
CBSE Sample Paper-05
SUMMATIVE ASSESSMENT -I
Class-IX MATHEMATICS

Time allowed: 3 hours

Maximum Marks: 90

General Instructions:

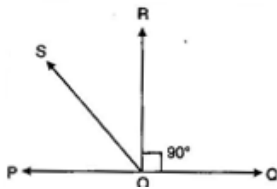
- a) All questions are compulsory.
 - b) The question paper consists of 31 questions divided into four sections – A, B, C and D.
 - c) Section A contains 4 questions of 1 mark each. Section B contains 6 questions of 2 marks each, Section C contains 10 questions of 3 marks each and Section D contains 11 questions of 4 marks each.
 - d) Use of calculator is not permitted.
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Section A

1. Evaluate: $(25)^{\frac{1}{3}} \times (5)^{\frac{1}{3}}$.
2. Find the zero of the polynomial $p(x) = 2x + 3$.
3. The distance of the point $(-6, -2)$ from y-axis is
4. Two angles of triangles are 65° and 45° respectively. Find third angles.

Section B

5. Show that $\sqrt{7} - 3$ is irrational.
6. If $x = 2k$ is a factor of $f(x) = x^5 - 4k^2x^3 + 2x + 2k + 3$, find k.
7. Find the remainder when $2x^4 + 6x^3 + 2x^2 - x + 2$ is divided by $(x + 2)$.
8. In figure, POQ is a line. Ray OR is perpendicular to line PQ. OS is another ray lying between rays OP and OR. Prove that: $\angle ROS = \frac{1}{2} (\angle QOS - \angle POS)$



9. In a $\triangle ABC$, $30A + 6B = 5C$. Determine $\angle A$, $\angle B$ and $\angle C$.
10. Draw a triangle ABC where vertices A, B and C are $(0, 2)$, $(2, -2)$ and $(-2, 2)$ respectively.

Section C

11. Express $2.4\overline{178}$ in the form $\frac{p}{q}$
 12. Classify the following numbers as rational or irrational: (i) $2 - \sqrt{5}$ (ii) $(3 + \sqrt{23}) - \sqrt{23}$
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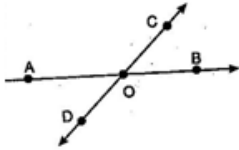
13. Factorise $\left(9x - \frac{1}{5}\right)^2 + \left(x + \frac{1}{3}\right)^2$

14. Without actual division, prove that $2x^4 - 6x^3 + 3x^2 + 3x - 2$ is exactly divisible by $x^2 - 3x + 2$.

15. If the polynomials $px^3 + 4x^2 + 3x - 4$ and $x^3 - 4x + p$ are divided by $x - 3$, then the remainder in each case is the same. Find the value of p .

16. If a point C lies between two points A and B such that $AC = BC$, then point C is called the mid-point of line segment AB. Prove that every line segment has one and only one mid-point.

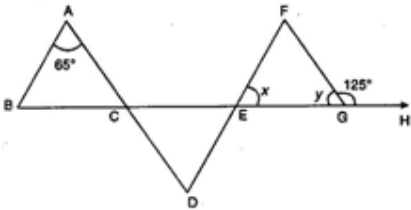
17. In the figure, if $\angle AOC + \angle BOD = 266^\circ$, then find all the four angles.



18. If a line is perpendicular to one of the two given parallel lines then prove that it is also perpendicular to the other line.

19. In a triangle ABC, $\angle A + \angle B = 84^\circ$ and $\angle B + \angle C = 146^\circ$. Find the measure of each of the angles of the triangle.

20. In the given figure, find x and y , if $AB \parallel DF$ and $AD \parallel FG$.



Section D

21. If $a = \frac{\sqrt{2}+1}{\sqrt{2}-1}$ and $b = \frac{\sqrt{2}-1}{\sqrt{2}+1}$, then find the value of $a^2 + b^2 - 4ab$.

