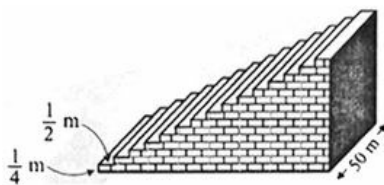


CBSE Test Paper 02
Chapter 05 Arithmetic Progression

1. Find the sum of first 'n' terms of an A.P.? **(1)**
 - a. $7n - 8$
 - b. $S = \frac{n}{2}[2a + (n - 1)d]$
 - c. $2n + 3$
 - d. $n^2 + 2$
2. The sum of first 24 terms of the list of numbers whose nth term is given by $a_n = 3 + 2n$ is **(1)**
 - a. 680
 - b. 672
 - c. 640
 - d. 600
3. The 17th term of an AP exceeds its 10th term by 7, then the common difference is **(1)**
 - a. -1
 - b. 1
 - c. 2
 - d. 0
4. The first term of an AP is 5, the last term is 45 and the sum is 400. The number of terms is **(1)**
 - a. 16
 - b. 20
 - c. 17
 - d. 18
5. If a, b and c are in A. P., then the value of $\frac{a-b}{b-c}$ is **(1)**
 - a. $\frac{a}{b}$
 - b. 1
 - c. $\frac{c}{a}$
 - d. $\frac{b}{c}$
6. Find the common difference of the A.P. and write the next two terms of A.P.
119,136,153,170,..... **(1)**

7. If $2x$, $x + 10$, $3x + 2$ are in A.P., find the value of x . **(1)**
8. Find 7th term from the end of the AP : 7, 10, 13, ..., 184. **(1)**
9. Find k , if the given value of x is the k^{th} term of the given AP 25, 50, 75, 100, ..., $x = 1000$. **(1)**
10. Find the 11th term from the end of the AP 10, 7, 4, ..., -62. **(1)**
11. Find the common difference d and write three more terms.
 $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, \dots$ **(2)**
12. The house of a row are numbered consecutively from 1 to 49. Show that there is a value of x such that the sum of the numbers of the houses preceding the house numbered x is equal to the sum of the numbers of the following it. Find this value of x . **(2)**
13. Divide 24 in three parts such that they are in AP and their product is 440. **(2)**
14. Determine an A.P. whose third term is 9 and when fifth term is subtracted from 8th term, we get 6. **(3)**
15. Find the second term and n^{th} term of an A.P. whose 6th term is 12 and the 8th term is 22. **(3)**
16. A small terrace at a football ground comprises of 15 steps each of which is 50 m long and built of solid concrete. Each step has a rise of $\frac{1}{4}$ m and a tread of $\frac{1}{2}$ m (see figure). Calculate the total volume of concrete required to build the terrace. **(3)**



17. If 9th term of an A.P. is zero, prove that its 29th term is double the 19th term. **(3)**
18. If the sum of Rs 1890 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is Rs 50 less than its preceding prize. Then find the value of each of the prizes. **(4)**
19. Let a sequence be defined by $a_1 = 1$, $a_2 = 1$ and, $a_n = a_{n-1} + a_{n-2}$ for all $n > 2$.
Find $\frac{a_{n+1}}{a_n}$ for $n = 1, 2, 3, 4$. **(4)**
20. Let there be an A.P. with first term ' a ', common difference ' d '. If a_n denotes its n^{th} term and S_n the sum of first n terms, find. n and S_n , if $a = 5$, $d = 3$ and $a_n = 50$. **(4)**

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Solution

1. b. $S = \frac{n}{2} [2a + (n - 1)d]$

Explanation: let $a = 1^{\text{st}}$ term, $d =$ common difference,

$S_n =$ sum of 1st n terms of an AP

then $S_n = (a) + (a + d) + (a + 2d) + \dots \{a + (n - 3)d\} + \{a + (n - 2)d\} + \{a + (n - 1)d\}$
..... (i)

