CHAPTER 15
Trigonometric Functions

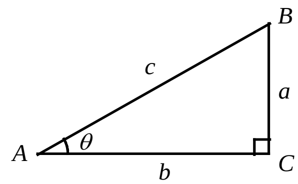
15-1. Trigonometric Ratios of Acute Angles

A ratio of the lengths of sides of a right triangle is called a **trigonometric ratio**. The six trigonometric ratios are **sine**, **cosine**, **tangent**, **cosecant**, **secant**, and **cotangent**.
Their abbreviations are sin, cos, tan, csc, sec, and cot, respectively. The six trigonometric ratios of any angle $0^\circ < \theta < 90^\circ$, sine, cosine, tangent, cosecant, secant, and cotangent, are defined as follows.

$$\sin\theta = \frac{\text{length of side opposite to } \theta}{\text{length of hypotenuse}} = \frac{a}{c} \qquad \csc\theta = \frac{1}{\sin\theta} = \frac{c}{a}$$

$$\cos\theta = \frac{\text{length of side adjacent to } \theta}{\text{length of hypotenuse}} = \frac{b}{c} \qquad \sec\theta = \frac{1}{\cos\theta} = \frac{c}{b}$$

$$\tan\theta = \frac{\text{length of side opposite to } \theta}{\text{length of side adjacent to } \theta} = \frac{a}{b} \qquad \cot\theta = \frac{1}{\tan\theta} = \frac{b}{a}$$



The sine and cosine are called **cofunctions**. In a right triangle ABC, $\angle A$ and $\angle B$ are complementary, that is, $m\angle A + m\angle B = 90$. Thus any trigonometric function of an acute angle is equal to the cofunction of the complement of the angle.

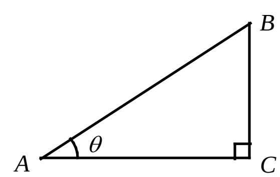
Complementary Angle Theorem

$\sin\theta = \cos(90^\circ - \theta)$ $\qquad$ $\cos\theta = \sin(90^\circ - \theta)$

If $\sin\angle A = \cos\angle B$, then $m\angle A + m\angle B = 90^\circ$.

Trigonometric Identities

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ $\qquad$ $\sin^2\theta + \cos^2\theta = 1$



Example 1 □ In the right triangle shown at the right, find $\cos\theta$ and $\tan\theta$ if $\sin\theta = \dfrac{2}{3}$.

Solution □

$\sin^2\theta + \cos^2\theta = 1$ $\qquad$ Trigonometric identity

$(\frac{2}{3})^2 + \cos^2\theta = 1$ $\qquad$ Substitute $\frac{2}{3}$ for $\sin\theta$.

$\cos^2\theta = 1 - (\frac{2}{3})^2 = 1 - \frac{4}{9} = \frac{5}{9}$

$\cos\theta = \sqrt{\frac{5}{9}} = \frac{\sqrt{5}}{\sqrt{9}} = \frac{\sqrt{5}}{3}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{2/3}{\sqrt{5}/3} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{\sqrt{5}\sqrt{5}} = \frac{2\sqrt{5}}{5}$

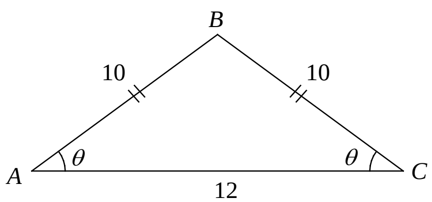
Example 2 □ In a right triangle, θ is an acute angle. If $\sin\theta = \dfrac{4}{9}$, what is $\cos(90^\circ - \theta)$?

Solution □ By the complementary angle property of sine and cosine,
$\cos(90^\circ - \theta) = \sin\theta = \dfrac{4}{9}$.

Exercises - Trigonometric Ratios of Acute Angles

Questions 1- 3 refer to the following information.

In the triangle shown below $AB = BC = 10$ and $AC = 12$.



1

What is the value of $\cos\theta$?

A) 0.4

B) 0.6

C) 0.8

D) 1.2

2

What is the value of $\sin\theta$?

A) 0.4

B) 0.6

C) 0.8

D) 1.2

3

What is the value of $\tan\theta$?

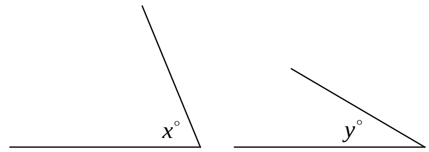
A) $\frac{3}{4}$

B) $\frac{4}{3}$

C) $\frac{5}{4}$

D) $\frac{5}{3}$

4



Note: Figures not drawn to scale.

In the figures above $y < x < 90$ and $\cos x^\circ = \sin y^\circ$. If $x = 3a - 14$ and $y = 50 - a$, what is the value of a ?

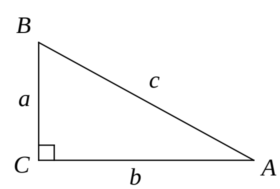
A) 16

B) 21

C) 24

D) 27

5



Given the right triangle ABC above, which of the following is equal to $\frac{a}{c}$?