22. If $a = 2 + \sqrt{3}$, find the value of:

(i) $a^2 + \frac{1}{a^2}$ (ii) $a^3 + \frac{1}{a^3}$

23. Using factor theorem, factorise the polynomial $x^3 + x^2 - 4x - 4$.

24. Factorise: $x^3 + 13x^2 + 32x + 20$.

25. Prove that: $(x+y)^3 + (y+z)^3 + (z+x)^3 - 3(x+y)(y+z)(z+x) = 2(x^3 + y^3 + z^3 - 3xyz)$

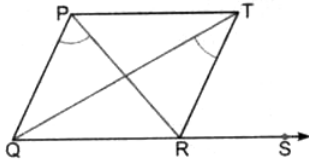
26. If $\left(x - \frac{1}{3}\right)$ and $(x - 3)$ are factors of $ax^2 + 5x + b$, prove that $a = b$.

27. If D is the midpoint of the hypotenuse AC of a right angled $\triangle ABC$, prove that $BD = \frac{1}{2} AC$.

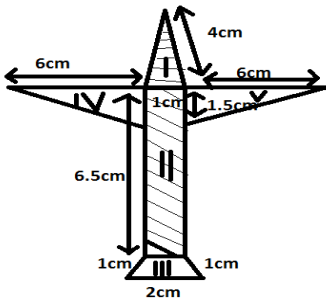
28. In a triangle, prove that the greater angle has the longer side opposite to it.

29. If the arms of one angle are respectively parallel to the arms of another angle, show that the two angles are either equal or supplementary.

30. In given figure, the side QR of $\triangle PQR$ is produced to point S. If the bisector of $\angle PQR$ and $\angle PRS$ meet at point T, then prove that $\angle QTR = \frac{1}{2} \angle QPR$.



31. Radha made a picture of an aeroplane with colored paper as shown in the following figure



Find the total area of the paper used.

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(Solutions)

SECTION-A

1. $(25)^{\frac{1}{3}} \times (5)^{\frac{1}{3}} = (25 \times 5)^{\frac{1}{3}} = (5^3)^{\frac{1}{3}}$
 $= 5.$
 2. For zero of the polynomial $p(x)$, we put $p(x) = 0 \Rightarrow 2x + 3 = 0$
 $\Rightarrow 2x = -3 \Rightarrow x = -\frac{3}{2}$
 3. 6 units
 4. 70°
 5. Suppose $\sqrt{7} - 3$ is rational
let $\sqrt{7} - 3 = x$ (x is a rational number)
 $\sqrt{7} = x + 3$
 x is a rational number 3 is also rational number
therefore $x + 3$ is rational number
but $\sqrt{7}$ is irrational number which is contradiction
therefore, $\sqrt{7} - 3$ is irrational number
 6. Here, $f(x) = x^5 - 4k^2x^3 + 2x + 2k + 3$
since $x + 2k$ is a factor of $f(x)$, so by factor theorem,
 $f(-2k) = 0$
 $(-2k)^5 - 4k^2(-2k)^3 + 2(-2k) + 2k + 3 = 0$
 $-32k^5 + 32k^5 - 4k + 2k + 3 = 0$
 $\Rightarrow -2k + 3 = 0 \Rightarrow -2k = -3 \Rightarrow k = \frac{3}{2}$
 7. By remainder theorem,
 $f(-2) = 2(-2)^4 + 6(-2)^3 + 2(-2)^2 - (-2) + 2$
 $\Rightarrow f(-2) = 32 - 48 + 8 + 2 + 2 = -4$
 8. $\angle QOS - \angle POS = (\angle QOR + \angle ROS) - \angle POS$
 $= 90^\circ + \angle ROS - \angle POS$
 $= (90^\circ - \angle POS) + \angle ROS$
 $= (\angle ROP - \angle POS) + \angle ROS$
 $= 2 \angle ROS$
-

Hence, $\angle ROS = \frac{1}{2} (\angle QOS - \angle POS)$

9. Given $30A = 6B = 5C$

$$\Rightarrow \frac{A}{1} = \frac{B}{5} = \frac{C}{6} \quad [\text{Dividing by } 30]$$

