Exercise 9.6

Answer 1E.

Consider the predator-prey system:

$$\frac{dx}{dt} = -0.05x + 0.0001xy$$

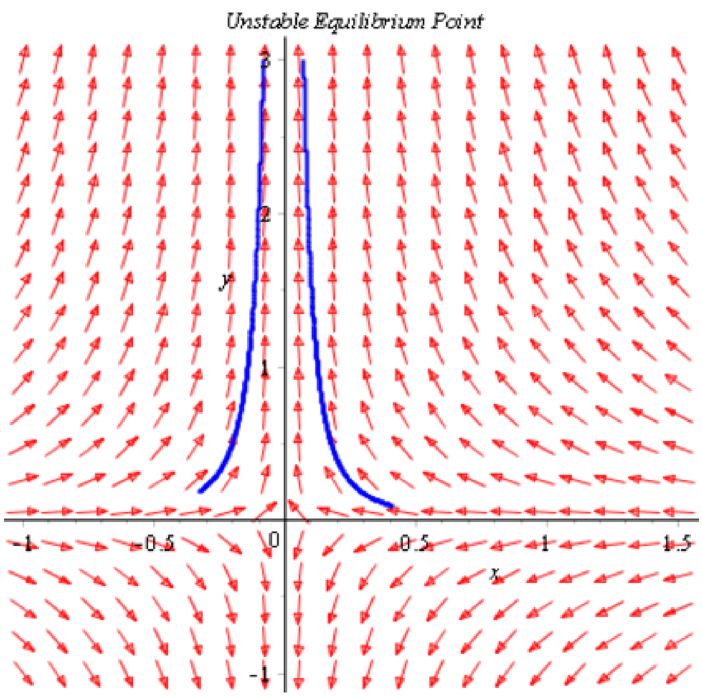
$$\frac{dy}{dt} = 0.1y - 0.005xy$$

```
with(DEtools);

Sys := {diff(x(t), t) = -0.5e-1*x(t)+0.1e-3*x(t)*y(t), diff(y(t), t) = .1*y(t)-0.5e-2*x(t)*y(t)};

DEplot(Sys, [x(t), y(t)], t = -10 .. 40, x = -1 .. 3/2, y = -1 .. 3, [[x(0) = -.2, y(0) = .5], [x(0) = .25, y(0) = .25]], stepsize = 0.5e-1, linecolor = blue, thickness = 2, arrows = slim, title = Unstable*Equilibrium*Point);
```

with(DEtools);

Sys := {*diff*(*x*(*t*), *t*) = −0.05 * *x*(*t*) + 0.0001 * *x*(*t*) * *y*(*t*), *diff*(*y*(*t*), *t*) = 0.1 * *y*(*t*) − 0.005 * *x*(*t*) * *y*(*t*)}

DEplot(*Sys*, [*x*(*t*), *y*(*t*)], *t* = −10 ..40, *x* = −1 ..3/2, *y* = −1 ..3, [[*x*(0) = −0.2, *y*(0) = 0.5], [*x*(0) = 0.25, *y*(0) = 0.25]], *stepsize* = 0.05, *linecolor* = *blue*, *thickness* = 2, *arrows* = *slim*, *title* = *Unstable Equilibrium Point*);



The term $-0.005xy$ decreases the natural growth rate of the prey and the term $0.0001xy$ increases the natural growth rate of the predators so for this system the variable x represents the predator population and the variable y represents the prey population.

When $y = 0$, that is the number of predators are zero then the prey population will increase exponentially so the growth of prey population is restricted only with predators.

And also the predators feed only on the prey.

When $x = 0$, that is the number of preys are zero then due to starvation the predator population will decrease exponentially to zero.

Consider the predator-prey system:

$$\frac{dx}{dt} = 0.2x - 0.0002x^2 - 0.006xy$$
$$\frac{dy}{dt} = -0.015y + 0.00008xy$$

with(DEtools);

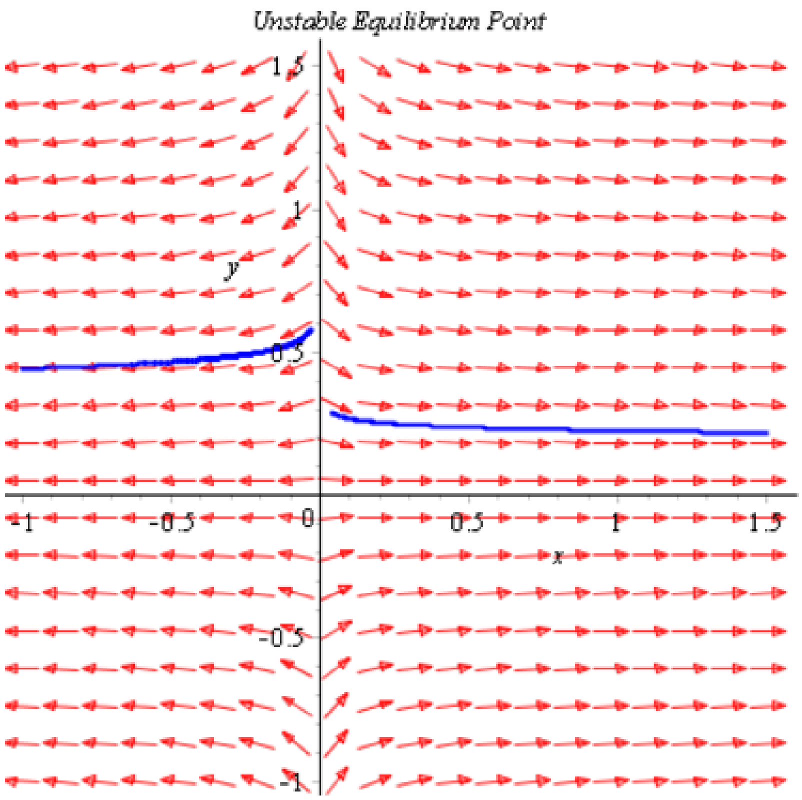
Sys := {diff(x(t), t) = .2*x(t)-0.2e-3*x(t)^2-0.6e-2*x(t)*y(t), diff(y(t), t) = -0.15e-1*y(t)+0.8e-4*x(t)*y(t)};

DEplot(Sys, [x(t), y(t)], t = -10 .. 40, x = -1 .. 3/2, y = -1 .. 1.5, [[x(0) = -.2, y(0) = .5], [x(0) = .25, y(0) = .25]], stepsize = 0.5e-1, linecolor = blue, thickness = 2, arrows = slim, title = Unstable*Equilibrium*Point)

with(*DEtools*);

$Sys := \{diff(x(t), t) = 0.2 * x(t) - 0.0002 * x(t)\text{^}2 - 0.006 * x(t) * y(t), diff(y(t), t) = -0.015 * y(t) + 0.00008 * x(t) * y(t)\}$

$DEplot(Sys, [x(t), y(t)], t = -10..40, x = -1..3/2, y = -1..1.5, [[x(0) = -0.2, y(0) = 0.5], [x(0) = 0.25, y(0) = 0.25]], stepsize = 0.05, linecolor = blue, thickness = 2, arrows = slim, title = Unstable\ Equilibrium\ Point)$;



The term $-0.006xy$ decreases the natural growth rate of the prey and the term $0.00008xy$ increases the natural growth rate of the predators so for this system the variable x represents the prey population and the variable y represents the predator population.

