MATRICES (XII, R. S. AGGARWAL)

EXERCISE 5A (Pg. No.: 154)

1. If $A=\begin{bmatrix} 5 & -2 & 6 & 1 \\ 7 & 0 & 8 & -3 \\ \sqrt{2} & 3/5 & 4 & 9 \end{bmatrix}$, then write

(i) The number of rows in A

(ii) The number of columns in A

(iii) The order of the matrix A

(iv) The number of all entries in A

(v) The element $a_{23}, a_{31}, a_{14}, a_{33}, a_{22}$ of A.

Sol. **(i)** The number of rows in $A=3$

(ii) The number of columns in $A=4$

(iii) The order of the matrix $A=3\times4$

(iv) The number of all entries in $A=12$

(v) The element $a_{23}=8,\ a_{31}=\sqrt{2},\ a_{14}=1,\ a_{33}=4,\ a_{22}=0$

2. Write the order of each of the following matrices :

(i) $A=\begin{bmatrix} 3 & 5 & 4 & -2 \\ 0 & \sqrt{3} & -1 & 4/9 \end{bmatrix}$

(ii) $B=\begin{bmatrix} 6 & -5 \\ 1/2 & 3/4 \\ -2 & -1 \end{bmatrix}$

(iii) $C=\begin{bmatrix} 7 & -\sqrt{2} & 5 & 0 \end{bmatrix}$

(iv) $D=\begin{bmatrix} 8 & -3 \end{bmatrix}$

(v) $E=\begin{bmatrix} -2 \\ 3 \\ 0 \end{bmatrix}$

(vi) $F=[6]$

Sol. **(i)** $A=\begin{bmatrix} 3 & 5 & 4 & -2 \\ 0 & \sqrt{3} & -1 & 4/9 \end{bmatrix}$, order $=(2\times4)$

(ii) $B=\begin{bmatrix} 6 & -5 \\ 1/2 & 3/4 \\ -2 & -1 \end{bmatrix}$, order $=(3\times2)$

(iii) $C=\begin{bmatrix} 7 & -\sqrt{2} & 5 & 0 \end{bmatrix}$, order $=(1\times4)$

(iv) $D=\begin{bmatrix} 8 & -3 \end{bmatrix}$, order $=(1\times2)$

(v) $E=\begin{bmatrix} -2 \\ 3 \\ 0 \end{bmatrix}$, order $=(3\times1)$

(vi) $F=[6]$, order $=(1\times1)$

3. If a matrix has 18 elements, what are the possible orders it can have?

Sol. We know that a matrix of order $m\times n$ has mn elements.

Hence, all possible orders of a matrix having 18 element are

$(18\times1), (1\times18), (9\times2), (2\times9), (6\times3), (3\times6)$

4. Find all possible orders of matrices having 7 elements.

Sol. We know that a matrix of order $m\times n$ has mn elements.

Hence, all possible orders of a matrix having 7 element are $(7\times1), (1\times7)$.

5. Construct a 3×2 matrix whose elements are given by $a_{ij}=(2i-j)$.

Sol. A 3×2 matrix has 3 rows and 2 columns.

In general, a 3×2 matrix is given by $A=\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}_{3\times 2}$,

Thus $a_{ij}=(2i-j)$ for $i=1, 2, 3$ and $j=1, 2$

$a_{11}=2-1=1$, $a_{12}=2(1)-2=2-2=0$, $a_{21}=2(2)-1=4-1=3$

$a_{22}=2(2)-2=4-2=2$, $a_{31}=2(3)-1=(6-1)=5$, $a_{32}=2(3)-2=6-2=4$

Hence, $A=\begin{bmatrix} 1 & 0 \\ 3 & 2 \\ 5 & 4 \end{bmatrix}$

6. Construct a 4×3 matrix whose elements are given by $a_{ij}=\frac{i}{j}$.

Sol. Let $A=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{41} & a_{42} & a_{43} \end{bmatrix}_{(4\times 3)}$. Thus $a_{ij}=\frac{i}{j}$ for $i=1, 2, 3, 4$ and $j=1, 2, 3$

$a_{11}=\frac{1}{1}=1$, $a_{12}=\frac{1}{2}$, $a_{13}=\frac{1}{3}$, $a_{21}=\frac{2}{1}=2$, $a_{22}=\frac{2}{2}=1$, $a_{23}=\frac{2}{3}$,

$a_{31}=\frac{3}{1}=3$, $a_{32}=\frac{3}{2}$, $a_{33}=\frac{3}{3}=1$, $a_{41}=\frac{4}{1}=4$, $a_{42}=\frac{4}{2}=2$, $a_{43}=\frac{4}{3}$

$$A=\begin{bmatrix} 1 & 1/2 & 1/3 \\ 2 & 1 & 2/3 \\ 3 & 3/2 & 1 \\ 4 & 2 & 4/3 \end{bmatrix}_{4\times 3}$$

7. Construct a 2×2 matrix whose elements are $a_{ij}=\frac{(i+2j)^2}{2}$.

Sol. Let, $A=\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_{(2\times 2)}$. Thus $a_{ij}=\frac{(i+2j)^2}{2}$ for $i=1, 2$ and $j=1, 2$.

$\therefore\ a_{11}=\frac{\{1+2(1)\}^2}{2}=\frac{(1+2)^2}{2}=\frac{(3)^2}{2}=\frac{9}{2}$, $a_{12}=\frac{\{1+2(2)\}^2}{2}=\frac{(1+4)^2}{2}=\frac{(5)^2}{2}=\frac{25}{2}$

$a_{21}=\frac{\{2+2(1)\}^2}{2}=\frac{(2+2)^2}{2}=\frac{(4)^2}{2}=\frac{16}{2}=8$, $a_{22}=\frac{\{2+2(2)\}^2}{2}=\frac{(2+4)^2}{2}=\frac{(6)^2}{2}=\frac{36}{2}=18$

Hence, $A=\begin{bmatrix} 9/2 & 25/2 \\ 8 & 18 \end{bmatrix}_{(2\times 2)}$

8. Construct a 2×3 matrix whose elements are $a_{ij}=\frac{(i-2j)^2}{2}$.

Sol. Let $A=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}_{(2\times 3)}$. Thus $a_{ij}=\frac{(i-2j)^2}{2}$ for $i=1, 2$ and $j=1, 2, 3$

$\therefore\ a_{11}=\frac{\{1-2(1)\}^2}{2}=\frac{(1-2)^2}{2}=\frac{(-1)^2}{2}=\frac{1}{2},\ a_{12}=\frac{\{1-2(2)\}^2}{2}=\frac{(1-4)^2}{2}=\frac{(-3)^2}{2}=\frac{9}{2}$

$a_{13}=\frac{\{1-2(3)\}^2}{2}=\frac{(1-6)^2}{2}=\frac{(-5)^2}{2}=\frac{25}{2},\ a_{21}=\frac{\{2-2(1)\}^2}{2}=\frac{(2-2)^2}{2}=0$

$a_{22}=\frac{\{2-2(2)\}^2}{2}=\frac{(2-4)^2}{2}=\frac{(-2)^2}{2}=\frac{4}{2}=2,\ a_{23}=\frac{\{2-2(3)\}^2}{2}=\frac{(2-6)^2}{2}=\frac{(-4)^2}{2}=\frac{16}{2}=8$

Hence, $A=\begin{bmatrix}1/2 & 9/2 & 25/2\\ 0 & 2 & 8\end{bmatrix}_{(2\times3)}$

9. Construct a 3×4 matrix whose elements are given by $a_{ij}=\frac{1}{2}|-3i+j|$.

Sol. Let $A=\begin{bmatrix}a_{11} & a_{12} & a_{13} & a_{14}\\ a_{21} & a_{22} & a_{23} & a_{24}\\ a_{31} & a_{32} & a_{33} & a_{34}\end{bmatrix}_{(3\times4)}$. Thus $a_{ij}=\frac{1}{2}|-3i+j|$ for $i=1,2,3$ and $j=1,2,3,4$

$\therefore\ a_{11}=\frac{1}{3}|-3(1)+1|=\frac{1}{2}|-3+1|=\frac{1}{2}\times2=1,\qquad a_{12}=\frac{1}{2}|-3(1)+2|=\frac{1}{2}|-3+2|=\frac{1}{2}\times1=\frac{1}{2}$

$a_{13}=\frac{1}{2}|-3(1)+3|=\frac{1}{2}|-3+3|=0,\qquad a_{14}=\frac{1}{2}|-3(1)+4|=\frac{1}{2}|-3+4|=\frac{1}{2}\times1=\frac{1}{2}$

$a_{21}=\frac{1}{2}|-3(2)+1|=\frac{1}{2}|-6+1|=\frac{1}{2}\times5=\frac{5}{2},\qquad a_{22}=\frac{1}{2}|-3(2)+2|=\frac{1}{2}|-6+2|=\frac{1}{2}\times4=2$

$a_{23}=\frac{1}{2}|-3(2)+3|=\frac{1}{2}|-6+3|=\frac{1}{2}\times3=\frac{3}{2},\qquad a_{24}=\frac{1}{2}|-3(2)+4|=\frac{1}{2}|-6+4|=\frac{1}{2}\times2=1$

$a_{31}=\frac{1}{2}|-3(3)+1|=\frac{1}{2}|-9+1|=\frac{1}{2}\times8=4,\qquad a_{32}=\frac{1}{2}|-3(3)+2|=\frac{1}{2}|-9+2|=\frac{1}{2}\times7=\frac{7}{2}$

$a_{33}=\frac{1}{2}|-3(3)+3|=\frac{1}{2}|-9+3|=\frac{1}{2}\times6=3,\qquad a_{34}=\frac{1}{2}|-3(3)+4|=\frac{1}{2}|-9+4|=\frac{1}{2}\times5=\frac{5}{2}$

Hence, $A=\begin{bmatrix}1 & 1/2 & 0 & 1/2\\ 5/2 & 2 & 3/2 & 1\\ 4 & 7/2 & 3 & 5/2\end{bmatrix}_{(3\times4)}$

EXERCISE 5B (Pg.No.: 167)

1. If $A=\begin{bmatrix}2 & -3 & 5\\ -1 & 0 & 3\end{bmatrix}$ and $B=\begin{bmatrix}3 & 2 & -2\\ 4 & -3 & 1\end{bmatrix}$, Verify that $(A+B)=(B+A)$.

Sol. $\text{LHS}=(A+B)=\begin{bmatrix}2 & -3 & 5\\ -1 & 0 & 3\end{bmatrix}+\begin{bmatrix}3 & 2 & -2\\ 4 & -3 & 1\end{bmatrix}=\begin{bmatrix}2+3 & -3+2 & 5+(-2)\\ -1+4 & 0+(-3) & 3+1\end{bmatrix}=\begin{bmatrix}5 & -1 & 3\\ 3 & -3 & 4\end{bmatrix}$

$\text{RHS}=(B+A)=\begin{bmatrix}3 & 2 & -2\\ 4 & -3 & 1\end{bmatrix}+\begin{bmatrix}2 & -3 & 5\\ -1 & 0 & 3\end{bmatrix}=\begin{bmatrix}3+2 & 2+(-3) & -2+5\\ 4+(-1) & -3+0 & 1+3\end{bmatrix}=\begin{bmatrix}5 & -1 & 3\\ 3 & -3 & 4\end{bmatrix}$

$\therefore$ LHS = RHS. $\Rightarrow (A+B)=(B+A)$. Hence proved.

2. If $A=\begin{bmatrix}3 & 5\\ -2 & 0\\ 6 & -1\end{bmatrix}, B=\begin{bmatrix}-1 & -3\\ 4 & 2\\ -2 & 3\end{bmatrix}$ and $C=\begin{bmatrix}0 & 2\\ 3 & -4\\ 1 & 6\end{bmatrix}$ verify that $(A+B)+C=A+(B+C)$.

Sol. $LHS = (A+B)+C = \left(\begin{bmatrix} 3 & 5 \\ -2 & 0 \\ 6 & -1 \end{bmatrix} + \begin{bmatrix} -1 & -3 \\ 4 & 2 \\ -2 & 3 \end{bmatrix}\right) + C = \begin{bmatrix} 3+(-1) & 5+(-3) \\ -2+4 & 0+2 \\ 6+(-2) & -1+3 \end{bmatrix} + C$

$$= \begin{bmatrix} 2 & 2 \\ 2 & 2 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ 3 & -4 \\ 1 & 6 \end{bmatrix} = \begin{bmatrix} 2+0 & 2+2 \\ 2+3 & 2+(-4) \\ 4+1 & 2+6 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 5 & -2 \\ 5 & 8 \end{bmatrix}$$

$RHS = A+(B+C) = A + \left(\begin{bmatrix} -1 & -3 \\ 4 & 2 \\ -2 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ 3 & -4 \\ 1 & 6 \end{bmatrix}\right) = A + \begin{bmatrix} -1+0 & -3+2 \\ 4+3 & 2+(-4) \\ -2+1 & 3+6 \end{bmatrix} = A + \begin{bmatrix} -1 & -1 \\ 7 & -2 \\ -1 & 9 \end{bmatrix}$

$$= \begin{bmatrix} 3 & 5 \\ -2 & 0 \\ 6 & -1 \end{bmatrix} + \begin{bmatrix} -1 & -1 \\ 7 & -2 \\ -1 & 9 \end{bmatrix} = \begin{bmatrix} 3+(-1) & 5+(-1) \\ -2+7 & 0+(-2) \\ 6+(-1) & -1+9 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 5 & -2 \\ 5 & 8 \end{bmatrix}$$

$\therefore$ LHS = RHS. $\Rightarrow (A+B)+C = A+(B+C)$. Hence proved.

3. If $A = \begin{bmatrix} 3 & 1 & 2 \\ 1 & 2 & -3 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 0 & 4 \\ 5 & -3 & 2 \end{bmatrix}$ find $(2A-B)$.

Sol. $2A-B = 2\begin{bmatrix} 3 & 1 & 2 \\ 1 & 2 & -3 \end{bmatrix} - \begin{bmatrix} -2 & 0 & 4 \\ 5 & -3 & 2 \end{bmatrix} = \begin{bmatrix} 6 & 2 & 4 \\ 2 & 4 & -6 \end{bmatrix} - \begin{bmatrix} -2 & 0 & 4 \\ 5 & -3 & 2 \end{bmatrix}$

$$= \begin{bmatrix} 6-(-2) & 2-0 & 4-4 \\ 2-5 & 4-(-3) & -6-2 \end{bmatrix} = \begin{bmatrix} 8 & 2 & 0 \\ -3 & 7 & -8 \end{bmatrix}$$

4. Let $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$ and $C = \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$. Find : (i) $A+2B$ (ii) $B-4C$ (iii) $A-2B+3C$.

Sol. **(i)** $A+2B = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} + 2\begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} 2 & 6 \\ -4 & 10 \end{bmatrix} = \begin{bmatrix} 2+2 & 4+6 \\ 3+(-4) & 2+10 \end{bmatrix} = \begin{bmatrix} 4 & 10 \\ -1 & 12 \end{bmatrix}$

(ii) $B-4C = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} - 4\begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} - \begin{bmatrix} -8 & 20 \\ 12 & 16 \end{bmatrix} = \begin{bmatrix} 1-(-8) & 3-20 \\ -2-12 & 5-16 \end{bmatrix} = \begin{bmatrix} 9 & -17 \\ -14 & -11 \end{bmatrix}$

(iii) $A-2B+3C = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} - 2\begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} + 3\begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 6 \\ -4 & 10 \end{bmatrix} + \begin{bmatrix} -6 & 15 \\ 9 & 12 \end{bmatrix}$

$$= \begin{bmatrix} 2-2+(-6) & 4-6+15 \\ 3-(-4)+9 & 2-10+12 \end{bmatrix} = \begin{bmatrix} -6 & 13 \\ 16 & 4 \end{bmatrix}$$

5. Let $A = \begin{bmatrix} 0 & 1 & -2 \\ 5 & -1 & -4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -3 & -1 \\ 0 & -2 & 5 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & -5 & 1 \\ -4 & 0 & 6 \end{bmatrix}$. Compute $5A-3B+4C$.

Sol. $5A-3B+4C = 5\begin{bmatrix} 0 & 1 & -2 \\ 5 & -1 & -4 \end{bmatrix} - 3\begin{bmatrix} 1 & -3 & -1 \\ 0 & -2 & 5 \end{bmatrix} + 4\begin{bmatrix} 2 & -5 & 1 \\ -4 & 0 & 6 \end{bmatrix}$

$$= \begin{bmatrix} 0 & 5 & -10 \\ 25 & -5 & -20 \end{bmatrix} - \begin{bmatrix} 3 & -9 & -3 \\ 0 & -6 & 15 \end{bmatrix} + \begin{bmatrix} 8 & -20 & 4 \\ -16 & 0 & 24 \end{bmatrix}$$

$$= \begin{bmatrix} 0-3+8 & 5-(-9)+(-20) & -10-(-3)+4 \\ 25-0+(-16) & -5-(-6)+0 & -20-15+24 \end{bmatrix} = \begin{bmatrix} 5 & -6 & -3 \\ 9 & 1 & -11 \end{bmatrix}$$

6. If $5A=\begin{bmatrix}5 & 10 & -15\\ 2 & 3 & 4\\ 1 & 0 & -5\end{bmatrix}$, find A.

Sol. $5A=\begin{bmatrix}5 & 10 & -15\\ 2 & 3 & 4\\ 1 & 0 & -5\end{bmatrix} \Rightarrow A=\frac{1}{5}\begin{bmatrix}5 & 10 & -15\\ 2 & 3 & 4\\ 1 & 0 & -5\end{bmatrix} \Rightarrow A=\begin{bmatrix}1 & 2 & -3\\ 2/5 & 3/5 & 4/5\\ 1/5 & 0 & -1\end{bmatrix}$

7. Find matrices A and B, if $A+B=\begin{bmatrix}1 & 0 & 2\\ 5 & 4 & -6\\ 7 & 3 & 8\end{bmatrix}$ and $A-B=\begin{bmatrix}-5 & -4 & 8\\ 11 & 2 & 0\\ -1 & 7 & 4\end{bmatrix}$.

Sol. $A+B=\begin{bmatrix}1 & 0 & 2\\ 5 & 4 & -6\\ 7 & 3 & 8\end{bmatrix}$ ...(i) $\quad A-B=\begin{bmatrix}-5 & -4 & 8\\ 11 & 2 & 0\\ -1 & 7 & 4\end{bmatrix}$ ...(ii)

Adding equation (i) and (ii), we get

$$(A+B)+(A-B)=\begin{bmatrix}1 & 0 & 2\\ 5 & 4 & -6\\ 7 & 3 & 5\end{bmatrix}+\begin{bmatrix}-5 & -4 & 8\\ 11 & 2 & 0\\ -1 & 7 & 4\end{bmatrix}$$

$$\Rightarrow 2A=\begin{bmatrix}1+(-5) & 0+(-4) & 2+8\\ 5+11 & 4+2 & -6+0\\ 7+(-1) & 3+7 & 8+4\end{bmatrix}=\begin{bmatrix}-4 & -4 & 10\\ 16 & 6 & -6\\ 6 & 10 & 12\end{bmatrix}$$

$$\Rightarrow A=\frac{1}{2}\begin{bmatrix}-4 & -4 & 10\\ 16 & 6 & -6\\ 6 & 10 & 12\end{bmatrix} \Rightarrow A=\begin{bmatrix}-2 & -2 & 5\\ 8 & 3 & -3\\ 3 & 5 & 6\end{bmatrix}$$

Putting the value of A in equation (i), we get

$$A+B=\begin{bmatrix}1 & 0 & 2\\ 5 & 4 & -6\\ 7 & 3 & 8\end{bmatrix} \Rightarrow B=\begin{bmatrix}1 & 0 & 2\\ 5 & 4 & -6\\ 7 & 3 & 8\end{bmatrix}-A \Rightarrow B=\begin{bmatrix}1 & 0 & 2\\ 5 & 4 & -6\\ 7 & 3 & 8\end{bmatrix}-\begin{bmatrix}-2 & -2 & 5\\ 8 & 3 & -3\\ 3 & 5 & 6\end{bmatrix}$$

$$\Rightarrow B=\begin{bmatrix}1-(-2) & 0-(-2) & 2-5\\ 5-8 & 4-3 & -6-(-3)\\ 7-3 & 3-5 & 8-6\end{bmatrix} \Rightarrow B=\begin{bmatrix}3 & 2 & -3\\ -3 & 1 & -3\\ 4 & -2 & 2\end{bmatrix}$$

Hence, matrix $A=\begin{bmatrix}-2 & -2 & 5\\ 8 & 3 & -3\\ 3 & 5 & 6\end{bmatrix}$ and $B=\begin{bmatrix}3 & 2 & -3\\ -3 & 1 & -3\\ 4 & -2 & 2\end{bmatrix}$

8. Find matrices A and B, if $2A-B=\begin{bmatrix}6 & -6 & 0\\ -4 & 2 & 1\end{bmatrix}$ and $2B+A=\begin{bmatrix}3 & 2 & 5\\ -2 & 1 & -7\end{bmatrix}$.

