Short Answer Type Questions – I

[2 marks]

Que 1. In Fig. 10.9, O is the centre of the circle. If ∠ACB = 30°, then find ∠ABC.

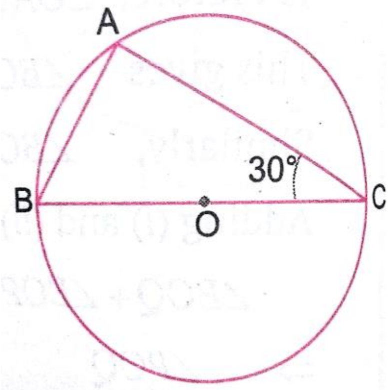

Fig. 10.9

Sol. ∠CAB = 90° (Angles in the semi-circle is 90°)

In ΔABC

∠ABC + ∠BCA + ∠CAB = 180°

∠ABC + 30° + 90° = 180°

∠ABC = 180° - 120°

⇒ ∠ABC = 60°

Que 2. In Fig. 10.10, if O is the centre of the circle then find ∠AOB.

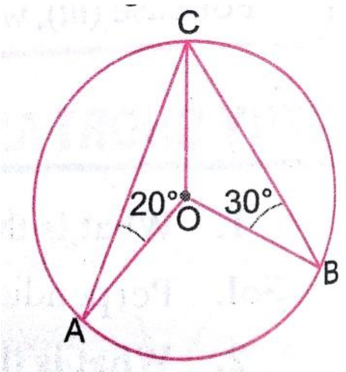

Fig. 10.10

Sol. OA = OC (radii of the same circle)

∴ ∠OCA = ∠OAC ⇒ ∠OCA = 20 °

Also, OB = OC

∴ ∠OCB = ∠OBC⇒ ∠OCB = 30°

Now, ∠ACB = 20° + 30° = 50°

∠AOB = 2 ∠ACB = 2 × 50° = 100°

Que 3. In Fig. 10.11, find the value of x and y.

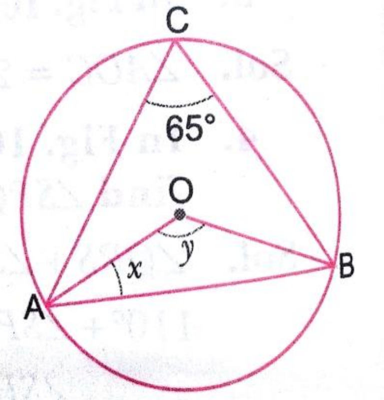

Fig. 10.11

Sol. $y = 2\angle ABC \Rightarrow y = 2 \times 65° \Rightarrow y = 130°$

$OA = OB$ (radii of the same circle)

$\therefore \angle OBA = \angle OAB \Rightarrow \angle OBA = x$

In ΔOAB,

$\angle OA + \angle OBA + y = 180°$

$x + x + 130° = 180°$

or $2x = 50°$ or $x = 25°$

Que 4. In Fig. 10.12, ABCD is a cyclic quadrilateral in which AB ||CD. If $\angle B$ = 65°, then find other angles.

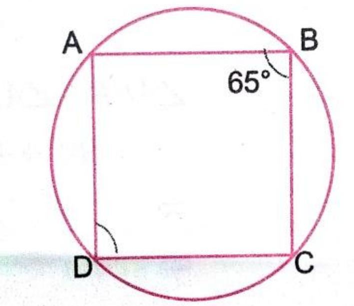

Fig. 10.12

Sol. $\angle B + \angle D = 180°$ (Opposite angles of cyclic quadrilateral)

$\Rightarrow 65° + \angle D = 180° - 65° = 115°$

Since AB ||C and BC is the transversal

$\therefore \angle B + \angle C = 180° \Rightarrow 65° + \angle C = 180°$

$\Rightarrow \angle C = 180° - 65°$

$\Rightarrow \angle C = 115°$

Now, $\angle A + 115° = 180°$ (Opposite angles of cyclin quadrilateral)

$60° + \angle AEC = 180°$

$\Rightarrow \angle AEC = 180° - 60° = 120°$

Que 5. In Fig. 10.15, $\angle ABC = 45°$, prove that $OA \perp OC$.

