Modern Physics

Fill in the Blanks

Q.1. To produce characteristic X-rays using a Tungsten target in an X-ray generator, the accelerating voltage should be greater than ___________ volts and the energy of the characteristic radiation is _____eV.

(The binding energy of the innermost electron in Tungsten is 40 keV). (1983 - 2 Marks)

Ans. 30,000, 30,000

Solution. For minimum accelerating voltage, the electron should jump from n = 2 to n = 1 level.

For characteristic X-rays

$$\frac{1}{\lambda} = R_{\alpha}(Z-1)^2\left[1-\frac{1}{n^2}\right] = = \frac{E}{hc}$$

$$\therefore \frac{E_1}{hc} = R_{\alpha}(Z-1)^2\left[1-\frac{1}{2^2}\right] \qquad \text{......(i)}$$

The binding energy of innermost electron = 40 keV

∴ Ionisation potential of tungsten = 40 kV = 40 × 10^3 V

$$\Rightarrow \frac{E_2}{hc} = R_{\alpha}(Z-1)^2\left[1-\frac{1}{\infty^2}\right] \qquad \text{.....(ii)}$$

$$\therefore \frac{E_1}{E_2} = \frac{\left[1-\frac{1}{2^2}\right]}{\left[1-\frac{1}{\infty^2}\right]}$$

$$\Rightarrow E_1 = \frac{3}{4}E_2 = \frac{3}{4} \times 40,000 \text{ eV} = 30,000 \text{ eV}$$

∴ Minimum accelerating voltage,

$$V_{\min} = \frac{E_1}{e} = 30,000\ V$$

Q.2. The radioactive decay rate of a radioactive element is found to be 10^3disintegration/second at a certain time. If the half life of the element is one second , the decay rate after one second is _______ and after three seconds is _______. (1983 - 2 Marks)

Ans. 500 disintegration/sec, 125 disintegration/sec

Solution. $A = A_0\left(\frac{1}{2}\right)^n$ where A_0 = Initial activity = 1000 dps (given)

A = Activity after n half lives

$$\text{At } t = 1, n = 1 \therefore A = 1000\left(\frac{1}{2}\right)^1 = 500 \text{ dps}$$

$$\text{At } t = 3, n = 3 \therefore A = 1000\left(\frac{1}{2}\right)^3 = 125 \text{ dps}$$

Q.3. The maximum kinetic energy of electrons emitted in the photoelectric effect is linearly dependent on the ...... of the incident radiation. (1984- 2 Marks)

Ans. frequency

Solution. Note : According to law of photoelectric effect

$(K.E.)_{max} = h_v - hv_0$

i.e., the maximum kinetic energy of electrons emitted in the photoelectric effect is linearly dependent on the frequency of incident radiation.

Q.4. In the Uranium radioactive series the initial nucleus is $^{238}_{92}U$ and the final nucleus is $^{206}_{82}Pb$. When the Uranium nucleus decays to lead, the number of a-particles emitted is ......and the number of b-particles emitted is ..... (1985 - 2 Marks)

Ans. eight, six

Solution. $^{238}_{92}U \rightarrow {}^{206}_{82}Pb + x\,{}^{4}_{2}He + y\,{}^{0}_{-1}e$

First we find the number of a- particles. The change in mass number during the decay from uranium to lead = 238 – 206 = 32. Therefore, the number of a-particles (with mass no. 4) = 32/4 = 8

The change in atomic number (i.e, number of protons) taking place when 8 a-particles are emitted and lead is formed is = 92 – (82 + 2 × 8) = 92 – (82 + 16) = 92 – 98 = – 6

This change will take place by emitting of six β-particles.

Q.5. When the number of electrons striking the anode of an X-ray tube is increased, the .... of the emitted X-rays increases, while when the speeds of the electrons striking the anode are increased, the cut-off wavelength of the emitted X-rays......... (1986 - 2 Marks)

Ans. intensity, decreases

Solution.

Note : More the number of electrons striking the anode, more is the intensity of X-rays.

When the speed of the striking electrons on anode is increased, the emitted X-rays have greater energy. We know that energy, E = hc/λ . Therefore, when E increases then l

decreases.

Q.6. When Boron nucleus $\left({}^{10}_{5}B\right)$ is bombarded by neutrons, a -particles are emitted. The resulting nucleus is of the element .... and has the mass number ...... (1986 - 2 Marks)

Ans. lithium, 7

Solution. ${}^{10}_{5}B + {}^{1}_{0}n \longrightarrow {}^{4}_{2}He + {}^{7}_{3}Li$

The resulting nucleus is of element lithium and mass number is 7.

Q.7. Atoms having the same .... but different .... are called isotopes. (1986 - 2 Marks)

Ans. atomic number, mass number

Solution. Atomic number, mass number

Q.8. The binding energies per nucleon for deuteron ($_1H^2$) and helium ($_2He^4$) are 1.1 MeV and 7.0 MeV respectively. The energy released when two deuterons fuse to form a helium nucleus ($_2He^4$) is .... (1988 - 2 Marks)

Ans. 23.6 MeV

Solution. $_1^2H + _1^2H \longrightarrow _2^4He$

Binding energy of two deuterons

= 2 [1.1 × 2] = 4.4 MeV

Binding energy of helium nucleus = 4 × 7.0 = 28 MeV

The energy released = 28 – 4.4 = 23.6 MeV

Q.9. In the forward bias arrangement of a p-n junction rectifier, the p end is connected to the ....terminal of the battery and the direction of the current is from ....to ....in the rectifier. (1988 - 2 Marks)

Ans. positive, p-part, n-part

Solution. Positive, p-part, n-part

Q.10. .... biasing of p-n junction offers high resistance to current flow across the junction. The biasing is obtained by connecting the p- side to the .... terminal of the battery. (1990 - 2 Marks)

Ans. reverse, negative terminal

Solution. Reverse, negative terminal.

Q.11. The wavelength of the characteristic X-ray K_α line emitted by a hydrogen like element is 0.32 Å. The wavelength of the K_β line emitted by the same

element will be ....   (1990 - 2 Marks)

Ans. 0.27Å

Solution. We know that

For K_α, $\frac{1}{\lambda} = C\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$, where C is a constant

$$\Rightarrow \frac{1}{0.32\text{Å}} = C\left[\frac{1}{1^2} - \frac{1}{2^2}\right] = \frac{3C}{4} \qquad \text{....(i)}$$

For K_β, $\frac{1}{\lambda} = C\left[\frac{1}{1^2} - \frac{1}{3^2}\right] = \frac{8C}{9}$

On dividing, we get λ = 0.27 Å

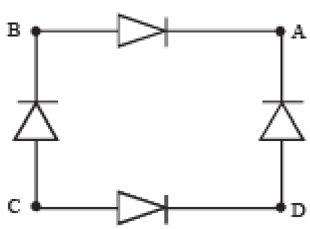
Q.12. The Bohr radius of the fifth electron of phosphorous atom (atomic number = 15) acting as a dopant in silicon (relative dielectric constant = 12) is .... Å   (1991 - 1 Mark)

Ans. 3.81Å

Solution. The fifth valence electron of phosphorous is in its third shell, i.e., n = 3. For phosphorous, Z = 15. The Bohr's radius for nth orbit

$$= \left(\frac{n^2}{Z}\varepsilon_r\right) r_0 = \frac{3^2}{15} \times 12 \times 0.529\ \text{Å} = 3.81\text{Å}$$

Q.13. For the given circuit shown in fig. to act as full wave rectifier, the a.c. input should be connected across .... and .... and the d.c. output would appear across .... and ....   (1991 - 1 Mark)



Ans. B and D, A and C

Solution. B and D is a.c. input and A and C is the d.c. output.

Case (i) When B is –ve and D is +ve Current passes from D → A → C → B

Case (ii) When B is + ve and D is – ve

Current passes from B → A → C → B
Thus curve is always from A to C in output (a d.c. current)

Q.14. In an X-ray tube, electrons accelerated through a potential difference of 15, 000 volts strike a copper target. The speed of the emitted X-ray inside the tube is .... m/s (1992 - 1 Mark)

Ans. 3×10^8

Solution. The speed of X-rays is always 3×10^8 m/s in vacuum. It does not depend on the potential differences through which electrons are accelerated in an X-ray tube.

Note : All electromagnetic waves propagate at 3×10^8 m/s in vacuum.

Q.15. In the Bohr model of the hydrogen atom, the ratio of the kinetic energy to the total energy of the electron in a quantum state n is .... (1992 - 1 Mark)

Ans. –1

Solution. $\text{K.E.} = \frac{kZe^2}{2r}$ and

Total energy $\text{T.E.} = \frac{-kZe^2}{2r}$ $\quad \therefore \frac{\text{K.E.}}{\text{T.E.}} = -1$

Q.16. In the nuclear process, $_6C^{11} \to {}_5B^{11} + \beta^+ + X$, X stands for .... (1992 - 1 Mark)

Ans. neutrino

Solution. ${}^{11}_{6}C \to {}^{11}_{5}B + \beta^+ + X \Rightarrow {}^{11}_{6}C \to {}^{11}_{5}B + {}^{0}_{+1}e + \nu$ (neutrino)

The balancing of atomic number and mass number is correct.

Therefore, X stands for neutrino.

Q.17. In a ..... biased p-n junction, the net flow of holes is from the n region to the p region. (1993 - 1 Mark)

Ans. reverse

Q.18. A potential difference of 20 kV is applied across an X-ray tube. The minimum wavelength of X-rays generated is...........Å. (1996 - 2 Marks)

Ans. 0.62Å

Solution. $\lambda_{min} = \frac{hc}{eV} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{1.6 \times 10^{-19} \times 20 \times 10^3} = 0.62 \text{Å}$

Q.19. The wavelength of K_α X-rays produced by an X-ray tube is 0.76Å. The atomic number of the anode material of the tube is..... (1996 - 2 Marks)

Ans. 41

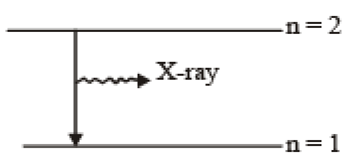
Solution.

$$\frac{1}{\lambda} = R(Z-1)^2 \left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

Since for K_α, $n_2 = 2$ and $n_1 = 1$

$$\therefore \frac{1}{0.76 \times 10^{-10}} = 1.097\,(Z-1)^2 \left[\frac{1}{1^2} - \frac{1}{2^2}\right]$$

$\Rightarrow z - 1 = 40 \quad \Rightarrow Z = 41$



Q.20. Consider the following reaction :

$${}_1^2H + {}_1^2H = {}_2^4He + Q$$

Mass of the deuterium atom = 2.0141 μ

Mass of helium atom = 4.0024 μ

This is a nuclear .... reaction in which the energy Q released is .... MeV. (1996 - 2 Marks)

Ans. fusion, 24.03
Solution. This is a nuclear fusion reaction

Energy released = (Δm) [931.5 MeV/u]

$= [2 \times 2.0141 - 4.0024] \times 931.5$ MeV

= 24.03 MeV

True/False

Q.1. The kinetic energy of photoelectrons emitted by a photosensitive surface depends on the intensity of the incident radiation. (1981- 2 Marks)

Ans. F

Solution. For photoelectric effect

$hv - hv_0 = (K.E.)max$

where h = Planck's constt.

v_0 = Threshold frequency

$\Rightarrow (K.E.)_{max} \propto n$

K.E. does not depend on the intensity of incident radiation.

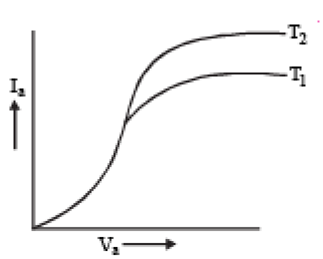
Q.2. In a photoelectric emission process the maximum energy of the photo-electrons increases with increasing intensity of the incident light. (1986 - 3 Marks)

Ans. F

Solution. $(K.E.)_{max} = hv - hv_0 \Rightarrow (K.E.)_{max} \propto v$

Thus maximum kinetic energy is proportional to frequency and not intensity.

Q.3. For a diode the variation of its anode current I_a with the anode voltage V_a at two different cathode temperatures T_1 and T_2 is shown in the figure. The temperature T_2 is greater than T_1. (1986 - 3 Marks)



Ans. T

Solution. Note : When the cathode temperature is higher, then more number of electrons will be emitted which in turn will increase the anode current.