Sol. $2A-B=\begin{bmatrix}6 & -6 & 0\\ -4 & 2 & 1\end{bmatrix}$ ...(i) $\times 2$

$2B+A=\begin{bmatrix}3 & 2 & 5\\ -2 & 1 & -7\end{bmatrix}$ ...(ii)

Adding (i) and (ii), we get, $(4A-2B)+(A+2B)=\begin{bmatrix}12 & -12 & 0\\ -8 & 4 & 2\end{bmatrix}+\begin{bmatrix}3 & 2 & 5\\ -2 & 1 & -7\end{bmatrix}$

$\Rightarrow 5A=\begin{bmatrix}12 & -12 & 0\\ -8 & 4 & 2\end{bmatrix}+\begin{bmatrix}3 & 2 & 5\\ -2 & 1 & -7\end{bmatrix} \Rightarrow 5A=\begin{bmatrix}12+3 & -12+2 & 0+5\\ -8+(-2) & 4+1 & 2+(-7)\end{bmatrix}$

$\Rightarrow 5A=\begin{bmatrix}15 & -10 & 5\\ -10 & 5 & -5\end{bmatrix} \Rightarrow A=\frac{1}{5}\begin{bmatrix}15 & -10 & 5\\ -10 & 5 & -5\end{bmatrix}=\begin{bmatrix}3 & -2 & 1\\ -2 & 1 & -1\end{bmatrix}$

Putting the value of A in equation (i), then $2A-B=\begin{bmatrix}6 & -6 & 0\\ -4 & 2 & 1\end{bmatrix}$

$\Rightarrow B=2A-\begin{bmatrix}6 & -6 & 0\\ -4 & 2 & 1\end{bmatrix} \Rightarrow B=2\begin{bmatrix}3 & -2 & 1\\ -2 & 1 & -1\end{bmatrix}-\begin{bmatrix}6 & -6 & 0\\ -4 & 2 & 1\end{bmatrix}$

$\Rightarrow B=\begin{bmatrix}6 & -4 & 2\\ -4 & 2 & -2\end{bmatrix}-\begin{bmatrix}6 & -6 & 0\\ -4 & 2 & 1\end{bmatrix} \Rightarrow B=\begin{bmatrix}6-6 & -4-(-6) & 2-0\\ -4-(-4) & 2-2 & -2-1\end{bmatrix}=\begin{bmatrix}0 & 2 & 2\\ 0 & 0 & -3\end{bmatrix}$

Hence, matrix $A=\begin{bmatrix}3 & -2 & 1\\ -2 & 1 & -1\end{bmatrix}$ and $B=\begin{bmatrix}0 & 2 & 2\\ 0 & 0 & -3\end{bmatrix}$

9. Find matrix X, if $\begin{bmatrix}3 & 5 & -9\\ -1 & 4 & -7\end{bmatrix}+X=\begin{bmatrix}6 & 2 & 3\\ 4 & 8 & 6\end{bmatrix}$.

Sol. $\begin{bmatrix}3 & 5 & -9\\ -1 & 4 & -7\end{bmatrix}+X=\begin{bmatrix}6 & 2 & 3\\ 4 & 8 & 6\end{bmatrix} \Rightarrow X=\begin{bmatrix}6 & 2 & 3\\ 4 & 8 & 6\end{bmatrix}-\begin{bmatrix}3 & 5 & -9\\ -1 & 4 & -7\end{bmatrix}$

$\Rightarrow X=\begin{bmatrix}6-3 & 2-5 & 3-(-9)\\ 4-(-1) & 8-4 & 6-(-7)\end{bmatrix} \Rightarrow X=\begin{bmatrix}3 & -3 & 12\\ 5 & 4 & 13\end{bmatrix}$

10. If $A=\begin{bmatrix}-2 & 3\\ 4 & 5\\ 1 & -6\end{bmatrix}$ and $B=\begin{bmatrix}5 & 2\\ -7 & 3\\ 6 & 4\end{bmatrix}$, find a matrix C such that $A+B-C=O$.

Sol. $A+B-C=O \Rightarrow C=A+B \Rightarrow C=\begin{bmatrix}-2 & 3\\ 4 & 5\\ 1 & -6\end{bmatrix}+\begin{bmatrix}5 & 2\\ -7 & 3\\ 6 & 4\end{bmatrix}$

$\Rightarrow C=\begin{bmatrix}-2+5 & 3+2\\ 4+(-7) & 5+3\\ 1+6 & -6+4\end{bmatrix} \Rightarrow C=\begin{bmatrix}3 & 5\\ -3 & 8\\ 7 & -2\end{bmatrix}$

11. Find the matrix X such that $2A-B+X=O$, where $A=\begin{bmatrix}3 & 1\\ 0 & 2\end{bmatrix}$ and $B=\begin{bmatrix}-2 & 1\\ 0 & 3\end{bmatrix}$.

Sol. $2A-B+X=O \Rightarrow X=B-2A \Rightarrow X=\begin{bmatrix}-2 & 1\\ 0 & 3\end{bmatrix}-2\begin{bmatrix}3 & 1\\ 0 & 2\end{bmatrix}$

$\Rightarrow X=\begin{bmatrix}-2 & 1\\ 0 & 3\end{bmatrix}-\begin{bmatrix}6 & 2\\ 0 & 4\end{bmatrix}=\begin{bmatrix}-2-6 & 1-2\\ 0-0 & 3-4\end{bmatrix} \Rightarrow X=\begin{bmatrix}-8 & -1\\ 0 & -1\end{bmatrix}$

12. If $A=\begin{bmatrix}1 & -3 & 2\\ 2 & 0 & 2\end{bmatrix}$, $B=\begin{bmatrix}2 & -1 & -1\\ 1 & 0 & -1\end{bmatrix}$, find a matrix C such that $(A+B+C)$ is a zero matrix.

Sol. $A+B+C=O \Rightarrow C=-A-B$

$$\Rightarrow C=-\begin{bmatrix}1 & -3 & 2\\ 2 & 0 & 2\end{bmatrix}-\begin{bmatrix}2 & -1 & -1\\ 1 & 0 & -1\end{bmatrix} \quad \Rightarrow C=\begin{bmatrix}-1-2 & +3+1 & -2+1\\ -2-1 & 0-0 & -2+1\end{bmatrix}=\begin{bmatrix}-3 & 4 & -1\\ -3 & 0 & -1\end{bmatrix}$$

13. If $A=diag\left[2, -5, 9\right]$, $B=diag\left[-3, 7, 14\right]$ and $C=diag\left[4, -6, 3\right]$ find

(i) $A+2B$ (ii) $B+C-A$ (iii) $2A+B-5C$

Sol. (i) $A+2B=\begin{bmatrix}2 & 0 & 0\\ 0 & -5 & 0\\ 0 & 0 & 9\end{bmatrix}+2\begin{bmatrix}-3 & 0 & 0\\ 0 & 7 & 0\\ 0 & 0 & 14\end{bmatrix}=\begin{bmatrix}2 & 0 & 0\\ 0 & -5 & 0\\ 0 & 0 & 9\end{bmatrix}+\begin{bmatrix}-6 & 0 & 0\\ 0 & 14 & 0\\ 0 & 0 & 28\end{bmatrix}$

$$=\begin{bmatrix}2-6 & 0 & 0\\ 0 & -5+14 & 0\\ 0 & 0 & 9+28\end{bmatrix}=\begin{bmatrix}-4 & 0 & 0\\ 0 & 9 & 0\\ 0 & 0 & 37\end{bmatrix}=diag\left[-4, 9, 37\right]$$

(ii) $B+C-A=\begin{bmatrix}-3 & 0 & 0\\ 0 & 7 & 0\\ 0 & 0 & 14\end{bmatrix}+\begin{bmatrix}4 & 0 & 0\\ 0 & -6 & 0\\ 0 & 0 & 3\end{bmatrix}-\begin{bmatrix}2 & 0 & 0\\ 0 & -5 & 0\\ 0 & 0 & 9\end{bmatrix}$

$$=\begin{bmatrix}-3+4-2 & 0 & 0\\ 0 & 7+(-6)-(-5) & 0\\ 0 & 0 & 14+3-9\end{bmatrix}=diag\left[-1, 6, 8\right]$$

(iii) $2A+B-5C=2\begin{bmatrix}2 & 0 & 0\\ 0 & -5 & 0\\ 0 & 0 & 9\end{bmatrix}+\begin{bmatrix}-3 & 0 & 0\\ 0 & 7 & 0\\ 0 & 0 & 14\end{bmatrix}-5\begin{bmatrix}4 & 0 & 0\\ 0 & -6 & 0\\ 0 & 0 & 3\end{bmatrix}$

$$=\begin{bmatrix}4 & 0 & 0\\ 0 & -10 & 0\\ 0 & 0 & 18\end{bmatrix}+\begin{bmatrix}-3 & 0 & 0\\ 0 & 7 & 0\\ 0 & 0 & 14\end{bmatrix}-\begin{bmatrix}20 & 0 & 0\\ 0 & -30 & 0\\ 0 & 0 & 15\end{bmatrix}$$

$$=\begin{bmatrix}4+(-3)-20 & 0 & 0\\ 0 & -10+7+30 & 0\\ 0 & 0 & 18+14-15\end{bmatrix}=\begin{bmatrix}-19 & 0 & 0\\ 0 & 27 & 0\\ 0 & 0 & 17\end{bmatrix}$$

$$=diag\left[-19, 27, 17\right]$$

14. Find the values of x and y, when

(i) $\begin{bmatrix}x+y\\ x-y\end{bmatrix}=\begin{bmatrix}8\\ 4\end{bmatrix}$ (ii) $\begin{bmatrix}2x+5 & 7\\ 0 & 3y-7\end{bmatrix}=\begin{bmatrix}x-3 & 7\\ 0 & -5\end{bmatrix}$

(iii) $2\begin{bmatrix}x & 5\\ 7 & y-3\end{bmatrix}+\begin{bmatrix}3 & -4\\ 1 & 2\end{bmatrix}=\begin{bmatrix}7 & 6\\ 15 & 4\end{bmatrix}$

Sol. (i) $\begin{bmatrix}x+y\\ x-y\end{bmatrix}=\begin{bmatrix}8\\ 4\end{bmatrix}$

$x+y=8 \quad \ldots(1) \qquad x-y=4 \quad \ldots(2)$

Adding equation (1) and (2), we get

$(x+y)+(x-y)=8+4 \quad \Rightarrow 2x=12 \quad \Rightarrow x=6$

Putting the value of x in equation (1), we get $x+y=8 \quad \Rightarrow y=8-x=8-6=2$

Hence, $x=6$ and $y=2$.

(ii) $\begin{bmatrix} 2x+5 & 7 \\ 0 & 3y-7 \end{bmatrix} = \begin{bmatrix} x-3 & 7 \\ 0 & -5 \end{bmatrix}$

$2x+5 = x-3$ and $3y-7=-5$

$\Rightarrow 2x-x=-3-5$ and $3y=-5+7$

$\Rightarrow x=-8$ and $3y=2 \quad \Rightarrow y=\frac{2}{3}$

Hence, $x=-8$ and $y=\frac{2}{3}$

(iii) $2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 4 \end{bmatrix} \Rightarrow \begin{bmatrix} 2x & 10 \\ 14 & 2y-6 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 4 \end{bmatrix}$

$2x+3=7$ and $2y-6+2=4$

$\Rightarrow 2x=7-3$ $\qquad \Rightarrow 2y=4-2+6$

$\Rightarrow 2x=4$ $\qquad \Rightarrow 2y=8$

$\Rightarrow x=2$ $\qquad \Rightarrow y=4$

Hence, $x=2$ and $y=4$.

15. Find the value of $(x+y)$ from the following equation

$2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

Sol. Given $2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2 & 6 \\ 0 & 2x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix} \Rightarrow \begin{bmatrix} 2+y & 6+0 \\ 0+1 & 2x+2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix} \Rightarrow \begin{bmatrix} 2+y & 6 \\ 1 & 2x+2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

$\Rightarrow 2+y=5$ and $2x+2=8 \quad \Rightarrow y=5-3$ and $2x=8-2$

$\Rightarrow y=2$ and $x=3 \qquad \therefore x=3$ and $y=2$

16. If $\begin{bmatrix} x-y & 2y \\ 2y+z & x+y \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 9 & 5 \end{bmatrix}$ then write the value of $(x+y+z)$

Sol. Given $\begin{bmatrix} x-y & 2y \\ 2y+z & x+y \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 9 & 5 \end{bmatrix}$

$\Rightarrow x-y=1,\ 2y=4,\ 2y+z=9$ and $x+y=5$

Now, $2y=4 \Rightarrow y=2$

$x-y=1 \Rightarrow x-2=1 \Rightarrow x=3$

$2y+z=9 \Rightarrow 4+z=9 \Rightarrow z=5$

$\therefore x+y+z=3+2+5=10$

EXERCISE 5C (Pg.No.: 186)

1. Compute AB and BA, whichever exists when

(i) $A=\begin{bmatrix} 2 & -1 \\ 3 & 0 \\ -1 & 4 \end{bmatrix}$ and $B=\begin{bmatrix} -2 & 3 \\ 0 & 4 \end{bmatrix}$ (ii) $A=\begin{bmatrix} -1 & 1 \\ -2 & 2 \\ -3 & 3 \end{bmatrix}$ and $B=\begin{bmatrix} 3 & -2 & 1 \\ 0 & 1 & 2 \\ -3 & 4 & -5 \end{bmatrix}$

(iii) $A=\begin{bmatrix}0 & 1 & -5\\ 2 & 4 & 0\end{bmatrix}$ and $B=\begin{bmatrix}1 & 3\\ -1 & 0\\ 0 & 5\end{bmatrix}$ (iv) $A=\begin{bmatrix}1 & 2 & 3 & 4\end{bmatrix}$ and $B=\begin{bmatrix}1\\ 2\\ 3\\ 4\end{bmatrix}$

(v) $A=\begin{bmatrix}2 & 1\\ 3 & 2\\ -1 & 1\end{bmatrix}$ and $B=\begin{bmatrix}1 & 0 & 1\\ -1 & 2 & 1\end{bmatrix}$

Sol. (i) $A=\begin{bmatrix}2 & -1\\ 3 & 0\\ -1 & 4\end{bmatrix}$ and $B=\begin{bmatrix}-2 & 3\\ 0 & 4\end{bmatrix}$

$$AB=\begin{bmatrix}2 & -1\\ 3 & 0\\ -1 & 4\end{bmatrix}\begin{bmatrix}-2 & 3\\ 0 & 4\end{bmatrix}=\begin{bmatrix}2(-2)-1(0) & 2(3)-1(4)\\ 3(-2)+0(0) & 3(3)+0(4)\\ -1(-2)+4(0) & -1(3)+4(4)\end{bmatrix}=\begin{bmatrix}-4-0 & 6-4\\ -6+0 & 9+0\\ 2+0 & -3+16\end{bmatrix}=\begin{bmatrix}-4 & 2\\ -6 & 9\\ 2 & 13\end{bmatrix}$$

$BA=\begin{bmatrix}-2 & 3\\ 0 & 4\end{bmatrix}\begin{bmatrix}2 & -1\\ 3 & 0\\ -1 & 4\end{bmatrix}$. B is 2×2, A is 3×2 column of B not equal to row of A

$\therefore$ BA does not exist.

(ii) $A=\begin{bmatrix}-1 & 1\\ -2 & 2\\ -3 & 3\end{bmatrix}$ and $B=\begin{bmatrix}3 & -2 & 1\\ 0 & 1 & 2\\ -3 & 4 & -5\end{bmatrix}$

$AB=\begin{bmatrix}-1 & 1\\ -2 & 2\\ -3 & 3\end{bmatrix}\begin{bmatrix}3 & -2 & 1\\ 0 & 1 & 2\\ -3 & 4 & -5\end{bmatrix}$. $\therefore$ AB does not exist.

$$BA=\begin{bmatrix}3 & -2 & 1\\ 0 & 1 & 2\\ -3 & 4 & -5\end{bmatrix}\begin{bmatrix}-1 & 1\\ -2 & 2\\ -3 & 3\end{bmatrix}=\begin{bmatrix}3(-1)+(-2)(-2)+1(-3) & 3(1)-2(2)+1(3)\\ 0(-1)+1(-2)+2(-3) & 0(1)+1(2)+2(3)\\ -3(-1)+4(-2)-5(-3) & -3(1)+4(2)-5(3)\end{bmatrix}$$

$$=\begin{bmatrix}-3+4-3 & 3-4+3\\ 0-2-6 & 0+2+6\\ 3-8+15 & -3+8-15\end{bmatrix}=\begin{bmatrix}-2 & 2\\ -8 & 8\\ 10 & -10\end{bmatrix}$$

(iii) $A=\begin{bmatrix}0 & 1 & -5\\ 2 & 4 & 0\end{bmatrix}$ and $B=\begin{bmatrix}1 & 3\\ -1 & 0\\ 0 & 5\end{bmatrix}$

$$AB=\begin{bmatrix}0 & 1 & -5\\ 2 & 4 & 0\end{bmatrix}\begin{bmatrix}1 & 3\\ -1 & 0\\ 0 & 5\end{bmatrix}=\begin{bmatrix}0(1)+1(-1)-5(0) & 0(3)+1(0)-5(5)\\ 2(1)+4(-1)+0(0) & 2(3)+4(0)+0(5)\end{bmatrix}$$

$$=\begin{bmatrix}0-1-0 & 0+0-25\\ 2-4+0 & 6+0+0\end{bmatrix}=\begin{bmatrix}-1 & -25\\ -2 & 6\end{bmatrix}$$

$$BA=\begin{bmatrix}1&3\\-1&0\\0&5\end{bmatrix}\begin{bmatrix}0&1&-5\\2&4&0\end{bmatrix}=\begin{bmatrix}1(0)+3(2)&1(1)+3(4)&1(-5)+3(0)\\-1(0)+0(2)&-1(1)+0(4)&-1(-5)+0(0)\\0(0)+5(2)&0(1)+5(4)&0(-5)+5(0)\end{bmatrix}$$

$$=\begin{bmatrix}0+6&1+12&-5+0\\0+0&-1+0&5+0\\0+10&0+20&0+0\end{bmatrix}=\begin{bmatrix}6&13&-5\\0&-1&5\\10&20&0\end{bmatrix}$$

(iv) $A=\begin{bmatrix}1&2&3&4\end{bmatrix}$ and $B=\begin{bmatrix}1\\2\\3\\4\end{bmatrix}$

$$AB=\begin{bmatrix}1(1)+2(2)+3(3)+4(4)\end{bmatrix}=\begin{bmatrix}1+4+9+16\end{bmatrix}=\begin{bmatrix}30\end{bmatrix}$$

$$BA=\begin{bmatrix}1\\2\\3\\4\end{bmatrix}\begin{bmatrix}1&2&3&4\end{bmatrix}=\begin{bmatrix}1(1)&1(2)&1(3)&1(4)\\2(1)&2(2)&2(3)&2(4)\\3(1)&3(2)&3(3)&3(4)\\4(1)&4(2)&4(3)&4(4)\end{bmatrix}=\begin{bmatrix}1&2&3&4\\2&4&6&8\\3&6&9&12\\4&8&12&16\end{bmatrix}$$

(v) $A=\begin{bmatrix}2&1\\3&2\\-1&1\end{bmatrix}$ and $B=\begin{bmatrix}1&0&1\\-1&2&1\end{bmatrix}$

$$AB=\begin{bmatrix}2&1\\3&2\\-1&1\end{bmatrix}\begin{bmatrix}1&0&1\\-1&2&1\end{bmatrix}=\begin{bmatrix}2(1)+1(-1)&2(0)+1(2)&2(1)+1(1)\\3(1)+2(-1)&3(0)+2(2)&3(1)+2(1)\\-1(1)+1(-1)&-1(0)+1(2)&-1(1)+1(1)\end{bmatrix}$$

$$=\begin{bmatrix}2-1&0+2&2+1\\3-2&0+4&3+2\\-1-1&0+2&-1+1\end{bmatrix}=\begin{bmatrix}1&2&3\\1&4&5\\-2&2&0\end{bmatrix}$$

$$BA=\begin{bmatrix}1&0&1\\-1&2&1\end{bmatrix}\begin{bmatrix}2&1\\3&2\\-1&1\end{bmatrix}=\begin{bmatrix}1(2)+0(3)+1(-1)&1(1)+0(2)+1(1)\\-1(2)+2(3)+1(-1)&-1(1)+2(2)+1(1)\end{bmatrix}$$

$$=\begin{bmatrix}2+0-1&1+0+1\\-2+6-1&-1+4+1\end{bmatrix}=\begin{bmatrix}1&2\\3&4\end{bmatrix}$$

2. Show that $AB\neq BA$ in each of the following cases.

(i) $A=\begin{bmatrix}5&-1\\6&7\end{bmatrix}$ and $B=\begin{bmatrix}2&1\\3&4\end{bmatrix}$ (ii) $A=\begin{bmatrix}1&2&3\\0&1&0\\1&1&0\end{bmatrix}$ and $B=\begin{bmatrix}-1&1&0\\0&-1&1\\2&3&4\end{bmatrix}$

Sol. (i) $A=\begin{bmatrix}5&-1\\6&7\end{bmatrix}$ and $B=\begin{bmatrix}2&1\\3&4\end{bmatrix}$

$$AB=\begin{bmatrix}5&-1\\6&7\end{bmatrix}\begin{bmatrix}2&1\\3&4\end{bmatrix}=\begin{bmatrix}5(2)+(-1)3&5(1)-1(4)\\6(2)+7(3)&6(1)+7(4)\end{bmatrix}=\begin{bmatrix}10-3&5-4\\12+21&6+28\end{bmatrix}=\begin{bmatrix}7&1\\33&34\end{bmatrix}$$

$$BA=\begin{bmatrix}2&1\\3&4\end{bmatrix}\begin{bmatrix}5&-1\\6&7\end{bmatrix}=\begin{bmatrix}2(5)+1(6)&2(-1)+1(7)\\3(5)+4(6)&3(-1)+4(7)\end{bmatrix}=\begin{bmatrix}10+6&-2+7\\15+24&-3+28\end{bmatrix}=\begin{bmatrix}16&5\\39&25\end{bmatrix}$$

Hence, $AB\neq BA$.