When $y=0$, that is the number of predators are zero then the prey population will increase exponentially so the growth of prey population is restricted only with predators.

And also the predators feed only on the prey.

When $x=0$, that is the number of preys are zero then due to starvation the predator population will decrease exponentially to zero.

The growth of prey population is restricted only with predators.

The predator population increases only by encounters with the prey and the predators feed only on the prey.

Answer 2E.

(a)

Consider the following differential equations:

$$\frac{dx}{dt}=0.12x-0.006x^2+0.00001xy$$
$$\frac{dy}{dt}=0.08x+0.00004xy$$

In the first differential equation, $\frac{dx}{dt}$ increases because $+0.00001xy$ gives a positive contribution to the model $\frac{dx}{dt}=0.12x-0.006x^2+0.00001xy.$

So, it's a cooperation model.

In the second equation, $\frac{dx}{dt}$ increases because $+0.00004xy$ gives a positive contribution to the model $\frac{dy}{dt}=0.08x+0.00004xy.$

So, it's also a cooperation model.

(b)

Consider the following differential equations:

$$\frac{dx}{dt}=0.15x-0.002x^2-0.0006xy$$
$$\frac{dy}{dt}=0.2y-0.00008y^2-0.0002xy$$

In the first differential equation, $\frac{dx}{dt}$ decreases because $-0.0006xy$ gives a negative contribution to the model $\frac{dx}{dt}=0.15x-0.002x^2-0.0006xy.$

So, it's a competition model.

In the second differential equation, $\frac{dx}{dt}$ decreases because $-0.0002xy$ gives a negative contribution to the model $\frac{dy}{dt}=0.2y-0.00008y^2-0.0002xy.$

So, it's also a competition model.

Answer 3E.

(a) Given differential equations are:

$$\frac{dx}{dt} = 0.5x - 0.004x^2 - 0.001xy$$

$$\frac{dy}{dt} = 0.4y - 0.001y^2 - 0.002xy$$

The rate $\frac{dx}{dt}$ decreases with increasing y.

The rate $\frac{dy}{dt}$ decreases with increasing x.

Therefore, the model describes competition.

(b)

$$\begin{Bmatrix} 0 = 0.5x - 0.004x^2 - 0.001xy \\ 0 = 0.4y - 0.001y^2 - 0.002xy \end{Bmatrix} \Leftrightarrow$$

$$\begin{Bmatrix} 0 = x(0.5 - 0.004x - 0.001y) \\ 0 = y(0.4 - 0.001y - 0.002x) \end{Bmatrix} \Leftrightarrow$$

This system of equations has 4 solutions.

They must satisfy the equations:

$$\begin{Bmatrix} x = 0 \\ y = 0 \end{Bmatrix} \vee \begin{Bmatrix} x = 0 \\ 0.4 = 0.001y + 0.002x \end{Bmatrix} \vee \begin{Bmatrix} 0.5 = 0.004x + 0.001y \\ y = 0 \end{Bmatrix} \vee$$

$$\begin{Bmatrix} 0.5 = 0.004x + 0.001y \\ 0.4 = 0.001y + 0.002x \end{Bmatrix}$$

$$\begin{array}{r} 0.5 = 0.004x + 0.001 \\ -(0.4 = 0.001y + 0.002x) \\ \hline 0.1 = 0.002x \end{array}$$

$x = 50$

$0.5 = 0.004x + 0.001y$

$0.5 = 0.004 \cdot 50 + 0.001y$

$0.3 = 0.001y$

$y = 300$

Solving the previous equations identifies the four equilibrium points as:

$\left\{ \begin{matrix} x = 0 \\ y = 0 \end{matrix} \right\}$ There are zero populations.

$\left\{ \begin{matrix} x = 0 \\ y = 400 \end{matrix} \right\}$ In the absence of an x- population, the y- population stabilizes to 400.

$\left\{ \begin{matrix} x = 125 \\ y = 0 \end{matrix} \right\}$ In the absence of an y- population, the x- population stabilizes to 125.

$\left\{ \begin{matrix} x = 50 \\ y = 300 \end{matrix} \right\}$ Both populations are stable.

Answer 4E.

Establish variables:

Flies $= P(t)$

Frogs $= Q(t)$

Crocodiles $= R(t)$

Since the frogs need to eat flies, and crocodiles need to eat frogs, the right hand side of the equation defining $\frac{dQ(t)}{dt}$ must contain two terms, one that depends on the flies and frogs, and the other that depends on crocodiles and frogs.

The first of these terms has therefore the form $K_1 f_1(P(t), Q(t))$ and the second term has the form $-K_2 f_2(Q(t), R(t))$, where K_1, K_2 are positive constants and $f_1(P(t), Q(t)), f_2(Q(t), R(t))$ are positive valued functions. The impact of these terms on the fly and crocodile growth is quantified by the similar terms $-K_3 f_1(P(t), Q(t))$ and $K_4 f_2(Q(t), R(t))$ respectively.

Because the population of flies will grow exponentially in the absence of frogs, the right hand side of the equation defining $\frac{dP(t)}{dt}$ also contains the term $K_5 P(t)$, where K_5 is some constant. Then

$$\frac{dP(t)}{dt} = K_5 P(t) - K_3 f_1(P(t), Q(t))$$

Because the population of crocodiles will decay exponentially in the absence of frogs, the right hand side of the equation defining $\frac{dR(t)}{dt}$ also contains the term $-K_6 R(t)$, where K_6 is some constant. Then