Now Rewrite S_n as follows

$S_n = \{a + (n-1)d\} + \{a + (n-2)d\} + \{a + (n-3)d\} \dots (a + 3d) + (a + 2d) + (a + d) + a$
..... (ii)

adding the terms i and ii vertically

adding 1st term of both we get $(a) + \{a + (n-1)d\} = 2a + (n-1)d$

adding 2nd term of both $(a + d) + \{a + (n - 2)d\} = 2a + (n-1)d$

adding 3rd terms of both $(a + 2d) + \{a + (n-3)d\} = 2a + (n-1)d$

since there are n terms in each of the equations i and ii , adding both the equations we get

$$2S_n = n\{2a + (n-1)d\}$$

$$S_n = \frac{n}{2} \{2a + (n-1)d\}.$$

2. b. 672

Explanation: Given: $a_n = 3 + 2n$

$$\therefore a_1 = 3 + 2 \times 1 = 3 + 2 = 5$$

$$a_2 = 3 + 2 \times 2 = 3 + 4 = 7$$

$$\therefore d = a_2 - a_1 = 7 - 5 = 2$$

$$\text{Now, } S_n = \frac{n}{2} [2a + (n - 1)d]$$

$$\Rightarrow S_{24} = \frac{24}{2} [2 \times 5 + (24 - 1) 2]$$

$$\Rightarrow S_{24} = 12 [10 + 23 \times 2] = 12 [10 + 46] = 672$$

3. b. 1

Explanation: According to question,

Given that the 17th term of an A.P exceeds its 10th term by 7 .

$$d = ?$$

$$\Rightarrow a + 16d = a + 9d + 7$$

$$\Rightarrow 16d - 9d = 7$$

$$\Rightarrow 7d = 7$$

$$\Rightarrow d = \frac{7}{7} = 1$$

Therefore , common difference = 1.

4. a. 16

Explanation: Given: $a = 5, l = 45, S_n = 400$

$$\therefore S_n = \frac{n}{2} (a + l)$$

$$\Rightarrow 400 = \frac{n}{2} (5 + 45)$$

$$\Rightarrow 800 = n \times 50$$

$$\Rightarrow n = 16$$

5. b. 1

Explanation: If a, b and c are in A.P.,

$$b - a = c - b$$

$$-(a - b) = -(b - c)$$

$$a - b = b - c$$

dividing both sides by $b - c$

$$\frac{a-b}{b-c} = \frac{b-c}{b-c}$$

$$\frac{a-b}{b-c} = 1$$

6. Given A.P is

119, 136, 153, 170.....

We know that common difference is difference between any consecutive terms of an A.P.

$$\text{So, common difference} = 136 - 119 = 17$$

$$\text{5th term} = 170 + 17 = 187 \text{ (} a_5 = a + 4d \text{)}$$

$$\text{6th term} = 187 + 17 = 204. \text{ (} a_6 = a + 5d \text{)}$$

7. If $2x, x + 10, 3x + 2$ are in A.P., we have to find the value of x .

Since, $2x, x + 10, 3x + 2$ are in A.P. therefore $2(x + 10) = 2x + 3x + 2$

$$\Rightarrow 2x + 20 = 5x + 2$$

$$\Rightarrow 3x = 18 \Rightarrow x = 6$$

8. Given, AP is 7, 10, 13, ..., 184.

we have to find 7th term from the end

reversing the AP, 184, ..., 13, 10, 7.

now, d = common difference = $7 - 10 = -3$

$$\begin{aligned}\therefore 7^{\text{th}} \text{ term from the beginning of AP} &= a + (7 - 1)d = a + 6d \\ &= 184 + (6 \times (-3)) \\ &= 184 - 18 = 166\end{aligned}$$

9. $a = 25$, $d = 50 - 25 = 25$, $x = 1000$

A.T.Q., $a_k = x$

$$\Rightarrow a + (k - 1)d = 1000$$

$$\Rightarrow 25 + (k - 1)25 = 1000$$

$$\Rightarrow (k - 1)25 = 975 \Rightarrow k - 1 = \frac{975}{25}$$

$$\Rightarrow k - 1 = 39 \Rightarrow k = 40$$

10. We have

$a = 10$, $d = (7 - 10) = -3$, $l = -62$ and $n = 11$.

\therefore 11th term from the end = $[l - (n - 1) \times d]$

$$= \{-62 - (11 - 1) \times (-3)\}$$

$$= (-62 + 30) = -32.$$

Hence, the 11th term from the end of the given AP is -32.

