inside the river downstream?

(b) The transmitter and the receiver are now pulled up into air. The air is blowing with a speed 5 m/s in the direction opposite the river stream. Determine the frequency of the sound detected by the receiver.

(Temperature of the air and water = 20°C; Density of river water = 10^3 kg/m^3;

Bulk modulus of the water = 2.088 x 10^9 Pa; Gas constant R = 8.31 J/mol-K; Mean molecular mass of air = 28.8 x 10^{-3} kg/mol; CP/CV for air = 1.4) (2001 - 10 Marks)

Ans. (a) 1.007×10^5 Hz,

(b) 1.03×10^5 Hz

Solution. KEY CONCEPT : The question is based on Doppler's effect where the medium through which the sound is travelling is also in motion.

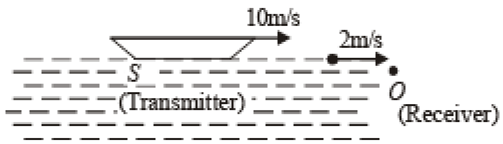
By Doppler's formula

$$v' = v\left[\frac{c + v_m \pm v_0}{c + v_m \pm v_s}\right] \qquad \text{...(1)}$$

NOTE : Sign convention for V_m is as follows : If medium is moving from s to O then + ve and vice versa.

Similarly v_0 and v_s are positive if these are directed from S to O and vice versa.

(a) Situation 1.



Velocity of sound in water $c = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{2.088 \times 10^9}{10^3}}$

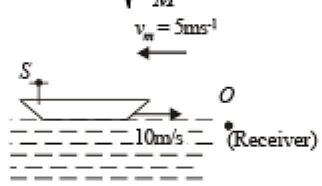
$c = 1445$ m/s; $v_m = +2$ m/s; $v_0 = 0$; $v_s = 10$ m/s

$$\therefore \quad v' = v\left[\frac{1445 + 2 - 0}{1445 + 2 - 10}\right] = v[1.007]$$

$\therefore n' = 1.007 \times 10^5$ Hz

(b) Situation 2.

In air $c = \sqrt{\frac{\gamma RT}{M}} = 344$ m/s



Applying formula (1)

$$v' = v\left[\frac{344 - 5 - 0}{344 - 5 - 10}\right] = 1.03 \times 10^5 \text{ Hz}$$

Q.21. Two narrow cylindrical pipes A and B have the same length.

Pipe A is open at both ends and is filled with a monoatomic gas of molar mass M_A. Pipe B is open at one end and closed at the other end, and is filled with a diatomic gas of molar mass M_B. Both gases are at the same temperature. (2002 - 5 Marks)

(a) If the fr equen cy of the second h armon ic of the fundamental mode in pipe A is equal to the frequency of the third harmonic of the fundamental mode in pipe B, determine the value of M_A/M_B.

(b) Now the open end of pipe B is also closed (so that the pipe is closed at both ends). Find the ratio of the fundamental frequency in pipe A to that in pipe B.

Ans. (a) 400/189

(b) 3/4

Solution. (a) Second harmonic in pipe A is

$$2(\nu_0)_A = 2\left[\frac{v}{2\ell}\right] = \frac{1}{\ell}\sqrt{\frac{\gamma_A RT}{M_A}}$$

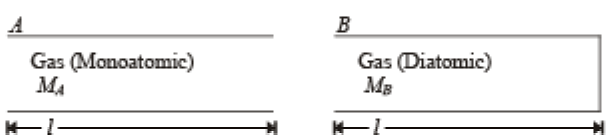
Third harmonic in pipe B is

$$3(\nu_0)_B = 3\left[\frac{v}{4\ell}\right] = \frac{3}{4\ell}\sqrt{\frac{\gamma_B RT}{M_B}}$$