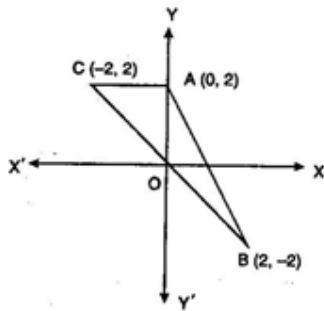
$$\Rightarrow \angle A : \angle B : \angle C = 1 : 5 : 6$$

Let $\angle A = x$, $\angle B = 5x$ and $\angle C = 6x$

$$\Rightarrow x + 5x + 6x = 180^\circ \Rightarrow 12x = 180^\circ \Rightarrow x = 15^\circ$$

Hence $\angle A = 15^\circ$, $\angle B = 75^\circ$ and $\angle C = 90^\circ$

10.



11. Let $\frac{p}{q} = 2.4\overline{178}$

$$\frac{p}{q} = 2.4178178178$$

Multiplying by 10

$$10\frac{p}{q} = 24.178178$$

Multiplying by 1000

$$10,000\frac{p}{q} = 1000 \times 24.178178$$

$$10,000\frac{p}{q} = 24178.178178$$

$$10000\frac{p}{q} - \frac{p}{q} = 24178.178178 - 2.4178178$$

$$9999\frac{p}{q} = 24154$$

$$\frac{p}{q} = \frac{24154}{9999}$$

12. (i) $2 - \sqrt{5}$

We know that $\sqrt{5} = 2.236\dots$, which is an irrational number.

$$2 - \sqrt{5} = 2 - 2.236\dots$$

$$= -0.236\dots, \text{ which is also an irrational number.}$$

Therefore, we conclude that $2 - \sqrt{5}$ is an irrational number.

$$(ii) (3 + \sqrt{23}) - \sqrt{23}$$

$$(3 + \sqrt{23}) - \sqrt{23} = 3 + \sqrt{23} - \sqrt{23}$$

$$= 3$$

Therefore, we conclude that $(3 + \sqrt{23}) - \sqrt{23}$ is a rational number.

13. We have, $\left(9x - \frac{1}{5}\right)^2 + \left(x + \frac{1}{3}\right)^2$

$$= \left[\left(9x - \frac{1}{5}\right) - \left(x + \frac{1}{3}\right) \right] \left[\left(9x - \frac{1}{5}\right) + \left(x + \frac{1}{3}\right) \right]$$

$$[\because a^2 - b^2 = (a - b)(a + b)]$$

$$= \left(9x - \frac{1}{5} - x - \frac{1}{3}\right) \left(9x - \frac{1}{5} + x + \frac{1}{3}\right)$$

$$= \left(8x - \frac{1}{5} - \frac{1}{3}\right) \left(10x - \frac{1}{5} + \frac{1}{3}\right)$$