Q.4. The order of magnitude of the density of nuclear matter is 10^4 kg m^{-2}. (1989 - 2 Marks)

Ans. F

Solution. $\text{Density} = \frac{m}{V} = \frac{A \times 1.67 \times 10^{-27}}{\frac{4}{3}\pi\left[R_0 A^{1/3}\right]^3}$

$$= \frac{1.67 \times 10^{-27}}{1.33 \times 3.14 \times (1.1 \times 10^{-15})} = 3 \times 10^{17}\,\text{kg/m}^3$$

where A = mass number.

Note : The order of nuclear density is 10^{17} kg/m^3.

Subjective Questions Part -1

Q.1. A single electron orbits around a stationary nucleus of charge + Ze. Where Z is a constant and e is the magnitude of the electronic charge. It requires 47.2 eV to excite the electron from the second Bohr orbit to the third Bohr orbit.

Find (1981- 10 Marks)

(i) The value of Z.

(ii) The energy required to excite the electron from the third to the fourth Bohr orbit.

(iii) The wavelength of the electromagnetic radiation required to remove the electron from the first Bohr orbit to infinity.

(iv) The kinetic energy, potential energy, potential energy and the angular momentum of the electron in the first Bohr orbit.

(v) The radius of the first Bohr orbit.

(The ionization energy of hydrogen atom = 13.6 eV, Bohr radius = 5.3×10^{-11} metre, velocity of light = 3×10^{8} m/sec.

Planck's constant = 6.6×10^{-34} joules - sec).

Ans. (i) 5

(ii) 16.53 eV

(iii) 3.65×10^{-9} m

(iv) 340 eV, –680 eV, $1.05 \times 10^{-3}4$ J.s

(v) 1.05×10^{-11}m

Solution.

$$E_2 = -\frac{13.6}{4}Z^2, \; E_3 = -\frac{13.6}{9}Z^2$$

$$E_3 - E_2 = -13.6Z^2\left(\frac{1}{9}-\frac{1}{4}\right) \; = +\frac{13.6\times 5}{36}Z^2$$

But $E_3 - E_2 = 47.2$ eV (Given)

$$\therefore \frac{13.6\times 5}{36}Z^2 = 47.2 \quad \therefore \; Z = \frac{\sqrt{47.2\times 36}}{13.6\times 5} = 5$$

$$\text{(ii)} \quad E_4 = \frac{-13.6}{16}Z^2$$

$$\therefore E_4 - E_3 = -13.6Z^2\left[\frac{1}{16}-\frac{1}{9}\right] = -13.6Z^2\left[\frac{9-16}{9\times 16}\right]$$

$$= \frac{+13.6\times 25\times 7}{9\times 16} = 16.53\,\text{eV}$$

$$\text{(iii)} \quad E_1 = -\frac{13.6}{1}\times 25 = -340\,\text{eV}$$

$$\therefore E = E_\infty - E_1 = 340\,\text{eV} = 340\times 1.6\times 10^{-19}\,\text{J} \quad [E_\infty = 0\,\text{eV}]$$

But $E = \frac{hc}{\lambda}$

$$\therefore \; \lambda = \frac{hc}{E} = \frac{6.6\times 10^{-34}\times 3\times 10^8}{340\times 10^{-19}\times 1.6} = 3.65\times 10^{-19}\,\text{m}$$

(iv) Total Energy of 1st orbit = – 340 eV We know that – (T.E). = K.E. [in case of electron revolving around nucleus] and 2T.E. = P.E.

∴ K.E. = 340 eV ; P.E. = – 680 eV

KEY CONCEPT :

Angular momentum in 1st orbit :

According to Bohr's postulate,

$$mvr = \frac{nh}{2\pi}$$

For $n = 1$,

$$mvr = \frac{h}{2\pi} = \frac{6.6\times 10^{-34}}{2\pi} = 1.05\times 10^{-34}\,\text{J-s.}$$

(v) Radius of first Bohr orbit

$$r_1 = \frac{5.3 \times 10^{-11}}{Z} = \frac{5.3 \times 10^{-11}}{5}$$

$= 1.06 \times 10^{-11}$ m

Q.2. Hydrogen atom in its ground state is excited by means of monochromatic radiation of wavelength 975Å. How many different lines are possible in the resulting spectrum ?

Calculate the longest wavelength amongst them. You may assume the ionization energy for hydrogen atom as 13.6 eV. (1982 - 5 Marks)

Ans. 6, 18.835×10^{-7} m

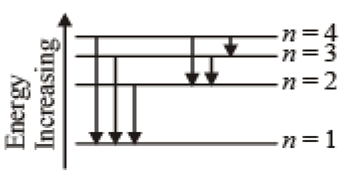
Solution.

$$E = \frac{12400}{\lambda(\text{in Å})}\text{ eV} = \frac{12400}{975} = 12.75\text{ eV} \qquad ...(i)$$

Also

$$13.6\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] = 12.75 \Rightarrow \left[\frac{1}{1} - \frac{1}{n_2^2}\right] = \frac{12.75}{13.6} \Rightarrow n_2 = 4$$

For every possible transition one downward arrow is shown therefore the possibilities are 6.



Note : For longest wavelength, the frequency should be smallest.

This corresponds to the transition from n = 4 to n = 3, the energy will be $E_4 = -\frac{13.6}{4^2}; E_3 = -\frac{13.6}{3^2}$

$$\therefore E_4 - E_3 = \frac{-13.6}{4^2} - \left(\frac{-13.6}{3^2}\right) = 13.6\left[\frac{1}{9} - \frac{1}{16}\right]$$

$$= 0.66\ \text{eV} = 0.66 \times 1.6 \times 10^{-19}\text{J} = 1.056 \times 10^{-19}\text{J}$$

$$\text{Now, } E = \frac{12400}{\lambda(\text{in Å})}\ \text{eV} \quad \therefore \quad \lambda = 18787\ \text{Å}$$

Q.3. How many electron, protons and neutr ons are th ere in a nucleus of atomic number 11 and mass number 24 ? (1982 - 2 Marks)

(i) number of electrons =

(ii) number of protons =

(iii) number of neutrons =

Ans. 0, 11, 13

Solution. (i) In a nucleus, number of electrons = 0 (∵electrons don't reside in the nucleus of atom).

(ii) number of protons = 11

(iii) number of neutrons = 24 – 11 = 13

Q.4. The ionization energy of a hydrogen like Bohr atom is 4 Rydbergs. (i) What is the wavelength of the radiation emitted when the electron jumps from the first excited state to the ground state ? (ii) What is the radius of the first orbit for this atom? (1984- 4 Marks)

Ans. (i) 300Å (ii) 2.5 × 10^{-1}1 m

Solution.

(i) $E_n = -\frac{\text{I.E.}}{n^2}$ for Bohr's hydrogen atom.

Here, I.E. = $4R$ $\therefore E_n = \frac{-4R}{n^2}$

$$\therefore E_2 - E_1 = \frac{-4R}{2^2} - \left(-\frac{4R}{1^2}\right) = 3R$$

..(i)

$$E_2 - E_1 = h\nu = \frac{hc}{\lambda} \qquad ..(ii)$$

From (i) and (ii)

$$\frac{hc}{\lambda} = 3R$$

$$\therefore \ \lambda = \frac{hc}{3R} = \frac{6.6 \times 10^{-34} \times 3 \times 10^{8}}{2.2 \times 10^{-18} \times 3} = 300 Å$$

(ii) The radius of the first orbit

Bohr's radius of hydrogen atom = 5×10^{-11} m (given)

$| E_n| = + 0.22 \times 10^{-17} Z^2 = 4R = 4 \times 2.2 \times 10–18$

$\therefore Z = 2$

$$\therefore \ r_n = \frac{r_0}{Z} = \frac{5 \times 10^{-11}}{Z} = \frac{5 \times 10^{-11}}{2} = 2.5 \times 10^{-11} \text{m}$$

Q.5. A double ionised Lithium atom is hydrogen-like with atomic number 3. (1985 - 6 Marks)

(i) Find the wavelength of the radiation required to excite the electron in Li^{++} from the first to the third Bohr orbit. (Ionisation energy of the hydrogen atom equals 13.6 eV.)

(ii) How many spectral lines ar e observed in the emission spectrum of the above excited system?

Ans. (i) 1.142×10^{-8}

(ii) 3

Solution.

$$E_n = -\frac{13.6}{n^2}Z^2 \text{ eV/atom}$$

$$\text{For } Li^{2+}, Z = 3 \quad \therefore \; E_n = \frac{-13.6\times 9}{n^2} \text{ eV/atom}$$

$$\therefore \; E_1 = -\frac{13.6\times 9}{1} \text{ and } E_3 = -\frac{13.6\times 9}{9} = -13.6$$

$\Delta E = E_3 - E_1 = -13.6 - (-13.6 \times 9)$

$= 13.6 \times 8 = 108.8$ eV/atom

$$\lambda = \frac{12400}{E \text{ (in eV)}} \text{Å} = \frac{12400}{108.8} = 114 \text{Å}$$

(ii) The spectral line observed will be three namely $3 \rightarrow 1$, $3 \rightarrow 2$, $2 \rightarrow 1$.

Q.6. A triode has plate characteristics in the form of parallel lines in the region of our interest. At a grid voltage of – 1 volt the anode current I (in milli amperes) is given in terms of plate voltage V (in volts) by the algebraic relation : I = 0.125V – 7.5

For grid voltage of – 3 volts, the current at anode voltage of 300 volts is 5 milliampere. Determine the plate resistance (r_p), transconductance (g_m) and the amplification factor (μ) for the triode. (1987 - 7 Marks)

Ans. 8 mΩ, 12.5×10^{-3} s,100

Solution. I = 0.125 V – 7.5

$$\Rightarrow dI = 0.125\, dV \qquad \text{or } \frac{dV}{dI} = \frac{1}{0.125} = 8$$

We know that plate resistance, $r_p = \frac{dV}{dI} = 8 \text{m}\Omega$

The transconductance, $g_m = \left[\frac{dI}{dV_g}\right]_{V=\text{constt}}$

At $V_g = -1$ volt, V = 300 volt, the plate current

I = [0.125 × 300 – 7.5] mA = 30 mA

Also it is given that $V_g = -3V$, $V = 300$ V and $I = 5mA$

$$\therefore g_m = \left[\frac{30-5}{-1-(-3)}\right] = \frac{25}{2} \times 10^{-3} = 12.5 \times 10^{-3} s$$

The characteristics are given in the form of parallel lines. Amplification factor

$= r_p\, g_m = 8 \times 10^3 \times 12.5 \times 10^{-3} = 100$

Q.7. A particle of charge equal to that of an electron, – e, and mass 208 times the mass of the electron (called a mu-meson) moves in a circular orbit around a nucleus of charge + 3e. (Take the mass of the nucleus to be infinite). Assuming that the Bohr model of the atom is applicable to this system. (1988 - 6 Marks)

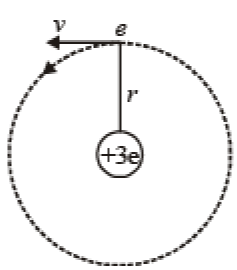
(i) Derive an expression for the radius of the nth Bohr orbit.

(ii) Find the value of n for which the radius of the orbit is approximately the same as that of the first Bohr orbit for the hydrogen atom.

(iii) Find the wavelength of the radiation emitted when the mu-meson jumps from the third orbit of the first orbit.

Ans. (i) $r = \dfrac{n^2 h^2 \epsilon_0}{624 \pi m e^2}$

Solution. (i) Let m be the mass of electron. Then the mass of mumeson is 208 m. According to Bohr's postulate, the angular momentum of mu-meson should be an integral multiple of h/2π.



$$\therefore \quad (208m)\, vr = \frac{nh}{2\pi}$$

$\therefore \quad v = \dfrac{nh}{2\pi \times 208mr} = \dfrac{nh}{416\pi mr}$ ....(i)

Note: Since mu-meson is moving in a circular path, therefore, it needs centripetal force which is provided by the electrostatic force between the nucleus and mumeson.

$$\therefore \quad \frac{(208m)v^2}{r} = \frac{1}{4\pi\varepsilon_0} \times \frac{3e \times e}{r^2}$$

$$\therefore \quad r = \frac{3e^2}{4\pi\varepsilon_0 \times 208mv^2}$$

Substituting the value of v from (1), we get

$$r = \frac{3e^2 \times 416\pi mr \times 416\pi mr}{4\pi\varepsilon_0 \times 208mn^2h^2}$$

$$\Rightarrow r = \frac{n^2h^2\varepsilon_0}{624\pi me^2} \qquad \text{....(i)}$$

(ii) The radius of the first orbit of the hydrogen atom

$$= \frac{\varepsilon_0 h^2}{\pi me^2} \qquad \text{...(ii)}$$

To find the value of n for which the radius of the orbit is approximately the same as that of the first Bohr orbit for hydrogen atom, we equate eq. (i) and (ii)

$$\frac{n^2h^2\varepsilon_0}{624\pi me^2} = \frac{\varepsilon_0 h^2}{\pi me^2} \quad \Rightarrow n = \sqrt{624} \approx 25$$

(iii) $\dfrac{1}{\lambda} = 208R \times Z^2\left[\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right]$

$$\Rightarrow \frac{1}{\lambda} = 208 \times 1.097 \times 10^7 \times 3^2\left[\frac{1}{1^2} - \frac{1}{3^2}\right]$$

$$\Rightarrow \lambda = 5.478 \times 10^{-11}\,\mathbf{m}$$

Q.8. A gas of identical hydrogen-like atoms has some atoms in the lowest (ground) energy level A and some atoms in a particular upper (excited) energy level B and there are no atoms in any other energy level. The atoms of the gas make transition to a higher energy level by absorbing monochromatic light of photon energy 2.7 eV. Subsequently, the atoms emit radiation of only six different photon energies. Some of the emitted photons have energy 2.7 eV, some have energy more and some have less than 2.7 eV. (1989 - 8 Marks)

(i) Find the prin cipal quantum number of the initially excited level B.