11. $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, \dots$

$$a_2 - a_1 = \sqrt{8} - \sqrt{2} = 2\sqrt{2} - \sqrt{2} = \sqrt{2}$$

$$a_3 - a_2 = \sqrt{18} - \sqrt{8} = 3\sqrt{2} - 2\sqrt{2} = \sqrt{2}$$

$$a_4 - a_3 = \sqrt{32} - \sqrt{18} = 4\sqrt{2} - 3\sqrt{2} = \sqrt{2}$$

i.e. $a_{k+1} - a_k$ is the same every time.

So, the given list of numbers forms an AP with the common difference $d = \sqrt{2}$.

The next three terms are:

$$\sqrt{32} + \sqrt{2} = 4\sqrt{2} + \sqrt{2} = 5\sqrt{2} = \sqrt{50}$$

$$5\sqrt{2} + \sqrt{2} = 6\sqrt{2} = \sqrt{72}$$

$$\text{and } 6\sqrt{2} + \sqrt{2} = 7\sqrt{2} = \sqrt{98}$$

12. The consecutive numbers on the houses of a row are 1, 2, 3, ..., 49

Clearly this list of number forming an AP.

Here, $a = 1$

$$d = 2 - 1 = 1$$

$$S_{x-1} = S_{49} - S_x$$

$$\Rightarrow \frac{x-1}{2} [2a + (x-1-1)d] - \frac{x}{2} [2a + (x-1)d]$$

$$\because S_n = \frac{n}{2} [2a + (n-1)d]$$

$$\Rightarrow \frac{x-1}{2} [2(1) + (x-2)(1)] = \frac{49}{2} [2(1) + (48)(1)] - \frac{x}{2} [2(1) + (x-1)(1)]$$

$$\Rightarrow \frac{x-1}{2} [x] = 1225 - \frac{x(x+1)}{2}$$

$$\Rightarrow \frac{(x-1)(x)}{2} + \frac{x(x+1)}{2} = 1225$$

$$\Rightarrow \frac{x}{2} (x-1+x+1) = 1225$$

$$\Rightarrow x^2 = 1225$$

$$\Rightarrow x = \sqrt{1225} \Rightarrow x = 35$$

Hence, the required value of x is 35.

13. Let the required numbers in A.P. are $(a - d)$, a and $(a + d)$.

$$\text{Sum of these numbers} = (a - d) + a + (a + d) = 3a$$

$$\text{Product of these numbers} = (a - d) \times a \times (a + d) = a(a^2 - d^2)$$

But given, sum = 24 and product = 440

$$\therefore 3a = 24 \Rightarrow a = 8$$

$$\text{and } a(a^2 - d^2) = 8(64 - d^2) = 440 \quad [\because a = 8]$$

$$\text{Or, } 64 - d^2 = 55$$

$$\text{Or, } d^2 = 64 - 55$$

$$\Rightarrow d^2 = 9$$

$$\Rightarrow d = \pm 3$$

When $a = 8$ and $d = 3$

The required numbers are (5, 8, 11).

When $a = 8$ and $d = -3$

The required numbers are (11, 8, 5).

14. Let the first term be a and the common difference be d .

$$a_n = a + (n - 1)d$$

Here given, $a_3 = 9$

$$\text{or, } a + 2d = 9 \dots(i)$$

$$a_8 - a_5 = 6$$

$$\text{or, } (a + 7d) - (a + 4d) = 6$$

$$a + 7d - a - 4d = 6$$

$$\text{or, } 3d = 6$$

$$\text{or, } d = 2 \dots(\text{ii})$$

Substituting this value of d from (ii) in (i), we get

$$\text{or, } a + 2(2) = 9$$

$$\text{or, } a + 4 = 9$$

$$\text{or } a = 9 - 4$$

$$\text{or, } a = 5$$

$$a = 5 \text{ and } d = 2$$

So, A.P. is 5,7,9,11,....