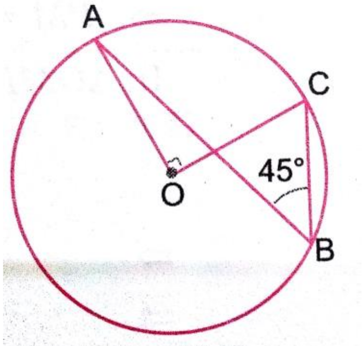

Fig. 10.15

Sol. As the angle subtended by an arc at the centre s twice the angle subtended by it at any point on the remaining part of the circle. Therefor,

$\angle AOC = 2\ \angle ABC$

$\Rightarrow \quad \angle AOC = 2 \times 45° = 90°$

Hence, $OA \perp OC$

Que 6. In Fig. 10.16, $\angle AOC = 120°$. Find $\angle BDC$.

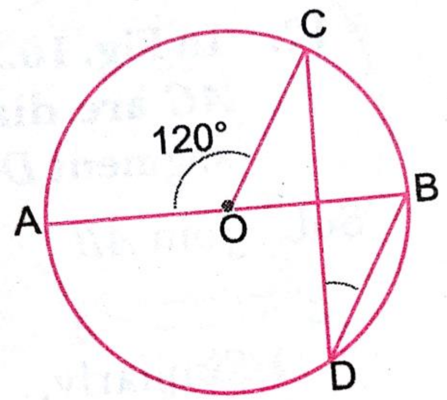

Fig. 10.16

Sol. $\angle AOC + \angle BOC = 180°$ (Linear pair)

$\Rightarrow \quad 120° + \angle BOC = 180°$

$\angle BOC = 180° - 120° = 60°$

Now, $\angle BOC = 2\ \angle BDC$

$\Rightarrow \quad 60° = 2\ \angle BDC$

$\Rightarrow \quad \angle BDC = 30°$

Que 7. In Fig. 10.17, O is the centre of a circle, find the value of x.

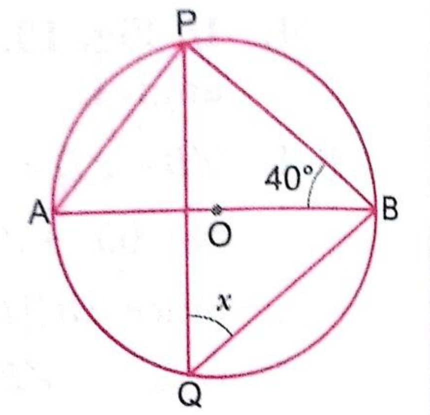

Fig. 10.17

Sol. In Δ APB,

$\angle APB = 90°$ (Angle in the semi-circle)

∠APB + ∠ABP + ∠BAP = 180°

90° + 40° + ∠BAP = 180°

⇒ ∠BAP = 180° - 130°

⇒ ∠ BAP = 50°

Now, ∠BQP = ∠BAP (Angles in the same segment)

∴ x = 50°

Que 8. Find the length of a chord which is at a distance of 12 cm from the centre of a circle of radius 13 cm.

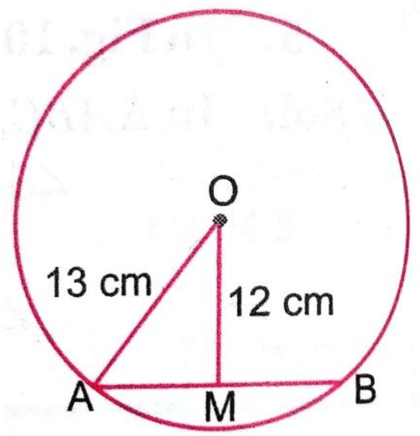

Fig. 10.18

Sol. Let AB be a chord of circle with centre O and radius 13 cm Draw OM ⊥ AB and join OA.