I. $\sin A$

II. $\cos B$

III. $\tan A$

A) I only

B) II only

C) I and II only

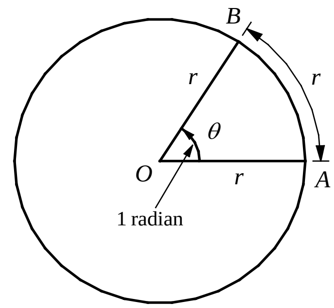
D) II and III only

15-2. The Radian Measure of an Angle

One **radian** is the measure of a central angle θ whose intercepted arc has a length equal to the circle's radius. In the figure at the right, if length of the arc $AB = OA$, then $m\angle AOB = 1$ radian .
Since the circumference of the circle is $2\pi r$ and a complete revolution has degree measure 360°,
2π radians $= 360^\circ$, or π radians $= 180^\circ$.

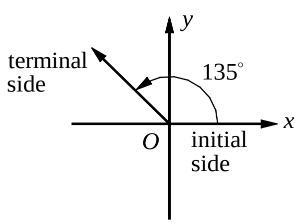
The conversion formula π radians $= 180^\circ$ can be used to convert radians to degrees and vice versa.

$$1 \text{ radian} = \frac{180^\circ}{\pi} \approx 57.3^\circ \quad \text{and} \quad 1^\circ = \frac{\pi}{180} \text{ radians}$$

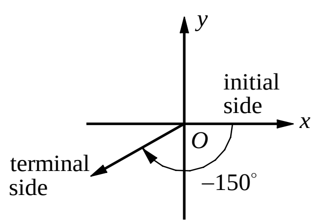


The measure of a central angle θ is 1 radian, if the length of the arc AB is equal to the radius of the circle.

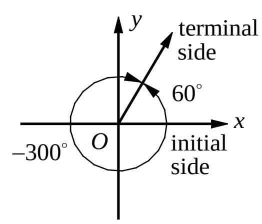
On a coordinate plane, an angle may be drawn by two rays that share a fixed endpoint at the origin. The beginning ray, called the **initial side** of the angle and the final position, is called the **terminal side** of the angle. An angle is in **standard position** if the vertex is located at the origin and the initial side lies along the positive x- axis. Counterclockwise rotations produce **positive angles** and clockwise rotations produce **negative angles**. When two angles have the same initial side and the same terminal side, they are called **coterminal angles**.



positive angle in standard position



negative angle in standard position



an angle of 60° is coterminal with an angle of -300°

You can find an angle that is coterminal to a given angle by adding or subtracting integer multiples of 360° or 2π radians. In fact, the sine and cosine functions repeat their values every 360° or 2π radians, and tangent functions repeat their values every 180° or π radians.

Periodic Properties of the Trigonometric Functions

$\sin(\theta \pm 360^\circ) = \sin\theta$ $\qquad$ $\cos(\theta \pm 360^\circ) = \cos\theta$ $\qquad$ $\tan(\theta \pm 180^\circ) = \tan\theta$

Example 1 □ Change the degree measure to radian measure and change the radian measure to degree measure.

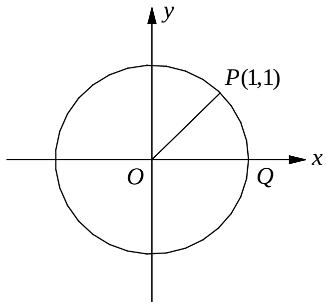
a. 45° $\qquad$ b. $\dfrac{2\pi}{3}$ radians

Solution □ a. $45^\circ = 45 \cdot \dfrac{\pi}{180}$ radians $= \dfrac{\pi}{4}$ radians

b. $\dfrac{2\pi}{3}$ radians $= \dfrac{2\pi}{3}$ radians$\left(\dfrac{180^\circ}{\pi \text{ radians}}\right) = 120^\circ$

Exercises - The Radian Measure of an Angle

1



In the xy-plane above, O is the center of the circle, and the measure of $\angle POQ$ is $k\pi$ radians. What is the value of k?

A) $\frac{1}{6}$

B) $\frac{1}{4}$

C) $\frac{1}{3}$

D) $\frac{1}{2}$

2

Which of the following is equal to $\cos(\frac{\pi}{8})$?

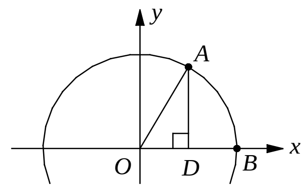
A) $\cos(\frac{3\pi}{8})$

B) $\cos(\frac{7\pi}{8})$

C) $\sin(\frac{3\pi}{8})$

D) $\sin(\frac{7\pi}{8})$

3



In the xy-plane above, O is the center of the circle and the measure of $\angle AOD$ is $\frac{\pi}{3}$. If the radius of circle O is 6 what is the length of AD?

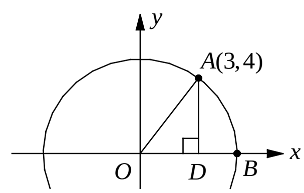
A) 3

B) $3\sqrt{2}$

C) 4.5

D) $3\sqrt{3}$

4



In the figure above, what is the value of $\cos\angle AOD$?

