☆ Bode Plot:

⇒ <u>Purpose:</u>

⇒ To draw the frequency response of OLTF.

⇒ To find the closed loop system stability.

⇒ To find the Gain margin, phase margin, gain cross over frequency, phase cross over frequency.

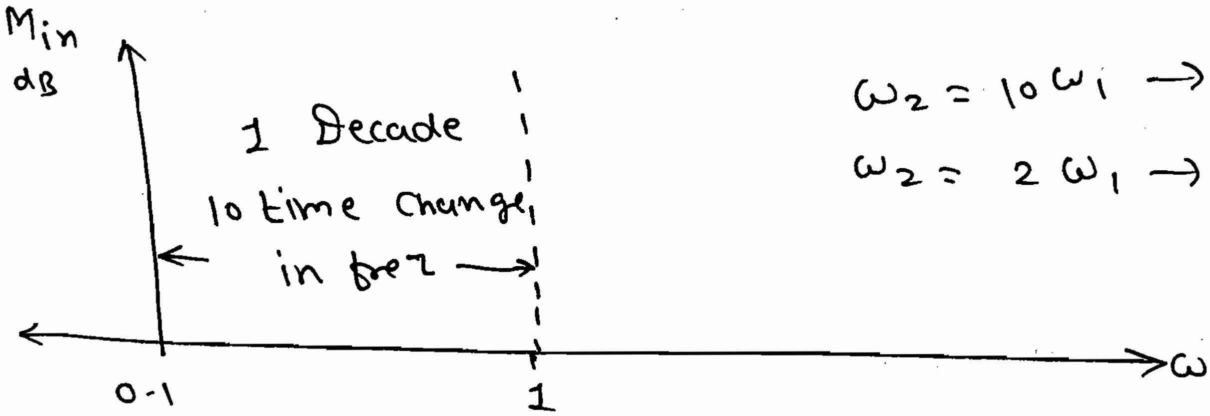
⇒ To find the relative stability by using GM & PM. If the GM & PM is very very large then the system is more relative stable. but the system response is slow. If GM & PM is very small, ~~and~~ ~~more~~ ~~osc~~ then the system become less relative stable and more oscillatory.

⇒ The optimum range of GM is 5 to 10 dB & PM is 30° to 40°.

⇒ The Bode plot consist the two plots. one is the magnitude plot and other is phase plot.

⇒ Bode Plot → (i) Magnitude plot.
→ (ii) Phase plot.

⇒ Magnitude Plot:



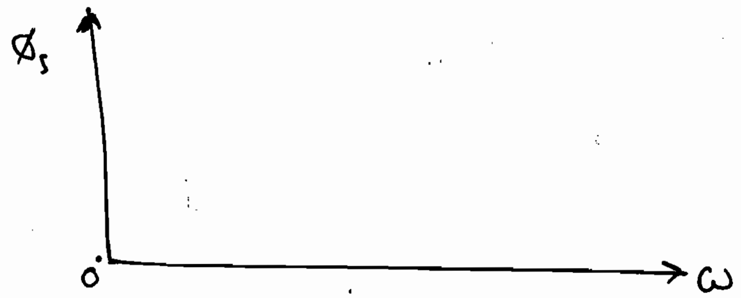
$\omega_2 = 10\omega_1 \rightarrow$ decade

$\omega_2 = 2\omega_1 \rightarrow$ octave.

⇒ $20\log 10 \longleftrightarrow 20\log 2$

20 dB | decade $\longleftrightarrow$ 6 dB | octave.

⇒ Phase plot:



* Procedure to draw the Bode Plot:

1) S is replaced by jω to convert it to freq. domain.

2) Write the magnitude and convert into (dB).

⇒ The Mag. in dB $M_{dB} = 20\log|GH(j\omega)|$.

3) Find the phase angle using;

$$\phi = \tan^{-1}\left(\frac{I_P}{R_P}\right).$$

4) Vary the 'ω' from min to max and draw the magnitude and phase plot approximately.

$\boxed{Q}$ Draw the Bode plot for $G(s)\cdot H(s) = K$.

Soln:

$$G(s).H(s) = K$$

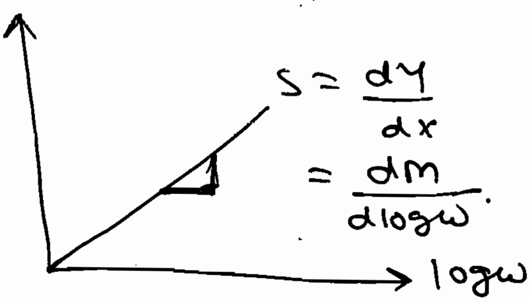
$$\Rightarrow \quad s \rightarrow j\omega$$

$$GH(j\omega) = K.$$

$$|GH(j\omega)| = K$$

$$M_{in\,dB} = 20\log|GH(j\omega)| = \boxed{20\log K}$$

$$\Rightarrow \text{Slope} = \frac{dm}{d\log\omega} = 0\ \text{dB/dec}$$



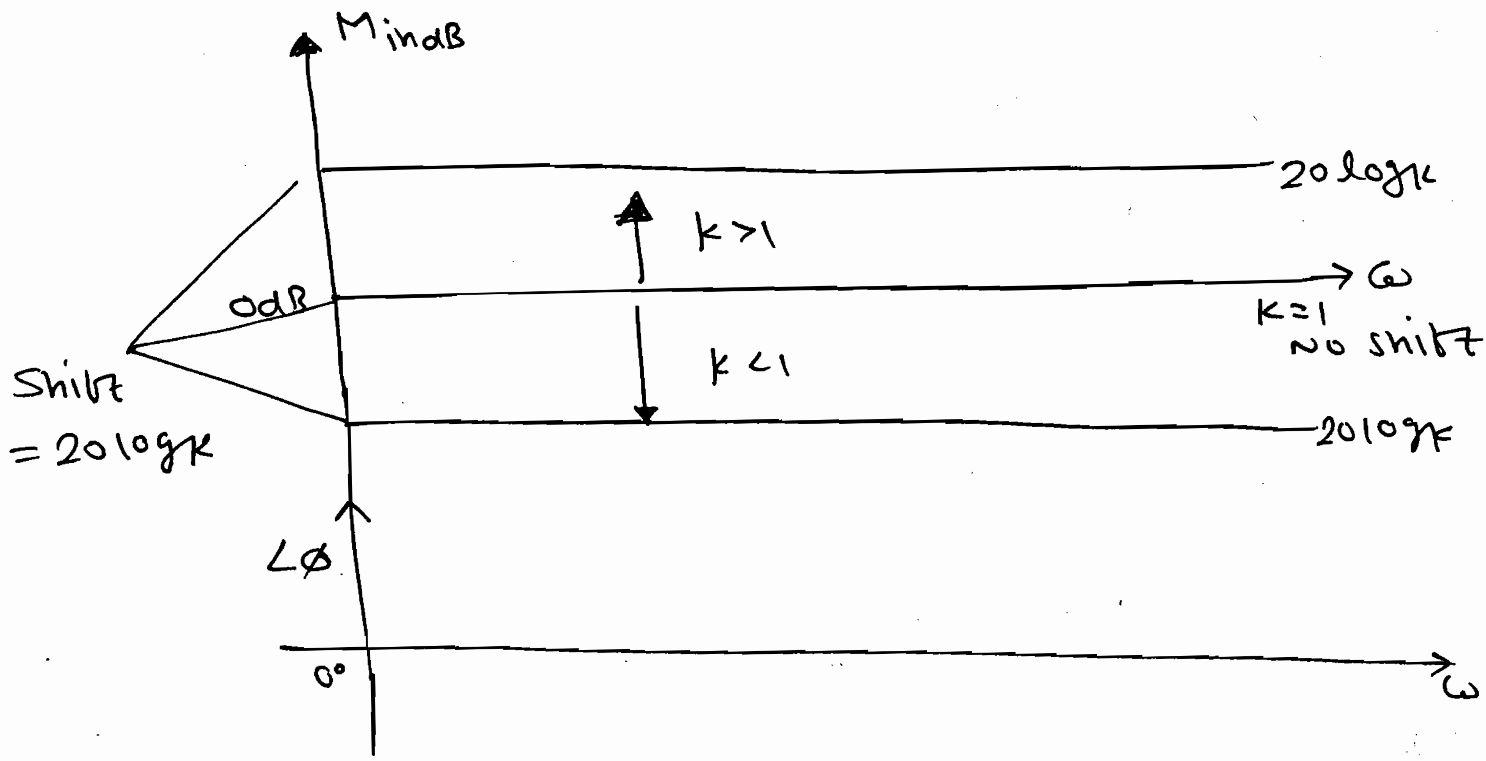
$M_{in\,dB} = 20\log K$

- $\rightarrow K = 1 \Rightarrow M_{dB} = 0$ dB
- $\rightarrow K > 1$ (10) $\Rightarrow M_{dB} = 20$ dB
- $\rightarrow K < 1$ (0.1) $\Rightarrow M_{dB} = -20$ dB.

$$\Rightarrow \angle\phi = \angle GH(j\omega) = \angle(K + j0)$$

$$= \tan^{-1}(0/K)$$

$$\boxed{\angle\phi = 0^\circ}$$



Note: The phase plot is independent of k value where as the shift in the magnitude plot depends on k value.

* n-Poles at origin | n-Zeros at origin

$GH(s) = 1/s^n$ | $GH(s) = s^n$

→ $s \to j\omega$

$GH(j\omega) = \frac{1}{(j\omega)^n}$

→ $M = \frac{1}{\omega^n}$

⇒ $M_{in\,dB} = 20\log\left(\frac{1}{\omega^n}\right)$

$= -20n\log(\omega)$

⇒ Slope $\frac{dM}{d\log\omega} = -20n$

⇒

$s \to j\omega$

$GH(j\omega) = (j\omega)^n$

→ $M = (\omega)^n$

→ $M_{in\,dB} = 20n\log(\omega)$

⇒ Slope

$\frac{dM}{d\log\omega} = 20n$

$\Rightarrow \angle\phi = \dfrac{1}{\angle j\omega \ldots n \text{ times}}$

$\Rightarrow \angle\phi = -90^\circ n$

*** Conclusion:**

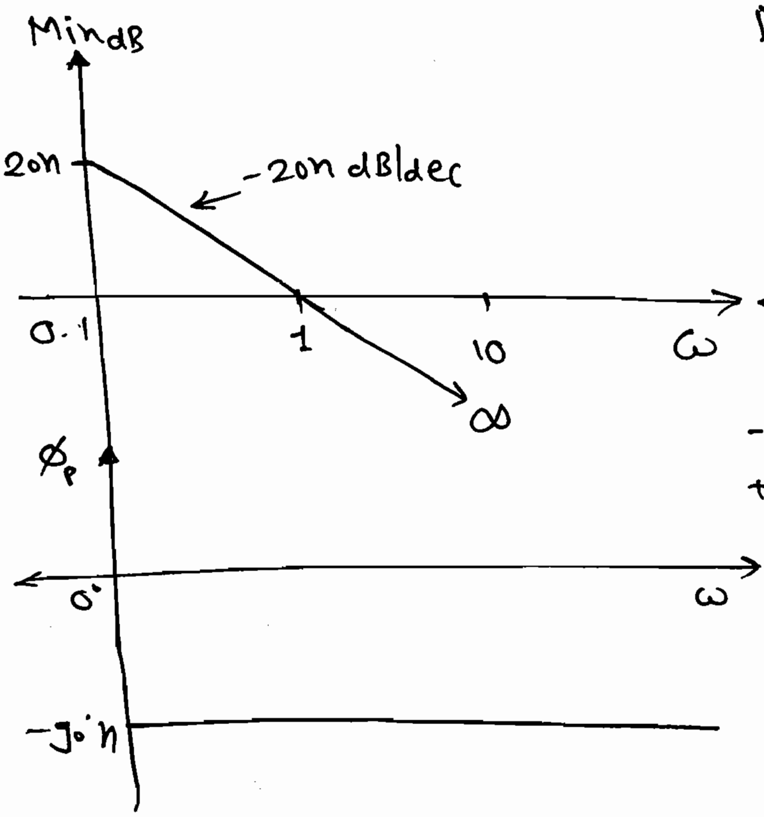
$$\boxed{\begin{aligned} S &= -20n \text{ dB/dec} \\ \phi &= -90^\circ n \end{aligned}}$$

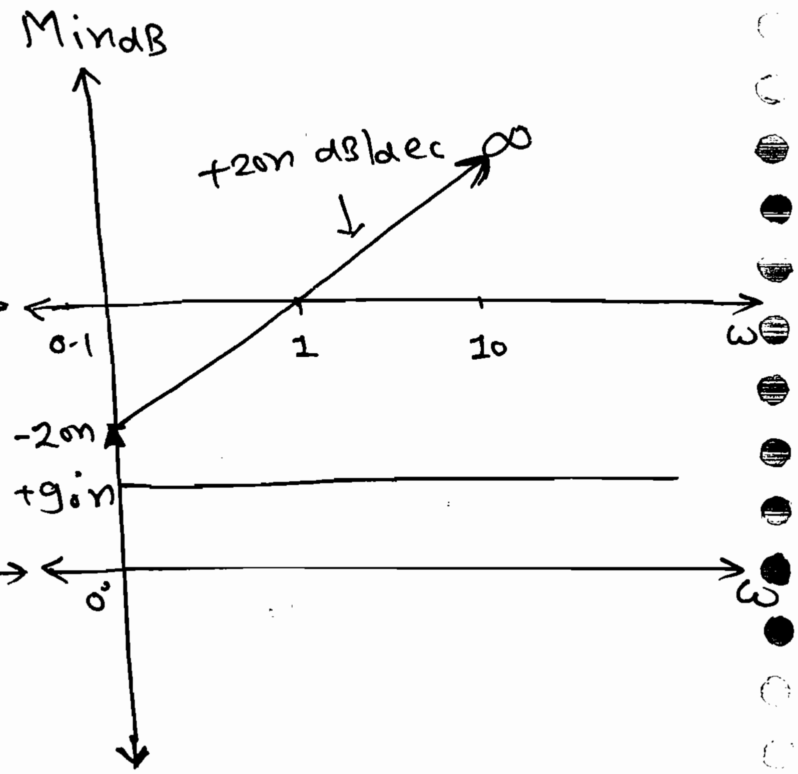
$\Rightarrow \angle\phi = \angle j\omega \ldots n \text{ times}$

$\Rightarrow \angle\phi = +90^\circ n.$

*** conclusion:**

$$\boxed{\begin{aligned} S &= +20n \text{ dB/dec} \\ \phi &= +90^\circ n. \end{aligned}}$$





$\Rightarrow$ Whenever the transfer f^n consist a poles and zeros at origin then the plot starts with a magnitude of opposite sign of slop at a freq of 0.1 and it should be passes through 0 db line intersect at $\omega = 1$ and extended upto ∞ if no corner freq exist when $K = 1$.

Q Draw the Bode Plot $G(s).H(s) = \frac{100}{s^8}$.

Soln: $G(s).H(s) = \frac{100}{s^8}$.

$\therefore M = \frac{100}{\omega^8}$.

$M_{in\,dB} = 20 \log \left(\frac{100}{\omega^8} \right)$.

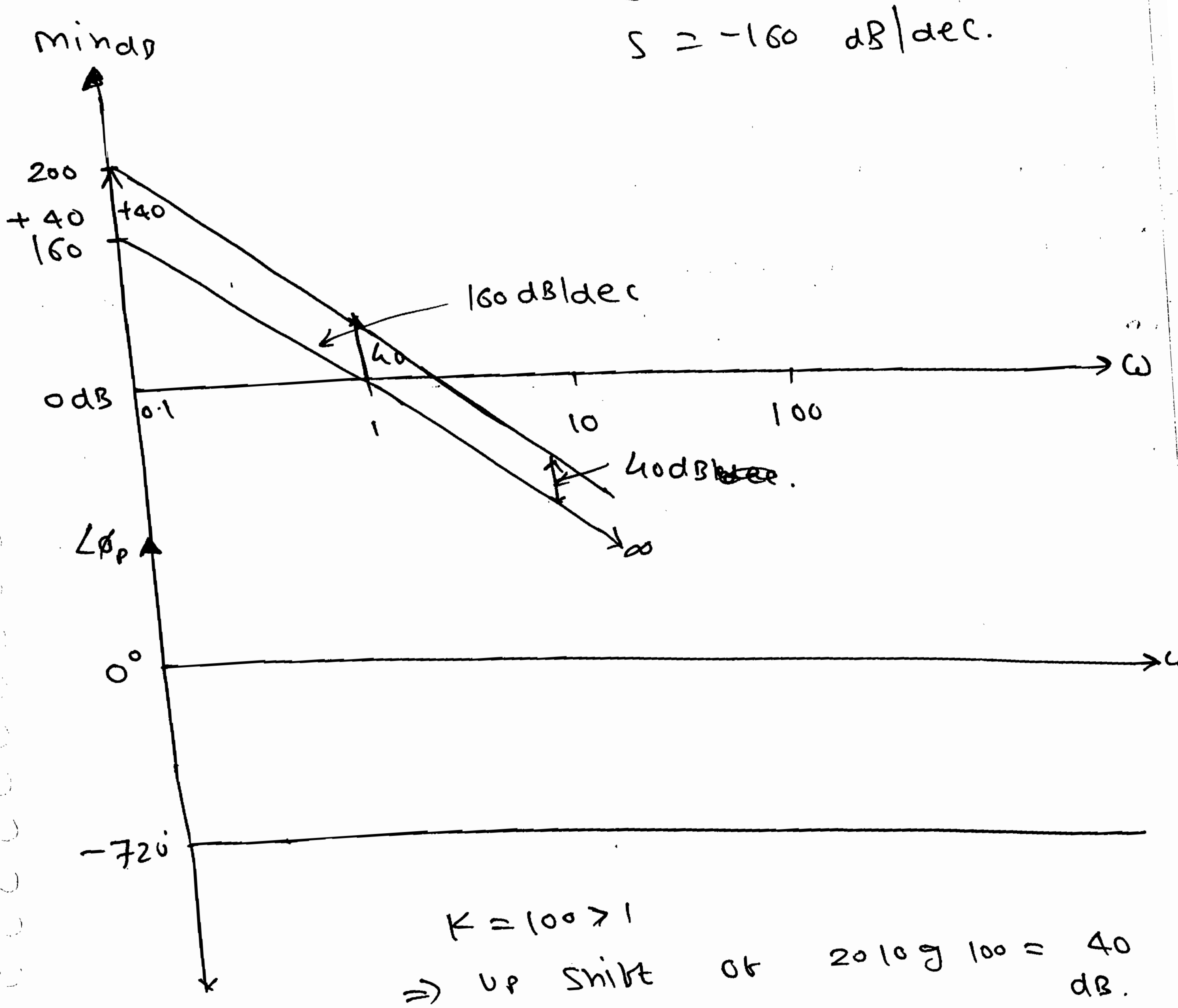
$\phi = -8 \times 90^\circ = -720^\circ$

$\Rightarrow$ 8 pole at origin

$s = -20n$

$s = -20 \times 8$

$s = -160$ dB/dec.