$$= \left(\frac{120 - 3 - 5}{15}\right) \left(\frac{150 + 3 + 5}{15}\right)$$

$$= \left(\frac{120x - 8}{15}\right) \left(\frac{150x + 2}{15}\right)$$

14. Let $p(x) = 2x^4 - 6x^3 + 3x^2 - 2$ and $g(x) = x^2 - 3x + 2$

Then, $g(x) = x^2 - 3x + 2$

$$= (x - 1)(x - 2)$$

Clearly $(x - 1)$ and $(x - 2)$ are factors of $g(x)$

Let $x - 1 = 0 \Rightarrow x = 1$

$$p(1) = 2(1)^4 - 6(1)^3 + 3(1)^2 + 3(1) - 2$$

$$= 2 - 6 + 3 + 3 - 2 = 0$$

Let $x - 2 = 0 \Rightarrow x = 2$

$$p(2) = 2(2)^4 - 6(2)^3 + 3(2)^2 + 3(2) - 2$$

$$= 32 - 48 + 12 + 6 - 2$$

$$= 50 - 50 = 0$$

$\therefore (x - 1)$ and $(x - 2)$ are factors of $p(x)$

$$\Rightarrow g(x) = (x-1)(x-2) \text{ is a factor of } p(x)$$

Hence, $p(x)$ is exactly divisible by $g(x)$

15. Let $A(x) = px^3 + 4x^2 + 3x - 4$

$$B(x) = x^3 - 4x + p$$

$$g(x) = x - 3$$

According to question, $A(3) = B(3)$

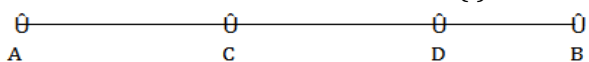
$$\Rightarrow p(3)^3 + 4(3)^2 + 3(3) - 4 = (3^3) - 4(3) + p$$

$$\Rightarrow 27p + 41 = 15 + p$$

$$\Rightarrow 27p - p = 15 - 41$$

$$\Rightarrow p = -1$$

16. Given $AC = BC$ (i)



If possible let D be another mid-point of AB

$$\therefore AD = DB$$
(ii)

Subtracting eq. (i) from eq. (ii), we get

$$AD - AC = DB - CB$$

$$\Rightarrow -CD = CD$$

$$\Rightarrow 2CD = 0$$

$$\Rightarrow CD = 0$$

\therefore C and D coincide.

Hence every line segment has one and only one mid-point.

17. $\angle AOC + \angle BOD = 266^\circ$ (i)

But $\angle BOD = \angle AOC$ [Vertically opposite]

$$\therefore \angle AOC + \angle AOC = 266^\circ$$

$$\Rightarrow \angle AOC = 133^\circ$$

Now $\angle AOC + \angle BOC = 180^\circ$ [Linear pair]

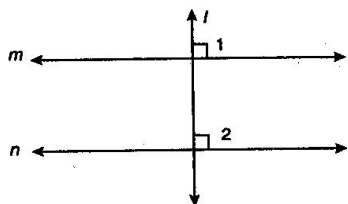
$$\Rightarrow 133^\circ + \angle BOC = 180^\circ$$

$$\Rightarrow \angle BOC = 47^\circ$$

$$\Rightarrow \angle AOD = \angle BOC$$

$$\Rightarrow \angle AOD = 47^\circ$$

18. Given : l, m, n are three lines such that $m \parallel n$ and $l \perp m$.



To prove: $l \perp n$

Proof : Since $l \perp m$

$$\Rightarrow \angle 1 = 90^\circ \quad \dots\dots\dots(i)$$

Now, $m \parallel n$ and transversal intersects them.

$$\Rightarrow \angle 2 = \angle 1 \quad \dots\dots\dots(ii) \quad [\text{Corresponding angles}]$$

From eq. (i) and (ii), we get,

$$\angle 2 = \angle 1 = 90^\circ \quad \Rightarrow \quad \angle 2 = 90^\circ$$

$$\therefore l \perp n$$

19. Given $\angle A + \angle B = 84^\circ \quad \dots\dots\dots(i)$

And $\angle B + \angle C = 146^\circ \quad \dots\dots\dots(ii)$

Adding eq. (i) and (ii), we get,

$$\angle A + \angle B + \angle B + \angle C = 230^\circ$$

$$\Rightarrow (\angle A + \angle B + \angle C) + \angle B = 230^\circ$$

$$\Rightarrow 180^\circ + \angle B = 230^\circ$$

$$\Rightarrow \angle B = 50^\circ$$

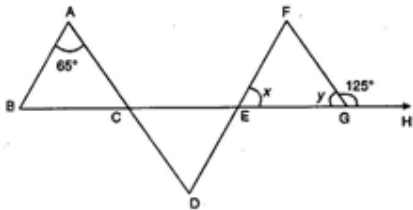
Putting the value of $\angle B$ in eq. (i), we get,

$$\angle A + 50^\circ = 84^\circ \quad \Rightarrow \quad \angle A = 34^\circ$$

Putting the value of $\angle B$ in eq. (ii), we get,

$$50^\circ + \angle C = 146^\circ \quad \Rightarrow \quad \angle C = 96^\circ$$

20.