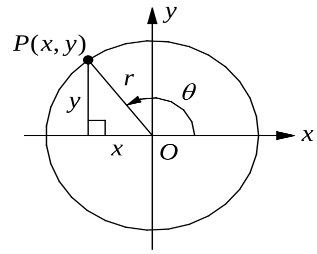
A) $\frac{3}{5}$

B) $\frac{3}{4}$

C) $\frac{4}{5}$

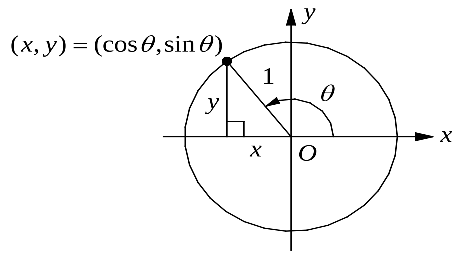
D) $\frac{4}{3}$

15-3. Trigonometric Functions and the Unit Circle



Suppose $P(x, y)$ is a point on the circle $x^2 + y^2 = r^2$ and θ is an angle in standard position with terminal side OP, as shown at the right. We define sine of θ and cosine of θ as

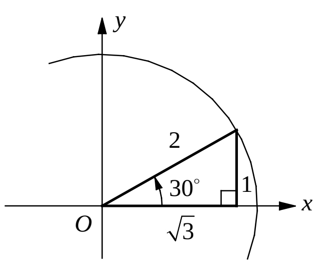
$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r}.$$



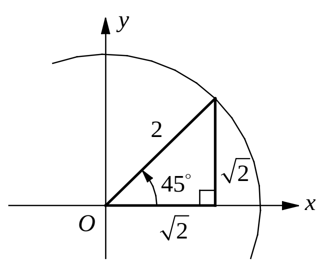
The circle $x^2 + y^2 = 1$ is called the **unit circle**. This circle is the easiest one to work with because $\sin\theta$ and $\cos\theta$ are simply the y- coordinates and the x- coordinates of the points where the terminal side of θ intersects the circle.

$$\sin\theta = \frac{y}{r} = \frac{y}{1} = y \qquad \cos\theta = \frac{x}{r} = \frac{x}{1} = x.$$

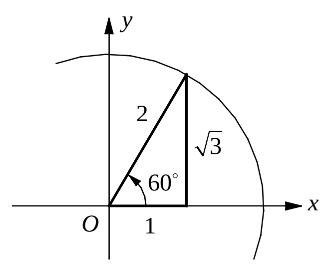
Angles in standard position whose measures are multiples of 30° ($\frac{\pi}{6}$ radians) or multiples of 45° ($\frac{\pi}{4}$ radians) are called **familiar angles**. To obtain the trigonometric values of sine, cosine, and tangent of the familiar angles, use 30°-60°-90° triangle ratio or the 45°-45°-90° triangle ratio.



$$\sin 30^\circ = \frac{y}{r} = \frac{1}{2}$$
$$\cos 30^\circ = \frac{x}{r} = \frac{\sqrt{3}}{2}$$

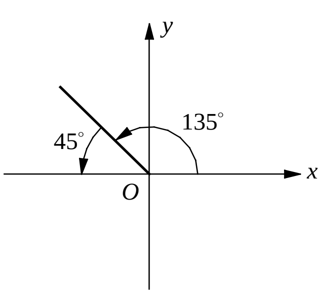


$$\sin 45^\circ = \frac{y}{r} = \frac{\sqrt{2}}{2}$$
$$\cos 45^\circ = \frac{x}{r} = \frac{\sqrt{2}}{2}$$

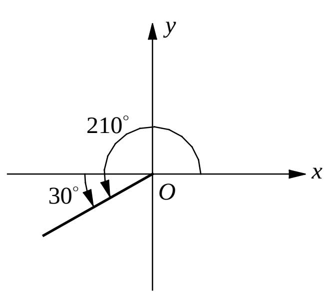


$$\sin 60^\circ = \frac{y}{r} = \frac{\sqrt{3}}{2}$$
$$\cos 30^\circ = \frac{x}{r} = \frac{1}{2}$$

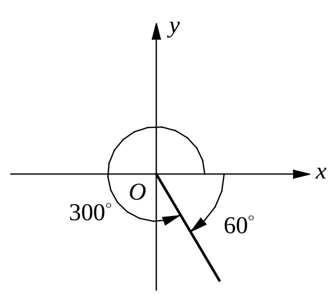
The **reference angle** associated with θ is the acute angle formed by the x- axis and the terminal side of the angle θ. A reference angle can be used to evaluate trigonometric functions for angles greater than 90°.



The reference angle for 135° is 45°.



The reference angle for 210° is 30°.



The reference angle for 300° is 60°.

Familiar Angles in a Coordinate Plane

Angles with a reference angle of $30^\circ (= \frac{\pi}{6})$ are $150^\circ (= \frac{5\pi}{6})$, $210^\circ (= \frac{7\pi}{6})$, and $330^\circ (= \frac{11\pi}{6})$.

Use the 30°-60°-90° triangle ratio to find the trigonometric values of these angles and put the appropriate signs. On the unit circle, $\sin\theta = y$ is positive in quadrant I and II and $\cos\theta = x$ is positive in quadrant I and IV.