$K = 100 > 1$

$\Rightarrow$ Up shift of $20 \log 100 = 40$ dB.

⇒ n-finite Poles

$$GH(s) = \frac{1}{(s\tau+1)^n}.$$

$s \to j\omega$

$$GH(j\omega) = \frac{1}{(j\omega\tau+1)^n}$$

$$M = \left(\frac{1}{\sqrt{(\omega\tau)^2+1}}\right)^n$$

$$M_{dB\,\text{actual}} = -20n\log\sqrt{(\omega\tau)^2+1}$$

$$\Rightarrow \phi_{\text{actual}} = \frac{\angle 1+j0}{\angle(1+j\omega\tau)\ldots n\text{ times}}$$

$$\Rightarrow \phi_{\text{actual}} = -n\tan^{-1}(\omega\tau).$$

*** Asymptotic / Approximate**

Case-1: $\omega\tau < 1$

neglect $\omega\tau$

$$M_{asy} = -20\log 1 = 0\,\text{dB/dec}$$

$$S = \frac{dM}{d\log\omega} = 0 \quad \text{dB/dec}$$

$$\phi_{asm} = \frac{\angle 1+j0}{\angle 1+j0\ldots n\text{ times}}$$

$$\phi_{asy} = 0^\circ.$$

Case-2: $\omega\tau > 1$

n-finite zeros.

$$GH(s) = (s\tau+1)^n.$$

$$GH(j\omega) = (j\omega\tau+1)^n.$$

$$M = \left(\sqrt{(\omega\tau)^2+1}\right)^n.$$

$$M_{dB\,\text{actual}} = +20\log\sqrt{(\omega\tau)^2+1}.$$

$$\phi_{\text{actual}} = \angle(1+j\omega\tau)\ldots n\text{ times}.$$

$$\phi_{\text{actual}} = +n\tan^{-1}(\omega\tau).$$

*** Asymptotic | Approximate:**

Case-1: $\omega\tau < 1$

neglect $\omega\tau$.

$$M_{asy} = 20\log 1 = 0\,\text{dB/dec}.$$

$$S = \frac{dm}{d\log\omega} = 0\ \text{dB/dec}.$$

$$\phi_{asm} = \angle 1+j0\ldots n\text{ times}$$

$$\phi_{asm} = 0^\circ.$$

Case-2: $\omega\tau > 1$.

Neg - 1

$$M_{asym} = -20n \log(\omega\tau)$$

$$M_{asym} = -20n\log\omega - 20n\log\tau$$

$$S = \frac{dm}{d\log\omega} = -20n \text{ db/dec}$$

$$\phi_{asm} = \frac{\angle 1 + j0}{\angle j\omega\tau \cdots n\text{ times}}$$

$$\phi_{asm} = -90^\circ n$$

Neg - 1

$$M_{asym} = +20n\log(\omega\tau).$$

$$M_{asym} = +20n\log\omega - 20\log\tau$$

$$S = \frac{dm}{d\log\omega} = +20n \text{ db/dec.}$$

$$\phi_{asm} = \angle j\omega\tau \cdots n\text{ times}$$

$$\phi_{asm} = +90^\circ n.$$

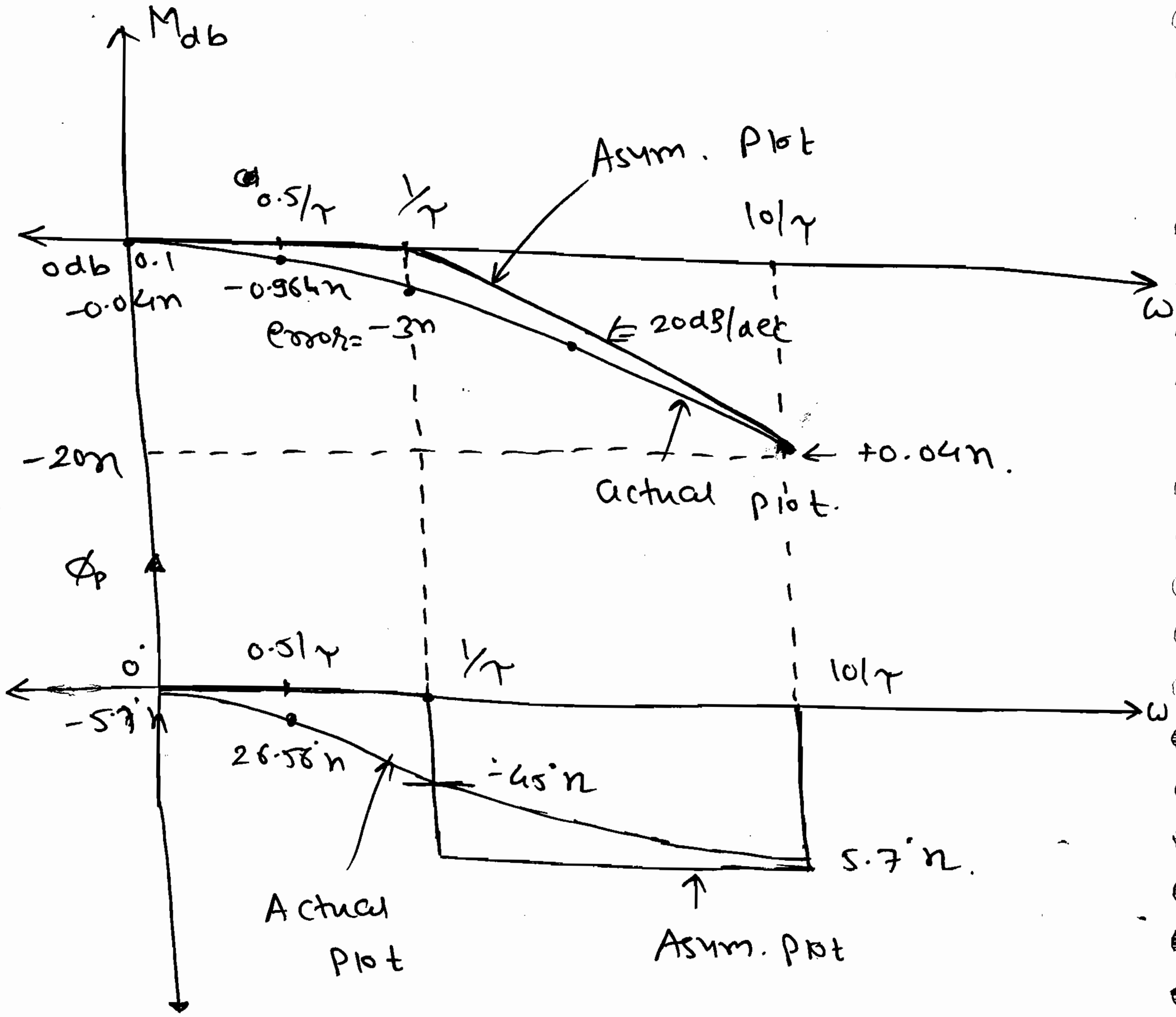
* Corner Frequency:

⇒ The freq. at which slope changing from one level to another level is called corner freqs.

⇒ The corner freq. is nothing but the finite poles and zeros location in the form of Magnitude.

	S	ϕ
< CF	0 db/dec	0
> CF	−20n db/dec	$-90^\circ n$.

⇒



⇒ $$\text{Error} = \underbrace{\text{Actual value}}_{\text{TF}} - \underbrace{\text{Asym. value}}_{\text{plot}}$$

⇒ To get the errors in the plot the Actual value is obtained from the transfer f^n and the asymptotic value is obtain from the Plot.

* Error at Corner Freq. :-

* Magnitude plot:

$\Rightarrow$ Error at CF $\omega = 1/\tau$ $= \left\{ \begin{matrix} M_{actual} \\ \omega = 1/\tau \end{matrix} \right\} - \left\{ \begin{matrix} M_{asym} \\ \omega = 1/\tau \end{matrix} \right\}$.

$$= -20^{n} \log \sqrt{\left(\frac{1}{\tau} \times \tau\right)^{2} + 1} \quad - 0$$

$$= -20 \log \sqrt{2} \cdot n \quad - 0$$

$$= -3n \quad - \quad 0$$

$$= -3n. \ dB.$$

$\Rightarrow$ error is Symm. about the Corner freq.

* Phase plot:

$\Rightarrow$ Error at CF $\omega = 1/\tau$ $= \left\{ \begin{matrix} \phi_{Actual} \\ \omega = 1/\tau \end{matrix} \right\} - \left\{ \begin{matrix} \phi_{Asymptotic} \\ \omega = 1/\tau \end{matrix} \right\}$.

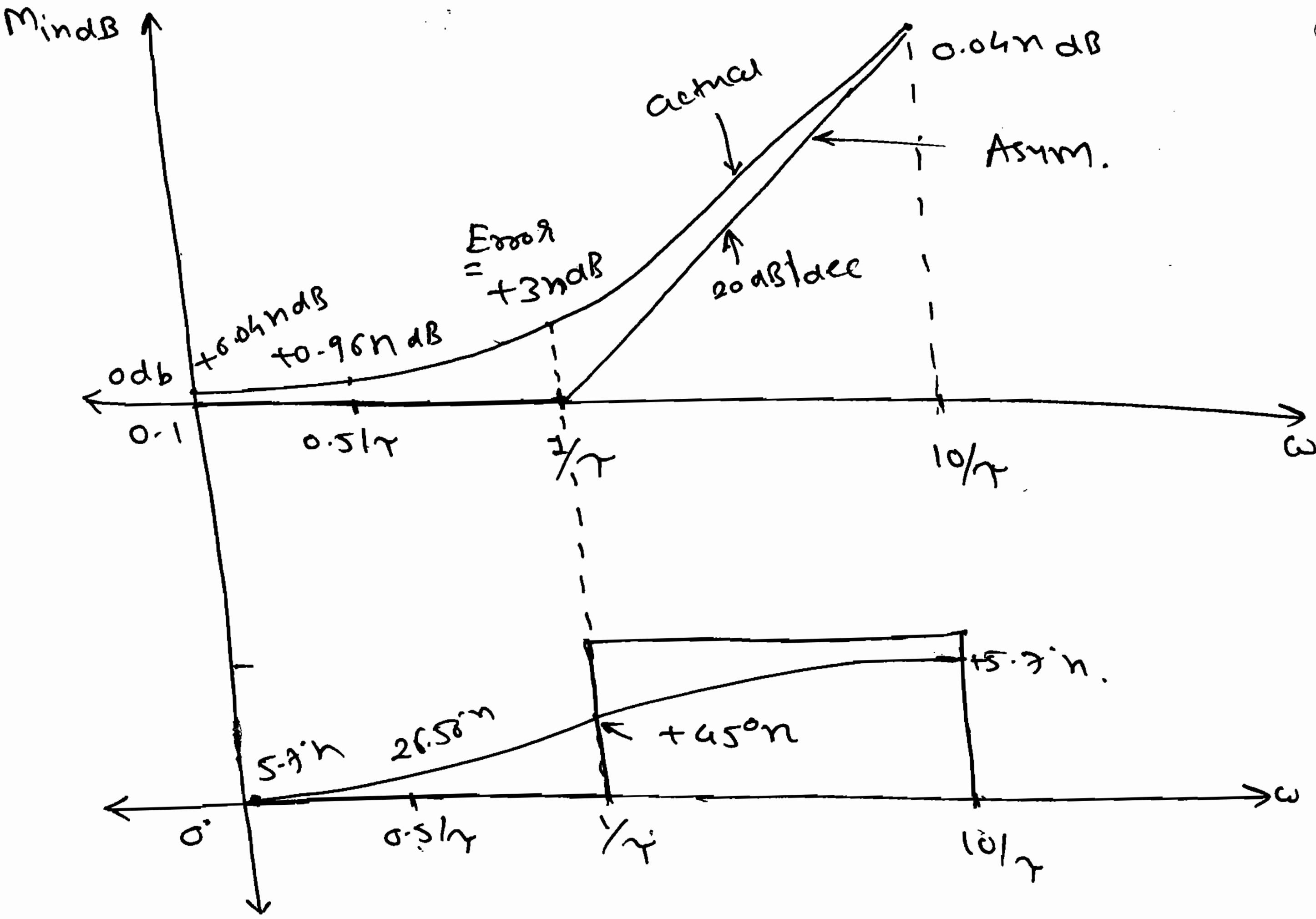
$$= -n \tan^{-1}(\omega\tau) \quad - 0$$

$$= -n \tan^{-1}(1). \quad - 0$$

$$= -45^{\circ} n \quad - 0$$

$$= -45^{\circ} n.$$

$\Rightarrow$ The error is max. at Corner freq.,
the error is Symm. about corner freq.
whatever the error exist below 1 decade
the same error exist after one decade
also.



$\boxed{a}$ Draw the Bode plot for

$$GH(s) = \frac{10\,(s+5)^2}{s\,(s+2)\,(s+10)}$$

Soln:

$$GH(j\omega) = \frac{10\,(j\omega+5)^2}{(j\omega)\,(j\omega+2)\,(j\omega+10)}$$

$$M = |GH(j\omega)| = \frac{10\cdot(\omega^2+5)}{\omega\cdot\sqrt{\omega^2+4}\cdot\sqrt{\omega^2+100}}$$

$$\angle GH = \phi = -90^\circ - \tan^{-1}(\omega/2) + 2\tan^{-1}(\omega/5) - \tan^{-1}(\omega/10)$$

$$M_{in\,dB} = 20\log_{10}\left[\frac{10\cdot(\omega^2+5)}{\omega\cdot\sqrt{\omega^2+4}\cdot\sqrt{\omega^2+100}}\right]$$

⇒ Time constant form:

$$GH(s) = \frac{\cancel{10} \times \cancel{25}^{\,12.5}\,(1+s/5)^2}{\cancel{2} \times \cancel{10}\; s\,(1+s/2)\,(1+s/10)} \rightarrow CF.$$

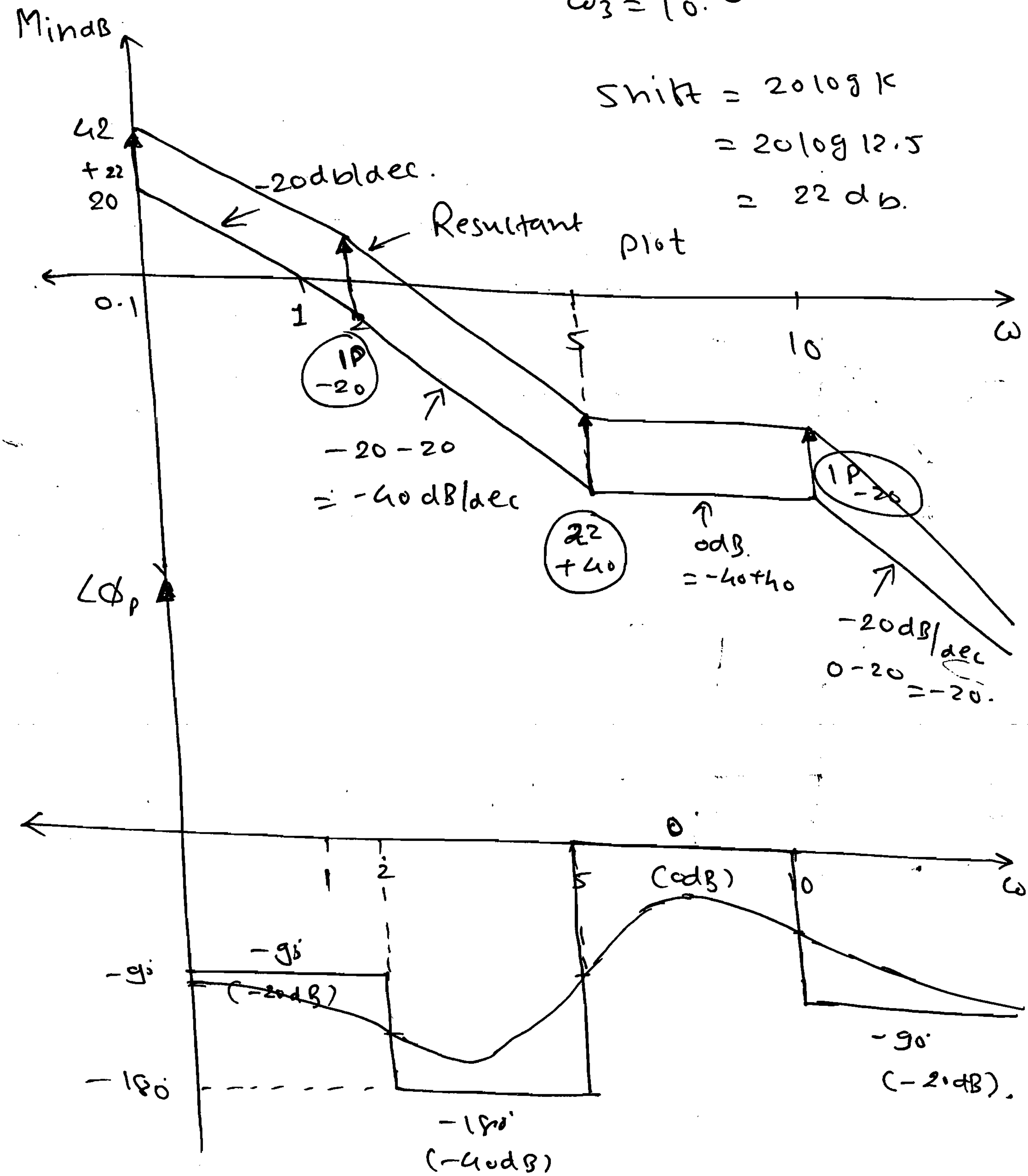
IPo $s \Rightarrow -20\,dB/dec.$

$$\left.\begin{array}{l}\omega_1 = 2,\\ \omega_2 = 5.\\ \omega_3 = 10.\end{array}\right\} \rightarrow CF.$$