In the right triangle OMA, we have

$OA^2 = OM^2 + AM^2$

⇒ $13^2 = 12^2 + AM^2$

⇒ $AM^2 = 169 – 144 = 25$

As the perpendicular from the centre of a chord bisects the chord. Therefore,

$$AB = 2AM = 2 \times 5 = 10 \text{ cm.}$$

Que 9. The radius of a circle is 13 cm and the length of one of its chords is 24 cm. Find the distance of the chord from the centre.

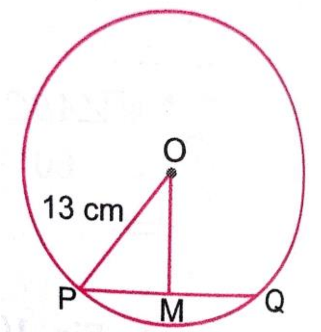

Fig. 10.19

Sol. Let PQ be a chord of a circle with centre O and radius 13 cm such that PQ = 24 cm.

From O, draw OM ⊥ PQ and join OP.

As, the Perpendicular from the centre of a circle to a chord bisects the chord.

∴ $PM = MQ = \frac{1}{2} PQ = \frac{1}{2} \times 24$

⇒ $PM = 12$ cm

In ΔPMP, we have

$$OP^2 = OM^2 + PM^2$$

$$13^2 = OM^2 + 12^2$$

⇒ $OM^2 = 169 - 144 = 25$

⇒ $OM = 5$ cm

Hence, the distance of the chord from the centre is 5 cm.

Que 10. In Fig. 10.20, two circles intersects at two points A and B. AD and AC are diameters to the circles. Prove that B lies on the line segment DC.

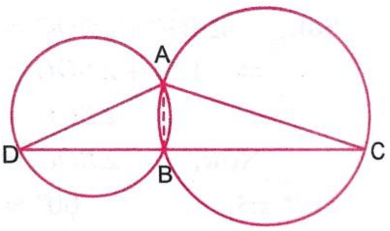

Fig. 10.20

Sol. Join AB ∠ABD = 90°

(Angles in a semicircle

Similarly, ∠ABC = 90°

So, $\angle ABD + \angle ABC = 90° + 90° = 180°$

Therefore, DBC is a line i.e., B lies on the light segment DC.

Que 11. In Fig. 10.21, AOB is a diameter of the circle and C, D, E are any three points on the semi-circle. Find the value of ∠ACD + ∠BED.

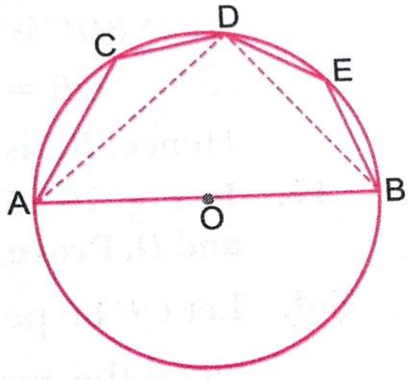

Fig. 10.21

Sol. Join BC,

Then, ∠ACB = 90° (Angle in the semi-circle)

Since DCBE is a cyclic quadrilateral.

∠BCD + ∠BED = 180°

Adding ∠ACB both the sides, we get

∠BCD + ∠BED + ∠ACB = ∠ACB + 180°

(∠BCD + ∠ACB) + ∠BED = 90° + 180°

∠ACD + ∠BED = 270°

Que 12. In Fig. 10.22, A, B, C and D are four points on a circle. AC and BD intersect at point E such that ∠BEC = 130° and ∠ECD = 20°. Find ∠BAC.