$\sin 150^\circ = \frac{1}{2}$

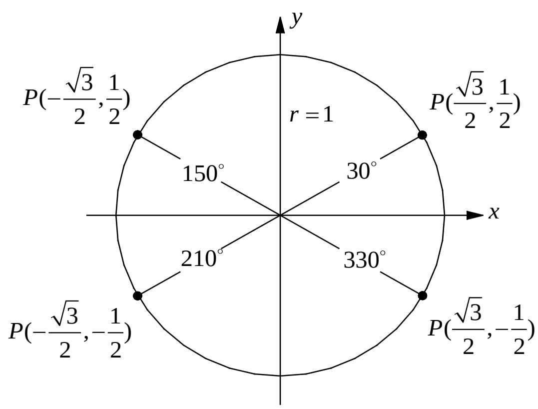
$\cos 150^\circ = -\frac{\sqrt{3}}{2}$

$\tan 150^\circ = -\frac{\sqrt{3}}{3}$

$\sin 210^\circ = -\frac{1}{2}$

$\cos 210^\circ = -\frac{\sqrt{3}}{2}$

$\tan 210^\circ = \frac{\sqrt{3}}{3}$



$\sin 30^\circ = \frac{1}{2}$

$\cos 30^\circ = \frac{\sqrt{3}}{2}$

$\tan 30^\circ = \frac{\sqrt{3}}{3}$

$\sin 330^\circ = -\frac{1}{2}$

$\cos 330^\circ = \frac{\sqrt{3}}{2}$

$\tan 330^\circ = -\frac{\sqrt{3}}{3}$

Angles with a reference angle of $60^\circ (= \frac{\pi}{3})$ are $120^\circ (= \frac{2\pi}{3})$, $240^\circ (= \frac{4\pi}{3})$, and $300^\circ (= \frac{5\pi}{3})$.

Use the 30°-60°-90° triangle ratio to find the trigonometric values of these angles and put the appropriate signs. On the unit circle, $\sin\theta = y$ is positive in quadrant I and II and $\cos\theta = x$ is positive in quadrant I and IV.

$\sin 120^\circ = \frac{\sqrt{3}}{2}$

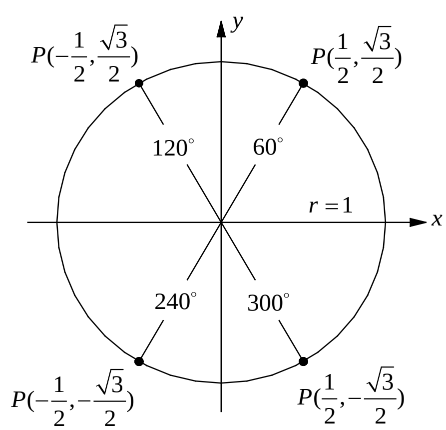
$\cos 120^\circ = -\frac{1}{2}$

$\tan 120^\circ = -\sqrt{3}$

$\sin 240^\circ = -\frac{\sqrt{3}}{2}$

$\cos 240^\circ = -\frac{1}{2}$

$\tan 240^\circ = \sqrt{3}$



$\sin 60^\circ = \frac{\sqrt{3}}{2}$

$\cos 60^\circ = \frac{1}{2}$

$\tan 60^\circ = \sqrt{3}$

$\sin 300^\circ = -\frac{\sqrt{3}}{2}$

$\cos 300^\circ = \frac{1}{2}$

$\tan 300^\circ = -\sqrt{3}$

Angles with a reference angle of $45^\circ (=\frac{\pi}{4})$ are $135^\circ (=\frac{3\pi}{4})$, $225^\circ (=\frac{5\pi}{4})$, and $315^\circ (=\frac{7\pi}{4})$,

Use the 45°-45°-90° triangle ratio to find the trigonometric values of these angles and put the appropriate signs. On the unit circle, $\sin\theta = y$ is positive in quadrant I and II and $\cos\theta = x$ is positive in quadrant I and IV.

$\sin 135^\circ = \frac{\sqrt{2}}{2}$

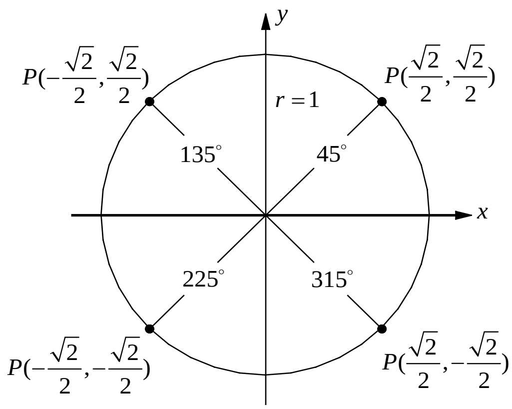
$\cos 135^\circ = -\frac{\sqrt{2}}{2}$

$\tan 135^\circ = -1$

$\sin 225^\circ = -\frac{\sqrt{2}}{2}$

$\cos 225^\circ = -\frac{\sqrt{2}}{2}$

$\tan 225^\circ = 1$



$\sin 45^\circ = \frac{\sqrt{2}}{2}$

$\cos 45^\circ = \frac{\sqrt{2}}{2}$

$\tan 45^\circ = 1$

$\sin 315^\circ = -\frac{\sqrt{2}}{2}$

$\cos 315^\circ = \frac{\sqrt{2}}{2}$

$\tan 315^\circ = -1$

For the angles 0°, $90^\circ = \frac{\pi}{2}$, $180^\circ = \pi$, and $270^\circ = \frac{3\pi}{2}$, $\sin\theta$ is equal to the y value of the point $P(x, y)$ and $\cos\theta$ is equal to the x value of the point $P(x, y)$. The points $P(1,0)$, $P(0,1)$, $P(-1,0)$, and $P(0,-1)$ on the unit circle corresponds to $\theta = 0^\circ = 0$, $\theta = 90^\circ = \frac{\pi}{2}$, $\theta = 180^\circ = \pi$, and $\theta = 270^\circ = \frac{3\pi}{2}$ respectively.