(ii) $A=\begin{bmatrix}1&2&3\\0&1&0\\1&1&0\end{bmatrix}$ and $B=\begin{bmatrix}-1&1&0\\0&-1&1\\2&3&4\end{bmatrix}$

$$AB=\begin{bmatrix}1&2&3\\0&1&0\\1&1&0\end{bmatrix}\begin{bmatrix}-1&1&0\\0&-1&1\\2&3&4\end{bmatrix}=\begin{bmatrix}1(-1)+2(0)+3(2)&1(1)+2(-1)+3(3)&1(0)+2(1)+3(4)\\0(-1)+1(0)+0(2)&0(1)+1(-1)+0(3)&0(0)+1(1)+0(4)\\1(-1)+1(0)+0(2)&1(1)+1(-1)+0(3)&1(0)+1(1)+0(4)\end{bmatrix}$$

$$=\begin{bmatrix}-1+0+6&1-2+9&0+2+12\\0+0+0&0-1+0&0+1+0\\-1+0+0&1-1+0&0+1+0\end{bmatrix}=\begin{bmatrix}5&8&14\\0&-1&1\\-1&0&1\end{bmatrix}$$

$$BA=\begin{bmatrix}-1&1&0\\0&-1&1\\2&3&4\end{bmatrix}\begin{bmatrix}1&2&3\\0&1&0\\1&1&0\end{bmatrix}=\begin{bmatrix}-1(1)+1(0)+0(1)&-1(2)+1(1)+0(1)&1(3)+1(0)+0(0)\\0(1)-1(0)+1(1)&0(2)-1(1)+1(1)&0(3)-1(0)+1(0)\\2(1)+3(0)+4(1)&2(2)+3(1)+4(1)&2(3)+3(0)+4(0)\end{bmatrix}$$

$$=\begin{bmatrix}1+0+0&-2+1+0&-3+0+0\\0+0+1&0-1+1&0-0+0\\2+0+4&4+3+4&6+0+0\end{bmatrix}=\begin{bmatrix}-1&-1&-3\\1&0&0\\6&11&6\end{bmatrix}$$

Hence, $AB\neq BA$.

3. Show that $AB=BA$ in each of the following cases.

(i) $A=\begin{bmatrix}\cos\theta&-\sin\theta\\\sin\theta&\cos\theta\end{bmatrix}$ and $B=\begin{bmatrix}\cos\phi&-\sin\phi\\\sin\phi&\cos\phi\end{bmatrix}$

(ii) $A=\begin{bmatrix}1&2&1\\3&4&2\\1&3&2\end{bmatrix}$ and $B=\begin{bmatrix}10&-4&-1\\-11&5&0\\9&-5&1\end{bmatrix}$ (iii) $A=\begin{bmatrix}1&3&-1\\2&2&-1\\3&0&-1\end{bmatrix}$ and $B=\begin{bmatrix}-2&3&-1\\-1&2&-1\\-6&9&-4\end{bmatrix}$

Sol. **(i)** $A=\begin{bmatrix}\cos\theta&-\sin\theta\\\sin\theta&\cos\theta\end{bmatrix}$ and $B=\begin{bmatrix}\cos\phi&-\sin\phi\\\sin\phi&\cos\phi\end{bmatrix}$

$$AB=\begin{bmatrix}\cos\theta\cos\phi-\sin\theta\sin\phi&-\cos\theta\sin\phi-\sin\theta\cos\phi\\\sin\theta\cos\phi+\cos\theta\sin\phi&-\sin\theta\sin\phi+\cos\theta\cos\phi\end{bmatrix}\Rightarrow AB=\begin{bmatrix}\cos(\theta+\phi)&-\sin(\theta+\phi)\\\sin(\theta+\phi)&\cos(\theta+\phi)\end{bmatrix}$$

and $BA=\begin{bmatrix}\cos\phi&-\sin\phi\\\sin\phi&\cos\phi\end{bmatrix}\begin{bmatrix}\cos\theta&-\sin\theta\\\sin\theta&\cos\theta\end{bmatrix}$

$$\Rightarrow BA=\begin{bmatrix}\cos\phi\cos\theta-\sin\phi\sin\theta&\cos\phi(-\sin\theta)-(-\sin\phi)\cos\theta\\\sin\phi\cos\theta+\cos\phi\sin\theta&\sin\phi(-\sin\theta)+\cos\phi\cos\theta\end{bmatrix}$$

$$\Rightarrow BA=\begin{bmatrix}\cos(\theta+\phi)&-\sin(\theta+\phi)\\\sin(\theta+\phi)&\cos(\theta+\phi)\end{bmatrix}$$. Hence, $AB=BA$.

(ii) $A=\begin{bmatrix}1&2&1\\3&4&2\\1&3&2\end{bmatrix}$ and $B=\begin{bmatrix}10&-4&-1\\-11&5&0\\9&-5&1\end{bmatrix}$

$$\therefore\quad AB=\begin{bmatrix}1&2&1\\3&4&2\\1&3&2\end{bmatrix}\begin{bmatrix}10&-4&-1\\-11&5&0\\9&-5&1\end{bmatrix}$$

$$=\begin{bmatrix}1(10)+2(-11)+1(9) & 1(-4)+2(5)+1(-5) & 1(-1)+2(0)+1(1)\\3(10)+4(-11)+2(9) & 3(-4)+4(5)+2(-5) & 3(-1)+4(0)+2(1)\\1(10)+3(-11)+2(9) & 1(-4)+3(5)+2(-5) & 1(-1)+3(0)+2(1)\end{bmatrix}$$

$$=\begin{bmatrix}10-22+9 & -4+10-5 & -1+0+1\\30-44+18 & -12+20-10 & -3+0+2\\10-33+18 & -4+15-10 & -1+0+2\end{bmatrix}=\begin{bmatrix}-3&1&0\\4&-2&-1\\-5&1&1\end{bmatrix}$$

$$BA=\begin{bmatrix}10&-4&-1\\-11&5&0\\9&-5&1\end{bmatrix}\begin{bmatrix}1&2&1\\3&4&2\\1&3&2\end{bmatrix}$$

$$=\begin{bmatrix}10(1)+(-4)(3)-1(1) & 10(2)-4(4)-1(3) & 10(1)-4(2)-1(2)\\-11(1)+5(3)+0(1) & -11(2)+5(4)+0(3) & -11(1)+5(2)+0(2)\\9(1)-5(3)+1(1) & 9(2)-5(4)+1(3) & 9(1)-5(2)+1(2)\end{bmatrix}$$

$$=\begin{bmatrix}10-12-1 & 20-16-3 & 10-8-2\\-11+15+0 & -22+20+0 & -11+10+0\\9-15+1 & 18-20+3 & 9-10+2\end{bmatrix}=\begin{bmatrix}-3&1&0\\4&-2&-1\\-5&1&1\end{bmatrix}$$

Hence, $AB=BA$.

(iii) $A=\begin{bmatrix}1&3&-1\\2&2&-1\\3&0&-1\end{bmatrix}$ and $B=\begin{bmatrix}-2&3&-1\\-1&2&-1\\-6&9&-4\end{bmatrix}$

$$AB=\begin{bmatrix}1&3&-1\\2&2&-1\\3&0&-1\end{bmatrix}\begin{bmatrix}-2&3&-1\\-1&2&-1\\-6&9&-4\end{bmatrix}$$

$$=\begin{bmatrix}1(-2)+3(-1)-1(-6) & 1(3)+3(2)-1(9) & 1(-1)+3(-1)-1(-4)\\2(-2)+2(-1)-1(-6) & 2(3)+2(2)-1(9) & 2(-1)+2(-1)-1(-4)\\3(-2)+0(-1)+(-1)(-6) & 3(3)+0(2)-1(9) & 3(-1)+0(-1)-1(-4)\end{bmatrix}$$

$$=\begin{bmatrix}-2-3+6 & 3+6-9 & -1-3+4\\-4-2+6 & 6+4-9 & -2-2+4\\-6-0+6 & 9+0-9 & -3-0+4\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}=I$$

$$BA=\begin{bmatrix}-2&3&-1\\-1&2&-1\\-6&9&-4\end{bmatrix}\begin{bmatrix}1&3&-1\\2&2&-1\\3&0&-1\end{bmatrix}$$

$$=\begin{bmatrix}-2(1)+3(2)-1(3) & -2(3)+3(2)-1(0) & -2(-1)+3(-1)-1(-1)\\-1(1)+2(2)-1(3) & -1(3)+2(2)-1(0) & -1(-1)+2(-1)-1(-1)\\-6(1)+9(2)-4(3) & -6(3)+9(2)+(-4)(0) & -6(-1)+9(-1)-4(-1)\end{bmatrix}$$

$$=\begin{bmatrix} -2+6-3 & -6+6-0 & 2-3+1 \\ -1+4-3 & -3+4-0 & 1-2+1 \\ -6+18-12 & -18+18-0 & 6-9+4 \end{bmatrix}=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}=I.\qquad \text{Hence, } AB=BA.$$

4. If $A=\begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$ and $B=\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$, show that $AB=A$ and $BA=B$.

Sol. $AB=\begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$

$$=\begin{bmatrix} 2(2)-3(-1)-5(1) & 2(-2)-3(3)-5(-2) & 2(-4)-3(4)-5(-3) \\ -1(2)+4(-1)+5(1) & -1(-2)+4(3)+5(-2) & -1(-4)+4(4)+5(-3) \\ 1(2)-3(-1)-4(1) & 1(-2)-3(3)-4(-2) & 1(-4)-3(4)-4(-3) \end{bmatrix}$$

$$=\begin{bmatrix} 4+3-5 & -4-9+10 & -8-12+15 \\ -2-4+5 & 2+12-10 & 4+16-15 \\ 2+3-4 & -2-9+8 & -4-12+12 \end{bmatrix}=\begin{bmatrix} 2 & -3 & -5 \\ -1 & 2 & 5 \\ 1 & -3 & -4 \end{bmatrix}=A.\ \text{Hence } AB=A.$$

and $BA=\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}\begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$

$$=\begin{bmatrix} 2(2)-2(-1)-4(1) & 2(-3)-2(4)-4(-3) & 2(-5)-2(5)-4(-4) \\ -1(2)+3(-1)+4(1) & -1(-3)+3(4)+4(-3) & -1(-5)+3(5)+4(-4) \\ 1(2)-2(-1)-3(1) & 1(-3)-2(4)-3(-3) & 1(-5)-2(5)-3(-4) \end{bmatrix}$$

$$=\begin{bmatrix} 4+2-4 & -6-8+12 & -10-10+16 \\ -2-3+4 & 3+12-12 & 5+15-16 \\ 2+2-3 & -3-8+9 & -5-10+12 \end{bmatrix}=\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}=B.\ \text{Hence } BA=B.$$

5. If $A=\begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix}$ and $B=\begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}$, show that AB is a zero matrix.

Sol. $AB=\begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix}\begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}$

$$=\begin{bmatrix} 0(a^2)+c(ab)-b(ac) & a(ab)+c(b^2)-b(bc) & 0(ac)+c(bc)-b(c^2) \\ -c(a^2)+0(ab)+a(ac) & -c(ab)+0(b^2)+a(bc) & c(ac)+0(bc)+a(c^2) \\ b(a^2)-a(ab)+0(ac) & b(ab)-a(b^2)+0(bc) & b(ac)-a(bc)+0(c^2) \end{bmatrix}$$

$$=\begin{bmatrix} 0+abc-abc & a^2b+cb^2-b^2c & 0+bc^2-bc^2 \\ -a^2c^2+0+a^2c & -abc+0+abc & -c^2a+0+ac^2 \\ a^2b-a^2b+0 & ab^2-ab^2+0 & abc-abc+0 \end{bmatrix}=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Hence, $AB=O$.

6. For the following matrices, verify that $A(BC)=(AB)C$.

(i) $A=\begin{bmatrix}1&2&5\\0&1&3\end{bmatrix}$, $B=\begin{bmatrix}2&3&0\\1&0&4\\1&-1&2\end{bmatrix}$ and $C=\begin{bmatrix}1\\4\\5\end{bmatrix}$

(ii) $A=\begin{bmatrix}2&3&-1\\3&0&2\end{bmatrix}$, $B=\begin{bmatrix}1\\1\\2\end{bmatrix}$ and $C=\begin{bmatrix}1&-2\end{bmatrix}$

Sol. **(i)** $\text{LHS}=A(BC)=A\left(\begin{bmatrix}2&3&0\\1&0&4\\1&-1&2\end{bmatrix}\begin{bmatrix}1\\4\\5\end{bmatrix}\right)=A\begin{bmatrix}2(1)+3(4)+0(5)\\1(1)+0(4)+4(5)\\1(1)-1(4)+2(5)\end{bmatrix}=A\begin{bmatrix}2+12+0\\1+0+20\\1-4+10\end{bmatrix}$

$$=\begin{bmatrix}1&2&5\\0&1&3\end{bmatrix}\begin{bmatrix}14\\21\\7\end{bmatrix}=\begin{bmatrix}1(14)+2(21)+5(7)\\0(14)+1(21)+3(7)\end{bmatrix}=\begin{bmatrix}14+42+35\\0+21+21\end{bmatrix}=\begin{bmatrix}91\\42\end{bmatrix}$$

$$\text{RHS}=(AB)C=\left(\begin{bmatrix}1&2&5\\0&1&3\end{bmatrix}\begin{bmatrix}2&3&0\\1&0&4\\1&-1&2\end{bmatrix}\right)C$$

$$=\begin{bmatrix}1(2)+2(1)+5(1)&1(3)+2(0)+5(-1)&1(0)+2(4)+5(2)\\0(2)+1(1)+3(1)&0(3)+1(0)+3(-1)&0(0)+1(4)+3(2)\end{bmatrix}$$

$$=\begin{bmatrix}2+2+5&3+0-5&0+8+10\\0+1+3&0+0-3&0+4+6\end{bmatrix}C=\begin{bmatrix}9&-2&18\\4&-3&10\end{bmatrix}\begin{bmatrix}1\\4\\5\end{bmatrix}$$

$$=\begin{bmatrix}9(1)-2(4)+18(5)\\4(1)-3(4)+10(5)\end{bmatrix}=\begin{bmatrix}9-8+90\\4-12+50\end{bmatrix}=\begin{bmatrix}91\\42\end{bmatrix}$$

Hence, $A(BC)=(AB)C$.

(ii) $\text{LHS}=A(BC)=A\left(\begin{bmatrix}1\\1\\2\end{bmatrix}\begin{bmatrix}1&-2\end{bmatrix}\right)=A\begin{bmatrix}1(1)&1(-2)\\1(1)&1(-2)\\2(1)&2(-2)\end{bmatrix}=A\begin{bmatrix}1&-2\\1&-2\\2&-4\end{bmatrix}=A\begin{bmatrix}1&-2\\1&-2\\2&-4\end{bmatrix}$

$$=\begin{bmatrix}2&3&-1\\3&0&2\end{bmatrix}\begin{bmatrix}1&-2\\1&-2\\2&-4\end{bmatrix}=\begin{bmatrix}2(1)+3(1)-1(2)&2(-2)+3(-2)-1(-4)\\3(1)+0(1)+2(2)&3(-2)+0(-2)+2(-4)\end{bmatrix}$$

$$=\begin{bmatrix}2+3-2&-4-6+4\\3+0+4&-6+0-8\end{bmatrix}=\begin{bmatrix}3&-6\\7&-14\end{bmatrix}$$

$$\text{RHS}=(AB)C=\left(\begin{bmatrix}2&3&-1\\3&0&2\end{bmatrix}\begin{bmatrix}1\\1\\2\end{bmatrix}\right)C=\begin{bmatrix}2(1)+3(1)-1(2)\\3(1)+0(1)+2(2)\end{bmatrix}C=\begin{bmatrix}2+3-2\\3+0+4\end{bmatrix}C$$

$$=\begin{bmatrix}3\\7\end{bmatrix}\begin{bmatrix}1&-2\end{bmatrix}=\begin{bmatrix}3(1)&3(-2)\\7(1)&7(-2)\end{bmatrix}=\begin{bmatrix}3&-6\\7&-14\end{bmatrix}$$

Hence, $A(BC)=(AB)C$.