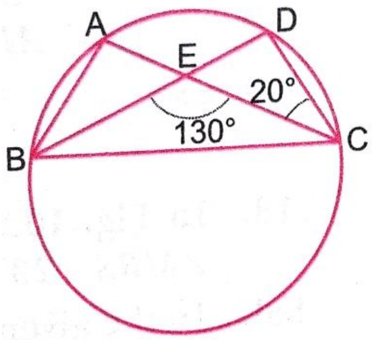

Fig. 10.22

Sol. Since the exterior angle of a triangle is equal to the sum of the interior opposite angles,

∴ ∠BEC = ∠ECD + ∠CDE

⇒ 130° = 20° + ∠CDE

⇒ ∠CDE = 130° - 20° = 110°

∠BDC = 110°

Now, ∠BAC = ∠BDC (Angles in the same segment)

∴ ∠BAC = 110°

Que 13. In Fig. 10.23, ∠ACB = 40°. Find ∠OAB.

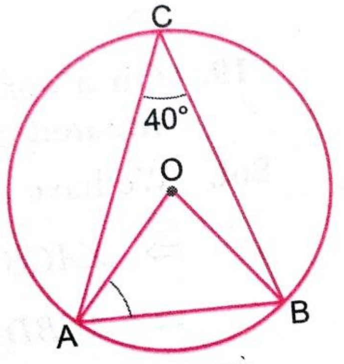

Fig. 10.23

Sol. Since OA = OB (Radii of the same circle)

∴ ∠OAB = ∠OBA

As the angle form by the act at the centre is twice the angle formed at any point in remaining part of the circle.

∴ ∠OAB = 2 ∠ACB = 2 × 40°

⇒ ∠AOB = 80°

In Δ AOB, we have

∠AOB + ∠OAB + ∠OBA = 180°

80° + ∠OAB + ∠OAB = 180° (∵ ∠OBA =$\angle OAB$)

⇒ 2 ∠OAB = 100° ⇒ ∠OAB = 50°

Que 14. In Fig. 10.24, ∠BAC = 30°. Show that BC is equal to the radius of the circumcircle of ΔABC whose centre is O.

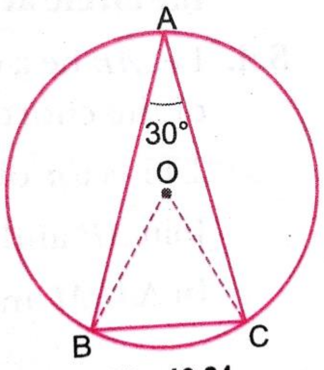

Fig. 10.24

Sol. ∠BOC = 2 ∠BAC

⇒ ∠BOC = 2 × 30° = 60°

Also, OC = OB (Radii of the same circle)

∴ ∠OCB = ∠OBC

In ΔOBC, we have

∠OBC + ∠OCB + ∠BOC = 180°

∠OBC + ∠OBC + 60° = 180°

⇒ 2∠OBC = 120° ⇒ ∠OBC = 60°

So, $\angle OBC = \angle OCB = \angle BOC = 60°$

∴ ΔBOC is an equilateral triangle.

∴ $OB = BC = OC$

Hence, BC is equal to the radius of the circumcircle.

Que 15. In Fig. 10.25, a line intersect two concentric circles with O at A, B, C and D, Prove that AB = CD.

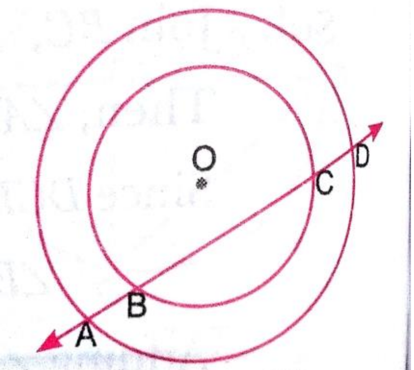

Fig. 10.25

Sol. Let OP be perpendicular from O on line l.