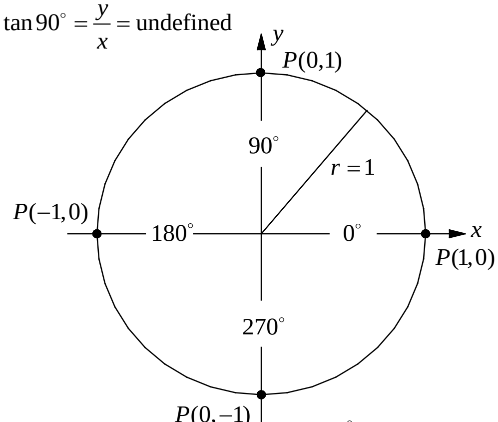
$\sin 90^\circ = y = 1 \quad \cos 90^\circ = x = 0$

$\tan 90^\circ = \frac{y}{x} = \text{undefined}$

$\sin 180^\circ = y = 0$

$\cos 180^\circ = x = -1$

$\tan 180^\circ = \frac{y}{x} = 0$



$\sin 0^\circ = y = 0$

$\cos 0^\circ = x = 1$

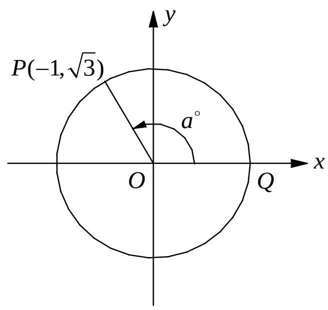
$\tan 0^\circ = \frac{y}{x} = 0$

$\sin 270^\circ = y = -1 \quad \cos 270^\circ = x = 0$

$\tan 270^\circ = \frac{y}{x} = \text{undefined}$

Exercises - The Trigonometric Functions and the Unit Circle

Questions 1 and 2 refer to the following information.



In the xy- plane above, O is the center of the circle, and the measure of $\angle POQ$ is a°.

1

What is the cosine of a°?

A) $-\frac{1}{2}$

B) $\sqrt{3}$

C) $-\frac{1}{\sqrt{3}}$

D) $\frac{\sqrt{3}}{2}$

2

What is the cosine of $(a+180)^\circ$?

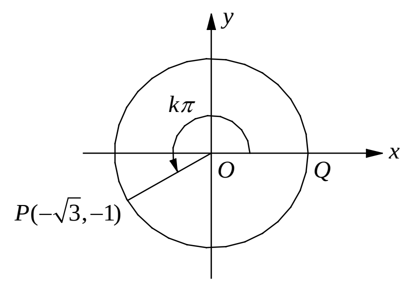
A) $-\sqrt{3}$

B) $-\frac{\sqrt{3}}{2}$

C) $\frac{1}{2}$

D) $\frac{1}{\sqrt{3}}$

Questions 3 and 4 refer to the following information.



In the xy- plane above, O is the center of the circle, and the measure of the angle shown is $k\pi$ radians.

3

What is the value of k?

A) $\frac{5}{6}$

B) $\frac{7}{6}$

C) $\frac{4}{3}$

D) $\frac{5}{3}$

4

What is the value of $\tan(k\pi)$?

A) $-\sqrt{3}$

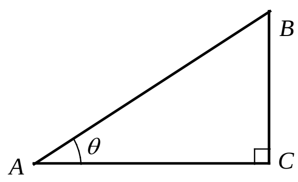
B) -1

C) $-\frac{1}{\sqrt{3}}$

D) $\frac{1}{\sqrt{3}}$

Chapter 15 Practice Test

1



Note: Figure not drawn to scale.

In the right triangle shown above, if $\tan\theta = \frac{3}{4}$, what is $\sin\theta$?

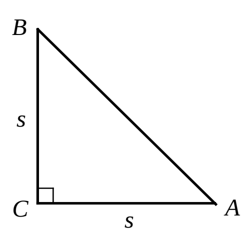
A) $\frac{1}{3}$

B) $\frac{1}{2}$

C) $\frac{4}{5}$

D) $\frac{3}{5}$

2



In the isosceles right triangle shown above, what is $\tan\angle A$?

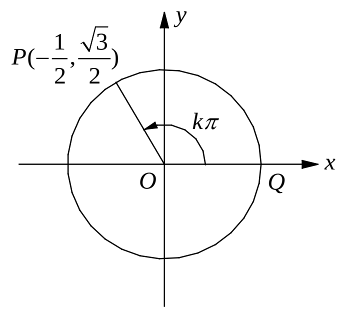
A) s

B) $\frac{1}{s}$

C) 1

D) $\frac{s}{\sqrt{2}}$

Questions 1 and 2 refer to the following information.



In the xy-plane above, O is the center of the circle, and the measure of $\angle POQ$ is $k\pi$ radians.

3

What is the value of k?

A) $\frac{1}{3}$

B) $\frac{1}{2}$

C) $\frac{2}{3}$

D) $\frac{3}{4}$

4

What is $\cos(k+1)\pi$?

