Linear Time Invariant Systems

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A system is a quantity which maps a set of input signal to a set of output signals. Linear time invariant (LTI) systems are used to represent signals as linear combinations of basic signals.

Continuous-time and Discrete-time Systems

A continuous time system (CTS) is one in which continuous time input signals are transformed into continuous time output signals.

e.g. integrator, differentiator, filters etc.

A discrete time system (DTS) is one which transform discrete time input signal into discrete time output signal.

$$y[n] = T\{x[n]\}$$

Moreover, a continuous time signal can be processed by a discrete time system. On the other hand processing of discrete time signal by a continuous time system is also possible because discrete time systems have several significant advantages over continuous time systems.

Classification of Systems

1. **Linear systems and non linear systems**

   When system satisfy principal of superposition and homogeneity, it is called linear system. Else non linear system.

   (i) Superposition (Additivity)

   if $x_1(t) \rightarrow y_1(t)$

   $x_2(t) \rightarrow y_2(t)$

   then $x_1(t) + x_2(t) \rightarrow y_1(t) + y_2(t)$

   (ii) Homogeneity (Scaling)

   $ax_1(t) + bx_2(t) \rightarrow ay_1(t) + by_2(t)$

2. **Time-invariant and time varying systems**

   A system is called time-variant if a time shift in the input signal causes the same time-shift in the output signal.

   If $x(t) \rightarrow y(t)$

   then $x(t - t_0) \rightarrow y(t - t_0)$

3. **Causal and non causal systems**

   The output of the causal system at the present time depends on only the present and/or past value of the input, not on its future values. A system is called non causal if it is not causal.

Note:

- All memory-less systems are causal, but not vice-versa.
- Causal system are referred as non-anticipative as the system output does not anticipate future values of input.

4. **Static and dynamic systems**

   System is said to be static or memory less if the output at any instant depends on only the input at that instant otherwise, the system is a dynamic system with memory.

5. **Stable and unstable systems**

   A system is bounded input bounded output i.e. BIBO stable if for any bounded input x the corresponding output y is also bounded.

Note:

A consequence of the homogeneity of linear system is that a zero input yield a zero output.

Convolution

The convolution integral or the superposition integral, represent a continuous-time linear time invariant (LTI) system in terms of its response to a unit impulse.

Continuous-time LTI system

- Convolution of two functions, $x_1(t)$ and $x_2(t)$

$$y(t) = x_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(\tau)x_2(t-\tau)d\tau$$

Note:

- If two analog signals get convolved:

  (i) The resultant of convolution of two signals will have a width equal to the sum of the individual width of the two signals being convolved

  (ii) Resultant of convolution has extent equal to the sum of the individual extents of the signals being convolved.

(iii) The area of resultant convolution is equal to the product of the area of the signals being convolved.

- If $f(t) * h(t) = y(t)$

  then $f(\alpha t) * h(\alpha t) = \frac{1}{|\alpha|} y(\alpha t)$

Some important result for convolution

$$\int_{-\infty}^{\infty} x_1(\tau)\, x_2(t-\tau) d\tau = \int_{-\infty}^{\infty} x_2(\tau)\, x_1(t-\tau) d\tau$$

Special Cases

$$\phi(t) * \delta(t) = \int_{-\infty}^{\infty} \phi(\tau)\, \delta(t-\tau)\, d\tau = \phi(0)$$

$$\delta(t) * \delta(t) = \int_{-\infty}^{\infty} \delta(\tau)\, \delta(t-\tau) d\tau = \delta(t)$$

Properties of the convolution integral

- Commutative property

$$y(t) = x_1(t) * x_2(t) = x_2(t) * x_1(t)$$

- Distribution property

$$x_1(t) * [x_2(t) + x_3(t)] = x_1(t) * x_2(t) + x_1(t) * x_3(t)$$

- Associative property

$$x_1(t) * (x_2(t) * x_3(t)) = (x_1(t) * x_2(t)) * x_3(t)$$

- Derivative of the convolution

$$\frac{dy(t)}{dt} = \frac{dx_1(t)}{dt} * x_2(t) = x_1(t) * \frac{dx_2}{dt}(t)$$

- Convolution of two delayed functions

  If $y(t) = x_1(t) * x_2(t)$

$$x_1(t-t_1) * x_2(t-t_2) = y\left((t-(t_1+t_2)\right)$$

- Time scaling property

  If $y(t) = x_1(t) * x_2(t)$

$$x_1(\alpha t) * x_2(\alpha t) = \frac{1}{|\alpha|} y(\alpha t) \;;\; \alpha \neq 0$$

Discrete-time LTI system

$$y[n] = \sum_{k=-\infty}^{\infty} x_1[k]\, x_2[n-k] = x_1[n] * x_2[n]$$

Note:

If two discrete signals get convolved:

(i) The resultant of convolution of two signals will have a length equal to the sum of the individual length of the two signals being convolved minus 1 i.e. $(L_1 + L_2 - 1)$.

(ii) Resultant of convolution has extent equal to the sum of the individual extents of the signals being convolved.

(iii) The sum of sampled values in resultant convolution is equal to the product of sum of the individual sampled values of the signals being convolved.

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