Since the perpendicular from the centre of a circle to a chord, bisects the chord. Therefore,

$$AP = DP \quad .....(i)$$

$$BP = CP \quad .....(ii)$$

Subtracting (ii) from (i), we get

$$AP - BP = DP - CP$$

⇒ $AP = CD$

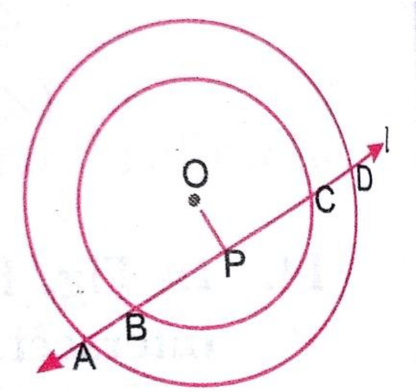

Fig. 10.26

Que 16. In Fig. 10.27, RS is diameter of the circle, NM is parallel to RS and ∠MRS = 29°, find ∠RNM.

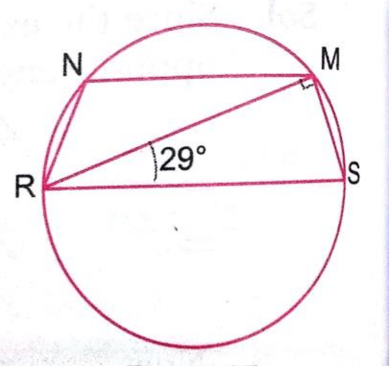

Fig. 10.27

Sol. In the Given figure ∠RMS = 90°

(Angle in the semi-circle as RS is diameter)

∴ ∠RSM = 180° - (29° + 90°) = 61°

∠RNM + ∠ ∠RSM = 180°

(Opposite angles if cyclin quadrilateral are supplementary)

∠RNM + 61° = 180°

⇒ ∠RNM = 119°

Que 17. On a common hypotenuse AB, two right triangle ACB and ADB are situated on opposite sides. Prove that ∠BAC = ∠BDC.

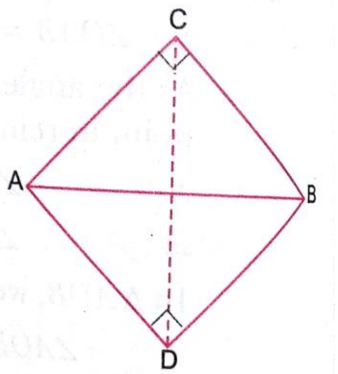

Fig. 10.28

We have ∠ACB = ∠ADB (Each 90°)

⇒ ∠ACB + ∠ADB = 90° + 90° 180°

⇒ ACBD is a cyclic quadrilateral

⇒ ∠BAC = ∠BDC (Angles in the same segment)

Que 18. If the perpendicular bisector of a chord AB of a circle PXAQBY intersects the circle at P and Q, then prove that arc PXA ≅ arc PYB.

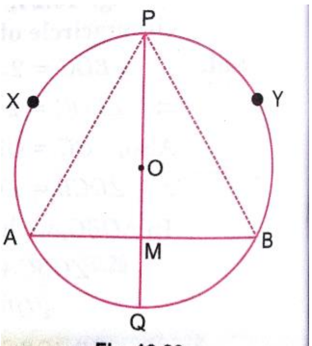

Fig. 10.29

Sol. Let AB be a chord of a circle having centre at O. Let PQ be the ⊥ bisector of the chord AB intersects it say at M.

⊥ Bisectors of the chord passes through the centre of the circle, i.e., O.

Join AP and BP.

In ΔAPM and ΔBPM

	AM= MB	(Given)
	∠PMA = ∠PMB	(90° each)
	PM = PM	(Common)
∴	ΔAPM ≅ ΔBPM	(SAS)
	PA = PB	(CPCT)

Hence. Arc PXA ≅ arc PYB

Que 19. In Fig. 10.30, P is the centre of the circle. Prove that ∠XPZ = 2(∠XYZ + ∠YXZ)

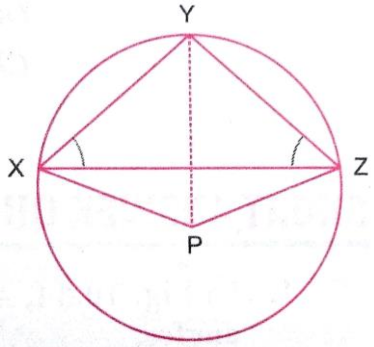

Fig. 10.30

Sol. Since arc XY subtends ∠XPY at the centre and ∠XZY at a point Z in the remaining part of the circle.