$$\angle y + 125^\circ = 180^\circ \quad [\text{Straight angle}]$$

$$\Rightarrow \angle y = 55^\circ \quad \dots\dots\dots(i)$$

Now AB is parallel to FD and transversal AD cuts them.

$$\angle D = \angle A \quad [\text{Alternate angles}]$$

$$\angle D = 65^\circ$$

Again $AD \parallel FG$, transversal FD cuts them.

$$\angle F = \angle D$$

$$\angle F = 65^\circ \quad \dots\dots\dots(ii)$$

In triangle EFG, $\angle x + \angle F + \angle y = 180^\circ$

$$\Rightarrow \angle x + 65^\circ + 55^\circ = 180^\circ$$

$$\Rightarrow \angle x = 60^\circ$$

21. Here, $a = \frac{\sqrt{2}+1}{\sqrt{2}-1} = \frac{\sqrt{2}+1}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} = \frac{(\sqrt{2}+1)^2}{(\sqrt{2})^2 - (1)^2}$

$$= \frac{(\sqrt{2})^2 + 1 + 2\sqrt{2}}{\sqrt{2}-1} = \frac{2+1+2\sqrt{2}}{1} = 3+2\sqrt{2}$$

$$\therefore a = 3 + 2\sqrt{2}$$

$$b = \frac{\sqrt{2}-1}{\sqrt{2}+1} = \frac{\sqrt{2}-1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1} = \frac{(\sqrt{2}-1)^2}{(\sqrt{2})^2 - (1)^2}$$

$$= \frac{(\sqrt{2})^2 + 1^2 - 2\sqrt{2}}{2-1} = \frac{2+1-2\sqrt{2}}{1} = 3-2\sqrt{2}$$

$$\therefore b = 3 - 2\sqrt{2}$$

From equation (i) and (ii)

$$a+b = 3+2\sqrt{2}+3-2\sqrt{2} = 6$$

$$ab = (3+2\sqrt{2})(3-2\sqrt{2}) = 3^2 - (2\sqrt{2})^2$$

$$= 9 - 4 \times 2 = 9 - 8 = 1$$

$$\therefore a^2 + b^2 - 4ab = a^2 + b^2 + 2ab - 6ab$$

$$= (a+b)^2 - 6ab$$

$$= 6^2 - 6 \times 1 = 36 - 6 = 30$$

22. (i) We have, $a = 3 + 2\sqrt{2}$ and $\frac{1}{a} = \frac{1}{3+2\sqrt{2}}$

$$\frac{1}{a} = \frac{1}{3+2\sqrt{2}} \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}} = \frac{3-2\sqrt{2}}{3^2 - (2\sqrt{2})^2} = \frac{3-2\sqrt{2}}{9-8}$$

$$\therefore \frac{1}{a} = 3 - 2\sqrt{2}$$

$$\left(a + \frac{1}{a}\right)^2 = a^2 + \frac{1}{a^2} + 2$$

Putting the value of $a + \frac{1}{a}$, we get

$$6^2 = a^2 + \frac{1}{a^2} + 2$$

$$\Rightarrow a^2 + \frac{1}{a^2} = 36 - 2 \quad \Rightarrow a^2 + \frac{1}{a^2} = 34$$

(ii) We have,

$$\left(a + \frac{1}{a}\right)^3 = a^3 + \frac{1}{a^3} + 3 \times a^2 \times \frac{1}{a} + 3 \times a \times \frac{1}{a^2}$$

$$\Rightarrow \left(a + \frac{1}{a}\right)^3 = \left(a^3 + \frac{1}{a^3}\right) + \left(a + \frac{1}{a}\right)$$

Putting the value of $a + \frac{1}{a}$, we get

$$6^3 = a^3 + \frac{1}{a^3} + 3 \times 6$$

$$\Rightarrow a^3 + \frac{1}{a^3} = 216 - 18 = 198$$

23. Let $p(x) = x^3 + x^2 - 4x - 4$

The constant term in $p(x)$ is equal - 4 and factors of - 4 are $\pm 1, \pm 2$,

Putting $x = -1$ in $p(x)$, we have

$$\begin{aligned} p(-1) &= (-1)^3 + (-1)^2 - 4 \times (-1) - 4 \\ &= -1 + 1 + 4 - 4 = 0 \end{aligned}$$

$\therefore (x+1)$ is a factor of $p(x)$.