15. Given $a_6 = 12$

$$\Rightarrow a + (6 - 1)d = 12$$

$$\Rightarrow a + 5d = 12 \dots\dots\dots(\text{i})$$

$$\text{and, } a_8 = 22$$

$$\Rightarrow a + (8 - 1)d = 22$$

$$\Rightarrow a + 7d = 22 \dots\dots\dots(\text{ii})$$

Subtracting equation (i) from (ii), we get

$$(a + 7d) - (a + 5d) = 22 - 12$$

$$\Rightarrow a + 7d - a - 5d = 10$$

$$\Rightarrow 2d = 10$$

$$\Rightarrow d = \frac{10}{2} = 5$$

Using value of d in equation (i), we get

$$a + 5 \times 5 = 12$$

$$\Rightarrow a = 12 - 25 = -13$$

$$\text{Second term}(a_2) = a + (2 - 1)d$$

$$= -13 + 1(5)$$

$$= -13 + 5 = -8$$

$$\text{nth term}(a_n) = a + (n - 1)d$$

$$= -13 + (n - 1)(5)$$

$$= 5n - 18$$

16. Volume of concrete required to build the first step, second step, third step, (in m²) are

$$\frac{1}{4} \times \frac{1}{2} \times 50, \left(2 \times \frac{1}{4}\right) \times \frac{1}{2} \times 50, \left(3 \times \frac{1}{4}\right) \times \frac{1}{2} \times 50$$

$$\Rightarrow \frac{50}{8}, 2 \times \frac{50}{8}, 3 \times \frac{50}{8}, \dots$$

$$\therefore \text{Total volume of concrete required} = \frac{50}{8} + 2 \times \frac{50}{8} + 3 \times \frac{50}{8} + \dots$$

$$= \frac{50}{8} [1 + 2 + 3 + \dots]$$

$$S_n = \frac{n}{2} [(2a + (n - 1)d)]$$

$$S_{15} = \frac{50}{8} \times \frac{15}{2} [2 \times 1 + (15 - 1) \times 1] [\because n = 15]$$

$$= \frac{50}{8} \times \frac{15}{2} \times 16 = 750 \text{ m}^3$$

17. We have,

$$a_9 = 0$$

$$\Rightarrow a + (9 - 1)d = 0$$

$$\Rightarrow a + 8d = 0$$

$$\Rightarrow a = -8d$$

$$\text{To prove: } a_{29} = 2a_{19}$$

$$\text{Proof: LHS} = a_{29}$$

$$= a + (29 - 1)d$$

$$= a + 28d$$

$$= -8d + 28d = 20d$$

$$\text{RHS} = 2a_{19}$$

$$= 2a + (19 - 1)d$$

$$= 2[-8d + 18d]$$

$$= 2 \times 10d$$

$$= 20d$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence, 29th term is double the 19th term.

18. Let 1st prize be Rs x.

The series in A.P. is x, x - 50, x - 100, x - 150,

Where a = x, d = - 50, S_n = 1890, n = 7.

As we know that

$$\begin{aligned}
S_n &= \frac{n}{2}[2a + (n-1)d] \\
\Rightarrow \frac{7}{2}[2x + (6)(-50)] &= 1890 \\
\Rightarrow \frac{7}{2}[2x - 300] &= 1890 \\
\Rightarrow 2x - 300 &= 1890(2/7) \\
\Rightarrow 2x &= 540 + 300 \\
\Rightarrow x &= \frac{840}{2} = 420
\end{aligned}$$

The prizes are: Rs 420, Rs 370, Rs 320, Rs 270, Rs 220, Rs 170, Rs 120.

19. Given $a_1 = 1$ and $a_2 = 1$ and $a_n = a_{n-1} + a_{n-2}$

$$\text{So } a_3 = a_2 + a_1 = 1 + 1 = 2$$

$$a_4 = a_3 + a_2 = 2 + 1 = 3$$

$$a_5 = a_4 + a_3 = 3 + 2 = 5$$

Now putting $n = 1, 2, 3$ and 4 in a_{n+1}/a_n we get

$$\begin{aligned}
\frac{a_2}{a_1} &= \frac{1}{1} = 1 \\
\frac{a_3}{a_2} &= \frac{2}{1} = 2 \\
\frac{a_4}{a_3} &= \frac{3}{2} = 1.5 \\
\frac{a_5}{a_4} &= \frac{5}{3} = 1.67
\end{aligned}$$

20. Given,

First term(a) = 5

Common difference(d) = 3

and, n th term (a_n) = 50

$$\Rightarrow a + (n-1)d = 50$$

$$\Rightarrow 5 + (n-1)(3) = 50$$

$$\Rightarrow 5 + 3n - 3 = 50$$

$$\Rightarrow 3n = 50 - 5 + 3$$

$$\Rightarrow 3n = 48$$

$$\Rightarrow n = \frac{48}{3} = 16$$

Therefore, $S_n = \frac{n}{2}[a + a_n]$

$$= \frac{16}{2}[5 + 50]$$

$$= 8 \times 55 = 440$$