$$\frac{dR(t)}{dt} = -K_6 R(t) + K_4 f_2(Q(t), R(t))$$

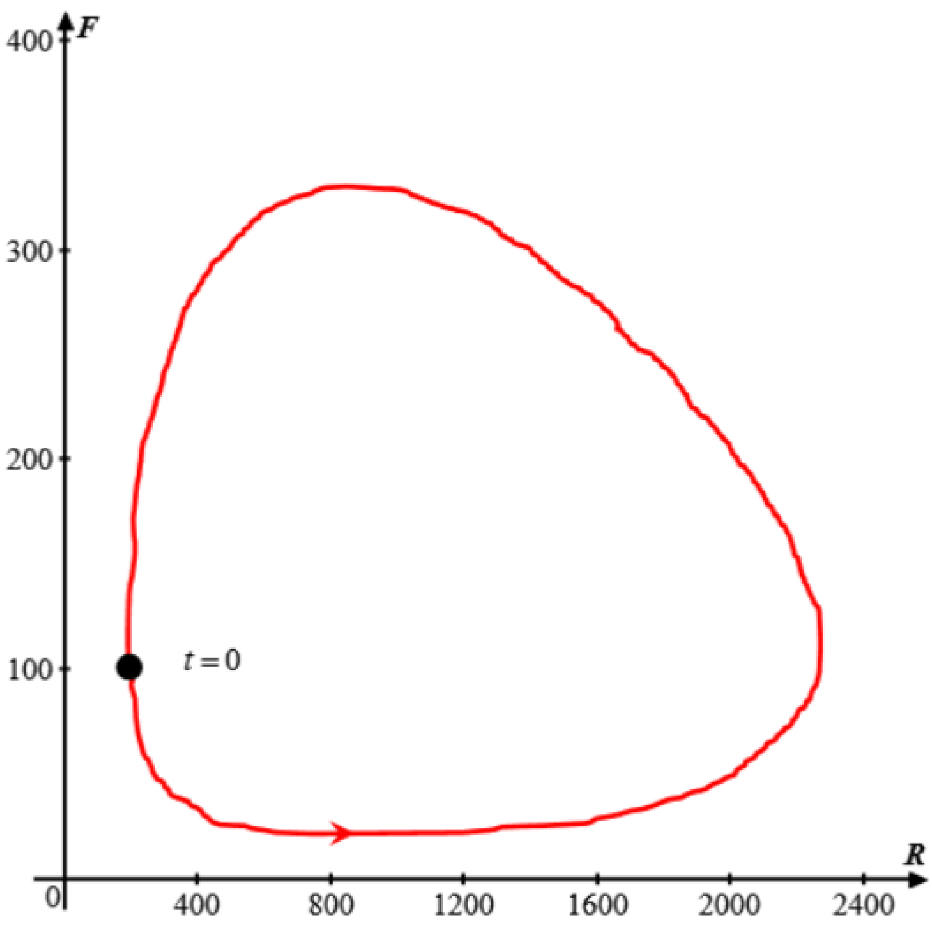
Because the population of frogs will decay exponentially in the absence of flies and crocodiles, the right hand side of the equation defining $\frac{dQ(t)}{dt}$ contains the term $-K_7Q(t)$, where K_7 is some constant. Then

$$\boxed{\frac{dQ(t)}{dt} = -K_7Q(t) + K_1f_1(P(t),Q(t)) - K_2f_2(Q(t),R(t))}$$

Answer 5E.

(a)

The phase trajectory for populations of rabbits (R) and foxes (F) is shown below



Carefully observe the above diagram

Initially the number of Foxes gradually decreases and the number of rabbits speedily increases

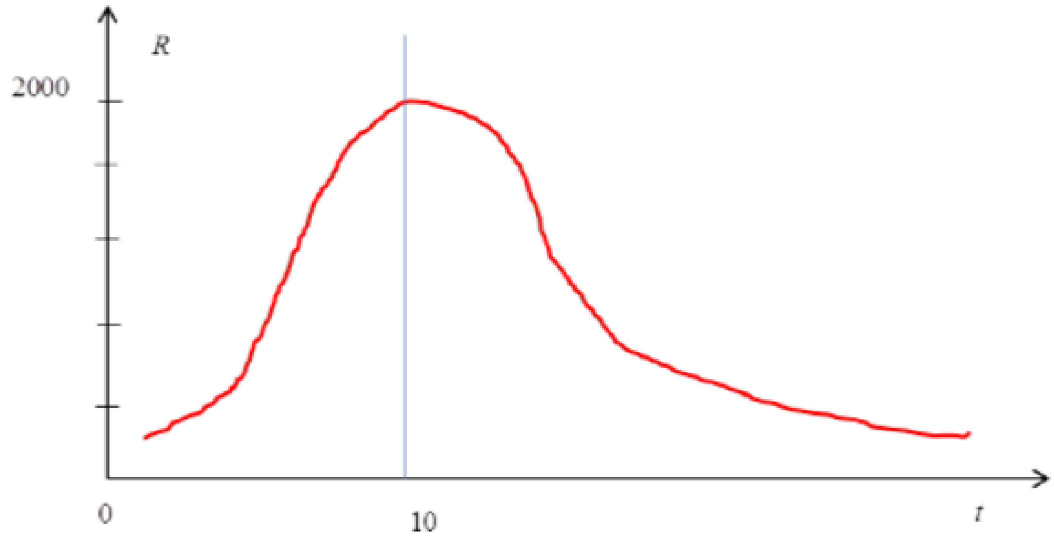
Then the numbers of foxes begin to stabilize, the number of rabbits increase very quickly

Then the number of foxes rise gradually, as the number of foxes begins to stabilize
Then the rabbits begin to decrease suddenly while the number of foxes increases at a reasonable rate.

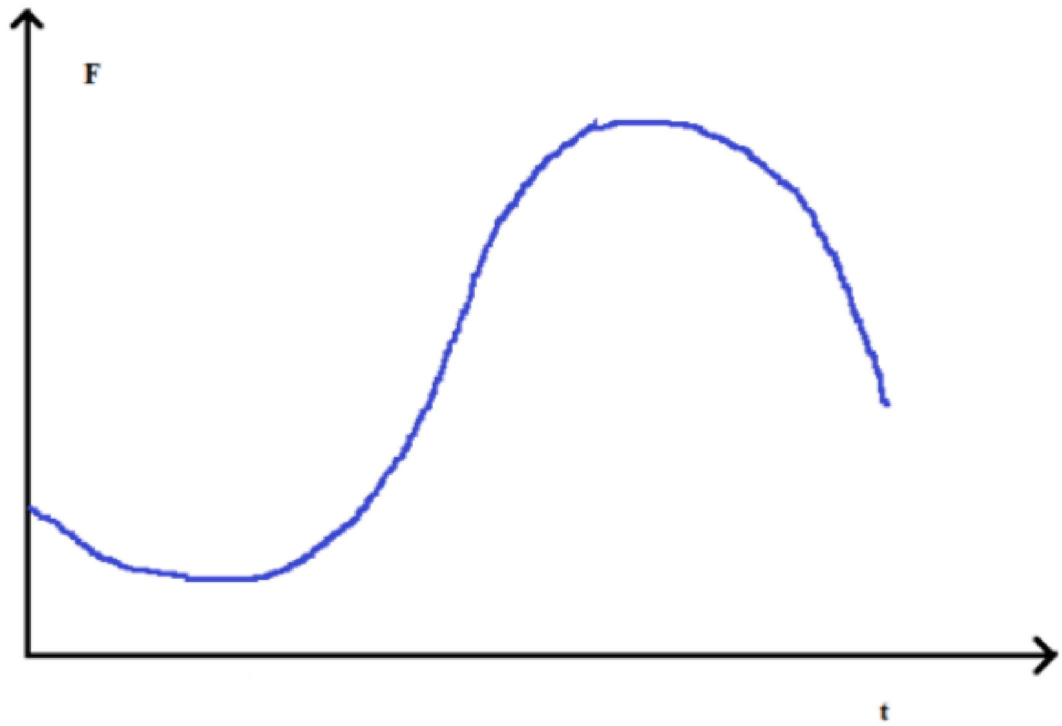
The number of foxes then stabilize while the number of rabbits keep decreasing , after which the number of rabbits drop sharply, while the number of foxes fall moderately

(b)

The rough sketch of graph of R as a time t



The rough sketch of graph of F as a time t



Answer 6E.

(a)

Consider the differential equation $\frac{dP}{dx} = 0.08P\left(1 - \frac{P}{1000}\right) - c$

As a model for a fish population, where t is measured in weeks and c is a constant.

Draw a direction fields for various values of c using CAC.