7. Verify that $A(B+C)=(AB+AC)$, when

(i) $A=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$, $B=\begin{bmatrix}2 & 0\\ 1 & -3\end{bmatrix}$ and $C=\begin{bmatrix}1 & -1\\ 0 & 1\end{bmatrix}$

(ii) $A=\begin{bmatrix}2 & 3\\ -1 & 4\\ 0 & 1\end{bmatrix}$, $B=\begin{bmatrix}5 & -3\\ 2 & 1\end{bmatrix}$ and $C=\begin{bmatrix}-1 & 2\\ 3 & 4\end{bmatrix}$

Sol. **(i)** $\text{LHS}=A(B+C)=A\left[\begin{bmatrix}2 & 0\\ 1 & -3\end{bmatrix}+\begin{bmatrix}1 & -1\\ 0 & 1\end{bmatrix}\right]=A\begin{bmatrix}2+1 & 0-1\\ 1+0 & -3+1\end{bmatrix}=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}\begin{bmatrix}3 & -1\\ 1 & -2\end{bmatrix}$

$$=\begin{bmatrix}1(3)+2(1) & 1(-1)+2(-2)\\ 3(3)+4(1) & 3(-1)+4(-2)\end{bmatrix}=\begin{bmatrix}3+2 & -1-4\\ 9+4 & -3-8\end{bmatrix}=\begin{bmatrix}5 & -5\\ 13 & -11\end{bmatrix}$$

$\text{RHS}=AB+AC=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}\begin{bmatrix}2 & 0\\ 1 & -3\end{bmatrix}+\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}\begin{bmatrix}1 & -1\\ 0 & 1\end{bmatrix}$

$$=\begin{bmatrix}1(2)+2(1) & 1(0)+2(-3)\\ 3(2)+4(1) & 3(0)+4(-3)\end{bmatrix}+\begin{bmatrix}1(1)+2(0) & 1(-1)+2(1)\\ 3(1)+4(0) & 3(-1)+4(1)\end{bmatrix}$$

$$=\begin{bmatrix}2+2 & 0-6\\ 6+4 & 0-12\end{bmatrix}+\begin{bmatrix}1+0 & -1+2\\ 3+0 & -3+4\end{bmatrix}=\begin{bmatrix}4 & -6\\ 10 & -12\end{bmatrix}+\begin{bmatrix}1 & 1\\ 3 & 1\end{bmatrix}=\begin{bmatrix}4+1 & -6+1\\ 10+3 & -12+1\end{bmatrix}=\begin{bmatrix}5 & -5\\ 13 & -11\end{bmatrix}$$

Hence, $A(B+C)=(AB+AC)$

(ii) $\text{LHS}=A(B+C)=A\left[\begin{bmatrix}5 & -3\\ 2 & 1\end{bmatrix}+\begin{bmatrix}-1 & 2\\ 3 & 4\end{bmatrix}\right]=A\begin{bmatrix}6+(-1) & -3+2\\ 2+3 & 1+4\end{bmatrix}=A\begin{bmatrix}5-1 & -1\\ 5 & 5\end{bmatrix}$

$$=\begin{bmatrix}2 & 3\\ -1 & 4\\ 0 & 1\end{bmatrix}\begin{bmatrix}4 & -1\\ 5 & 5\end{bmatrix}=\begin{bmatrix}2(4)+3(5) & 2(-1)+3(5)\\ -1(4)+4(5) & -1(-1)+4(5)\\ 0(4)+1(5) & 0(-1)+1(5)\end{bmatrix}=\begin{bmatrix}8+15 & -2+15\\ -4+20 & 1+20\\ 0+5 & 0+5\end{bmatrix}=\begin{bmatrix}23 & 13\\ 16 & 21\\ 5 & 5\end{bmatrix}$$

$\text{RHS}=AB+BC=\begin{bmatrix}2 & 3\\ -1 & 4\\ 0 & 1\end{bmatrix}\begin{bmatrix}5 & -3\\ 2 & 1\end{bmatrix}+\begin{bmatrix}2 & 3\\ -1 & 4\\ 0 & 1\end{bmatrix}\begin{bmatrix}-1 & 2\\ 3 & 4\end{bmatrix}$

$$=\begin{bmatrix}2(5)+3(2) & 2(3)+3(1)\\ -1(5)+4(2) & -1(-3)+4(1)\\ 0(5)+1(2) & 0(-3)+1(1)\end{bmatrix}+\begin{bmatrix}2(-1)+3(3) & 2(2)+3(4)\\ -1(-1)+4(3) & -1(2)+4(4)\\ 0(-1)+1(3) & 0(2)+1(4)\end{bmatrix}$$

$$=\begin{bmatrix}10+6 & -6+3\\ -5+8 & 3+4\\ 0+2 & 0+1\end{bmatrix}+\begin{bmatrix}-2+9 & 4+12\\ 1+12 & -2+16\\ 0+3 & 0+4\end{bmatrix}$$

$$=\begin{bmatrix}16 & -3\\ 3 & 7\\ 2 & 1\end{bmatrix}+\begin{bmatrix}7 & 16\\ 13 & 14\\ 3 & 4\end{bmatrix}=\begin{bmatrix}16+7 & -3+16\\ 3+13 & 7+14\\ 2+3 & 1+4\end{bmatrix}=\begin{bmatrix}23 & 13\\ 16 & 21\\ 5 & 5\end{bmatrix}$$

Hence $A(B+C)=AB+AC$.

8. If $A=\begin{bmatrix}1 & 0 & -2\\ 3 & -1 & 0\\ -2 & 1 & 1\end{bmatrix}$, $B=\begin{bmatrix}0 & 5 & -4\\ -2 & 1 & 3\\ -1 & 0 & 2\end{bmatrix}$ and $C=\begin{bmatrix}1 & 5 & 2\\ -1 & 1 & 0\\ 0 & -1 & 1\end{bmatrix}$; verity that $A(B-C)=(AB-AC)$.

Sol. LHS $=A(B-C)=A\left(\begin{bmatrix}0 & 5 & -4\\ -2 & 1 & 3\\ -1 & 0 & 2\end{bmatrix}-\begin{bmatrix}1 & 5 & 2\\ -1 & 1 & 0\\ 0 & -1 & 1\end{bmatrix}\right)=A\begin{bmatrix}0-1 & 5-5 & -4-2\\ -2+1 & 1-1 & 3-0\\ -1-0 & 0+1 & 2-1\end{bmatrix}$

$$=\begin{bmatrix}1 & 0 & -2\\ 3 & -1 & 0\\ -2 & 1 & 1\end{bmatrix}\begin{bmatrix}-1 & 0 & -6\\ -1 & 0 & 3\\ -1 & 1 & 1\end{bmatrix}$$

$$=\begin{bmatrix}1(-1)+0(-1)-2(-1) & 1(0)+0(0)-2(1) & 1(-6)+0(3)-2(1)\\ 3(-1)-1(-1)+0(-1) & 3(0)-1(0)+0(1) & 3(-6)-1(3)+0(1)\\ -2(-1)+1(-1)+1(-1) & -2(0)+1(0)+1(1) & -2(-6)+1(3)+1(1)\end{bmatrix}$$

$$=\begin{bmatrix}-1-0+2 & 0+0-2 & -6+0-2\\ -3+1-0 & 0-0+0 & -18-3+0\\ 2-1-1 & 0+0+1 & 12+3+1\end{bmatrix}=\begin{bmatrix}1 & -2 & -8\\ -2 & 0 & -21\\ 0 & 1 & 16\end{bmatrix}$$

RHS $=AB-AC=\left(\begin{bmatrix}1 & 0 & -2\\ 3 & -1 & 0\\ -2 & 1 & 1\end{bmatrix}\begin{bmatrix}0 & 5 & -4\\ -2 & 1 & 3\\ -1 & 0 & 2\end{bmatrix}\right)-\left(\begin{bmatrix}1 & 0 & -2\\ 3 & -1 & 0\\ -2 & 1 & 1\end{bmatrix}\begin{bmatrix}1 & 5 & 2\\ -1 & 1 & 0\\ 0 & -1 & 1\end{bmatrix}\right)$

$$=\begin{bmatrix}1(0)+0(-2)-2(-1) & 1(5)+0(1)-2(0) & 1(-4)+0(3)-2(3)\\ 3(0)-1(-2)+0(-1) & 3(5)-1(1)+0(0) & 3(-4)-1(3)+0(2)\\ -2(0)+1(-2)+1(-1) & -2(5)+1(1)+1(0) & -2(-4)+1(3)+1(2)\end{bmatrix}$$

$$-\begin{bmatrix}1(1)+0(-1)-2(0) & 1(5)+0(1)-2(-1) & 1(2)+0(0)+(-2)1\\ 3(1)-1(-1)+0(0) & 3(5)-1(1)+0(-1) & 3(2)-1(0)+0(1)\\ -2(1)+1(-1)+1(0) & -2(5)+1(1)+1(-1) & -2(2)+1(0)+1(1)\end{bmatrix}$$

$$=\begin{bmatrix}0-0+2 & 5+0-0 & -4+0-4\\ 0+2-0 & 15-1+0 & -12-3+0\\ 0-2-1 & -10+1+0 & 8+3+2\end{bmatrix}-\begin{bmatrix}1-0-0 & 5+0+2 & 2+0-2\\ 3+1+0 & 15-1+0 & 6-0+0\\ -2-1+0 & -10+1-1 & -4+0+1\end{bmatrix}$$

$$=\begin{bmatrix}2 & 5 & -8\\ 2 & 14 & -15\\ -3 & -9 & 13\end{bmatrix}-\begin{bmatrix}1 & 7 & 0\\ 4 & 14 & 6\\ -3 & -10 & -3\end{bmatrix}=\begin{bmatrix}2-1 & 5-7 & -8-0\\ 2-4 & 14-14 & -15-6\\ -3+3 & -9+10 & 13+3\end{bmatrix}=\begin{bmatrix}1 & -2 & -8\\ -2 & 0 & -21\\ 0 & 1 & 16\end{bmatrix}$$

$\therefore$ LHS = RHS. Hence, $A(B-C)=AB-AC$.

9. If $A=\begin{bmatrix}ab & b^2\\ -a^2 & -ab\end{bmatrix}$, show that $A^2=O$.

Sol. $A^2=A.A=\begin{bmatrix}ab & b^2\\ -a^2 & -ab\end{bmatrix}\begin{bmatrix}ab & b^2\\ -a^2 & -ab\end{bmatrix}=\begin{bmatrix}ab(ab)+b^2(-a^2) & ab(b^2)+b^2(-ab)\\ -a^2(ab)-ab(-a^2) & -a^2(b^2)-ab(-ab)\end{bmatrix}$

$$=\begin{bmatrix}a^2b^2-a^2b^2 & ab^3-ab^3\\ -a^3b+a^3b & -a^2b^2+a^2b^2\end{bmatrix}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}=O$$

Hence, $A^2 = O$.

10. If $A=\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$, show that $A^2 = A$.

Sol. $A^2 = A.A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$

$$= \begin{bmatrix} 2(2)-2(-1)-4(1) & 2(-2)-2(3)-4(-2) & 2(-4)-2(4)-4(-3) \\ -1(2)+3(-1)+4(1) & -1(-2)+3(3)+4(-2) & -1(-4)+3(4)+4(-3) \\ 1(2)-2(-1)-3(1) & 1(-2)-2(3)-3(-2) & 1(-4)-2(4)-3(-3) \end{bmatrix}$$

$$= \begin{bmatrix} 4+2-4 & -4-6+8 & -8-8+12 \\ -2-3+4 & 2+9-8 & 4+12-12 \\ 2+2-3 & -2-6+6 & -4-8+9 \end{bmatrix} = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$$

Hence, $A^2 = A$.

11. If $A=\begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix}$, show that $A^2 = I$.

Sol. $A^2 = A.A = \begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix}\begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix}$

$$= \begin{bmatrix} 4(4)-1(3)-4(3) & 4(-1)-1(0)-4(-1) & 4(-4)-1(-4)-4(-3) \\ 3(4)+0(3)-4(3) & 3(-1)+0(0)-4(-1) & 3(-4)+0(-4)-4(-3) \\ 3(4)-1(3)-3(3) & 3(-1)-1(0)-3(-1) & 3(-4)-1(-4)-3(-3) \end{bmatrix}$$

$$= \begin{bmatrix} 16-3-12 & -4-0+4 & -16+4+12 \\ 12+0-12 & -3+0+4 & -12-0+12 \\ 12-3-9 & -3-0+3 & -12+4+9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

Hence, $A^2 = I$.

12. If $A=\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ and $B=\begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}$, find $(3A^2 - 2B + I)$.

Sol. $3A^2 - 2B + I = 3\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix} - 2\begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$= 3\begin{bmatrix} 2(2)-1(3) & 2(-1)-1(2) \\ 3(2)+2(3) & 3(-1)+2(2) \end{bmatrix} - \begin{bmatrix} 0 & 8 \\ -2 & 14 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= 3\begin{bmatrix} 4-3 & -2-2 \\ 6+6 & -3+4 \end{bmatrix} - \begin{bmatrix} 0 & 8 \\ -2 & 14 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= 3\begin{bmatrix} 1 & -4 \\ 12 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 8 \\ -2 & 14 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -12 \\ 36 & 3 \end{bmatrix} - \begin{bmatrix} 0 & 8 \\ -2 & 14 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$=\begin{bmatrix}3 & -20\\ 38 & -11\end{bmatrix}+\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=\begin{bmatrix}4 & -20\\ 38 & -10\end{bmatrix}$$

13. If $A=\begin{bmatrix}2 & -2\\ -3 & 4\end{bmatrix}$ then find $\left(-A^2+6A\right)$.

Sol. $-A^2+6A=-\begin{bmatrix}2 & -2\\ -3 & 4\end{bmatrix}\begin{bmatrix}2 & -2\\ -3 & 4\end{bmatrix}+6\begin{bmatrix}2 & -2\\ -3 & 4\end{bmatrix}$

$$=-\begin{bmatrix}2(2)-2(-3) & 2(-2)-2(4)\\ -3(2)+4(-3) & -3(-2)+4(4)\end{bmatrix}+\begin{bmatrix}12 & -12\\ -18 & 24\end{bmatrix}$$

$$=\begin{bmatrix}-10 & 12\\ 18 & -22\end{bmatrix}+\begin{bmatrix}12 & -12\\ -18 & 24\end{bmatrix}=\begin{bmatrix}-10+12 & 12+(-12)\\ 18+(-18) & -22+24\end{bmatrix}=\begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix}$$

14. If $A=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$, show that $A^2-5A+7I=O$.

Sol. LHS $=A^2-5A+7I$

$$=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}-5\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}+7\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}3(3)+1(-1) & 3(1)+1(2)\\ -1(3)+2(-1) & -1(1)+2(2)\end{bmatrix}-\begin{bmatrix}15 & 5\\ -5 & 10\end{bmatrix}+\begin{bmatrix}7 & 0\\ 0 & 7\end{bmatrix}$$

$$=\begin{bmatrix}9-1 & 3+2\\ -3-2 & -1+4\end{bmatrix}-\begin{bmatrix}15 & 5\\ -5 & 10\end{bmatrix}+\begin{bmatrix}7 & 0\\ 0 & 7\end{bmatrix}$$

$$=\begin{bmatrix}8 & 5\\ -5 & 3\end{bmatrix}-\begin{bmatrix}15 & 5\\ -5 & 10\end{bmatrix}+\begin{bmatrix}7 & 0\\ 0 & 7\end{bmatrix}=\begin{bmatrix}8-15+7 & 5-5+0\\ -5+5+0 & 3-10+7\end{bmatrix}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}=0$$

Hence, $A^2-5A+7I=O$.

15. Show that the matrix $A=\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}$ satisfies the equation $A^3-4A^2+A=O$.

Sol. LHS $=A^3-4A^2+A$

$$=\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}-4\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}+\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}$$

$$=\begin{bmatrix}2(2)+3(1) & 2(3)+3(2)\\ 1(2)+2(1) & 1(3)+2(2)\end{bmatrix}\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}-4\begin{bmatrix}2(2)+3(1) & 2(3)+3(2)\\ 1(2)+2(1) & 1(3)+2(2)\end{bmatrix}+\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}$$

$$=\begin{bmatrix}4+3 & 6+6\\ 2+2 & 3+4\end{bmatrix}\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}-4\begin{bmatrix}4+3 & 6+6\\ 2+2 & 3+4\end{bmatrix}+\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}$$

$$=\begin{bmatrix}7 & 12\\ 4 & 7\end{bmatrix}\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}-4\begin{bmatrix}7 & 12\\ 4 & 7\end{bmatrix}+\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}$$

$$=\begin{bmatrix}7(2)+12(1) & 7(3)+12(2)\\ 4(2)+7(1) & 4(3)+7(2)\end{bmatrix}-\begin{bmatrix}28 & 48\\ 16 & 28\end{bmatrix}+\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}$$

$$=\begin{bmatrix}14+12 & 21+24\\ 8+7 & 12+14\end{bmatrix}-\begin{bmatrix}28 & 48\\ 16 & 28\end{bmatrix}+\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}$$

$$= \begin{bmatrix} 26 & 45 \\ 15 & 26 \end{bmatrix} - \begin{bmatrix} 28 & 48 \\ 16 & 28 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 26-28+2 & 45-48+3 \\ 15-16+1 & 26-28+2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0. \text{ Hence, } A^3 - 4A^2 + A = O.$$

16. If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$, find k so that $A^2 = kA - 2I$.

Sol. $A^2 = kA - 2I \quad \Rightarrow kA = A^2 + 2I$

$$\Rightarrow kA = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} + 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow kA = \begin{bmatrix} 3(3)-2(4) & 3(-2)-2(-2) \\ 4(3)-2(4) & 4(-2)-2(-2) \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\Rightarrow kA = \begin{bmatrix} 9-8 & -6+4 \\ 12-8 & -8+4 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \Rightarrow kA = \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \Rightarrow kA = \begin{bmatrix} 1+2 & -2+0 \\ 4+0 & -4+2 \end{bmatrix}$$

$$\Rightarrow kA = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} \Rightarrow k\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} \Rightarrow \begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$

$3k = 3,\ -2k = -2,\ 4k = 4,\ -2k = -2$. Hence, $k = 1$.

17. If $A = \begin{bmatrix} -1 & 2 \\ 3 & 1 \end{bmatrix}$, find $f(A)$, where $f(x) = x^2 - 2x + 3$.

Sol. $f(x) = x^2 - 2x + 3 \quad \Rightarrow f(A) = A^2 - 2A + 3I$

$$\Rightarrow f(A) = \begin{bmatrix} -1 & 2 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} -1 & 2 \\ 3 & 1 \end{bmatrix} - 2\begin{bmatrix} -1 & 2 \\ 3 & 1 \end{bmatrix} + 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow f(A) = \begin{bmatrix} -1(-1)+2(3) & -1(2)+2(1) \\ 3(-1)+1(3) & 3(2)+1(1) \end{bmatrix} - \begin{bmatrix} -2 & 4 \\ 6 & 2 \end{bmatrix} + 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow f(A) = \begin{bmatrix} 1+6 & -2+2 \\ -3+3 & 6+1 \end{bmatrix} - \begin{bmatrix} -2 & 4 \\ 6 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$\Rightarrow f(A) = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} - \begin{bmatrix} -2 & 4 \\ 6 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \Rightarrow f(A) = \begin{bmatrix} 7+2+3 & 0-4+0 \\ 0-6+0 & 7-2+3 \end{bmatrix} = \begin{bmatrix} 12 & -4 \\ -6 & 8 \end{bmatrix}$$

18. If $A = \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix}$ and $f(x) = 2x^3 + 4x + 5$, find $f(A)$.

Sol. $f(x) = 2x^3 + 4x + 5 \quad \Rightarrow f(A) = 2A^3 + 4A + 5I$

$$\Rightarrow f(A) = 2\begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix} + 4\begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix} + 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow f(A) = 2\begin{bmatrix} 1(1)+2(4) & 1(2)+2(-3) \\ 4(1)-3(4) & 4(2)+(-3)(-3) \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix} + \begin{bmatrix} 4 & 8 \\ 16 & -12 \end{bmatrix} + \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$\Rightarrow f(A) = 2\begin{bmatrix} 1+8 & 2-6 \\ 4-12 & 8+9 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix} + \begin{bmatrix} 4+5 & 8+0 \\ 16+0 & -12+5 \end{bmatrix}$$

$$\Rightarrow f(A) = 2\begin{bmatrix} 9 & -4 \\ -8 & 17 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix} + \begin{bmatrix} 9 & 8 \\ 16 & -7 \end{bmatrix}$$

$$\Rightarrow f(A)=2\begin{bmatrix} 9(1)-4(4) & 9(2)-4(-3) \\ -8(1)+17(4) & -8(2)+17(-3) \end{bmatrix}+\begin{bmatrix} 9 & 8 \\ 16 & -7 \end{bmatrix}$$

$$\Rightarrow f(A)=2\begin{bmatrix} 9-16 & 18+12 \\ -8+68 & -16-51 \end{bmatrix}+\begin{bmatrix} 9 & 8 \\ 16 & -7 \end{bmatrix} \Rightarrow f(A)=2\begin{bmatrix} -7 & 30 \\ 60 & -67 \end{bmatrix}+\begin{bmatrix} 9 & 8 \\ 16 & -7 \end{bmatrix}$$

$$\Rightarrow f(A)=\begin{bmatrix} -14 & 60 \\ 120 & -134 \end{bmatrix}+\begin{bmatrix} 9 & 8 \\ 16 & -7 \end{bmatrix} \Rightarrow f(A)=\begin{bmatrix} -14+9 & 60+8 \\ 120+16 & -134-7 \end{bmatrix}=\begin{bmatrix} -5 & 68 \\ 136 & -141 \end{bmatrix}$$

19. Find the value of x and y when $\begin{bmatrix} 2 & -3 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 1 \\ 3 \end{bmatrix}$.