Shift $= 20\log K$

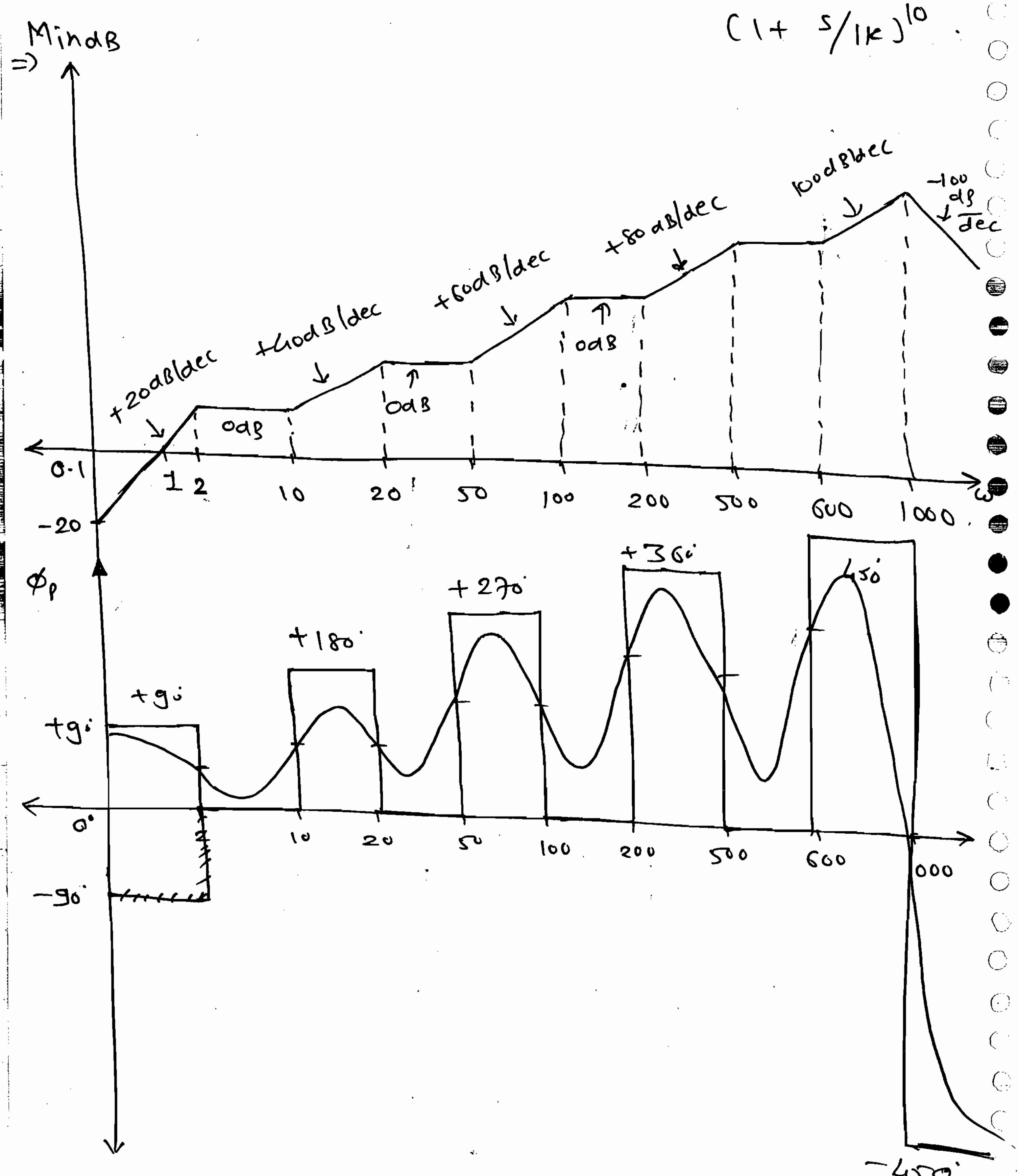
$= 20\log 12.5$

$= 22\,db.$



⇒ The Initial slope of the plot is given by poles and zeros located at origin.

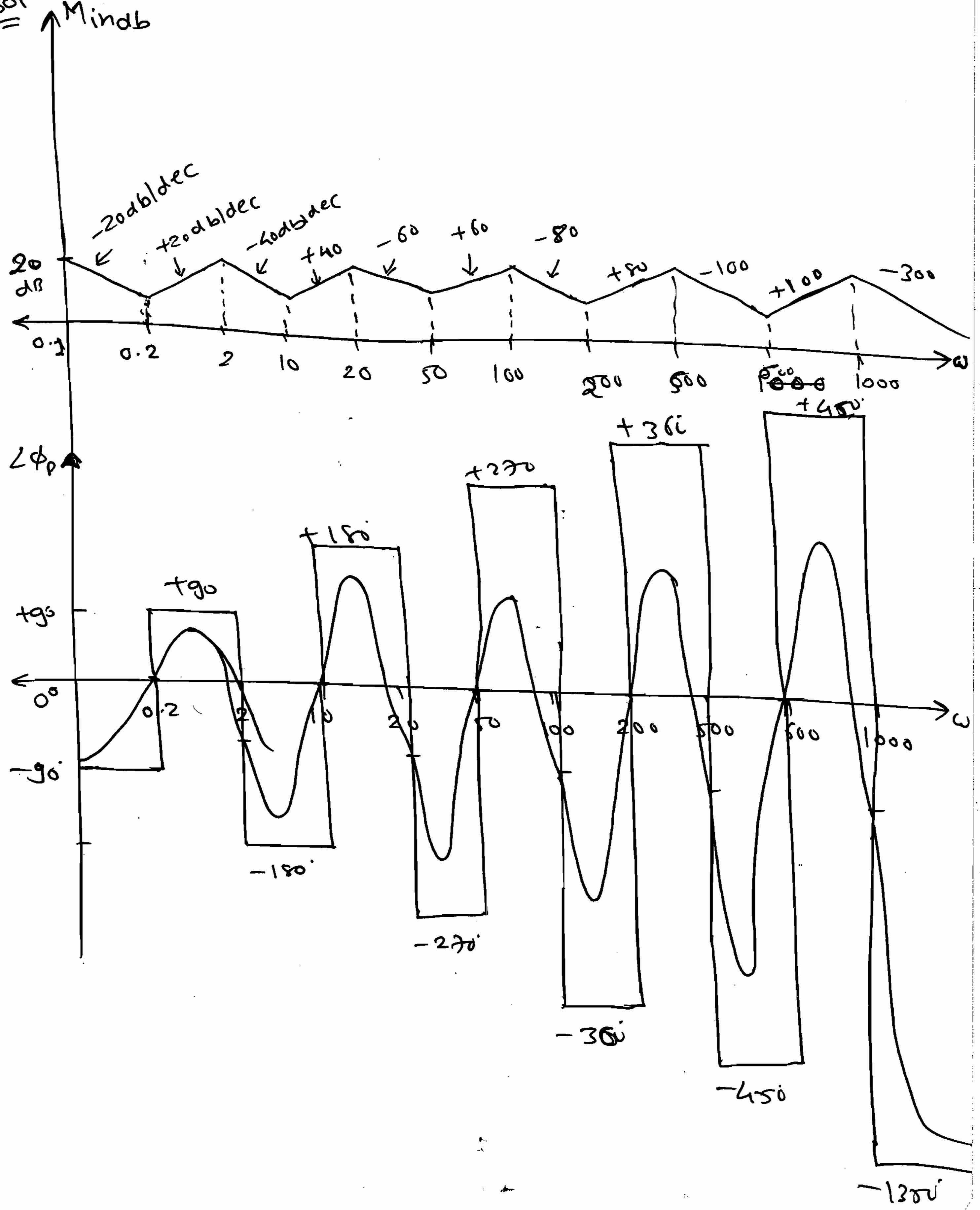
[a] $$G_H(s) = \frac{s(1+s/10)^2 (1+s/50)^3 (1+s/200)^4 (1+s/600)^5}{(1+s/2)(1+s/20)^2(1+s/100)^3(1+s/500)^4 (1+s/1k)^{10}}$$



Q

$$G H(s) = \frac{(1+s/0.2)^2 (1+s/10)^4 (1+s/50)^6 (1+s/200)^8 \times (1+s/600)^{10}}{s(1+s/2)^3 (1+s/20)^5 (1+s/100)^7 (1+s/500)^9 \times (1+s/1000)^{20}}$$

Soln:



Q) Find the change in slope at the following corner freqs

1) $\omega = 2$ 4) 50 7) $\omega = 500$
2) $\omega = 10$ 5) $\omega = 100$ 8) $\omega = 1K$.
3) $\omega = 20$ 6) $\omega = 200$

→ Find the slope of the line betn two corner freqs.

1) $\omega =$ 2 to 10. 4) High freq asymptote
2) $\omega =$ 20 to 50.
3) $\omega =$ 200 to 500

→ Find the slopes around the corner freq. for

1) $\omega = 2$, 20, 200, 1K.

$$G(s)\cdot H(s) = \frac{s^5 (1+s/10)^{20} (1+s/50)^{50} (1+s/200)^{200} (1+s/500)^{500}}{(1+s/2)^{10} (1+s/20)^{30} (1+s/100)^{100} (1+s/500)^{500} (1+s/1K)^{1K}}$$

Soln:

$$\boxed{\text{Change in slope} = \text{New slope} - \text{Previous slope.}}$$

CF	-20 P | +20 Z	CS
2	10P	-200
10	20Z	+400
20	30P	-600
50	50Z	+1000
100	-100P	-2000
200	200Z	+4000
500	500P	-20000
1000	1000P	~~+2~~ -20000

⇒ Slope- betn ω_1 & ω_2:

Note :→ To get a slope of line betn two freqs from ω_1 to ω_2 then consider all the terms in TF up to ω_1 only, get the no. of poles and zeros. and resultant slope.

1) $\omega = 2$ to 10

⇒ >2 ✓ IN $\quad$ <10 ✗ out

$\therefore$ $P = 10$, $Z = 5$

⇒ 5 P ⇒ -100.

2) $\omega = 20$ to 50.

⇒ >20 IN $\quad$ <50 out ⇒ $P = 40$, $Z = 25$

⇒ $40 - 25 = 15P$ ⇒ -300.

3) $\omega = 200$ to 500

⇒ >200 IN $\quad$ <500 out. ⇒ $P = 140$, $Z = 275$

⇒ $P - Z = 40 - 275 = 235\,Z$

235 zero ⇒ $+2700$.

4) ∞

$P = 1640$
$Z = 875$ $\Big\}$ ⇒ $P = 765$ ⇒ -15300.

(3) Find ~~frq~~ Slopes around the freq.

(i) $\omega = 2$

>2	<2
IN	out
↓	↓
$P = 10$	$z = 5$
$z = 5$	$p = 0$
$p = 5$	$z = 5$
$\Rightarrow -100$	$\Rightarrow +100$

(ii) $\omega = 200$

>200	<200
IN	out
↓	↓
$p = 140$	$p = 140$
$z = 275$	$z = 75$
$z = 135$	$p = +65$
$\Rightarrow 2700$	$\Rightarrow -1300$

(iii) $\omega = 20$.

>20	<20
IN	out
↓	↓
$P = 40$	$P = 10$
$z = 25$	$z = 25$
$p = 15$	$z = 15$
$\Rightarrow -300$	$\Rightarrow +300$

(iv) $\omega = 1K$.

>1000	<1000
IN	out
↓	↓
$p = 1640$	$p = 640$
$z = 875$	$z = 875$
$p = 765$	$z = 235$
$\Rightarrow -15{,}300$	$+4700$

* Transfer Function from the magnitude plot:

⇒ **Procedure:**

1) observe the initial slope it gives the no. of zeros and poles at origin.

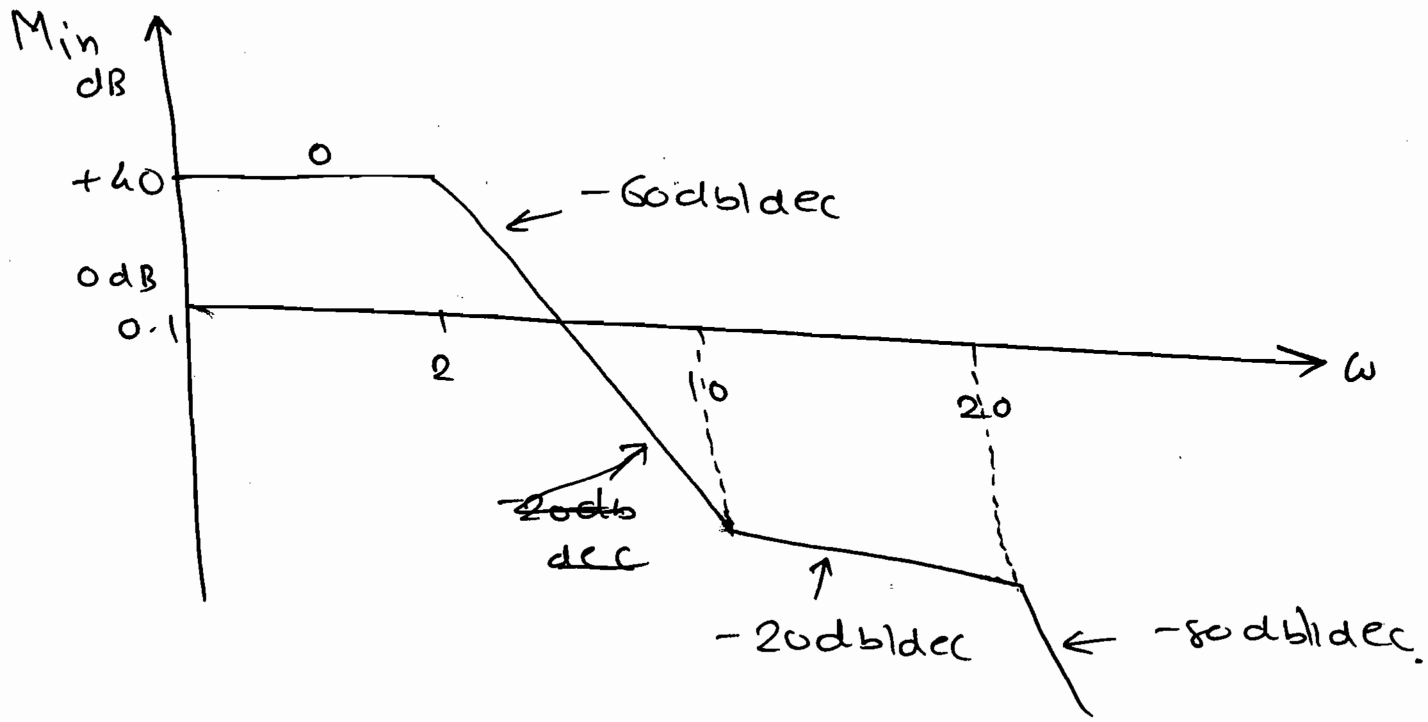
2) Find the change slope at each and every corner freq. The change

in Slope is (+ve), consider finite zero.

If change in slope is (−ve) consider the finite poles.

3) Find the K value by using known magnitude at known freq.

$\boxed{Q}$ Find the TF to the given asymptotic magnitude plot of a minimum phase system.



$\underline{\underline{\text{Soln:}}}$ The initial slope is zero so no poles and zeros at origin.

$\Rightarrow$ CS = $-60 - 0 = \ominus 60$

poles 20 × ③ → 3 poles at $\omega = 2$.