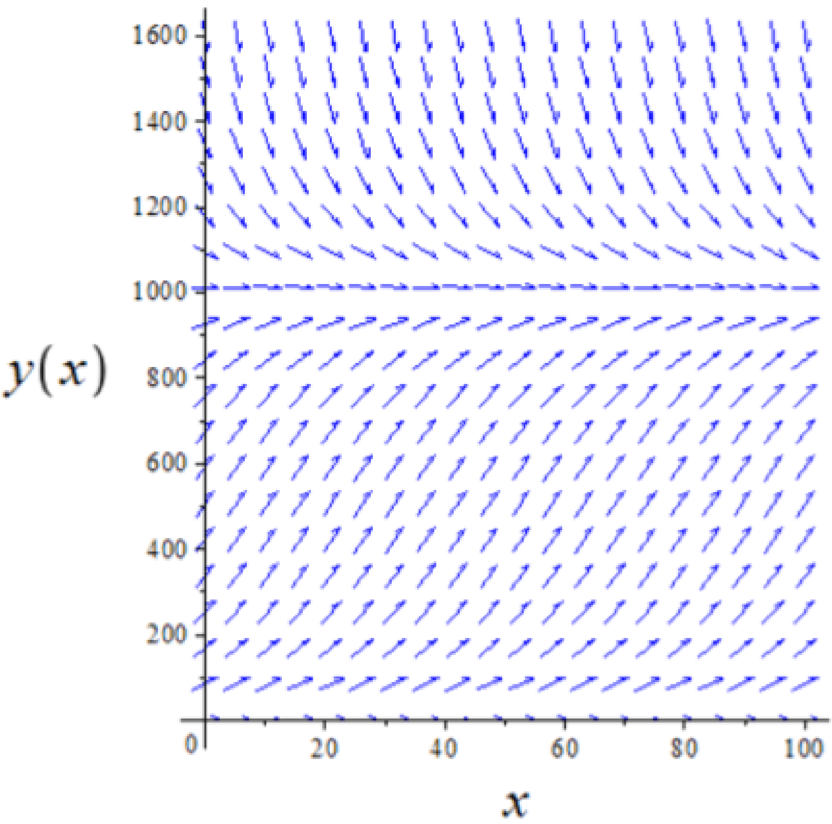
For $c = 0$, the maple command is

```
with(DEtools):
with(plot):
dfieldplot(diff(y(x),x)=0.08*y(x)*(1-y(x)/1000)-y(x),x=0..100,y=0..1600,color=blue);
```

> $with(DEtools):$
$with(plots):$

> $dfieldplot\left(diff(y(x), x) = 0.08 \cdot y(x) \cdot \left(1 - \frac{y(x)}{1000}\right), y(x), x = 0..100, y = 0..1600, color = blue\right);$

Sketch the direction field of the differential equation $\frac{dP}{dx} = 0.08P\left(1 - \frac{P}{1000}\right)$



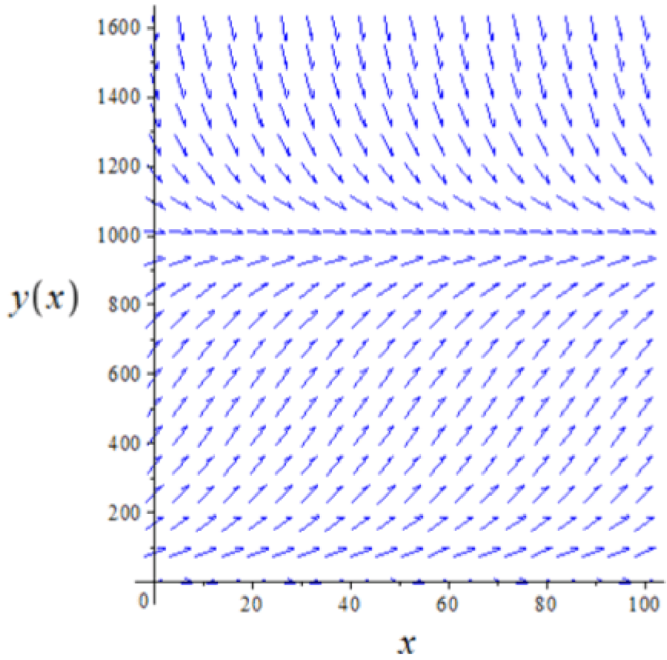
For $c = 1$, the maple command is

```
with(DEtools):
with(plot):
dfieldplot(diff(y(x),x)=0.08*y(x)*(1-y(x)/1000)-1,y(x),x=0..100,y=0..1600,color=blue);
```

> $with(DEtools):$
$with(plots):$

> $dfieldplot\left(diff(y(x), x) = 0.08 \cdot y(x) \cdot \left(1 - \frac{y(x)}{1000}\right) - 1, y(x), x = 0..100, y = 0..1600, color = blue\right);$

Sketch the direction field of the differential equation $\frac{dP}{dx} = 0.08P\left(1 - \frac{P}{1000}\right) - 1$



For $c = -1$, the maple command is

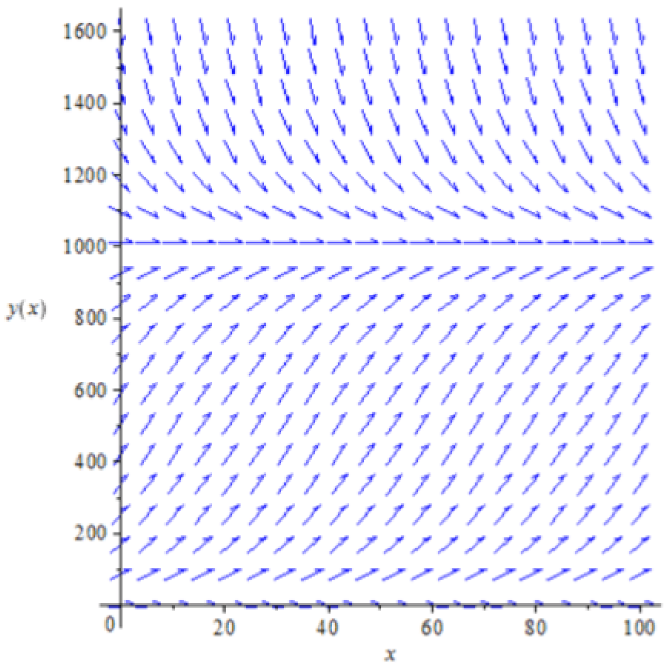
with(DEtools):

with(plot):

dfieldplot(diff(y(x),x)=0.08*y(x)*(1-y(x)/1000)+1,y(x),x=0..100,y=0..1600,color=blue);

```
> with(DEtools) :
with(plots) :
> dfieldplot(diff(y(x), x) = 0.08·y(x)·(1 − y(x)/1000) − 1, y(x), x = 0..100, y = 0..1600, color
    = blue);
```

Sketch the direction field of the differential equation $\frac{dP}{dx} = 0.08P\left(1 - \frac{P}{1000}\right) + 1$