Given $\nu_A = \nu_B$

$$\frac{1}{\ell}\sqrt{\frac{\gamma_A RT}{M_A}} = \frac{3}{4\ell}\sqrt{\frac{\gamma_B RT}{M_B}}$$

or, $$\frac{M_A}{M_B} = \frac{\gamma_A}{\gamma_B}\times\left(\frac{4}{3}\right)^2 = \frac{5/3}{7/5}\times\frac{16}{9} = \frac{400}{189}$$



Now, $$\frac{(\nu_0)_A}{(\nu_0)_B} = \sqrt{\frac{\gamma_A}{\gamma_A}\times\frac{M_B}{M_B}} = \frac{3}{4}$$

Q.22. A tuning fork of frequency 480 Hz resonates with a tube closed at one end of length 16 cm and diameter 5 cm in fundamental mode. Calculate velocity of sound in air. (2003 - 2 Marks)

Ans. 336 m/s

Solution. KEY CONCEPT : In the fundamental mode

$$(\ell + 0.6r) = \frac{\lambda}{4} = \frac{v}{4f} \Rightarrow v = 4f(\ell + 0.6r) = 336 \text{ m/s.}$$

Q.23. A string tied between x = 0 and x = ℓ vibrates in fundamental mode. The amplitude A, tension T and mass per unit length μ is given. Find the total energy of the string. (2003 - 4 Marks)

Ans. $\frac{\pi^2 T a^2}{4\ell}$

Solution. Here $\ell = \frac{\lambda}{2}$ or $\lambda = 2\ell$ Since, $k = \frac{2\pi}{\lambda} = \frac{2\pi}{2\ell} = \frac{\pi}{\ell}$

The amplitude of vibration at a distance x from x = 0 is given by A = a sin k x
Mechanical energy at x of length dx is

$$dE = \frac{1}{2}(dm)A^2\omega^2 = \frac{1}{2}(\mu dx)(a \sin kx)^2(2\pi \nu)^2$$

$= 2\pi^2 \mu \nu^2 a^2 \sin^2 kx \, dx$

But $v = \nu\lambda$

$$\therefore \quad \nu = \frac{v}{\lambda} \Rightarrow \nu^2 = \frac{v^2}{\lambda^2} = \frac{T/\mu}{4\ell^2} \qquad \left[\because v = \sqrt{T/\mu}\right]$$

$$\therefore \quad dE = 2\pi^2 \mu \frac{T/\mu}{4\ell^2} a^2 \sin^2\left\{\left(\frac{\pi}{\ell}\right)x\right\} dx$$

∴ Total energy of the string

$$E = \int dE = \int_0^\ell 2\pi^2 \mu \frac{T/\mu}{4\ell^2} a^2 \sin^2\left(\frac{\pi x}{\ell}\right) dx$$

$$= \frac{\pi^2 T a^2}{4\ell}$$

Q.24. A whistling train approaches a junction. An observer standing at junction observes the frequency to be 2.2 KHz and 1.8 KHz of the approaching and the receding train respectively. Find the speed of the train (speed of sound = 300 m/s) (2005 - 2 Marks)

Ans. 30 m/s

Solution. Let the speed of the train be v_T

While the train is approaching

Let v be the actual frequency of the whistle. Then

$$v' = v\frac{v_S}{v_S - v_T}$$

where v_S = Speed of sound = 300 m/s (given)

v' = 2.2 K Hz. = 2200 Hz (given)

$$\therefore \quad 2200 = v\frac{300}{300 - v_T} \quad \ldots(i)$$

While the train is receding

$$v'' = v\frac{v_S}{v_S + v_T}$$

Here, v' = 1.8 KHz = 1800 Hz (given)

$$\therefore \quad 1800 = v\frac{300}{300 + v_T} \quad \ldots(ii)$$

Dividing (i) and (ii)

$$\frac{2200}{1800} = \frac{300}{300 - v_T} \times \frac{300 + v_T}{300} \quad \Rightarrow \quad v_T = 30 \text{ m/s}$$

Q.25. A transverse harmonic disturbance is produced in a string.