Sol. $\begin{bmatrix} 2(x)-3(y) \\ 1(x)+1(y) \end{bmatrix}=\begin{bmatrix} 1 \\ 3 \end{bmatrix} \Rightarrow \begin{bmatrix} 2x-3y \\ x+y \end{bmatrix}=\begin{bmatrix} 1 \\ 3 \end{bmatrix}$

$2x-3y=1$ ...(1)

and $x+y=3$ ...(2) × 3

$\Rightarrow 5x=10 \Rightarrow x=2$

Putting the value of x in equation (2), we get $2x-3y=1 \Rightarrow 2(2)-3y=1$

$\Rightarrow 3y=4-1 \Rightarrow 3y=3 \Rightarrow y=1$ Hence, $x=2$ and $y=1$.

20. Solve for x and y when $\begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 3 \\ 11 \end{bmatrix}$.

Sol. $\begin{bmatrix} 3(x)-4(y) \\ 1(x)+2(y) \end{bmatrix}=\begin{bmatrix} 3 \\ 11 \end{bmatrix} \Rightarrow \begin{bmatrix} 3x-4y \\ x+2y \end{bmatrix}=\begin{bmatrix} 3 \\ 11 \end{bmatrix}$

$3x-4y=3$ ...(1)

$x+2y=11$ ...(2) × 2

Solving equations (1) and (2), we get $5x=25 \Rightarrow x=5$

Putting the value of x in equation (1), we get $3x-4y=3 \Rightarrow 4y=3x-3$

$\Rightarrow 4y=3(5)-3 \Rightarrow 4y=15-3 \Rightarrow 4y=12 \Rightarrow y=3$. Hence, $x=5$ and $y=3$.

21. If $A=\begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$ find x and y such that $A^2+xI=yA$.

Sol. $A^2+xI=yA$

$$\Rightarrow \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}+x\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}=y\begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3(3)+1(7) & 3(1)+1(5) \\ 7(3)+5(7) & 7(1)+5(5) \end{bmatrix}+\begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix}=\begin{bmatrix} 3y & y \\ 7y & 5y \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 9+7 & 3+5 \\ 21+35 & 7+25 \end{bmatrix}+\begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix}=\begin{bmatrix} 3y & y \\ 7y & 5y \end{bmatrix} \Rightarrow \begin{bmatrix} 16 & 8 \\ 56 & 32 \end{bmatrix}+\begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix}=\begin{bmatrix} 3y & y \\ 7y & 5y \end{bmatrix}$$

$16+x=3y$ ...(1)

$8+0=y$ ...(2)

$\Rightarrow y=8$, putting the value of y in equation (1), we get, $16+x=3y$

$\Rightarrow 16+x=3(8) \Rightarrow x=24-16 \Rightarrow x=8$. Hence, $x=8$ and $y=8$.

22. If $A=\begin{bmatrix}3&2\\1&1\end{bmatrix}$ find the values of a and b such that $A^2+aA+bI=O$.

Sol. $A^2+aA+bI=O \Rightarrow \begin{bmatrix}3&2\\1&1\end{bmatrix}\begin{bmatrix}3&2\\1&1\end{bmatrix}+a\begin{bmatrix}3&2\\1&1\end{bmatrix}+b\begin{bmatrix}1&0\\0&1\end{bmatrix}=0$

$\Rightarrow \begin{bmatrix}3(3)+2(1)&3(2)+2(1)\\1(3)+1(1)&1(2)+1(1)\end{bmatrix}+\begin{bmatrix}3a&2a\\a&a\end{bmatrix}+\begin{bmatrix}b&0\\0&b\end{bmatrix}=0$

$\Rightarrow \begin{bmatrix}9+2&6+2\\3+1&2+1\end{bmatrix}+\begin{bmatrix}3a&2a\\a&a\end{bmatrix}+\begin{bmatrix}b&0\\0&b\end{bmatrix}=0 \quad \Rightarrow \begin{bmatrix}11&8\\4&3\end{bmatrix}+\begin{bmatrix}3a&2a\\a&a\end{bmatrix}+\begin{bmatrix}b&0\\0&b\end{bmatrix}=0$

$11+3a+b=0 \qquad \ldots(1)$

$4+a+0=0 \quad \Rightarrow a=-4$

Putting the value of a in equation (1), we get $11+3(-4)+b=0 \quad \Rightarrow b=12-11=1$

Hence, $a=-4$ and $b=1$.

23. Find the matrix A such that $\begin{bmatrix}5&-7\\-2&3\end{bmatrix}.A=\begin{bmatrix}-16&-6\\7&2\end{bmatrix}$.

Sol. Let matrix $A=\begin{bmatrix}a&b\\x&y\end{bmatrix}$

$\Rightarrow \begin{bmatrix}5&-7\\-2&3\end{bmatrix}\begin{bmatrix}a&b\\x&y\end{bmatrix}=\begin{bmatrix}-16&-6\\7&2\end{bmatrix} \quad \Rightarrow \begin{bmatrix}5a-7x&5b-7y\\-2a+3x&-2b+3y\end{bmatrix}=\begin{bmatrix}-16&-6\\7&2\end{bmatrix}$

$5a-7x=-16 \qquad \ldots(1)\times 3$

$-2a+3x=7 \qquad \ldots(2)\times 7$

$5b-7y=-6 \qquad \ldots(3)\times 3$

$-2b+3y=2 \qquad \ldots(4)\times 7$

Solving equations (1) and (2), we get $a=1$

Putting the value of a in equation (1), we get

$5a-7x=-16 \quad \Rightarrow 5(1)-7x=-16 \quad \Rightarrow 5-7x=-16 \quad \Rightarrow 7x=5+16$

$\Rightarrow 7x=21 \quad \Rightarrow x=3$

Now, solving the equation (3) and (4), we get $b=-4$

Putting the value of b in equation (3), we get $5b-7y=-6 \quad \Rightarrow 7y=5b+6$

$\Rightarrow 7y=5(-4)+6 \quad \Rightarrow 7y=-20+6 \quad \Rightarrow 7y=-14 \quad \Rightarrow y=-2$

$\therefore \quad a=1, b=-4, x=3$ and $y=-2$. Hence, matrix $A=\begin{bmatrix}1&-4\\3&-2\end{bmatrix}$.

24. Find the matrix A such that $A.\begin{bmatrix}2&3\\4&5\end{bmatrix}=\begin{bmatrix}0&-4\\10&3\end{bmatrix}$.

Sol. Let $A=\begin{bmatrix}a&x\\b&y\end{bmatrix} \quad \Rightarrow \begin{bmatrix}a&x\\b&y\end{bmatrix}\begin{bmatrix}2&3\\4&5\end{bmatrix}=\begin{bmatrix}0&-4\\10&3\end{bmatrix} \quad \Rightarrow \begin{bmatrix}2a+4x&3a+5x\\2b+4y&3b+5y\end{bmatrix}=\begin{bmatrix}0&-4\\10&3\end{bmatrix}$

$2a+4x=0 \qquad \ldots(1)\times 5$

$3a+5x=-4 \qquad \ldots(2)\times 4$

$2b+4y=10 \qquad \ldots(3)\times 5$

$3b+5y=3 \qquad \ldots(4)\times 4$

Solving the equation (1) and (2), we get $-2a=16 \Rightarrow a=-8$

Putting the value of a in equation (1), we get

$2a+4x=0 \Rightarrow 2(-8)+4x=0 \Rightarrow 4x=16 \Rightarrow x=4$

Now solve the equation (3) and (4), we get $-2b=38 \Rightarrow b=-19$

Putting the value of b in equation (iii), we get $2b+4y=10$

$\Rightarrow 2(-19)+4y=10 \Rightarrow 4y=10+38 \Rightarrow 4y=48 \Rightarrow y=12$

$\therefore$ $a=-8, b=-19, x=4$ and $y=12$. Hence, $A=\begin{bmatrix}-8 & 4\\ -19 & 12\end{bmatrix}$.

Or alternatively : $A=\begin{bmatrix}0 & -4\\ 10 & 3\end{bmatrix}\begin{bmatrix}2 & 3\\ 4 & 5\end{bmatrix}^{-1}$

25. If $A=\begin{bmatrix}1 & -1\\ 2 & -1\end{bmatrix}, B=\begin{bmatrix}a & 1\\ b & -1\end{bmatrix}$ and $(A+B)^2=(A^2+B^2)$ then find the values of a and b.

Sol. Let $(A+B)^2=A^2+B^2=(A+B)(A+B)=A^2+B^2$

$\Rightarrow A^2+AB+BA+B^2=A^2+B^2 \qquad \Rightarrow AB+BA=0$

$\Rightarrow AB=-BA$ **and try it yourself, then find** $a=1, b=4$

26. If $F(x)=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}$, show that $F(x).F(y)=F(x+y)$

Sol. Let $F(x)=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}$ and $F(y)=\begin{bmatrix}\cos y & -\sin y & 0\\ \sin y & \cos y & 0\\ 0 & 0 & 1\end{bmatrix}$

$$\therefore\ F(x).F(y)=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\cos y & -\sin y & 0\\ \sin y & \cos y & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}\cos x\cos y-\sin x\sin y & -\cos x\sin y-\sin x\cos y & 0\\ \sin x\cos y+\cos x\sin y & -\sin x\sin y+\cos x\cos y & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}\cos(x+y) & -\sin(x+y) & 0\\ \sin(x+y) & \cos(x+y) & 0\\ 0 & 0 & 1\end{bmatrix}=F(x+y)$$

Hence, $F(x).F(y)=F(x+y)$

27. If $A=\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$, show that $A^2=\begin{bmatrix}\cos 2\alpha & \sin 2\alpha\\ -\sin 2\alpha & \cos 2\alpha\end{bmatrix}$.

Sol. Let $A=\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$

$A^2=A.A=\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$

$$= \begin{bmatrix} \cos\alpha.\cos\alpha + \sin\alpha(-\sin\alpha) & \cos\alpha.\sin\alpha + \sin\alpha.\cos\alpha \\ -\sin\alpha.\cos\alpha + \cos\alpha(-\sin\alpha) & -\sin\alpha.\sin\alpha + \cos\alpha.\cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\alpha - \sin^2\alpha & 2\sin\alpha\cos\alpha \\ -2\sin\alpha\cos\alpha & \cos^2\alpha - \sin^2\alpha \end{bmatrix} = \begin{bmatrix} \cos 2\alpha & \sin 2\alpha \\ -\sin 2\alpha & \cos 2\alpha \end{bmatrix}.$$ Hence proved.

28. If $\begin{bmatrix} 1 & x & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 3 & 2 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} = O$, find x.

Sol. $\begin{bmatrix} 1 & x & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 3 & 2 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} = 0$

$$\Rightarrow \begin{bmatrix} 1(1) + x(4) + 1(3) & 1(2) + x(5) + 1(2) & 1(3) + x(6) + 1(5) \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 1 + 4x + 3 & 2 + 5x + 2 & 3 + 6x + 5 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 4 + 4x & 5x + 4 & 6x + 8 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} = 0 \quad \Rightarrow \left[(4x+4)(1) + (5x+4)(-2) + (6x+8)3\right] = 0$$

$$\Rightarrow \left[4x + 4 - 10x - 8 + 18x + 24\right] = 0 \quad \Rightarrow 12x + 20 = 0 \quad \Rightarrow 12x = -20 \quad \Rightarrow x = \frac{-20}{12} = \frac{-5}{3}.$$

29. If $\begin{bmatrix} x & 4 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 2 \\ 1 & 0 & 2 \\ 0 & 2 & -4 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ -1 \end{bmatrix} = O$, find x.

Sol. $\begin{bmatrix} x & 4 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 2 \\ 1 & 0 & 2 \\ 0 & 2 & -4 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ -1 \end{bmatrix} = O$

$$\Rightarrow \begin{bmatrix} x(2) + 4(1) + 1(0) & x(1) + 4(0) + 1(2) & x(2) + 4(2) + 1(-4) \end{bmatrix} \begin{bmatrix} x \\ 4 \\ -1 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 2x + 4 + 0 & x + 0 + 2 & 2x + 8 - 4 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ -1 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 2x + 4 & x + 2 & 2x + 4 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ -1 \end{bmatrix} = 0 \quad \Rightarrow \left[(2x+4)x + (x+2)4 + (2x+4)(-1)\right] = 0$$

$$\Rightarrow \left[2x^2 + 4x + 4x + 8 - 2x - 4\right] = 0 \quad \Rightarrow 2x^2 + 6x + 4 = 0 \quad \Rightarrow 2x^2 + 4x + 2x + 4 = 0$$

$$\Rightarrow 2x(x+2) + 2(x+2) = 0 \quad \Rightarrow (x+2)(2x+2) = 0 \quad \Rightarrow x + 2 = 0 \text{ or } 2x + 2 = 0$$

$\therefore\ x=-2$ or $x=\frac{-2}{2}=-1$. Hence, $x=-2$ or -1.

30. Find the values of a and b for which $\begin{bmatrix} a & b \\ -a & 2b \end{bmatrix}\begin{bmatrix} 2 \\ -1 \end{bmatrix}=\begin{bmatrix} 5 \\ 4 \end{bmatrix}$.

Sol. $\begin{bmatrix} a & b \\ -a & 2b \end{bmatrix}\begin{bmatrix} 2 \\ -1 \end{bmatrix}=\begin{bmatrix} 5 \\ 4 \end{bmatrix} \Rightarrow \begin{bmatrix} a(2)+b(-1) \\ -a(2)+2b(-1) \end{bmatrix}=\begin{bmatrix} 5 \\ 4 \end{bmatrix}$

$2a-b=5 \qquad \ldots(1)$

$-2a-2b=4 \qquad \ldots(2)$

Solving equations (1) and (2), we get $-3b=9 \quad \Rightarrow b=-3$

Putting the value of b in equation (1), we get $2a-b=5 \quad \Rightarrow 2a=5+b$

$\Rightarrow 2a=5+(-3) \quad \Rightarrow 2a=2 \quad \Rightarrow a=1$. Hence, $a=1$ and $b=-3$.

31. If $A=\begin{bmatrix} 3 & 4 \\ -4 & -3 \end{bmatrix}$ find $f(A)$, where $f(x)=x^2-5x+7$.

Sol. $f(x)=x^2-5x+7 \ \Rightarrow f(A)=A^2-5A+7I$

$\Rightarrow f(A)=\begin{bmatrix} 3 & 4 \\ -4 & -3 \end{bmatrix}\begin{bmatrix} 3 & 4 \\ -4 & -3 \end{bmatrix}-5\begin{bmatrix} 3 & 4 \\ -4 & -3 \end{bmatrix}+7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow f(A)=\begin{bmatrix} 3(3)+4(-4) & 3(4)+4(-3) \\ -4(3)-3(-4) & -4(4)-3(-3) \end{bmatrix}-\begin{bmatrix} 15 & 20 \\ -20 & -15 \end{bmatrix}+\begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$\Rightarrow f(A)=\begin{bmatrix} 9-16 & 12-12 \\ -12+12 & -16+9 \end{bmatrix}-\begin{bmatrix} 15 & 20 \\ -20 & -15 \end{bmatrix}+\begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$\Rightarrow f(A)=\begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix}-\begin{bmatrix} 15 & 20 \\ -20 & -15 \end{bmatrix}+\begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$\Rightarrow f(A)=\begin{bmatrix} -7-15+7 & 0-20+0 \\ 0+20+0 & -7+15+7 \end{bmatrix} \quad \Rightarrow f(A)=\begin{bmatrix} -15 & -20 \\ 20 & 15 \end{bmatrix}$

32. If $A=\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, prove that $A^n=\begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$ for all $n \in N$.

Sol. We shall prove the result by using the principle of mathematical induction.

When $n=1$, we have $A^1=\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$. Thus, the result is true for $n=1$.

Let the result be true for $n=k$. then $A^k=\begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}$

$\therefore\ A^{k+1}=A.A^k=\begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \quad \Rightarrow A^{k+1}=\begin{bmatrix} 1(1)+k(0) & 1(1)+k(1) \\ 0(1)+1(0) & 0(1)+1(1) \end{bmatrix}$

$\Rightarrow A^{k+1}=\begin{bmatrix} 1+0 & 1+k \\ 0 & 0+1 \end{bmatrix} \quad \Rightarrow A^{k+1}=\begin{bmatrix} 1 & 1+k \\ 0 & 1 \end{bmatrix}$

Thus, the result is true for $n=(k+1)$, whenever it is true for $n=k$.

So, the result is true for $n \in N$. Hence, $A^n=\begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$ for all values of $n \in N$.

33. Give an example of two matrices A and B such that $A \neq O,\ B \neq O,\ AB = O$ and $BA \neq O$.

Sol. Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix};$

$$AB = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \Rightarrow AB = \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \Rightarrow AB = O$$

Again let $BA = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \Rightarrow BA = \begin{bmatrix} 0+0 & 0+0 \\ 1+0 & 0+0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \Rightarrow BA \neq O$

34. Give an example of three matrices A, B, C such that $AB = AC$ but $B \neq C$.

Sol. Let the three matrices be $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

$$AB = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \Rightarrow AB = \begin{bmatrix} 1+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$AC = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \Rightarrow AC = \begin{bmatrix} 1+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

Hence, $AB = AC$

35. If $A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}, B = \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}$, find $\left(3A^2 - 2B + I\right)$.

Sol. $3A^2 - 2B + I \Rightarrow 3\begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} - 2\begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$= 3\begin{bmatrix} 1(1)+0(-1) & 1(0)+0(7) \\ -1(1)+7(-1) & -1(0)+7(7) \end{bmatrix} - \begin{bmatrix} 0 & 8 \\ -2 & 14 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= 3\begin{bmatrix} 1-0 & 0+0 \\ -1-7 & 0+49 \end{bmatrix} - \begin{bmatrix} 0 & 8 \\ -2 & 14 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= 3\begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix} - \begin{bmatrix} 0 & 8 \\ -2 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ -24 & 147 \end{bmatrix} - \begin{bmatrix} 0 & 8 \\ -2 & 14 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3-0+1 & 0-8+0 \\ -24+2+0 & 147-14+1 \end{bmatrix} = \begin{bmatrix} 4 & -8 \\ -22 & 134 \end{bmatrix}$$

36. If $\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}\begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$, find the value of x

Sol. Given $\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}\begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 2-6 & -6+12 \\ 5-14 & -15+28 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix} \Rightarrow \begin{bmatrix} -4 & -6 \\ -9 & 13 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix} \Rightarrow x = 13$$

EXERCISE 5D (Pg.No.: 201)

1. If $A=\begin{bmatrix}2 & -3 & 5\\ 0 & 7 & -4\end{bmatrix}$, verify that $(A')'=A$.

Sol. $A=\begin{bmatrix}2 & -3 & 5\\ 0 & 7 & -4\end{bmatrix} \Rightarrow A'=\begin{bmatrix}2 & 0\\ -3 & 7\\ 5 & -4\end{bmatrix} \Rightarrow (A')'=\begin{bmatrix}2 & -3 & 5\\ 0 & 7 & -4\end{bmatrix}$ $\therefore$ $(A')'=A$. Hence proved.

2. If $A=\begin{bmatrix}3 & 5\\ -2 & 0\\ 4 & -6\end{bmatrix}$, verify that $(2A)'=2A'$.

Sol. $A=\begin{bmatrix}3 & 5\\ -2 & 0\\ 4 & -6\end{bmatrix}$.