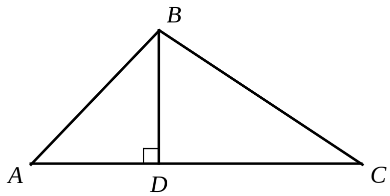
A) $\frac{1}{\sqrt{3}}$

B) $\frac{1}{2}$

C) $\frac{\sqrt{3}}{2}$

D) $\sqrt{3}$

5



In triangle ABC above, $\overline{AC} \perp \overline{BD}$. Which of the following does not represent the area of triangle ABC?

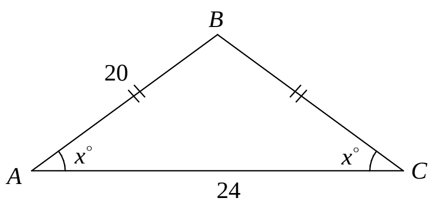
A) $\frac{1}{2}(AB\cos\angle A + BC\cos\angle C)(AB\cos\angle ABD)$

B) $\frac{1}{2}(AB\cos\angle A + BC\cos\angle C)(BC\sin\angle C)$

C) $\frac{1}{2}(AB\sin\angle ABD + BC\sin\angle CBD)(AB\sin\angle A)$

D) $\frac{1}{2}(AB\sin\angle ABD + BC\sin\angle CBD)(BC\cos\angle C)$

6



In the isosceles triangle above, what is the value of $\sin x^\circ$?

A) $\frac{1}{2}$

B) $\frac{3}{5}$

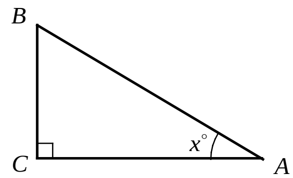
C) $\frac{2}{3}$

D) $\frac{4}{5}$

7

In triangle ABC, the measure of $\angle C$ is 90°, $AC = 24$, and $BC = 10$. What is the value of $\sin A$?

8



In the right triangle ABC above, the cosine of x° is $\frac{3}{5}$. If $BC = 12$, what is the length of AC?

9

If $\sin(5x-10)^\circ = \cos(3x+16)^\circ$, what is the value of x?

Answer Key

Section 15-1

1. B	2. C	3. B	4. D	5. C

Section 15-2

1. B	2. C	3. D	4. A

Section 15-3

1. A	2. C	3. B	4. D

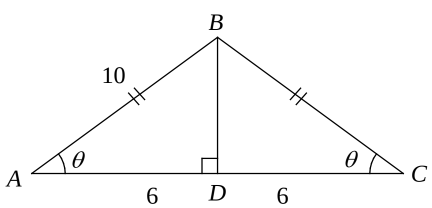
Chapter 15 Practice Test

1. D	2. C	3.C	4. B	5. D
6. D	7. $\frac{5}{13}$	8. 9	9. 10.5	

Answers and Explanations

Section 15-1

1. B



Draw a perpendicular segment from B to the opposite side AC. Let the perpendicular segment intersect side AC at D. Because the triangle is isosceles, a perpendicular segment from the vertex to the opposite side bisects the base and creates two congruent right triangles.

Therefore, $AD = \frac{1}{2}AC = \frac{1}{2}(12) = 6$.

In right ΔABD,

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{AD}{AB} = \frac{6}{10} = 0.6.$$

2. C

$AB^2 = BD^2 + AD^2$ Pythagorean Theorem

$10^2 = BD^2 + 6^2$

$100 = BD^2 + 36$

$64 = BD^2$

$8 = BD$

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{BD}{AB} = \frac{8}{10} = 0.8$$

3. B

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{BD}{AD} = \frac{8}{6} = \frac{4}{3}$$

4 . D

If x and y are acute angles and $\cos x^\circ = \sin y^\circ$, $x + y = 90$ by the complementary angle theorem.

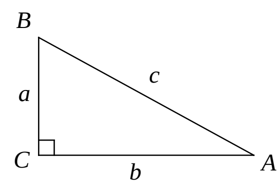
$(3a - 14) + (50 - a) = 90$ $x = 3a - 14$, $y = 50 - a$

$2a + 36 = 90$ Simplify.

$2a = 54$

$a = 27$

5. C



I. $\sin A = \frac{\text{opposite of } \angle A}{\text{hypotenuse}} = \frac{a}{c}$

Roman numeral I is true.

II. $\cos B = \frac{\text{adjacent of } \angle B}{\text{hypotenuse}} = \frac{a}{c}$

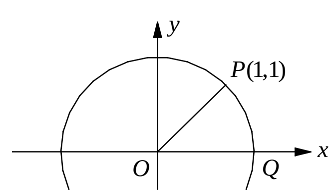
Roman numeral II is true.

III. $\tan A = \frac{\text{opposite of } \angle A}{\text{adjacent of } \angle A} = \frac{a}{b}$

Roman numeral III is false.

Section 15-2

1. B



The graph shows $P(x, y) = P(1,1)$. Thus, $x = 1$ and $y = 1$. Use the distance formula to find the length of radius OA.

$OA = \sqrt{x^2 + y^2} = \sqrt{1^2 + 1^2} = \sqrt{2}$

$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{1}{\sqrt{2}}$ or $\sin\theta = \frac{\sqrt{2}}{2}$

Therefore, the measure of $\angle POQ$ is 45°, which is equal to $45(\frac{\pi}{180}) = \frac{\pi}{4}$ radians.

Thus, $k = \frac{1}{4}$.

2. C

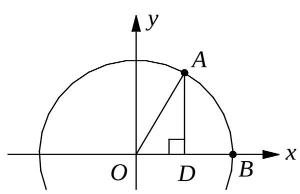
Use the complementary angle theorem.