$\Rightarrow$ Cs = $-20 - (-60) = \oplus 40 \to$ ②×20 at $\omega = 10$.

zero → 2 zeros

$$\Rightarrow \quad Cs = -80 - (-20) = -60 \Rightarrow \text{3 poles at } \omega = 20.$$

$$\text{So,} \quad GH(s) = \frac{K\left(\frac{s}{10}+1\right)^2}{\left(\frac{s}{2}+1\right)^3 \left(\frac{s}{20}+1\right)^3}.$$

$$\text{at} \quad \omega = 0.1 \qquad M_{indB} = 0.1.$$

$$M = |GH(j\omega)| = \frac{K \left[\left(\frac{\omega}{10}\right)^2+1\right]}{\left[\left(\frac{\omega}{2}\right)^2+1\right]^{3/2} \left[\left(\omega/20\right)^2+1\right]^{3/2}}$$

$$\Rightarrow M_{indB} = 20\log K + 40 \log\sqrt{1+\cancelto{0}{\left(\frac{0.1}{10}\right)^2}} - 60\log\sqrt{\cancelto{0}{\left(\frac{0.1}{2}\right)^2}+1} - 60\log\sqrt{\cancelto{0}{\left(\frac{0.1}{20}\right)^2}+1}$$

$$M_{db}\Big|_{\omega=0.1} = 40$$

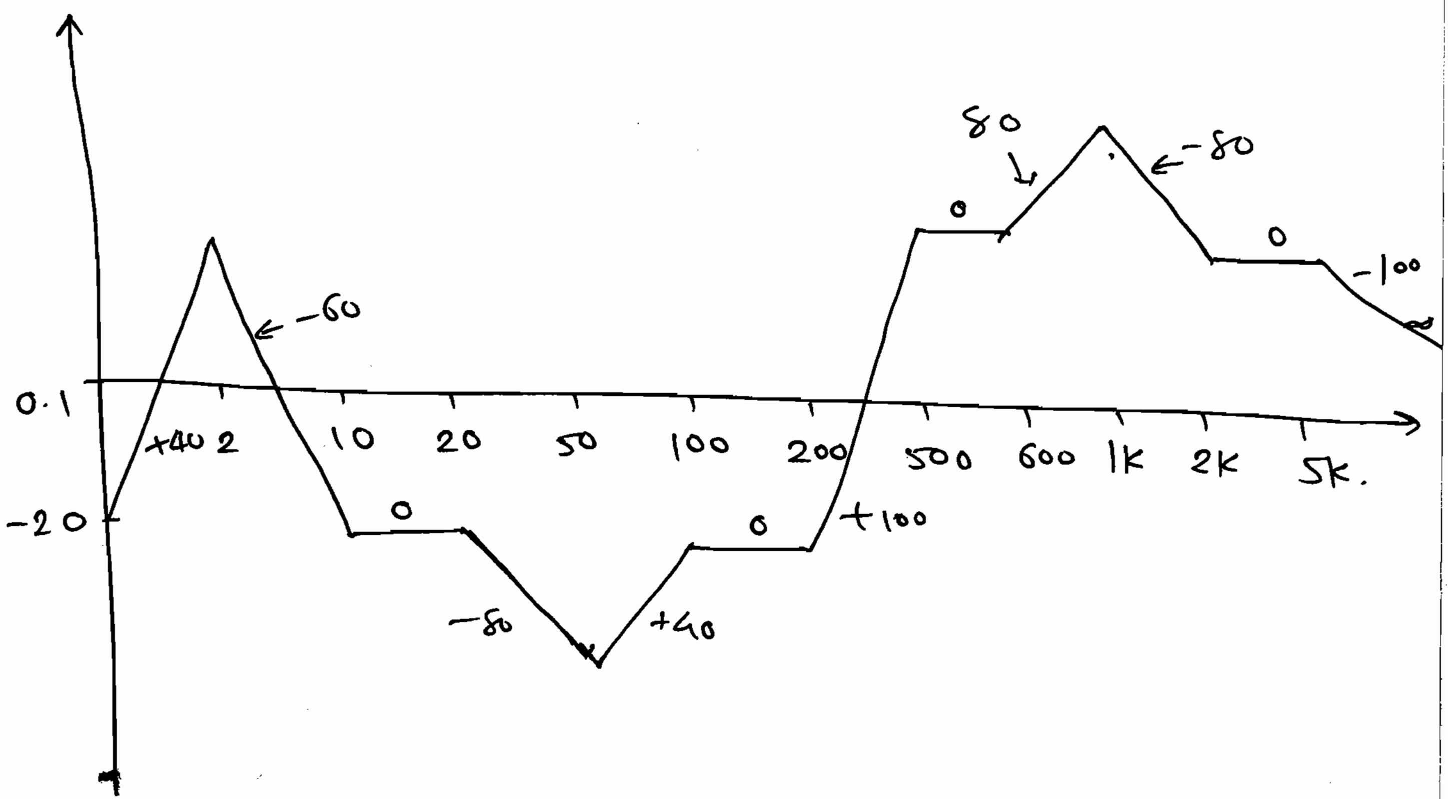
$$\therefore \quad \overset{2}{\cancel{40}} = \cancel{20}\log_{10} K.$$

$$\Rightarrow \quad \log_{10} K = 2$$

$$K = 10^2 \Rightarrow \boxed{K = 100}$$

__Note:__ To get the K value compare the corner freq. with freq. where the magnitude is known if the corner freq. is greater than (or) equal to one (CF ≥ 1). then neglect the corner freq.

Q Find the TF:



Soln:

$$G H(s) = \frac{K s^2 \left(\frac{s}{10}+1\right)^3 \left(\frac{s}{50}+1\right)^6 \left(\frac{s}{200}+1\right)^5 \left(\frac{s}{600}+1\right)^4 \left(\frac{s}{2k}+1\right)^4}{\left(\frac{s}{2}+1\right)^3 \left(\frac{s}{20}+1\right)^4 \left(\frac{s}{100}+1\right)^2 \left(\frac{s}{500}+1\right)^{+5} \left(\frac{s}{1k}+1\right)^8 \times \left(\frac{s}{5k}+1\right)^5}.$$

$$M_{in\,dB}\Big|_{\omega=0.1} = -20.$$

$$\therefore -20 = 20\log K + 40\log\omega.$$

$$\Rightarrow -20 = 20\log K + 40\log 0.1.$$

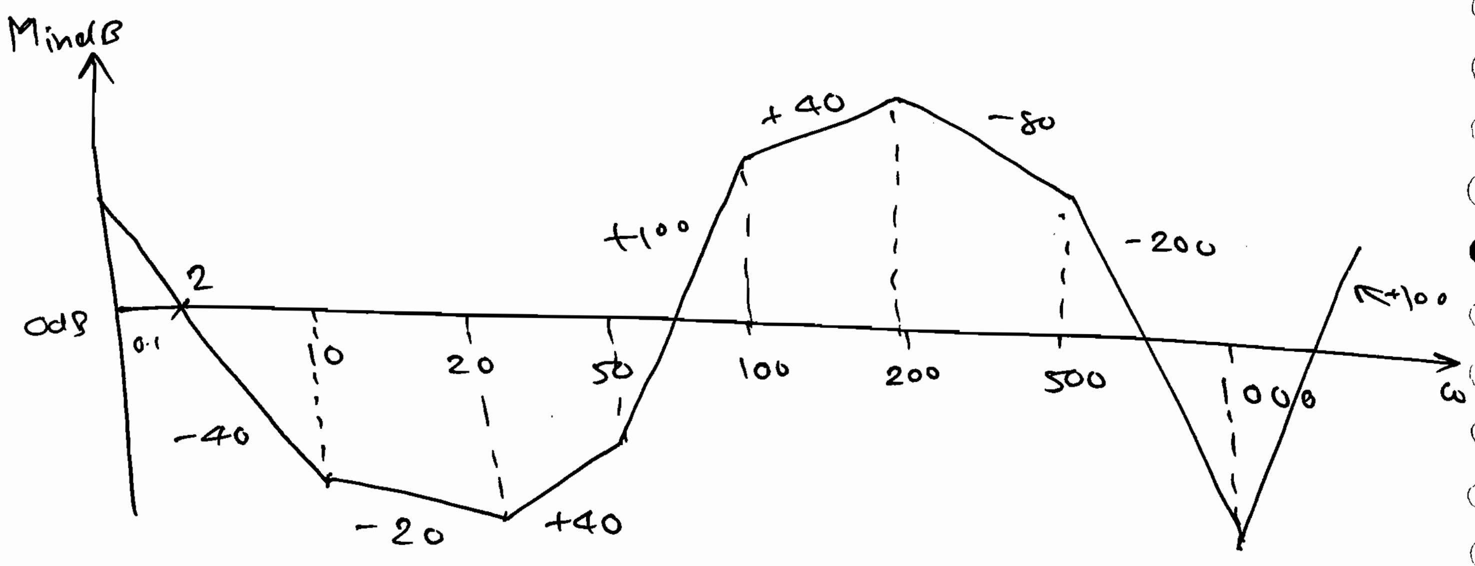
$$-20 = 20\log K - 40.$$

$$20\log K = 20$$

$$\Rightarrow \log K = 1$$

$$\Rightarrow \boxed{K = 10}$$

Q Find K



Sol^n:

$$GH(s) = \frac{K\left(1+\frac{s}{10}\right)^1 \left(1+\frac{s}{20}\right)^3 \left(1+s/_{50}\right)^3 \left(1+s/_{1k}\right)^{15}}{s^2 \left(1+\frac{s}{100}\right)^3 \left(1+\frac{s}{200}\right)^6 \left(1+s/_{500}\right)^6}$$

Now, at $\omega = 2$, $M_{db} = 0\,dB$.

$$\therefore \quad 0\,dB = \cancel{K}\ 20 \log_{10}\left(\frac{K}{\omega^2}\right).$$

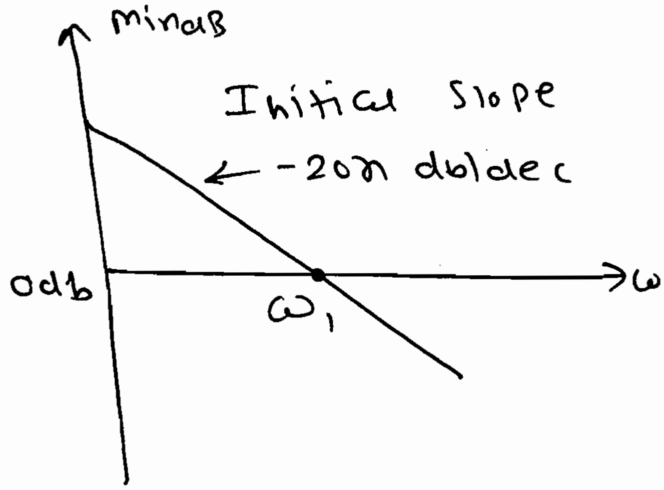
$$\Rightarrow \quad 0\,dB = 20 \log_{10} K - 40 \log \omega. \Big|_{\omega=2}$$

$$\cancel{20} \log K = \overset{2}{\cancel{40}} \log \omega$$

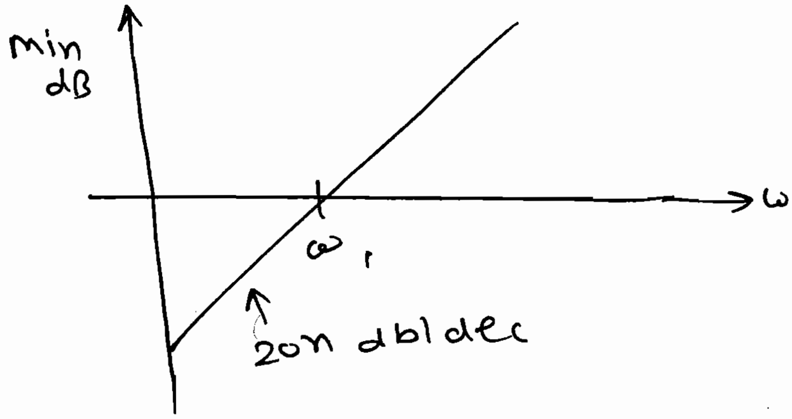
$$\log K = 2 \log 2$$

$$\log K = \log 2^2$$

$$\boxed{K = 4}$$

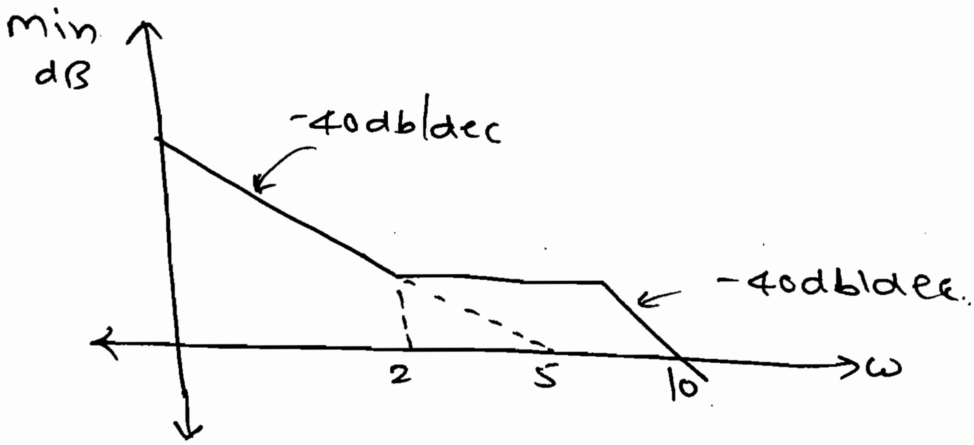
* 

$\Rightarrow \quad K = (\omega_1)^n.$

* 

$\Rightarrow \quad K = (\omega_1)^{-n}.$

$\boxed{Q}$ Find TF.



Solⁿ: Initial slope $= -40$ dB/dec $= -2 \times 20$ dB/dec
$\boxed{n=2}$

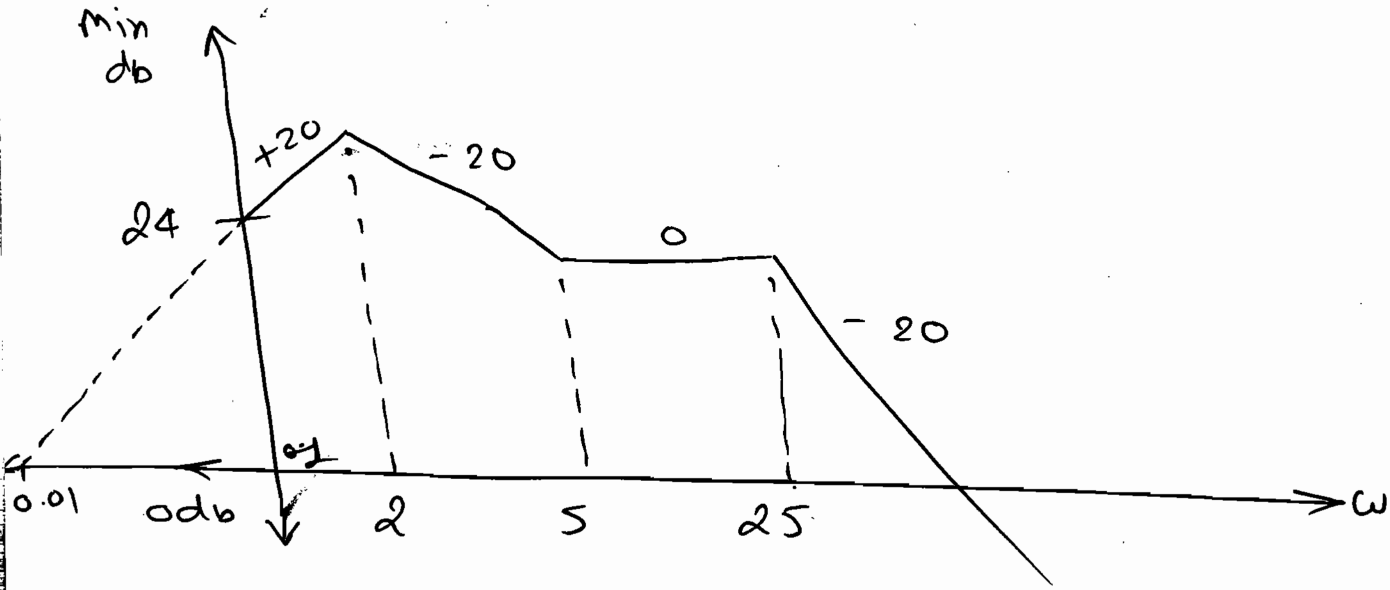
$\Rightarrow$ Initial slope crosses the ω axis at $\omega = 5$ rad/sec. & $n = 2$

$\Rightarrow \quad K = (\omega)^n$

$K = (5)^2 = 25.$

$$\Rightarrow \boxed{TF = \frac{25\,(1+s/2)^2}{s^2\,(1+s/10)^2}}.$$

Q Find the TF.



Soln:

$$G_H(s) = \frac{K s^2 \times (1+s/5)}{(1+s/2)^2\,(1+s/25)}.$$

Now $\quad M\text{in db}\big|_{\omega=0.1} = 24.\text{db}$

$\therefore \quad 24 = 20\log(K\cancel{s}\cdot\omega).$

$\Rightarrow 24 = 20\log_{10}K + 20\log(0.1).$

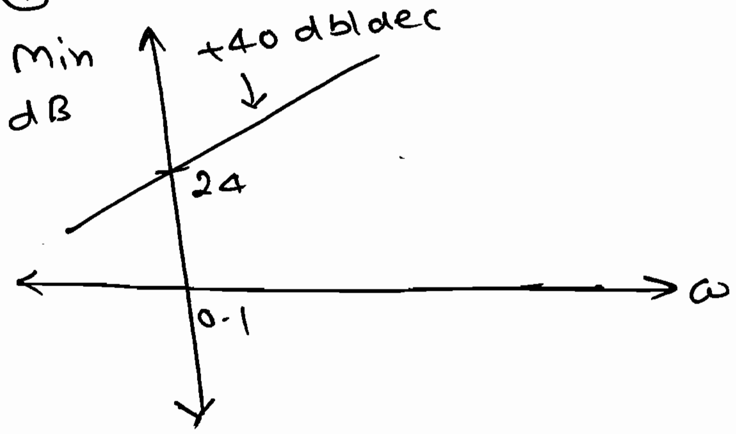
$24 + \cancel{1/20} = 20\log_{10}K.$

$\log_{10}K = 1.2$

$\Rightarrow \boxed{K = 15.85}$

Q

① Find K value for the following Bode plots:



Soln:

$$GH(s) = K \cdot s^2.$$

at $\omega = 0.1$, $M_{db} = 24$ dB.

$$\Rightarrow 24 = 20 \log (K \cdot \omega^2).$$

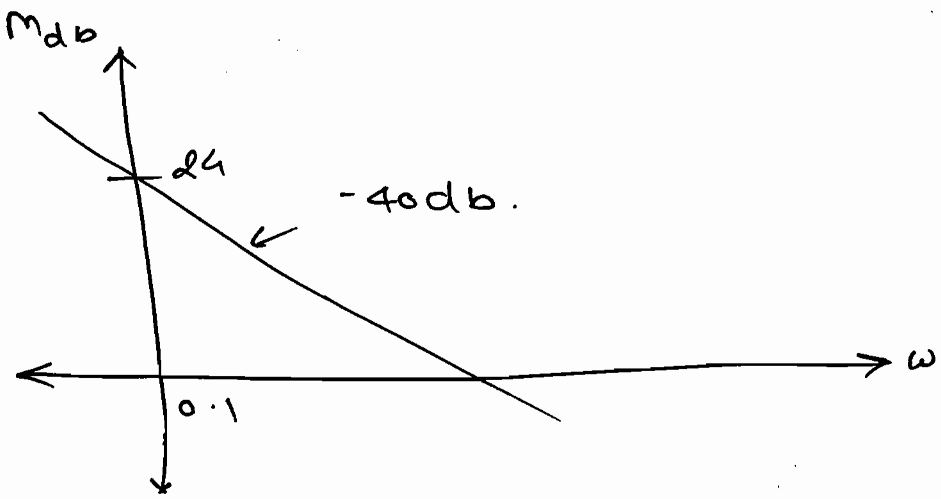
$$\Rightarrow 24 = 20 \log K + 40 \log (0.1).$$

$$24 = 20 \log K - 40.$$

$$64 = 20 \log K$$

$$\Rightarrow \boxed{K = 1584.89}$$

②



Soln:

$$GH(s) = \frac{K}{s^2}.$$

$$M_{db}\Big|_{\omega=0.1} = 24 \text{ dB}.$$

$$\Rightarrow \quad 24 = 20\log K - 40\log(\omega).$$

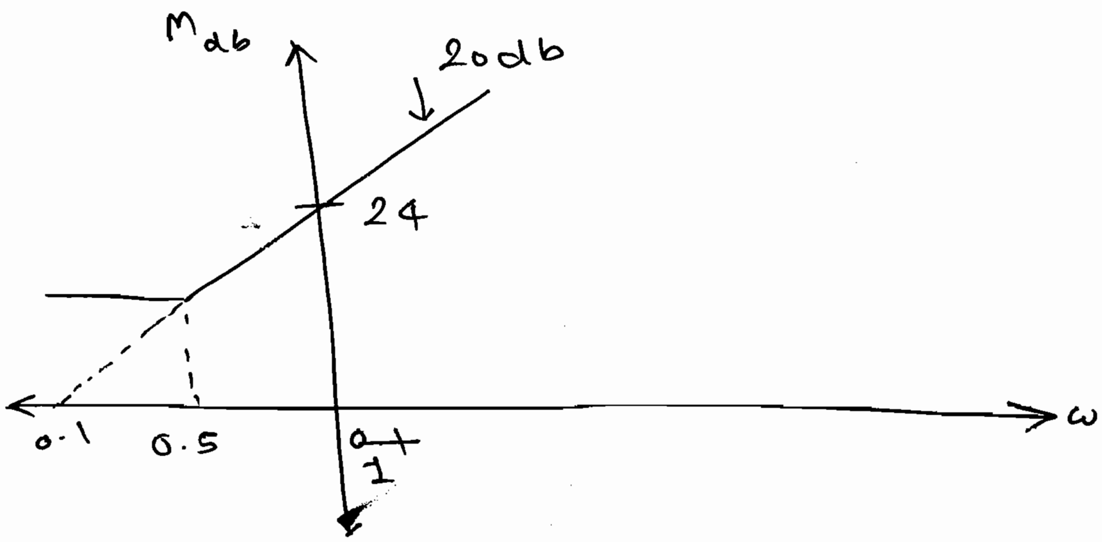
$$\therefore \quad 24 = 20\log K - 40\log(0.1).$$

$$\therefore \quad 24 - 40 = 20\log K$$

$$\therefore \quad \log K = \frac{-16}{20}.$$

$$\Rightarrow \quad \boxed{K = 0.15848}$$

3



Soln:

$$G H(s) = \frac{K \times (1 + s/0.5)}{1}.$$

at → $M_{db}\Big|_{\omega=1} = 24$ db.