$\text{LHS}=2A=2\begin{bmatrix}3 & 5\\ -2 & 0\\ 4 & -6\end{bmatrix}=\begin{bmatrix}6 & 10\\ -4 & 0\\ 8 & -12\end{bmatrix} \Rightarrow (2A)'=\begin{bmatrix}6 & -4 & 8\\ 10 & 0 & -12\end{bmatrix}$

$\text{RHS}=2A'=2\begin{bmatrix}3 & -2 & 4\\ 5 & 0 & -6\end{bmatrix}=\begin{bmatrix}6 & -4 & 8\\ 10 & 0 & -12\end{bmatrix}$ $\therefore$ $(2A)'=2A'$. Hence proved.

3. If $A=\begin{bmatrix}3 & 2 & -1\\ -5 & 0 & -6\end{bmatrix}$ and $B=\begin{bmatrix}-4 & -5 & -2\\ 3 & 1 & 8\end{bmatrix}$, verity that $(A+B)'=(A'+B')$.

Sol. $A'=\begin{bmatrix}3 & -5\\ 2 & 0\\ -1 & -6\end{bmatrix}$ and $B'=\begin{bmatrix}-4 & 3\\ -5 & 1\\ -2 & 8\end{bmatrix}$

$\Rightarrow (A+B)=\begin{bmatrix}3 & 2 & -1\\ -5 & 0 & -6\end{bmatrix}+\begin{bmatrix}-4 & -5 & -2\\ 3 & 1 & 8\end{bmatrix}=\begin{bmatrix}3-4 & 2-5 & -1-2\\ -5+3 & 0+1 & -6+8\end{bmatrix}=\begin{bmatrix}-1 & -3 & -3\\ -2 & 1 & 2\end{bmatrix}$

$\text{LHS}=(A+B)'=\begin{bmatrix}-1 & -2\\ -3 & 1\\ -3 & 2\end{bmatrix}$

$\text{RHS}=(A'+B')=\begin{bmatrix}3 & -5\\ 2 & 0\\ -1 & -6\end{bmatrix}+\begin{bmatrix}-4 & 3\\ -5 & 1\\ -2 & 8\end{bmatrix}=\begin{bmatrix}3-4 & -5+3\\ 2-5 & 0+1\\ -1-2 & -6+8\end{bmatrix}=\begin{bmatrix}-1 & -2\\ -3 & 1\\ -3 & 2\end{bmatrix}$

$\therefore$ LHS = RHS. Hence proved that $(A+B)'=(A'+B')$.

4. If $P=\begin{bmatrix}3 & 4\\ 2 & -1\\ 0 & 5\end{bmatrix}$ and $Q=\begin{bmatrix}7 & -5\\ -4 & 0\\ 2 & 6\end{bmatrix}$, verify that $(P+Q)'=(P'+Q')$.

Sol. $\text{LHS}=(P+Q)'=\left(\begin{bmatrix}3 & 4\\ 2 & -1\\ 0 & 5\end{bmatrix}+\begin{bmatrix}7 & -5\\ -4 & 0\\ 2 & 6\end{bmatrix}\right)'=\begin{bmatrix}3+7 & 4-5\\ 2-4 & -1+0\\ 0+2 & 5+6\end{bmatrix}'=\begin{bmatrix}10 & -1\\ -2 & -1\\ 2 & 11\end{bmatrix}'$

$$\Rightarrow (P+Q)' = \begin{bmatrix} 10 & -2 & 2 \\ -1 & -1 & 11 \end{bmatrix}$$

RHS, $P' = \begin{bmatrix} 3 & 2 & 0 \\ 4 & -1 & 5 \end{bmatrix}, Q' = \begin{bmatrix} 7 & -4 & 2 \\ -5 & 0 & 6 \end{bmatrix}$

$$\Rightarrow (P'+Q') = \begin{bmatrix} 3 & 2 & 0 \\ 4 & -1 & 5 \end{bmatrix} + \begin{bmatrix} 7 & -4 & 2 \\ -5 & 0 & 6 \end{bmatrix} = \begin{bmatrix} 3+7 & 2-4 & 0+2 \\ 4-5 & -1+0 & 5+6 \end{bmatrix} = \begin{bmatrix} 10 & -2 & 2 \\ -1 & -1 & 11 \end{bmatrix}$$

$\therefore$ LHS = RHS. Hence proved that $(P+Q)' = (P'+Q')$.

5. If $A = \begin{bmatrix} 4 & 1 \\ 5 & 8 \end{bmatrix}$, show that $(A+A')$ is symmetric.

Sol. $A = \begin{bmatrix} 4 & 1 \\ 5 & 8 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 4 & 5 \\ 1 & 8 \end{bmatrix}$

$$(A+A') = \begin{bmatrix} 4 & 1 \\ 5 & 8 \end{bmatrix} + \begin{bmatrix} 4 & 5 \\ 1 & 8 \end{bmatrix} = \begin{bmatrix} 4+4 & 1+5 \\ 5+1 & 8+8 \end{bmatrix} = \begin{bmatrix} 8 & 6 \\ 6 & 16 \end{bmatrix}$$

$\Rightarrow (A+A')' = \begin{bmatrix} 8 & 6 \\ 6 & 16 \end{bmatrix}$ $\quad \therefore (A+A') = (A+A')'$. Hence, $(A+A')$ is a symmetric.

6. If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, show that $(A-A')$ is skew-symmetric.

Sol. $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}$

$$(A-A') = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix} = \begin{bmatrix} 3-3 & -4-1 \\ 1+4 & -1+1 \end{bmatrix} = \begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix}$$

$$\Rightarrow (A-A')' = \begin{bmatrix} 0 & 5 \\ -5 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix}$$

$\therefore$ $(A-A') = -(A-A')'$. Hence, $(A-A')'$ is a skew symmetric.

7. Show that the matrix $A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$ is skew symmetric.

Sol. $A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix} \Rightarrow A' = -\begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$

$\Rightarrow A' = -A$. Hence, A is a skew symmetric.

8. Express the matrix $A = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}$, as the sum of a symmetric matrix and a skew-symmetric matrix.

Sol. $A = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$

$$(A+A') = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix} + \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 2+2 & 3-1 \\ -1+3 & 4+4 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 2 & 8 \end{bmatrix}$$

$$(A-A')=\begin{bmatrix}2&3\\-1&4\end{bmatrix}-\begin{bmatrix}2&-1\\3&4\end{bmatrix}=\begin{bmatrix}2-2&3+1\\-1-3&4-4\end{bmatrix}=\begin{bmatrix}0&4\\-4&0\end{bmatrix}$$

$\therefore$ Sum of the symmetric and skew symmetric matrix

$$=\frac{1}{2}(A+A')+\frac{1}{2}(A-A')=\frac{1}{2}\begin{bmatrix}4&2\\2&8\end{bmatrix}+\frac{1}{2}\begin{bmatrix}0&4\\-4&0\end{bmatrix}=\begin{bmatrix}2&1\\1&4\end{bmatrix}+\begin{bmatrix}0&2\\-2&0\end{bmatrix}$$

9. Express the matrix $A=\begin{bmatrix}3&-4\\1&-1\end{bmatrix}$ as the sum of a symmetric matrix and a skew-symmetric matrix.

Sol. $A=\begin{bmatrix}3&-4\\1&-1\end{bmatrix} \Rightarrow A'=\begin{bmatrix}3&1\\-4&-1\end{bmatrix}$

$$(A+A')=\begin{bmatrix}3&-4\\1&-1\end{bmatrix}+\begin{bmatrix}3&1\\-4&-1\end{bmatrix}=\begin{bmatrix}3+3&-4+1\\1-4&-1-1\end{bmatrix}=\begin{bmatrix}6&-3\\-3&-2\end{bmatrix}$$

$$(A-A')=\begin{bmatrix}3&-4\\1&-1\end{bmatrix}-\begin{bmatrix}3&1\\-4&-1\end{bmatrix}=\begin{bmatrix}3-3&-4-1\\1+4&-1+1\end{bmatrix}=\begin{bmatrix}0&-5\\5&0\end{bmatrix}$$

$\therefore$ Sum of the symmetric and a skew-symmetric matrix,

$$A=\frac{1}{2}(A+A')+\frac{1}{2}(A-A')=\frac{1}{2}\begin{bmatrix}6&-3\\-3&-2\end{bmatrix}+\frac{1}{2}\begin{bmatrix}0&-5\\5&0\end{bmatrix}=\begin{bmatrix}3&-3/2\\-3/2&1\end{bmatrix}+\begin{bmatrix}0&-5/2\\5/2&0\end{bmatrix}$$

10. Express the matrix $A=\begin{bmatrix}-1&5&1\\2&3&4\\7&0&9\end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrix.

Sol. $A=\begin{bmatrix}-1&5&1\\2&3&4\\7&0&9\end{bmatrix} \Rightarrow A'=\begin{bmatrix}-1&2&7\\5&3&0\\1&4&9\end{bmatrix}$

$$(A+A')=\begin{bmatrix}-1&5&1\\2&3&4\\7&0&9\end{bmatrix}+\begin{bmatrix}-1&2&7\\5&3&0\\1&4&9\end{bmatrix}=\begin{bmatrix}-1-1&5+2&1+7\\2+5&3+3&4+0\\7+1&0+4&9+9\end{bmatrix}=\begin{bmatrix}-2&7&8\\7&6&4\\8&4&18\end{bmatrix}$$

$$(A-A')=\begin{bmatrix}-1&5&1\\2&3&4\\7&0&9\end{bmatrix}-\begin{bmatrix}-1&2&7\\5&3&0\\1&4&9\end{bmatrix}=\begin{bmatrix}-1+1&5-2&1-7\\2-5&3-3&4-0\\7-1&0-4&9-9\end{bmatrix}=\begin{bmatrix}0&3&-6\\-3&0&4\\6&-4&0\end{bmatrix}$$

$\therefore$ Sum of a symmetric and a skew-symmetric matrix

$$A=\frac{1}{2}(A+A')+\frac{1}{2}(A-A')$$

$$=\frac{1}{2}\begin{bmatrix}-2&7&8\\7&6&4\\8&4&18\end{bmatrix}+\frac{1}{2}\begin{bmatrix}0&3&-6\\-3&0&4\\6&-4&0\end{bmatrix}=\begin{bmatrix}-1&7/2&4\\7/2&3&2\\4&2&9\end{bmatrix}+\begin{bmatrix}0&3/2&-3\\-3/2&0&2\\3&-2&0\end{bmatrix}$$

11. Express the matrix A as the sum of a symmetric and a skew-symmetric matrix, where $A=\begin{bmatrix}3&-1&0\\2&0&3\\1&-1&2\end{bmatrix}$.

Sol. $A=\begin{bmatrix}3 & -1 & 0\\ 2 & 0 & 3\\ 1 & -1 & 2\end{bmatrix} \Rightarrow A'=\begin{bmatrix}3 & 2 & 1\\ -1 & 0 & -1\\ 0 & 3 & 2\end{bmatrix}$

$$(A+A')=\begin{bmatrix}3 & -1 & 0\\ 2 & 0 & 3\\ 1 & -1 & 2\end{bmatrix}+\begin{bmatrix}3 & 2 & 1\\ -1 & 0 & -1\\ 0 & 3 & 2\end{bmatrix}=\begin{bmatrix}3+3 & -1+2 & 0+1\\ 2-1 & 0+0 & 3-1\\ 1+0 & -1+3 & 2+2\end{bmatrix}=\begin{bmatrix}6 & 1 & 1\\ 1 & 0 & 2\\ 1 & 2 & 4\end{bmatrix}$$

$$(A-A')=\begin{bmatrix}3 & -1 & 0\\ 2 & 0 & 3\\ 1 & -1 & 2\end{bmatrix}-\begin{bmatrix}3 & 2 & 1\\ -1 & 0 & -1\\ 0 & 3 & 2\end{bmatrix}=\begin{bmatrix}3-3 & -1-2 & 0-1\\ 2+1 & 0-0 & 3+1\\ 1-0 & -1-3 & 2-2\end{bmatrix}=\begin{bmatrix}0 & -3 & -1\\ 3 & 0 & 4\\ 1 & -4 & 0\end{bmatrix}$$

$\therefore$ Sum of a symmetric and a skew-symmetric matrix, $A=\frac{1}{2}(A+A')+\frac{1}{2}(A-A')$

$$=\frac{1}{2}\begin{bmatrix}6 & 1 & 1\\ 1 & 0 & 2\\ 1 & 2 & 4\end{bmatrix}+\frac{1}{2}\begin{bmatrix}0 & -3 & -1\\ 3 & 0 & 4\\ 1 & -4 & 0\end{bmatrix}=\begin{bmatrix}3 & 1/2 & 1/2\\ 1/2 & 0 & 1\\ 1/2 & 1 & 2\end{bmatrix}+\begin{bmatrix}0 & -3/2 & -1/2\\ 3/2 & 0 & 2\\ 1/2 & -2 & 0\end{bmatrix}$$

12. Express the matrix $A=\begin{bmatrix}3 & 2 & 5\\ 4 & 1 & 3\\ 0 & 6 & 7\end{bmatrix}$ as sum of two matrices such that one is symmetric and the other is skew-symmetric.

Sol. $A=\begin{bmatrix}3 & 2 & 5\\ 4 & 1 & 3\\ 0 & 6 & 7\end{bmatrix} \Rightarrow A'=\begin{bmatrix}3 & 4 & 0\\ 2 & 1 & 6\\ 5 & 3 & 7\end{bmatrix}$

$$(A+A')=\begin{bmatrix}3 & 2 & 5\\ 4 & 1 & 3\\ 0 & 6 & 7\end{bmatrix}+\begin{bmatrix}3 & 4 & 0\\ 2 & 1 & 6\\ 5 & 3 & 7\end{bmatrix}=\begin{bmatrix}3+3 & 2+4 & 5+0\\ 4+2 & 1+1 & 3+6\\ 0+5 & 6+3 & 7+7\end{bmatrix}=\begin{bmatrix}6 & 6 & 5\\ 6 & 2 & 9\\ 5 & 9 & 14\end{bmatrix}$$

$$(A-A')=\begin{bmatrix}3 & 2 & 5\\ 4 & 1 & 3\\ 0 & 6 & 7\end{bmatrix}-\begin{bmatrix}3 & 4 & 0\\ 2 & 1 & 6\\ 5 & 3 & 7\end{bmatrix}=\begin{bmatrix}3-3 & 2-4 & 5-0\\ 4-2 & 1-1 & 3-6\\ 0-5 & 6-3 & 7-7\end{bmatrix}=\begin{bmatrix}0 & -2 & 5\\ 2 & 0 & -3\\ -5 & 3 & 0\end{bmatrix}$$

$\therefore$ Sum of a symmetric and skew-symmetric matrix $A=\frac{1}{2}(A+A')+\frac{1}{2}(A-A')$

$$=\frac{1}{2}\begin{bmatrix}6 & 6 & 5\\ 6 & 2 & 9\\ 5 & 9 & 14\end{bmatrix}+\frac{1}{2}\begin{bmatrix}0 & -2 & 5\\ 2 & 0 & -3\\ -5 & 3 & 0\end{bmatrix}=\begin{bmatrix}3 & 3 & 5/2\\ 3 & 1 & 9/2\\ 5/2 & 9/2 & 7\end{bmatrix}+\begin{bmatrix}0 & -1 & 5/2\\ 1 & 0 & -3/2\\ -5/2 & 3/2 & 0\end{bmatrix}$$

13. For each of the following pairs of matrices A and B, verity that $(AB)'=B'A'$.

(i) $A=\begin{bmatrix}1 & 3\\ 2 & 4\end{bmatrix}$ and $B=\begin{bmatrix}1 & 4\\ 2 & 5\end{bmatrix}$ (ii) $A=\begin{bmatrix}3 & -1\\ 2 & -2\end{bmatrix}$ and $B=\begin{bmatrix}1 & -3\\ 2 & -1\end{bmatrix}$

(iii) $A=\begin{bmatrix}-1\\ 2\\ 3\end{bmatrix}$ and $B=\begin{bmatrix}-2 & -1 & -4\end{bmatrix}$. (iv) $A=\begin{bmatrix}-1 & 2 & -3\\ 4 & -5 & 6\end{bmatrix}$ and $B=\begin{bmatrix}3 & -4\\ 2 & 1\\ -1 & 0\end{bmatrix}$

Sol. **(i)** $A=\begin{bmatrix}1&3\\2&4\end{bmatrix}$ and $B=\begin{bmatrix}1&4\\2&5\end{bmatrix}$

$\text{LHS}=(AB)'$

$\Rightarrow AB=\begin{bmatrix}1&3\\2&4\end{bmatrix}\begin{bmatrix}1&4\\2&5\end{bmatrix}=\begin{bmatrix}1(1)+3(2)&1(4)+3(5)\\2(1)+4(2)&2(4)+4(5)\end{bmatrix}=\begin{bmatrix}1+6&4+15\\2+8&8+20\end{bmatrix}=\begin{bmatrix}7&19\\10&28\end{bmatrix}$

$\Rightarrow (AB)'=\begin{bmatrix}7&10\\19&28\end{bmatrix}$

$\text{RHS}=B'A'$, $B'=\begin{bmatrix}1&2\\4&5\end{bmatrix}$ and $A'=\begin{bmatrix}1&2\\3&4\end{bmatrix}$

$\Rightarrow B'A'=\begin{bmatrix}1&2\\4&5\end{bmatrix}\begin{bmatrix}1&2\\3&4\end{bmatrix}=\begin{bmatrix}1(1)+2(3)&1(2)+2(4)\\4(1)+5(3)&4(2)+5(4)\end{bmatrix}=\begin{bmatrix}1+6&2+8\\4+15&8+20\end{bmatrix}=\begin{bmatrix}7&10\\19&28\end{bmatrix}$

$\therefore$ LHS = RHS. Hence proved that $(AB)'=B'A'$.

(ii) $A=\begin{bmatrix}3&-1\\2&-2\end{bmatrix}$ and $B=\begin{bmatrix}1&-3\\2&-1\end{bmatrix}$

$\text{LHS}=(AB)'$

$\Rightarrow AB=\begin{bmatrix}3&-1\\2&-2\end{bmatrix}\begin{bmatrix}1&-3\\2&-1\end{bmatrix}=\begin{bmatrix}3(1)-1(2)&3(-3)-1(-1)\\2(1)-2(2)&2(-3)-2(-1)\end{bmatrix}=\begin{bmatrix}3-2&-9+1\\2-4&-6+2\end{bmatrix}=\begin{bmatrix}1&-8\\-2&-4\end{bmatrix}$

$\Rightarrow (AB)'=\begin{bmatrix}1&-2\\-8&-4\end{bmatrix}$

$\text{RHS}=B'A'$, $B'=\begin{bmatrix}1&2\\-3&-1\end{bmatrix}$ and $A'=\begin{bmatrix}3&2\\-1&-2\end{bmatrix}$

$\Rightarrow B'A'=\begin{bmatrix}1&2\\-3&-1\end{bmatrix}\begin{bmatrix}3&2\\-1&-2\end{bmatrix}=\begin{bmatrix}1(3)+2(-1)&1(2)+2(-2)\\-3(3)-1(-1)&-3(2)-1(-2)\end{bmatrix}=\begin{bmatrix}3-2&2-4\\-9+1&-6+2\end{bmatrix}=\begin{bmatrix}1&-2\\-8&-4\end{bmatrix}$

$\therefore$ LHS = RHS. Hence proved that $(AB)'=(B'A')$.

(iii) $A=\begin{bmatrix}-1\\2\\3\end{bmatrix}$ and $B=\begin{bmatrix}-2&-1&-4\end{bmatrix}$.