∴ ∠XPY = 2∠XZY …....(i)

Similarly, arc YZ subtends ∠YPZ at the centre and ∠YXZ at a point Y in the remaining part of the circle.

∴ ∠YPZ = 2∠YXZ ……..(ii)

Adding (i) and (ii),

$\angle XPY + \angle YPZ = 2\angle XYZ + 2\angle YXZ$

$\angle XPZ = 2(\angle XZY + \angle YXZ)$

Hence Prove.

Que 20. If BM and CN are the perpendiculars, drawn on the sides AB and AC of the ΔABC, then prove that the points B, C, M and N are cyclic.

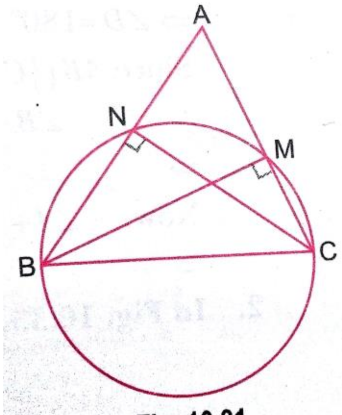

Fig. 10.31

Sol. Let us consider BC as a diameter of the circle.
Angles subtended by the diameter in a semicircle is 90°.
Given, $\angle BNC = \angle BMC = 90°$
So, the points M and N should be on the same circle.
Hence, BCMN form a cyclin quadrilateral.

Que 21. In Fig. 10.32, ∠ADC = 130° and chord BC = chord BE. Find ∠CBE.

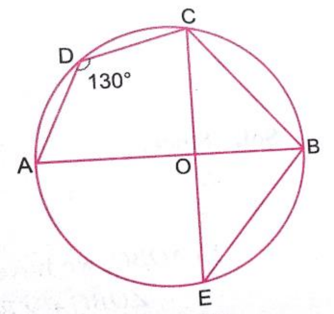

Fig. 10.32

Sol. Consider the points A, B, C and D. They formed a cyclin quadrilateral.

∴ $\angle ADC + \angle ABC = 180°$ (Opposite angles of cyclin quadrilateral)

$130° + \angle ABC = 180°$

$\angle ABC = 50°$

In ΔBOC and ΔBOE,

BC = BE (Equal chords)

OC = OE (Radii)

OB = OB (Common)

$\Delta BOC \cong \Delta BOE$ (SSS rule)

∴ $\angle OBC = \angle OBE = 50°$ (CPCT)

∴ $\angle CBE = \angle CBO + \angle EBO$

$= 50° + 50° = 100°$

Que 22. In Fig. 10.33, if OA = 10 cm, AB = 16 cm and OD ⊥ to AB. Find the value of CD.

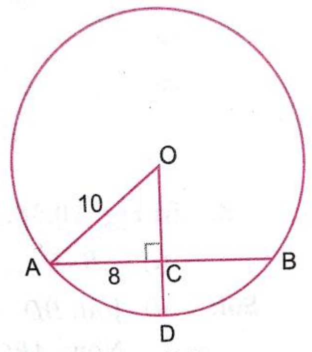

Fig. 10.33

Sol. As OD **is** ⊥ to AB

⇒ AC = CB

(⊥ from the centre to the chord bisects the chord)

∴ $AC = \frac{AB}{2} = 8cm$

In right ΔOCA,

$OA^2 = AC^2 + OC^2$

$(10)^2 = 8^2 + OC^2$

$OC^2 = 100 - 64$

$OC^2 = 36$

∴ $OC = \sqrt{36}$

OC = 6 cm

CD = OD – OC

= 10 – 6 = 4 cm. [∵ OA = OD = 10 cm (radii)]