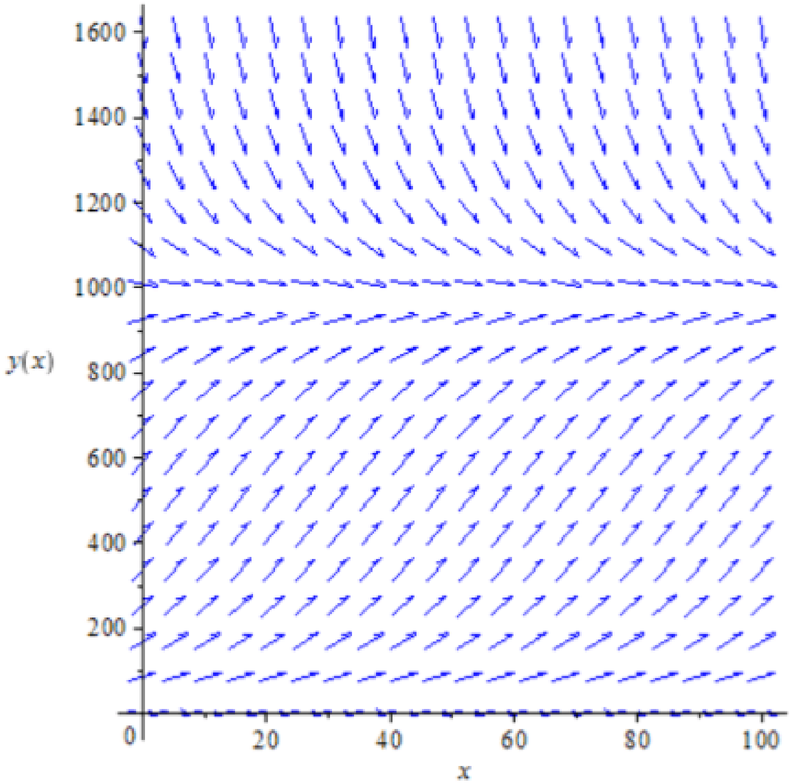
For $c = 2$, the maple command is

```
with(DEtools):
with(plot):
dfieldplot(diff(y(x),x)=0.08*y(x)*(1-y(x)/1000)-2,y(x),x=0..100,y=0..1600,color=blue);
```

> *with(DEtools)* :
with(plots) :

> $dfieldplot\left(diff(y(x), x) = 0.08 \cdot y(x) \cdot \left(1 - \frac{y(x)}{1000}\right) - 2, y(x), x = 0..100, y = 0..1600, color = blue\right);$

Sketch the direction field of the differential equation $\frac{dP}{dx} = 0.08P\left(1 - \frac{P}{1000}\right) - 2$



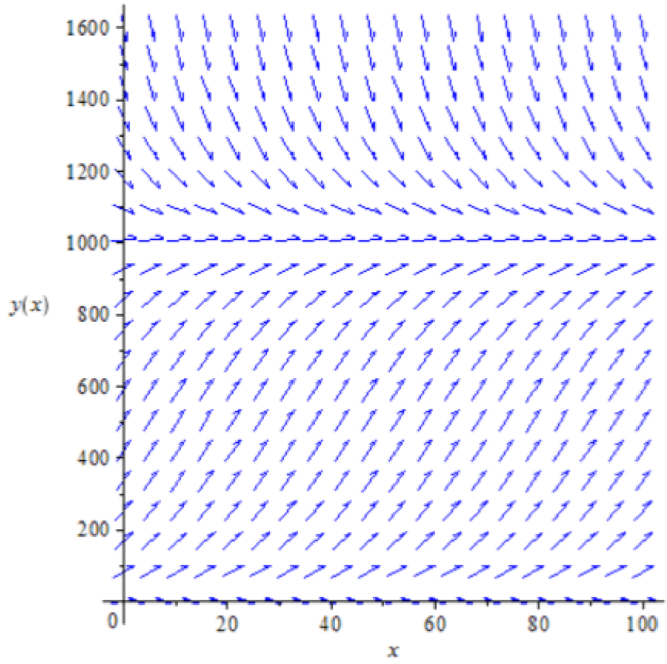
For $c = -2$, the maple command is

```
with(DEtools):
with(plot):
dfieldplot(diff(y(x),x)=0.08*y(x)*(1-y(x)/1000)+2,y(x),x=0..100,y=0..1600,color=blue);
```

> *with(DEtools)* :
with(plots) :

> $dfieldplot\left(diff(y(x), x) = 0.08 \cdot y(x) \cdot \left(1 - \frac{y(x)}{1000}\right) + 2, y(x), x = 0..100, y = 0..1600, color = blue\right);$

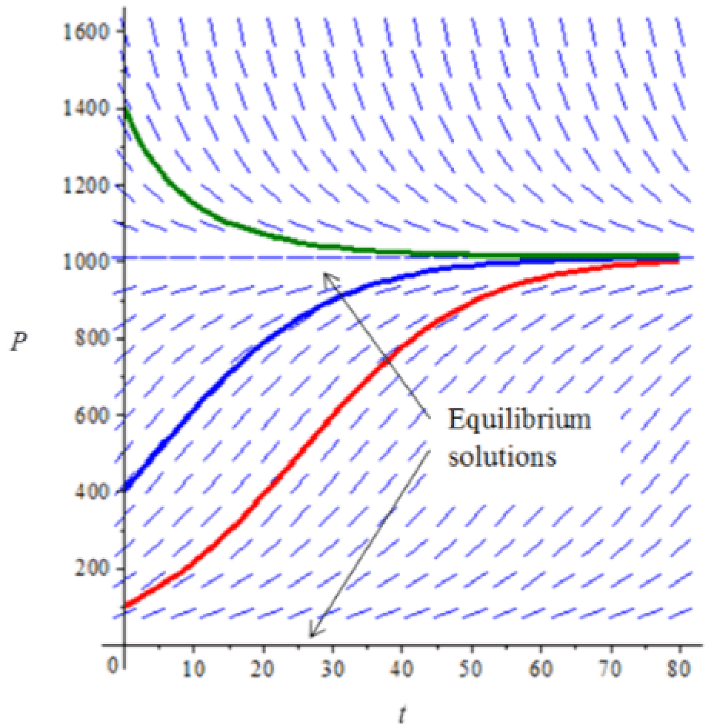
Sketch the direction field of the differential equation $\frac{dP}{dx}=0.08P\left(1-\frac{P}{1000}\right)+2$



(b)

From the direction fields in part (a), the equation $\frac{dP}{dx}=0.08P\left(1-\frac{P}{1000}\right)-c$ is autonomous, so the slopes are same any horizontal line.

Sketch the solution curves with initial population $P(0)=100, P(0)=400, P(0)=1400$



From the above graph, the differential equation $\frac{dP}{dx}=0.08P\left(1-\frac{P}{1000}\right)-c$ has the equilibrium solution for $c=1$.