(iii) Find the ionization energy for the gas atoms.

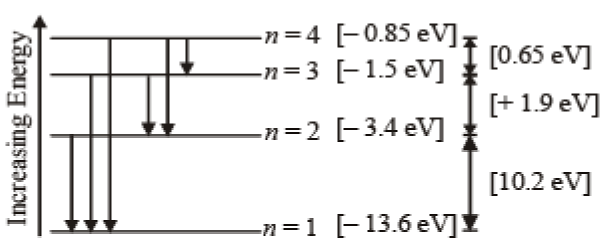
(iii) Find the maximum and the minimum energies of the emitted photons.

Ans. (i) 2

(ii) 14.46 eV

(iii) 13.5 eV, 0.7 eV

Solution. (i) The transition state of six different photon energies are shown.



Since after absorbing monochromatic light, some of the emitted photons have energy more and some have less than 2.7 eV, this indicates that the excited level B is n = 2. (This is because if n = 3 is the excited level then energy less than 2.7 eV is not possible).

(ii) For hydrogen like atoms we have

$$E_n = \frac{-13.6}{n^2} Z^2 \text{ eV/atom}$$

$$E_4 - E_2 = \frac{-13.6}{16} Z^2 - \left(\frac{-13.6}{4}\right) Z^2 = 2.7$$

$$\Rightarrow Z^2 \times 13.6 \left[\frac{1}{4} - \frac{1}{16}\right] = 2.7$$

$$\Rightarrow Z^2 = \frac{2.7}{13.6} \times \frac{4 \times 16}{12} \Rightarrow \text{I.E.} = 13.6 Z^2 \left(\frac{1}{1^2} - \frac{1}{\infty^2}\right)$$

$$= 13.6 \times \frac{2.7}{13.6} \times \frac{4 \times 16}{12} = 14.46 \text{ eV}$$

(iii) Max. Energy

$$E_4 - E_3 = -13.6 Z^2 \left(\frac{1}{4^2} - \frac{1}{1^2}\right)$$

$$= 13.6 \times \frac{2.7}{13.6} \times \frac{4 \times 16}{12} \times \frac{15}{16} = 13.5 \text{ eV}$$

Min. Energy

$$E_4 - E_3 = -13.6 Z^2 \left(\frac{1}{4^2} - \frac{1}{3^2}\right)$$

$$= 13.6 \times \frac{2.7}{13.6} \times \frac{4 \times 16}{12} \times \frac{7}{9 \times 16} = 0.7 \text{eV}$$

Q.9. Electron s in hydrogen like atom (Z = 3) make transitions from the fifth to the fourth orbit and from the fourth to the third orbit. The resulting radiations are incident normally on a metal plate and eject photoelectrons. The stopping potential for the photoelectrons ejected by the shorter wavelength is 3.95 volts. Calculate the work function of the metal and the stopping potential for the photoelectrons ejected by the longer wavelength. (1990 - 7 Marks)

(Rydberg constant = 1.094 × 10 7 m -1)

Ans. 2 eV, 0.754 V

Solution. For hydrogen like atom energy of the nth orbit is

$$E_n = -\frac{13.6}{n^2} Z^2 \text{ eV/atom}$$

For transition from n = 5 to n = 4,

$$hv = 13.6 \times 9\left[\frac{1}{16} - \frac{1}{25}\right] = \frac{13.6 \times 9 \times 9}{16 \times 25} = 2.754\,\text{eV}$$

For transition from n = 4 to n = 3,

$$hv' = 13.6 \times 9\left[\frac{1}{9} - \frac{1}{16}\right] = \frac{13.6 \times 9 \times 7}{9 \times 16} = 5.95\text{eV}$$

For transition n = 4 to n = 3, the frequency is high and hence wavelength is short.

For photoelectric effect, hn' – W = eV_0, where W = work function

$5.95 \times 1.6 \times 10^{-19}$ – W = 1.6 × 10–19 × 3.95

⇒ W = $2 \times 1.6 \times 10^{-19}$ = 2 eV
Again applying hn – W = eV'_0

We get, $2.754 \times 1.6 \times 10^{-19} - 2 \times 1.6 \times 10^{-19}$ = 1.6 × 10–19 V'_0

⇒ V'_0 = 0.754 V
Q.10. It is proposed to use the nuclear fusion reaction $_1^2H + _1^2H \rightarrow _2^4He$ in a nuclear reactor of 200 MW rating. If the energy from the above reaction is used with a 25 per cent efficiency in the reactor, how many grams of deuterium fuel will be needed per day. (The masses of $_1^2H$ and $_2^4He$ are 2.0141 atomic mass units and 4.0026 atomic mass units respectively)

Ans. 119.6 gm

Solution. Energy required per day

E = P × t = $200 \times 10^6 \times 24 \times 60 \times 60$

= 1.728×10^{13}J

Energy released per fusion reaction

= [2 (2.0141) – 4.0026] × 931.5 MeV

= 23.85 MeV = 23.85 × 106 × 1.6 × 10^{-19}

= 38.15 × 10–13J

∴No. of fusion reactions required

$$= \frac{1.728 \times 10^{13}}{38.15 \times 10^{-13}} = 0.045 \times 10^{26}$$

∴ No. of deuterium atoms required

$= 2 \times 0.045 \times 10^{26} = 0.09 \times 10^{26}$

Number of moles of deuterium atoms

$$= \frac{0.09 \times 10^{26}}{6.02 \times 10^{23}} = 14.95$$

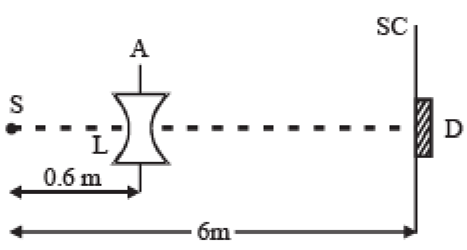
∴ Mass in gram of deuterium atoms

= 14.95 × 2 = 29.9 g

But the efficiency is 25%.

Therefore, the actual mass required = 119.6 g

Q.11. A monochromatic point source radiating wavelength 6000 Å, with power 2 watt, an aperture A of diameter 0.1 m and a large screen SC are placed as shown in fig. A photoemissive detector D of surface area 0.5 cm2 is placed at the centre of the screen. The efficiency of the detector for the photoelectron generation per incident photon is 0.9. (1991 - 2 + 4 + 2 Marks)



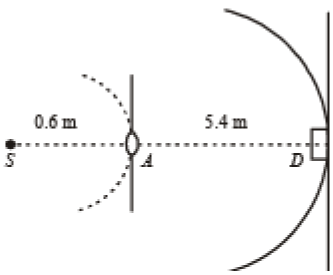
(a) Calculate the photon flux at the centre of the screen and the photocurrent in the detector.

(b) If the concave lens L of focal length 0.6 m is inserted in the aperture as shown, find the new values of photon flux and photocurrent. Assume a uniform average transmission of 80% from the lens.

(c) If the work function of the photoemissive sur face is 1eV, calculate the values of the stopping potential in the two cases (without and with the lens in the aperture).

Solution. Energy of one photon $E = \frac{hc}{\lambda} = \frac{(6.6 \times 10^{-34})(3.0 \times 10^{8})}{6000 \times 10^{-10}}$

$= 3.3 \times 10^{-19}$ J



Power of the source is 2 W or 2 J/s. Therefore, number of photons emitting per second,

$$n_1 = \frac{2}{3.3 \times 10^{-19}} = 6.06 \times 10^{18} \,/\,\mathrm{s}$$

At distance 0.6 m, number of photons incident per unit area per unit time :

$$n_2 = \frac{n_1}{4\pi(0.6)^2} = 1.34 \times 10^{18} \,/\,\mathrm{m}^2 \,/\,\mathrm{s}$$

Area of aperture is,

$$S_1 = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.1)^2 = 7.85 \times 10^{-3}\,\mathrm{m}^2$$

∴ Total number of photons incident per unit time on the aperture,

$n_3 = n_2 s_1 = (1.34 \times 10^{18})\ (7.85 \times 10_{-3})$ / s

$= 1.052 \times 10^{1}6$ / s

The aperture will become new source of light.

Now these photons are further distributed in all directions.

Hence, at the location of detector, photons incident per unit area per unit time :

$$n_4 = \frac{n_3}{4\pi(6-0.6)^2} = \frac{1.052\times10^{16}}{4\pi(5.4)^2}$$
$$= 2.87 \times 10^{13}\ \mathrm{s^{-1}\,m^{-2}}$$

This is the photon flux at the centre of the screen. Area of detector is 0.5 cm2 or 0.5 × 10^{-4} m^2. Therefore, total number of photons incident on the detector per unit time :

$$n_5 = (0.5\times10^{-4})(2.87\times10^{13}d) = 1.435\times10^{9}\,\mathrm{s^{-1}}$$

The efficiency of photoelectron generation is 0.9. Hence, total photoelectrons generated per unit time :

$n_6 = 0.9n_5 = 1.2915 \times 10^9\ s^{-1}$

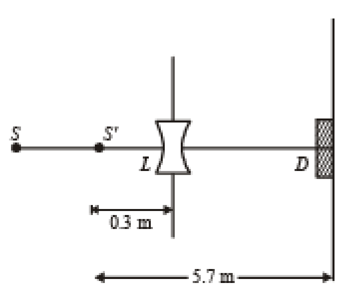
or, photocurrent in the detector :

$i = (e)n_6 = (1.6 \times 10^{-19})\ (1.2915 \times 10^9) = 2.07 \times 10^{-10}$ A

(b) Using the lens formula :

$$\frac{1}{v} - \frac{1}{-0.6} = \frac{1}{-0.6} \text{ or } v = -0.3\ \mathrm{m}$$

i.e., image of source (say S', is formed at 0.3 m from the lens.)



Total number of photons incident per unit time on the lens are still n_3 or 1.052 × 10^{16}/s. 80% of it transmits to second medium. Therefore, at a distance of 5.7 m from

S' number of photons incident per unit are per unit time will be :

$$n_7 = \frac{(80/100)(1.05 \times 10^{16})}{(4\pi)(5.7)^2} = 2.06 \times 10^{13} s^{-1} m^{-2}$$

This is the photon flux at the detector.

New value of photocurrent is :

i = $(2.06 \times 10^{13})(0.5 \times 10^{-4})(0.9)(1.6 \times 10^{-19})$

= 1.483×10^{-10} A

(c) For stopping potential

$$\frac{hc}{\lambda} = (E_K)_{max} + W = eV_0 + W$$

$$\therefore \quad eV_0 = \frac{hc}{\lambda} - W = \frac{3.315 \times 10^{-19}}{1.6 \times 10^{-19}} - 1 = 1.07 \text{eV}$$

$$\therefore \quad V_0 = 1.07 \text{ Volt}$$

Note : The value of stopping potential is not affected by the presence of concave lens as it changes the intensity and not the frequency of photons. The stopping potential depends on the frequency of photons.

Q.12. A nucleus X, initially at rest, undergoes alpha decay according to the equation, $^{A}_{92}X \rightarrow ^{228}_{Z}Y + \alpha$

(a) Find the values of A and Z in the above process.

(b) The alpha particle pr oduced in the above process is found to move in a circular track of radius 0.11 m in a uniform magnetic field of 3 Tesla. Find the energy (In MeV) released during the process and the binding energy of the parent nucleus X.