The maximum transverse velocity is 3 m/s and maximum transverse acceleration is 90 m/s^2. If the wave velocity is 20 m/s then find the waveform. (2005 - 4 Marks)

Ans. $y = 0.1 \sin\left[30t \pm \frac{3}{2}x \pm \phi\right]$

Solution. KEY CONCEPT : The wave form of a transverse harmonic disturbance is

$y = a \sin(\omega t \pm kx \pm \varphi)$

Given $v_{max} = a\omega = 3$ m/s ... (i)

$A_{max} = a\omega^2 = 90$ m/s^2 ... (ii)

Velocity of wave v = 20 m/s ... (iii)

Dividing (ii) by (i)

$$\frac{a\omega^2}{a\omega} = \frac{90}{3} \Rightarrow \omega = 30 \text{ rad/s} \quad \text{... (iv)}$$

Substituting the value of w in (i), we get

$$a = \frac{3}{30} = 0.1\text{m} \quad \text{... (v)}$$

$$\text{Now, } k = \frac{2\pi}{\lambda} = \frac{2\pi}{v/\nu} = \frac{\omega}{v} = \frac{30}{20} = \frac{3}{2} \quad \text{... (vi)}$$

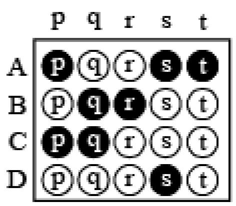
From (iv), (v) and (vi) the wave form is

$$y = 0.1 \sin\left[30t \pm \frac{3}{2}x \pm \phi\right]$$

Match the Following

DIRECTIONS (Q. No. 1-2) : Each question contains statements given in two columns, which have to be matched. The statements in Column-I are labelled A, B, C and D, while the statements in ColumnII are labelled p, q, r and s. Any given statement in Column-I can have correct matching with ONE OR MORE statement(s) in Column-II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example :

If the correct matches are A-p, s and t; B-q and r; C-p and q; and D-s then the correct darkening of bubbles will look like the given.



Q.1. Each of the properties of sound listed in the column A primarily depends on one of the quantities in column B. Write down the matching pairs from the two columns. (1980)

Column A	**Column B**
A. pitch	**p.** Waveform
B. quality	**q.** fr equency
C. loudness	**r.** intensity

Ans. A → q; B → p; C → r

Solution.

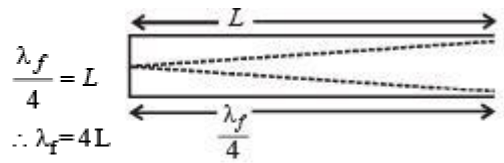
(A) Pitch	**q.** fr equency
(B) quality	**p.** waveform
(C) loudn ess	**r.** intensity

Q.2. Column I shows four systems, each of the same length L, for producing standing waves. The lowest possible natural frequency of a system is called its fundamental frequency, whose wavelength is denoted as λ_f. Match each system with statements given in Column II describing the nature and wavelength of the standing waves. (2011)

Ans. A → p, t; B → p, s; C → q, s; D → q, r

Solution. (A) Pipe closed at one end

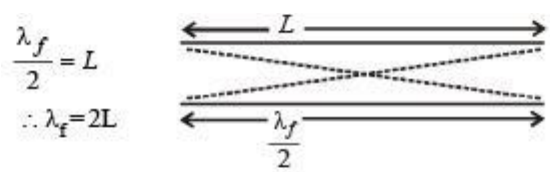
Waves produced are longitudinal (sound waves)



(p, t) are correct matching

(B) Pipe open at both ends

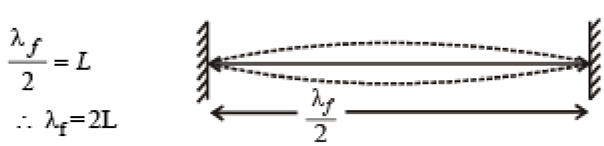
waves produced are longitudinal (sound waves)



(p, s) are correct matching.