$\text{LHS}=(AB)'$

$\Rightarrow AB=\begin{bmatrix}-1\\2\\3\end{bmatrix}\begin{bmatrix}-2&-1&-4\end{bmatrix}=\begin{bmatrix}-1(-2)&-1(-1)&-1(-4)\\2(-2)&2(-1)&2(-4)\\3(-2)&3(-1)&3(-4)\end{bmatrix}=\begin{bmatrix}2&1&4\\-4&-2&-8\\-6&-3&-12\end{bmatrix}$

$\Rightarrow (AB)'=\begin{bmatrix}2&-4&-6\\1&-2&-3\\4&-8&-12\end{bmatrix}$

$$\text{RHS} = B'A',\ B' = \begin{bmatrix} -2 \\ -1 \\ -4 \end{bmatrix} \text{ and } A' = \begin{bmatrix} -1 & 2 & 3 \end{bmatrix}$$

$$\Rightarrow B'A' = \begin{bmatrix} -2 \\ -1 \\ -4 \end{bmatrix}\begin{bmatrix} -1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} -2(-1) & -2(2) & -2(3) \\ -1(-1) & -1(2) & -1(3) \\ -4(-1) & -4(2) & -4(3) \end{bmatrix} = \begin{bmatrix} 2 & -4 & -6 \\ 1 & -2 & -3 \\ 4 & -8 & -12 \end{bmatrix}$$

$\therefore$ LHS = RHS. Hence proved that $(AB)' = B'A'$

(iv) $A = \begin{bmatrix} -1 & 2 & -3 \\ 4 & -5 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -4 \\ 2 & 1 \\ -1 & 0 \end{bmatrix}$

$\text{LHS} = (AB)'$

$$\Rightarrow AB = \begin{bmatrix} -1 & 2 & -3 \\ 4 & -5 & 6 \end{bmatrix}\begin{bmatrix} 3 & -4 \\ 2 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} -1(3)+2(2)-3(-1) & -1(-4)+2(1)-3(0) \\ 4(3)-5(2)+6(-1) & 4(-4)-5(1)+6(0) \end{bmatrix}$$

$$= \begin{bmatrix} -3+4+3 & 4+2-0 \\ 12-10-6 & -16-5+0 \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ -4 & -21 \end{bmatrix} \Rightarrow (AB)' = \begin{bmatrix} 4 & -4 \\ 6 & -21 \end{bmatrix}$$

$$\text{RHS} = B'A',\ B' = \begin{bmatrix} 3 & 2 & -1 \\ -4 & 1 & 0 \end{bmatrix} \text{ and } A' = \begin{bmatrix} -1 & 4 \\ 2 & -5 \\ -3 & 6 \end{bmatrix}$$

$$\Rightarrow B'A' = \begin{bmatrix} 3 & 2 & -1 \\ -4 & 1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 4 \\ 2 & -5 \\ -3 & 6 \end{bmatrix} = \begin{bmatrix} 3(-1)+2(2)-1(-3) & 3(4)+2(-5)-1(6) \\ -4(-1)+1(2)+0(-3) & -4(4)+1(-5)+0(6) \end{bmatrix}$$

$$= \begin{bmatrix} -3+4+3 & 12-10-6 \\ 4+2+0 & -16-5+0 \end{bmatrix} = \begin{bmatrix} 4 & -4 \\ 6 & -21 \end{bmatrix}$$

$\therefore$ LHS = RHS. Hence, proved that $(AB)' = B'A'$.

14. If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, show that $A'A = I$.

Sol. $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} \Rightarrow A' = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

$$A'A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\alpha + \sin^2\alpha & \cos\alpha\sin\alpha - \sin\alpha\cos\alpha \\ \sin\alpha\cos\alpha - \sin\alpha\cos\alpha & \sin^2\alpha + \cos^2\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I.$$

Hence, proved that $A'A = I$.

15. If matrix $A = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$, write AA'.

Sol. $A=\begin{bmatrix}1 & 2 & 3\end{bmatrix} \Rightarrow A'=\begin{bmatrix}1\\2\\3\end{bmatrix}$; $AA'=\begin{bmatrix}1 & 2 & 3\end{bmatrix}\begin{bmatrix}1\\2\\3\end{bmatrix}=\begin{bmatrix}1(1)+2(2)+3(3)\end{bmatrix}=[1+4+9]=[14]$

EXERCISE 5E (Pg.No.: 211)

Using elementary row transformations, find the inverse of each of the following matrices.

1. $\begin{bmatrix}1 & 2\\3 & 7\end{bmatrix}$

Sol. We have, $\begin{bmatrix}1 & 2\\3 & 7\end{bmatrix}=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}A$

Applying $R_2 \to R_2-3R_1$, $\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}=\begin{bmatrix}1 & 0\\-3 & 1\end{bmatrix}A$

Applying $R_1 \to R_1-2R_2$, $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}=\begin{bmatrix}7 & -2\\-3 & 1\end{bmatrix}A$. Hence, $A^{-1}=\begin{bmatrix}7 & -2\\-3 & 1\end{bmatrix}$

2. $\begin{bmatrix}1 & 2\\2 & -1\end{bmatrix}$

Sol. We have, $\begin{bmatrix}1 & 2\\2 & -1\end{bmatrix}=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}A$

Applying $R_2 \to R_2-2R_1$, $\begin{bmatrix}1 & 2\\0 & -5\end{bmatrix}=\begin{bmatrix}1 & 0\\-2 & 1\end{bmatrix}A$

Applying $R_1 \to 5R_1+2R_2$, $\begin{bmatrix}5 & 0\\0 & -5\end{bmatrix}=\begin{bmatrix}1 & 2\\-2 & 1\end{bmatrix}A$

Applying $R_2 \to -R_2$, $\begin{bmatrix}5 & 0\\0 & 5\end{bmatrix}=\begin{bmatrix}1 & 2\\2 & -1\end{bmatrix}A \Rightarrow 5\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}=\begin{bmatrix}1 & 2\\2 & -1\end{bmatrix}A$

$\Rightarrow \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}=\begin{bmatrix}\frac{1}{5} & \frac{2}{5}\\ \frac{2}{5} & -\frac{1}{5}\end{bmatrix}A$. Hence $A^{-1}=\begin{bmatrix}\frac{1}{5} & \frac{2}{5}\\ \frac{2}{5} & -\frac{1}{5}\end{bmatrix}$

3. $\begin{bmatrix}2 & 5\\-3 & 1\end{bmatrix}$

Sol. We have, $\begin{bmatrix}2 & 5\\-3 & 1\end{bmatrix}=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}A$

Applying $R_2 \to 2R_2+3R_1$, $\begin{bmatrix}2 & 5\\0 & 17\end{bmatrix}=\begin{bmatrix}1 & 0\\3 & 2\end{bmatrix}A$

Applying $R_1 \to 17R_1-5R_2$, $\begin{bmatrix}34 & 0\\0 & 17\end{bmatrix}=\begin{bmatrix}2 & -10\\3 & 2\end{bmatrix}A$

Applying $R_1 \to \frac{1}{34}R_1$ and $R_2 \to \frac{1}{17}R_2$, $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}=\begin{bmatrix}\frac{1}{17} & -\frac{5}{17}\\ \frac{3}{17} & \frac{2}{17}\end{bmatrix}A$ Hence, $A^{-1}=\begin{bmatrix}\frac{1}{17} & -\frac{5}{17}\\ \frac{3}{17} & \frac{2}{17}\end{bmatrix}$.

4. $\begin{bmatrix} 2 & -3 \\ 4 & 5 \end{bmatrix}$

Sol. $\begin{bmatrix} 2 & -3 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

Applying $R_2 \to R_2 - 2R_1$, $\begin{bmatrix} 2 & -3 \\ 0 & 11 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A$

Applying $R_1 \to 11R_1 + 3R_2$, $\begin{bmatrix} 22 & 0 \\ 0 & 11 \end{bmatrix} = \begin{bmatrix} 5 & 3 \\ -2 & 1 \end{bmatrix} A$

Applying $R_1 \to \frac{1}{22}R_1$ and $R_2 \to \frac{1}{11}R_2$, $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{5}{22} & \frac{3}{22} \\ -\frac{2}{11} & \frac{1}{11} \end{bmatrix} A$ Hence, $A^{-1} = \begin{bmatrix} \frac{5}{22} & \frac{3}{22} \\ -\frac{2}{11} & \frac{1}{11} \end{bmatrix}$

5. $\begin{bmatrix} 4 & 0 \\ 2 & 5 \end{bmatrix}$

Sol. We have $\begin{bmatrix} 4 & 0 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

Applying $R_2 \to 2R_2 - R_1$, $\begin{bmatrix} 4 & 0 \\ 0 & 10 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix} A$

Applying $R_1 \to \frac{1}{4}R_1$ and $R_2 \to \frac{1}{10}R_2$, $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & 0 \\ -\frac{1}{10} & \frac{1}{5} \end{bmatrix} A$. Hence $A^{-1} = \begin{bmatrix} \frac{1}{4} & 0 \\ -\frac{1}{10} & \frac{1}{5} \end{bmatrix}$.

6. $\begin{bmatrix} 6 & 7 \\ 8 & 9 \end{bmatrix}$

Sol. We have, $\begin{bmatrix} 6 & 7 \\ 8 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

Applying $R_2 \to 6R_2 - 8R_1$, $\begin{bmatrix} 6 & 7 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -8 & 6 \end{bmatrix} A$

Applying $R_1 \to 2R_1 + 7R_2$, $\begin{bmatrix} 12 & 0 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} -54 & 42 \\ -8 & 6 \end{bmatrix} A$

Applying $R_1 \to \frac{1}{12}R_1$ and $R_2 \to -\frac{1}{2}R_2$, $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{9}{2} & \frac{7}{2} \\ -4 & 3 \end{bmatrix} A$. Hence, $A^{-1} = \begin{bmatrix} -\frac{9}{2} & \frac{7}{2} \\ -4 & 3 \end{bmatrix}$

7. $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

Sol. We have, $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$

Applying $R_1 \leftrightarrow R_2$, $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$

Applying $R_3 \to R_3 - 3R_1$, $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & -5 & -8 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & -3 & 1 \end{bmatrix} A$

Applying $R_3 \to R_3 + 5R_2$, $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 5 & -3 & 1 \end{bmatrix} A$

Applying $R_1 \to R_1 - 2R_2$, $R_3 \to \frac{1}{2}R_3$, $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 5/2 & -3/2 & 1/2 \end{bmatrix} A$

Applying $R_1 \to R_1 + R_3$, $R_2 \to R_2 - 2R_3$, $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -4 & 3 & -1 \\ \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{bmatrix} A$

Hence, $A^{-1} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -4 & 3 & -1 \\ \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{bmatrix} \Rightarrow A^{-1} = \frac{1}{2}\begin{bmatrix} 1 & -1 & 1 \\ -8 & 6 & -2 \\ 5 & -3 & 1 \end{bmatrix}$

8. $\begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$

Sol. Let $A = \begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$, $A = IA \Rightarrow \begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$

Applying $R_1 \leftrightarrow R_3$, $\begin{bmatrix} 3 & -2 & 2 \\ 2 & 2 & 3 \\ 2 & -3 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

Applying $R_1 \to R_1 - R_2$, $R_3 \to R_3 - R_2$, $\begin{bmatrix} 1 & -4 & -1 \\ 2 & 2 & 3 \\ 0 & -5 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 1 \\ 0 & 1 & 0 \\ 1 & -1 & 0 \end{bmatrix} A$

Applying $R_2 \to R_2 - 2R_1$, $\begin{bmatrix} 1 & -4 & -1 \\ 0 & 10 & 5 \\ 0 & -5 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 1 \\ 0 & 3 & -2 \\ 1 & -1 & 0 \end{bmatrix} A$

Applying $R_2 \leftrightarrow R_3$, $\begin{bmatrix} 1 & -4 & -1 \\ 0 & -5 & 0 \\ 0 & 10 & 5 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 1 \\ 1 & -1 & 0 \\ 0 & 3 & -2 \end{bmatrix} A$

Applying $R_3 \to R_3 + 2R_2$, $R_2 \to -\frac{1}{5}R_2$, $\begin{bmatrix} 1 & -4 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 5 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 1 \\ -\frac{1}{5} & \frac{1}{5} & 0 \\ 2 & 1 & -2 \end{bmatrix} A$

Applying $R_1 \to R_1 + 4R_2$, $R_3 \to \frac{1}{5}R_3$, $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{4}{5} & -\frac{1}{5} & 1 \\ -\frac{1}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{1}{5} & -\frac{2}{5} \end{bmatrix} A$

Applying $R_1 \to R_1 + R_3$, $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{2}{5} & 0 & \frac{3}{5} \\ -\frac{1}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{1}{5} & -\frac{2}{5} \end{bmatrix} A$

Hence, $A^{-1} = \frac{1}{5}\begin{bmatrix} -2 & 0 & 3 \\ -1 & 1 & 0 \\ 2 & 1 & -2 \end{bmatrix} \Rightarrow A^{-1} = -\frac{1}{5}\begin{bmatrix} 2 & 0 & -3 \\ 1 & -1 & 0 \\ -2 & -1 & 2 \end{bmatrix}$

9. $\begin{bmatrix} 3 & 0 & 2 \\ 1 & 5 & 9 \\ 6 & 4 & 7 \end{bmatrix}$

Sol. Let $A = \begin{bmatrix} 3 & 0 & 2 \\ 1 & 5 & 9 \\ 6 & 4 & 7 \end{bmatrix}$, $A = IA \Rightarrow \begin{bmatrix} 3 & 0 & 2 \\ 1 & 5 & 9 \\ 6 & 4 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$

Applying $R_3 \to R_3 - 2R_1$, $\begin{bmatrix} 3 & 0 & 2 \\ 1 & 5 & 9 \\ 0 & 4 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} A$

Applying $R_2 \to 3R_2 - R_1$, $\begin{bmatrix} 3 & 0 & 2 \\ 0 & 15 & 25 \\ 0 & 4 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 3 & 0 \\ -2 & 0 & 1 \end{bmatrix} A$

Applying $R_2 \to \frac{1}{5}R_2$, $\begin{bmatrix} 3 & 0 & 2 \\ 0 & 3 & 5 \\ 0 & 4 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{5} & \frac{3}{5} & 0 \\ -2 & 0 & 1 \end{bmatrix} A$

Applying $R_3 \to 3R_3 - 4R_2$, $\begin{bmatrix} 3 & 0 & 2 \\ 0 & 3 & 5 \\ 0 & 0 & -11 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{5} & \frac{3}{5} & 0 \\ -\frac{26}{5} & -\frac{12}{5} & 3 \end{bmatrix} A$

Applying $R_3 \to -\frac{1}{11}R_3$, $\begin{bmatrix} 3 & 0 & 2 \\ 0 & 3 & 5 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{5} & \frac{3}{5} & 0 \\ \frac{26}{55} & \frac{12}{55} & -\frac{3}{11} \end{bmatrix} A$

Applying $R_2 \to R_2 - 5R_3$ and $R_1 \to R_1 - 2R_3$, $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{3}{55} & -\frac{25}{55} & \frac{6}{11} \\ -\frac{141}{55} & -\frac{27}{55} & \frac{15}{11} \\ \frac{26}{55} & \frac{12}{55} & -\frac{3}{11} \end{bmatrix} A$

Applying $R_1 \to \frac{1}{3}R_1$, $R_2 \to \frac{1}{3}R_1$, $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{55} & -\frac{8}{55} & \frac{2}{11} \\ -\frac{47}{55} & -\frac{9}{55} & \frac{5}{11} \\ \frac{26}{55} & \frac{12}{55} & -\frac{3}{11} \end{bmatrix} A$

Hence, $A^{-1} = -\frac{1}{55}\begin{bmatrix} -1 & 8 & -10 \\ 47 & 9 & -25 \\ -26 & -12 & 15 \end{bmatrix}$

10. $\begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$

Sol. Let $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$, $A = IA \Rightarrow \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$

Applying $R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 3R_1$, $\begin{bmatrix} 1 & 2 & -3 \\ 0 & -1 & 8 \\ 0 & -9 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix} A$

Applying $R_1 \to R_1 + 2R_2$, $R_3 \to R_3 - 9R_2$, $\begin{bmatrix} 1 & 0 & 13 \\ 0 & -1 & 8 \\ 0 & 0 & -67 \end{bmatrix} = \begin{bmatrix} -3 & 2 & 0 \\ -2 & 1 & 0 \\ 15 & -9 & 1 \end{bmatrix} A$

Applying $R_3 \to -\frac{1}{67}R_3$, $\begin{bmatrix} 1 & 0 & 13 \\ 0 & -1 & 8 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -3 & 2 & 0 \\ -2 & 1 & 0 \\ -\frac{15}{67} & \frac{9}{67} & -\frac{1}{67} \end{bmatrix} A$

Applying $R_1 \to R_1 - 13R_3,\ R_2 \to R_2 - 8R_3,$ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{6}{67} & \frac{17}{67} & \frac{13}{67} \\ -\frac{14}{67} & -\frac{5}{67} & \frac{8}{67} \\ -\frac{15}{67} & \frac{9}{67} & -\frac{1}{67} \end{bmatrix} A$

Applying $R_2 \to (-1)R_2,$ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{6}{67} & \frac{17}{67} & \frac{13}{67} \\ \frac{14}{67} & \frac{5}{67} & -\frac{8}{67} \\ -\frac{15}{67} & \frac{9}{67} & -\frac{1}{67} \end{bmatrix}$ A Hence, $A^{-1} = \frac{1}{67}\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$

11. $\begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & -1 \\ 3 & -5 & 0 \end{bmatrix}$

Sol. We have $\begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & -1 \\ 3 & -5 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.A$

Applying $R_1 \to R_1 - R_2,$ $\begin{bmatrix} 1 & -1 & -1 \\ 2 & 0 & -1 \\ 3 & -5 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.A$

Applying $R_2 \to R_2 - 2R_1, R_3 \to R_3 - 3R_1,$ $\begin{bmatrix} 1 & -1 & -1 \\ 0 & 2 & 1 \\ 0 & -2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & 0 \\ -3 & 3 & 1 \end{bmatrix}.A$

Applying $R_2 \to \frac{1}{2}R_2,$ $\begin{bmatrix} 1 & -1 & -1 \\ 0 & 1 & \frac{1}{2} \\ 0 & -2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -1 & \frac{3}{2} & 0 \\ -3 & 3 & 1 \end{bmatrix}.A$

Applying $R_1 \to R_1 + R_2, R_3 \to R_3 + 2R_2,$ $\begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{1}{2} \\ 0 & 0 & 4 \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{2} & 0 \\ -1 & \frac{3}{2} & 0 \\ -5 & 6 & 1 \end{bmatrix}.A$

Applying $R_3 + \frac{1}{4}R_3,$ $\begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{1}{2} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{2} & 0 \\ -1 & \frac{3}{2} & 0 \\ -\frac{5}{4} & \frac{3}{2} & \frac{1}{4} \end{bmatrix}.A$

Applying $R_1 \to R_1 + \frac{1}{2}R_3,\ R_2 \to R_2 + \frac{1}{2}R_3,$ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{-5}{8} & \frac{5}{4} & \frac{1}{8} \\ \frac{-3}{8} & \frac{3}{4} & \frac{-1}{8} \\ \frac{-5}{4} & \frac{3}{2} & \frac{1}{4} \end{bmatrix}.A$

$$\therefore\ A^{-1} = -\frac{1}{8}\begin{bmatrix} -5 & 5(2) & 1 \\ -3 & 3(2) & -1 \\ -5 & 3(4) & 1(2) \end{bmatrix} \Rightarrow A^{-1} = -\frac{1}{8}\begin{bmatrix} 5 & -10 & -1 \\ 3 & -6 & 1 \\ 10 & -12 & -2 \end{bmatrix}$$

12. $\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$

Sol. Let $A = \begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$

$\because A = IA$

$$\Rightarrow \begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A$$

$$\Rightarrow \begin{bmatrix} 1 & 3 & -2 \\ 0 & 9 & -7 \\ 0 & -5 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} A \begin{cases} R_2 \to R_2 + 3R_1 \\ R_3 \to R_3 - 2R_1 \end{cases}$$

$$\Rightarrow \begin{bmatrix} 1 & 3 & -2 \\ 0 & -1 & 1 \\ 0 & -5 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 2 \\ -2 & 0 & 1 \end{bmatrix} A \ \{R_2 + R_2 + 2R_3$$

$$\Rightarrow \begin{bmatrix} 1 & 3 & -2 \\ 0 & 1 & -1 \\ 0 & -5 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & -2 \\ -2 & 0 & 1 \end{bmatrix} A \ \{R_2 \to -R_2$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} -2 & 3 & 6 \\ 1 & -1 & -2 \\ 3 & -5 & -9 \end{bmatrix} A \begin{cases} R_1 \to R_1 - 3R_2 \\ R_3 + R_3 + 5R_2 \end{cases}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 3 & 6 \\ 1 & -1 & -2 \\ -3 & 5 & 9 \end{bmatrix} A \ \{R_3 \to -R_3$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -2 & -3 \\ -2 & 4 & 7 \\ -3 & 5 & 9 \end{bmatrix} A \begin{cases} R_1 \to R_1 - R_3 \\ R_2 + R_2 + R_3 \end{cases}$$

$$\Rightarrow I = \begin{bmatrix} 1 & -2 & -3 \\ -2 & 4 & 7 \\ -3 & 5 & 9 \end{bmatrix} \cdot A \quad \therefore A^{-1} = \begin{bmatrix} 1 & -2 & -3 \\ -2 & 4 & 7 \\ -3 & 5 & 9 \end{bmatrix}$$

13. $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$

Sol. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$, $A = IA \Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$

Applying $R_2 \to R_2 - 2R_1$, $R_3 \to R_3 + 2R_1$, $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} A$

Applying $R_1 \to R_1 - 2R_2$, $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 & -2 & 0 \\ -2 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} A$

Applying $R_1 \to R_1 - R_3$, $R_2 \to R_2 - R_3$, $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix} A$

Hence, $A^{-1} = \begin{bmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix}$

14. $\begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix}$

Sol. We have $\begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} .A \Rightarrow \begin{bmatrix} 1 & -3 & 1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} .A$

Applying $R_1 \to R_1 - R_2$, $\begin{bmatrix} 1 & -3 & -1 \\ 0 & 9 & 2 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} .A$

Applying $R_2 \to R_2 - 2R_1$, $\begin{bmatrix} 1 & -3 & -1 \\ 0 & 1 & 0 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -2 \\ 0 & 1 & 1 \end{bmatrix} .A$

Applying $R_1 \to R_1 + 3R_2, R_3 \to R_3 - 4R_2$, $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -5 & 8 & -6 \\ -2 & 3 & -2 \\ 8 & -12 & 9 \end{bmatrix} .A$

Applying $R_1 \to R_1 + R_3$, $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -4 & 3 \\ -2 & 3 & -2 \\ 8 & -12 & 9 \end{bmatrix} . A$. Hence, $A^{-1} = \begin{bmatrix} 3 & -4 & 3 \\ -2 & 3 & -2 \\ 8 & -12 & 9 \end{bmatrix}$.