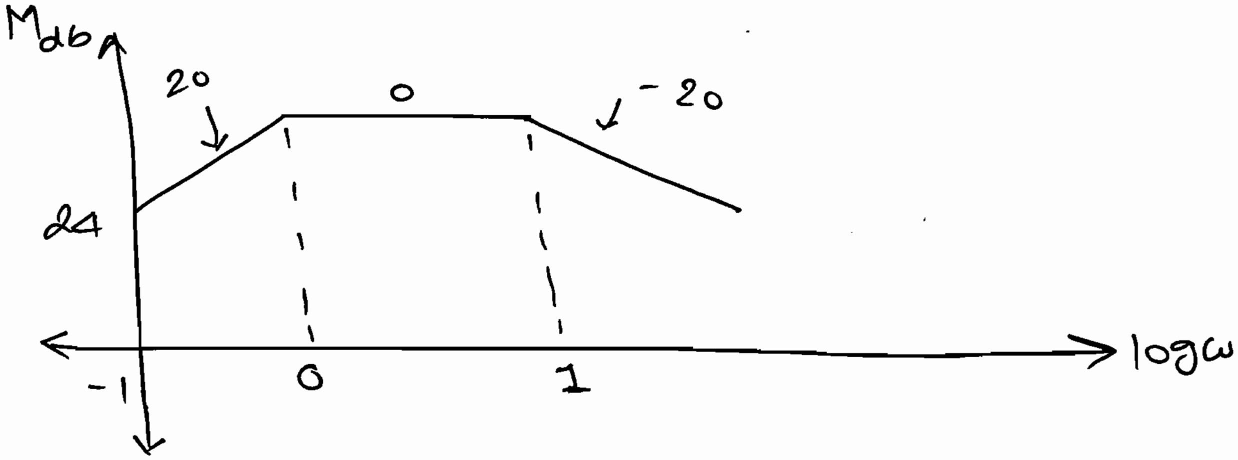
$$\therefore \quad 24 = 20\log K + 20\log\left(\sqrt{1 + (\omega/0.5)^2}\right).$$

$$\therefore \quad 24 = 20\log_{10} K + 20\log\sqrt{1+4}$$

$$\therefore \quad 24 - 6.989 = 20\log_{10} K$$

$$\Rightarrow \quad \boxed{K = 7.0876}$$

④



Sol^n:

$\log\omega = -1 \Rightarrow \omega = 0.1$

$\log\omega = 0 \Rightarrow \omega = 1.$

$\log\omega = 1 \Rightarrow \omega = 10.$

So, $$GH(s) = \frac{Ks}{\left(\frac{s}{1}+1\right)^1\left(\frac{s}{10}+1\right)^1}$$

Now, $$M_{db}\Big|_{\omega=0.1} = 24.$$

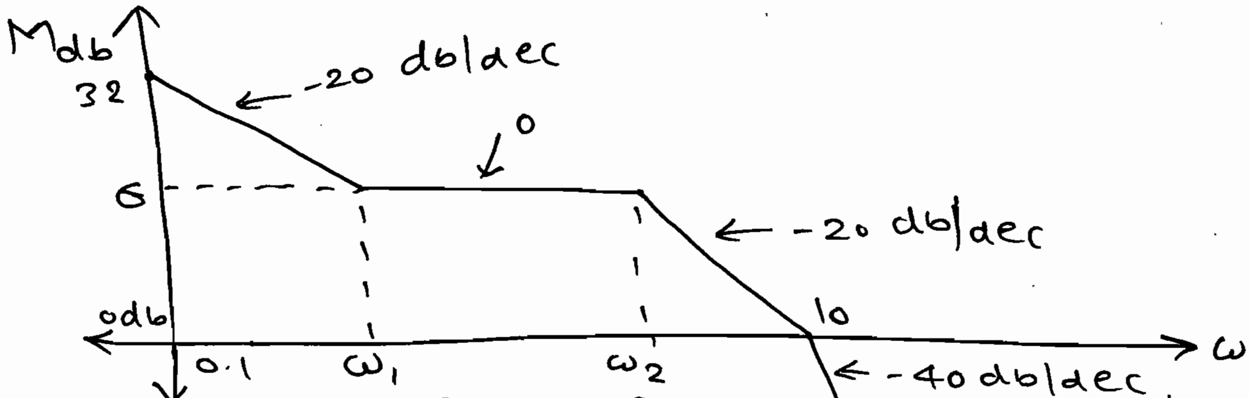
$$\therefore\ 24 = 20\log K + 20\log\omega - 20\log\sqrt{\cancel{\omega^2}^{\,0}+1} - 20\log\sqrt{\cancel{\frac{\omega^2}{100}}^{\,0}+1}.$$

$$\therefore\ 24 = 20\log K - 20$$

$$\frac{24+20}{20} = \log_{10}K$$

$$\Rightarrow \boxed{K = 158.487}$$

Q Find the ω_1, ω_2, TF



Soln: $\rightarrow$ Slope $= \dfrac{dM}{d\log\omega}$.

$$\therefore \quad -20\,db/dec = \frac{6-0}{\cancel{\log\omega_1} \; \log\omega_2 - \log 10}.$$

$$\therefore \quad -20 = \frac{6}{\log\omega_2 - 1}$$

$$\therefore \quad -20\log\omega_2 + 20 = 6$$

$$20\log\omega_2 = +14$$

$$\log\omega_2 = 14/20$$

$$\Rightarrow \boxed{\omega_2 = 5 \text{ rad/sec}}$$

$\rightarrow$ Slope $= \dfrac{dM}{d\log\omega}$

$$\therefore \quad -20 = \frac{32-6}{\log 0.1 - \log\omega_1}.$$

$$\therefore \quad -20 = \frac{26}{-1 - \log\omega_1}.$$

$$\therefore \quad 20 + 20\log\omega_1 = 26.$$

$$20\log\omega_1 = 6.$$

$$\boxed{\omega_1 = 2 \text{ rad/sec}}$$

$\Rightarrow$ TF $\quad G_H(s) = \dfrac{K\,(1+s/2)^1}{s\,(1+s/5)^1\,(1)\,(1+s/10)^1}$

$$M_{db}\big|_{\omega=0.1} = 32.$$

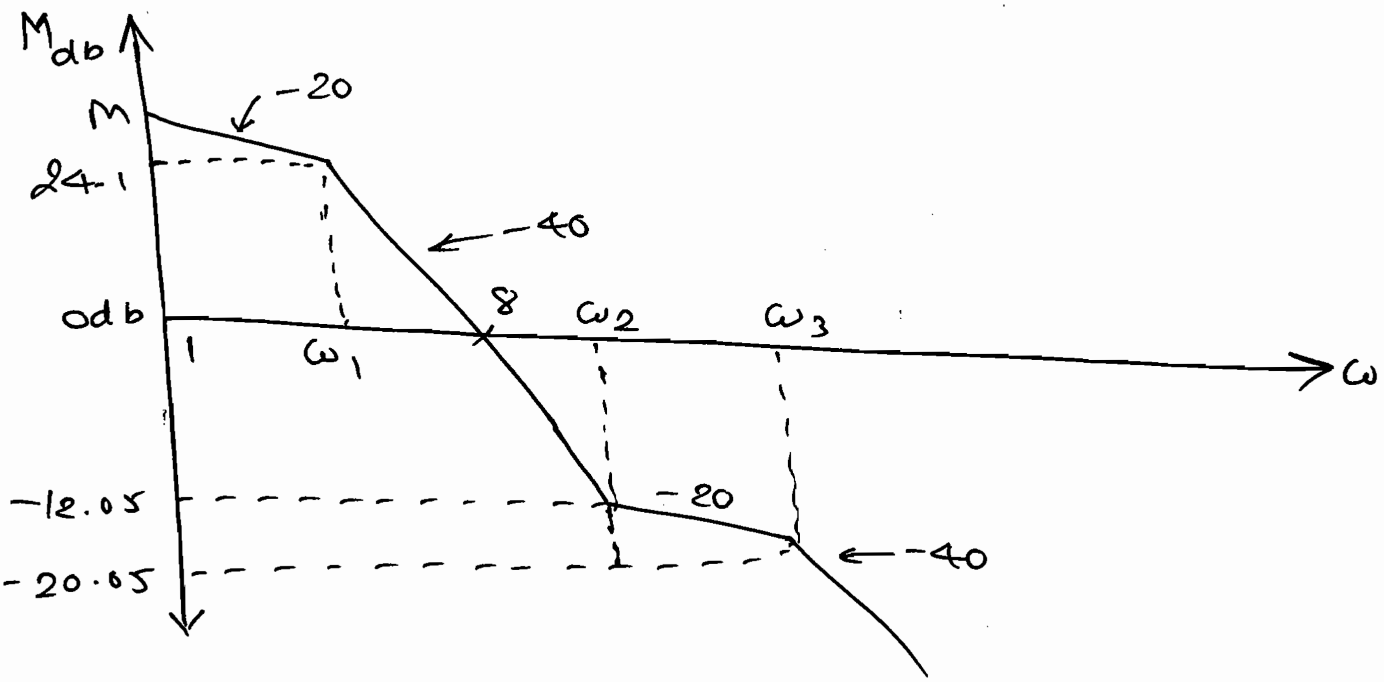
$$\Rightarrow \quad 32 = 20\log k - 20\log(0.1).$$

$$\therefore \quad \frac{32-20}{20} = \log_{10} K$$

$$\Rightarrow \quad \boxed{K = 3.98 \simeq 4}$$

So, TF $\boxed{GH(s) = \dfrac{4(1+S/2)}{S(1+S/5)(1+S/10)}}$.

* ⓐ Find the Magnitude M, ω_1, ω_2 and ω_3.



Soln:

$$\text{Slope} = \frac{dM}{d\log_{10}\omega}.$$

$$\therefore \quad +40 = \frac{+12.05 - 0}{\log\omega_2 - \log 8}.$$

$$\therefore \quad +40\log\omega_2 - 40\log 8 = 12.05.$$

$$40\log\omega_2 = 48.17$$

$$\Rightarrow \quad \omega_2 = 15.948$$

$$\boxed{\omega_2 \simeq 16 \text{ rad/sec}}$$

$$\Rightarrow \text{Slope} = \frac{dM}{d\log\omega}$$

$$\Rightarrow -40 = \frac{24.1 - 0}{\log\omega_1 - \log 8}$$

$$\therefore -40\log\omega_1 + 40\log 8 = 24.1$$

$$40\log\omega_1 = 12.12$$

$$\Rightarrow \boxed{\omega_1 = 2 \text{ rad/sec}}$$

$$\Rightarrow \text{Slope} = \frac{dM}{d\log\omega}$$

$$\Rightarrow -20 = \frac{M - 24.1}{\log\cancel{\omega}\,1 - \log\omega_1}$$

$$2\cancel{0} + 20\log\omega_1 = M - 24.1$$

$$2\cancel{0} + 20\log 2 = M - 24.1$$

$$\Rightarrow \cancel{M}/2 \quad \boxed{M = 30.1 \text{ dB}}$$

$$\Rightarrow \text{Slope} = \frac{dM}{d\log\omega}$$

$$\therefore -20 = \frac{-12.05 + 20.05}{\log\omega_2 - \log\omega_3}$$

$$\therefore -20\log 6 + 20\log\omega_3 = 8$$

$$20\log\omega_3 = 34.02/1.$$

$$\Rightarrow \boxed{\omega_3 \simeq 40 \text{ rad/sec}}$$

Now,

TF $$GH(s) = \frac{K\,(1+s/16)}{s\left(1+\frac{s}{2}\right)\left(1+s/40\right)}.$$

$$M_{db}\Big|_{\omega=1} = 30.1\ db.$$

$$\therefore\ 30.1\ db = 20\log k - 20\log(1).$$

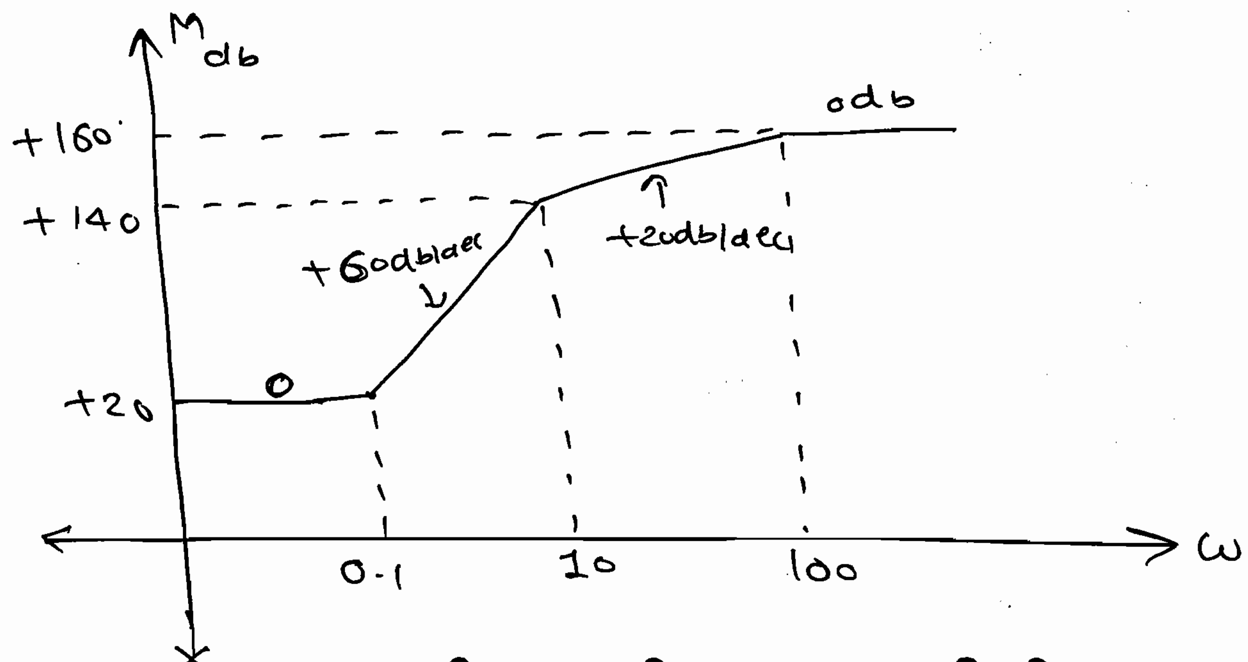
$$\Rightarrow\ 20\log K = 30.1.$$

$$\log K = \frac{30.1}{20}$$

$$\Rightarrow\ \boxed{K \simeq 32}$$

$$\Rightarrow\ \boxed{GH(s) = \frac{32\left(1+\frac{s}{16}\right)}{s(1+s/2)(1+s/40)}}$$

Q The Asymptotic Aproximation of the logM Vs freq plot of a minimum phase system is shown in figure its TF is — ?



Soln:

Slope-1 $\Rightarrow$ 0 db/dec.

Slope-2 $\Rightarrow$ $\dfrac{140 - 20}{-\log 0.1 + \log 10} = \dfrac{120}{+1+1} = +60$ db/dec

Slope-3 $\Rightarrow$ $\dfrac{dM}{d\log\omega} = \dfrac{160-140}{\log 100 - \log 10} = \dfrac{20}{2-1}$

$= 20$ db/dec.

So, TF $G_H(s) = \dfrac{K\left(1+\frac{s}{0.1}\right)^3}{\left(1+s/10\right)^2\left(1+s/100\right)^1}$.

$$M_{indB}\Big|_{\omega=0.1} = 20.$$

$\therefore\ 20 = 20\log K + 60\log\sqrt{1+\left(\frac{0.1}{0.1}\right)^2}$.

$\therefore\ 20 = 20\log K + 60 \times \log\sqrt{2}$.