Putting $x = 2$ in $p(x)$, we have

$$\begin{aligned} p(2) &= 2^3 + 2^2 - 4 \times 2 - 4 \\ &= 8 + 4 - 8 - 4 \end{aligned}$$

$$p(2) = 0$$

$\therefore (x-2)$ is a factor of $p(x)$

Putting $x = -2$ in $p(x)$, we have

$$\begin{aligned} p(-2) &= (-2)^3 + (-2)^2 - 4(-2) - 4 \\ &= -8 + 4 + 8 - 4 \end{aligned}$$

$$p(-2) = 0$$

$\therefore (x+2)$ is a factor of $p(x)$

As $p(x)$ is a polynomial of degree 3, so it cannot have more than three linear factors.

$$\therefore p(x) = k(x+1)(x+2)(x-2)$$

$$x^3 + x^2 - 4x - 4 = k(x+1)(x+2)(x-2)$$

Putting $x = 0$ on both the sides, we get

$$0 + 0 - 4 \times 0 - 4 = k(0+1)(0+2)(0-2)$$

$$-4 = -4k \Rightarrow k = \frac{4}{4} = 1$$

Putting $k = 1$, we get

$$\begin{aligned} x^3 + x^2 - 4x - 4 &= 1(x+1)(x+2)(x-2) \\ &= (x+1)(x+2)(x-2) \end{aligned}$$

24. Let $p(x) = x^3 + 13x^2 + 31x + 20$

The constant term in $p(x)$ is equal to 20 and the factors of 20 are $\pm 1, \pm 2, \pm 4, \pm 5, \pm 10$.

Putting $x = -2$ in $p(x)$, we have

$$p(-2) = (-2)^3 + 13(-2)^2 + 31(-2) + 20$$

$$= -8 + 52 - 64 + 20 = -72 + 72 = 0$$

$$p(-2) = 0$$

As $p(-2) = 0$, so $(x + 2)$ is a factor of $p(x)$. Now, divided $P(x)$ by $(x + 2)$

$$\begin{array}{r} x^2 + 11x + 10 \\ x + 2 \overline{) x^3 + 13x^2 + 32x + 20} \\ \underline{-x^3 + 2x^2} \\ 11x^2 + 32x + 20 \\ \underline{-11x^2 + 22x} \\ 10x + 20 \\ \underline{-10x + 20} \\ 0 \end{array}$$

$$\therefore p(x) = (x + 2)(x^2 + 11x + 10) = (x + 2)[x^2 + 10x + x + 10]$$

$$= (x + 2)[x(x + 10) + 1(x + 10)] = (x + 2)[(x + 10)(x + 1)]$$

$$= (x + 1)(x + 2)(x + 10)$$

25. Let $x + y = p, y + z = q, z + x = r$

$$LHS = p^3 + q^3 + r^3 - 3pqr$$

$$= (p + q + r)(p^2 + q^2 + r^2 - pq - pr - rp)$$

Now, $p + q + r = 2(x + y + z)$

$$p^2 + q^2 + r^2 - pq - pr - rp = (x + y)^2 + (y + z)^2 + (z + x)(y + z) - (y + z)(z + x) - (z + x)(x + y)$$