Note the differential equation has at least one equilibrium solution for $c<20$

Consider the differential equation $\frac{dP}{dx} = 0.08P\left(1 - \frac{P}{1000}\right) - c$

Let $f(P) = 0.08P\left(1 - \frac{P}{1000}\right)$

The term $f(P) = 0.08P\left(1 - \frac{P}{1000}\right)$ represents the rate of fish population and c represents the harvesting rate

Rewrite the equation as

$$\begin{aligned} f(P) &= 0.08P\left(1 - \frac{P}{1000}\right) \\ &= \left(0.08P - \frac{0.08P^2}{1000}\right) \\ &= \left(\frac{0.08P \times 1000 - 0.08P^2}{1000}\right) \\ &= \left(\frac{0.08}{1000}\right)(1000P - P^2) \end{aligned}$$

The reproduction rate approaches maximum value at $f'(P) = 0$, so

$$f'(P) = \left(\frac{0.08}{1000}\right)(1000 - 2P) = 0$$

(d)

Since the solution of the differential equation $\frac{dP}{dx} = 0.08P\left(1 - \frac{P}{1000}\right) - c$ is

$P(x) = \frac{1000}{1 + Ae^{-kt}}$ where $A = \frac{K - P_0}{P_0}$

Using the above expression $P(x)$, $\lim_{x \to \infty} P(x) = 1000$ this is to be expected.

Solve the equation $\left(\frac{0.08}{1000}\right)(1000 - 2P) = 0$ for P

$$\begin{aligned} \left(\frac{0.08}{1000}\right)(1000 - 2P) &= 0 \\ 1000 - 2P &= 0 \\ P &= 500 \end{aligned}$$

Find the second derivative of $f(P) = 0.08P\left(1 - \frac{P}{1000}\right)$

$$f''(P) = \left(\frac{0.08}{1000}\right)(-2) < 0$$

The second derivative is negative, so the maximum reproduction rate is

$$\begin{aligned} f(500) &= 0.08 \times 500\left(1 - \frac{500}{1000}\right) \\ &= 20 \end{aligned}$$

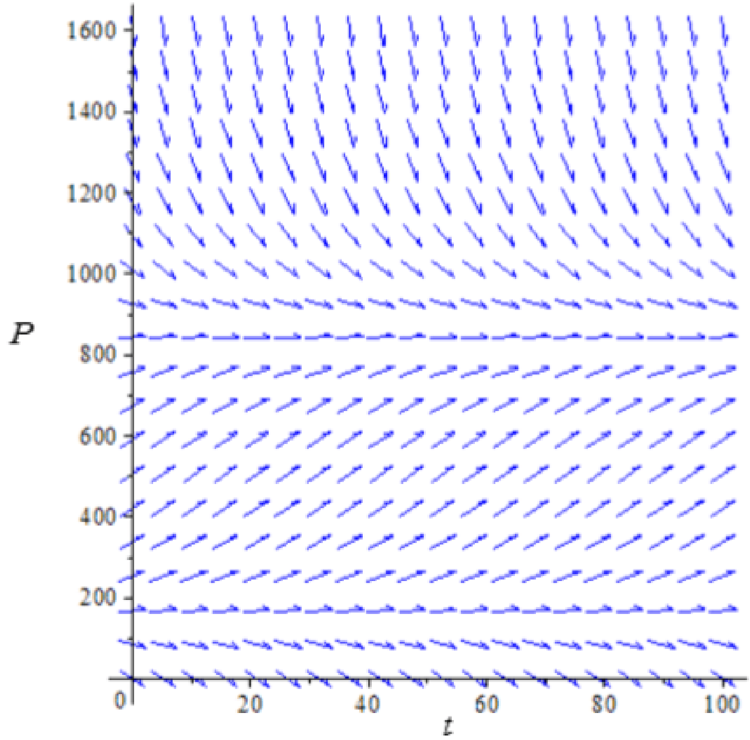
If the value of c exceed this value 20, the production rate P will be negative for every value of P

Therefore, fish population will always die out for $c > 20$.

(c)

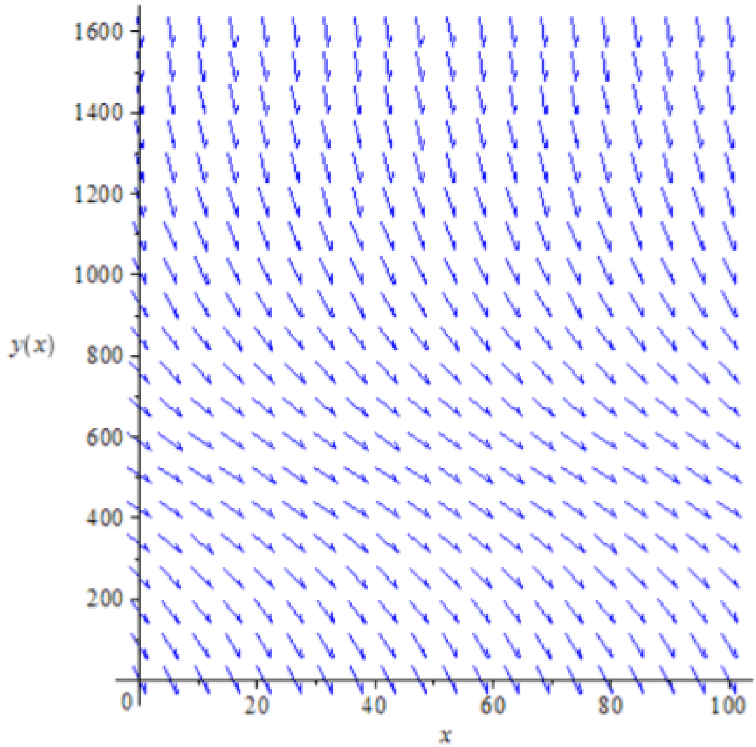
For $c < 20$,

Sketch the direction field for the differential equation $\frac{dP}{dx} = 0.08P\left(1 - \frac{P}{1000}\right) - c$



For $c > 20$,

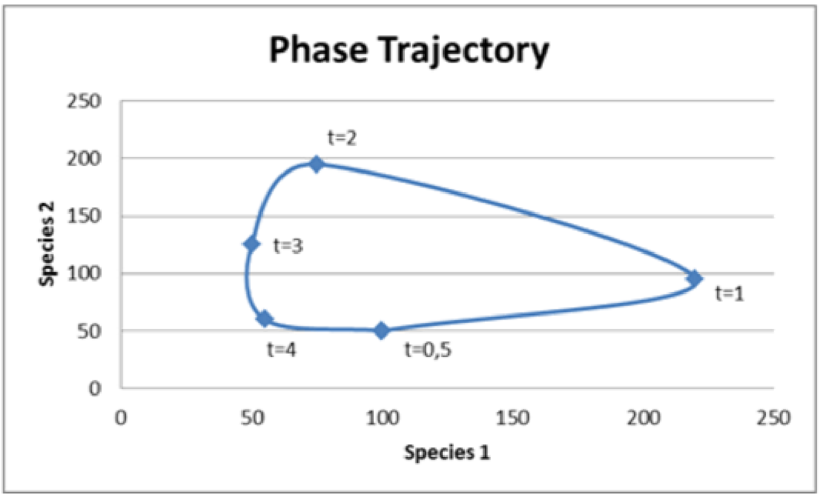
Sketch the direction field for the differential equation $\frac{dP}{dx} = 0.08P\left(1 - \frac{P}{1000}\right) - c$



Answer 7E.