Given that : m(Y) = 228.03 u; m $\left(^{1}_{0}n\right) = 1.009\,u.$

$m\left({}^{4}_{2}\mathrm{He}\right) = 4.003\ u;\ m\left({}^{1}_{1}\mathrm{H}\right) = 1.008\ u$

Ans. (a) 232, 90

(b) 5.34 MeV, 1823 MeV

Solution. **(a)** ${}^{A}_{92}\mathrm{X} \rightarrow {}^{228}_{X}\mathrm{Y} + {}^{4}_{2}\mathrm{He}$

A = 228 + 4 = 232; 92 = Z + 2 ⇒ Z = 90

(b) Let v be the velocity with which α - particle is emitted.

Then

$$\frac{mv^2}{r} = qvB \Rightarrow v = \frac{qrB}{m} = \frac{2 \times 1.6 \times 10^{-19} \times 0.11 \times 3}{4.003 \times 10^{-27}}$$
$$\Rightarrow v = 1.59 \times 10^{7}\,\mathrm{ms}^{-1}.$$

Applying law of conservation of linear momentum during α-decay we get

$m_Y v_Y = m\alpha v\alpha$ ...(1)

The total kinetic energy of α-particle and Y is

$$E = \mathrm{K.E.}_{\alpha} + \mathrm{K.E.}_{Y} = \frac{1}{2} m_\alpha {v_\alpha}^2 + \frac{1}{2} m_Y {v_Y}^2$$

$$= \frac{1}{2} m_\alpha v_\alpha^2 + \frac{1}{2} m_Y \left[\frac{m_\alpha v_\alpha}{m_Y}\right]^2 = \frac{1}{2} m_\alpha {v_\alpha}^2 + m_\alpha v_\alpha^2 + \frac{m_\alpha^2 v_\alpha^2}{2m_Y}$$

$$= \frac{1}{2} m_\alpha v_\alpha^2 \left[1 + \frac{m_\alpha}{m_Y}\right]$$

$$= \frac{1}{2} \times 4.033 \times 1.6 \times 10^{-27} \times (1.59 \times 10^{7})^2 \left[1 + \frac{4.003}{228.03}\right] \mathrm{J}$$

= 8.55 × 10–13 J

= 5.34 MeV

∴ Mass equivalent of this energy

$= \frac{5.34}{931.5} = 0.0051$ a.m.u.

Also, $m_X = m_Y + m_\alpha$ + mass equivalent of energy (E)

= 228.03 + 4.003 + 0.0057 = 232.0387 u.

The number of nucleus = 92 protons + 140 neutron.

∴ Binding energy of nucleus X

= [92 × 1.008 + 140 × 1.009] – 232.0387

= 1.9571 u = 1.9571 × 931.5

= 1823 MeV.

Q.13. Light from a discharge tube containing hydrogen atoms falls on the surface of a piece of sodium. The kinetic energy of the fastest photoelectrons emitted from sodium is 0.73 eV. The work function for sodium is 1.82 eV. Find (1992 - 10 Marks)

(a) the energy of the photons causing the photoelectric emission,

(b) the quantum numbers of the two levels involved in the emission of these photons,

(c) the change in the angular momentum of the electron in the hydrogen atom in the above transition, and

(d) the recoil speed of the emitting atom assuming it to be at rest before the transition. (Ionization potential of hydrogen is 13.6 eV)

Ans. (a) 2.55 eV (b) 4 → 2

(c) $-\frac{h}{\pi}$

(d) 0.814 m/s

Solution. (a) The energy of photon causing photoelectric emission

= Work function of sodium metal + KE of the fastest photoelectron

= 1.82 + 0.73 = 2.55 eV

(b) We know that $E_n = \frac{-13.6}{n^2}\frac{\text{eV}}{\text{atom}}$ for hydrogen atom.

Let electron jump from n_2 to n_1 then

$$E_{n_2} - E_{n_1} = \frac{-13.6}{n_2^2} - \left(\frac{-13.6}{n_1^2}\right)$$

$$\Rightarrow 2.55 = 13.6\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

By hit and trial we get $n_2 = 4$ and $n_1 = 2$

[angular momentum mvr = nh/2π]

(c) Change in angular momentum

$$= \frac{n_1 h}{2\pi} - \frac{n_2 h}{2\pi} = \frac{h}{2\pi}(2-4) = \frac{h}{2\pi} \times (-2) = -\frac{h}{\pi}$$

(d) The momentum of emitted photon can be found by de Broglie relationship

$$\lambda = \frac{h}{p} \Rightarrow p = \frac{h}{\lambda} = \frac{hv}{c} = \frac{E}{c} \quad \therefore p = \frac{2.55 \times 1.6 \times 10^{-19}}{3 \times 10^8}$$

Note : The atom was initially at rest the recoil momentum of the atom will be same as emitted photon (according to the conservation of angular momentum).

Let m be the mass and v be the recoil velocity of hydrogen atom then

$$m \times v = \frac{2.55 \times 1.6 \times 10^{-19}}{3 \times 10^8}$$

$$\Rightarrow v = \frac{2.55 \times 1.6 \times 10^{-19}}{3 \times 10^8 \times 1.67 \times 10^{-27}} = 0.814 \text{ m/s}$$

Q.14. A small quantity of solution containing Na^{24} radio nuclide (half life = 15 hour) of activity 1.0 microcurie is injected into the blood of a person. A sample of the blood of volume 1 cm^3 taken after 5 hour show an activity of 296 disintegrations per minute. Determine the total volume of the blood in the body of the person. Assume that radioactive solution mixes uniformly in the blood of the person. (1 curie = 3.7×10^{10} disintegrations per second) (1994 - 6 Marks)

Ans. 5.95 ℓ

Solution. $t_{1/2}$ = 15 hours

Activity initially $A_0 = 10^{-6}$ Curie (in small quantity of solution of ^{24}Na) = 3.7×10^4dps

Observation of blood of volume 1 cm^3

After 5 hours, A = 296 dpm

The initial activity can be found by the formula

$$t = \frac{2.303}{\lambda}\log_{10}\frac{A_0}{A} \Rightarrow 5 = \frac{2.303}{0.693/15} \times \log_{10}\frac{A_0}{296}$$

$$\Rightarrow \log_{10}\frac{A_0}{296} = \frac{5 \times 0.693}{2.303 \times 15} = \frac{0.3010}{3} = 0.10033$$

$$\Rightarrow \frac{A_0}{296} = 1.26 \Rightarrow A_0 = 373 \text{ dpm} = \frac{373}{60} \text{ dps}$$

This is the activity level in 1 cm^3. Comparing it with the initial activity level of 3.7×10^4 dps we find the volume of blood.

$$V = \frac{3.7 \times 10^4}{373/60} = 5951.7 \text{ cm}^3 = 5.951 \text{ litre}$$

Q.15. A **hydrogen like atom (atomic number Z) is in a higher excited state of quantum number n. The excited atom can make a transition to the first excited state by successively emitting two photons of energy 10.2 and 17.0 eV respectively.**

Alternately, the atom from the same excited state can make a transition to the second excited state by successively emitting two photons of energies 4.25 eV and 5.95 eV respectively. (1994 - 6 Marks)
Determine the values of n and Z. (Ionization energy of Hatom = 13.6 eV)

Ans. 6, 3

Solution. For hydrogen like atoms

$$E_n = -\frac{13.6}{n^2}Z^2 \text{ eV/atom}$$

Given $E_n - E_2 = 10.2 + 17 = 27.2$ eV ..(i)

$E_n - E_3 = 4.24 + 5.95 = 10.2$ eV

$\therefore E_3 - E_2 = 17$

$$\text{But } E_3 - E_2 = -\frac{13.6}{9}Z^2 - \left(-\frac{13.6}{4}Z^2\right)$$

$$= -13.6Z^2\left[\frac{1}{9} - \frac{1}{4}\right]$$

$$= -13.6Z^2\left[\frac{4-9}{36}\right] = \frac{13.6\times 5}{36}Z^2$$

$$\therefore \frac{13.6\times 5}{36}Z^2 = 17 \Rightarrow Z = 3$$

$$E_n - E_2 = -\frac{13.6}{n_2}\times 3^2 - \left[-\frac{13.6}{2^2}\times 3^2\right]$$

$$= -13.6\left[\frac{9}{n^2} - \frac{9}{4}\right] = -13.6\times 9\left[\frac{4-n^2}{4n^2}\right] \quad ...(ii)$$

From eq. (i) and (ii),

$$-13.6 \times 9\left[\frac{4-n^2}{4n^2}\right] = 27.2$$

$$\Rightarrow -122.4\,(4-n^2) = 108.8n^2$$

$$\Rightarrow n^2 = \frac{489.6}{13.6} = 36 \Rightarrow n = 6$$

Q.16. An electron, in a hydrogen-like atom, is in an excited state.

It has a total energy of –3.4 eV. Calculate (i) the kinetic energy and (ii) the de Broglie wavelength of the electron. (1996 - 3 Marks)

Ans. (i) 3.4 eV

(ii) 0.66×10^{-9} m

Solution. (i) En = – 3.4 eV

The kinetic energy is equal to the magnitude of total energy in this case.

∴ K.E. = + 3.4 eV

(ii) The de Broglie wavelength of electron

$$\lambda = \frac{h}{\sqrt{2mK}} = \frac{6.64 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 3.4 \times 1.6 \times 10^{-19}}}\ \text{eV}$$

$$= 0.66 \times 10^{-9}\ \text{m}$$

Q.17. At a given instant there are 25% undecayed radio-active nuclei in a sample. After 10 seconds the number of undecayed nuclei reduces to 12.5%. Calculate (i) mean-life of the nuclei, and (ii) the time in which the number of undecayed nuclei will further reduce to 6.25% of the reduced number. (1996 - 3 Marks)

Ans. (i) 14.43 sec.

(ii) 40 sec.

Solution. (i) From the given information, it is clear that half life of the radioactive nuclei is 10 sec (since half the amount is consumed in 10 second. 12.5% is half of 25% pls. note).

Mean life

$$\tau = \frac{1}{\lambda} = \frac{1}{0.693/t_{1/2}} = \frac{t_{1/2}}{0.693} = \frac{10}{0.693} = 14.43 \text{ sec.}$$

(ii) $N = N_0 e^{-\lambda t} \Rightarrow \frac{N}{N_0} = \frac{6.25}{100}$

$$\lambda = 0.0693\,\text{s}^{-1}$$

$$\frac{6.25}{100} = e^{-0.0693t} \Rightarrow e^{+0.0693t} = \frac{100}{6.25} = 16$$

$$0.0693t = \ln 16 = 2.773 \text{ or } t = \frac{2.733}{0.0693} = 40 \text{ sec.}$$

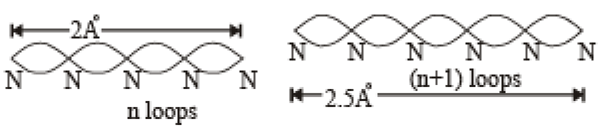
Q.18. Assume that the de Broglie wave associated with an electron can form a standing wave between the atoms arranged in a one dimensional array with nodes at each of the atomic sites. It is found that one such standing wave is formed if the distance d between the atoms of the array is 2Å. A similar standing wave is again formed if d is increased to 2.5 Å but not for any intermediate value of d. Find the energy of the electrons in electron volts and the least value of d for which the standing wave of the type described above can form. **(1997 - 5 Marks)**

Ans. 151 eV, 0.5 Å

Solution. As nodes are formed at each of the atomic sites, hence

$$2\text{Å} = n\left(\frac{\lambda}{2}\right) \qquad ...(1)$$

[∵Distance between successive nodes = λ/2]



$$\text{and } 2.5\text{ Å} = (n+1)\frac{\lambda}{2}$$

$$\therefore \frac{2.5}{2} = \frac{n+1}{n}, \frac{5}{4} = \frac{n+1}{n} \text{ or } n = 4$$

Hence, from equation (1),

$$2\text{Å} = 4\frac{\lambda}{2} \text{ i.e., } \lambda = 1\text{Å}$$

Now, de broglie wavelength is given by

$$\lambda = \frac{h}{\sqrt{2mK}} \text{ or } K = \frac{h^2}{\lambda^2 .2m}$$

$$\therefore \quad K = \frac{(6.63\times10^{-34})^2}{(1\times10^{-10})^2\times2\times9.1\times10^{-31}\times1.6\times10^{-19}} \text{ eV}$$

$$= \frac{(6.63)^2}{8\times9.1\times1.6}\times10^2 \text{ eV} = 151 \text{ eV}$$

d will be minimum, when

$$n = 1,\ d_{\min} = \frac{\lambda}{2} = \frac{1\text{Å}}{2} = 0.5\text{Å}$$

Subjective Questions Part -2

Q.19. The element Curium $^{248}_{96}$ Cm has a mean life of 10^{13} seconds.