Solving we get, $= x^2 + y^2 + z^2 - xy + yz - xz$

$$\therefore (p + q + r)(p^2 + q^2 + r^2 - pq - rq - rp) = 2(x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$$

$$= 2(x^3 + y^3 + z^3 - 3xyz)$$

26. Let $f(x) = ax^2 + 5x + b$

$$f(3) = 0 \Rightarrow 9a + 15 + b = 0 \Rightarrow 9a + b = -15 \dots\dots\dots(1)$$

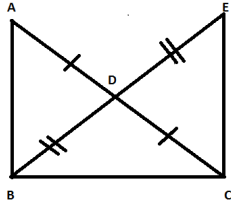
Similarly, $f\left(\frac{1}{3}\right) = 0 \Rightarrow \frac{a}{9} + \frac{5}{3} + b = 0 \Rightarrow \frac{a}{3} + b = -\frac{5}{3} \dots\dots\dots(2)$

$$(1) = (2) \Rightarrow a = b$$

27. **Given:** $\triangle ABC$ in which $\angle B = 90^\circ$ and D is the mid-point of AC .

To prove: $BD = \frac{1}{2} AC$

Construction: Produce BD to E so that $BD = DE$. Join EC



Proof: In $\triangle ADB$ and $\triangle CDE$, we have

$$AD = DC \quad \triangle ABC \text{ (Given)}$$

$$BD = DE \quad \text{(By construction)}$$

$$\angle ADB = \angle CDE \quad \text{(Vertically opp. Angles)}$$

$$\therefore \triangle ADB \cong \triangle CDE \quad \text{(By SAS congruence criterion)}$$

$$\Rightarrow AB = CE \text{ and } \angle CED = \angle ABD \quad \dots\dots(1) \quad \text{(CPCT)}$$

Thus, transversal BE cuts AB and CE such that the alternate angles $\angle CED$ and $\angle ABD$ are equal. So, $CE \parallel AB$

$$\Rightarrow \angle ABC + \angle ECB = 180^\circ \quad \text{(co-interior angles)}$$

$$\Rightarrow \angle ECB = 90^\circ \quad (\because \angle ABC = 90^\circ)$$

Thus, in $\triangle ABC$ and $\triangle ECB$, we have

$$AB = EC \quad \text{(From (1))}$$

$$BC = CB \quad \text{(common)}$$

$$\angle ABC = \angle ECB = 90^\circ$$

$$\therefore \triangle ABC \cong \triangle ECB \quad \text{(By SAS)}$$

$$\Rightarrow AC = BE \quad \text{(CPCT)}$$

$$\Rightarrow \frac{1}{2} AC = \frac{1}{2} BE \Rightarrow \frac{1}{2} AC = BD$$

28. **Given:** A triangle ABC in which $\angle ABC > \angle ACB$

To prove: $AC > AB$

Proof: There are three possibilities

(1) $AB > AC$

(2) $AB = AC$

(3) $AB < AC$

CASE 1: $AB > AC$

$\angle C > \angle B$, as the greater side has greater angle opposite to it.

It is not possible as we are given that $\angle B > \angle C$

CASE 2: $AB = AC$

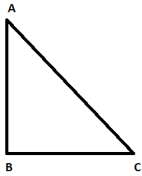
Then $\angle C = \angle B$ as angles opposite to equal sides are equal.

But $\angle B > \angle C$ is given. So it is also not possible.

CASE 3: $AB < AC$

As only one case is left, it has to be true.

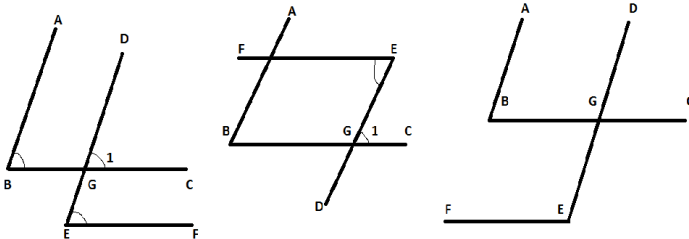
Hence, if two sides of a triangle are unequal, the greater side has greater angle opposite to it.



29. **Given:** Two angles $\angle ABC$ and $\angle DEF$ such that $BA \parallel ED$ and $BC \parallel EF$.