If we plot the points found at $t = 0, 1, 2, 3, 4, 5$ we can roughly sketch the phase trajectory.

t	Species 1	Species 2
0	100	50
1	220	95
2	75	195
3	50	125
4	55	60
5	100	50



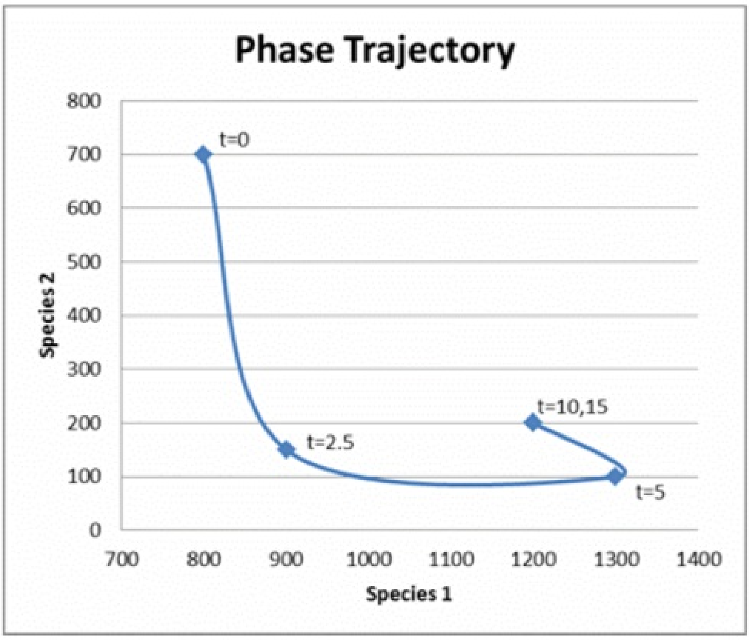
Answer 8E.

If we plot the points found at $t = 0, 2.5, 5, 10, 15$ we can roughly sketch the phase trajectory.

Table showing the populations of two species:

t	Species 1	Species 2
0	800	700
2.5	900	150
5	1300	100
10	1200	200
15	1200	200

Figure showing the phase trajectory of given species:



Answer 9E.

We need to solve the given differentiable equation
$\frac{dW}{dR} = \frac{-0.02W + 0.00002RW}{0.08R - 0.001RW}$, to show $\frac{R^{0.02}W^{0.08}}{e^{0.00002R}e^{0.001W}} = C$

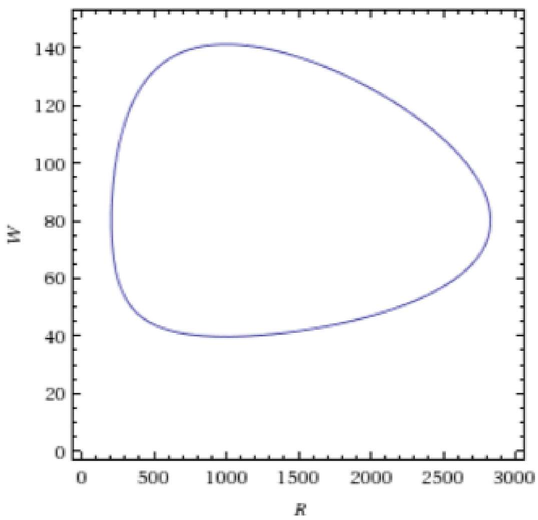
Now
$$\frac{dW}{dR} = \frac{-0.02W + 0.00002RW}{0.08R - 0.001RW}$$
$$= \frac{W(-0.02 + 0.00002R)}{R(0.08 - 0.001W)}$$
Separating the variables we get
$$R(0.08 - 0.001W)dW = W(-0.02 + 0.00002R)dR$$
$$\Rightarrow \frac{(0.08 - 0.001W)dW}{W} = \frac{(-0.02 + 0.00002R)dR}{R}$$

On integrating we get
$$\int \frac{(0.08 - 0.001W)dW}{W} = \int \frac{(-0.02 + 0.00002R)dR}{R}$$
$$\Rightarrow 0.08\ln W - 0.001W = -0.02\ln R + 0.00002R + C_1$$
Here C_1 is the constant of integration.

We have
$$0.08\ln W - 0.001W = -0.02\ln R + 0.00002R + c$$
$$\Rightarrow 0.08\ln W + 0.02\ln R = 0.00002R + 0.001W + C_1$$
$$\Rightarrow \ln W^{0.08} + \ln R^{0.02} = 0.00002R + 0.001W + C_1$$
$$\Rightarrow \ln\left(W^{0.08}R^{0.02}\right) = 0.00002R + 0.001W + C_1$$
$$\Rightarrow W^{0.08}R^{0.02} = e^{0.00002R + 0.001W + C_1}$$

$$= e^{0.00002R}e^{0.001W}e^{C_1}$$
$$= e^{0.00002R}e^{0.001W}C \qquad \left(\text{Let } e^{C_1} = C\right)$$
So $W^{0.08}R^{0.02} = e^{0.00002R}e^{0.001W}C$
$$\Rightarrow C = \frac{W^{0.08}R^{0.02}}{e^{0.00002R}e^{0.001W}}$$

Figure showing the solution curve that pass through $(1000, 40)$:



Answer 10E.

(a)

Consider the equations

$$\frac{dA}{dt} = 2A - 0.01AL$$

$$\frac{dL}{dt} = -0.5L + 0.0001AL$$

Find the equilibrium solution

Both A and L will be constant if both derivatives are 0, that is

$$\left.\begin{aligned} 2A - 0.01AL = 0 \\ -0.5L + 0.0001AL = 0 \end{aligned}\right\} \quad \text{......(1)}$$

Solve the system (1)

One solution is $A = 0, L = 0$

Thus, $(0,0)$ is one equilibrium solution

The other constant solution is

$$2A - 0.01AL = 0$$

$$0.01AL = 2A$$

$$L = \frac{2A}{0.01A}$$

$$L = 200$$

And

$$0.0001AL = 0.5L$$

$$A = \frac{0.5L}{0.0001L}$$

$$A = 5000$$

So, the other equilibrium solution is $(A, L) = (5000, 200)$

This means that 5000 aphids and 200 lady bugs are the right size that there are no changes in the size of either population.

(b)

Find an expression for $\frac{dL}{dA}$

Use the Chain Rule to eliminate t :

$$\frac{dL}{dA} = \frac{dL}{dt} \cdot \frac{dt}{dA}$$

$$= (-0.5L + 0.0001AL)\left(\frac{1}{2A - 0.01AL}\right)$$

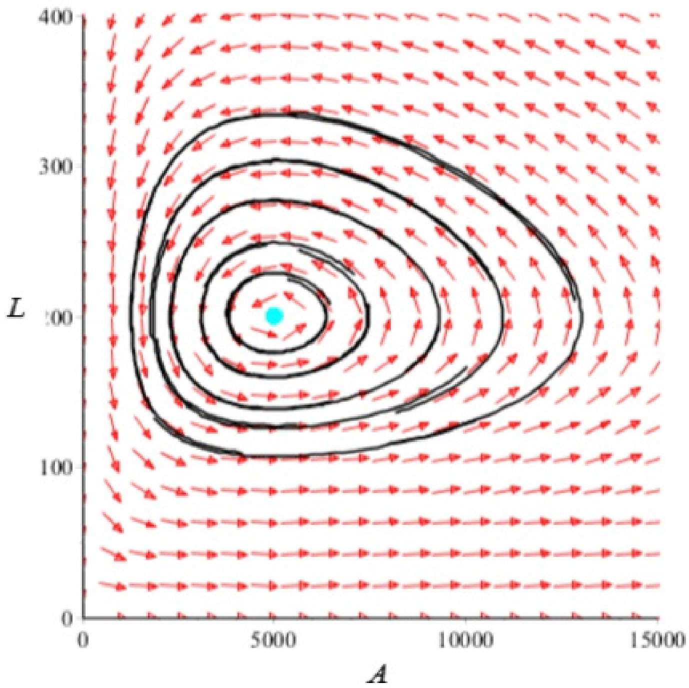
$$= \frac{-0.5L + 0.0001AL}{2A - 0.01AL}$$