15. $\begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

Sol. Let $A = \begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

Now, $A = I \cdot A$

$$\Rightarrow \begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A$$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ -1 & 1 & 2 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A \{R_1 \leftrightarrow R_2$$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 3 & 5 \\ 0 & -5 & -8 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & -3 & 1 \end{bmatrix} \cdot A \begin{cases} R_2 \to R_2 + R_1 \\ R_3 \to R_3 - 3R_1 \end{cases}$$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 3 & 5 \\ 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 2 & -1 & 1 \end{bmatrix} \cdot A \{R_3 \to R_3 + 2R_2$$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 2 & -1 & 1 \\ 1 & 1 & 0 \end{bmatrix} \cdot A \{R_2 \leftrightarrow R_3$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} -4 & 3 & -2 \\ 2 & -1 & 1 \\ -5 & 4 & -3 \end{bmatrix} \cdot A \begin{cases} R_1 + R_1 - 2R_2 \\ R_3 + R_3 - 3R_2 \end{cases}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -4 & 3 & -2 \\ 2 & -1 & 1 \\ 5 & -4 & 3 \end{bmatrix} \cdot A \{R_3 \leftrightarrow -R_3$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ 5 & -7 & 3 \end{bmatrix} \cdot A \begin{cases} R_1 \to R_1 + R_3 \\ R_2 + R_2 - 2R_3 \end{cases}$$

$$\Rightarrow I = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ 5 & -4 & 3 \end{bmatrix} \cdot A \quad \text{Hence } A^{-1} = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ 5 & -4 & 3 \end{bmatrix}$$

EXERCISE 5F (Pg.No.: 213)

Very Short Answer Questions.

1. Construct a 3×2 matrix whose elements are given by $a_{ij}=\frac{1}{2}(i-2j)^2$.

Sol. Let matrix $A=\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}_{3\times 2}$. Thus $a_{ij}=\frac{1}{2}(i-2j)^2$ for $i=1,\ 2$ for $j=1,\ 2,\ 3$.

$\therefore\ a_{11}=\frac{1}{2}\{1-2(1)\}^2=\frac{1}{2}(1-2)^2=\frac{1}{2}(-1)^2=\frac{1}{2}\times 1=\frac{1}{2}$

$a_{12}=\frac{1}{2}\{1-2(2)\}^2=\frac{1}{2}(1-4)^2=\frac{1}{2}(-3)^2=\frac{1}{2}\times 9=\frac{9}{2}$, $a_{21}=\frac{1}{2}\{2-2(1)\}^2=\frac{1}{2}(2-2)^2=\frac{1}{2}\times 0=0$

$a_{22}=\frac{1}{2}\{2-2(2)\}^2=\frac{1}{2}(2-4)^2=\frac{1}{2}(-2)^2=\frac{1}{2}\times 4=2$, $a_{31}=\frac{1}{2}\{3-2(1)\}^2=\frac{1}{2}(3-2)^2=\frac{1}{2}(1)^2=\frac{1}{2}$

$a_{32}=\frac{1}{2}\{3-2(2)\}^2=\frac{1}{2}(3-4)^2=\frac{1}{2}(-1)^2=\frac{1}{2}$

Hence, $A=\begin{bmatrix} 1/2 & 9/2 \\ 0 & 2 \\ 1/2 & 1/2 \end{bmatrix}$

2. Construct a 2×3 matrix whose elements are given by $a_{ij}=\frac{1}{2}|-3i+j|$.

Sol. Let $A=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$, Thus $a_{ij}=\frac{1}{2}|-3i+j|$ for $i=1,\ 2$ and $j=1,\ 2,\ 3$

$\therefore\ a_{11}=\frac{1}{2}|-3(1)+1|=\frac{1}{2}|-3+1|=\frac{1}{2}\times 2=1$, $a_{12}=\frac{1}{2}|-3(1)+2|=\frac{1}{2}|-3+2|=\frac{1}{2}\times 1=\frac{1}{2}$

$a_{13}=\frac{1}{2}|-3(1)+3|=\frac{1}{2}|-3+3|=\frac{1}{2}\times 0=0$, $a_{21}=\frac{1}{3}|-3(2)+1|=\frac{1}{2}|-6+1|=\frac{1}{2}\times 5=\frac{5}{2}$

$a_{22}=\frac{1}{2}|-3(2)+2|=\frac{1}{2}|-6+2|=\frac{1}{2}\times 4=2$, $a_{23}=\frac{1}{2}|-3(2)+3|=\frac{1}{2}|-6+3|=\frac{1}{2}\times 3=\frac{3}{2}$

Hence, $A=\begin{bmatrix} 1 & 1/2 & 0 \\ 5/2 & 2 & 3/2 \end{bmatrix}$

3. If $\begin{bmatrix} x+2y & -y \\ 3x & 4 \end{bmatrix}=\begin{bmatrix} -4 & 3 \\ 6 & 4 \end{bmatrix}$, find the values of x and y.

Sol. $\begin{bmatrix} x+2y & -y \\ 3x & 4 \end{bmatrix}=\begin{bmatrix} -4 & 3 \\ 6 & 4 \end{bmatrix}$

$x+2y=-4 \quad \ldots(1)$

$3x=6 \ \Rightarrow x=2$

Putting the value of x in equation (1), we get $x+2y=-4\ \Rightarrow 2+2y=-4$

$\Rightarrow 2y=-4-2\ \Rightarrow 2y=-6\ \Rightarrow y=-3$. Hence, $x=2$ and $y=-3$.

4. Find the values of x and y, if $2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix}+\begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix}=\begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$.

Sol. $2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix}+\begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix}=\begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 & 6 \\ 0 & 2x \end{bmatrix}+\begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix}=\begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

$2+y=5$ and $2x+2=8$

$\Rightarrow y=5-2$ and $\Rightarrow 2x=8-2$

$\Rightarrow y=3$ and $\Rightarrow x=3$ Hence, $x=3$ and $y=3$.

5. If $x.\begin{bmatrix} 2 \\ 3 \end{bmatrix}+y.\begin{bmatrix} -1 \\ 1 \end{bmatrix}=\begin{bmatrix} 10 \\ 5 \end{bmatrix}$, find the values of x and y.

Sol. $\begin{bmatrix} 2x \\ 3x \end{bmatrix}+\begin{bmatrix} -y \\ y \end{bmatrix}=\begin{bmatrix} 10 \\ 5 \end{bmatrix}$

$2x-y=10 \quad \ldots(1)$

$3x+y=5 \quad \ldots(2)$

Solving the equations (1) and (2), we get $5x=15 \Rightarrow x=3$

Putting the value of x in equation (1), we get $2x-y=10 \Rightarrow 2(3)-y=10$

$\Rightarrow y=6-10 \Rightarrow y=-4$. Hence, $x=3$ and $y=-4$.

6. If $\begin{bmatrix} x & 3x-y \\ 2x+z & 3y-\omega \end{bmatrix}=\begin{bmatrix} 3 & 2 \\ 4 & 7 \end{bmatrix}$, find the values of x, y, z, ω.

Sol. $\begin{bmatrix} x & 3x-y \\ 2x+z & 3y-\omega \end{bmatrix}=\begin{bmatrix} 3 & 2 \\ 4 & 7 \end{bmatrix}$

$x=3 \quad \ldots(1)$

and $2x+z=4 \quad \ldots(2)$

$\Rightarrow z=4-2x \Rightarrow z=4-2(3) \Rightarrow z=4-6=-2$

Now, $3y-\omega=7 \Rightarrow \omega=3y-7 \Rightarrow \omega=3(7)-7=21-7=14$. Hence, $x=3, y=7, z=-2, \omega=1$

7. If $\begin{bmatrix} x & 6 \\ -1 & 2\omega \end{bmatrix}+\begin{bmatrix} 4 & x+y \\ z+\omega & 3 \end{bmatrix}=3\begin{bmatrix} x & y \\ z & \omega \end{bmatrix}$, find the values of x, y, z, ω.

Sol. $\begin{bmatrix} x+4 & 6+x+y \\ -1+z+\omega & 2\omega+3 \end{bmatrix}=\begin{bmatrix} 3x & 3y \\ 3z & 3\omega \end{bmatrix}$

$x+4=3x \Rightarrow 4=2x \Rightarrow x=2$

$6+x+y=3y \Rightarrow 6+x=2y \Rightarrow 6+2=2y \Rightarrow y=4$

$\therefore 2\omega+3=3\omega \Rightarrow \omega=3$

$-1+z+\omega=3z \Rightarrow -1+\omega=2z \Rightarrow -1+3=2z \Rightarrow 2=2z \Rightarrow z=1$

Hence, $x=2, y=4, z=1, \omega=3$.

8. If $A=diag(3 \quad -2 \quad 5)$ and $B=diag(1 \quad 3 \quad -4)$, find $(A+B)$

Sol. $A=\begin{bmatrix} 3 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ and $B=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -4 \end{bmatrix}$

$\Rightarrow (A+B)=\begin{bmatrix} 3 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 5 \end{bmatrix}+\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -4 \end{bmatrix} \Rightarrow (A+B)=\begin{bmatrix} 3+1 & 0 & 0 \\ 0 & -2+3 & 0 \\ 0 & 0 & 5-4 \end{bmatrix}=\begin{bmatrix} 4 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow (A+B) = diag\begin{pmatrix} 4 & 1 & 1 \end{pmatrix}$

9. Show that $\cos\theta\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \sin\theta\begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix} = I$.

Sol. LHS $= \cos\theta\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \sin\theta\begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$

$= \begin{bmatrix} \cos^2\theta & \sin\theta\cos\theta \\ -\sin\theta\cos\theta & \cos^2\theta \end{bmatrix} + \begin{bmatrix} \sin^2\theta & -\sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{bmatrix}$

$= \begin{bmatrix} \sin^2\theta + \cos^2\theta & \sin\theta\cos\theta - \sin\theta\cos\theta \\ -\sin\theta\cos\theta + \sin\theta\cos\theta & \sin^2\theta + \cos^2\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$ = RHS. Hence proved.

10. If $A = \begin{bmatrix} 1 & -5 \\ -3 & 2 \\ 4 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 1 \\ 2 & -1 \\ -2 & 3 \end{bmatrix}$, find the matrix C such that $A+B+C$ is a zero matrix.

Sol. $A+B+C = O \Rightarrow C = -A - B \Rightarrow C = -\begin{bmatrix} 1 & -5 \\ -3 & 2 \\ 4 & -2 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ 2 & -1 \\ -2 & 3 \end{bmatrix}$

$\Rightarrow C = \begin{bmatrix} -1-3 & 5-1 \\ 3-2 & -2+1 \\ -4+2 & 2-3 \end{bmatrix} = \begin{bmatrix} -4 & 4 \\ 1 & -1 \\ -2 & -1 \end{bmatrix}$

11. If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, then find the least value of α for which $A + A' = I$.

Sol. Let $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \Rightarrow A' = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

$A + A' = I \Rightarrow \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} + \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2\cos\alpha & 0 \\ 0 & 2\cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\therefore\ 2\cos\alpha = 1 \Rightarrow \cos\alpha = \frac{1}{2} \Rightarrow \alpha = \frac{\pi}{3}$

12. Find the values of x and y for which $\begin{bmatrix} 2 & -3 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$.

Sol. $\begin{bmatrix} 2 & -3 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix} \Rightarrow \begin{bmatrix} 2x-3y \\ x+y \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

$2x - 3y = 1 \quad \ldots(1)$

and $x + y = 3 \quad \ldots(2) \times 3$

Solving equations (1) and (2), we get $5x = 10 \Rightarrow x = 2$

Putting the value of x in equation (1), we get $2x - 3y = 1 \Rightarrow 2(2) - 3y = 1$

$\Rightarrow 4 - 1 = 3y \Rightarrow 3y = 3 \Rightarrow y = 1$. Hence, $x = 2$ and $y = 1$.

13. Find the values of x and y for which $\begin{bmatrix} x & y \\ 3y & x \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix}=\begin{bmatrix} 3 \\ 5 \end{bmatrix}$.

Sol. $\begin{bmatrix} x & y \\ 3y & x \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix}=\begin{bmatrix} 3 \\ 5 \end{bmatrix} \Rightarrow \begin{bmatrix} x+2y \\ 3y+2x \end{bmatrix}=\begin{bmatrix} 3 \\ 5 \end{bmatrix}$

$x+2y=3 \quad \ldots(1)\times 2$

and $2x+3y=5 \quad \ldots(2)$

Solving equations (1) and (2), we get $y=1$

Putting the value of y in equation (1), we get $x+2y=3 \Rightarrow x=3-2y$

$\Rightarrow x=3-2(1)=3-2=1$. Hence, $x=1$ and $y=1$.

14. If $A=\begin{bmatrix} 4 & 5 \\ 1 & 8 \end{bmatrix}$, show that $(A+A')$ is symmetric.

Sol. $A=\begin{bmatrix} 4 & 5 \\ 1 & 8 \end{bmatrix} \Rightarrow A'=\begin{bmatrix} 4 & 1 \\ 5 & 8 \end{bmatrix}$

$\Rightarrow (A+A')=\begin{bmatrix} 4 & 5 \\ 1 & 8 \end{bmatrix}+\begin{bmatrix} 4 & 1 \\ 5 & 8 \end{bmatrix}=\begin{bmatrix} 4+4 & 5+1 \\ 1+5 & 8+8 \end{bmatrix}=\begin{bmatrix} 8 & 6 \\ 6 & 16 \end{bmatrix} \Rightarrow (A+A')'=\begin{bmatrix} 8 & 6 \\ 6 & 16 \end{bmatrix}$

$\therefore \quad (A+A')=(A+A)'$. Hence, proved that $(A+A')$ is a symmetric matrix.

15. If $A=\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$, show that $(A-A')$ is skew-symmetric.

Sol. $A=\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} \Rightarrow A'=\begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix}$

$(A-A')=\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}-\begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix}=\begin{bmatrix} 2-2 & 3-4 \\ 4-3 & 5-5 \end{bmatrix}=\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

Since, in resultant matrix diagonal values are just opposite. Hence prove that $(A-A')$ is a ske symmetric.

16. If $A=\begin{bmatrix} 2 & -3 \\ 4 & 5 \end{bmatrix}$ and $B=\begin{bmatrix} -1 & 2 \\ 0 & 3 \end{bmatrix}$, find a matrix X such that $A+2B+X=O$.

Sol. $A+2B+X=O \Rightarrow X=-A-2B \Rightarrow X=-\begin{bmatrix} 2 & -3 \\ 4 & 5 \end{bmatrix}-2\begin{bmatrix} -1 & 2 \\ 0 & 3 \end{bmatrix}$

$\Rightarrow X=\begin{bmatrix} -2 & 3 \\ -4 & -5 \end{bmatrix}-\begin{bmatrix} -2 & 4 \\ 0 & 6 \end{bmatrix} \Rightarrow X=\begin{bmatrix} -2+2 & 3-4 \\ -4-0 & -5-6 \end{bmatrix} \Rightarrow X=\begin{bmatrix} 0 & -1 \\ -4 & -11 \end{bmatrix}$

17. If $A=\begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$ and $B=\begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix}$, find a matrix X such that $3A-2B+X=O$.

Sol. $3A-2B+X=O \Rightarrow X=2B-3A \Rightarrow X=2\begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix}-3\begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$

$\Rightarrow X=\begin{bmatrix} -4 & 2 \\ 6 & 4 \end{bmatrix}-\begin{bmatrix} 12 & 6 \\ 3 & 9 \end{bmatrix}=\begin{bmatrix} -4-12 & 2-6 \\ 6-3 & 4-9 \end{bmatrix}=\begin{bmatrix} -16 & -4 \\ 3 & -5 \end{bmatrix}$

18. If $A=\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, show that $A'A=I$.

Sol. $A=\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix} \Rightarrow A'=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$

$$A'A=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}=\begin{bmatrix}\cos^2\alpha+\sin^2\alpha & \sin\alpha\cos\alpha-\cos\alpha\sin\alpha\\ \sin\alpha\cos\alpha-\cos\alpha\sin\alpha & \sin^2\alpha+\cos^2\alpha\end{bmatrix}$$

$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=I$. Hence proved.

19. If A and B are symmetric matrices of the same order, show that $(AB-BA)$ is a skew-symmetric matrix.

Sol. If A and B are symmetry matrix, then $A'=A,\ B'=B$

Then, $(AB-BA)'=(AB)'-(BA)'=B'A'-A'B'=BA-AB$

$\therefore\ A'=A,\ B'=B=-AB+BA \quad \Rightarrow (AB-BA)'=-(AB-BA)$

Hence, $AB-BA$ is skew-symmetry matrix.

20. If $A=\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}$ and $f(x)=x^2-4x+1$, find $f(A)$.

Sol. Given, $f(x)=x^2-4x+1 \quad \Rightarrow f(A)=A^2-4A+I$

$$A^2=A.A=\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}=\begin{bmatrix}4+3 & 6+6\\ 2+2 & 3+4\end{bmatrix}=\begin{bmatrix}7 & 12\\ 4 & 7\end{bmatrix}$$

$$\therefore\ f(A)=\begin{bmatrix}7 & 12\\ 4 & 7\end{bmatrix}-4\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}+\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=\begin{bmatrix}7 & 12\\ 4 & 7\end{bmatrix}-\begin{bmatrix}8 & 12\\ 4 & 8\end{bmatrix}+\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}+\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

21. If the matrix A is both symmetric and skew-symmetric, show that A is a zero matrix.

Sol. Given A is symmetric matrix. Hence, $A'=A$ ...(1)

Also, A is skew-symmetric. Hence, $A'=-A$ ...(2)

Subtracting (1) and (2), we have, $0=2A \quad \Rightarrow A=0$

Hence, A is zero matrix.