To prove: $\angle ABC = \angle DEF$ or $\angle ABC + \angle DEF = 180^\circ$

Proof: The arms of the angles may be parallel in the same sense or in the opposite sense. So, three cases arise.



Case 1: When both pairs of arms are parallel in the same sense.

In this case, $BA \parallel ED$ and BC is the transversal.

$\therefore \angle ABC = \angle 1$ (corresp. angles)

Again, $BC \parallel EF$ and DE is the transversal.

$\therefore \angle 1 = \angle DEF$ (corresp. Angles)

Hence, $\angle ABC = \angle DEF$

CASE 2: When both pairs of arms are parallel in opposite sense.

In this case, $BA \parallel ED$ and BC is transversal.

$\therefore \angle ABC = \angle 1$ (corresp. Angles)

Again, $FE \parallel BC$ and ED is the transversal

$\therefore \angle DEF = \angle 1$ (alternate int. angles)

Hence, $\angle ABC = \angle DEF$

CASE 3: When one pair of arms are parallel in same sense and other pair parallel in opposite sense.

In this case, $BA \parallel ED$ and BC is the transversal.

$\therefore \angle EGB = \angle ABC$ (alternate int. angles)

Now, $BC \parallel EF$ and DE is the transversal

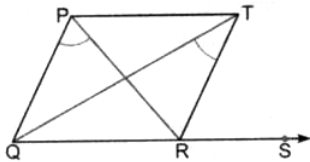
$\therefore \angle DEF + \angle EGB = 180^\circ$ (co-int. angles)

$\Rightarrow \angle DEF + \angle ABC = 180^\circ$ ($\because \angle EGB = \angle ABC$)

Hence, $\angle ABC$ and $\angle DEF$ are supplementary.

30. **Given:** A ΔPQR , whose side QR is produced to S.
The bisectors of $\angle PQR$ and $\angle PRS$ meet at point T.

To prove: $\angle QTR = \frac{1}{2} \angle QPR$



Proof: Side QR of ΔPQR is produced to S.

therefore, $\angle PRS = \angle P + \angle Q$

$$\Rightarrow \frac{1}{2} \angle PRS = \angle P + \frac{1}{2} \angle Q$$

$$\Rightarrow \angle TRS = \frac{1}{2} \angle P + \frac{1}{2} \angle Q \quad \dots\dots\dots (i)$$

Again, side OR of TQR is produced to S

Therefore, $\angle TRS = \angle QTR + \angle RQT$

$$\Rightarrow \frac{1}{2} \angle TRS = \angle T + \frac{1}{2} \angle Q \quad \dots\dots\dots (ii)$$

From (i) and (ii), we get

$$\frac{1}{2} \angle P + \frac{1}{2} \angle Q = \angle T + \frac{1}{2} \angle Q$$

$$\Rightarrow \angle T = \frac{1}{2} \angle P \text{ or } \angle QTR = \frac{1}{2} \angle QPR$$

31. **Area of region 1:**

Region 1 is enclosed by a triangle of sides $a = 4\text{cm}$, $b = 5\text{cm}$ and $c = 1\text{cm}$

Let $2s$ be the perimeter of the triangle. Then,

$$2s = 4 + 4 + 1 \Rightarrow s = \left(\frac{9}{2} \text{ cm}\right)$$

Using Heron's formula, area of region 1 = 19.875 sq.cm

Area of region 2:

Region 2 is a rectangle of length 6.5 cm and breadth 1 cm

\therefore Area of region 2 = $6.5 * 1 \text{ sq.cm}$

Area of region 3:

Region 3 is an isos. Trapezium

Using pythagoras theorem for $\triangle ABC$, find BE

Area of region 3 = 1.3 sq.cm

Area of region 4 = 4.5 sq.cm using area of triangle

Area of region 5: Region 4 and 5 are congruent, so , area = 4.5 sq.cm

Hence, the total area = 18.7875 sq.cm