Thus, $\boxed{\frac{dL}{dA} = \frac{-0.5L + 0.0001AL}{2A - 0.01AL}}$

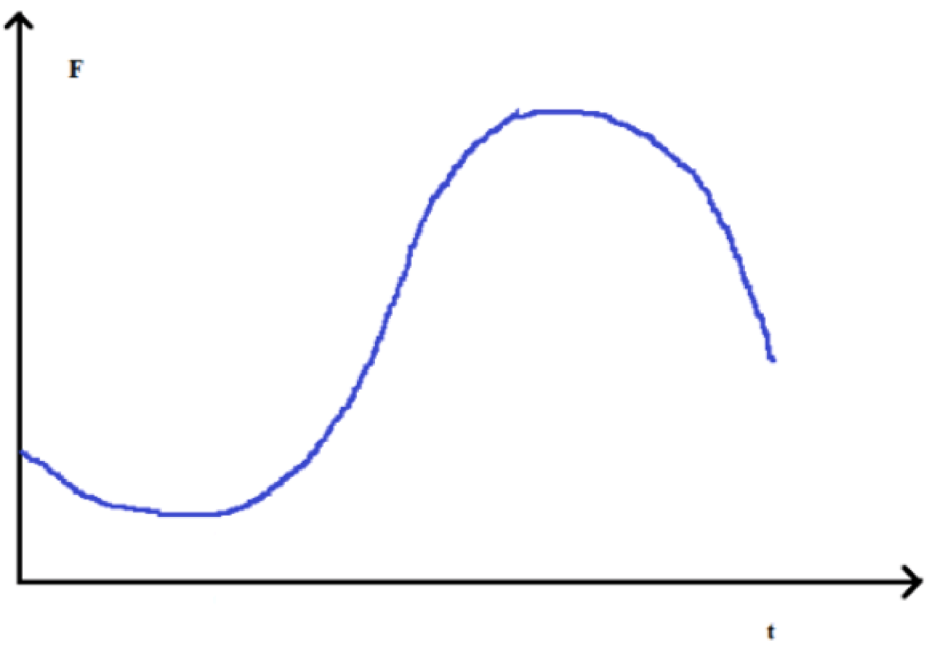
(c)

Consider the differential equation $\frac{dL}{dA} = \frac{-0.5L + 0.0001AL}{2A - 0.01AL}$

The phase portrait is shown below, the point inside the solution curve is the equilibrium point



The rough sketch of graph of F as a time t



Answer 11E.

We will examine the following Lotka-Volterra equations:

$$\frac{dR}{dt} = 0.08R(1 - 0.0002R) - 0.001RW$$

$$\frac{dW}{dt} = -0.02W + 0.00002RW$$

Where R is the population of rabbits and W is the population of wolves.

(a)In the absence of wolves the rabbit population satisfies the equation:

$$\frac{dR}{dt} = 0.08R(1-0.0002R)$$
$$\Rightarrow \frac{dR}{0.08R(1-0.0002R)} = dt$$

We use partial fractions to prepare for integration:

$$\frac{1}{0.08R(1-0.0002R)} = \frac{A}{0.08R} + \frac{B}{1-0.0002R}$$
$$= \frac{A-0.0002AR+0.08BR}{0.08R(1-0.0002R)}$$
$$\Rightarrow A=1,\ B=\frac{0.0002}{0.08} = 0.0025$$

We can plug these values back into the equation:

$$\frac{dR}{0.08R} + 0.0025\frac{dR}{1-0.0002R} = dt$$

Integrate:

$$\int_{R(0t)}^{R(t)} \frac{dR}{0.08R} + \int_{R(0)}^{R(t)} 0.0025\frac{dR}{1-0.0002R} = \int_0^t dt$$
$$\Rightarrow 12.5\ln\left(\frac{R(t)}{R(0)}\right) - \frac{0.0025}{0.0002}\ln\left(\frac{1-0.0002R(t)}{1-0.0002R(0)}\right) = t$$

Simplify:

$$\ln\left(\frac{R(t)^{12.5}(1-0.0002R(0))^{12.5}}{R(0)^{12.5}(1-0.0002R(t))^{12.5}}\right) = t$$
$$\Rightarrow \frac{R(t)(1-0.0002R(0))}{R(0)(1-0.0002R(t))} = (e^t)^{\frac{1}{12.5}}$$
$$\Rightarrow \frac{(1-0.0002R(t))}{R(t)} = \frac{(1-0.0002R(0))}{R(0)}(e^t)^{\frac{-1}{12.5}}$$
$$\Rightarrow \frac{1}{R(t)} = 0.0002 + \frac{(1-0.0002R(0))}{R(0)}(e^t)^{\frac{-1}{12.5}}$$

$$\Rightarrow \frac{1}{R(t)} = 0.0002 + \frac{(1-0.0002R(0))}{R(0)}e^{-0.08t}$$
$$\Rightarrow R(t) = \frac{R(0)}{0.0002R(0)+(1-0.0002R(0))e^{-0.08t}}$$

Taking the limit of $R(t)$ will tell us what happens to the rabbit population in the absence of wolves.

$$\boxed{\lim_{t\to+\infty} R(t) = \lim_{t\to+\infty} \frac{R(0)}{0.0002R(0)+(1-0.0002R(0))e^{-0.08t}} = \frac{1}{0.0002} = 5000}$$

(b)Both R and W will be constant if both derivatives are zero:

Equilibrium occurs when $\frac{dR}{dt} = \frac{dW}{dt} = 0$:

$$\begin{cases} 0.08R(1-0.0002R) - 0.001RW = 0 \\ -0.02W + 0.00002RW = 0 \end{cases}$$

$$\Rightarrow \begin{cases} 0.08R - 1.6\times 10^{-5}R^2 - 0.001RW = 0 \\ -0.02W + 0.00002RW = 0 \end{cases}$$

$$\Rightarrow \begin{cases} 0.08R - 1.6\times 10^{-5}R^2 - 0.001RW = 0 \\ W(-0.02 + 0.00002R) = 0 \end{cases}$$

(i) $W = 0$, $R = 0$, zero populations

(ii) $W = 0$, $R = 5000$, in the absence of wolves the rabbit population will be 5000.

(iii) $W = 64$, $R = 1000$, both populations are stable

(c)The population stabilizes at 1000 rabbits and 64 wolves.

(d)