$\cos(\theta) = \sin(90^\circ - \theta)$, or $\cos(\theta) = \sin(\frac{\pi}{2} - \theta)$

Therefore, $\cos(\frac{\pi}{8}) = \sin(\frac{\pi}{2} - \frac{\pi}{8}) = \sin(\frac{3\pi}{8})$.

All the other answer choices have values different from $\cos(\frac{\pi}{8})$.

3. D

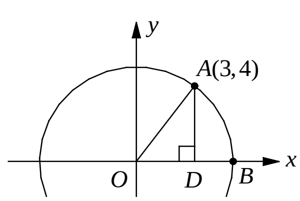


In ΔOAD, $\sin\frac{\pi}{3} = \sin 60^\circ = \frac{AD}{OA} = \frac{AD}{6}$.

Since $\sin 60^\circ = \frac{\sqrt{3}}{2}$, you get $\frac{AD}{6} = \frac{\sqrt{3}}{2}$.

Therefore, $2AD = 6\sqrt{3}$ and $AD = 3\sqrt{3}$.

4. A



Use the distance formula to find the length of OA.

$OA = \sqrt{x^2 + y^2} = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$

$\cos\angle AOD = \frac{OD}{OA} = \frac{3}{5}$

Section 15-3

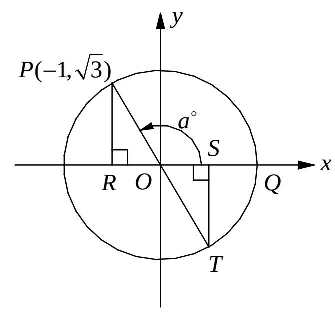
1. A

Draw segment PR, which is perpendicular to the x-axis. In right triangle POR, $x = -1$ and $y = \sqrt{3}$. To find the length of OP, use the Pythagorean theorem.

$OP^2 = PR^2 + OR^2 = (\sqrt{3})^2 + (-1)^2 = 4$

Which gives $OP = 2$.

$\cos a^\circ = \frac{x}{OP} = \frac{-1}{2}$

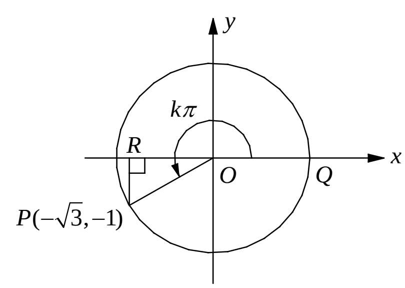


2. C

Since the terminal side of $(a+180)^\circ$ is OT, the value of $\cos(a+180)^\circ$ is equal to $\frac{OS}{OT}$.

$\frac{OS}{OT} = \frac{1}{2}$

3. B



Draw segment PR, which is perpendicular to the x-axis. In right triangle POR, $x = -\sqrt{3}$ and $y = -1$. To find the length of OP, use the Pythagorean theorem.

$OP^2 = PR^2 + OR^2 = (-1)^2 + (\sqrt{3})^2 = 4$

Which gives $OP = 2$.

Since $\sin\angle POR = \frac{y}{OP} = \frac{-1}{2}$, the measure of $\angle POR$ is equal to 30°, or $\frac{\pi}{6}$ radian.

$k\pi = \pi + \frac{\pi}{6} = \frac{7}{6}\pi$

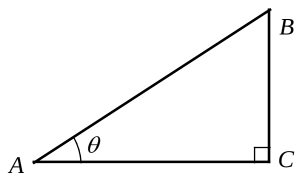
Therefore, $k = \frac{7}{6}$

4. D

$$\tan(k\pi) = \tan(\frac{7}{6}\pi) = \frac{y}{x} = \frac{-1}{-\sqrt{3}} = \frac{1}{\sqrt{3}}$$

Chapter 15 practice Test

1. D



Note: Figure not drawn to scale.

In ΔABC, $\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{BC}{AC}$.

If $\tan\theta = \frac{3}{4}$, then $BC = 3$ and $AC = 4$.

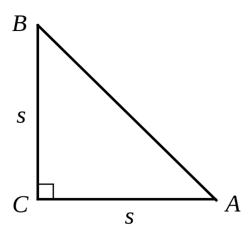
By the Pythagorean theorem,

$AB^2 = AC^2 + BC^2 = 4^2 + 3^2 = 25$, thus

$AB = \sqrt{25} = 5$.

$$\sin\theta = \frac{BC}{AB} = \frac{3}{5}$$

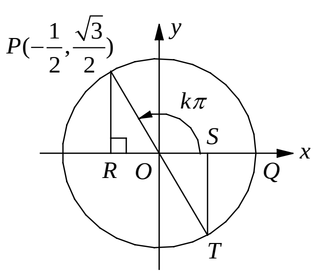
2. C



$$\tan\angle A = \frac{\text{opposite side of } \angle A}{\text{adjacent side of } \angle A} = \frac{s}{s} = 1$$

$$= \frac{s}{s} = 1$$

3. C



Draw segment PR, which is perpendicular to the x-axis. In right triangle POR, $x = -\frac{1}{2}$ and $y = \frac{\sqrt{3}}{2}$. To find the length of OP, use the Pythagorean theorem.