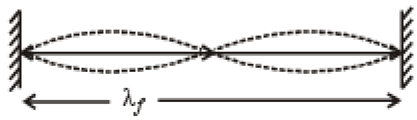
(c) Stretched wire clamped at both ends

Waves produced are transverse in nature. (waves on string)



(q, s) are correct matching. (D) Stretched wave clamped at both ends & mid point Waves produced are transverse in nature (waves on string)

$\lambda_f = L$



(q, r) are correct matching.

Integer Value Correct Type

Q.1. A 20 cm long string, having a mass of 1.0 g, is fixed at both the ends. The tension in the string is 0.5 N. The string is set into vibrations using an external vibrator of frequency 100 Hz. Find the separation (in cm) between the successive nodes on the string. (2009)

Ans. 5

Solution. We know that, $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{0.5}{10^{-3}/0.2}} = 10\text{m/s}$

The wavelength of the wave established

$$\lambda = \frac{v}{f} = \frac{10}{100} = 0.1m = 10\text{cm}$$

∴ The distance between two successive nodes

$$= \frac{\lambda}{2} = \frac{10}{2} = 5\text{cm}$$

Q.2. A stationary source is emitting sound at a fixed frequency f_0, which is reflected by two cars approaching the source.

The difference between the frequencies of sound reflected from the cars is 1.2% of f_0. What is the difference in the speeds of the cars (in km per hour) to the nearest integer ?

The cars are moving at constant speeds much smaller than the speed of sound which is 330 ms^{-1}. (2010)

Ans. 7

Solution.

Let v be the speed of sound and v_c and f_0 the speed and frequency of car.

The frequency of sound reflected by the car is

$$\therefore f_1' = f_0\left[\frac{v+v_c}{v-v_c}\right]$$

Differentiating the above equation w.r.t. v_c ,we get

$$\frac{d\,f_1'}{dv_c} = f_0\left[\frac{(v-v_c)\frac{d}{dv_c}(v+v_c)-(v+v_c)\frac{d}{dv_c}(v-v_c)}{(v-v_c)^2}\right]$$

$$\therefore \frac{df_1'}{dv_c} = f_0\left[\frac{2v}{(v-v_c)^2}\right] = f_0\frac{2v}{v^2} \quad (\because v_c << v)$$

$$\therefore \frac{df_1'}{f_0}\times 100 = \frac{2}{v}\times dv_c$$

$$\therefore 0.012 = \frac{2\times dv_c}{330}$$

$\therefore dv_c = 0.198$ m/s ≈ 7 km/h

Q.3. Wh en two progressive waves $y_1 = 4\sin(2x - 6t)$ and $y_2 = 3\sin\left(2x - 6t - \frac{\pi}{2}\right)$ are superimposed, the amplitude of the resultant wave is

Ans. 5

Solution.

Resultant amplitude, $A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$

$$= \sqrt{4^2 + 3^2 + 2\times 4\times 3\times\cos\frac{\pi}{2}} = \sqrt{16+9+0} = \mathbf{5}$$

Q.4. Four harmonic waves of equal frequencies and equal intensities I_0 have phase angles $0, \frac{\pi}{3}, \frac{2\pi}{3}$ and π. **When theyare superposed, the intensity of the resulting wave is nI_0. The value of n is (JEE Adv. 2015)**

Ans. 3

Solution.

$$y = \sqrt{I_0}\left[\sin O + \sin\frac{\pi}{3} + \sin\frac{2\pi}{3} + \sin\pi\right]$$

$$y = \sqrt{I_0}\left[\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\right] = \sqrt{3}\sqrt{I_0}$$

$$\therefore \quad I_r = y^2 = 3I_0 \quad \Rightarrow \quad n = 3$$