Its pirmary decay modes are spontaneous fission and adecay, the former with a probability of 8% and the latter with a probability of 92%. Each fission releases 200 MeV of energy. The masses involved in a-decay are as follows: $^{248}_{96}\text{Cm} = 248.072220u$, $^{244}_{94}\text{Pu}$ = 244.064100 u and $^{4}_{2}\text{He}=4.002603\,u$. Calculate the power output from a sample of 10^{20} Cm atoms. (1 u = 931 MeV/c^2.) (1997 - 5 Marks)

Ans. 3.32×10^{-5} W

Solution. The reaction involved in a-decay is

$$^{248}_{96}\text{Cm} \rightarrow {}^{244}_{94}\text{Pu} + {}^{4}_{2}\text{He}$$

Mass defect

$$\Delta m = \text{Mass of } {}^{248}_{96}\text{Cm} - \text{Mass of } {}^{244}_{94}\text{Pu} - \text{Mass of } {}^{4}_{2}\text{He}$$

= (248.072220 – 244.064100 – 4.002603) u

= 0.005517u

Therefore, energy released in a-decay will be

E_α = (0.005517 × 931) MeV = 5.136 MeV

Similarly, $E_{fission}$ = 200 MeV (given)

Mean life is given as $t_{mean} = 10^{13}\text{s} = \frac{1}{\lambda}$

∴ Disintegration constant $\lambda = 10^{-13}\ \text{s}^{-1}$

Rate of decay at the moment when number of nuclei are 10^{20} is

$$\frac{dN}{dt} = \lambda N = (10^{-13})(10^{20}) = 10^{7}\,\text{dps}$$

Of these distintegrations, 8% are in fission and 92% are in a-decay.

Therefore, energy released per second

$= (0.08 \times 10^7 \times 200 + 0.92 \times 10^7 \times 5.136)$ MeV

$= 2.074 \times 10^8$ MeV

$\therefore$ Power output (in watt) = Energy released per second (J/s)

$= (2.074 \times 10^8)(1.6 \times 10^{-13})$

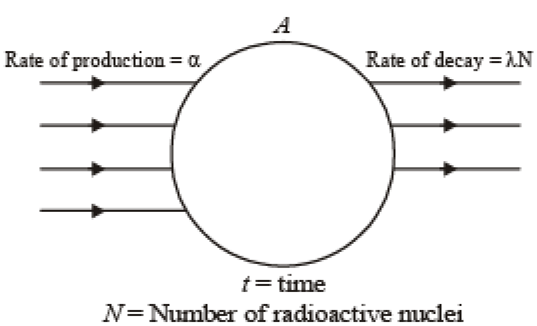
Q.20. Nuclei of a radioactive element A are being produced at a constant rate α. The element has a decay constant λ. At time t = 0, there are N_0 nuclei of the element. (1998 - 8 Marks)

(a) Calculate the number N of nuclei of A at time t.

(b) If $\alpha = 2N_0\lambda$, calculate the number of nuclei of A after one half-life of A, and also the limiting value of N as $t \to \infty$.

Ans. (a) $\frac{1}{\lambda}\left[\alpha - (\alpha - \lambda N_0)e^{-\lambda t}\right]$ (b) $\frac{3}{2}N_0, 2N_0$

Solution.



(a) Let at time ‘t’ number of radioactive nuclei are N.

Net rate of formation of nuclei of A.

$$\frac{dN}{dt} = \alpha - \lambda N \text{ or } \frac{dN}{\alpha - \lambda N} = dt$$

$$\text{or } \int_{N_0}^{N} \frac{dN}{\alpha - \lambda N} = \int_0^t dt$$

Solving this equation, we get

$$N = \frac{1}{\lambda}\left[\alpha - (\alpha - \lambda N_0)e^{-\lambda t}\right] \quad \text{........(1)}$$

(b) Substituting a = $2\lambda N_0$ and

$t = t_{1/2} = \frac{\ln(2)}{\lambda}$ in equation (1),

we get, $N = \frac{3}{2}N_0$

(ii) Substituting $\alpha = 2\lambda N_0$ and $t \to \infty$ in equation (1), we get

$$N = \frac{\alpha}{\lambda} = 2N_0.$$

Q.21. Photoelectrons are emitted when 40 nm radiation is incident on a surface of work function 1.9 eV. These photoelectrons pass through a region containing α-particles. A maximum energy electron combines with an α–particle to form a He^+ ion, emitting a single photon in this process. He+ ions thus formed are in their fourth excited state. Find the energies in eV of the photons, lying in the 2 to 4 eV range, that are likely to be emitted during and after the combination. [Take h = 4.14×10^{-15} eV.s.] (1999 - 5 Marks)

Ans. 3.4 eV, 3.84 eV and 2.64 eV

Solution. The energy of the incident photon is

$$E_1 = \frac{hc}{\lambda} = \frac{(4.14\times10^{-15}\text{eVs})(3\times10^{8}\text{m/s})}{(400\times10^{-9}\text{m})} = 3.1\text{eV}$$

The maximum kinetic energy of the emitted electrons is $E_{max} = E_1 - W$ = 3.1 eV – 1.9 eV = 1.2eV

It is given that,

$$\begin{pmatrix}\text{Emitted electrons} \\ \text{of maximum energy}\end{pmatrix} + {}_2He^{2+} \longrightarrow \underset{\text{in 4th excited state}}{He^{+}} + \text{photon}$$

The fourth excited state implies that the electron enters in the n = 5 state.

In this state its energy is

$$E_5 = -\frac{(13.6\text{eV})Z^2}{n^2} = -\frac{(13.6\text{eV})(2)^2}{5^2}$$
$$= -2.18\text{ eV.}$$

The energy of the emitted photon in the above combination reaction is $E = E_{max} + (-E_5) = 1.2\text{ eV} + 2.18\text{ eV} = 3.4\text{ eV}$

Note : After the recombination reaction, the electron may undergo transition from a higher level to a lower level thereby emitting photons.

The energies in the electronic levels of He^+ are

$$E_4 = \frac{(-13.6\text{eV})(2^2)}{4^2} = -3.4\text{eV}$$

$$E_3 = \frac{(-13.6\text{eV})(2^2)}{3^2} = -6.04\text{eV}$$

$$E_2 = \frac{(-13.6\text{eV})(2^2)}{2^2} = -13.6\text{eV}$$

The possible transitions are n = 5 → n = 4

$\Delta E = E_5 - E_4 = [-2.18 - (-3.4)]\text{ eV} = 1.28\text{ eV}$

n = 5 → n = 3

$\Delta E = E_5 - E_3 = [-2.18 - (-6.04)]\text{ eV} = 3.84\text{ eV}$

n = 5 → n = 2

$\Delta E = E_5 - E_2 = [-2.18 - (-13.6)]\text{ eV} = 11.4\text{ eV}$

$n = 4 \rightarrow n = 3$

$\Delta E = E_4 - E_3 = [-3.4 - (-6.04)]$ eV $= 2.64$ eV

Hence, the photons that are likely to be emitted in the range of 2 eV to 4 eV are 3.4 eV, 3.84 eV and 2.64 eV.

Q.22. A hydrogen-like atom of atomic number Z is in an excited state of quantum number 2n. It can emit a maximum energy photon of 204 eV. If it makes a transition to quantum state n, a photon of energy 40.8 eV is emitted. Find n, Z and the ground state energy (in eV) for this atom. Also calculate the minimum energy (in eV) that can be emitted by this atom during de-excitation. Ground state energy of hydrogen atom is –13.6 eV. (2000 - 6 Marks)

Ans. 2, 4, 10.5 eV

Solution. Energy for an orbit of hydrogen like atoms is

$$E_n = -\frac{13.6Z^2}{n^2}$$

For transition from 2n orbit to 1 orbit

$$\text{Maximum energy} = 13.6Z^2\left(\frac{1}{1} - \frac{1}{(2n)^2}\right)$$

$$\Rightarrow 204 = 13.6Z^2\left(\frac{1}{1} - \frac{1}{4n^2}\right) \quad \text{...(i)}$$

Also for transition $2n \rightarrow n$.

$$40.8 = 13.6Z^2\left(\frac{1}{n^2} - \frac{1}{4n^2}\right) \Rightarrow 40.8 = 13.6Z^2\left(\frac{3}{4n^2}\right)$$

$$\Rightarrow 40.8 = 40.8\,\frac{Z^2}{4n^2} \Rightarrow 4n^2 = Z^2 \text{ or } 2n = Z \text{ ....(ii)}$$

From (i) and (ii)

$$204 = 13.6Z^2\left(1 - \frac{1}{Z^2}\right) = 13.6Z^2 - 13.6$$

$$13.6Z^2 = 204 + 13.6 = 217.6$$

$$Z^2 = \frac{217.6}{13.6} = 16,\ Z = 4,\ n = \frac{Z}{2} = \frac{4}{2} = 2$$

$$\text{orbit no.} = 2n = 4$$

For minimum energy = Transition from 4 to 3.

$$E = 13.6 \times 4^2\left(\frac{1}{3^2} - \frac{1}{4^2}\right) = 13.6 \times 4^2\left(\frac{7}{9 \times 16}\right)$$

$$= 10.5 \text{ eV.}$$

Hence n = 2, Z = 4, E_{min} = 10.5 eV

Q.23. When a beam of 10.6 eV photons of intensity 2.0 W/m^2 falls on a platinum surface of area 1.0 × 10^{-4} m^2 and work function 5.6 eV, 0.53% of the incident photons eject photoelectrons.

Find the number of photoelectrons emitted per second and their minimum and maximum energies (in eV). Take 1eV = 1.6 × 10^{-19} J. (2000 - 4 Marks)

Ans. 6.25 × 10^{11}, 0 eV, 5 eV

Solution. No. of photons/sec

$$= \frac{\text{Energy incident on platinum surface per second}}{\text{Energy of one photon}}$$

No. of photons incident per second

$$= \frac{2 \times 10 \times 10^{-4}}{10.6 \times 1.6 \times 10^{-19}} = 1.18 \times 10^{14}$$

As 0.53% of incident photon can eject photoelectrons

∴ No. of photoelectrons ejected per second

$$= 1.18 \times 10^{14} \times \frac{0.53}{100} = 6.25 \times 10^{11}$$

Minimum energy = 0 eV,

Maximum energy = (10.6 – 5.6) eV = 5 eV

Q.24. In a nuclear reaction ^{235}U undergoes fission liberating 200 MeV of energy. The reactor has a 10% efficiency and produces 1000 MW power. If the reactor is to function for 10 years, find the total mass of uranium required. (2001 - 5 Marks)

Ans. 38451 Kg

Solution. The formula for η of power will be

$$\eta = \frac{P_{out}}{P_{in}}$$

$$\therefore \quad P_{in} = \frac{P_{out}}{\eta} = \frac{1000 \times 10^6}{0.1} = 10^{10}\,\mathrm{W}$$

Energy required for this power is given by

E = P × t

= 10^{10} × 86,400 × 365 × 10

= 3.1536 × 10^{18} J

200 × 1.6 × 10–13 J of energy is released by 1 fission

∴ 3.1536 × 10^{18} J of energy is released by

$$\frac{3.1536 \times 10^{18}}{200 \times 1.6 \times 10^{-13}} \text{ fission}$$

= 0.9855 × 10^{29} fission

= 0.985 × 10^{29} of U^{235} atoms.