~~20 - 9.03 = 20 log K~~

$26 = 20\log K$

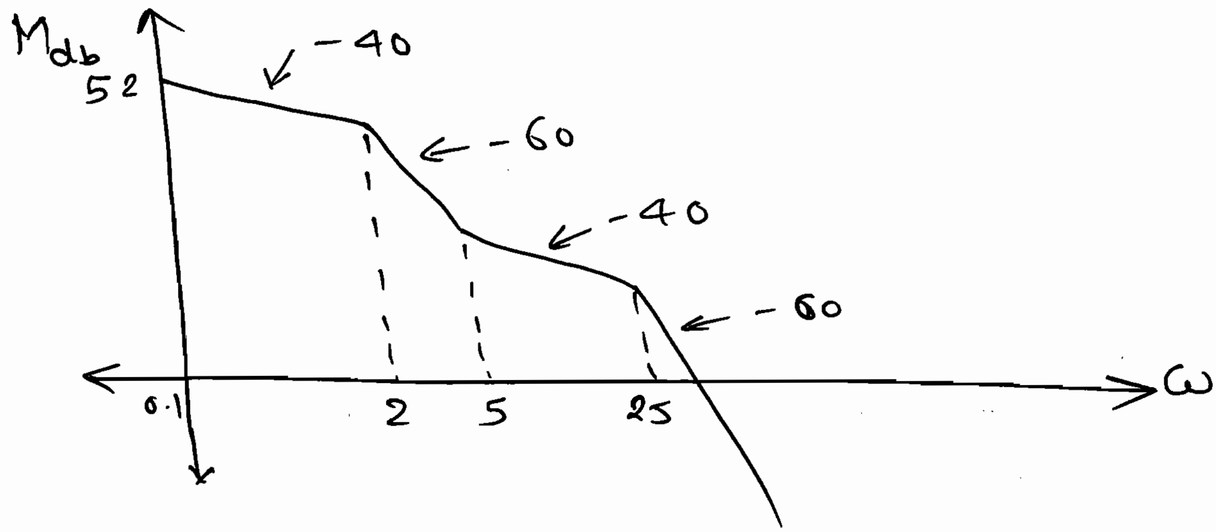
$\therefore \Rightarrow \boxed{K=10}$

$\therefore\ G_H(s) = \dfrac{10\left(1+s/0.1\right)^3}{\left(1+\frac{s}{10}\right)^2\left(1+s/100\right)^1}$.

$\therefore\ G_H(s) = \dfrac{10\times\left(\frac{1}{0.1}\right)^3\left(s+0.1\right)^3\times 10^2\times 100}{(s+10)(s+100)}$.

$$\Rightarrow \boxed{GH(s) = \frac{10^8 (s+0.1)^3}{(s+10)^2 (s+100)}}$$

Q



Soln:

$$TF = GH(s) = \frac{k\,(1+s/5)^1}{s^2 (1+s/2)^1 (1+s/25)^1}$$

Now, $M_{db}\big|_{\omega=0.1} = 52\ db$

$$\Rightarrow 52 = 20\log k - 40\log \omega.$$

$$\therefore 52 = 20\log k - 40\log(0.1).$$

$$\frac{\cancel{102}\ 12}{20} = \log k.$$

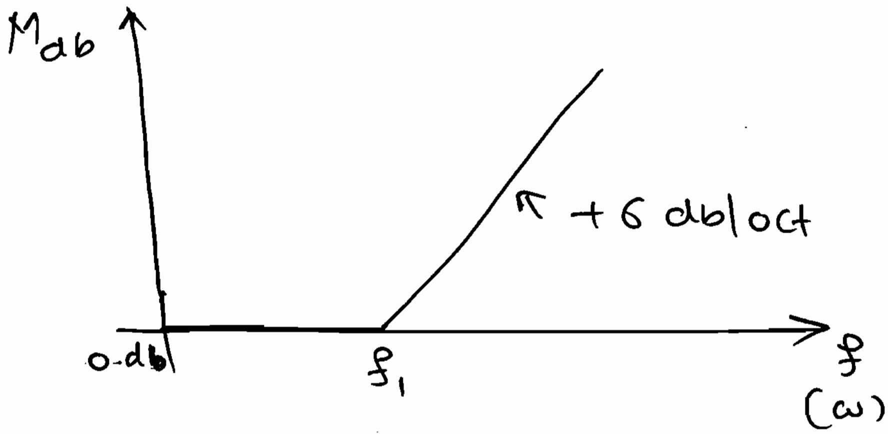
$$\therefore \Rightarrow \boxed{k \simeq 4.}$$

$$\Rightarrow TF = \frac{4\,(1+s/5)}{s^2 (1+s/2)(1+s/25)}.$$

$$= \frac{4}{\cancel{8}} \frac{\times 2 \times \cancel{25}^{5} (s+5)}{s^2 (s+2)(s+25)}.$$

$$\Rightarrow \boxed{TF = \frac{40(s+5)}{s^2(s+2)(s+25)}}.$$

Q Find the TF



Sol^n:

$$GH(s) = K\left(1+\frac{s}{r_1}\right).$$

$$GH(j\omega) = K\left(1+\frac{j\cancel{2\pi} f}{\cancel{2\pi} f_1}\right)$$

$$GH(f) = K\left(1+j\frac{f}{f_1}\right).$$

Here, $M_{in\,dB}\big|_{\omega=0.1} = 0\ db$

$$\Rightarrow K = 1.$$

$$\therefore \boxed{GH(f) = \left(1+j\frac{f}{f_1}\right)}.$$

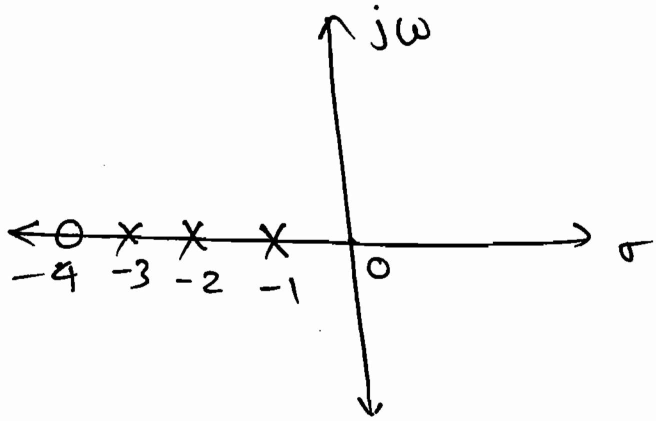
* Classification of System :-

1) Minimum phase system :-

⇒ A system in which all the finite poles and finite zeros lies in the left of s-plane then it is called minimum phase system.

⇒ The minimum phase ~~sm~~ system gives the angle $\leq 90^\circ$.

e.g :



$$MPS = \frac{(S+4)}{(S+2)(S+3)(S+1)}.$$

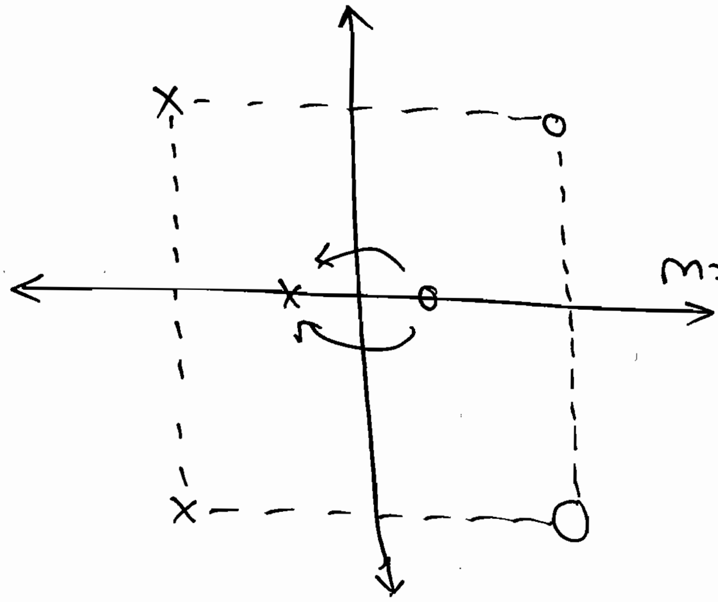
2) All Pass System:-

⇒ A system in which all poles are lies in the left of s-plane and all the zeros are lies in Right of the s-plane which are symmetrical about imaginary axis then it is called All pass system.

⇒ All pass system gives Magnitude of

'1', and the Phase angle varies b/w $\pm 180°$.

=)



e.g.

$$\boxed{ALPS = \frac{S-1}{S+1}}$$

$M = 1$, $\phi = \angle \pm 180°$.

3) **Non- Minimum phase System** :-

=> A system in which one (or) more zero (or) one (or) more pole (or) Both poles and zeros lies in the Right of the s-plane then it is called Non - minimum phase system.

=> The NMP gives the more -ve angle at $\omega = \infty$.

=> The NMP system may be Unstable.

e.g. $NMPS = \frac{(S+1)(S-2)}{(S+3)(S+5)}$.

$$NMPS = \left[\frac{(S+1)(S+2)}{(S+3)(S+5)}\right] \times \left(\frac{S-2}{S+2}\right)$$

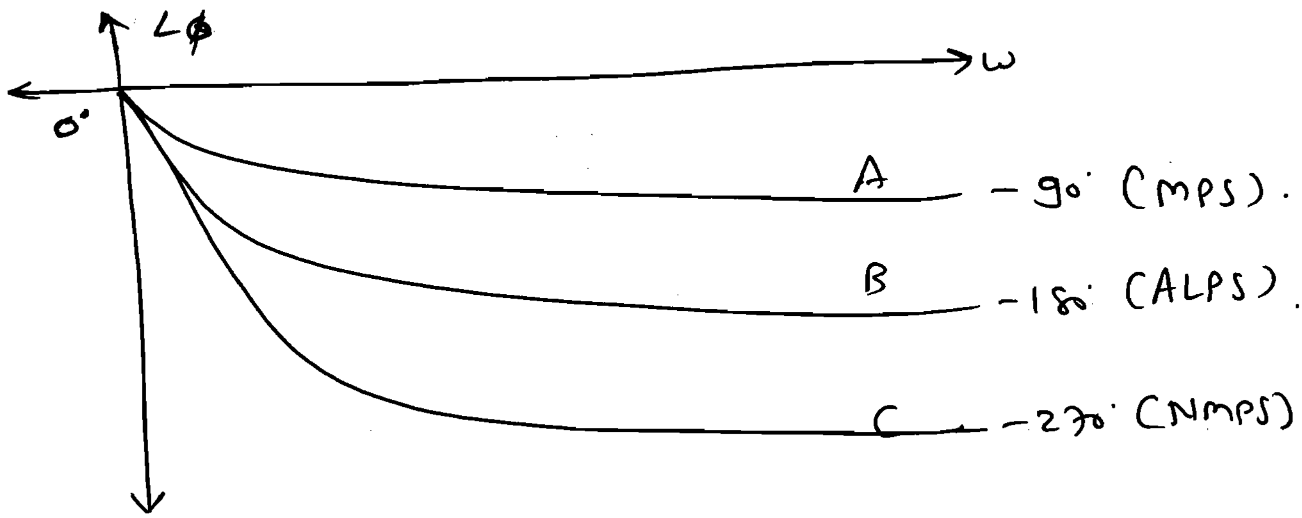
$$\underbrace{\left[\frac{(S+1)(S+2)}{(S+3)(S+5)}\right]}_{MPS} \qquad \underbrace{\left(\frac{S-2}{S+2}\right)}_{APS}$$

$\Rightarrow$ $\boxed{NMPS = MPS \times APS.}$

$\Rightarrow$ Nmps is nothing but the Product of MPS and ALPS.

$\Rightarrow$ $\phi_{Nmps} = \phi_{MPS} + \phi_{ALPS}.$

$\boxed{a}$ Identify the curves A, B, C in the given phase plot.



* Stability Condition :-

$\rightarrow$ The Stability conditions are to find the CL sys stability.

$\rightarrow$ The CL system Stability is given by char. eq^n i.e. $1 + G(s) \cdot H(s) = 0.$

then s is replaced by $j\omega$.

$\rightarrow \quad 1 + G(j\omega) \cdot H(j\omega) = 0.$

$G(j\omega) \cdot H(j\omega) = -1 + j0.$

Mg $\longrightarrow$ $|G(j\omega) \cdot H(j\omega)| = 1$ in linear

$M_{in\,dB} = 20 \log 1 = 0\,dB.$

$\boxed{M_{in\,dB} = 0\,dB}$

⇒ Gain Crossover frequency: (ω_{gc}):

⇒ The freq. at which the magnitude equal to '1' in linear and '0' in db is called Gain cross over freq.

⇒ Write the phase angle.

$$\boxed{\angle GH(j\omega) = \phi = \angle -1 + j0 = -180^\circ.}$$

=> Phase Cross over frequency:: (ω_{pc}):

⇒ The freq. at which the ~~magnitude~~ phase angle equal to -180°, is called phase cross over frequency. (ω_{pc}).

⇒ <u>Gain</u> <u>Margin</u>:-

⇒ Inverse of Magnitude at ω_{pc} gives the Gain Margin.

⇒ The Gain margin is the factor by which the system gain is increased to bring the system verge of the stability.

$$\therefore \quad \boxed{GM = \frac{1}{|GH(j\omega)|_{\omega=\omega_{pc}}}}$$

$$\Rightarrow \quad \boxed{GM_{db} = -20 \log |GH(j\omega)|_{\omega=\omega_{pc}}}$$

⇒ <u>Phase</u> - <u>Margin</u>:

⇒ It is a additional phase lag required to add to the system to bring the system at verge of the Stability.

$$\boxed{PM = 180^\circ + \angle GH(j\omega)\Big|_{\omega=\omega_{gc}}}$$

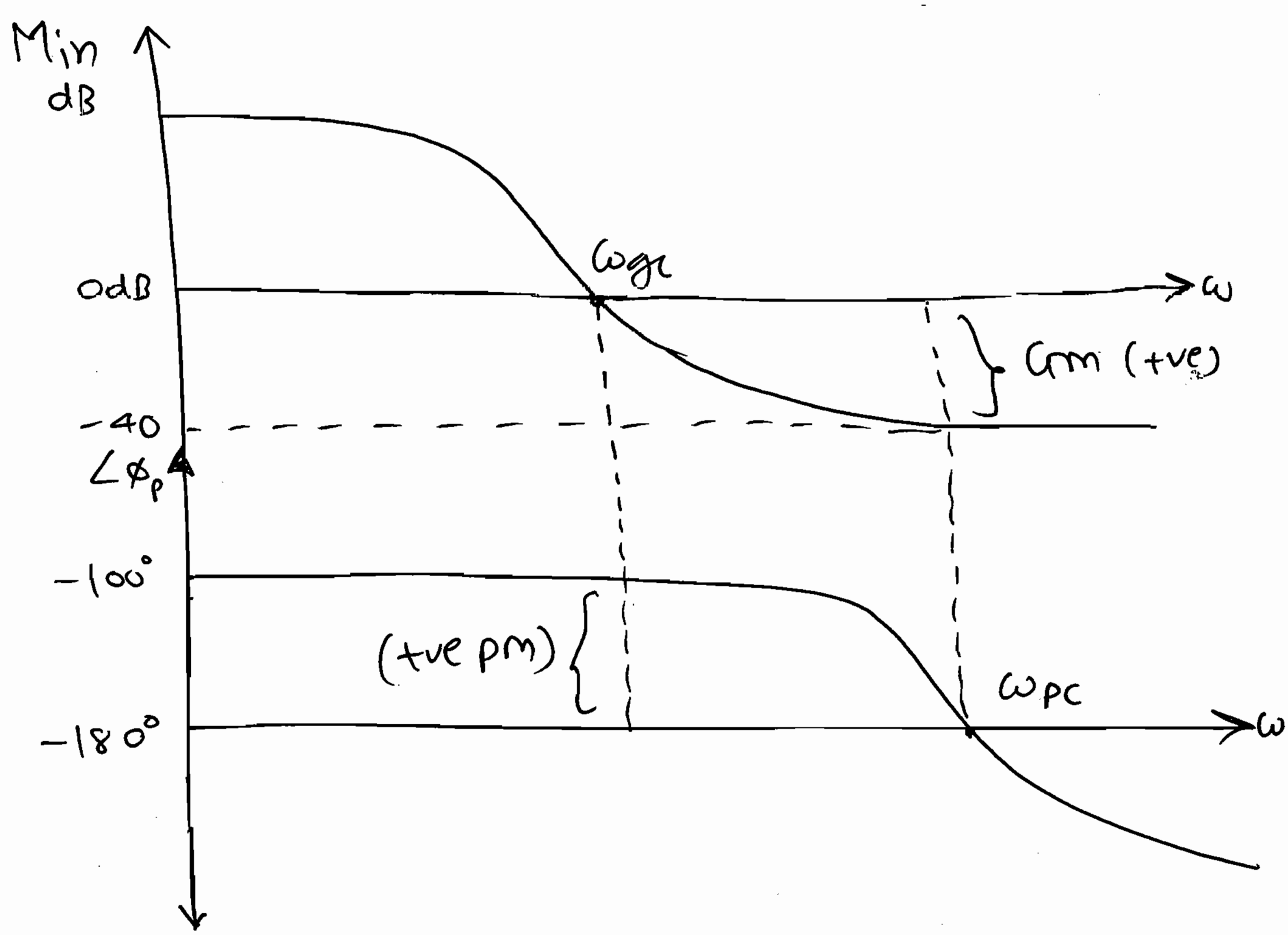
*

⇒ $\omega_{pc} > \omega_{gc} \longrightarrow$ S Gm $\begin{cases} \to +ve \text{ in dB} \\ \to >1 \text{ in } L \end{cases}$ & {pm +ve}

⇒ $\omega_{pc} = \omega_{gc} \longrightarrow$ MS Gm $\begin{cases} \to 0 \text{ db} \\ \to 1 \ L \end{cases}$ & {pm = 0°}

⇒ $\omega_{pc} < \omega_{gc} \longrightarrow$ US Gm $\begin{cases} \to -ve \text{ in dB} \\ \to <1 \text{ in } L \end{cases}$ & {pm -ve}

$\boxed{Q}$ Identify the Stability to the given Bode plot:



⇒ $\omega_{pc} > \omega_{gc}$ ⇒ S

$\Rightarrow \quad GM = -(M_{in\,dB})\Big|_{\omega=\omega_{pc}}$

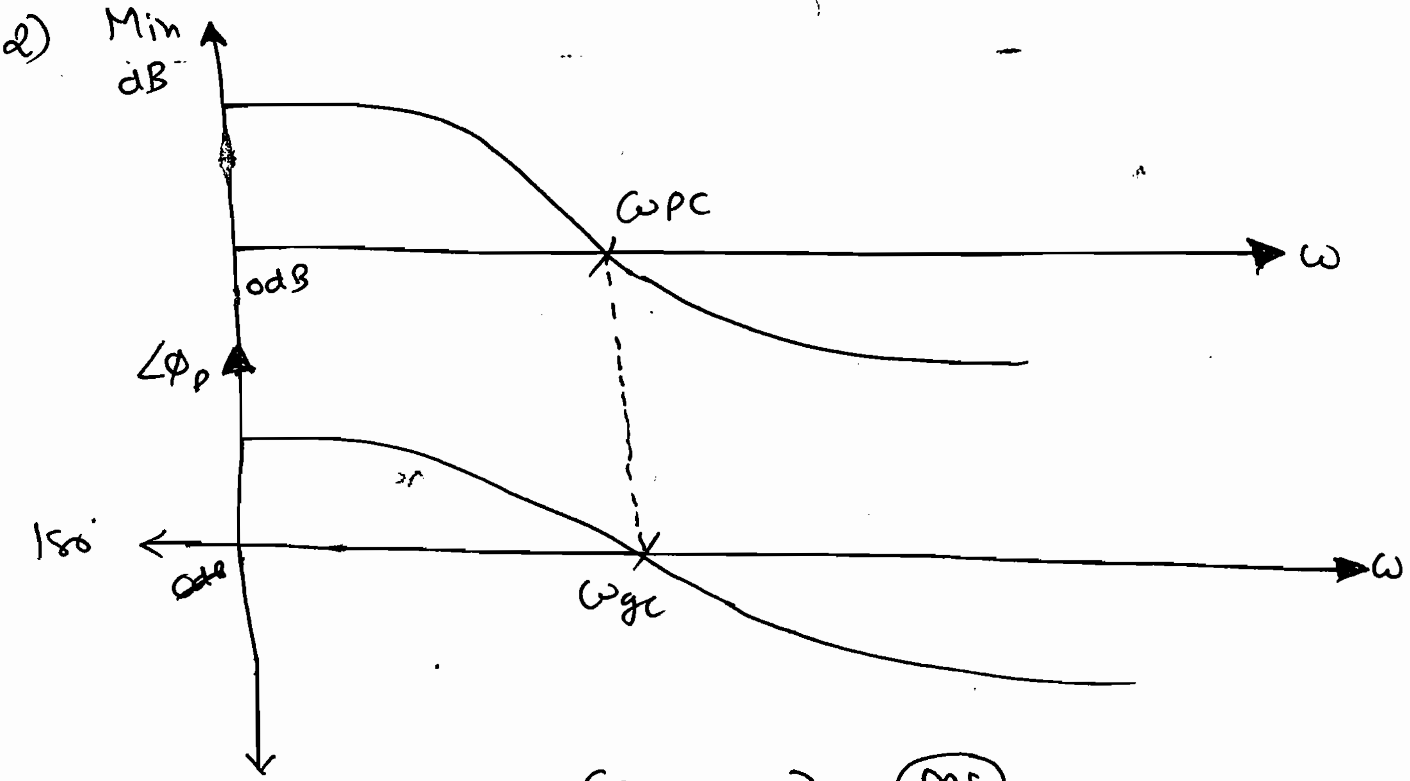
$= -(-40\,db).$

$= +40\,db$

$\Rightarrow$ Phase margin $pm = 180^\circ + \angle GH\Big|_{\omega=\omega_{gc}}$

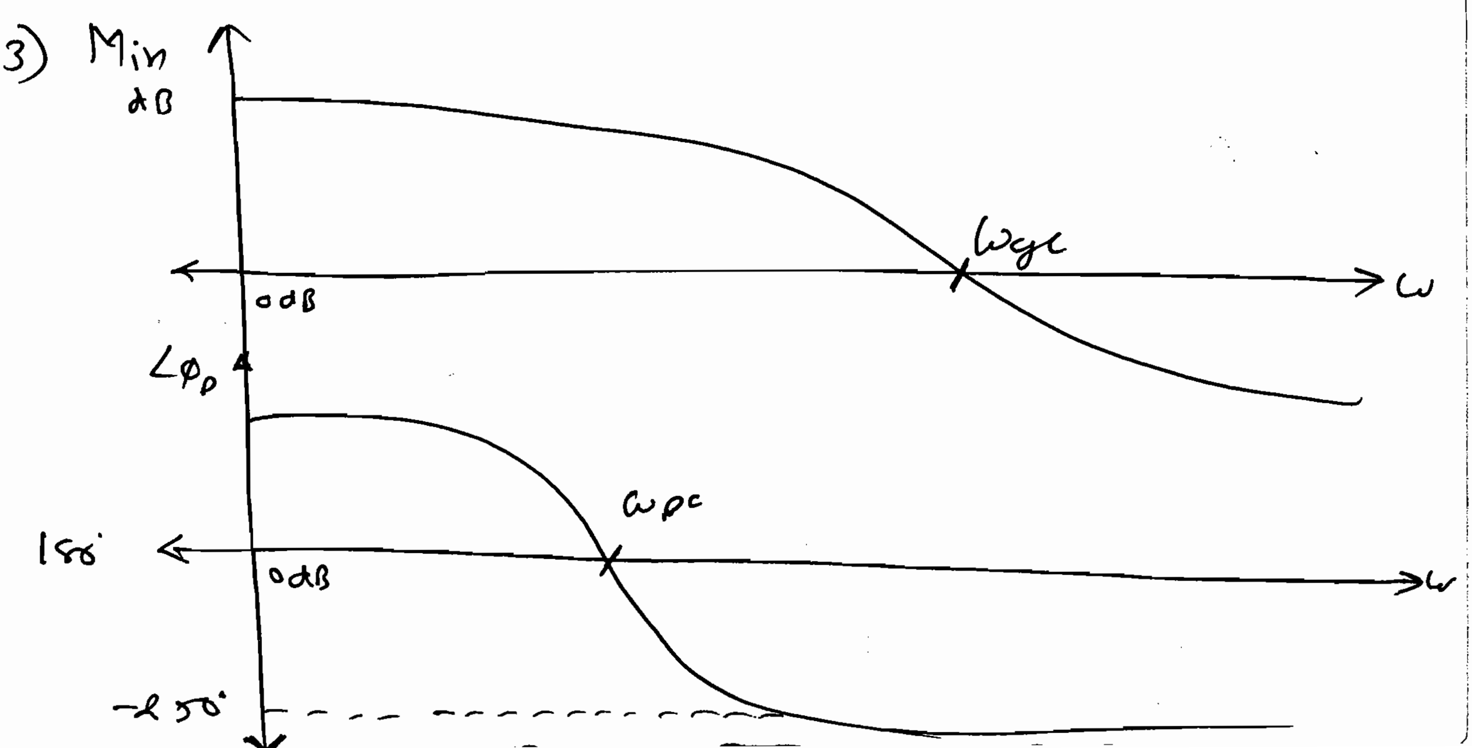
$\therefore PM = 180^\circ - 100 = +80^\circ > 0 \Rightarrow +ve.$

2)



$\omega_{pc} = \omega_{gc} \Rightarrow$ (MS).

3)



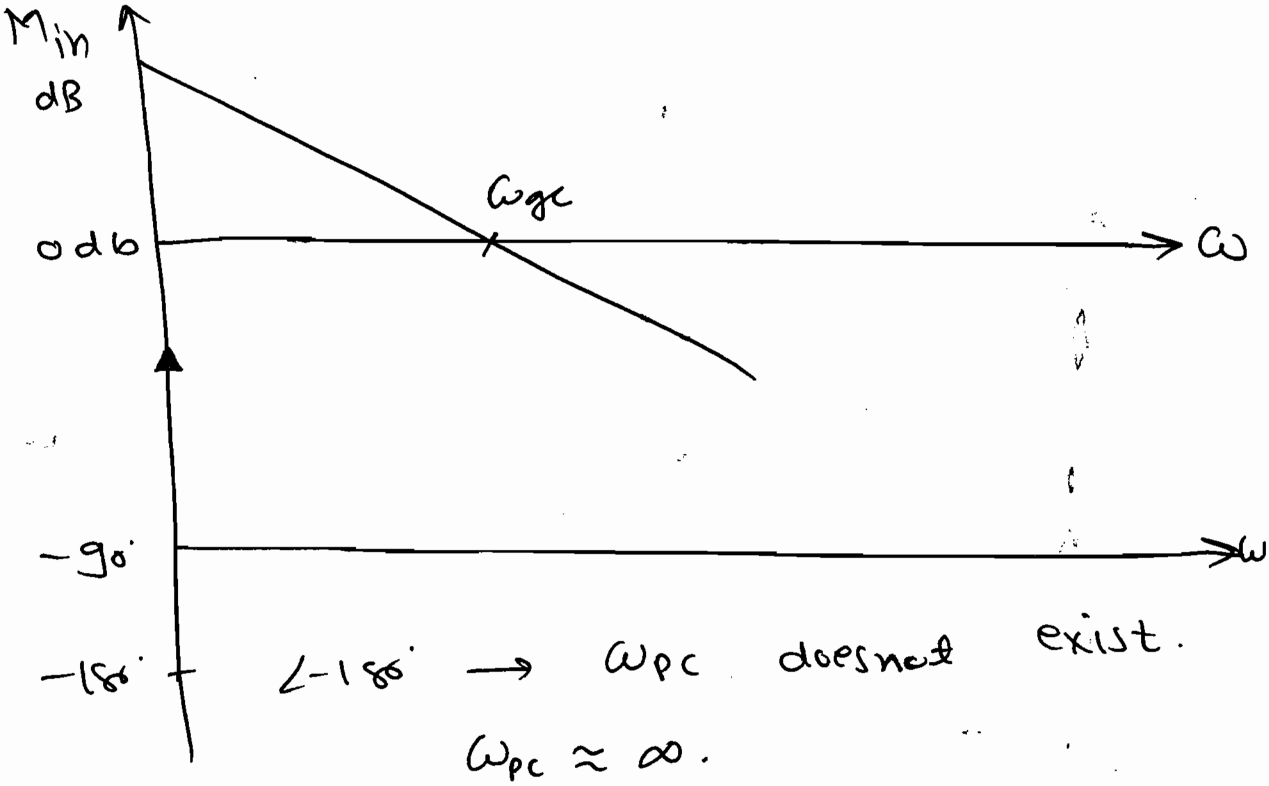
$\Rightarrow \quad \omega_{pc} < \omega_{gc} \longrightarrow$ US

$GM = -50\ (dB)$

$PM = 180^\circ - 250^\circ$

$PM = -70^\circ \quad \Rightarrow$ US.

4)



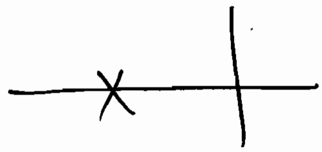
$\angle -180^\circ \longrightarrow \omega_{pc}$ doesnot exist.

$\omega_{pc} \approx \infty$.

$\omega_{pc} \gg \omega_{gc} \Rightarrow$ S.

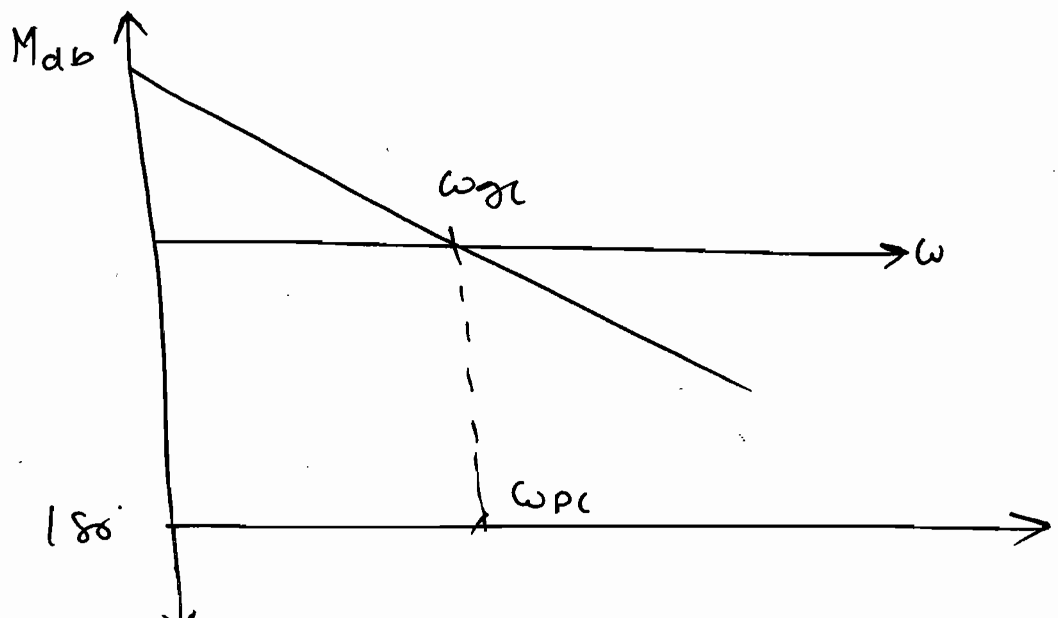
$G(s) = 1/s, \quad H(s) = 1.$

$CLTF = \dfrac{1}{s+1}.$



CL $\longrightarrow$ S.

5)

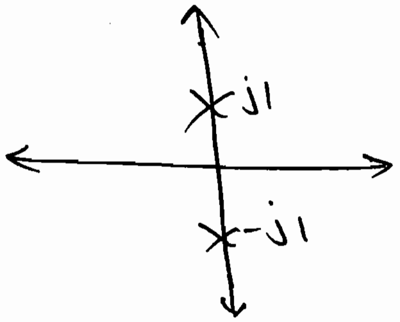


$\Rightarrow \omega_{pc} = 0$ to ∞

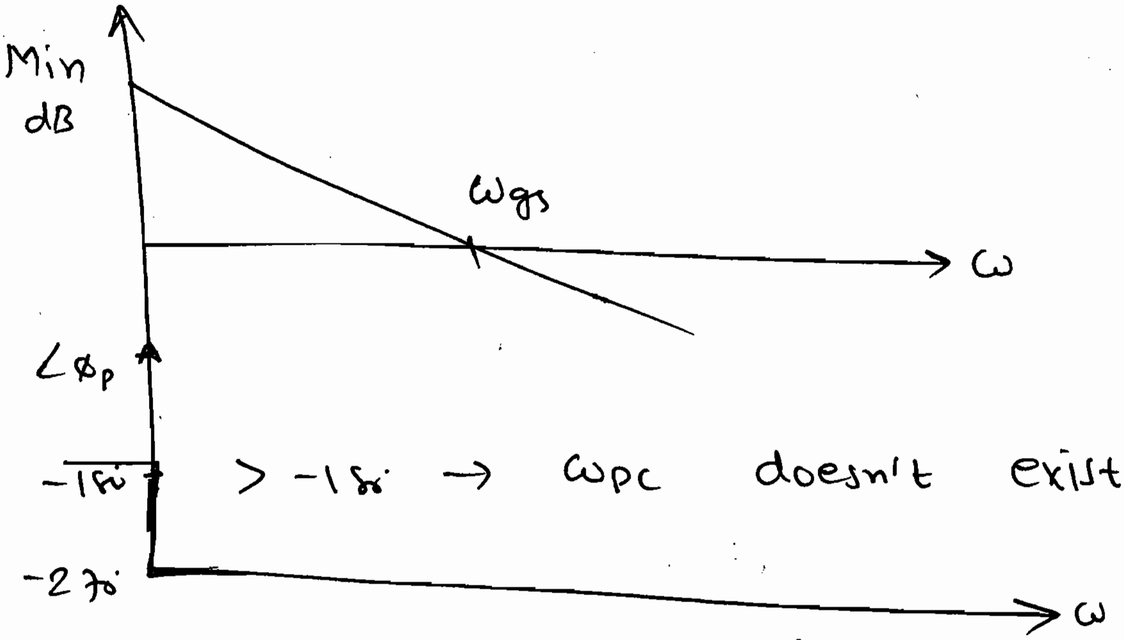
$\omega_{pc} = \omega_{gc} \Rightarrow$ MS.

e.g. $GH(s) = 1/s^2 \Rightarrow CLTF = \frac{1}{s^2+1}$.

CL $\Rightarrow$ MS



5)



$> -180° \rightarrow \omega_{pc}$ doesn't exist

$\omega_{pc} << \omega_{gc}$ $\omega_{pc} \approx 0$.

$\longrightarrow$ US.

e.g.

$\rightarrow G(s) = \frac{1}{s^3}$, $H(s) = 1$.

$CLTF = \frac{1}{s^3+1}$ terms missing $\Rightarrow$ US.

Note: whenever the plot ~~are~~ (or) TF maintains less (-ve) than 180° at all the freq. range then the system is stable because here

$\omega_{pc} >> \omega_{gc}$. (Actually ω_{pc} does not exist but approximatelly infinity.)

⇒ Whenever the Plot (or) TF maintains exactly equal to $-180°$ at all the Freq. range then the system is Marginal Stable because here $\omega_{pc} = \omega_{gc}$.

⇒ Whenever the plot (or) TF maintains more (-ve) than 180° at all the freq. range then the system is unstable because here $\omega_{pc} < \omega_{gc}$. (Actually ω_{pc} does not exist but Approximatelly 0).