$$OP^2 = PR^2 + OR^2 = (\frac{\sqrt{3}}{2})^2 + (\frac{-1}{2})^2 = \frac{3}{4} + \frac{1}{4} = 1$$

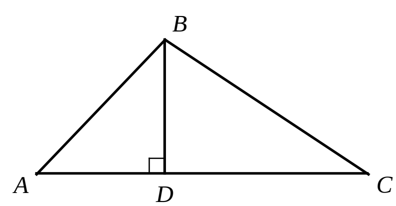
Which gives $OP = 1$. Thus, triangle OPR is 30°-60°-90° triangle and the measure of $\angle POR$ is 60°, which is $\frac{\pi}{3}$ radian. Therefore, the measure of $\angle POQ$ is $\pi - \frac{\pi}{3}$, or $\frac{2\pi}{3}$ radian. If $\angle POQ$ is $k\pi$ radians then k is equal to $\frac{2}{3}$.

4. B

Since the terminal side of $(k+1)\pi$ is OT, the value of $\cos(k+1)\pi$ is equal to $\frac{OS}{OT}$.

$$\frac{OS}{OT} = \frac{1}{2}$$

5. D



Area of triangle $ABC = \frac{1}{2}(AC)(BD)$

Check each answer choice.

A) $\frac{1}{2}(AB\cos\angle A + BC\cos\angle C)(AB\cos\angle ABD)$

$$= \frac{1}{2}(AB \cdot \frac{AD}{AB} + BC \cdot \frac{CD}{BC})(AB \cdot \frac{BD}{AB})$$

$$= \frac{1}{2}(AD + CD)(BD) = \frac{1}{2}(AC)(BD)$$

B) $\frac{1}{2}(AB\cos\angle A + BC\cos\angle C)(BC\sin\angle C)$

$$= \frac{1}{2}(AB \cdot \frac{AD}{AB} + BC \cdot \frac{CD}{BC})(BC \cdot \frac{BD}{BC})$$

$$= \frac{1}{2}(AD + CD)(BD) = \frac{1}{2}(AC)(BD)$$

C) $\frac{1}{2}(AB\sin\angle ABD + BC\sin\angle CBD)(AB\sin\angle A)$

$= \frac{1}{2}(AB\cdot\frac{AD}{AB} + BC\cdot\frac{CD}{BC})(AB\cdot\frac{BD}{AB})$

$= \frac{1}{2}(AD + CD)(BD) = \frac{1}{2}(AC)(BD)$

D) $\frac{1}{2}(AB\sin\angle ABD + BC\sin\angle CBD)(BC\cos\angle C)$

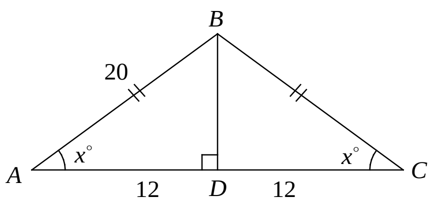
$= \frac{1}{2}(AB\cdot\frac{AD}{AB} + BC\frac{CD}{BC})(BC\cdot\frac{CD}{BC})$

$= \frac{1}{2}(AD + CD)(CD) = \frac{1}{2}(AC)(CD)$

Which does not represent the area of triangle ABC.

Choice D is correct.

6. D



Draw segment BD, which is perpendicular to side AC. Because the triangle is isosceles, a perpendicular segment from the vertex to the opposite side bisects the base and creates two congruent right triangles.

Therefore, $AD = \frac{1}{2}AC = \frac{1}{2}(24) = 12$.

By the Pythagorean theorem, $AB^2 = BD^2 + AD^2$

Thus, $20^2 = BD^2 + 12^2$.

$BD^2 = 20^2 - 12^2 = 256$

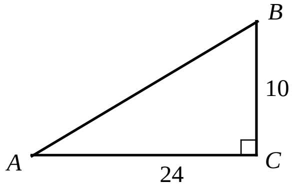
$BD = \sqrt{256} = 16$

In right ΔABD,

$\sin x^\circ = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{BD}{AB} = \frac{16}{20} = \frac{4}{5}$.

7. $\frac{5}{13}$

Sketch triangle ABC.



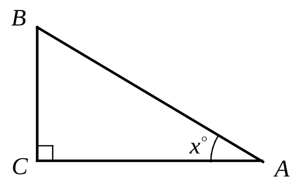
$AB^2 = BC^2 + AC^2$

$AB^2 = 10^2 + 24^2 = 676$

$AB = \sqrt{676} = 26$

$\sin A = \frac{10}{26} = \frac{5}{13}$

8. 9



$\cos x^\circ = \frac{AC}{AB} = \frac{3}{5}$

Let $AC = 3x$ and $AB = 5x$.

$AB^2 = BC^2 + AC^2$ Pythagorean Theorem

$(5x)^2 = 12^2 + (3x)^2$ $BC = 12$

$25x^2 = 144 + 9x^2$

$16x^2 = 144$

$x^2 = 9$

$x = \sqrt{9} = 3$

Therefore, $AC = 3x = 3(3) = 9$

9. 10.5

According to the complementary angle theorem, $\sin\theta = \cos(90 - \theta)$.

If $\sin(5x - 10)^\circ = \cos(3x + 16)^\circ$,

$3x + 16 = 90 - (5x - 10)$.

$3x + 16 = 90 - 5x + 10$

$3x + 16 = 100 - 5x$

$8x = 84$

$x = 10.5$