6.023 × 10^{23} atoms of Uranium has mass 235g

∴ 0.9855 × 10^{29} atoms of Uranium has

$$\frac{235 \times 0.9855 \times 10^{29}}{6.023 \times 10^{23}} \text{ g} = 38451 \text{ kg}$$

Q.25. A nucleus at rest undergoes a decay emitting an a particle of de-Broglie wavelength $\lambda = 5.76 \times 10^{-15}$ m. If the mass of the daughter nucleus is 223.610 amu and that of the a particles is 4.002 amu, determine the total kinetic energy in the final state. Hence, obtain the mass of the parent nucleus in amu. (1 amu = 931.470 MeV/c^2) (2001-5 Marks)

Ans. 10^{-12} J, 227.62 amu

Solution. Let the reaction be

$$^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He$$

Here, m_y = 223.61 amu and m_α = 4.002 amu

We know that

$$\lambda = \frac{h}{mv} \Rightarrow m^2v^2 = \frac{h^2}{\lambda^2} = p^2$$

$$\Rightarrow \text{But K.E.} = \frac{p^2}{2m}. \text{ Therefore K.E.} = \frac{h^2}{2m\lambda^2} \qquad ...(i)$$

Applying eq. (i) for Y and α, we get

$$\text{K.E.}_\alpha = \frac{(6.6 \times 10^{-34})^2}{2 \times 4.002 \times 1.67 \times 10^{-27} \times 5.76 \times 10^{-15} \times 5.76 \times 10^{-15}}$$

= 0.0982243 × 10–11 = 0.982 × 10–12J

Similarly (K.E.)Y = 0.0178×10^{-12} J

Total energy = 10^{-12} J

We know that $E = \Delta mc^2$

$$\therefore \Delta m = \frac{E}{c^2} = \frac{10^{-12}}{(3\times10^8)^2}\,\text{kg}$$

$1.65 \times 10^{-27}\text{kg} = 1\text{ amu}$

$$\because \frac{10^{-12}}{(3\times10^8)^2}\ \text{kg} = \frac{10^{-12}\,\text{amu}}{1.67\times10^{-27}\times(3\times10^8)^2}$$

$$= \frac{10^{-12}\,\text{amu}}{1.67\times9\times10^{-27}\times10^{16}} = 0.00665\text{ amu}$$

The mass of the parent nucleus X will be

$m_x = m_y + m_a + \Delta_m$

= 223.61 + 4.002 + 0.00665 = 227.62 amu

Q.26. A radioactive nucleus X decays to a nucleus Y with a decay constant λ_x = 0.1 s^{-1}. Y further decays to a stable nucleus Z with a decay constant λ_Y = 1/30 s^{-1}. Initially, there are only X nuclei and their number is $N_o = 10^{20}$. Set up the rate equations for the populations of X, Y and Z. The population of Y nucleus as a function of time is given by $N_Y(t) = (N_0\lambda_X/ (\lambda_X - \lambda_Y))\{\exp(-\lambda_Y t)-\exp(-\lambda_x t)\}$. Find the time at which N_Y is maximum and determine the populations X and Z at that instant. (2001-5 Marks)

Ans. $15\log_e 3, \frac{10^{20}}{3\sqrt{3}}, 10^{20}\left(\frac{3\sqrt{3}-4}{3\sqrt{3}}\right)$

Solution. $X \xrightarrow[\lambda_x = 0.1\text{s}^{-1}]{T_{1/2} = 10\text{sec}} Y \xrightarrow[\lambda_y = \frac{1}{30}\text{s}^{-1}]{T_{1/2} = 30\text{sec}} Z$

The rate of equation for the population of X, Y and Z will be

$$\frac{dN_x}{dt} = -\lambda_x N_x \qquad \text{...(i)}$$

$$\frac{dN_y}{dt} = -\lambda_y N_y + \lambda_x N_x \qquad \text{...(ii)}$$

$$\frac{dN_z}{dt} = -\lambda_y N_y \qquad \text{...(iii)}$$

⇒ On integration, we get

$N_x = N_0e^{-\lambda xt}$ ...(iv)

Given

$$N_y = \frac{\lambda_x N_0}{\lambda_x - \lambda_Y}\left[e^{-\lambda_y t} - e^{\lambda_x t}\right]$$

To determine the maximum N_Y, we find

$$\frac{dN_Y}{dt} = 0$$

From (ii)

$-\lambda_y N_y + \lambda_x N_x = 0$

$\Rightarrow \lambda_x N_x = \lambda_y N_y$ ....(v)

$$\Rightarrow \lambda_x(N_0 e^{-\lambda_x t}) = \lambda_y\left[\frac{\lambda_x N_0}{\lambda_x - \lambda_y}\left(e^{-\lambda_y t} - e^{\lambda_x t}\right)\right]$$

$$\Rightarrow \frac{\lambda_x - \lambda_y}{\lambda_y} = \frac{e^{-\lambda_y t} - e^{-\lambda_x t}}{e^{-\lambda_x t}} \quad \Rightarrow \frac{\lambda x}{\lambda_y} = e^{(\lambda_x - \lambda_y)t}$$

$$\Rightarrow \log_e \frac{\lambda_x}{\lambda_v} = \left(\lambda_x - \lambda_y\right)t$$

$$\Rightarrow t = \frac{\log_e\left(\lambda_x / \lambda\gamma\right)}{\lambda_x - \lambda_\gamma} = \frac{\log_e\left[0.1/\left(\frac{1}{30}\right)\right]}{0.1 - \frac{1}{30}} = 15\log_e 3$$

$$\therefore\ N_x = N_0 e^{-0.1(15\log_e 3)} = N_0 e^{\log_e(3^{-1.5})}$$

$$\Rightarrow\ N_x = N_0 3^{-1.5} = \frac{10^{20}}{3\sqrt{3}}$$

$$\text{Since, } \frac{dN_y}{dt} = 0 \text{ at } t = 15\log_e 3, \quad \therefore\ N_y = \frac{\lambda_x N_x}{\lambda_y} = \frac{10^{20}}{\sqrt{3}}$$

and $N_z = N_0 - N_x - N_y$

$$= 10^{20} - \left(\frac{10^{20}}{3\sqrt{3}}\right) - \frac{10^{20}}{\sqrt{3}} \quad = 10^{20}\left(\frac{3\sqrt{3}-4}{3\sqrt{3}}\right)$$

Q.27. A hydrogen-like atom (descr ibed by the Bohr model) is observed to emit six wavelengths, originating from all possible transitions between a group of levels. These levels have energies between -0.85 eV and -0.544 eV (including both th ese values). (2002 - 5 Marks)

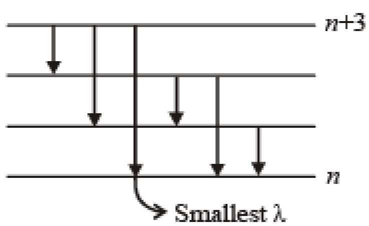
(a) Find the atomic number of the atom.

(b) Calculate the smallest wavelength emitted in th ese transitions.

(Take hc = 1240 eV-nm, ground state energy of hydrogen atom = –13.6 eV)

Ans. (a) 3 (b) 4052 nm

Solution. If x is the difference in quantum number of the states than ${}^{x+1}C_2 = 6 \Rightarrow x = 3$



Now, we have $\dfrac{-z^2(13.6\text{eV})}{n^2} = -0.85\text{eV}$ ...(i)

and $\dfrac{-z^2(13.6\text{eV})}{(n+3)^2} = -0.544\text{ eV}$ ...(ii)

Solving (i) and (ii) we get n = 12 and z = 3

(b) Smallest wavelength λ is given by

$$\frac{hc}{\lambda} = (0.85 - 0.544)\text{ eV}$$

Solving, we get λ = 4052 nm.

Q.28. Two metallic plates A an d B, each of area 5×10^{-4} m^2, are placed parallel to each other at a separation of 1 cm. Plate B carries a positive charge of 33.7×10^{-12} C. A monochromatic beam of light, with photons of energy 5 eV each, starts falling on plate A at t = 0 so that 10^{16} photons fall on it per square meter per second. Assume that one photoelectron is emitted for every 106 incident photons. Also assume that all the emitted photoelectrons are collected by plate B and the work function of plate A remains constant at the value 2eV. Determine (2002 - 5 Marks)

(a) the number of photoelectrons emitted up to t = 10 s,

(b) the magnitude of the electric field between the plates A and B at t = 10 s, and

(c) the kinetic energy of the most energetic photoelectron emitted at t = 10 s when it reaches plate B.

Neglect the time taken by the photoelectron to reach plate B.

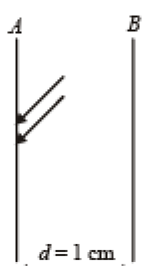
Take $\varepsilon_0 = 8.85 \times 10^{-12}$ C^2/N-m^2

Ans. (a) 5×10^7

(b) 2000 N/C

(c) 23 eV

Solution. Number of electrons falling on the metal plate A = $10^{16} \times (5 \times 10^{-4})$



$\therefore$ Number of photoelectrons emitted from metal plate A upto 10 seconds is

$$n_e = \frac{(5\times10^4)\times10^{16}}{10^6}\times10 = 5\times10^7$$

(b) Charge on plate B at t = 10 sec

$Q_b = 33.7 \times 10^{-12} - 5 \times 10^7 \times 1.6 \times 10^{-19} = 25.7 \times 10^{-12}$ C

also $Q_a = 8 \times 10^{-12}$ C

$$E = \frac{\sigma_B}{2\varepsilon_0} - \frac{\sigma_A}{2\varepsilon_0} = \frac{1}{2A\varepsilon_0}(Q_B - Q_A)$$

$$= \frac{17.7\times10^{-12}}{5\times10^{-4}\times8.85\times10^{-12}} = 2000\ N/C$$

(c) K.E. of most energetic particles

$= (hv - \varphi) + e\,(Ed) = 23$ eV

Note : $(hv - \varphi)$ is energy of photoelectrons due to light e (Ed) is the energy of photoelectrons due to work done by photoelectrons between the plates.

Q.29. Frequency of a photon emitted due to transition of electron of a certain element from L to K shell is found to be 4.2×10^{18} Hz. Using Moseley's law, find the atomic number of the element, given that the Rydberg's constant R = 1.1×10^7 m^{-1}. (2003 - 2 Marks)

Ans. 42

Solution. According to Bohr 's model, the energy released during transition from n_2 to n_1 is given by

$$\Delta E = hv = Rhc(Z-b)^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

For transition from L shell to K shell

$b = 1$, $n_2 = 2$, $n_1 = 1$

$$\therefore\ (Z-1)^2 Rhc\left[\frac{1}{1} - \frac{1}{4}\right] = hv$$

On putting the value of $R = 1.1 \times 10^7\ m^{-1}$ (given),

$c = 3 \times 10^8$ m/s, we get $Z = 42$

Q.30. A radioactive sample emits n β-particles in 2 sec. In next 2 sec it emits 0.75 n β-particle, what is the mean life of the sample? (2003 - 2 Marks)

Ans. $\frac{2}{\log_e(4/3)}$

Solution.

$$\lambda = \frac{\log_e \frac{A_0}{A}}{t} = \frac{1}{2}\log_e \frac{n}{0.75n}$$

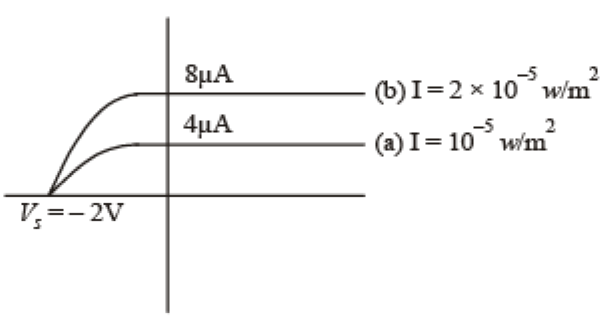
$$\Rightarrow \text{Mean Life} = \frac{1}{\lambda} = \frac{2}{\log_e 4/3}$$

Q.31. In a photoelectric experiment set up, photons of energy 5 eV falls on the cathode having work function 3 eV. (a) If the saturation current is $i_A = 4\mu A$ for intensity $10^{-5} W/m^2$, then plot a graph between anode potential and current. (b) Also draw a graph for intensity of incident radiation $2 \times 10^{-5}\ W/m^2$. (2003 - 2 Marks)

Solution. (a) $eV_0 = h\nu - h\nu_0 = 5 - 3 = 2$ eV

$\therefore V_0 = 2$ volt

(b) Note : When the intensity is doubled, the saturation current is also doubled.



Q.32. A radioactive sample of ^{238}U decays to Pb through a process for which the half-life is 4.5×10^9 years. Find the ratio of number of nuclei of Pb to ^{238}U after a

time of 1.5×10^9years.