* Complex Bode Plot:

① n - Poles (complex):

$$\Rightarrow \quad G(s) \cdot H(s) = \left(\frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \right)^n .$$

$s \to j\omega$

$$GH(j\omega) = \left(\frac{\omega_n^2}{(j\omega)^2 + 2\xi\omega_n j\omega + \omega_n^2} \right)^n .$$

$$GH(j\omega) = \left(\frac{\omega_n^2}{-\omega^2 + j2\xi\omega\omega_n + \omega_n^2} \right)^n$$

$$GH(j\omega) = \left(\frac{1}{1 - (\omega/\omega_n)^2 + j2\xi(\omega/\omega_n)} \right)^n$$

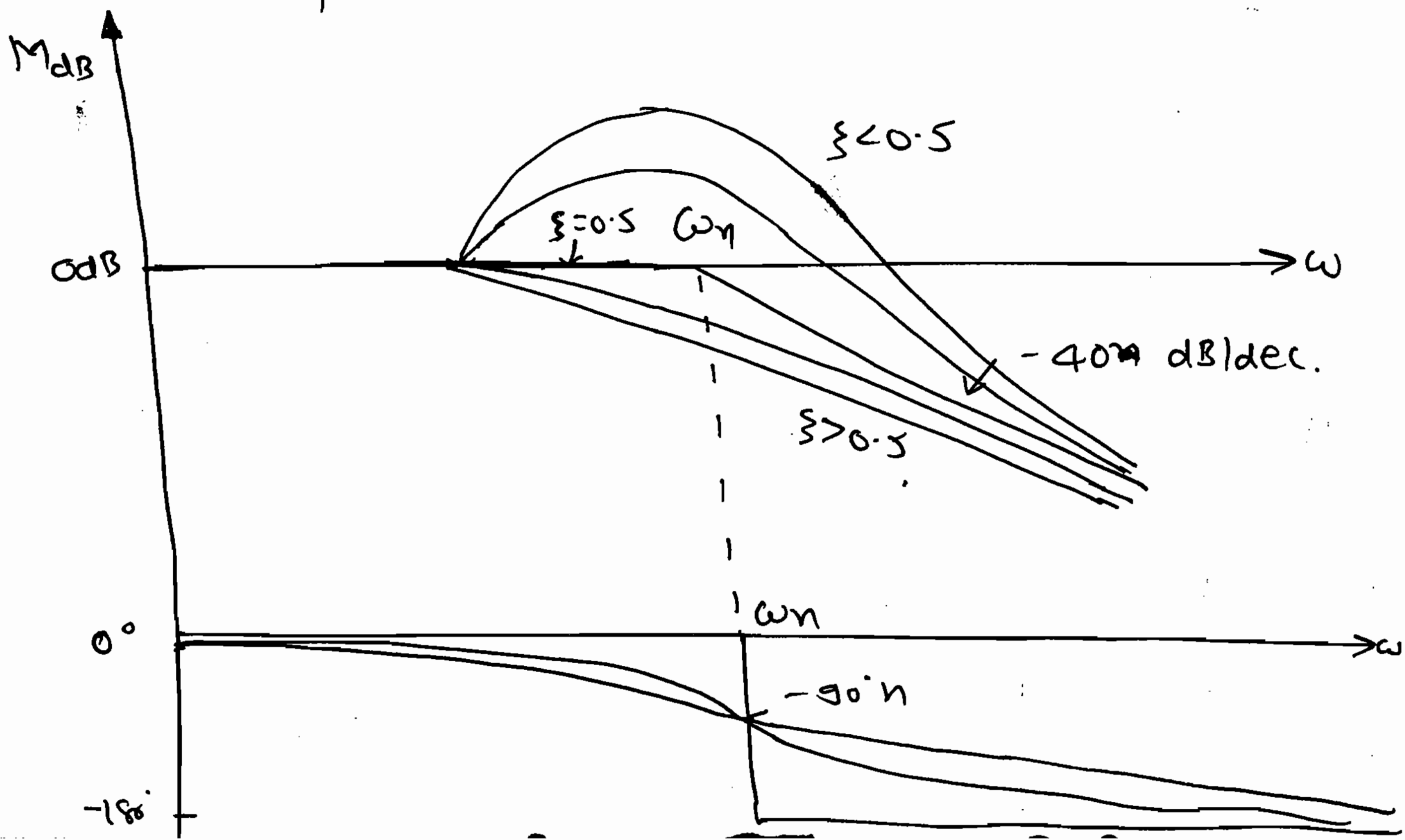
$$\therefore M = |GH(j\omega)| = \left(\frac{1}{\sqrt{\{1-(\omega/\omega_n)^2\}^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}} \right)^n$$

$$\therefore M_{\text{in dB actual}} = -20^n \log \sqrt{\{1-(\omega/\omega_n)^2\}^2 + (2\xi\,\omega/\omega_n)^2}$$

$$\phi_{\text{actual}} = -n \tan^{-1}\left[\frac{2\xi\,\omega/\omega_n}{1-(\omega/\omega_n)^2}\right].$$

$\omega_n \longrightarrow$ Corner freq.

	S	ϕ
< CF	0 dB	0°
> CF	-40^n dB/dec	$-180°n$



⇒ Correction at corner freq.

$$M_{\substack{correction \\ (\omega=\omega_n)}} = -20n\log 2\xi.$$

$$\phi_{\substack{correction \\ (\omega=\omega_n)}} = -90^\circ n.$$

⇒ The correction at corner freq. depends on ξ in the Magnitude plot where in the phase plot the correction at corner freq. is other than ~~the~~ ~~correction~~ ~~depends~~ ~~on~~ ~~$\xi\omega_n$~~ is constant other than corner freq. the correction depends on $\xi\omega_n$.

② $\underline{\underline{n}}$- Complex Zeros:

$$\Rightarrow \quad G_H(s) = \left(\frac{s^2 + 2\xi\omega_n s + \omega_n^2}{\omega_n^2}\right)^n$$

$s \to j\omega$

$$G_H(j\omega) = \left(1 - (\omega/\omega_n)^2 + j\,2\xi(\omega/\omega_n)\right)^n.$$

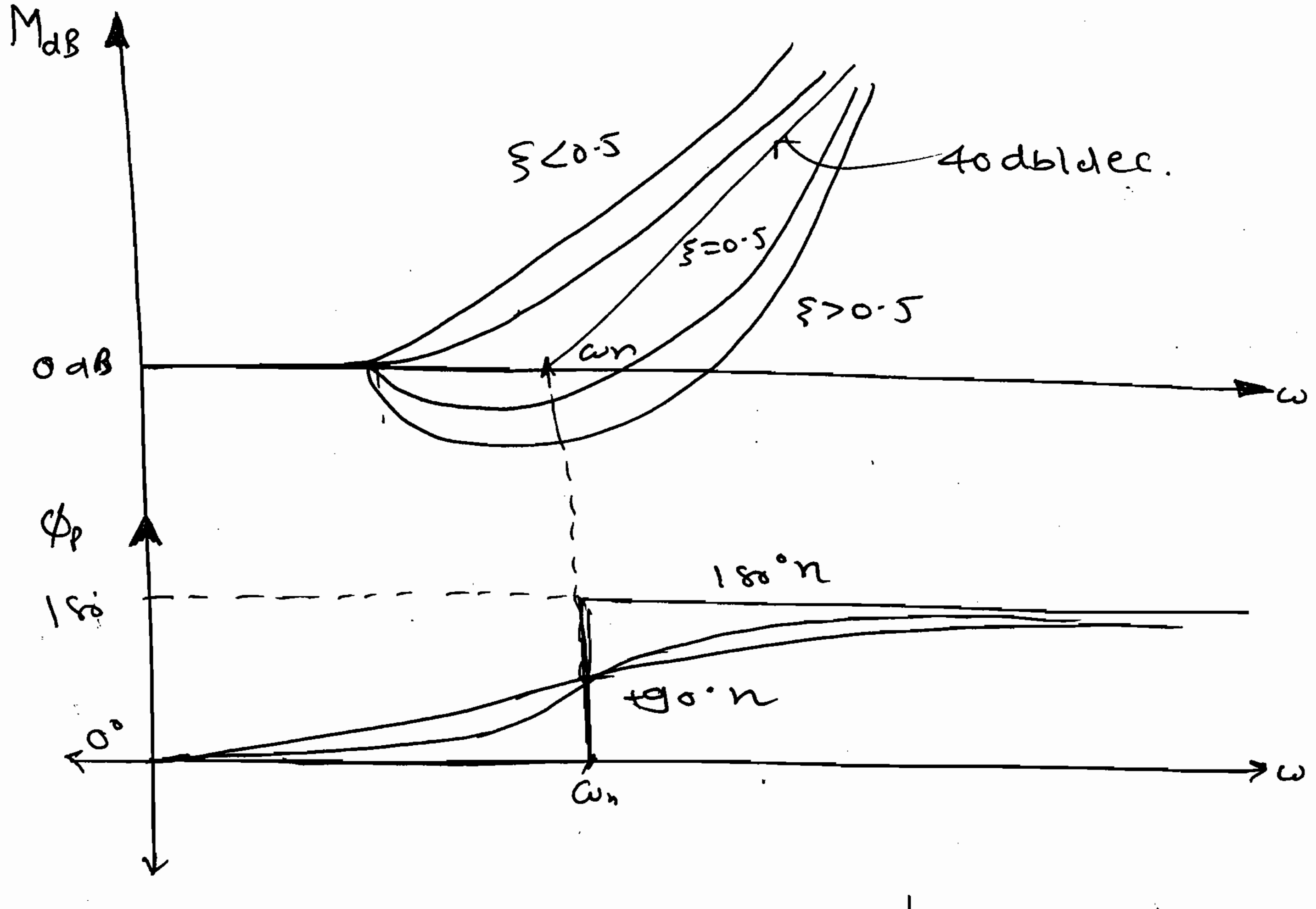
$$\therefore M = |G_H(j\omega)| = \left(\sqrt{\{1-(\omega/\omega_n)^2\}^2 + (2\xi\,\omega/\omega_n)^2}\right)^n$$

$$M_{dB} = 20n\log\sqrt{\{1-(\omega/\omega_n)^2\}^2 + (2\xi\omega/\omega_n)^2}$$

$$\Rightarrow \quad \phi_{actual} = n \tan^{-1}\left[\frac{2\xi\omega/\omega_n}{1-(\omega/\omega_n)^2}\right].$$

$\omega_n \longrightarrow$ corner freq.

	M	ϕ
< CF	0 dB/dec	0°
> CF	+ 40n	+ 180° n



$\Rightarrow$ <u>Correction at Corner freq.</u> :-

$M_{correction}$ at CF ($\omega=\omega_n$) $= +20n\log(2\xi)$.

ϕ correction at CF ($\omega=\omega_n$) $= +90° n$.

[Q] Draw the Bode plot to the given system.

$$GH(s) = \frac{s^2\left(1 + s/20 + \frac{s^2}{100}\right)^4}{\left(1 + s/3 + s^2/9\right)^3 \left(1 + s/50\right)^4}.$$

Sol^n:

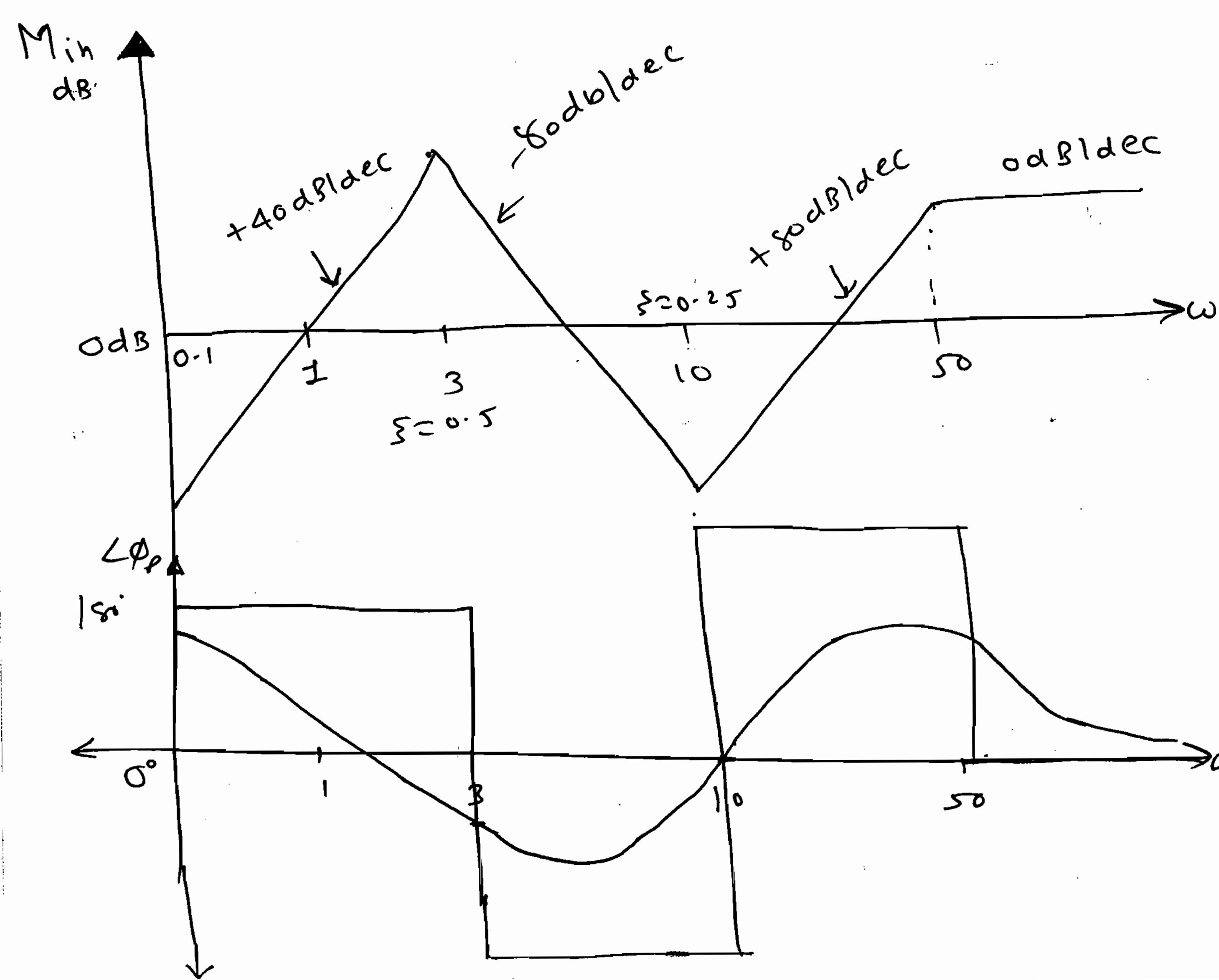
$\omega_{n_1}^2 = 9$ $\qquad$ $\omega_{n_2}^2 = 100$

$\omega_{n_1} = 3$ rad/sec. $\qquad$ $\omega_{n_2} = 10$ rad/sec.

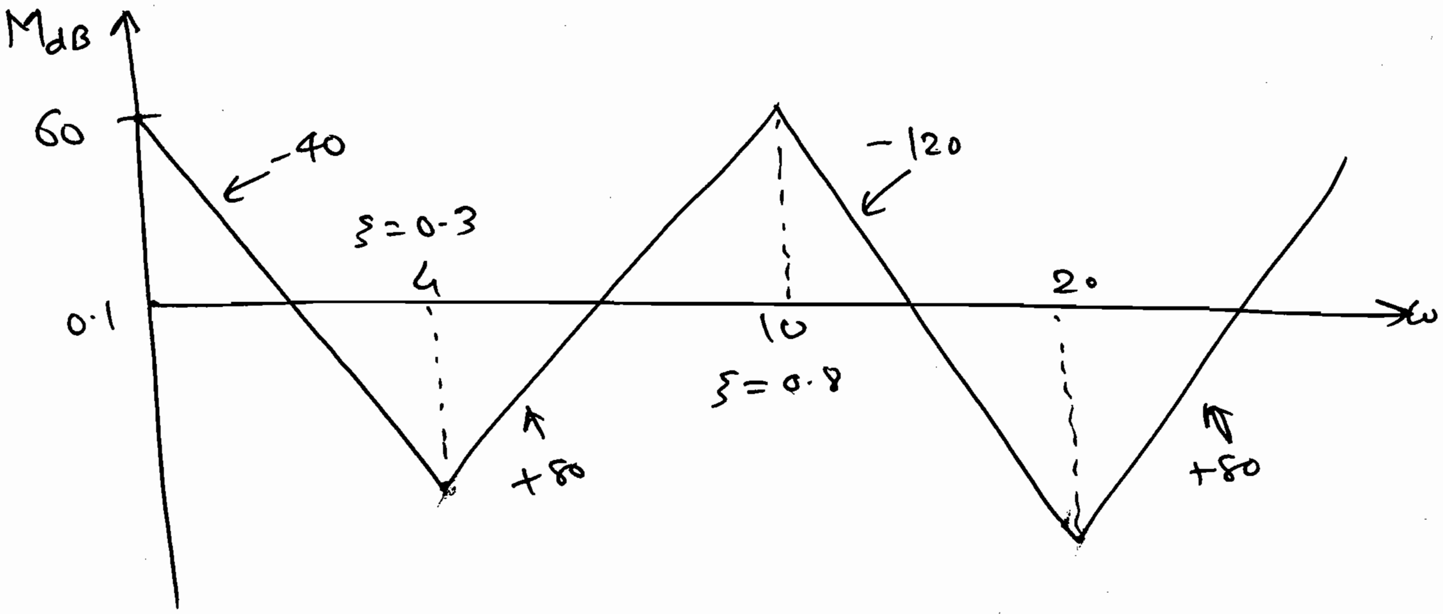
$2\xi_1 \frac{\cancel{\omega}}{\omega_n} = \cancel{\omega}/3$ $\qquad$ $\frac{2\xi_2}{\omega_n} = \frac{1}{20}$

$\frac{2\xi_1}{3} = 1/3$ $\qquad$ $\therefore \boxed{\xi_2 = 0.25}$

$\boxed{\xi_1 = 0.5}$ $\qquad$ $\boxed{\omega_{n_3} = 50 \text{ rad/sec}}$



Q Find the TF to the given asymptotic Mag. plot.



$$GH(s) = K \frac{\left(1 + \frac{2(0.3)s}{4} + \frac{s^2}{16}\right)^3 \left(1 + s/20\right)^{10}}{s^2\left(\frac{s^2}{100} + \frac{2(0.8)s}{10} + 1\right)^5}$$

$$M_{dB}\Big|_{\omega = 0.1} = 60 \text{ dB.}$$

$$\Rightarrow \quad 60 \text{ dB} = 20 \log K - 40 \log (0.1).$$

$$20 = 20 \log K$$

$$\log = 1$$

$$\Rightarrow \quad \boxed{K = 10}$$

$$\Rightarrow$$

$$\boxed{TF = GH(s) = \frac{10\left(\frac{s^2}{16} + \frac{0.6}{4}s + 1\right)^3 \left(1 + s/20\right)^{10}}{s^2\left(\frac{s^2}{100} + 1.6s + 1\right)^5}}$$