Given $(2)^{1/3} = 1.26$. **(2004 - 2 Marks)**

Ans. 0.26

Solution. a = Initial Uranium atom

(a – x) = Uranium atoms left

$$(a-x) = a\left(\frac{1}{2}\right)^n$$

$$\text{and } n = \frac{t}{t_{1/2}} = \frac{1.5\times10^9}{4.5\times10^9} = \frac{1}{3}$$

$$\therefore\ a - x = a\left(\frac{1}{2}\right)^{1/3}$$

$$\Rightarrow \frac{a}{a-x} = \frac{1}{(1/2)^{1/3}} = \frac{2^{1/3}}{1} = 1.26$$

$$\Rightarrow \frac{x}{a-x} = 1.26 - 1 = 0.26$$

Q.33. The photons from the Balmer series in Hydrogen spectrum having wavelength between 450 nm to 700 nm are incident on a metal surface of work function 2 eV. Find the maximum kinetic energy of ejected electron. (Given hc = 1242 eV nm) (2004 - 4 Marks)

Ans. 0.55 eV

Solution. KEY CONCEPT :

The wavelength λ, of photon for different lines of Balmer series is given by

$$\frac{hc}{\lambda} = 13.6\left[\frac{1}{2^2} - \frac{1}{n^2}\right] \text{eV, where } n = 3, 4, 5$$

Using above relation, we get the value of λ = 657nm, 487 nm between 450 nm and 700 nm. Since 487 nm, is smaller than 657 nm, electron of max. K.E. will be emitted for photon corresponding to wavelength 487 nm with

$$(\text{K.E.}) = \frac{hc}{\lambda} - W = \left(\frac{1242}{487} - 2\right) = 0.55 \text{ eV}$$

Q.34. The potential energy of a particle of mass m is given by $V(x) = \begin{cases} E_0; & 0 \le x \le 1 \\ 0; & x > 1 \end{cases}$ **λ_1 and λ_2 are the de-Broglie wavelengths of the particle, when $0 \le x \le 1$ and $x > 1$ respectively. If the total energy of particle is $2E_0$, find λ_1 / λ_2. (2005 - 2 Marks)**

Ans. $\sqrt{2}$

Solution. The de Broglie wave length is given by

$$\lambda = \frac{h}{mv} \Rightarrow \lambda = \frac{h}{\sqrt{2mK}}$$

Case (i) $0 \le x \le 1$

For this, potential energy is E_0 (given)

Total energy = $2E_0$ (given)

∴ Kinetic energy = $2E_0 - E_0 = E_0$

$$\lambda_1 = \frac{h}{\sqrt{2mE_0}} \quad \text{...(i)}$$

Case (ii) x > 1

For this, potential energy = 0 (given)

Here also total energy = $2E_0$ (given)

∴ Kinetic energy = $2E_0$

$$\lambda_1 = \frac{h}{\sqrt{2mE_0}} \quad \text{...(i)}$$

Dividing (i) and (ii)

$$\frac{\lambda_1}{\lambda_2} = \sqrt{\frac{2E_0}{E_0}} \Rightarrow \frac{\lambda_1}{\lambda_2} = \sqrt{2}$$

Q.35. Highly energetic electrons are bombarded on a target of an element containing 30 neutrons. The ratio of radii of nucleus to that of Helium nucleus is $(14)^{1/3}$. Find (a) atomic number of the nucleus. (b) the frequency of K_α line of the X-ray produced. ($R = 1.1 \times 10^7 m^{-1}$ and $c = 3 \times 10^8$ m/s) (2005 - 4 Marks)

Ans. (a) 56

(b) 1.546×10^{18} Hz.

Solution. (a) KEY CONCEPT : We know that radius of nucleus is given by formula

$r = r_0 A^{1/3}$ where r_0 = constt, and A = mass number.

For the nucleus $r_1 = r_0 4^{1/3}$

For unknown nucleus $r_2 = r_0 (A)^{1/3}$

$$\therefore \frac{r_2}{r_1} = \left(\frac{A}{4}\right)^{1/3}, \ (14)^{1/3} = \left(\frac{A}{4}\right)^{1/3} \Rightarrow A = 56$$

∴ No of proton = A – no. of neutrons = 56 – 30 = 26

∴ Atomic number = 26

(b) We know that $\nu = Rc(Z-b)^2 \left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

Here, $R = 1.1 \times 10^7$, $c = 3 \times 10^8$, Z = 26

b = 1 (for K_α), $n_1 = 1$, $n_2 = 2$

$$\therefore \nu = 1.1 \times 10^7 \times 3 \times 10^8 [26-1]^2 \left[\frac{1}{1} - \frac{1}{4}\right]$$

$$= 3.3 \times 10^{15} \times 25 \times 25 \times \frac{3}{4} = 1.546 \times 10^{18} \text{ Hz}$$

Q.36. In hydrogen-like atom (z = 11), nth line of Lyman series has wavelength λ. The de-Broglie's wavelength of electron in the level from which it originated is also λ. Find the value of n ? (2006 - 6M)

Ans. 24

Solution. Note : nth line of Lyman series means electron jumping from (n + 1)th orbit to 1st orbit.

For an electron to revolve in (n + 1)th orbit. $2\pi r = (n + 1)\lambda$

$$\Rightarrow \lambda = \frac{2\pi}{(n+1)} \times r = \frac{2\pi}{(n+1)}\left[0.529 \times 10^{-10}\right]\frac{(n+1)^2}{Z}$$

$$\Rightarrow \frac{1}{\lambda} = \frac{Z}{2\pi\left[0.529 \times 10^{-10}\right](n+1)} \quad ...(i)$$

Also we know that when electron jumps from (n + 1)th orbit to 1st orbit.

$$\frac{1}{\lambda} = RZ^2\left[\frac{1}{1^2} - \frac{1}{(n+1)^2}\right] = 1.09 \times 10^7 Z^2\left[1 - \frac{1}{(n+1)^2}\right] \quad ...(ii)$$

From (i) and (ii)

$$\frac{Z}{2\pi(0.529 \times 10^{-10})(n+1)} = 1.09 \times 10^7 Z^2\left[1 - \frac{1}{(n+1)^2}\right]$$

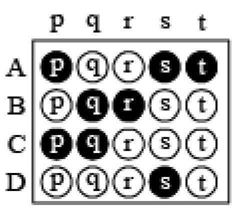
On solving, we get n = 24

Match the Following

DIRECTIONS (Q. No. 1 to 4 & 6) : Each question contains statements given in

two columns, which have to be matched. The statements in Column-I are labelled A, B, C and D, while the statements in Column-II are labelled p, q, r and s. Any given statement in Column-I can have correct matching with ONE OR MORE statement(s) in Column-II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example :

If the correct matches are A-p, s and t; B-q and r; C-p and q; and D-s then the correct darkening of bubbles will look like the given.



Q.1. In the following, column I lists some physical quantities and the column II gives approximate energy values associated with some of them. Choose the appropriate value of energy from column II for each of the physical quantities in column I and write the corresponding letter p, q, r, etc. against the number (A), (B), (C), (D) etc. of the physical quantity in the answer book. In your answer, the sequence of column I should be maintained. (1997 - 4 Marks)

Column I	**Column II**
(A) Energy of thermal neutrons	(p) 0.025 eV
(B) Energy of X-rays	(q) 0.5 eV
(C) Binding energy per nucleon	(r) 3 eV
(D) Photoelectric threshold of a metal	(s) 20 eV
	(t) 10 k eV
	(u) 8 M eV

Ans. A-p; B-t; C-u; D-r

Solution. A → p; B → t; C → u; D → r

The correct match is as follows :

(A) Energy of thermal neutrons (p) 0.025 eV

(B) Energy of X-rays (t) 10 k eV

(C) Binding energy per nucleon (u) 8 M eV

(D) Photoelectric threshold (r) 3 eV of a metal

Q.2. Given below are certain matching type questions, where two columns (each having 4 items) are given. Immediately after the columns the matching grid is given, where each item of Column I has to be matched with the items of Column II, by encircling the correct match(es). Note that an item of column I can match with more than one item of column II. All the items of column II must be matched. Match the following : (2006 - 6M)

Column I	**Column II**
(A) Nuclear fusion	(p) Converts some matter into energy
(B) Nuclear fission	(q) Generally possible for nuclei with low atomic number
(C) β-decay	(r) Generally possible for nuclei with higher atomic number
(D) Exothermic nuclear reaction	(s) Essentially proceeds by weak nuclear forces

Ans. A-p, q; B-p, r; C-p, s; D-p, q, r

Solution. A → p, q; B → p, r; C → p, s; D → p, q, r

In a nuclear fusion reaction matter is converted into energy and nuclei of low atomic number generally given this reaction.

In a nuclear fission reaction matter is converted into energy and nuclei of high atomic number generally given this reaction.

Q.3. Some laws / processes are given in Column I. Match these with the physical phenomena given in Column II and indicate your answer by darkening appropriate bubbles in the 4 × 4 matrix given in the ORS.

Column I	**Column II**
(A) Transition between two atomic energy levels	(p) Characteristic X-rays
(B) Electron emission from a material	(q) Photoelectric effect
(C) Mosley's law	(r) Hydrogen spectrum
(D) Change of photon energy into kinetic energy of electrons	(s) β-decay

Ans. A-p, r; B-q, s; C-p; D-q

Solution. A → p, r

Reason : Characteristic X-ray are produced due to transition of electrons from one energy level to another.

Similarly the lines in the hydrogen spectrum is obtained due to transition of electrons from one energy level to another.

B → q, s

Reason : In photoelectric effect electrons from the metal surface are emitted out upon the incidence of light of appropriate frequency.

Note : In β-decay, electrons are emitted from the nucleus of an atom.

C → p

Moseley gave a law which related frequency of emitted Xray with the atomic number of the target material $\sqrt{v} = a(Z - b)$

D → q

In photoelectric effect, energy of photons of incident ray gets converted into kinetic energy of emitted electrons.

Q.4. Column-II gives certain systems undergoing a process. Column-I suggests changes in some of the parameters related to the system. Match the statements in Column-I to the approapriate process(es) from Column-II.

Column-I	**Column-II**
(A) The energy of the system is increased	(p) System : A capacitor, initially uncharged Process : It is connected to a battery
(B) Mechanical energy is provided to the system, which is converted into energy of random motion of its parts	(q) System : A gas in an adiabatic container fitted with an adiabatic piston Process: The gas is compressed by pushing the piston
(C) Internal energy of the system is converted into its mechanical energy	(r) System: A gas in a rigid container Process: The gas gets cooled due to colder atmosphere surrounding it
(D) Mass of the system is decreased	(s) System: A heavy nucleus, initially at rest Process: The nucleus fissions into two fragments of nearly equal masses and some neutrons are emitted
	(t) System: A resistive wire loop Process: The loop is placed in a time varying magnetic field perpendicular to its plane

Ans. A-p, q, t; B-q; C-s; D-s

Solution. A → p, q, t; B → q; C → s; D → s

(p) When an uncharged capacitor is connected to a battery, it becomes charged and energy is stored in the capacitor. (A) is the correct option.

(q) When a gas in an adiabatic container fitted with an adiabatic piston is compressed by pushing the piston

(i) the internal energy of the system increases

$\Delta U = Q - W = 0 - (-PdV) =+ PdV$

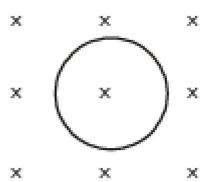
(ii) Mechanical energy is proceeded to the piston which is converted into kinetic energy of the gas molecules.

(r) None of the options in column I matches. As the gas in a rigid container gets cooled, the internal energy of the system will decrease. The average kinetic energy per molecule will decrease.

(s) When a heavy nucleus initially at rest splits into two nuclei of nearly equal masses and some neutrons are emitted then

(i) Internal energy of the system is converted into mechanical energy (precisely speaking kinetic energy) and

(ii) Mass of the system decr eases which converts into energy.



(t) When a resistive wire loops is placed in a time varying magnetic field perpendicular to its palne.

(i) Induced current shows in the loop due to which the energy of system is increased.

DIRECTION (Q.No. 5) : Following question has matching list I and II. The codes for the lists have choices (a), (b), (c) and (d) out of which ONLY ONE is correct.

Q.5. Match List I of the nuclear processes with List II containing parent nucleus and one of the end products of each process and then select the correct answer using the codes given below the lists: (JEE Adv. 2013-II)

List I	**List II**
P. Alpha decay	1. $^{15}_{8}O \rightarrow {}^{15}_{7}O + ...$
Q. b+ decay	2. $^{138}_{92}U \rightarrow {}^{234}_{90}Th + ...$

R. Fission	3. ${}^{185}_{83}Bi \rightarrow {}^{184}_{82}Pb + ...$
S. Proton emission	4. ${}^{239}_{94}Pu \rightarrow {}^{140}_{57}La + ...$

Codes:

	P	Q	R	S
(a)	4	2	1	3
(b)	1	3	2	4
(c)	2	1	4	3
(d)	4	3	2	1

Ans. (c)

Solution.

$$ {}^{15}_{8}O \longrightarrow {}^{15}_{7}N + \underset{\beta^+ \text{ particle}}{{}^{0}_{1}\beta} $$

$$ {}^{238}_{92}U \longrightarrow {}^{234}_{90}Th + \underset{\alpha-\text{particle}}{{}^{4}_{2}He} $$

$$ {}^{185}_{83}Bi \longrightarrow {}^{184}_{82}Pb + \underset{\text{proton}}{{}^{1}_{1}H} $$

$$ {}^{239}_{94}Pu \longrightarrow {}^{140}_{57}La + {}^{99}_{37}X $$

(c) is the correct option.

Q.6. Match the nuclear processes given in column I with the appropriate option(s) in column II. (JEE Adv. 2015)

Column I	**Column II**
(A) Nuclear fusion	(p) Absorption of thermal neutrons by ${}^{235}_{92}U$
(B) Fission in a nuclear reactor	(q) ${}^{60}_{27}Co$ nucleus
(C) β-decay	(r) Energy production in stars via hydrogen conversion to helium

(D) γ-ray emission	(s) Heavy water
	(t) Neutrino emission

Ans. A-r, t; B-p, s; C-p, q, r, t; D-p, q, r, t

Solution. A → r, t; B → p, s; C → p, q, r, t; D → p, q, r, t

Based on facts

Integer Value Correct

Q.1. An α-particle and a proton are accelerated from rest by a potential difference of 100 V. After this, their de Broglie wavelengths are λ_α and λ_p respectively. The ratio $\frac{\lambda_p}{\lambda_\alpha}$, to the nearest integer, is (2010)

Ans. 3

Solution. We know that, $\lambda = \frac{h}{\sqrt{2mqV}}$

$$\therefore \frac{\lambda_1}{\lambda_\alpha} = \sqrt{\frac{m_\alpha q_\alpha}{m_p q_p}} = \sqrt{\frac{4}{1} \times \frac{2}{1}} = \sqrt{8} \approx 3$$

Q.2. To determine the half life of a radioactive element, a student plots a graph of $ln\left|\frac{dN(t)}{dt}\right|$ versus t. Here $\left|\frac{dN(t)}{dt}\right|$ is the rate of radioactive decay at time t. If the number of radioactive nuclei of this element decreases by a factor of p after 4.16 years, the value of p is (2010)

Ans. 8

Solution. We know that $N = N_0e^{-\lambda t}$

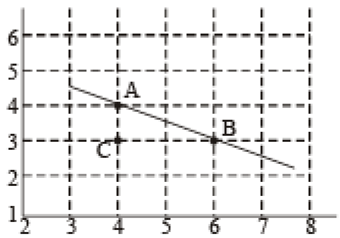
$$\therefore \frac{dN}{dt} = N_0 e^{-\lambda t}(-\lambda) = -N_0 \lambda e^{-\lambda t}$$

Taking log on both sides

$$\log_e \frac{dN}{dt} = \log_e(-N_0\lambda) - \lambda t$$

Comparing it with the graph line,

we get $\lambda = \frac{1}{2}\text{yr}^{-1}$ $\left[\frac{AC}{BC} = \frac{1}{2}\right]$



$$\therefore T_{1/2} = \frac{0.693}{\lambda} = 0.693 \times 2 = 1.386 \text{ years}$$

$$\text{Now } \frac{N}{N_0} = \left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}} \Rightarrow \frac{1}{p} = \left(\frac{1}{2}\right)^{\frac{4.16}{1.386}} = \frac{1}{8}$$

Q.3. The activity of a freshly prepared radioactive sample is 10^{10} disintegrations per second, whose mean life is 10^9 s. The mass of an atom of this radioisotope is 10^{-25} kg. The mass (in mg) of the radioactive sample is (2011)

Ans. 1

Solution.

$$\text{We know that, } \left|\frac{dN}{dt}\right| = \lambda N = \frac{1}{T_{mean}} N$$

$$\therefore \quad 10^{10} = \frac{1}{10^9} \times N$$

$\therefore$ N = 10^{19}

i.e. 1019 radioactive atoms are present in the freshly prepared sample.

The mass of the sample = $10^{19} \times 10^{-25}$ kg = 10^{-6}kg = 1 mg

Q.4. A silver sphere of radius 1 cm and work function 4.7 eV is suspended from an insulating thread in freespace. It is under continuous illumination of 200 nm wavelength light. As photoelectrons are emitted, the sphere gets charged and acquires a potential. The maximum number of photoelectrons emitted from the sphere is A × 10^z (where 1 < A < 10). The value of 'z' is (2011)

Ans. 7

Solution. Stopping potential $= \frac{1}{e}\left[\frac{hc}{\lambda} - \phi\right]$ where hc = 1240eV –nm

$$= \frac{1}{e}\left[\frac{1240}{200} - 4.7\right] = \frac{1}{e}[6.2 - 4.7]$$
$$= \frac{1}{e} \times 1.5eV = 1.5\text{V}$$
$$\text{But V} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} = \frac{1}{4\pi\varepsilon_0}\frac{ne}{r}$$
$$\therefore \text{n} = \frac{Vr(4\pi\varepsilon_0)}{e} = \frac{1.5 \times 10^{-2}}{9 \times 10^9 \times 1.6 \times 10^{-19}}$$
$$\therefore \text{n} = 1.04 \times 10^7$$

Comparing it with A × 10z we get, z = 7

Q.5. A proton is fired from very far away towards a nucleus with charge Q = 120 e, where e is the electronic charge.

It makes a closest approach of 10 fm to the nucleus. The de Broglie wavelength (in units of fm) of the proton at its start is: (take the proton mass, m_p = (5/3) × 10^{-27} kg; h/e = 4.2 × 10^{-15} J.s / C; $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ m/F; 1 fm $= 10^{-15}$ m **(2012-I)**

Ans. 7

Solution. Loss in K.E. of proton = Gain in potential energy of the proton – nucleus system

$$\frac{1}{2}mv^2 = \frac{1}{4\pi\,\varepsilon_0}\frac{q_1\,q_2}{r}$$

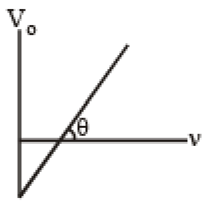
$$\therefore \quad \frac{p^2}{2m} = \frac{1}{4\pi\,\epsilon_0}\frac{q_1\,q_2}{r}$$
$$\therefore \quad \frac{1}{2m}\left(\frac{h^2}{\lambda^2}\right) = \frac{1}{4\pi\,\epsilon_0}\frac{q_1\,q_2}{r}$$
$$\therefore \quad \lambda = \sqrt{\frac{4\pi\,\epsilon_0\,r\,.\,h^2}{q_1\,q_2\,(2\,m)}} = 7\text{ fm}$$

Q.6. The work functions of Silver and Sodium are 4.6 and 2.3 eV, respectively. The ratio of the slope of the stopping potential versus frequency plot for Silver to that of Sodium is (JEE Adv. 2013-I)

Ans. 1

Solution. For photoelectric effect

$$\frac{h\nu}{e}-\frac{\phi_o}{e}=V_o$$



The slope is

tan θ = h/e = constant

∴ The ratio will be 1.

Q.7. A freshly prepared sample of a radioisotope of half-life 1386 s has activity 10^3disintegrations per second. Given that ln2 = 0.693, the fraction of the initial number of nuclei (expressed in nearest integer percentage) that will decay in the first 80 s after preparation of the sample is (JEE Adv. 2013-I)

Ans. 4

Solution. For a radioactive decay

$$N = N_0 \ e^{-\lambda t}$$

$$\therefore \frac{N}{N_0}=e^{-\lambda t} \therefore 1-\frac{N}{N_0}=1-e^{-\lambda t}$$

$$\therefore \frac{N_0-N}{N}=1-e^{-\frac{0.693}{t_{1/2}}\times t}=1-e^{-0.04}=1-(1-0.04)$$

$$\approx 0.04 = 4\% \qquad [\because e^{-x}=1-x \ x<<1]$$

Q.8. A nuclear power plant supplying electrical power to a village uses a radioactive material of half life T years as the fuel. The amount of fuel at the beginning is such that the total power requirement of the village is 12.5% of the electrical power available from the plant at that time. If the plant is able to meet the total power needs of the village for a maximum period of nT years, then the value of n is (JEE Adv. 2015)

Ans. 3

Solution.

$$\begin{array}{ccccccc} N_o & \xrightarrow{T} & \dfrac{N_o}{2} & \xrightarrow{T} & \dfrac{N_o}{4} & \xrightarrow{T} & \dfrac{N_o}{8} \\ 100\% & & 50\% & & 25\% & & 12.5\% \end{array}$$

Three half life are required. Therefore n = 3

Q.9. Consider a hydrogen atom with its electron in the n^{th} orbital.
An electromagnetic radiation of wavelength 90 nm is used to ionize the atom. If the kinetic energy of the ejected electron is 10.4 eV, then the value of n is (hc = 1242 eV nm) (JEE Adv. 2015)

Ans. 2

Solution. $\frac{hc}{\lambda}=\frac{13.6}{n^2}+10.2$

$\therefore \quad \frac{1242}{90}=\frac{13.6}{n^2}+10.2 \therefore \quad n^2=4 \therefore n=2$

Q.10. For a radioactive material, its activity A and rate of change of its activity R are defined as $A=-\frac{dN}{dt}$ and $R=-\frac{dA}{dt}$, **where N(t) is the number of nuclei at time t. Two radioactive sources P (mean life τ) and Q (mean life 2τ) have the same activity at t = 0. Their rates of change of activities at t = 2τ are R_P and R_Q, respectively.** If $\frac{R_P}{R_Q}=\frac{n}{e}$, **then the value of n is (JEE Adv. 2015)**

Ans. 2

Solution.

$$R = -\frac{dA}{dt} = -\frac{d}{dt}\left[-\frac{dN}{dt}\right] = \frac{d^2N}{dt^2} = \frac{d^2\left(N_o e^{-\lambda t}\right)}{dt^2}$$

$$\therefore \quad R = N_o \lambda^2 e^{-\lambda t} = (N_o\lambda)\,\lambda\, e^{-\lambda t} = A_o \lambda e^{-\lambda t}$$

$$[\because A_o = N_o\lambda]$$

$$\therefore \quad \frac{R_P}{R_Q} = \frac{\lambda_P\, e^{-\lambda_P t}}{\lambda_Q\, e^{-\lambda_Q t}} = \frac{\lambda P}{\lambda Q} \times \frac{e^{\lambda_Q t}}{e^{\lambda_P t}} = \frac{2\tau}{\tau}\,\frac{e^{\frac{2\tau}{2\tau}}}{e^{\frac{2\tau}{\tau}}} = \frac{2}{e}$$

$$\therefore \quad n = 2$$

Q.11. An electron is an excited state of Li^{2+} ion has angular momentum 3h/2π. The de Broglie wavelength of the electron in this state is pπ a_0 (where a_0 is the Bohr radius). The value of p is (JEE Adv. 2015)

Ans. 2

Solution. Given mvr = 3h/2π ⇒ n = 3

$$\therefore \quad \frac{h\,r}{\lambda} = \frac{3\,h}{2\pi} \qquad \left[\because \lambda = \frac{h}{mv}\right]$$

$$\therefore \quad \lambda = \frac{2\pi r}{3} = \frac{2}{3}\pi\left[a_0\frac{n^2}{z}\right] \qquad \left[\because r = a_0\frac{n^2}{z}\right]$$

$$\therefore \quad \lambda = \frac{2}{3}\pi\, a_0\left[\frac{3\times 3}{3}\right] = 2\pi a_0$$

$$\therefore \quad p = 2$$

Q.12. The isotope ${}^{12}_{5}B$ having a mass 12.014 u undergoes β–decay to ${}^{12}_{6}C$. ${}^{12}_{6}C$ has an excited state of the nucleus $\left({}^{12}_{6}C^*\right)$ at 4.041 MeV above its ground state. If ${}^{12}_{5}E$ decays to ${}^{12}_{6}C^*$, the maximum kinetic energy of the β–particle in units of MeV is (1 u = 931.5 MeV/c^2, where c is the speed of light in vacuum). (JEE Adv. 2016)

Ans. 9

Solution. Maximum kinetic energy of β-particle

$= [\text{mass of } {}^{12}_{5}B - \text{mass of } {}^{12}_{6}C] \times 931.5 - 4.041$

= [12.014 – 12] × 931.5 – 4.041] = 9MeV

Q.13. A hydrogen atom in its ground state is irradiated by light of wavelength 970 Å. Taking hc/e = 1.237 × 10^{-6} eV m and the ground state energy of hydrogen atom as –13.6 eV, the number of lines present in the emission spectrum is (JEE Adv. 2016)

Ans. 6

Solution. $E = \frac{hc}{\lambda} = \frac{1.237 \times 10^{-6}}{970 \times 10^{-10}} eV = 12.75 eV$

∴ The energy of electron after absorbing this photon

= –13.6 + 12.75 = – 0.85eV

This corresponds to n = 4

Number of spectral line $= \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$